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AN MILP APPROACH TO MULTI-LOCATION, MULTI-PERIOD EQUIPMENT SELECTION FOR SURFACE MINING WITH CASE STUDIES

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Abstract. In the surface mining industry, the Equipment Selection Problem involves choosing an appropriate fleet of trucks and loaders such that the long-term mine plan can be satisfied. An important characteristic for multi-location (multi-location and multi-dumpsite) mines is that the underlying problem is a multi-commodity flow problem. The problem is therefore at least as difficult as the fixed-charge, capacitated multi-commodity flow problem. For long-term schedules it is useful to consider both the purchase and salvage of the equipment, since equipment may be superseded, and there is the possibility of used pre-existing equipment. This may also lead to heterogeneous fleets and arising compatibility considerations. In this paper, we consider two case studies provided by our industry partner. We develop a mixed-integer linear programming model for heterogeneous equipment selection in a surface mine with multiple locations and a multiple period schedule. Encoded in the solution is an allocation scheme in addition to a purchase and salvage policy. We develop a solution approach, including variable preprocessing, to tackle this large-scale problem. We illustrate the computational effectiveness of the resulting model on the two case studies for large sets of equipment and long-term schedule scenarios.

1. **Introduction.** The selection of an appropriate fleet of trucks and loaders to operate in a surface mine is an important problem for the mining industry due to the large cost of purchasing and operating the equipment over many years. A poor choice in the truck and loader fleet, in either types or fleet size, can lead to unnecessary expense and, in some cases, the inability to satisfy capacity constraints. As a problem in itself, equipment selection brings together the selection, purchase, replacement and allocation problems into one optimization problem. It becomes more interesting still when we consider a mine with multiple locations or routes, where the compatibility of the interacting fleets is enforced. Although the purchase

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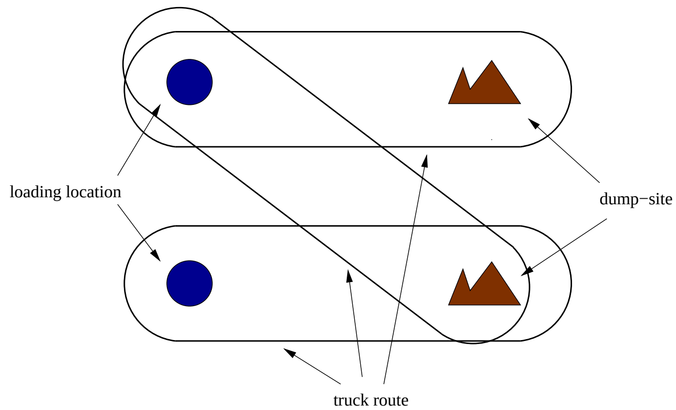

FIGURE 1. A multiple-location mine model with 2 loading locations, 2 dump-sites and 3 truck routes.

of equipment is made on a strategic (yearly) time scale, the equipment must be capable of fulfilling production requirements on a tactical (daily) time scale. This mismatch in time fidelity can lead to poorly dimensioned models, which in turn are computationally cumbersome. The additional dimension of the multiple routes and locations also exacerbates what is already a large-scale problem.

In this paper, we address this practical optimization problem of choosing equipment for a surface mine plan. In particular, we wish to select a compatible fleet of trucks and loaders to move mined materials between multiple mining and dumping sites, at minimum cost. As far as we know, there has been no literature that addresses multiple period schedules or multiple locations or allows for pre-existing equipment (and subsequent heterogeneous fleets). Multiple periods and locations increase the dimension of the problem, but it is the pre-existing equipment that requires compatible fleet constraints that makes the problem difficult. We seek to address these deficiencies in the literature. The main features of our model approach are:

- the consideration of a multiple location and multiple period mining plan;
- the consideration of heterogeneous fleets and subsequent compatibility requirements;
- simultaneously optimizing the purchase and salvage policy, and equipment scheduling policy (i.e. allocation policy);
- providing correction for discretization error;
- variable preprocessing based on a 'staircase' structure in the solution;
- development of solving approaches that help to reduce the total number of constraints in the model;
- illustration of computation effectiveness in a real-world context through two case studies.

In the remainder of this section, we will outline some important background to the equipment selection problem and discuss relevant related literature.

A typical surface mine may have several mining locations, several dump-sites or several routes from a location to a dump-site, as illustrated in Figure 1. Different mining locations may have capacity requirements that affect the type of loader selected to excavate the material, and consequently alter the rate of production at that site. Along the routes the trucks may alternate between the mill and dump sites; the routes themselves may also vary significantly in terms of time required to perform a full cycle. Furthermore, the consideration of multiple locations or routes introduces the need to allocate equipment to locations. Since the movement of equipment around a network must be consistent, this feature leads to an underlying problem which can be described as a multi-commodity flow problem [4]. Purchase may occur in any period. The problem is therefore at least as difficult as the fixed-charge, capacitated multi-commodity flow problem. To see this, consider Figure 2. In a multi-commodity flow problem defined on a network, there may be several sources, here s, and several sinks, here t. If we consider trucks moving along routes between sources and sinks in a single commodity context, then this would be equivalent to fixing homogeneous equipment to routes. However, in a multi-commodity flow problem, there may also be more than one type of *commodity*, or flow, leaving from any source. Furthermore, the flow can split at a node. Carrying the analogue to equipment selection, this is equivalent to allocating heterogeneous truck flow on routes, where the flow may separate and switch to other sinks or sources. We obtain *fixed-charge* and *capacitated* from charging a fixed price for equipment at the time of purchase, and limiting the number of equipment we may use in flow by fleetsize.

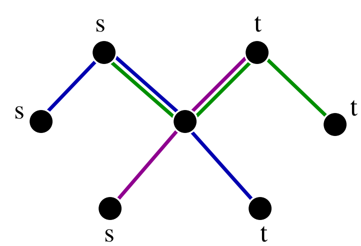

Figure 2. A simple multi-commodity flow network.

We restrict the focus of this paper to mining equipment selection under the following assumptions:

- Multiple locations and multiple routes exist on which the selected fleet may move about;
- Multiple truck cycle times—truck cycle time is fixed for a given route, where the route is defined as a pair of loading location and dumping destination;
- Known mine plan—an acceptable mine plan has already been derived (including selection of mining method), and is fixed for optimization period;
- Salvage—all equipment is salvageable at the start of each period at some depreciated value of the original capital expense;
- No auxiliary equipment—wheel loaders and small trucks are not considered in this model, although can be easily included if the cost and maintenance data is available;
- Known operating hours—the operating hours of the mine are estimated by taking planned downtime, blasting and weather delays into account;
- Heterogeneous fleets—different types of equipment may work within one fleet, so long as compatibility requirements are satisfied;
- Fleet retention—all equipment is retained at the end of the last period;

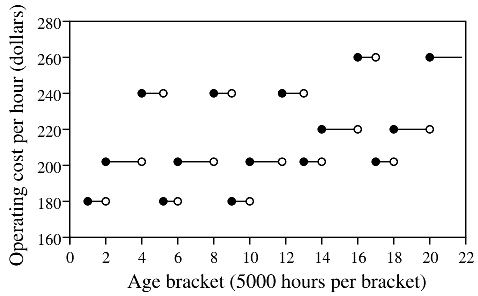

Figure 3. Discretized operating cost function against age brackets. The rise in operating cost reflects the increased maintenance expense; large drops in the expense occur when a significant maintenance, such as overhaul, has taken place.

- Full period utilization—operating costs are charged as though the equipment has been fully utilized for each entire period in which it is owned.
- Age bracket size—the size of an age bracket, B_0, used to discretize the availability function, is strictly larger than the size of a period, i.e. $B_0 > \max\{H^k\}$;
- Equipment availability—equipment availability, maintenance requirements and equipment utilization change over time, and can be effectively approximated using age brackets.

The first mine plan must use approximate transportation costs, since the equipment selection solution will not be known. Over the life of the mine, the generation of mine plans at planning intervals will depend on the selected equipment fleet. Therefore, new plans should be generated if changes in the equipment fleet are generated at an equipment planning interval.

In the surface mining industry, the generally non-linear non-convex operating cost and availability (as functions of the age of the equipment) are commonly discretized to step-wise functions that are divided into age brackets of size B_0, as illustrated in Figure 3. The operating expense reflects the cost of operating and maintaining the equipment. It takes into account varying maintenance expenses, availability and productivity levels, which are known to vary with the age of the equipment. In our case studies, we use an age bracket size of 5000 hours (as in [25]). We have similar factors for the availability of the equipment (the proportion of time it is available to work), utilization (the proportion of time it is effective) and maintenance (the proportion of time the equipment is available after maintenance). All of these factors are functions of the age bracket the equipment is in, indicating that the performance of the equipment changes with its use in a non-linear way.

