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*Research article*

## **Fuzzy adaptive event-triggered distributed control for a class of nonlinear multi-agent systems**

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**Abstract:** In this work, we examine an adaptive and event-triggered distributed controller for nonlinear multi-agent systems (MASs). Second, we present a fuzzy adaptive event-triggered distributed control approach using a Lyapunov-based filter and the backstepping recursion technique. Next, the controller and adaptive rule presented guarantee that all tracking errors between the leader and the follower converge in a limited area close to the origin. According to the Lyapunov stability theory, this demonstrates that all other signals inside the closed loop are assured to be semi-globally, uniformly and finally constrained. Finally, simulation tests are conducted to illustrate the effectiveness of the control mechanism.

**Keywords:** nonlinear multi-agent systems; fuzzy logic system; event-triggered distributed control; backstepping method

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### **1. Introduction**

In nature, many organisms can accomplish many complex and routine tasks through the division of labor in groups and group cooperation. Examples include the cooperative division of labor in pigeon and goose flocks, the wolf pack effect and the migration of fish populations in formation. How each organism cooperates with each other to divide labor has attracted scholars in many fields of study to explore [1]. Without internal information exchange and centralized control from the outside world, these organisms can only accomplish what they cannot do autonomously through division of labor and cooperation with their peers. By studying the behavior of division of labor and cooperation among these organisms, the scholar step by step proposed the concept of multi-agent systems (MASs) [2–5], MASs are collections of multiple intelligence intended to turn huge, complicated systems into small, manageable, communicable and coordinate able systems [6–8]. In practical applications, both the computation of an intelligent body's own processor and the communication between intelligent bodies

consume energy, and the energy consumption of communication is generally much greater than that of computation [9]. In this context, considering how to reduce the performance requirements of MASs becomes an important research topic [10–12]. Because the communication energy consumption of the system is large and most of the existing work requires uninterrupted communication. This has led to the introduction of a periodic sampling mechanism to limit the communication frequency between the intelligence, which effectively reduces the energy consumption. Astrom and the other scholars have proposed a special sampling mechanism, event-triggered control [13–17].

The notion of event triggering is fundamental in contemporary industrial 1999 saw the introduction of event-triggered control by Astrom and Arzen [18]. Traditional time-triggered control schemes demand regular sampling even when the system is working properly, which may increase both energy consumption and network congestion with the progress of communication. Furthermore, will only the control job will be executed if a parameter or status error has occurred. In an event-triggered control system, control tasks will only be executed when an error of a system parameter or system indicator surpasses a certain threshold [19]. In recent years, these benefits have sparked the interest of academics in event-triggered control. Thus, several approaches [20–23] have been developed to address these issues in event-triggered control. These results give essential theoretical guidance for the research and implementation of an event-triggered distributed control approach in MASs [24–29]. Event-triggered control can decide when to trigger a control action based on changes in the system state or error, and in combination with fuzzy control, it can be controlled using fuzzy logic at the time of triggering, which reduces computational overhead and improves the efficiency of the control system.

Fuzzy control is a method for regulating items for which accurate mathematical models are difficult to develop [30–32]. This method of control is based on fuzzy logic, which resembles the way humans think. In lieu of developing a mathematical model, fuzzy control may make choices in real time using the system's actual input and output data in combination with the knowledge of trained operators. However, there are a number of intrinsic disadvantages to fuzzy control: First, they are less accurate; second, they cannot adapt effectively to novel conditions; and third, they are susceptible to oscillation phenomena [33–36]. Adaptive control is the solution provided by fuzzy control. Researchers may be able to make even greater progress by combining fuzzy control with adaptive control. The 1979 research article titled "Language Self-Organizing Controller" published discoveries that opened the path for the creation of fuzzy adaptive control. In 1982, a revolutionary method to fuzzy adaptive control employing fuzzy control rules that permitted self-adjustment was introduced. In 1993, researchers investigated the approximation accuracy of the fuzzy system as a function approximation [37–40], integrating the learning ability of the adaptive method with the universal approximation capability of the fuzzy logic system to create a fuzzy adaptive controller. These findings have paved the way for further research in the area of fuzzy adaptive control. Since then, several scientists have created fuzzy adaptive controllers that use the universal approximation provided by fuzzy logic. A finite-time L2 gain asynchronous control strategy based on the T-S fuzzy model approach is proposed in [41], which provides a new perspective and methodology for dealing with continuous-time control problems and accelerates the development of fuzzy adaptive control systems. Based on the above this paper can reduce the computation and execution frequency of the control system by introducing a relative threshold event triggering strategy. This is particularly important in systems where fuzzy adaptive distributed control involves a large number of computations and iterations. Therefore, the introduction of event-triggered control accelerates the development of fuzzy adaptive control systems.

In the first section, we examine the current state and future direction of the fields of MASs, the event-triggered distributed control and the fuzzy adaptive control. The second section of the article presents the fundamental theory of fuzzy adaptive event-triggered distributed control for nonlinear systems, including graph theory theorems, issue descriptions and fuzzy logic system principles. Using the backstepping approach and the given Lyapunov function, an adaptive event-triggered distributed controller is constructed in the third section. The findings of the stability study conducted in the fourth section indicate that the adaptive event-triggered distributed controller can keep the system stable. In the fifth segment, numerical simulation studies are utilized to validate the performance of the distributed controller.

When designing event triggering rules, it is necessary to select appropriate parameters, such as threshold value and trigger interval, which have a great impact on the stability and performance of the system. However, the event triggering mechanism proposed in this paper based on the relative threshold strategy provides an effective way to solve the problem. Moreover, in most distributed systems, the communication load may become a significant bottleneck, leading to delays, packet loss and other problems that may degrade the system performance. In this context, we propose an event-triggered distributed controller based on a relative threshold policy that reduces the amount of communication required between multiple agents, thereby increasing the efficiency of the overall system and reducing the computational burden. The use of a fuzzy logic system with filters further enhances the controller's ability to handle complex and nonlinear system dynamics, making it suitable for a wide range of applications. In addition to confirming the feasibility and effectiveness of the control mechanism, Lyapunov's stability theorem proves that all closed-loop system signals are semi-global, uniform and ultimately bounded.

## 2. System descriptions and preliminaries

### 2.1. Graph theory

In this part, the communication relationships between agents are described. The following is the introduction of the relevant knowledge: the Laplacian matrix is  $G = E - D$ , directed communication topology  $\zeta = (\varepsilon, \delta, E)$  shows the communication information between agent  $i$  and  $j$ .  $\delta = (1, 2, \dots, M)$  are represented as a set of  $M$  nodes. The set of edges between nodes are  $\delta \subseteq \varepsilon \times \varepsilon$ . Each agent in the graph can be represented by a node. There is no self-loop in topology  $e_{i,i} = 0, \forall i \in \varepsilon$ . The node  $i$  to  $j$  of edge is  $(i, j) \in \delta$ , indicating that node  $i$  is said to be adjacent to  $j$ . The set of adjacent edges  $i$  is  $M_i = \{j \in \varepsilon | (i, j) \in \delta, i \neq j\}$ . The adjacency matrix is designed as  $E = (e_{i,j}) \in R^{M_i \times M_i}$ . If  $(i, j) \in \delta$ , the weight  $e_{i,j} > 0$ . Or else, the weight  $e_{i,j} = 0$ . Then the entry of node  $i$  is  $d_i = \sum_{j \in M_i} e_{i,j}$ ,  $D = \text{diag}\{d_1, d_2, \dots, d_{M_i}\}$  is the diagonal matrix. Since only part of the agent in MASs can receive the tracking signal directly, the output of the leader can be seen as the output of the given tracking signal leader.

