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Research article

Practical exponential stability with respect to $h-$manifolds of discontinuous delayed Cohen–Grossberg neural networks with variable impulsive perturbations

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Abstract: In the present work, we study discontinuous impulsive systems of the type of Cohen-Grossberg Neural Networks (CGNNs) with time-varying delays. The impulsive perturbations are realized not at fixed moments of time, and can be considered as control inputs. The hybrid concept of practical exponential stability with respect to specific manifolds defined by a function is introduced and studied analytically. The established results are applied to the case of Bidirectional Associative Memory (BAM) CGNNs. Lyapunov function method and the Razumikhin technique are the base of the proofs. A numerical example is also presented to demonstrate the applicability and effectiveness of the obtained stability conditions. The proposed results extend and complement some existing stability criteria for impulsive CGNNs with time-varying delays.

Keywords: Cohen–Grossberg neural networks; practical exponential stability; $h-$manifold; variable impulsive perturbations; time-varying delays

1. Introduction

The concept of the specific type of CGNNs introduced in [1] and the wide classes of their applications make this type of neural network systems powerful resources to study numerous real world phenomena. Some of these phenomena studied in engineering and computer sciences include problems such as intelligent recognition processes, handwriting recognition, image recognition, pattern and sequence recognition, data processing, blind signal separation, email spam filtering, signal processing, control problems, fixed point computations, function approximation, optimization problems and many other real life phenomena. The requirement of the existence of stable states in most of the applied problems motivates and justifies the great deal of attention to the stability analysis of CGNNs [2–4].

Also, time delays in the states trajectories may have some significant effects on the stability behaviors of these systems. That is why recently there exist many stability results for CGNNs in which the effects of constant delays, time-varying delays, distributed delays, mixed time-varying delays have been investigated. See, for example [5–10] and the references therein.

In addition, systems with discontinuous solutions extensively arise in modelling of many phenomena by CGNNs. Such systems are well represented by impulsive differential equations [11–13]. Since impulses can distabilize the behavior of a system and also can be considered as control inputs [14–17], the stability analysis of CGNNs under impulsive perturbations and/or control has been a very hot topic of interest [18–22].

However, in most of the contributions to the stability theory of impulsive CGNNs, impulses at fixed instants are considered. The more general case of variable impulsive perturbations in delayed CGNNs started to gain an attention recently [23–26]. In [23] the existence and stability of almost periodic solutions of such neural network models are investigated. A generalized robust stability of CGNNs with variable impulsive perturbations and mixed delays is studied in [24]. The paper [25] deals with the stability with respect to $h-$manifolds for Cohen–Grossberg bidirectional associative memory neural networks with variable impulsive perturbations and time-varying delays. The paper [26] is devoted to global stability of integral manifolds for reaction-diffusion delayed CGNNs with variable impulsive perturbations. Indeed, systems with variable impulsive perturbations that include as a special case impulsive systems with fixed moments of impulses, are more general and more appropriate for practical applications. However, their investigations are related to strains such as bifurcation, "merging" of solutions, "beating" of solutions, etc., since different solutions have different impulsive inputs and the time at which impulsive perturbations occur depend on the current state [27–30].

Since different types of neural network models behave differently, a number of stability concepts have been introduced in their qualitative analysis. The most studied is the exponential stability concept, and recently, a large number of exponential stability criteria for impulsive delayed CGNNs have been proposed. See, for example, [19–21] and the references therein. In fact, it is well known that the global exponential stability guarantees the fast convergent rate for the neural network's states.

The notion of practical stability that is quite independent of that of Lyapunov stability is of a significant importance in numerical research and practical engineering problems [31–33]. It is well known [34–37] that in some cases, though a system is stable or asymptotically stable in the Lyapunov sense, it's actually useless in practice because of undesirable transient characteristics (e.g., the stability domain or the attraction domain is not large enough to allow the desired deviation to cancel out). Despite the high importance of the practical stability notion it is not developed for impulsive delayed CGNNs with variable impulsive perturbations and this is the basic aim of the paper.

In addition, in recent years more researchers are interested in the investigation of practical stability behavior with respect to manifolds, instead of single system's states, [38–41]. The manifold under consideration is defined mainly by specific functions. The main advantages of the stability with respect to manifolds concept are related to the ability of allowing the state trajectories to converge to a neighborhood of the equilibrium or even to a set of solutions without disrupting their stability properties.

In this paper, motivated by the above discussion we will introduce the hybrid concept of practical exponential stability with respect to a manifold, defined by a continuous function for a class of CGNNs with time-varying delays and variable impulsive perturbations. By using the Lyapunov function method and differential inequalities, some sufficient conditions that guarantee the defined stability behavior are established. The obtained results are applied to the case of BAM CGNNs. An example is also presented to demonstrate the applicability and effectiveness of the obtained stability conditions. The proposed results extend and complement some existing stability criteria for impulsive CGNNs with time-varying delays.

2. Preliminaries

We will use the following notations: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space endowed with the norm $||x|| = \sum_{i=1}^{n} |x_i|$ of a $x \in \mathbb{R}^n$ and $\mathbb{R}_+ = [0, \infty)$.