The full period utilization assumption is really about the granularity of our data—we have set up this problem with time windows of annual periods, and there are considerations such as budgets and labour flexibility which are typically annual in nature. Purchase and salvage of equipment typically occur periodically, so accordingly we define a period to be 1 year in length as a reasonable timeframe with which to consider purchase and salvage of equipment and any cost benefits associated with bulk purchases. Smaller periods may be considered, such as quarterly, to generate finer fidelity schedules and allocation solutions, but purchase and salvage would generally not occur at these smaller intervals. For this reason, and also to aid the tractability of the problem, we will not consider periods smaller than 1 year in length in our experiments.

Often the data for costing and even projected demand is imperfect. Since the projection period itself can be quite long (up to 20 years), a pragmatic approach is to formulate the problem as a deterministic problem to match the provided mine plan, and perform sensitivity analysis to understand the robustness of the obtained solutions. Furthermore, since the problem is already large-scale, the additional consideration of stochasticity and uncertainty would only exacerbate the difficulty of the problem. In this sense, obtaining solutions via a deterministic modeling approach such as mixed-integer programming is appropriate for the multi-location equipment selection problem where truck and loader units are integral and the capacity constraints can be captured linearly.

This problem is closely related to facility material handling equipment selection and machine selection in manufacturing systems (see, for e.g.,[1, 3, 9, 19, 21, 22]) and equipment replacement (see, for e.g., [21]). Integer programming and decomposition techniques have been used to model these problems, such as in [3, 9]. Recently, the equipment selection problem has been considered in the forestry harvesting industry, also with a mixed-integer programming approach (see [2]). This problem is essentially the same as the surface mining problem, whereby the models must select the equipment and the number of hours of operation for a given harvesting schedule. In [2] the decision variables do not capture the age of the equipment as an index, which initially reduces the overall number of integer or binary variables. However, in order to relate the appropriate operating cost to equipment activity, the authors developed a piece-wise linear approximation model which increased the quantity of binary variables dramatically. Another key difference is that, for the foresting industry, equipment are subdivided into compatible sets: compatible fleets do not need to be enforced in the model.

Many modelling approaches have been applied to the surface mining equipment selection problem, including genetic algorithms [19] and queuing theory [22]. These approaches aim to obtain fast and robust solutions respectively, for short time horizons, but lack the capabilities of obtaining exact solutions for long-term horizon instances of their mixed-integer program counterparts. The use of integer programming methods to solve operations research problems in surface mining is well established in both the mining and construction literature [15]. For example, there is extensive literature that details mixed-integer programming approaches to finding mine plans [10, 11, 16, 17, 23]. Mixed-integer programming has also been considered for the equipment selection problem [13, 18, 25], however, much of the focus is on project completion and dispatching or allocation. This literature in particular commonly has the assumption that equipment types are given, rather than allowing the models to select these with the fleet size. Fleet homogeneity is also a common assumption (e.g., see [7]) that is too restrictive for the surface mining application. In one paper [8], the authors describe heterogeneous fleets as "unacceptable or even unthinkable", although only anecdotal evidence has supported these claims to date. A common belief is that the cost of training artisans and storing spare parts far exceeds the benefit obtained through a mixed fleet that better matches the production schedule. However, to the knowledge of the authors, there is no published study into these costs. The cost comparison from our case studies provide interesting discussion points for this debate.

In other equipment selection literature, some solution methods look at optimizing productivity [24] and equipment matching [20] rather than mining cost. Since maximising productivity is different to minimizing cost (and can inadvertently lead to higher costs), such objectives are most useful in the construction industry where earlier completion can be more profitable than finishing on time. To restrain these higher costs, "budgeting constraints" have been considered where the maximum permissible budget cash outlay for a given time period is an upper bound [8]. In surface mining, however, we select equipment for a mine plan that must be met. In this sense, there is no advantage to having a fleet that is more productive than required. In view of this, we choose the alternate approach—to optimize the cost of the operation in our model.

The development of key equipment selection literature is as follows:

- In [26], the authors formulated the general materials handling problem as an assignment problem in which they assume all equipment is compatible and the optimization horizon is one period (i.e., no purchase occurs in this model, and the model is unhindered by the time dimension). In their objective function they minimized the cost of utilization (defined here as the amount of time the equipment has been used) by using a predetermined utilization matrix—this objective function is ideal for the mining equipment selection problem but is currently not achievable due to the dependency of costs and equipment availability on equipment age. That is, it leads to a state-dependent objective function.
- In [14], the authors extended the Webster and Reed model to combine the equipment selection problem with the allocation problem. They developed this model such that the selected equipment perform a set of tasks (rather than satisfying productivity requirements) within a nominated time-frame, thus making it better suited to construction than mining equipment selection.
- In [8], the authors developed a systematic decision making model for the selection of equipment types under the assumption of homogeneous fleets and single period time horizon. Another key assumption in the model is that equipment is kept for its entire life, and that we pay the cost of running it for its entire life regardless of utilization or the number of periods it is actually required.
- In [18], the authors developed a binary integer programming model to select equipment for a mine, which was paired with a layer stratification model. A solution to this model determines equipment types that suit the surrounding environment, rather than fleets to meet a production schedule. It is intended to reduce a large set of available equipment types to a satisfactory set from which optimal equipment selection can take place using other methods.
- In [12], the authors used a linear programming model to select the optimum loader type for construction use over a short time horizon. Salvage of equipment and productivity changes with the age of the equipment are therefore not considered;
- In [25], the authors developed a mixed-integer programming model that allocates mining equipment to a schedule such that the maintenance cost is minimized. This is one of the first papers to consider how the equipment is utilized across the schedule and the impact this has on operating cost. The paper does not seek, however, to select the equipment types or fleet sizes.

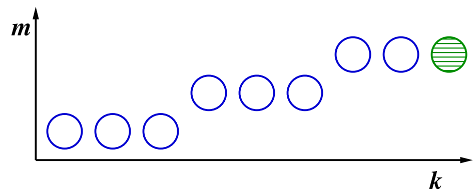

FIGURE 4. As time progresses, the $x_{t,k,m}$ variables move along the 'steps' of age-brackets, ending with a single instance of the variable $s_{t,k,m}$. This staircase structure is important for preprocessing.

We derive our model in Section 2. We begin by describing the model setting, decision variables and reductions, before deriving the objective function with corrector for discretization error (Section 2.1) and constraints (Section 2.2). We describe two surface mining case studies with varying mining locations and routes in Section 3 with a note on practical implications of our solutions (Section 3.3). We conclude with a discussion of the research and possible further advancements in Section 4.

2. **Problem formulation.** We define the problem using an arc-based representation of the mining locations ($i \in \mathcal{I}$) and routes, (i, j), to dumpsites. To simplify notation, we denote a route by $j \in \mathcal{J}$. We denote the set of all truck types by $\mathcal{T}$ and loader types by $\mathcal{L}$. Here we adopt three indexes to represent type ($t \in \mathcal{T}$), period ($k \in \{1, 2, \dots, K\}$) and age bracket ($m \in \{1, 2, \dots, M\}$). We use integer variables to track whole equipment units while continuous variables allocate equipment to routes:

- **:** $x_{t,k,m}$: number of trucks of type t owned in period k which are in age bracket m (integer variable);
- **:** $y_{l,k,m}$: number of loaders of type l owned in period k which are in age bracket m (integer variable);
- **:** $s_{t,k,m}$: number of trucks of type t salvaged in period k which are in age bracket m (integer variable);
- **:** $s_{l,k,m}$: number of loaders of type l salvaged in period k which are in age bracket m (integer variable);
- **:** $f_{t,j,k,m}$: portion of trucks of type t, in age bracket m, that are allocated to route j in period k, where $f_{t,j,k,m} \in [0, x_{t,k,m}]$ (continuous variable),
- **:** $f_{l,i,k,m}$: portion of loaders of type l, in age bracket m, that are allocated to location i in period k, where $f_{l,i,k,m} \in [0, y_{l,k,m}]$ (continuous variable).

To simplify prose in this paper, we sometimes just describe constraints for the trucks if a corresponding, and identical, constraint also exists for the loaders in our model. The complete model (Section 2.3) contains all constraints. The relationship between $x_{t,k,m}$ and $s_{t,k,m}$ is illustrated in Figure 4. The $f_{t,j,k,m}$ variable will trace the 'staircase' and allocate portions of the total time to particular routes, as described in the following section. The analog exists for the loader variables, though for brevity we restrict the description to trucks.

The precedence characteristic of the solutions elicit a natural 'staircase' structure in the possible values of the decision variables. The possible height and depth of the staircase is limited by the maximum number of hours the equipment can be used per period in combination with the age of the pre-existing equipment. We use $M_t^{\max}$ to denote the maximum age bracket of truck type t—this value may vary depending on the equipment type. Since $B_0 > H_k$ (the age-bracket size is greater than the size of the period), $M_t^{\max}$ can also be restricted by the time period as equipment cannot age more than one age bracket in one period. However, we consider the possibility of pre-existing equipment (which is known *a priori*), with the starting age $M_t^{\max}$. The maximum age bracket for truck type t in time period k is as follows:

$$M_k(t) = \min\{M_t^{\max} + k - 1, M_t^{\max}\}.$$

This becomes a reduced limit for index l in any relevant constraint. We note that as salvage occurs at the start of the period, salvage variables extend to $M_k(t) + 1$.