Among them, four lemmas and one assumption are given in this paper.

**Lemma 1 [16].** If directed graph  $\zeta$  is defined as a tree, after that there is the directional path from the root node to all the other nodes. If this root of the spanning tree is node 0, then  $G + E$  is a non-singular matrix.

## 2.2. Problem formulation

We examine a system with followers, each of which may be characterized as a class of  $n$ -order tight feedback systems with unknown parameters:

$$\begin{cases} \dot{x}_{i,m} = x_{i,m+1} + f_{i,m}(\bar{x}_{i,m}) + w_{i,m}(t) \\ \dot{x}_{i,n_i} = u_i + f_{i,n_i}(\bar{x}_i) + w_{i,n_i}(t) \\ y = x_1 \end{cases} \quad (2.1)$$

To facilitate derivation, where  $i = 1, \dots, N_i$ ,  $f_{i,1}(x_i)$ ,  $f_{i,n_i}(x_i)$  stand for unknown non-linear functions,  $w_{i,n_i}(t)$ ,  $w_{i,1}(t)$  stand for unknown external disturbance,  $\bar{x}_i = [x_{i,1}, x_{i,2}, \dots, x_{i,n_i}]^T \in R_i^n$  indicates system states,  $y \in R_i$  and  $u_i \in R_i$  are the system's input and output.  $f_{i,m}(\bar{x}_{i,m})$  and  $f_{i,n_i}(x_i)$  are respectively abbreviated as  $f_{i,m}$  and  $f_{i,n_i}$ .

The control objectives of this paper are as follows:

- 1) the follower's output signal is in a finite neighborhood of the leader's output signal.
- 2) in the closed loop system, an adaptive event triggering controller limits all signals.

**Assumption 1.**  $w_{i,n_i}$  is the unknown external disturbance,  $\dot{w}_{i,n_i}(t)$  are bounded, such that  $|\dot{w}_{i,n_i}(t)| \leq \bar{w}_{i,n_i}$ ,  $\bar{w}_{i,n_i}$  is a positive constant.

**Lemma 2 [16].** (Young's inequality) Suppose  $x, y, c$  and  $a$  are the non-negative real numbers, and  $c, a$  satisfies  $1/c + 1/a = 1$ , then we have

$$xy \leq \frac{1}{c}x^c + \frac{1}{a}y^a$$

the equality sign holds when and only when  $m^c = n^a$ .

**Lemma 3 [17].** First, define  $\hat{x} = [\hat{x}_{j,1}, \dots, \hat{x}_{j,N}]^T$ ,  $\tilde{y} = [y_1, \dots, y_M]^T$ ,  $\tilde{y}_0 = 1_M \otimes y_0$ , we have  $\|\tilde{y} - \tilde{y}_0\| \leq \|\hat{x}\|/\zeta(G + D)$ , where  $\zeta(G + D)$  is the least odd value of the matrix.

## 2.3. Fuzzy logic system

There are the fuzzy rule bases, fuzzy inference and the fuzzification operator. The defuzzification operator is the fundamental components of a fuzzy logic system or fuzzy control. Combining several sets of fuzzy inference rules with fuzzification, fuzzy inference synthesis and defuzzification produces a novel flavor of fuzzy logic. The technique to fuzzy logic employed in this work is detailed below. First, fuzzy rules are represented as follow:  $R^l: x_i, y \in R^l$  are the system's input and the output. If  $x_i$  is  $F_i^l$ ,  $x_{n_i}$  is  $F_n^l$ ,  $y$  is  $G_i^l$ ,  $i = 1, 2, \dots, N_i$ ,  $l = 1, 2, \dots, n$ .  $N_i$  is represented as the number of fuzzy logic rules,  $G_i^l$  and  $F_i^l$  are fuzzy sets. By solving fuzzy rules and using fuzzy methods to define (including multi-point product definition reasoning, centralized weighted multi-point average method to solve multi-point fuzzy, single point average method to solve fuzzy rules), the mathematical expression of a fuzzy logic system is

$$y(x) = \frac{\sum_{l=1}^n \bar{y}_l \prod_{i=1}^{N+1} \mu_{F_i^l}(x_i)}{\sum_{l=1}^n \left[ \prod_{i=1}^{N+1} \mu_{F_i^l}(x_i) \right]} \quad (2.2)$$

where  $\bar{y}_l = \max \mu_{G_l^l}(y)$ ,  $\mu_{G_l^l}(y)$  and  $\mu_{F_l^l}(x_i)$  are defined as membership functions of fuzzy sets  $G_l^l$  and  $F_l^l$ , respectively.

The fuzzy basis function:

$$\theta_l = \frac{\prod_{i=1}^{N+1} \mu_{F_i^l}(x_i)}{\sum_{l=1}^n \left[ \prod_{i=1}^{N+1} \mu_{F_i^l}(x_i) \right]} \quad (2.3)$$

Let  $W^T = [\bar{y}_1, \bar{y}_2, \dots, \bar{y}_N] = [W_1, W_2, \dots, W_N]$ ,  $\theta(X) = [\theta_1(x), \dots, \theta_N(x)]^T$  and  $y(x) = W^T \theta(X)$ .

**Lemma 4 [16].**  $f(\mathcal{X})$  is a continuous function defined  $\Psi$  on a compact set  $\theta(\mathcal{X}) = [\theta_1(\mathcal{X}), \theta_2(\mathcal{X}), \dots, \theta_L(\mathcal{X})]^T$ ,  $\xi^* = [\xi_1, \xi_2, \dots, \xi_L]^T \in R^l$ , given  $\forall \vartheta > 0$ , the inequality holds:

$$\xi^* := \arg \min_{\xi \in R^l} \left\{ \sup_{x \in \Psi} |f(\mathcal{X}) - W^T \theta(\mathcal{X})| \right\} \leq \vartheta \quad (2.4)$$

### 3. Event-triggered distributed control

#### 3.1. Design the event trigger controller

In this section, based on system (2.1), the adaptive dynamic surface control is designed, the error surfaces of  $i$ th agents are represented as:

$$\tilde{x}_{i,1} = \sum_{j \in N_i} e_{i,0}(y_i - y_0) + e_{i,j}(y_i - y_j) \quad (3.1)$$

$$\tilde{x}_{i,m} = x_{i,m} - \Phi_{i,m-1}, m = 2, 3, \dots, n \quad (3.2)$$

$$\vartheta_{i,k} = -\rho_{i,k} + \Phi_{i,k}, k = 1, \dots, n \quad (3.3)$$

where  $y_0$  represents the reference signal,  $\rho_{i,k}$  indicates the virtual controller,  $\Phi_{i,k}$  represents the output of the first-order filter.