In this paper we will investigate the stability behavior of the states of the following system of CGNNs with time-varying delays and variable impulsive perturbations:

$$\begin{cases} \dot{\mathcal{W}}_i(t) = -A_i(\mathcal{W}_i(t))\Big[B_i(\mathcal{W}_i(t)) - \sum_{j=1}^{n} C_{ij}F_j(\mathcal{W}_j(t)) \\ \qquad - \sum_{j=1}^{n} D_{ij}G_j(\mathcal{W}_j(t - s_j(t))) - V_i\Big], \ t \neq \tau_k(\mathcal{W}), \\ \mathcal{W}_i(t^+) = \mathcal{W}_i(t) + P_{ik}(\mathcal{W}_i(t)), \ t = \tau_k(\mathcal{W}), \end{cases} \tag{2.1}$$

where $\mathcal{W}(t)$ is an $n-$dimensional vector-function that represents the state vector at time t, $t \geq t_0$, $t_0 \in \mathbb{R}_+$. Furthermore the model parameters that denote the connection weights $C_{ij}, D_{ij} \in \mathbb{R}$, $V_i \in \mathbb{R}$ denotes the external input of the node i at time t, the functions $A_i \in C[\mathbb{R}, \mathbb{R}_+]$, $B_i, F_j, G_j, s_j, P_{ik} \in C[\mathbb{R}, \mathbb{R}]$, $0 \leq s_j(t) \leq \nu$, $\nu = const > 0$, $t > s_j$, $i, j = 1, \ldots, n$, and the continuous functions

$\tau_k : \mathbb{R}^n \to \mathbb{R}$, $k = 1, 2, \ldots$. For the meanings of the systems parameters A_i, B_i, $F_j, G_j, s_j, P_{ik}, \tau_k$ we refer to [2–10, 23, 24, 26].

The impulsive jumps for the states of the defined model (2.1) occur at some times t_{l_k} when the integral curve of a state $\mathcal{W}(t)$ meets hypersurfaces defined by the equations

$$t = \tau_k(\mathcal{W}),\ t \geq t_0,\ \mathcal{W} \in \mathbb{R}^n,\ k = 1, 2, \ldots.$$

CGNNs with variable impulsive perturbations of the type (2.1) generalize the impulsive systems of CGNNs with fixed impulsive instants studied in [18–22]. For more detailed information about systems with variable-time impulses, see [23–30].

We study the model (2.1) under the following initial conditions

$$\begin{cases} \mathcal{W}(t) = \varphi_0(t - t_0),\ t_0 - \nu \leq t \leq t_0, \\ \mathcal{W}(t_0^+) = \varphi_0(0), \end{cases} \quad (2.2)$$

where the initial function φ_0 is piecewise continuous with finite number of points of discontinuity of the first kind and bounded on $[-\nu, 0]$. The family of all such functions will be denoted by $\mathcal{PC}$.

Throughout this paper, we will need the following assumptions:

A_1. The functions $\tau_k(\mathcal{W})$ are continuous, $k = 1, 2, \ldots$, $\tau_0(\mathcal{W}) \equiv t_0$ for $\mathcal{W} \in \mathbb{R}^n$ and

$$t_0 < \tau_1(\mathcal{W}) < \tau_2(\mathcal{W}) < \ldots,\ \tau_k(\mathcal{W}) \to \infty \text{ as } k \to \infty$$

uniformly on $\mathcal{W} \in \mathbb{R}^n$.

A_2. The functions A_i, $i = 1, 2, \ldots, n$ are continuous on $\mathbb{R}$ and there exist positive constants $\underline{A}_i$ and $\overline{A}_i$ such that $1 < \underline{A}_i \leq A_i(\chi) \leq \overline{A}_i$ for any $\chi \in \mathbb{R}$ and $i = 1, 2, \ldots, n$.

A_3. The functions B_i, $i = 1, 2, \ldots, n$ are continuous on $\mathbb{R}$, $B_i(0) = 0$ and there exist positive constants $\bar{B}_i$ such that

$$\frac{B_i(\chi_1) - B_i(\chi_2)}{\chi_1 - \chi_2} \geq \bar{B}_i,$$

for all $\chi_1, \chi_2 \in \mathbb{R}$, $\chi_1 \neq \chi_2$ and $i = 1, 2, \ldots, n$.

A_4. The activation functions F_i, G_i $i = 1, 2, \ldots, n$ are continuous on $\mathbb{R}$, $F_j(0) = 0$, $G_j(0) = 0$ and there exist constants $K_j > 0$ and $L_j > 0$ such that

$$|F_j(\chi_1) - F_j(\chi_2)| \leq K_j|\chi_1 - \chi_2|,\quad |G_j(\chi_1) - G_j(\chi_2)| \leq L_j|\chi_1 - \chi_2|$$

for all $\chi_1, \chi_2 \in \mathbb{R}, \chi_1 \neq \chi_2$, $j = 1, 2, \ldots, n$.

A_5. Any solution of the initial value problem (IVP) (2.1), (2.2), satisfied the following relations

$$\mathcal{W}_i(t_{l_k}^-) = \mathcal{W}_i(t_{l_k}),\ \mathcal{W}_i(t_{l_k}^+) = \mathcal{W}_i(t_{l_k}) + P_{il_k}(\mathcal{W}_i(t_{l_k})),$$

where the instants $t_{l_1},\ t_{l_2}, \ldots$ ($t_0 < t_{l_1} < t_{l_2} < \ldots$) are the impulsive moments, $l_k = 1, 2, \ldots$.