At implementation, we do not permit over-age variables from existing, thus effecting forced salvage. That is, restricting variable creation in this way is equivalent to the following constraint:

$$x_{t,k,m} = s_{t,k+1,m+1} \qquad \forall \quad m > M_k(t), t \in \mathcal{T}, k \in \{1, \ldots, K-1\}.$$

If we consider the set of pre-existing equipment types, $\mathcal{P}$, there is the possibility of immediate salvage in the age bracket $m = 1$. However, for all other equipment types we can prevent the unbounded salvage variables from dominating:

$$s_{t,k,1} = 0 \qquad \forall \quad t \notin \mathcal{P}, k \in \{1, \ldots, K\}.$$

This too can be effected during variable creation, thereby reducing the overall number of variables and constraints required to represent the problem.

2.1. **Objective function.** We must satisfy demand either by location or route, depending on the nature of the mine plan. For this formulation, the availability of the equipment throughout the period is determined by the age bracket. The *production capability* is determined by its availability ($A_{t,m}$), capacity (C_t) and cycle time ($\tau_{t,j}$) (where the cycle time for a truck is the route cycle time and the cycle time for a loader is the time required to fill a particular truck type):

$$P_{t,j,k,m} = \frac{A_{t,m} C_t}{\tau_{t,j}}. \tag{1}$$

In our objective for the mixed-integer program, we minimize the cost of running the fleet for the entire mine plan, including purchase, operating expense and salvage. We represent the fixed cost of purchasing truck of type t by F_t and discount this purchase to the present using a discount factor,

$$D_k^1 = \frac{1}{(1+I)^k}$$

(where k is the current period, starting from 1, and I is the interest rate). Thus, the *total capital expense* for a truck of type t is

$$\sum_{t,k} F_t D_k^1 x_{t,k,1},$$

with a corresponding term for loaders.

The discretization of the variable costs over age brackets can lead to misleading operating costs, as the equipment may start the period in one age bracket (and corresponding cost) and move into another age bracket for the remainder of the period. To illustrate this, consider Figure 5. The best case is case (a), where the age bracket of the equipment is correct for the entire period. However, in case (b) the equipment moves into a new age bracket during the period. This will result in a discretization error in the model.

FIGURE 5. The two cases of equipment age landing between periods. For case (a), the equipment stays in the same age bracket for the entire period. In case (b), the equipment steps over into the next age bracket within the period.

In order to provide the most accurate costing possible for this model, we must determine the proportion of time that the equipment remains in age bracket m within the period, and the proportion it lies in the proceeding age bracket, $m+1$. The process to achieve this in a way that obtains a constant coefficient for variables, thereby maintaining linearity in the model, is described over the next page. Although it appears tedious and cumbersome, in practice it is very simple to implement in a computer program.

To calculate the portion of time that each set of equipment spends in each age bracket, we first calculate the age of the equipment (in age brackets) in any given period. To do this, we need to know when the equipment was purchased. We denote the purchase period by k'. The equipment must also be owned in the current period for this calculation to take place. The availability of equipment is the proportion of the period that the equipment is available to operate—unavailability is often due to planned maintenance. Availability as it is used here is calculated using the availability, utilization and maintenance factors discussed above, and is dependent on the current age bracket of the equipment. Let H_k be the operating hours for period k and $A_{t,m}$ be the availability of truck type t in age bracket m. We obtain the age of equipment in operated hours using a recursive formula since the availability of the equipment is a function of equipment age itself. The base of the recursion is:

$$\beta(k') = A_{t,1} H_{k'}.$$

Then, the age of the equipment in operated hours can be obtained for any period k by:

$$\beta(k) = \sum_{k' \le h' < k} A_{t,\left\lfloor \frac{\beta(h'-1)}{B_0} \right\rfloor} H_{h'}.$$

We obtain the age bracket in which equipment lies at the beginning of a period by $b(k)$:

$$b(k) = \left\lfloor \frac{\beta(k)}{B_0} \right\rfloor.$$

Since $B_0 > H_k$, the equipment may only lie in $h \in \{1,2\}$ age brackets within one period. To begin, we define $B^h_{t,k,m}$ to be the proportion of total operated hours that truck t spends in age bracket $m+h-1$ in period k (where the incumbent age bracket is m).

Theorem 2.1. *The proportion of time that any group of machinery spends in any one age bracket can be represented by the following two expressions:*

$$B^1_{t,k,m} = \begin{cases} 1, & \text{if } (m+1)B_0 - \beta(k) > A_{t,b(k)} H_k \\ \dfrac{(m+1)B_0 - \beta(k)}{A_{t,b(k)} H_k}, & \text{otherwise} \end{cases}$$

and

$$B^2_{t,k,m} = 1 - B^1_{t,k,m}.$$

Proof. The age at the start of the period is given by $\beta(t)$ as defined above. The quantity of hours worked in the current period is given by:

$$A_{t,m}H_k.$$

If the equipment stays in the same age bracket for the entire period, we require that the difference between the marker for the next age bracket, $(m+1)B_0$, and the age at the start of the period exceeds the quantity of hours worked in the current period. That is, if:

$$(m+1)B_0 - \beta(k) > A_{t,b(k)}H_k.$$

Similarly, if the equipment moves into another age bracket for part of the period, we can simply look at the difference between the marker for the next age bracket and the age at the start of the period. Dividing by the operated hours for the current period gives the proportion of total operated hours, as required. □

We can easily adjust these formulas for the case of pre-existing equipment, but for the sake of clarity omit this here. We can now use $B^1_{t,k,m}$ and $B^2_{t,k,m}$ to correct the operating cost (denoted by $V_{t,k,m}$) in the objective function as follows (also with a discount factor):

$$\sum_{t,k,m,h} \frac{B^h_{t,k,m}}{(1+m)^k} V_{t,k,b(k)+h-1} x_{t,k,m}.$$

Lastly, we consider the income from salvaging old equipment. We apply a combined depreciation (at rate J per period) and discount factor (at rate I per period):

$$D^2_{k,m} = \frac{(1-J)^l}{(1+I)^k},$$

where l is the age of the equipment at the start of period t. Since F_e is the original capital expense, the salvage 'cost' is:

$$-\sum_{t,k,m} F_t D^2_{k,m} \mathbf{s}_{t,k,m},$$

with a corresponding term for loaders.

2.2. **Constraints.** We require the loaders to satisfy the productivity demand, $D_{i,k}$, at location $i \in \mathcal{I}$, giving us the following *capacity demand constraints*:

$$\sum_{l,m} P_{l,k,m} f_{l,i,k,m} \geq D_{i,k} \qquad \forall \quad k \in \{1,\ldots,K\}, i \in \mathcal{I}, \tag{2}$$

where $P_{l,k,m}$ is the maximum possible productivity of loader l that is aged m in period k. This expression is obtained by considering equipment capacity, swing time (time to deliver one load to the truck), number of required swings (truck to loader capacity ratio), and downtime due to maintenance and other factors. Similarly, we require the trucks to satisfy the demand for each route or dump site, $j \in \mathcal{J}$:

$$\sum_{t,m} P_{t,k,m} f_{t,j,k,m} \geq D_{j,k} \qquad \forall \quad k \in \{1,\ldots,K\}, j \in \mathcal{J}. \tag{3}$$

The trucks must also match the capacity demand of the mining locations. For each location i, we are only interested in the routes, j, that connect to the location. We denote the set of routes that connect to location i by $\mathcal{J}(i)$. Then we have:

$$\sum_{t,m;j\in\mathcal{J}(i)} P_{t,k,m} f_{t,j,k,m} \geq D_{i,k} \qquad \forall \quad k \in \{1,\dots,K\}, i \in \mathcal{I}. \tag{4}$$

The capacity constraints must be satisfied with the set of compatible trucks and loaders for each location. That is, from the chosen fleet of trucks we must consider whether the set of trucks that are *compatible* with the loaders are capable of fulfilling the capacity constraints.

Theorem 2.2. *Suppose we model the equipment selection problem as a mixed-integer linear program with a minimizing cost objective function. Then production feasibility is not guaranteed with constraints 2 and 4 alone.*

Proof. Consider one loading location. Let there be exactly two types of loaders operating at this location, λ_1 and λ_2. From constraint 2, we have:

$$P_{\lambda_1} + P_{\lambda_2} \geq D.$$

That is, the productivity of the loader fleets of type λ_1 and λ_2 meet the productivity requirements at the location. Suppose we have two types of trucks servicing the location, τ_1 and τ_2. From constraint 4 we have:

$$P_{\tau_1} + P_{\tau_2} \geq D.$$

That is, the productivity of the truck fleets of type τ_1 and τ_2 meet the productivity requirements of the location.

Next, suppose that the compatibility sets of loader types with truck types are different for each type of loader. Specifically, loader λ_1 is only compatible with truck τ_1, and loader λ_2 is only compatible with truck τ_2.