*Step 1:* The Lyapunov function candidate is defined as:

$$V_{i,1} = \frac{1}{2} \tilde{x}_{i,1}^2 + \frac{1}{2} \tilde{W}_{i,1}^T H_{i,1}^{-1} \tilde{W}_{i,1} \quad (3.4)$$

where  $H_i$  is given as a positive definite matrix,  $\hat{W}_i$  is an estimate of parameter  $W_i$ .

The derivative of (3.4), it has

$$\begin{aligned} \dot{V}_{i,1} &= \tilde{x}_{i,1} \dot{\tilde{x}}_{i,1} + \tilde{W}_{i,1}^T H_{i,1}^{-1} \dot{\tilde{W}}_{i,1} \\ &= \tilde{x}_{i,1} [(e_{i,0} + d_i)(\tilde{x}_{i,2} + \vartheta_{i,1} + \rho_{i,1} + W_{i,1}^T \varphi_{i,1} + \delta_{i,1} + w_{i,1}) \\ &\quad - d_i(x_{j,2} + W_{j,1}^T \theta_{j,1} + \delta_{j,1} + w_{j,1}) - e_{i,0} \dot{y}_0] + \tilde{W}_{i,1}^T H_{i,1}^{-1} \dot{\tilde{W}}_{i,1} \end{aligned} \quad (3.5)$$

where  $\tilde{W}_i = -\hat{W}_i + W_i$ .

$$\begin{aligned} \dot{V}_{i,1} &= \tilde{x}_{i,1} [(e_{i,0} + d_i)(\tilde{x}_{i,2} + \vartheta_{i,1} + \rho_{i,1} + W_{i,1}^T \varphi_{i,1} + \delta_{i,1} + w_{i,1}) \\ &\quad - d_i(x_{j,2} + \tilde{W}_{j,1}^T \theta_{j,1} + \hat{W}_{j,1}^T \theta_{j,1} + \delta_{j,1} + w_{j,1}) - e_{i,0} \dot{y}_0] \\ &\quad - \tilde{W}_{i,1}^T H_{i,1}^{-1} \dot{\hat{W}}_{i,1} \end{aligned} \quad (3.6)$$

Using the Young's inequality, it is

$$\begin{aligned} & \tilde{x}_{i,1}(e_{i,0} + d_i)(\vartheta_{i,1} + \delta_{i,1} + w_{i,1}) \\ & \leq \frac{3}{2}(e_{i,0} + d_i)^2 \tilde{x}_{i,1}^2 + \frac{1}{2}\vartheta_{i,1}^2 + \frac{1}{2}\bar{\delta}_{i,1}^2 + \frac{1}{2}\bar{w}_{i,1}^2 \end{aligned} \quad (3.7)$$

$$\begin{aligned} & - \tilde{x}_{i,1} \sum_{j \in N_i} e_{i,j}(\tilde{W}_{j,1}^T \theta_{j,1} + \delta_{j,1} + w_{j,1}) \\ & \leq \sum_{j \in N_i} \frac{e_{i,j}}{2}(\tilde{W}_{j,1}^T \tilde{W}_{j,1} \theta_{j,1}^2 + \bar{\delta}_{j,1}^2 + \bar{w}_{i,1}^2) + \frac{3}{2} \sum_{j \in N_i} e_{i,j} \tilde{x}_{i,1}^2 \end{aligned} \quad (3.8)$$

where  $\bar{w}_{i,1}$  is a positive constant.

Substituting (3.7) and (3.8) into (3.6), it has

$$\begin{aligned} \dot{V}_{i,k_i} = & \dot{V}_{i,k_i-1} + \tilde{x}_{i,k_i}(\tilde{x}_{i,k_i+1} - \dot{\Phi}_{i,k_i-1} + \vartheta_{i,k_i} + \rho_{i,k_i} + \delta_{i,k_i} + w_{i,k_i} + W_{i,k_i}^T \theta_{i,k_i}) \\ & + \vartheta_{i,k_i-1} \left( -\frac{\vartheta_{i,k_i-1}}{\tau_{i,k_i-1}} + \psi_{i,k_i-1} \right) - \tilde{W}_{i,k_i}^T H_{i,k_i}^{-1} \hat{W}_{i,k_i} \end{aligned} \quad (3.9)$$

the virtual controller  $\rho_{i,1}$  is represented as:

$$\begin{aligned} \rho_{i,1} = & -\hat{W}_{i,1}^T \theta_{i,1} - \frac{3}{2}(d_i + e_{i,0})\tilde{x}_{i,1} + \frac{1}{d_i + e_{i,0}} \\ & \times \left[ \sum_{j \in N_i} e_{i,j}(x_{j,2} + \hat{W}_{j,1}^T \theta_{j,1} - \frac{3}{2}\tilde{x}_{i,1}) + e_{i,0}\dot{y}_0 - c_{i,1}\tilde{x}_{i,1} \right] \end{aligned} \quad (3.10)$$

where  $c_{i,1} > 0$  is a design parameter, the adaptive law  $\hat{W}_{i,1}$  is represented as:

$$\dot{\hat{W}}_{i,1} = H_{i,1} \left[ -\sigma_{i,1} \hat{W}_{i,1} + (e_{i,0} + d_i) \theta_{i,1} \tilde{x}_{i,1} \right] \quad (3.11)$$

Substituting (3.10) and (3.11) into (3.9), it has

$$\begin{aligned} \dot{V}_{i,1} \leq & -c_{i,1} \tilde{x}_{i,1}^2 + (e_{i,0} + d_i) \tilde{x}_{i,1} \tilde{x}_{i,2} + \frac{1}{2}\vartheta_{i,1}^2 + \frac{1}{2}\bar{\delta}_{i,1}^2 + \frac{\bar{\omega}^2}{2} \\ & + \sum_{j \in N_i} \frac{e_{i,j}}{2}(\tilde{W}_{j,1}^T \tilde{W}_{j,1} \theta_{i,1}^2 + \bar{\delta}_{j,1}^2 + \bar{w}_{i,1}^2) + \sigma_1 \tilde{W}_{i,1}^T \hat{W}_{i,1} \end{aligned} \quad (3.12)$$

Step  $k_i$  ( $k_i = 2, 3, \dots, n_i - 1$ ): the virtual control signal is  $\rho_{i,k_i-1}$ , the time constant is  $\tau_{i,k_i-1}$ , we obtain

$$\begin{aligned} \rho_{i,k_i-1} & = \tau_{i,k_i-1} \dot{\Phi}_{i,k_i-1} + \Phi_{i,k_i-1} \\ \Phi_{i,k_i-1}(0) & = \rho_{i,k_i-1}(0) \end{aligned} \quad (3.13)$$

