NETWORKS AND HETEROGENEOUS MEDIA Website: http://aimSciences.org
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Volume **1**, Number **3**, September **2006** pp. **513–514**

ERRATUM

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Abstract. In this erratum, we correct a mistake that has propagated in the error analysis of [4].

We use the notation and numerotion of [4]. There is a mistake in the exponent of Lemma 4, which should read:

Lemma 4. *Assume Hypotheses 1 and properties (9) and (10). Let* $A \in \mathcal{M}_3(\mathbb{R})$, $u_A \in W^{1,p}_{\#}((0,1)^3, \mathbb{R}^3)$ *be the solution of (42) and* u^h_A *be the solution of (43). Then there exists a constant* $C > 0$ *independent of* h *such that*

$$\|u_A - u^h_A\|_{1,p} \leq C \inf \left\{ \|u_A - v_h\|^s_{1,p}, v_h \in V_h \right\},$$

with $s = \frac{1}{\beta \wedge p - \alpha}$.

The proof of this result is the same as in [4] provided the correction of inequality (45) and the use of [2, Lemma 23.9] for the case $\beta > p$.

Starting from Lemma 4, Theorems 9 and 10 then read

Theorem 9. *Assume Hypotheses 1, (10) with* $\beta \leq p$ *and in addition* $\alpha > 0$ *in (9). Let* $\mathcal{T}_h$ *be a regular triangulation of* $(0,1)^3$, $\mathcal{T}_H$ *be a regular triangulation of* Ω *and* V_h *and* V_H *be linear finite element subspaces of* $W^{1,p}_{\#}((0,1)^3, \mathbb{R}^3)$ *and of* $W^{1,p}_0(\Omega, \mathbb{R}^3)$ *respectively associated to* $\mathcal{T}_h$ *and* $\mathcal{T}_H$. *Let* $\bar{u} \in W^{1+2/p,p}(\Omega, \mathbb{R}^3)$ *and* u *be the minimizer of (22). Theorem 8 and formula (41) provide a minimizer* $u^h_H = u^{1,h}_H$ *of (23). Then there exist positive constants* C_1 *and* C_2 *independent of* h *and* H, *such that*

$$\|u - u^h_H\|_{1,p} \leq C_1 h^{\frac{2}{p}\frac{\alpha}{(p-1)(p-\alpha)}} + C_2 H^{\frac{2}{p}\frac{\beta-\alpha}{p(\beta-\alpha)-\alpha}}.$$

Theorem 10. *With the notation of Theorem 9, assume Hypotheses 1 with* $\alpha \geq 0$ *and* $\beta \leq p$ *in (9) and (10). Then there exist positive constants* C_1 *and* C_2 *independent of* h *and* H, *such that*

$$\|u - u^h_H\|_{1,p} \leq C_1 h^{\frac{2}{p}\frac{1}{p(p-\alpha)}} + C_2 H^{\frac{2}{p}\frac{\beta-\alpha}{p(\beta-\alpha)-\alpha}}.$$

The proofs of the previous theorems are the same as in [4], provided the good version of Lemma 4. It should also be noticed that the error estimate of Theorem 9

2000 *Mathematics Subject Classification.* Primary: 74Q05; Secondary: 74B20.

Key words and phrases. Homogenization, determination of effective properties, nonlinear elasticity.

is not optimal in the linear case, for which one can prove (using the symmetry of the linear operator, see [1])

$$\|u - u_H^h\|_{1,p} \leq C_1 h^2 + C_2 H.$$

When dealing with energies with p-structure (namely polynomial of degree p in ξ, with $\alpha = 1$ and $\beta = 2$), Ebmeyer and Liu have proved in [3], using the interpolation theory in Nikolskij spaces, the following optimal error estimate

Lemma 4 bis. *Assume Hypotheses 1 and properties (9) and (10) with $a(x,\cdot)$ polynomial of degree $p-1$ for all $x \in (0,1)^3$, $\alpha = 1$ and $\beta = 2$. Let $A \in \mathcal{M}_3(\mathbb{R})$, $u_A \in W^{1,p}_{\#}((0,1)^3, \mathbb{R}^3)$ be the solution of (42) and u_A^h be the solution of (43) in a P1-finite element subspace of $W^{1,p}_{\#}((0,1)^3, \mathbb{R}^3)$. Then there exists a constant $C > 0$ independent of h such that*

$$\|u_A - u_A^h\|_{1,p} \leq Ch.$$

Using Lemma 4 bis, one can improve the error estimate of Theorem 9 (for the p-structure energy, $\beta = 2$, $\alpha = 1$), obtaining

$$\|u - u_H^h\|_{1,p} \leq C_1 h^{\frac{1}{p-1}} + C_2 H.$$

For non periodic materials, this error analysis has been extended in [5].

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Received for publication July 2006.

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