Let $P_{\tau_2} = D$ and $P_{\lambda_1} = P_{\lambda_2} = \frac{D}{2}$. Then, since it is a minimization problem, the constraints 2 and 4 are met minimally and the actual productivity capability of the trucks and loaders (when working together) is $\frac{D}{2}$ and the productivity requirements of the mine are not met. □

Therefore, we must ensure that the weak productivity constraint is satisfied for every possible subset of truck and loader fleets. To capture this in a constraint set, we first recall that the set of loader types is denoted by $\mathcal{L}$. Next, we define the set $\mathcal{T}(\mathcal{L}')$ to be the set of truck types that are compatible with the subset of loader types $\mathcal{L}' \subset \mathcal{L}$.

Theorem 2.3. *The compatibility of the selected fleets is ensured, in combination with constraint 2, by the following superset constraint set:*

$$\sum_{t\in\mathcal{T}(\mathcal{L}'),m} P_{t,k,m} f_{t,j,k,m} \geq \sum_{l\in\mathcal{L}',m} P_{l,k,m} f_{l,i,k,m} \qquad \forall \quad \mathcal{L}' \subset \mathcal{L}, k, i. \tag{5}$$

Proof. Suppose it is not sufficient. Then there exists some combination of equipment such that the compatibility prevents satisfaction of productivity requirements. Let this set of equipment be represented by $\mathcal{L}^1$ and $\mathcal{T}(\mathcal{L}^1)$. From constraint 5 we know that:

$$\sum_{t\in\mathcal{T}(\mathcal{L}^1),m} P_{t,k,m} f_{t,j,k,m} \geq \sum_{l\in\mathcal{L}^1,m} P_{l,k,m} f_{l,i,k,m}.$$

That is, the productivity of the set of trucks at least matches the productivity of its compatible loader set. This truck set cannot be the only equipment, otherwise

$$\sum_{t,m} P_{t,k,m} f_{t,j,k,m} \geq D_{t,k}$$

and we are done. Therefore the selected truck set includes the subset $\mathcal{T}(\mathcal{L}^1)$ and some other subset $\mathcal{T}(\mathcal{L}^2)$.

That is, we have

$$\sum_{l \in \mathcal{L}^1, m} P_{l,k,m} f_{l,i,k,m} + \sum_{l \in \mathcal{L}^2, m} P_{l,k,m} f_{l,i,k,m} \geq D_{k,i},$$

for all i, k. That is, we have

$$\begin{aligned} \sum_{\substack{t \in \mathcal{T}(\mathcal{L}^1), \\ m}} P_{t,k,m} f_{t,j,k,m} + \sum_{\substack{t \in \mathcal{T}(\mathcal{L}^2), \\ m}} P_{t,k,m} f_{t,j,k,m} &\geq \sum_{\substack{l \in \mathcal{L}^1, \\ m}} P_{l,k,m} f_{l,i,k,m} + \sum_{\substack{l \in \mathcal{L}^2, \\ m}} P_{l,k,m} f_{l,i,k,m} \\ &\geq D, \end{aligned}$$

and the requirements are met. $\Rightarrow\Leftarrow$ □

Since $\mathcal{L}'$ comes from the power set of $\mathcal{L}$, the compatibility constraint set will generate $K|\mathcal{I}|(2^{|\mathcal{L}|}-1)$ constraints (where K is the total number of periods and $|\mathcal{I}|$ is the total number of locations). As a power set constraint, it should be implemented using a separation algorithm. However, for a given case study the number of loaders possible in the final solution will generally be much lower than the complete set. In this case we can limit the generation of constraints to a maximum of α loader types. This will produce $K|\mathcal{I}| \sum_{a=1}^{\alpha} (\frac{|\mathcal{L}|!}{a!(|\mathcal{L}|-a)!})$ constraints. To further reduce the overall number of these constraints in the solver, we use a separation algorithm. With this branch-and-cut method, we begin with no compatibility constraints in the model. We iteratively solve the model and check for feasibility—any violated constraints are then added into the model before it is resolved.

We link the equipment tracking variables, $x_{t,k,m}$, to the allocation variables $f_{t,j,k,m}$ by placing an upper bound on the allocation in the following *coupling constraints* (with a corresponding constraint for loaders):

$$x_{t,k,m} \geq \sum_j f_{t,j,k,m} \qquad \forall \quad t \in \mathcal{T}, k \in \{1, \dots, K\}, m \in \{1, \dots, M\}. \tag{6}$$

We ensure that, in each period, we can only own non-new equipment if we owned it in the previous period, as captured in the following *precedence constraints* (with a corresponding constraint for loaders):

$$x_{t,k,m} = x_{t,k-1,m-1} - s_{t,k,m} \quad \forall\ t \in \mathcal{T}, k \in \{2, \dots, K\}, m \in \{2, \dots, M\}, \tag{7}$$

$$x_{t,k-1,m-1} \geq s_{t,k,m} \qquad \forall\ t \in \mathcal{T}, k \in \{2, \dots, K\}, m \in \{2, \dots, M\}. \tag{8}$$

In this model we consider pre-existing equipment. We only need to consider pre-existing trucks and loaders which are drawn from the subset $\mathcal{P} \subset \mathcal{T} \cup \mathcal{L}$. Recall that $b(1)$ is the starting age of the pre-existing truck type t. Then, if $\bar{x}_{t,1,b(1)}$ is the number of pre-existing truck of type t, with age $b(1)$, we have (with a corresponding constraint for loaders):

$$x_{t,1,b(1)} + s_{t,1,b(1)} = \bar{x}_{t,1,b(1)} \qquad \forall \quad t \in \mathcal{P}. \tag{9}$$

2.3. **Complete model.**

$$\min \quad \sum_{t,k} F_t D_k^1 x_{t,k,1} + \sum_{l,k} F_t D_k^1 y_{l,k,1} + \sum_{t,k,m,h} B_{t,k,m}^h D_k^1 V_{t,k,b(k)+h-1} f_{t,j,k,m}$$
$$+ \sum_{l,k,m,h} B_{t,k,m}^h D_k^1 V_{l,k,b(k)+h-1} f_{l,i,k,m} - \sum_{t,k,m} F_t D_{k,m}^2 s_{t,k,m} - \sum_{l,k,m} F_l D_{k,m}^2 s_{l,k,m}$$

$$\text{subject to} \quad \sum_{l,m} P_{l,k,m} f_{l,i,k,m} \geq D_{i,k} \qquad \forall\ i, k, \tag{10}$$

$$\sum_{t,m;j\in\mathcal{J}(i)} P_{t,k,m} f_{t,j,k,m} \geq D_{i,k} \qquad \forall\ i, k, \tag{11}$$

$$\sum_{t\in\mathcal{T}(\mathcal{L}'),m} P_{t,k,m} f_{t,j,k,m} \geq \sum_{l\in\mathcal{L}',m} P_{l,k,m} f_{l,i,k,m} \qquad \forall \quad \mathcal{L}' \subset \mathcal{L}, i, k, \tag{12}$$

$$x_{t,k,m} \geq \sum_j f_{t,j,k,m} \qquad \forall\ t, k, m, \tag{13}$$

$$y_{l,k,m} \geq \sum_i f_{l,i,k,m} \qquad \forall\ l, k, m, \tag{14}$$

$$x_{t,k,m} = x_{t,k-1,m-1} - s_{t,k,m} \qquad \forall\ t, k > 1, m > 1, \tag{15}$$

$$x_{t,k-1,m-1} \geq s_{t,k,m} \qquad \forall\ t, k > 1, m > 1, \tag{16}$$

$$y_{l,k,m} = y_{t,k-1,m-1} - s_{l,k,m} \qquad \forall\ l, k > 1, m > 1, \tag{17}$$

$$y_{t,k-1,m-1} \geq s_{l,k,m} \qquad \forall\ l, k > 1, m > 1, \tag{18}$$

$$x_{t,1,b(1)} + s_{t,1,b(1)} = \bar{x}_{t,1,b(1)} \qquad \forall\ t \in \mathcal{P}, \tag{19}$$

$$x_{t,k,m}, y_{l,k,m}, s_{t,k,m}, s_{l,k,m} \in \mathbb{Z}^+,$$

$$f_{t,j,k,m}, f_{l,i,k,m} \in \mathcal{R}^+.$$

3. **Results.** We consider two case studies provided by our industry partner. The first is for a planned mine with no comparative solution. The second is for a mine with pre-existing equipment and a comparative solution. These case studies illustrate that the difficulty in solving the model is not wholly influenced by the size of K or the number of routes in the mine plan. In both case studies, we deal with mine progression over time by allocating a cycle time of zero to those routes that are inaccessible in a given time period.

3.1. **Case study 1.** Our industry partner wishes to select a fleet of trucks and loaders for an open pit iron ore mine operating under a truck-loader hauling system. This mine is in the planning stages and begins with no pre-existing equipment.