Combining (3.3) and (3.12), it has

$$\dot{\vartheta}_{i,k_i-1} = -\dot{\rho}_{i,k_i-1} - \frac{\vartheta_{i,k_i-1}}{\tau_{i,k_i-1}} = \psi_{i,k_i-1} - \frac{\vartheta_{i,k_i-1}}{\tau_{i,k_i-1}} \quad (3.14)$$

where  $\psi_{i,k_i-1}$  is continuous function. The Lyapunov function candidate is defined as follow:

$$V_{i,k_i} = V_{i,k_i-1} + \frac{1}{2} \tilde{x}_{i,k_i}^2 + \frac{1}{2} \vartheta_{i,k_i-1}^2 + \frac{1}{2} \tilde{W}_{i,k_i}^T H_{i,k_i}^{-1} \tilde{W}_{i,k_i} \quad (3.15)$$

Take the derivative of both ends of (3.15), it has

$$\begin{aligned} \dot{V}_{i,k_i} = & \dot{V}_{i,k_i-1} + \tilde{x}_{i,k_i}(\tilde{x}_{i,k_i+1} - \dot{\Phi}_{i,k_i-1} + \vartheta_{i,k_i} + \rho_{i,k_i} + \delta_{i,k_i} + w_{i,k_i} + W_{i,k_i}^T \theta_{i,k_i}) \\ & + \vartheta_{i,k_i-1} \left( -\frac{\vartheta_{i,k_i-1}}{\tau_{i,k_i-1}} + \psi_{i,k_i-1} \right) - \tilde{W}_{i,k_i}^T H_{i,k_i}^{-1} \dot{\hat{W}}_{i,k_i} \end{aligned} \quad (3.16)$$

Based on Young's inequality, we get

$$\vartheta_{i,k_i-1} \psi_{i,k_i-1} \leq \frac{1}{2} \psi_{i,k_i-1}^2 + \frac{1}{2} \vartheta_{i,k_i-1}^2 \quad (3.17)$$

$$\begin{aligned} & \tilde{x}_{i,k_i}(\vartheta_{i,k_i} + \delta_{i,k_i} + w_{i,k_i}) \\ & \leq \frac{3}{2} \tilde{x}_{i,k_i}^2 + \frac{1}{2} \vartheta_{i,k_i}^2 + \frac{1}{2} \bar{\delta}_{i,k_i}^2 + \frac{\bar{w}_{i,k_i}^2}{2} \end{aligned} \quad (3.18)$$

Combining (3.17) and (3.18) into (3.16), we can obtain

$$\begin{aligned} \dot{V}_{i,k_i} \leq & \dot{V}_{i,k_i-1} + \tilde{x}_{i,k_i}(\tilde{x}_{i,k_i+1} + \rho_{i,k_i} + W_{i,k_i}^T \theta_{i,k_i} - \dot{\Phi}_{i,k_i-1}) \\ & - \frac{\vartheta_{i,k_i-1}^2}{\tau_{i,k_i-1}} + \frac{1}{2} \psi_{i,k_i-1}^2 + \frac{1}{2} \vartheta_{i,k_i-1}^2 + \frac{3}{2} \tilde{x}_{i,k_i}^2 + \frac{1}{2} \vartheta_{i,k_i}^2 \\ & + \frac{1}{2} \bar{\delta}_{i,k_i}^2 - \tilde{W}_{i,k_i}^T H_{i,k_i}^{-1} \dot{\hat{W}}_{i,k_i} + \frac{\bar{w}_{i,k_i}^2}{2} \end{aligned} \quad (3.19)$$

Based on (3.19), the virtual controller  $\rho_{i,k_i}$ :

$$\begin{aligned} \rho_{i,k_i} = & -c_{i,k_i} \tilde{x}_{i,k_i} - \frac{3}{2} \tilde{x}_{i,k_i} - \hat{W}_{i,k_i}^T \theta_{i,k_i} \\ & - \ell_{i,k_i-1} \tilde{x}_{i,k_i-1} + \dot{\Phi}_{i,k_i-1} \end{aligned} \quad (3.20)$$

the adaptive law  $\hat{W}_{i,k_i}$  is:

$$\dot{\hat{W}}_{i,k_i} = H_{i,k_i}(\theta_{i,k_i} \tilde{x}_{i,k_i} - \sigma_{i,k_i} \hat{W}_{i,k_i}) \quad (3.21)$$

where  $c_{i,k_i} > 0$  is designed as a design parameter.

Define  $\ell_{i,k_i} = \begin{cases} e_{i,0} + d_i, k_i = 1 \\ 1, k_i \neq 1 \end{cases}$ , we obtain

$$\begin{aligned} \dot{V}_{i,k_i} \leq & - \sum_{\alpha=1}^{k_i} c_{i,\alpha} \tilde{x}_{i,\alpha}^2 + \sum_{j \in N_i} \frac{e_{i,j}}{2} (\tilde{W}_{j,1}^T \tilde{W}_{j,1} \theta_{j,1}^2 + \bar{\delta}_{j,1}^2 + \bar{w}_{i,k_i}^2) \\ & - \sum_{\alpha=1}^{k_i} \vartheta_{i,\alpha}^2 \left( \frac{1}{\tau_{i,\alpha}} - 1 \right) + \tilde{x}_{i,k_i} \tilde{x}_{i,k_i+1} + \frac{1}{2} \sum_{\alpha=1}^{k_i} \bar{\delta}_{i,\alpha}^2 \\ & + \frac{1}{2} \sum_{\alpha=1}^{k_i-1} \psi_{i,\alpha}^2 + \frac{1}{2} \vartheta_{i,k_i}^2 + \sum_{\alpha=1}^{k_i} \sigma_{\alpha} \tilde{W}_{i,\alpha}^T \hat{W}_{i,\alpha} + \frac{1}{2} \sum_{\alpha=1}^{k_i} \bar{w}_{i,k_i}^2 \end{aligned} \quad (3.22)$$

*Step n<sub>i</sub>*: the virtual control signal is  $\rho_{i,n_i-1}$ , the time constant is  $\tau_{i,n_i-1}$ , it has

$$\begin{aligned}\rho_{i,n_i-1} &= \tau_{i,n_i-1} \dot{\Phi}_{i,n_i-1} + \Phi_{i,n_i-1} \\ \rho_{i,n_i-1}(0) &= \Phi_{i,n_i-1}(0)\end{aligned}\quad (3.23)$$

$$\dot{\vartheta}_{i,n_i-1} = -\frac{\vartheta_{i,n_i-1}}{\tau_{i,n_i-1}} - \dot{\rho}_{i,n_i-1} = -\frac{\vartheta_{i,n_i-1}}{\tau_{i,n_i-1}} + \psi_{i,n_i-1}\quad (3.24)$$

where  $\psi_{i,n_i-1}$  is devised as a continuous function.