In order to introduce the hybrid concept of practical exponential stability with respect to a manifold, we consider a continuous function $h = h(t, \mathcal{W})$, $h : [t_0 - \nu, \infty) \times \mathbb{R}^n \to \mathbb{R}$ and the following sets that will be called $h-manifolds$ defined by the function h:

$$M_t(n-1)(\varepsilon) = \{x \in \mathbb{R}^n : \ ||h(t, \mathcal{W})|| < \varepsilon,\ t \in [t_0, \infty)\},\ \varepsilon > 0,$$

$$M_{t,\nu}(n-1)(\varepsilon) = \{\varphi \in \mathcal{PC} : \ ||h(t, \varphi)||_\nu < \varepsilon,\ t \in [t_0 - \nu, t_0]\},$$

where $||h(t, \varphi)||_\nu = \sup_{-\nu \leq \xi \leq 0} ||h(t, \varphi(\xi))||$.

We also assume that the sets $M_t(n-1)(\varepsilon)$, $M_{t,\nu}(n-1)(\varepsilon)$ are $(n-1)$-dimensional manifolds in $\mathbb{R}^n$, and any solution $\mathcal{W}(t) = \mathcal{W}(t; t_0, \varphi_0)$ of the IVP (2.1), (2.2) satisfying

$$||h(t, \mathcal{W}(t; t_0, \varphi_0))|| \leq \mathcal{H} < \infty$$

is defined on $[t_0, \infty)$.

Definition 2.1. The system (2.1) is said to be *practically globally exponentially stable* with respect to the function h, if given $(\lambda, \mathcal{A})$, $0 < \lambda < \mathcal{A}$, $t_0 \in \mathbb{R}_+$ and $\varphi_0 \in M_{t_0,\nu}(n-1)(\lambda)$ imply the existence of positive constants γ, p, such that

$$\mathcal{W}(t; t_0, \varphi_0) \in M_t(n-1)\Big(\mathcal{A} + \gamma||h(t_0^+, \varphi_0)||_\nu e^{-p(t-t_0)}\Big),$$

where $\mathcal{W}(t; t_0, \varphi_0)$ is a solution of the IVP (2.1), (2.2).

The class of piecewise-continuous Lyapunov-type functions will be introduced as follows [13, 23–26, 29].

Definition 2.2. A function $V : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}_+$ *belongs to the class* Ω^n if:

1. $V(t, \mathcal{W})$ is continuous in $(\tau_{k-1}(\mathcal{W}), \tau_k(\mathcal{W})) \times \mathbb{R}^n$, $k = 1, 2, \ldots$, locally Lipschitz continuous with respect to its second argument $\mathcal{W}$, and $V(t, 0) = 0$ for $t \geq 0$;

2. For each $(t^*, \mathcal{W}^*)$ such that $\tau_k(\mathcal{W}^*) = t^*$, $t^* \in \mathbb{R}_+$, $\mathcal{W}^* \in \mathbb{R}^n$, $k = 1, 2, \ldots$ there exist the finite limits

$$V(t^{*-}, \mathcal{W}^*) = \lim_{\substack{(t,\mathcal{W}) \to (t^*, \mathcal{W}^*) \\ t < t^*}} V(t, \mathcal{W}),\ V(t^{*+}, \mathcal{W}^*) = \lim_{\substack{(t,\mathcal{W}) \to (t^*, \mathcal{W}^*) \\ t > t^*}} V(t, \mathcal{W})$$

and $V(t^{*-}, \mathcal{W}^*) = V(t^*, \mathcal{W}^*)$.

For a function $V \in \Omega^n$, we will consider the following derivative with respect to system (2.1), defined by [13]

$$D^+V(t, \varphi(0)) = \lim_{\chi \to 0^+} \sup \frac{1}{\chi}[V(t+\chi, \mathcal{W}(t+\chi; t_0, \varphi)) - V(t, \varphi(0))],$$

where $(t, \varphi) \in \mathbb{R}_+ \times \mathcal{PC}$.

The following lemma is necessary in the proof of our main results.

Lemma 2.1. *Assume that the function $V \in \Omega^n$ is such that for $t \in [t_0, \infty)$, $\varphi \in \mathcal{PC}$,*

$$V(t^+, \varphi(0) + \Delta\varphi) \leq V(t, \varphi(0)), \quad t = \tau_k(\varphi), \quad k = 1, 2, \ldots, \tag{2.3}$$

and the inequality

$$D^+V(t, \varphi(0)) \leq -pV(t, \varphi(0)) + d, \ t \neq \tau_k(\varphi), \ k = 1, 2, \ldots \tag{2.4}$$

is valid whenever $V(t + \xi, \varphi(\xi)) \leq V(t, \varphi(0))$, for $-\nu \leq \xi \leq 0$, $p > 0$ and $d > 0$.

Then

$$V(t, \mathcal{W}(t; t_0, \varphi_0))$$

$$\leq \sup_{-\nu \leq \xi \leq 0} V(t_0^+, \varphi_0(\xi)) \exp(-p(t - t_0)) + \frac{d}{p}, \tag{2.5}$$

for $t \in [t_0, \infty)$.

Proof Let t_{l_1}, $t_{l_1} > t_0$, be the first impulsive moment, i.e. the first moment when the integral curve of the IVP (2.1), (2.2) meets some of the hypersurfaces with equations $t = \tau_k(\mathcal{W}(t))$, $t \geq t_0$, $\mathcal{W} \in \mathbb{R}^n$, $k = 1, 2, \ldots$.