3.1.1. *Locations and routes.* For this mine, there are only two mining locations. Each location produces ore which should be delivered to the mill, and waste which should be delivered to a dump site. Figure 6 describes the two mining locations and four routes which connect them to the waste dump site and the mill.

3.1.2. *Production requirements.* This case study considers a mine operating under a truck-loader hauling system and mines ore and waste in an open pit. Our industry partner provided the production requirement data for both the mining locations and the truck routes. For this case study, we consider $K = 9$ periods in total, each of length 1 year. This new mine is simple in terms of the number of mining locations. The overall production requirements change drastically as the overburden

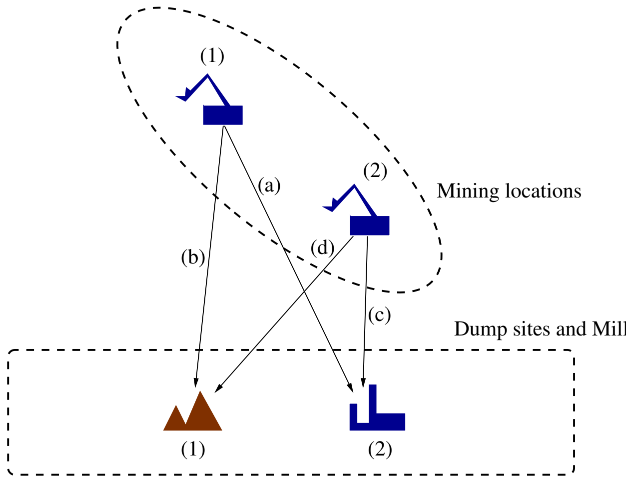

FIGURE 6. The locations for case study two.

	Period								
Route	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
(a)	51701	668713	1602777	1463289	2227402	1657740	2031283	2230192	2474503
(b)	2082800	8843227	7294449	5837564	5356417	8051374	4582733	4666783	2001808
(c)	0	0	1189184	1120125	337101	1073245	1106596	455511	235810
(c)	0	9412391	9063476	10593225	11355932	4091566	530276	84354	94315
Total (MT)	2.13	18.92	19.14	19.01	19.27	14.87	8.25	7.43	4.80

Key for routes:
(a) $M1$ to Mill
(b) $M1$ to Dump site (1)
(c) $M2$ to Mill
(d) $M2$ to Dump site (1)

TABLE 1. The production requirements for the routes for the second case study.

is removed, and as the routes become longer (Table 1). For example, in period one, the production requirements are only 2.13 million tonnes. This grows to around 19 million tonnes for the subsequent 3 years. The estimated truck cycle times (also provided by the industry partner) also demonstrates great variability from period to period, and between locations (Table 2). For example, the smallest truck cycle time is 2.64 minutes, while the longest is 23.82 minutes.

3.1.3. *Case specific parameters.* We have the following parameters supplied by our industry partner:

- The mine is removing ore and waste, and operates under a shovel-truck system.
- The mine operates for 7604 hours in each period (accounting for blasting days, holidays and other non-operational days);
- The loaders are selected from a set of 20 loader types.
- The trucks are selected from a set of 8 truck types.
- There are K = 9 periods in total, each of length 1 year.
- The cost-bracket length, B_0, is 5000 hours.
- The interest rate for all periods is 8%.

	Route			
Period	**(a)**	**(b)**	**(c)**	**(d)**
1	8.24	2.64	0.0	0.0
2	8.3	3.48	0.0	8.24
3	9.28	3.84	5.74	10.23
4	10.52	4.88	8.73	10.45
5	11.16	6.01	10.38	12.6
6	12.47	7.23	11.71	16.72
7	12.05	8.49	13.82	19.6
8	15.77	10.11	15.49	21.37
9	17.74	12.05	16.52	22.82

TABLE 2. The truck cycle times for the second case study.

Periods	*Variables*	*Constraints*	*Time (seconds)*	*Quality*	*Solution*
7	9100	4242 + 2044	5331	Optimal	1.88599×10^7
8	11648	5473 + 1484	12049	Optimal	1.97785×10^7
9	14502	6858 + 2380	19477	3%	2.05244×10^7

TABLE 3. The results summary for the first case study solutions with varying periods.

- The depreciation rate is set to 50%.
- The maximum value for any truck variable is 30, i.e., maximum 30 trucks of a given type, in any age bracket, in any period;
- The maximum value for any loader variable is 10, i.e., maximum 10 loaders of a given type, in any age bracket, in any period.

Note that the depreciation value is used to estimate the sale (or salvage) value of used equipment, rather than for tax offset purposes. We choose a high depreciation value to lessen the impact of a sale on the decisions, as the second hand market is unreliable.

Our industry partner also provided Utilisation Factors, which are reducing factors to account for lost hours due to inefficiency, maintenance, and availability of the equipment; and, a compatibility matrix between all equipment types. However, we choose to suppress this information, along with the truck and loader types, to protect our industry partners contractual relationships.

3.1.4. *Computational results.* We implemented this case study on a Pentium 4 PC with 3.0GHz and 2.5GB of RAM. The model was implemented in C++ using *Ilog Concert Technology v 2.5* objects and *Ilog Cplex v 11.0* libraries to solve the problem with default settings. The constraints (12) were implemented as lazy constraints—i.e., we separated the constraints from the model and only added those which were violated by feasible MIP solutions. We implemented this problem with 14502 variables (3672 integers) and 6858 constraints. The separation algorithm added a further 2380 compatibility constraints. We ran the 9-period problem for 34 hours before the computer memory was exhausted and achieved a solution within 3% of optimality. The computation times for single solves of the problem with 7, 8 and 9 periods is presented in Table 3. The solution obtained is given in Figure 7. In this figure, a black circle depicts a purchase, while an empty circle depicts a salvage. For this case study, the optimal solution was to purchase one loader of type three, and operate these for the entire nine periods. Only five trucks were purchased for

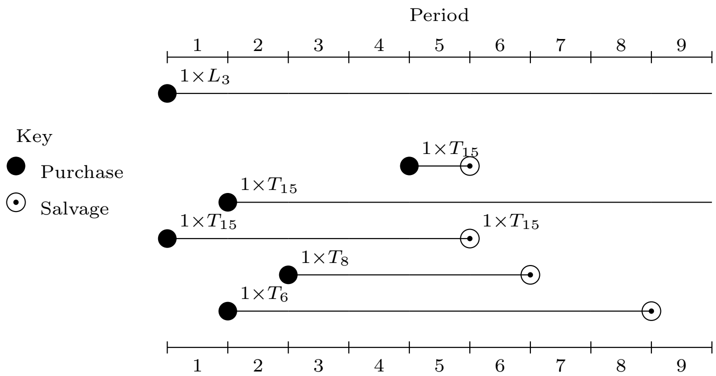

FIGURE 7. The first case study 9-period purchase and salvage policy with depreciation 50%.

the entire 9 periods. The purchasing pattern is indicative of the dramatic increase in production requirements in the first few periods—in the beginning, one truck is sufficient. In reality, this would not be a suitable solution, because if that truck is in maintenance, the mine no longer produces. Also, it means that the loader is not utilized while the truck is delivering its load. In the subsequent two periods, three further trucks are purchased. By period four, there are three types of trucks working in the fleet. In period five, the productivity of the existing trucks has fallen sufficiently so that a new purchase is worthwhile. The fifth truck is then salvaged only 1 year later. While is it not realistic that the truck is sold after only one year, it is conceivable that the truck is relocated to another mine in need of a relatively new truck. The remaining trucks are gradually salvaged as the needs of the mine decrease, leaving only one truck until the end of the final period.

The truck allocation solution is presented in Table 4. It is difficult to identify the slack in the allocation of equipment because the model did not motivate a minimum utilization value—i.e., it does not cost more (in terms of the objective function value) to allocate trucks to locations for more time than necessary. Some routes receive every type of truck in the fleet. This could have an impact on cycle time, as some trucks will be faster than others or will be served faster at the loader. However, this level of detail is not captured in our model. The loader allocation solution splits one loader across two mining locations [Table 5]. In some mining scenarios where loader movement across the mine is prevented, this solution would be unrealistic. It is likely that one loader at each location is preferred. In this case, it is easy to add constraints that reflect node-disjointed flow for the loaders—for example, this can be achieved by forcing $f_{l,i,k,m} \in \mathbb{Z}$.

This case study seems small with respect to the number of periods and the number of routes and locations. However, the symmetry in this problem—arising because of the number of identical equipment which can be allocated to the same decisions—makes it difficult to solve with default settings. We will defer comparisons with the second case study to the Discussion section (3.3).