The Lyapunov function candidate is defined as follow:

$$V_{i,n_i} = V_{i,n_i-1} + \frac{1}{2} \tilde{x}_{i,n_i}^2 + \frac{1}{2} \vartheta_{i,n_i-1}^2 + \frac{1}{2} \tilde{W}_{i,n_i}^T H_{i,n_i}^{-1} \tilde{W}_{i,n_i}\quad (3.25)$$

Take the derivative of both ends of (3.25)

$$\begin{aligned}\dot{V}_{i,n_i} &= \dot{V}_{i,n_i-1} + \tilde{x}_{i,n_i} \dot{\tilde{x}}_{i,n_i} + \vartheta_{i,n_i-1} \dot{\vartheta}_{i,n_i-1} + \tilde{W}_{i,n_i}^T H_{i,n_i}^{-1} \dot{\tilde{W}}_{i,n_i} \\ &= \dot{V}_{i,n_i-1} + \tilde{x}_{i,n_i} (u + W_{i,n_i}^T \theta_{i,n_i} + \delta_{i,n_i} - \dot{\Phi}_{i,n_i-1} + w_{i,n_i}) \\ &\quad + \vartheta_{i,n_i-1} \left( -\frac{\vartheta_{i,n_i-1}}{\tau_{i,n_i-1}} + \psi_{i,n_i-1} \right) - \tilde{W}_{i,n_i}^T H_{i,n_i}^{-1} \dot{\tilde{W}}_{i,n_i}\end{aligned}\quad (3.26)$$

The control signal  $\varpi_i$  is desired as follow:

$$\begin{aligned}\varpi_i &= - (1 + \Upsilon_i) [\rho_{i,n_i} \tanh\left(\frac{\tilde{x}_{i,n_i} \rho_{i,n_i}}{\varphi_i}\right) \\ &\quad + \bar{m}_i \tanh\left(\frac{\tilde{x}_{i,n_i} \bar{m}_i}{\varphi_i}\right)]\end{aligned}\quad (3.27)$$

where virtual controller  $\rho_{i,n_i}$  will be given later,  $\bar{m}_i > m_i / (1 - \Upsilon_i)$ ,  $\varpi_i$  is a positive design parameter,  $0 < \Upsilon_i < 1$ ,  $m_i > 0$ .

We adopt an event-triggered control method based on relative threshold strategy, which is designed as follows

$$\begin{cases} u_i = \varpi_i(T_{i,r}), \quad \forall T_i \in [T_{i,r}, T_{i,r+1}), \\ T_{i,r+1} = \inf \{t \in \mathbb{R}_+ \mid |T_i(t)| \geq \Upsilon_i |u_i| + m_i\}, \quad T_{i,1} = 0 \end{cases}\quad (3.28)$$

**Remark 1:** The relative threshold strategy is a method of dynamically adjusting the threshold value according to the amplitude of  $u_i$ . By correlating with the amplitude of the control signal  $u_i$ , a suitable threshold can be determined according to the stability and control performance requirements of the system. When the amplitude of  $u_i$  is large, the system may be in an unstable or highly perturbed situation, and the use of a larger threshold can avoid frequent triggering events and reduce the communication load. When the system is stable, the amplitude of  $u_i$  is usually small. At this point, we can choose a smaller threshold value to obtain more accurate control performance.

According to the above formula, we can get



$$\begin{aligned}
\tilde{x}_{i,n_i} u_i &= \frac{\tilde{x}_{i,n_i} (\varpi_i - \varepsilon_{i,2} m_i)}{1 + \varepsilon_{i,1} \Upsilon_i} \\
&\leq \frac{\tilde{x}_{i,n_i} \varpi_i}{1 + \Upsilon_i} + \frac{|\tilde{x}_{i,n_i} m_i|}{1 - \Upsilon_i} \\
&\leq \frac{\tilde{x}_{i,n_i} \varpi_i}{1 + \Upsilon_i} + |\tilde{x}_{i,n_i} \bar{m}_i|
\end{aligned} \tag{3.29}$$

According to (3.28),  $u_i \geq 0$ ,  $u_i = (\varpi_i - \varepsilon_{i,2} m_i)/(1 + \varepsilon_{i,1} \Upsilon_i)$ , when  $u_i < 0$ ,  $u_i = (\varpi_i - \varepsilon_{i,2} m_i)/(1 - \varepsilon_{i,1} \Upsilon_i)$ , time-varying functions  $\varepsilon_{i,1}(t)$  satisfies  $|\varepsilon_{i,1}(t)| \leq 1$ , where  $\varepsilon_{i,2}(t) = \text{sign}(u_i) \times \varepsilon_{i,1}(t)$ .

According to Lemma 2, we can get

$$\vartheta_{i,n_i-1} \psi_{i,n_i-1} \leq \frac{1}{2} \psi_{i,n_i-1}^2 + \frac{1}{2} \vartheta_{i,n_i-1}^2 \tag{3.30}$$

$$\tilde{x}_{i,n_i} (\delta_{i,n_i} + w_{i,n_i}) \leq \tilde{x}_{i,n_i}^2 + \frac{1}{2} \delta_{i,n_i}^2 + \frac{1}{2} \bar{w}_{i,n_i}^2 \tag{3.31}$$

By substituting (3.29)–(3.31) into (3.26), we can get

$$\begin{aligned}
\dot{V}_{i,n_i} &\leq \dot{V}_{i,n_i-1} + \tilde{x}_{i,n_i} \left( \frac{\varpi_i}{1 + \Upsilon_i} + W_{i,n_i}^T \theta_{i,n_i} + \tilde{x}_{i,n_i} - \dot{\Phi}_{i,n_i-1} \right) \\
&\quad + |\tilde{x}_{i,n_i} \bar{m}_i| - \tilde{W}_{i,n_i}^T H_{i,n_i}^{-1} \hat{W}_{i,n_i} + \frac{1}{2} \delta_{i,n_i}^2 + \frac{1}{2} \bar{w}_{i,n_i}^2 + \frac{1}{2} \psi_{i,n_i-1}^2 \\
&\quad - \vartheta_{i,n_i-1}^2 \left( \frac{1}{\tau_{i,\beta}} - 1 \right)
\end{aligned} \tag{3.32}$$

where the  $\tanh(\cdot)$  function has the following property

$$0 \leq |z| - z \tanh\left(\frac{z}{\varphi}\right) \leq 0.2785\varphi \tag{3.33}$$

where  $\varphi > 0$  and  $z \in R$ , we can get

$$\begin{aligned}
\frac{\tilde{x}_{i,n_i} \varpi_i}{1 + \Upsilon_i} &= |\tilde{x}_{i,n_i} \rho_{i,n_i}| - \tilde{x}_{i,n_i} \rho_{i,n_i} \tanh\left(\frac{\tilde{x}_{i,n_i} \rho_{i,n_i}}{\varphi_i}\right) \\
&\quad + |\tilde{x}_{i,n_i} \bar{m}_i| - \tilde{x}_{i,n_i} \bar{m}_i \tanh\left(\frac{\tilde{x}_{i,n_i} \bar{m}_i}{\varphi_i}\right) \\
&\quad - |\tilde{x}_{i,n_i} \rho_{i,n_i}| - |\tilde{x}_{i,n_i} \bar{m}_i| \\
&\leq 0.557V_i + \tilde{x}_{i,n_i} \rho_{i,n_i} - |\tilde{x}_{i,n_i} \bar{m}_i|
\end{aligned} \tag{3.34}$$

the virtual controller is  $\rho_{i,n_i}$ :

$$\begin{aligned}
\rho_{i,n_i} &= -c_{i,n_i} \tilde{x}_{i,n_i} - \tilde{x}_{i,n_i-1} - \tilde{x}_{i,n_i} + \frac{1}{2(1 + \Upsilon_i)} \tilde{x}_{i,n_i} \\
&\quad - \hat{W}_{i,n_i}^T \theta_{i,n_i} + \dot{\Phi}_{i,n_i-1}
\end{aligned} \tag{3.35}$$