First, we consider the case $t \in [t_0, t_{l_1}]$. From the properties of the derivative $D^+V(t, \varphi(0))$, we obtain

$$D^+\Big\{V(t, \varphi(0)) \exp(p(t - t_0))\Big\}$$

$$= \Big\{D^+V(t, \varphi(0)) + pV(t, \varphi(0))\Big\} \exp(p(t - t_0)), \ t \in [t_0, t_{l_1}]. \tag{2.6}$$

Now, we integrate both sides of (2.6), and using (2.4) we obtain

$$V(t, \mathcal{W}(t; t_0, \varphi)) \exp(p(t - t_0)) - \sup_{-\nu \leq \xi \leq 0} V(t_0^+, \varphi_0(\xi))$$

$$\leq d \int_{t_0}^{t} \exp(p(\tau - t_0)) d\tau$$

whenever $V(t + \xi, \varphi(\xi)) \leq V(t, \varphi(0))$ for $-\nu \leq \xi \leq 0$.

The last inequality leads to (2.5) for $t \in [t_0, t_{l_1}]$.

Since $t_{l_1} = \min\{t : t = \tau_k(\mathcal{W}(t)), \ t > t_0\}$, we get from (2.3) that

$$V(t_{l_1}^+, \mathcal{W}(t_{l_1}^+; t_0, \varphi_0)) \leq V(t_{l_1}, \mathcal{W}(t_{l_1}; t_0, \varphi_0))$$

$$\leq \sup_{-\nu \leq \xi \leq 0} V(t_0^+, \varphi_0(\xi)) \exp(-p(t - t_0)) + \frac{d}{p}.$$

Let $t_{l_k} = \min\{t : t = \tau_k(\mathcal{W}(t)), \ t > t_{l_{k-1}}\}$, and suppose that (2.5) is satisfied for $t \in (t_{l_{k-1}}, t_{l_k}]$, $l_k > 1$. Then we will get as above

$$V(t_{l_k}^+, \mathcal{W}(t_{l_k}^+; t_0, \varphi_0)) \leq V(t_{l_k}, \mathcal{W}(t_{l_k}; t_0, \varphi_0))$$

$$\leq \sup_{-\nu \leq \xi \leq 0} V(t_0^+, \varphi_0(\xi)) \exp(-p(t - t_0)) + \frac{d}{p}.$$

The assertion of Lemma 2.1 follows by induction.

3. Main results

We will state our main practical exponential stability results here.

Theorem 3.1. *Assume that for system* (2.1) *assumptions A_1–A_5 hold and, in addition:*

1. *$0 < \lambda < \mathcal{A}$ are given and there exists a positive constant q such that $\sum_{i=1}^{n} |V_i| < \mathcal{A}q$;*

2. *For the system parameters, there exists a $p > q$ such that $p = p_1 - p_2$, where*

$$p_1 = \underline{A} \min_{1 \leq i \leq n} \Big(\bar{B}_i - \sum_{j=1}^{n} |C_{ji}| K_i\Big) > p_2 = \overline{A} \max_{1 \leq i \leq n} \sum_{j=1}^{n} |D_{ji}| L_i,$$

$\underline{A} = \min_{1 \leq i \leq n} \underline{A}_i$, $\overline{A} = \max_{1 \leq i \leq n} \overline{A}_i$;

3. *The impulsive functions P_{ik} are such that*

$$P_{ik}(\mathcal{W}_i(t)) = -\gamma_{ik} \mathcal{W}_i(t), \ |1 - \gamma_{ik}| \leq \frac{\underline{A}}{\overline{A}},$$

where $i = 1, 2, \ldots, n$, $k = 1, 2, \ldots$;

4. *For the function $h(t, x)$ there exists a function $V \in \Omega^n$ such that the next inequalities hold*

$$||h(t, \mathcal{W})|| \leq V(t, \mathcal{W}) \leq \Lambda(H) ||h(t, \mathcal{W})||, \ t \in [t_0, \infty), \ \mathcal{W} \in \mathbb{R}^n,$$

where $\Lambda(H) \geq 1$ exists for any $0 < H \leq \infty$.

Then system (2.1) *is practically globally exponentially stable with respect to the function h.*

Proof Consider an arbitrary solution $\mathcal{W}(t) = (\mathcal{W}_1(t), \mathcal{W}_2(t), \dots, \mathcal{W}_n(t))^T$ of (2.1) with an initial function $\varphi_0 \in \mathcal{PC}$.

We define a Lyapunov function as

$$V(t, \mathcal{W}(t)) = \sum_{i=1}^{n} \int_0^{\mathcal{W}_i(t)} \frac{sgn(s)}{A_i(s)} ds, \ t \in \mathbb{R}_+.$$

We have from the above definition that

$$\frac{1}{\bar{A}} \|\mathcal{W}(t)\| \leq V(t, \mathcal{W}(t)) \leq \frac{1}{\underline{A}} \|\mathcal{W}(t)\|. \tag{3.1}$$

First, we can easily show that condition (2.3) of Lemma 2.1 is satisfied. For $t = \tau_k(\varphi)$, $\varphi \in \mathcal{PC}$, $k = 1, 2, \dots$, using (3.1) and condition 1 of Theorm 3.1, we have

$$V(t^+, \varphi(0) + \Delta\varphi) \leq \frac{1}{\underline{A}} \|\varphi(0) + \Delta\varphi\| = \frac{1}{\underline{A}} \sum_{i=1}^{n} |1 - \gamma_{ik}\| \varphi_i(0)|$$

$$\leq \frac{1}{\bar{A}} \sum_{i=1}^{n} |\varphi_i(0)| = \frac{1}{\bar{A}} \|\varphi(0)\| \leq V(t, \varphi(0)). \tag{3.2}$$