	Period								
Route	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
(a)	0.01 $T_{15}(1)$	0.14 $T_{15}(2)$	1.00 $T_6(2)$	0.44 $T_{15}(4)$	0.51 $T_8(3)$	0.51 $T_{15}(5)$	0.62 $T_{15}(6)$	0.89 $T_{15}(8)$	0.11 $T_{15}(8)$ 1.00 $T_{15}(9)$
(b)	0.99 $T_{15}(1)$	0.97 $T_6(1)$	0.58 $T_8(1)$	0.14 $T_{15}(3)$ 0.56 $T_{15}(4)$	0.88 $T_6(4)$	1.00 $T_8(4)$ 0.26 $T_{15}(5)$	1.00 $T_6(6)$ 0.74 $T_{15}(7)$	0.07 $T_6(7)$ 1.00 $T_{15}(7)$ 0.11 $T_{15}(8)$	0.74 $T_{15}(8)$
(c)			0.23 $T_{15}(2)$	0.24 $T_{15}(3)$	0.07 $T_8(3)$	0.23 $T_{15}(5)$ 0.21 $T_{15}(6)$	0.38 $T_{15}(6)$	0.19 $T_6(7)$	0.10 $T_{15}(8)$
(d)		0.03 $T_6(1)$ 1.00 $T_{15}(1)$ 0.86 $T_{15}(2)$	0.42 $T_8(1)$ 0.77 $T_{15}(2)$ 1.00 $T_{15}(3)$	0.62 $T_{15}(3)$ 1.00 $T_8(2)$ 0.44 $T_9(3)$	0.12 $T_6(4)$ 0.41 $T_8(3)$ 1.00 $T_{15}(1)$ 1.00 $T_{15}(4)$ 1.00 $T_{15}(5)$	1.00 $T_6(5)$ 0.79 $T_{15}(6)$	0.26 $T_{15}(7)$	0.73 $T_6(7)$	0.06 $T_{15}(8)$

Key: $xT_t(m)$ indicates that x trucks of type t and age m operate on the route in the given period.

TABLE 4. The 9-period truck allocation policy for the first case study, 50% depreciation.

	Period								
Route	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
(a)	0.01 $L_3(1)$	0.03 $L_3(2)$	0.07 $L_3(3)$	0.06 $L_3(4)$	0.09 $L_3(5)$	0.07 $L_3(6)$	0.08 $L_3(7)$	0.09 $L_3(8)$	0.10 $L_3(9)$
(b)	0.09 $L_3(1)$	0.36 $L_3(2)$	0.30 $L_3(3)$	0.24 $L_3(4)$	0.23 $L_3(5)$	0.33 $L_3(6)$	0.19 $L_3(7)$	0.20 $L_3(8)$	0.08 $L_3(9)$
(c)			0.05 $L_3(3)$	0.05 $L_3(4)$	0.01 $L_3(5)$	0.04 $L_3(6)$	0.05 $L_3(7)$	0.02 $L_3(8)$	0.01 $L_3(9)$
(d)		0.38 $L_3(2)$	0.39 $L_3(3)$	0.44 $L_3(4)$	0.47 $L_3(5)$	0.17 $L_3(6)$	0.02 $L_3(7)$	0.01 $L_3(8)$	0.01 $L_3(9)$

Key: $xL_l(m)$ indicates that x loaders of type l and age m operate on the route in the given period.

TABLE 5. The 9-period loader allocation policy for the first case study, 50% depreciation.

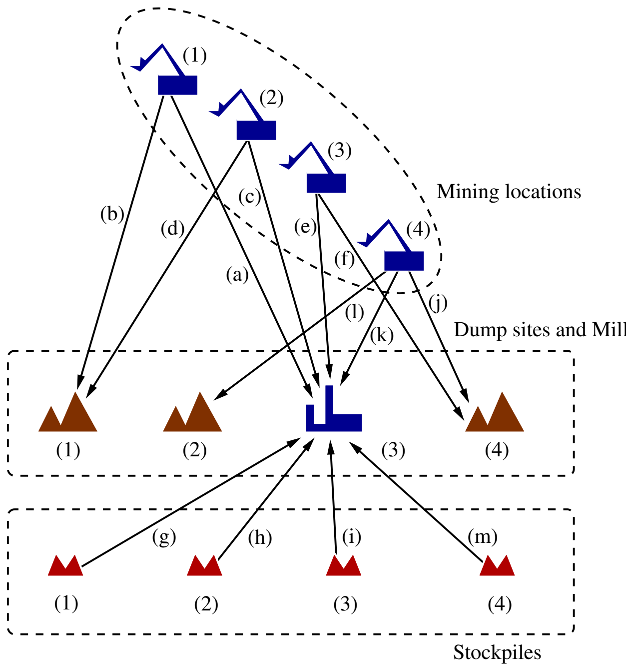

FIGURE 8. Routes from mining locations to dump sites for the second case study.

3.2. **Case Study 2.** Our industry partner provided a second case study from an ongoing mining operation with pre-existing equipment and a more complex route structure. This case study considers a surface mine operating under a truck-loader hauling system, and mines ore and waste in an open pit.

3.2.1. *Locations and routes.* The mine for this case study has eight loading locations—four mining locations and four stockpiles, as depicted in Figure 8. Mixing constraints are the quantity of different grades of ore required to make up the final grade. Typically the final grade is determined by market demand. The mixing constraints are not considered in this model, as they are assumed to be pre-optimized when the mine plan is produced. In this case study the stockpiles are old, and newly mined ore or waste is not dumped in these locations. Instead, they are used to create the appropriate mix at the mill—so the flow from these locations is unidirectional. There are also four dump sites including one mill. Connecting these mining locations, stockpiles and dump site, are 13 routes in total (route key provided in Table 6).

3.2.2. *Production requirements.* Our industry partner provided the production requirement data for both the mining locations and the truck routes. Tables 6 and 7 describe the quantity of material (tonnes) to be moved from each location, to each dump site. Our industry partner also provided pre-estimated truck cycle times for each route, presented in Table 8.

3.2.3. *Pre-existing equipment.* We begin the mine plan with some pre-existing equipment, as listed in Table 9. This includes eleven 172 tonne trucks of varying age in

Route	**Period** 1	2	3	4	5	6	7	8	9	10	11	12	13
(a)	2598	0	0	0	0	0	0	0	0	0	0	0	0
(b)	455	0	0	0	0	0	0	0	0	0	0	0	0
(c)	203	10141	7659	0	0	0	0	0	0	0	0	0	0
(d)	741	7060	2964	0	0	0	0	0	0	0	0	0	0
(e)	221	827	5928	12797	9919	0	0	0	13990	13990	13990	5184	11
(f)	2592	22935	41035	26948	19873	0	0	0	8901	18191	12589	5572	32
(g)	0	650	0	0	0	0	0	0	0	0	0	0	0
(h)	0	270	809	0	0	0	0	0	0	0	0	0	0
(i)	0	1142	0	0	0	0	0	0	0	0	0	0	0
(j)	0	0	0	737	4071	12990	12890	13990	0	0	0	7713	0
(k)	0	0	0	17051	24126	8984	8583	12836	0	0	0	6251	0
(l)	0	0	0	0	0	799	0	0	0	0	0	0	0
(m)	0	0	0	0	0	0	798	0	0	0	0	0	0
Req. kT	23125	28102	36102	38484	42450	43329	45000	43329	42450	38484	36102	28102	23125

Key for routes

(a) Mining location (1) to Dump site (3)
(b) Mining location (1) to Dump site (1)
(c) Mining location (2) to Dump site (3)
(d) Mining location (2) to Dump site (1)
(e) Mining location (3) to Dump site (3)
(f) Mining location (3) to Dump site (4)
(g) Stockpile (1) to Dump site (3)
(h) Stockpile (2) to Dump site (3)
(i) Stockpile (3) to Dump site (3)
(j) Mining location (4) to Dump site (1)
(k) Mining location (4) to Dump site (2)
(l) Mining location (4) to Dump site (3)
(m) Stockpile (4) to Dump site (3)

TABLE 6. The production requirements (tonnes) for the truck routes for case study two. Dump site (3) is the mill—we treat this dump site as the others.

Location	**Period** 1	2	3	4	5	6	7	8	9	10	11	12	13
M(1)	3053	0	0	0	0	0	0	0	0	0	0	0	0
M(2)	944	17201	10624	0	0	0	0	0	0	0	0	0	0
M(3)	2813	23762	46963	39746	29792	0	0	0	22891	32181	26579	10756	43
M(4)	0	650	0	0	0	0	0	0	0	0	0	0	0
S(1)	0	270	809	0	0	0	0	0	0	0	0	0	0
S(2)	0	1142	0	0	0	0	0	0	0	0	0	0	0
S(3)	0	0	0	17789	28197	22773	21474	26826	0	0	0	13964	0
S(4)	0	0	0	0	0	0	798	0	0	0	0	0	0
Req. kT	23125	28102	36102	38484	42450	43329	45000	43329	42450	38484	36102	28102	23125

Key for locations
M(i) Mining location *i* *S(j)* Stockpile *j*

TABLE 7. The production requirements (tonnes) for the mining locations for case study two.