According to the above formula, the adaptive law  $\hat{W}_{i,n_i}$  is devised as:

$$\dot{\hat{W}}_{i,n_i} = H_{i,n_i}(\theta_{i,n_i}\tilde{x}_{i,n_i} - \sigma_{i,n_i}\hat{W}_{i,n_i}) \quad (3.36)$$

Combined with the above formula, we can obtain

$$\begin{aligned} \dot{V}_{i,n_i} \leq & - \sum_{\alpha=1}^{n_i} c_{i,\alpha}\tilde{x}_{i,\alpha}^2 + \frac{1}{2} \sum_{\alpha=1}^{n_i-1} \psi_{i,\alpha}^2 + \frac{1}{2} \sum_{\alpha=1}^{n_i} \tilde{w}_{i,n_i}^2 - \sum_{\alpha=1}^{n_i-1} \vartheta_{i,\alpha}^2 \left(\frac{1}{\tau_{i,\alpha}} - 1\right) \\ & + \sum_{\alpha=1}^{n_i} \sigma_{\alpha} \tilde{W}_{i,\alpha}^T \hat{W}_{i,\alpha} + \frac{1}{2} \sum_{\alpha=1}^{n_i} \bar{\delta}_{i,\alpha}^2 + 0.557V_i + \frac{1}{2(1 + \Upsilon_i)} \tilde{x}_{i,n_i} \\ & + \sum_{j \in N_i} \frac{e_{i,j}}{2} (\tilde{W}_{j,1}^T \tilde{W}_{j,1} \theta_{j,1}^2 + \bar{\delta}_{j,1}^2) \end{aligned} \quad (3.37)$$

### 3.2. Stabilization analysis

**Theorem 1.** For MASs (2.1), when assumption 1 is satisfied, the closed-loop system is stabilized by the adaptive law (3.36), the events designed by Eqs (3.27), (3.28) and (3.35) to trigger the distributed controller and select appropriate design parameters  $c_{i,1}, \dots, c_{i,k_I}, \dots, c_{i,n_i}, \tau_{i,k_I-1}, \dots, \tau_{i,n_i-1}, \sigma_{i,1}, \dots, \sigma_{i,k_I}, \dots, \sigma_{i,n_i}, H_{i,1}, \dots, H_{i,k_I}, \dots, H_{i,n_i}, \Upsilon_i, \delta_i, m_i, \bar{m}_i$ . Ensure that all signals on the system are bounded. The errors converge is in a small neighborhood.

**Proof of the Theorem 1.** We choose the following Lyapunov function

$$\begin{aligned} V &= \sum_{i=1}^{N_i} V_{i,n_i} \\ &= \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i} \frac{1}{2} \tilde{x}_{i,\alpha}^2 + \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i-1} \frac{1}{2} \vartheta_{i,\alpha}^2 \\ &\quad + \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i} \frac{1}{2} \tilde{W}_{i,\alpha}^T H_i^{-1} \tilde{W}_{i,\alpha} \end{aligned} \quad (3.38)$$

According to the inequalities

$$\begin{aligned} & \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i} \sigma_i \tilde{W}_{i,\alpha}^T \hat{W}_{i,\alpha} \\ & \leq - \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i} \frac{\sigma_i}{2} \tilde{W}_{i,\alpha}^T \tilde{W}_{i,\alpha} + W_{i,\alpha}^T W_{i,\alpha} \end{aligned} \quad (3.39)$$

$$\begin{aligned} & \sum_{i=1}^{N_i} \sum_{j \in N_i} \frac{e_{i,j}}{2} (\tilde{W}_{j,1}^T \tilde{W}_{j,1} \theta_{j,1}^2 + \bar{\delta}_{j,1}^2) \\ & = \sum_{j=1}^{N_i} \sum_{i=1}^{N_i} \frac{e_{i,j}}{2} (\tilde{W}_{i,1}^T \tilde{W}_{i,1} \theta_{i,1}^2 + \bar{\delta}_{i,1}^2) \end{aligned} \quad (3.40)$$

Thus,  $\dot{V}$  is derived in the following form:

$$\begin{aligned}
 \dot{V} \leq & - \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i} c_{i,\alpha} \tilde{x}_{i,\alpha}^2 + \sum_{i=1}^{N_i} 0.557V_i + \frac{1}{2} \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i} \bar{\delta}_{i,\alpha}^2 \\
 & - \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i-1} \left( \frac{1}{\tau_{i,\alpha}} - 1 \right) \theta_{i,\alpha}^2 + \sum_{i=1}^{N_i} \frac{1}{2(1 + \Upsilon_i)} \tilde{x}_{i,n_i} \\
 & + \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i} \bar{\delta}_{i,1}^2 + \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i} \frac{\sigma_i}{2} W_{i,\alpha}^T W_{i,\alpha} + \frac{1}{2} \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i-1} \psi_{i,\alpha}^2 \\
 & + \frac{1}{2} \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i} \bar{w}_{i,\alpha}^2 - \sum_{i=1}^{N_i} \frac{\sigma_i - \sum_{j=1}^{n_i} e_{j,1} \theta_{i,1}^2}{2\lambda_{\max}(H_i^{-1})} \sum_{\beta=1}^{n_i} \tilde{W}_{i,\alpha}^T H_i^{-1} \tilde{W}_{i,\alpha}
 \end{aligned} \tag{3.41}$$

where  $c_{i,\beta} > 0$ ,  $\alpha = 1, \dots, n$ ,  $1/\tau_{i,\alpha} - 1 > 0$ ,  $1/[2(1 + \Upsilon_i)] > 0$ ,  $\sigma_i - \sum_{j=1}^{n_i} e_{j,1} \theta_{i,1}^2 > 0$ . The maximum eigen-value of the matrix is expressed as  $\Phi_{\max}(\cdot)$ . On a bounded compact set  $A_{i,k}$ ,  $|\psi_{i,k}|$  has a maximum.

According to the above formula, it has

$$\begin{aligned}
 \bar{D} = & \frac{1}{2} \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i} \bar{w}_{i,\alpha}^2 + \sum_{i=1}^{N_i} 0.557V_i + \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i} \frac{\sigma_i}{2} W_{i,\alpha}^T W_{i,\alpha} \\
 & + \frac{1}{2} \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i} \bar{\delta}_{i,1}^2 + \frac{1}{2} \sum_{i=1}^{N_i} \sum_{\alpha=1}^{n_i-1} E_{i,\alpha}^2
 \end{aligned} \tag{3.42}$$

$$\text{Select } C = \min \left\{ 2c_{i,n_i}, \frac{2}{\tau_{i,\alpha}} - 2, \frac{1}{1+\Upsilon_i}, \frac{\sigma_i - \sum_{j=1}^{n_i} e_{j,1} \theta_{i,1}^2}{\Phi_{\max}(H_i^{-1})} \right\}.$$

Therefore, (3.41) is rewritten as  $\dot{V}(t) \leq -CV(t) + \bar{D}$ .