Next, we will prove (2.4). If $t \geq t_0$ and $t \neq \tau_k(\varphi)$, $\varphi \in \mathcal{PC}$, then, using again (3.1) and the conditions of Theorem 3.1, we get

$$D^+ V(t, \varphi(0)) \leq -\sum_{i=1}^{n} \Bigg(\bar{B}_i |\varphi_i(0)| - \sum_{j=1}^{n} |C_{ij}| K_j |\varphi_j(0)|$$

$$- \sum_{j=1}^{n} |D_{ij}| L_j |\varphi_j(-s_j(0))| - sgn(\mathcal{W}_i(t)) V_i \Bigg)$$

$$\leq \Bigg(-\bar{B}_i + \sum_{j=1}^{n} |C_{ji}| K_i \Bigg) V(t, \varphi(0))$$

$$+ \sum_{j=1}^{n} |D_{ji}| L_i \sup_{-\nu \leq \xi \leq 0} V(t + \xi, \varphi(\xi)) + \sum_{i=1}^{n} |V_i|$$

$$\leq -p_1 V(t, \varphi(0)) + p_2 \sup_{-\nu \leq \xi \leq 0} V(t + \xi, \varphi(\xi)) + \mathcal{A}q.$$

It follows from the last estimate and condition 2 of Theorem 3.1 that, there exists a positive constant $p > q$, such that

$$D^+ V(t, \varphi(0)) \leq -p V(t, \varphi(0)) + \mathcal{A}q, \tag{3.3}$$

whenever $V(t + \xi, \varphi(\xi)) \leq V(t, \varphi(0))$ for $-\nu \leq \xi \leq 0$.

Now, we apply (3.2), (3.3) and Lemma 2.1 to get

$$V(t, \mathcal{W}(t)) \leq \mathcal{A} + \sup_{-\nu \leq \xi \leq 0} V(t_0^+, \varphi_0(\xi)) e^{-p(t - t_0)} \tag{3.3}$$

for $t \in [t_0, \infty)$.

Let $\varphi_0 \in M_{t,\nu}(\lambda)$. Then from condition 4 of Theorem 3.1 and (3.4) it follows

$$\|h(t, \mathcal{W}(t; t_0, \varphi_0))\| \leq V(t, \mathcal{W}(t; t_0, \varphi_0))$$

$$\leq \mathcal{A} + \sup_{-\nu \leq \xi \leq 0} V(t_0^+, \varphi_0(\xi)) e^{-p(t - t_0)}$$

$$< \mathcal{A} + \Lambda(H) \|h(t_0^+, \varphi_0)\|_\nu e^{-p(t - t_0)}, \ t \in [t_0, \infty).$$

Hence,

$$\mathcal{W}(t; t_0, \varphi_0) \in M_t \left(\mathcal{A} + \Lambda(H) \|h(t_0^+, \varphi_0)\|_\nu e^{-p(t - t_0)} \right)$$

for $t \geq t_0$, i.e., the system (2.1) is practically globally exponentially stable with respect to the function h and the proof of Theorem 3.1. is complete.

Among the different types of CGNNs, the type of BAM neural network models attracts more attention [19, 20, 22, 25]. The goal of the next part of our paper is to investigate the practical global exponential stability of BAM CGNNs with time-varying delays and variable impulsive perturbations.

We will apply our result to the following BAM neural network model with time-varying delays and variable impulsive perturbations of Cohen–Grossberg type:

$$\begin{cases} \dot{\mathcal{W}}_{\mathcal{K}}(t) = -A_{\mathcal{K}}(\mathcal{W}_{\mathcal{K}}(t)) \Bigg[B_{\mathcal{K}}(\mathcal{W}_{\mathcal{K}}(t)) - \sum_{\mathcal{L}=1}^{m} C_{\mathcal{K}\mathcal{L}} F_{\mathcal{L}}(\mathcal{U}_{\mathcal{L}}(t)) \\ \qquad - \sum_{\mathcal{L}=1}^{m} D_{\mathcal{K}\mathcal{L}} G_{\mathcal{L}}(\mathcal{U}_{\mathcal{L}}(t - s_{\mathcal{L}}(t))) - V_{\mathcal{K}} \Bigg], \ t \neq \theta_k(\mathcal{W}, \mathcal{U}), \\ \dot{\mathcal{U}}_{\mathcal{L}}(t) = -\hat{A}_{\mathcal{L}}(\mathcal{U}_{\mathcal{L}}(t)) \Bigg[\hat{B}_{\mathcal{L}}(\mathcal{U}_{\mathcal{L}}(t)) - \sum_{\mathcal{K}=1}^{n} \hat{C}_{\mathcal{L}\mathcal{K}} \hat{F}_{\mathcal{K}}(\mathcal{W}_{\mathcal{K}}(t)) \\ \qquad - \sum_{\mathcal{K}=1}^{n} \hat{D}_{\mathcal{L}\mathcal{K}} \hat{G}_{\mathcal{K}}(\mathcal{W}_{\mathcal{K}}(t - \hat{s}_{\mathcal{K}}(t))) - \hat{V}_{\mathcal{L}} \Bigg], \ t \neq \theta_k(\mathcal{W}, \mathcal{U}), \\ \mathcal{W}_{\mathcal{K}}(t^+) = \mathcal{W}_{\mathcal{K}}(t) + P_{\mathcal{K}k}(\mathcal{W}_{\mathcal{K}}(t)), \ t = \theta_k(\mathcal{W}, \mathcal{U}), \\ \mathcal{U}_{\mathcal{L}}(t^+) = \mathcal{U}_{\mathcal{L}}(t) + Q_{\mathcal{L}k}(\mathcal{U}_{\mathcal{L}}(t)), \ t = \theta_k(\mathcal{W}, \mathcal{U}), \end{cases} \tag{3.4}$$