Period	**Routes** (a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)
1	35.56	35.17	38.49	37.89	11.23	10.62	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	38.03	37.42	14.97	17.23	16.05	60.00	20.00	0.0	0.0	0.0	0.0
3	0.0	0.0	40.09	40.77	25.02	22.08	0.0	60.00	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	27.18	25.09	0.0	0.0	0.0	34.39	34.27	0.0	0.0
5	0.0	0.0	0.0	0.0	28.58	26.25	0.0	0.0	0.0	26.57	27.38	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	32.20	33.51	37.43	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	34.48	35.71	0.0	15.00
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	37.66	38.76	0.0	0.0
9	0.0	0.0	0.0	0.0	32.64	29.41	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	36.42	32.47	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	38.53	36.49	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	43.54	43.57	0.0	0.0	0.0	39.52	46.67	0.0	0.0
13	0.0	0.0	0.0	0.0	46.93	45.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE 8. Truck cycle times for case study two.

Equipment i.d.	L_7^P	L_7^P	L_{17}^P	T_{12}^P	T_{12}^P	T_{12}^P
Quantity	1	1	1	3	2	6
Capacity (tonnes)	34	34	42	172	172	172
Age (years)	16	17	16	7	8	11

TABLE 9. Pre-existing equipment for case study two.

hours; and three loaders, namely two 34 tonne hydraulic shovels and one 42 tonne hydraulic shovel.

3.2.4. *Case-specific parameters.* Some parameters are defined by the industry partner:

- The mine operates for 7604 hours in each period (accounting for blasting days, holidays and other non-operational days);
- The loaders are selected from a set of 20 loader types;
- The trucks are selected from a set of 8 truck types;
- The cost-bracket partition, B_0, is 5000 hours;
- The schedule is 13 years long;
- The discount rate for all periods is 8% (the approximate interest rate obtainable on investments).

We define the following parameters:

- There are 13 periods, K, each of length 1 year;
- The depreciation rate is set to 50% (a rate of 40-60% is common for this application due to unreliability in the second hand equipment market);
- The maximum value for any truck variable is 30, i.e., maximum 30 trucks of a given type, in any age bracket, in any period;
- The maximum value for any loader variable is 10, i.e., maximum 10 loaders of a given type, in any age bracket, in any period.

3.2.5. *Computational results.* We implemented this case study on a Pentium 4 PC with 3.0GHz and 2.5GB of RAM. The model was implemented in C++ using *Ilog Concert Technology v 2.5* objects and *Ilog Cplex v 11.0* libraries to solve the problem. The mixed-integer program contained 63433 variables (5304 integer) and 19366 constraints; we set up the 13-period problem with just 15571 constraints before the compatibility constraints were taken into account.

Table 10 shows the number of constraints that were added by the separation algorithm overall for the full problem and versions of the problem with a reduced schedule length (namely, 10, 11 and 12 years).

Periods	*Variables*	*Constraints*	*Time (seconds)*	*Quality*	*Solution*
10	39855	9814 + 3319	5643	3%	1.26292×10^8
11	47166	11599 + 3043	3979	3%	1.31263×10^8
12	55032	13521 + 4043	17656	3%	1.37168×10^8
13	63433	15571 + 3795	26662	3%	1.37249×10^8

TABLE 10. The results summary for the second case study with depreciation 50% and 13 periods.

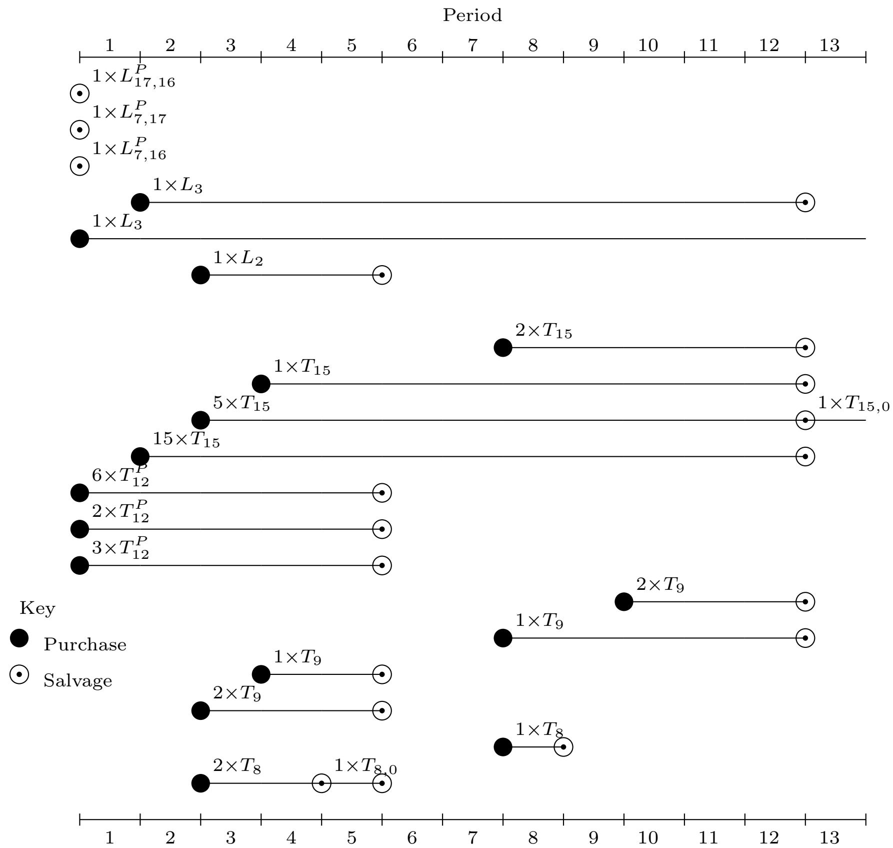

FIGURE 9. The 3% optimal 13-period solution for case study two with the depreciation parameter set to 50%.

After 7.5 hours of algorithm run-time, we obtained a solution within 3% of the optimal solution for the 50% depreciation case study. When the algorithm was permitted to run for a longer period, the computer memory was exhausted. However, this is a satisfactory optimality gap for this application, as we will illustrate later with a retrospective solution comparison.

The purchase and salvage policy for this multi-location mine is complicated by the capacity requirements, which contain several significant changes from period to period. This leads to short-term ownership of some trucks. For example a type-8 truck was purchased in period 8 and salvaged at the start of period 9. The complete purchase and salvage policy is sketched in Figure 9, where pre-existing equipment is indicated by the P index.

The allocation policy is shown in Tables 11–14. In the allocation policy for this case study, we represent the age of the equipment in parentheses as an equipment tracking tool. Since the age of the equipment is a factor in the cost of operating the equipment, it is important to allocate the correct age equipment as dictated by the policy.

For the purposes of mine management, it would be a simple task to create a spreadsheet that can reflect the flexibilities in the policy and allow dynamic changes in the policy without affecting the objective function value.

Routes	Period 1	2	3	4	5	6	7
(a)	1.81 $T_{12}(8)$ 6.00 $T_{12}(12)$						
(b)	0.35 $T_{12}(8)$						
(c)	0.17 $T_{12}(8)$	9.42 $T_{15}(1)$	6.00 $T_{12}(14)$ 0.97 $T_{15}(1)$				
(d)	2.00 $T_{12}(9)$	5.90 $T_{12}(13)$	0.68 $T_{12}(10)$ 2.00 $T_{12}(11)$				
(e)	0.05 $T_{12}(8)$	0.18 $T_{12}(9)$ 0.09 $T_{12}(13)$	3.70 $T_{15}(1)$	2.65 $T_9(2)$ 1.50 $T_8(2)$ 6.73 $T_{15}(3)$	0.42 $T_{12}(13)$ 6.00 $T_{12}(16)$		
(f)	0.61 $T_{12}(8)$	2.32 $T_{12}(9)$ 2.00 $T_{12}(10)$ 4.87 $T_{15}(1)$	2.00 $T_9(1)$ 2.32 $T_{12}(10)$ 0.32 $T_{15}(1)$ 15.0 $T_{15}(2)$ 0.99 $T_8(1)$	1.00 $T_9(1)$ 2.00 $T_9(2)$ 3.00 $T_{12}(11)$ 1.97 $T_{12}(12)$ 6.00 $T_{12}(15)$	3.00 $T_{12}(12)$ 1.58 $T_{12}(13)$ 1.00 $T_{15}(2)$ 5.00 $T_{15}(3)$ 0.67 $T_{15}(4)$		
(g)		0.31 $T_{15}(1)$					
(h)		0.40 $T_{15}(1)$	1.00 $T_8(1)$				
(i)		0.50 $T_{12}(9)$					
(j)				0.50 $T_8(2)$ 0.03 $T_{12}(12)$	2.67 $T_{15}(4)$	12.84 $T_{15}(5)$	13.115 $T_{15}(6)$
(k)				1.00 $T_{15}(1)$ 5.00 $T_{15}(2)$ 8.27 $T_{15}(3)$	11.7 $T_{15}(4)$	1.00 $T_{15}(3)$ 4.26 $T_{15}(4)$ 2.16 $T_{15}(5)$	1.00 $T_{15}(4)$ 5.00 $T_{15}(5)$ 1.59 $T_{15}(6)$
(l)						0.74 $T_{15}(4)$	
(m)							0.30 $T_{15}(6)$

Key: $xT_t(m)$ indicates that x trucks of type t and age m operate on the route in the given period.