Besides, we get that

$$\frac{1}{2} \tilde{x}_{i,1}^2 \leq V(t) \leq e^{-Ct} V(0) + \frac{\bar{D}}{C} (1 - e^{-Ct}) \tag{3.43}$$

According to (3.1), it has

$$\tilde{x}_{i,1} = \sum_{j \in N_i} e_{i,j} [(y_i - y_j) - (y_j - y_0)] + e_{i,0} (y_i - y_0) \tag{3.44}$$

From Lemma 1, we get that  $\lim_{t \rightarrow \infty} \|\bar{y} - \bar{y}_0\| \leq \frac{1}{\zeta(E+D)} \sqrt{\frac{2\bar{D}}{C}}$ .

The parameters are then adjusted until the tracking error settles inside a small area close to the origin. Zeno behavior in the context of event triggering control is the occurrence of an infinite number of triggers in a finite amount of time. Due to the planned event trigger condition, which is indicated by an increase in event trigger frequency and a decrease in trigger interval, the controller is unable to

modify the trigger before a certain time period [42–44]. The subsequent data reveals that the suggested control mechanism to prevent Zeno behavior needs a minimum time interval.

$$\frac{d}{dt} |o_i| = \text{sign}(o_i) \frac{d(o_i)}{dt} \leq |\dot{\omega}_i| \tag{3.45}$$

Because  $\omega_i$  is differentiability and the independent variable of  $\dot{\omega}_i$  is a bounded closed-loop signal. Then, there must be a positive constant  $W_i$  making  $|\dot{\omega}_i| \leq W_i$ . Because  $o_i(t_{i,r}) = 0$  and  $\lim_{t \rightarrow t_{i,r+1}} |o_i(t)| > m_i$ , this trigger time interval must be greater than  $m_i/\theta_i$ , avoiding the Zeno behavior.

#### 4. Simulation study

This section proves that the suggested control algorithm is effective. The system is comprised of a leader and four successively numbered followers: 0, 1, 2, 3 and 4. Figure 1 depicts a boss and four subordinates. Each follower’s system dynamics equations are defined as follow:

$$\begin{aligned} \dot{x}_{i,1} &= x_{i,2} + x_{i,1}^2 \sin(x_{i,1}) + w_{i,1} \\ \dot{x}_{i,2} &= u_i + x_{i,2} \sin(x_{i,1}) + w_{i,2} \\ y &= x_{i,1}, \quad i = 1, \dots, 4 \end{aligned} \tag{4.1}$$

where  $x_{i,1}, x_{i,2}$  are the states of systems,  $w_{i,1} = x_{i,1} \sin(x_{i,2}) \cos(t)$ ,  $w_{i,2} = 1 + x_{i,1}x_{i,2}\sin^2(t)$ , the output signal of leader is  $y_0 = 0.5 \sin(t)$ .

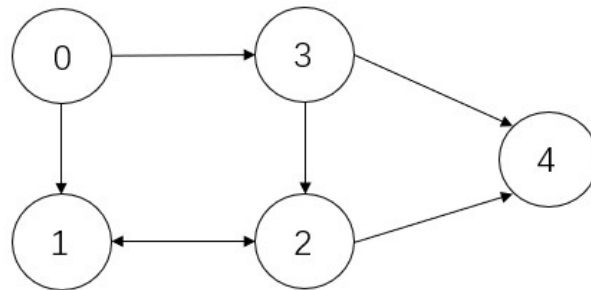


Figure 1. Communication topology.

From Figure 1, the weight adjacency matrix  $E$  and Laplacian matrix  $G$  of the tracker are:

$$E = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

In the simulation, select the design constants for the entire control scheme  $\xi_{i,1} = 55, \xi_{i,2} = 65, \sigma_{i,1} = \sigma_{i,2} = 10, H_{i,1} = 0.4, H_{i,2} = 0.1, \varphi_{i,1} = \varphi_{i,2} = 2.6, \tau_{i,1} = \tau_{i,2} = 0.1, \Upsilon_i = 0.29, c_{i,1} = 80, c_{i,2} = 100$ . The state initial value design constants are chosen as  $x_1 = [0, 0]^T$ ,

$x_2 = [0, 0]^T$ ,  $x_3 = [0, 0]^T$ ,  $x_4 = [0, 0]^T$ . The membership functions of fuzzy sets are chosen as follows  $\mu_{F_j^1} = \exp[-(x_{i,m} + 2)^2/4]$ ,  $\mu_{F_j^2} = \exp[-(x_{i,m} + 1.5)^2/4]$ ,  $\mu_{F_j^3} = \exp[-(x_{i,m} + 0.5)^2/4]$ ,  $\mu_{F_j^4} = \exp[-(x_{i,m} - 0.5)^2/4]$ ,  $\mu_{F_j^5} = \exp[-(x_{i,m} - 1.5)^2/4]$ ,  $\mu_{F_j^6} = \exp[-(x_{i,m} - 2)^2/4]$ .

Remark 2: A distributed tracking algorithm based on event triggering and adaptive fuzzy control for uncertain nonlinear multi-intelligent body systems is proposed. The algorithm has high requirements on the continuity of control signals, and the system’s event-based triggering and sampling may result in incoherent control signals, thus affecting the control effect [45]. In this paper, by introducing a relative threshold strategy, the system can set an error threshold between neighboring intelligence and trigger only when the error exceeds the threshold, thus reducing the number of event triggers. This ensures that the system can update the control signal in time, which improves the control accuracy and stability, and reduces the computational burden of the system.

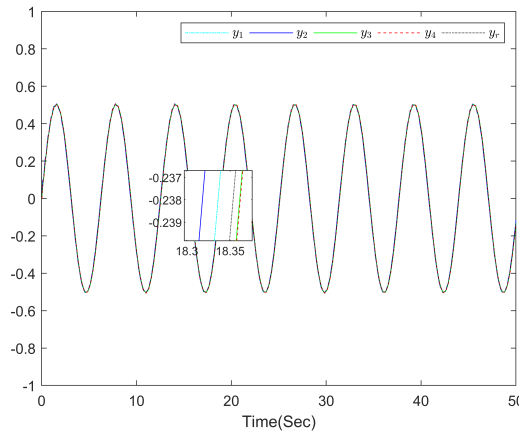


Figure 2. Leader’s signal and follower’s output.