where $\mathcal{K} = 1, 2, \dots, n$, $\mathcal{L} = 1, 2, \dots, m$, the parameters $C_{\mathcal{K}\mathcal{L}}, D_{\mathcal{K}\mathcal{L}} \in \mathbb{R}$, $V_{\mathcal{K}} \in \mathbb{R}$, $A_{\mathcal{K}} \in C[\mathbb{R}, \mathbb{R}_+]$,

$B_{\mathcal{K}}, F_{\mathcal{L}}, G_{\mathcal{L}}, s_{\mathcal{L}}, P_{\mathcal{K}k} \in C[\mathbb{R},\mathbb{R}]$, $0 \le s_{\mathcal{L}}(t) \le \nu$, $\nu = const > 0$, $t > s_{\mathcal{L}}$ are the same as in (2.1), and $\hat{A}_{\mathcal{L}} \in C[\mathbb{R},\mathbb{R}_+]$, $\hat{C}_{\mathcal{L}\mathcal{K}}, \hat{D}_{\mathcal{L}\mathcal{K}} \in \mathbb{R}$, $\hat{V}_{\mathcal{L}} \in \mathbb{R}$, $\hat{B}_{\mathcal{L}}, \hat{F}_{\mathcal{K}}, \hat{G}_{\mathcal{K}}, Q_{\mathcal{L}k} \in C[\mathbb{R},\mathbb{R}]$, and the continuous functions $\theta_k : \mathbb{R}^{n+m} \to \mathbb{R}$, $k = 1,2,\ldots$ are with identical properties.

Let $\varphi_0 \in PC[[-\nu,0],\mathbb{R}^n], \psi_0 \in PC[[-\nu,0],\mathbb{R}^m]$. We will study the model (3.4) with an initial function $\phi_0 = (\varphi_0,\psi_0)^T$.

Define a continuous function $\hat{h} = \hat{h}(t,\mathcal{Z})$, $\hat{h} : [t_0-\nu,\infty)\times\mathbb{R}^{n+m} \to \mathbb{R}$ and consider the $\hat{h}-$manifolds

$$M_t(n+m-1)(\varepsilon) = \{\mathcal{Z} \in \mathbb{R}^{n+m} : \ ||\hat{h}(t,\mathcal{Z})|| < \varepsilon,\ t \in [t_0,\infty)\},$$

$$M_{t,\nu}(n+m-1)(\varepsilon) = \Big\{\phi \in PC[[-\nu,0],\mathbb{R}^n]\times PC[[-\nu,0],\mathbb{R}^m] : \sup_{-\nu\le\xi\le 0} ||\hat{h}(t,\phi(\xi))|| < \varepsilon,\ t \in [t_0-\nu,t_0]\Big\}.$$

Introduce the following assumptions:

A_6. The functions $\theta_k(\mathcal{W},\mathcal{U})$ are continuous, $k = 1,2,\ldots$, $\theta_0(\mathcal{W},\mathcal{U}) \equiv t_0$ for $\mathcal{W},\mathcal{U} \in \mathbb{R}^m$ and

$$t_0 < \theta_1(\mathcal{W},\mathcal{U}) < \theta_2(\mathcal{W},\mathcal{U}) < \ldots,\ \theta_k(\mathcal{W},\mathcal{U}) \to \infty$$

as $k \to \infty$ uniformly on $\mathcal{W} \in \mathbb{R}^n$, $\mathcal{U} \in \mathbb{R}^m$.

A_7. The functions $\hat{A}_{\mathcal{L}}$, $\mathcal{L} = 1,2,\ldots,m$ are continuous on $\mathbb{R}$ and there exist positive constants $\underline{\hat{A}}_{\mathcal{L}}$ and $\overline{\hat{A}}_{\mathcal{L}}$ such that $1 < \underline{\hat{A}}_{\mathcal{L}} \le \hat{A}_{\mathcal{L}}(\chi) \le \overline{\hat{A}}_{\mathcal{L}}$ for any $\chi \in \mathbb{R}$ and $\mathcal{L} = 1,2,\ldots,m$.

A_8. The functions $\hat{B}_{\mathcal{L}}$, $\mathcal{L} = 1,2,\ldots,m$ are continuous on $\mathbb{R}$, $\hat{B}_{\mathcal{L}}(0) = 0$ and there exist positive constants $\bar{\hat{B}}_{\mathcal{L}}$ such that

$$\frac{\hat{B}_{\mathcal{L}}(\chi_1) - \hat{B}_{\mathcal{L}}(\chi_2)}{\chi_1 - \chi_2} \ge \bar{\hat{B}}_{\mathcal{L}},$$

for all $\chi_1,\chi_2 \in \mathbb{R}$, $\chi_1 \ne \chi_2$ and $\mathcal{L} = 1,2,\ldots,m$.