TABLE 11. The truck allocation policy for case study two with 13 periods and 50% depreciation (first 7 periods).

3.3. **Discussion.** In the final solutions for the case studies, the model has selected 3 and 4 truck types respectively. However, it is considered unusual in industry to have more than three types of trucks, and generally this number would only arise due to pre-existing equipment. This suggests that the models are not reflecting the true penalties associated with the fixed costs of owning different types of equipment, or conversely, that the reasoning behind homogeneous or small mixes of fleet needs further justification. Also, operating different truck types on the same route may influence the accuracy of the cycle times due to bunching of equipment. This issue is usually addressed during dispatching of equipment, but ideally should be considered during equipment selection—i.e., to account for the interactive effect of the selected equipment. It is not obvious how to incorporate this into the current deterministic mixed-integer program, but it makes an interesting question for future research.

As some of the data for the case studies was limited or unknown (such as the requirement for stockpiles to have their own loaders, or when locations are mined simultaneously within a period) the solutions generated by our model sometimes requires that one loader move from one location to another. This may not be realistic and in this case can be amended by enforcing integrality constraints on the loader decision variables.

	Period					
Routes	**8**	**9**	**10**	**11**	**12**	**13**
(a)						
(b)						
(c)						
(d)						
(e)		1.00 $T_9(2)$ 15.00 $T_{15}(8)$	1.00 $T_{15}(7)$ 5.00 $T_{15}(8)$ 6.85 $T_{15}(9)$	2.00 $T_9(2)$ 1.00 $T_9(4)$ 2.00 $T_{15}(4)$ 1.00 $T_{15}(8)$ 5.00 $T_{15}(9)$ 0.83 $T_{15}(10)$	2.00 $T_9(3)$ 1.00 $T_9(5)$ 0.72 $T_{15}(5)$ 1.00 $T_{15}(9)$	0.01 $T_{15}(11)$
(f)		2.00 $T_{15}(2)$ 1.00 $T_{15}(6)$ 5.00 $T_{15}(7)$	2.00 $T_9(1)$ 1.00 $T_9(3)$ 2.00 $T_{15}(3)$ 8.15 $T_{15}(9)$	14.2 $T_{15}(10)$	6.16 $T_{15}(11)$	0.99 $T_{15}(11)$
(g)						
(h)						
(i)						
(j)	1.00 $T_8(1)$ 1.00 $T_9(1)$ 10.6 $T_{15}(7)$				1.28 $T_{15}(5)$ 6.43 $T_{15}(11)$	
(k)	2.00 $T_{15}(1)$ 1.00 $T_{15}(5)$ 5.00 $T_{15}(6)$ 4.37 $T_{15}(7)$				5.00 $T_{15}(10)$ 2.41 $T_{15}(11)$	
(l)						
(m)						

Key: $xT_t(m)$ indicates that x trucks of type t and age m operate on the route in the given period.

TABLE 12. The truck allocation policy for the second case study solution with 13-periods and 50% depreciation (last 6 periods).

The first case study had many less routes than the first, which made the overall size of the problem significantly smaller. However this problem was still difficult to solve, demonstrating the difficulty in differentiating between similar pieces of equipment over long-term schedules. The problem exhibits a lot of symmetry, which, if addressed, would lead to faster solving times.

One form of validation is to compare our solutions with the actual solution implemented in the mine. We are fortunate to have obtained this information for the second of the two case studies. We do not know the complete process behind the derivation of the industry solutions we present here. However, from discussions with our partner combined with our knowledge of the literature, we will describe our understanding of the process.

For the second case study, three equipment selection alternative solutions were given to us. The first and cheapest solution was created on an in-house equipment

	Period						
Locations	**1**	**2**	**3**	**4**	**5**	**6**	**7**
M1	0.13 $L_3(1)$						
M2	0.04 $L_3(1)$	0.70 $L_3(1)$	0.43 $L_3(3)$				
M3	0.12 $L_3(1)$	0.21 $L_3(1)$ 0.76 $L_3(2)$	0.51 $L_2(1)$ 1.00 $L_3(2)$ 0.56 $L_3(3)$	1.00 $L_2(2)$ 0.93 $L_3(4)$	0.33 $L_2(3)$ 1.00 $L_3(4)$		
M4		0.26 $L_3(1)$					
S1		0.01 $L_3(1)$	0.05 $L_2(1)$				
S2		0.05 $L_3(1)$					
S3				0.73 $L_3(3)$	0.67 $L_2(3)$ 0.68 $L_3(5)$	0.93 $L_3(5)$ 0.13 $L_3(6)$	0.89 $L_3(6)$
S4							0.03 $L_3(6)$

Key: $xL_l(m)$ indicates that x loaders of type l and age m operate on the route in the given period.

TABLE 13. The loader allocation policy for the second case study with 13 periods and 50% depreciation (first 7 periods).

	Period					
Locations	**8**	**9**	**10**	**11**	**12**	**13**
M1						
M2						
M3		0.34 $L_3(8)$ 0.97 $L_3(9)$	0.38 $L_3(9)$ 1.00 $L_3(10)$	0.26 $L_3(10)$ 1.00 $L_3(11)$	0.47 $L_3(12)$	0.01 $L_3(13)$
M4						
S1						
S2						
S3	0.13 $L_3(7)$ 1.00 $L_3(8)$				0.07 $L_3(11)$ 0.53 $L_3(12)$	
S4						

Key: $xL_l(m)$ indicates that x loaders of type l and age m operate on the route in the given period.

TABLE 14. The loader allocation policy for the second case study with 13 periods and 50% depreciation (last 6 periods).

selection spreadsheet. This spreadsheet was not designed to consider mixed fleets—i.e. all trucks selected must be compatible with all loaders. Furthermore, the truck fleet must be homogeneous. The spreadsheet functioned by first selecting the loaders by minimizing the cost of operation such that the capacity constraints were met. Then a *match factor equation* (such as that discussed in [6]) was used to determine the best fleet size for trucks, whose type was pre-determined by the pre-existing truck fleet. This solution kept one $34T$ loader but salvaged the other two pre-existing loaders. Two new $40T$ loaders were purchased to meet the capacity constraints. The existing truck fleet was kept, and further trucks (of the same type) were added in period one (five trucks), period two (two trucks) and period five (one truck). This selection policy cost $\$1.51483 \times 10^8$ (using our objective function) for the full 13 period schedule. In comparison, our policy yielded a saving of $\$14,234,000$—a saving of 9.4%.

The second solution provided by the industry partner contained the same truck purchase and salvage policy. However, an equipment selection manager believed that all loaders should be salvaged, and that a new $42T$ and two $57T$ loaders should be purchased instead. In this solution, the choice of loaders is restricted in that they must be compatible with the existing truck fleet. This policy yielded a cost of $\$1.55241 \times 10^8$. Comparatively, our solution saved $\$17,992,000$ or 11.6%.

Finally, the third solution purchased a loader fleet that maintained full homogeneity—three $57T$ loaders. The truck purchase and salvage policy was the same as above. This was the actual solution adopted for the mine, and cost $\$1.66550 \times 10^8$. Comparatively our solution saved $\$29,301,000$ or 17.6%.

It is interesting to observe that the industry partner selected the most expensive of their three solutions. This indicates that they attribute a value to having homogeneous fleets. Since we did not account for the cost of ancillary equipment, on-costs for spares and the training of artisans for maintenance and equipment use, it is difficult to make a true comparison of these solutions. However, it is clear that there is an advantage to using an integer programming approach if all the relevant data is available.

4. **Conclusions.** In this paper, we have captured the truck and loader equipment selection problem for surface mines in a large-scale mixed-integer program, developing preprocessing and adopting a separation algorithm to improve the tractability. We used a continuous allocation variable that suggested the optimal portion of the fleet that should work at each location. These variables created a flexible allocation policy alongside the purchase and salvage policy generated by the model. This is a useful tool for mining engineers, who may take the allocation policy as further evidence that the selected fleet would be able to perform the required tasks under uncertainty, or simply use the policy as a guide to manage the fleet.

In our formulation, we paid particular attention to reducing the discretization error in the objective function. We provided a formula that accurately accounts for cost shifts that occur when equipment moves from one age bracket to the next within a period. We solved two case studies over the entire mine plan, demonstrating the model is computationally effective for real world problems.

We have adopted similar assumptions for this model as our work in [5]—a full discussion of the validity of the assumptions can be found there. One important assumption is that the mine plan is deterministic. However, the mine plan is dependent on future demand. Ideally, the equipment selection model would be robust to uncertainty in the market and other stochastic elements, such as machine breakdown, maintenance and truck cycle times. Modelled as a stochastic mixed-integer program, this would decrease the tractability of the model as it stands. However, capturing the uncertainty, stochasticity and interactive effects of truck types are important starting points for future research on this topic.

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