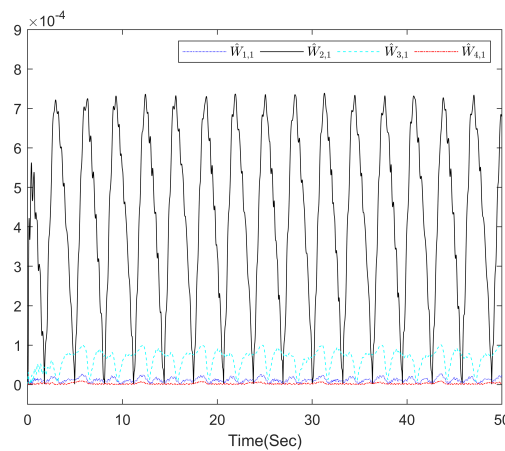
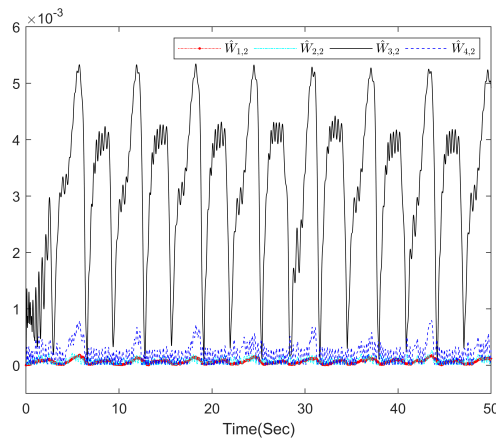
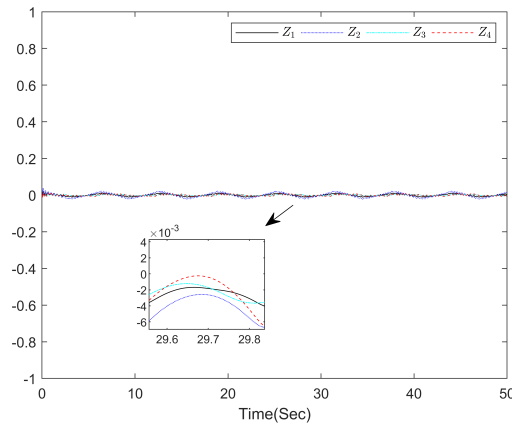


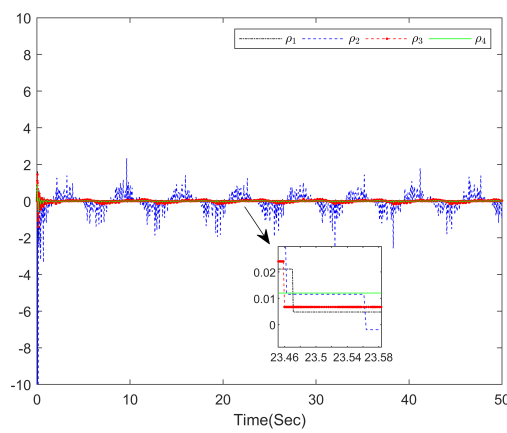
Figure 3. The changing trajectory of  $\hat{\theta}_1$ .



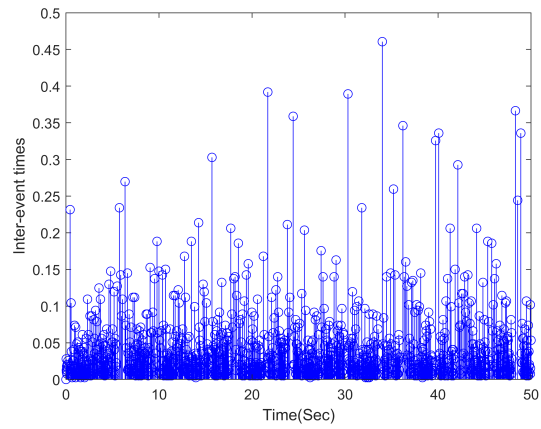
**Figure 4.** The changing trajectory of  $\hat{\theta}_2$ .



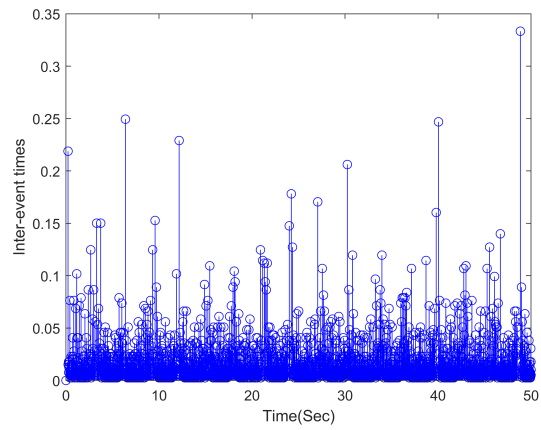
**Figure 5.** The trajectory of the error.



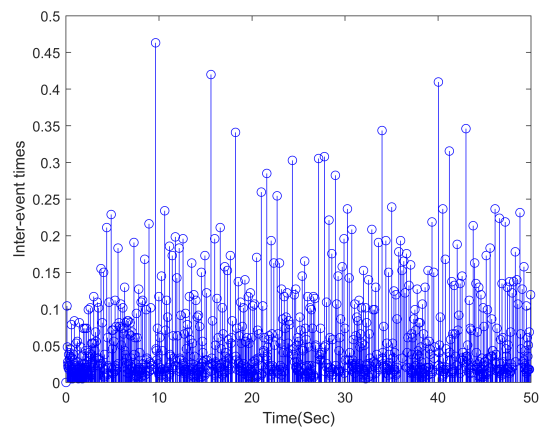
**Figure 6.** Event trigger distributed controller.



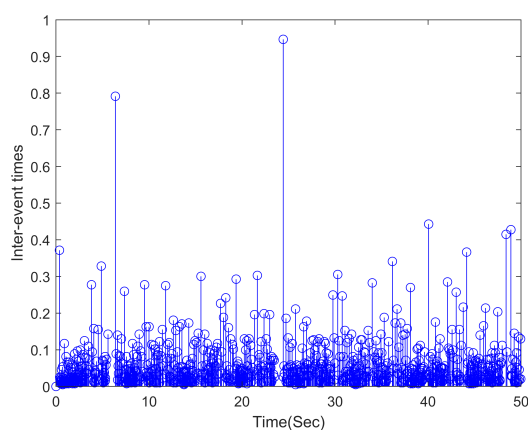
**Figure 7.** The interval at which an event is triggered.



**Figure 8.** The interval at which an event is triggered.



**Figure 9.** The interval at which an event is triggered.



**Figure 10.** The interval at which an event is triggered.

Figures 2–10 illustrate the simulation results. The signal route and final output are presented in Figure 2. Figures 3 and 4 depict the outcomes of a simulated application of the adaptive rule. The image clearly demonstrates the superior quality of the simulation results. The region to which the tracking error tends to converge is seen in Figure 5. The event-triggered control is preferable to the time-triggered control, as shown in Figure 6. This study provides a strategy for distributed control that uses fuzzy events to drastically reduce the amount of communication infrastructure needed. Figure 10 illustrates the control input event firing interval, which demonstrates that there was no Zeno behavior. Consequently, numerical simulation is used to validate the efficacy of the suggested control mechanism.

## 5. Conclusions

For a category of nonlinear MASs, a non-strict feedback fuzzy adaptive event-triggered distributed control approach is given. On the basis of the described function and using Lyapunov stability theory and the universal approximation of a fuzzy logic system, a technique for output feedback control is given. This work offers an adaptive event-triggered distributed controller that gets updates only when an event happens to prevent wasting precious communication resources. Simulation is done to test the efficacy of the management plan. The results of this research can be applied not only to the specific system described in the current paper, but can also be generalized to other nonlinear systems with output or state constraints, and can therefore be discussed and investigated in the next step.

## Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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### Conflict of interest

The authors declare there are no conflicts of interest.

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