A_9. The activation functions $\hat{F}_{\mathcal{K}}, \hat{G}_{\mathcal{K}}$ $\mathcal{K} = 1,2,\ldots,n$ are continuous on $\mathbb{R}$, $\hat{F}_{\mathcal{K}}(0) = 0$, $\hat{G}_{\mathcal{K}}(0) = 0$ and there exist constants $\hat{K}_{\mathcal{K}} > 0$ and $\hat{L}_{\mathcal{K}} > 0$ such that

$$|\hat{F}_{\mathcal{K}}(\chi_1) - \hat{F}_{\mathcal{K}}(\chi_2)| \le \hat{K}_{\mathcal{K}}|\chi_1 - \chi_2|,$$

$$|\hat{G}_{\mathcal{K}}(\chi_1) - \hat{G}_{\mathcal{K}}(\chi_2)| \le \hat{L}_{\mathcal{K}}|\chi_1 - \chi_2|$$

for all $\chi_1,\chi_2 \in \mathbb{R}, \chi_1 \ne \chi_2$, $\mathcal{K} = 1,2,\ldots,n$.

With the useof a Lyapunov function $V \in \Omega^{n+m}$

$$V(t,\mathcal{Z}(t)) = \sum_{\mathcal{K}=1}^{n}\int_0^{\mathcal{W}_{\mathcal{K}}(t)} \frac{sgn(s)}{A_{\mathcal{K}}(s)}ds + \sum_{\mathcal{L}=1}^{m}\int_0^{\mathcal{U}_{\mathcal{L}}(t)} \frac{sgn(s)}{\hat{A}_{\mathcal{L}}(s)}ds,$$

the proof of the next theorem is similar to the proof of Theorem 3.1.

Theorem 3.2. *Assume that for system* (3.4) *assumptions* A_1–A_9 *hold and, in addition:*

1. $0 < \lambda < \mathcal{A}$ *are given and there exists a positive constant* $\hat{q}$ *such that* $\sum_{\mathcal{K}=1}^{n}|V_{\mathcal{K}}| + \sum_{\mathcal{L}=1}^{m}|\hat{V}_{\mathcal{L}}| < \mathcal{A}\hat{q}$;

2. *For the system parameters, there exists a* $\hat{p} > \hat{q}$ *such that* $\hat{p} = \hat{p}_1 - \hat{p}_2$, *where*

$$\hat{p}_1 = \underline{A}\Big(\min_{1\le\mathcal{K}\le n}\Big(\bar{B}_{\mathcal{K}} - \sum_{\mathcal{L}=1}^{m}|\hat{C}_{\mathcal{L}\mathcal{K}}|\hat{K}_{\mathcal{K}}\Big) + \min_{1\le\mathcal{L}\le m}\Big(\bar{\hat{B}}_{\mathcal{L}} - \sum_{\mathcal{K}=1}^{n}|C_{\mathcal{K}\mathcal{L}}|K_{\mathcal{L}}\Big)\Big)$$

$$> \hat{p}_2 = \overline{A}\Big(\max_{1\le\mathcal{L}\le m}\sum_{\mathcal{K}=1}^{n}|D_{\mathcal{L}\mathcal{K}}|L_{\mathcal{L}} + \max_{1\le\mathcal{K}\le n}\sum_{\mathcal{L}=1}^{m}|\hat{D}_{\mathcal{K}\mathcal{L}}|\hat{L}_{\mathcal{K}}\Big),$$

where

$$\underline{A} = \min(\min_{1\le\mathcal{K}\le n}\underline{A}_{\mathcal{K}}, \min_{1\le\mathcal{L}\le m}\underline{\hat{A}}_{\mathcal{L}}),$$

$$\overline{A} = \max(\max_{1\le\mathcal{K}\le n}\overline{A}_{\mathcal{K}}, \max_{1\le\mathcal{L}\le m}\overline{\hat{A}}_{\mathcal{L}});$$

3. *The functions* $Q_{\mathcal{L}k}$ *are such that*

$$Q_{\mathcal{L}k}(\mathcal{U}_{\mathcal{L}}(t)) = -\delta_{\mathcal{L}k}\mathcal{U}_{\mathcal{L}}(t),$$

and

$$\max\left(|1-\gamma_{\mathcal{K}k}|, |1-\delta_{\mathcal{L}k}|\right) \le \frac{\underline{A}}{\overline{A}},$$

where $\hat{\gamma}_k = \min_{1\le\mathcal{K}\le n}\gamma_{\mathcal{K}k}$, $\hat{\delta}_k = \min_{1\le\mathcal{L}\le m}\delta_{\mathcal{L}k}$, $\mathcal{K} = 1,2,\ldots,n$, $\mathcal{L} = 1,2,\ldots,m$, $k = 1,2,\ldots$;

4. *For the function* $\hat{h}(t,\mathcal{Z})$ *there exists a function* $V \in \Omega^{n+m}$ *such that the next inequalities hold*

$$||\hat{h}(t,\mathcal{Z})|| \le V(t,\mathcal{Z}) \le \Lambda(\hat{H})||\hat{h}(t,\mathcal{Z})||, \quad t \in [t_0,\infty),$$

where $\Lambda(H) \ge 1$ *exists for any* $0 < \hat{H} \le \infty$.

Then system (3.4) *is practically globally exponentially stable with respect to the function* $\hat{h}$.

Remark 3.1. Since the hybrid stability concept of practical global exponential stability with respect to a function h generalizes the exponential stability and the practical stability notions, with Theorem 3.1 we extend and complement the existing stability results for CGNNs with impulses and delays. For example, our results extend the results in [24–26] considering the practical stability case. Also, we extend numerous exponential stability results such as [19–21, 23], considering stability with respect to a function h instead a separate state (steady state or an almost periodic solution).

Remark 3.2. The reported stability results can be applied as control strategies, using appropriate impulsive functions. In fact, the impulsive control is essential in many applied problems [13–17]. In addition, since we consider the most general case of variable impulsive perturbations, we extend and generalize the results in [18–20, 22].

4. An example

As an example, let consider the following impulsive Cohen–Grossberg-type neural network with time-varying delays

$$\begin{cases} \dot{\mathcal{W}}_i(t) = -A_i(\mathcal{W}_i(t))\Bigg[B_i(\mathcal{W}_i(t)) - \sum_{j=1}^{2} C_{ij}F_j(\mathcal{W}_j(t)) \\ \qquad - \sum_{j=1}^{2} D_{ij}G_j(\mathcal{W}_j(t - s_j(t))) - V_i\Bigg],\ t \neq \tau_k(\mathcal{W}(t)), \\ \mathcal{W}(t^+) = \begin{pmatrix} \frac{1}{3k} & 0 \\ 0 & \frac{1}{3k} \end{pmatrix}\mathcal{W}(t),\ t = \tau_k(\mathcal{W}(t)), \\ i = 1, 2, \quad k = 1, 2, \ldots, \end{cases} \tag{4.1}$$

where

$$\mathcal{W}(t) = \begin{pmatrix} \mathcal{W}_1(t) \\ \\ \mathcal{W}_2(t) \end{pmatrix}, \quad V_1 = V_2 = 0.03,$$

$$A_1(\mathcal{W}_1) = 3 + 0.2\sin(\mathcal{W}_1), \quad A_2(\mathcal{W}_2) = 4 - 0.1\cos(\mathcal{W}_2),$$

$$B_1(\mathcal{W}_1) = B_2(\mathcal{W}_2) = 3\mathcal{W}_i,\ i = 1, 2,$$

$$F_i(\mathcal{W}_i) = G_i(\mathcal{W}_i) = \frac{|\mathcal{W}_i + 1| - |\mathcal{W}_i - 1|}{2}, \quad 0 \leq s_i(t) \leq 1,$$

$$C_{11} = 1, \quad C_{12} = 0.5, \quad C_{21} = 0.6, \quad C_{22} = 0.5,$$

$$D_{11} = 0.2, \quad D_{12} = 0.1, \quad D_{21} = 0.15, \quad D_{22} = 0.1,$$

the functions $\tau_k(\mathcal{W}) = |\mathcal{W}| + 2k$, $k = 1, 2, \ldots$ are continuous and satisfy

$$t_0 < \tau_1(\mathcal{W}) < \tau_2(\mathcal{W}) < \ldots < \tau_k(\mathcal{W}) \to \infty \text{ as } k \to \infty$$

uniformly on $\mathbb{R}^2$.

We can check that assumptions $A_2 - A_4$ are satisfied and $\bar{B}_1 = \bar{B}_2 = 3$, $\quad K_1 = K_2 = L_1 = L_2 = 1$.

Also, condition 1 of Theorem 3.1 is satisfied for $\mathcal{A} > 0.025$ and condition 2 of Theorem 3.1 holds for $\underline{A} = 2.8$, $\overline{A} = 4.1$ $p_1 = 3.92 > p_2 = 1.435$. In addition, the constants γ_{ik}, $i = 1, 2$, $k = 1, 2, \ldots$ are such that

$$|1 - \gamma_{ik}| = \frac{2.8}{4.1} < 1.$$

Consider the continuous function $h = |\mathcal{W}_1| + |\mathcal{W}_2|$ for which condition 4 of Theorem 3.1 is achieved.

Since all conditions of Theorem 3.1 are satisfied, the system (4.1) is practically globally exponentially stable with respect to the function h. The stable behavior is represented in Figure 1.

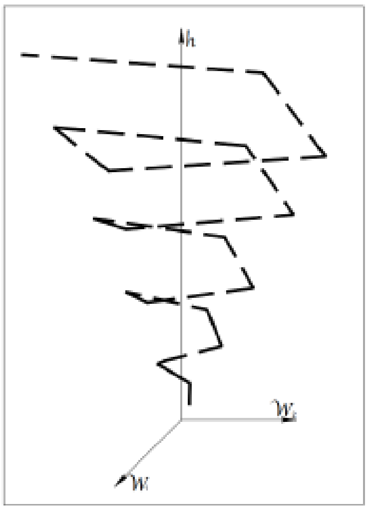

Figure 1. The practically exponentially stable behavior of the model (4.1) with respect to the function h.

5. Conclusions

In this paper, a discontinuous CGNN system with time-varying delays and variable impulsive instants is investigated. The generalized notion of practical global exponential stability with respect to a manifold is introduced for the model under consideration. Applying the Lyapunov function approach, criteria are obtained. The new results are applied to the case of BAM CGNNs. An example is presented to illustrate the established criteria. With this research we contribute to the development of the stability theory of impulsive delayed CGNNs. The proposed concept can be applied to all specific types of CGNNs, as well as, to different types of models of diverse interest.

Conflict of interest

The authors declare no conflict of interest.

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