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Theory article

Mean-square consensus of a semi-Markov jump multi-agent system based on event-triggered stochastic sampling

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Abstract: This paper focuses on achieving leader-follower mean square consensus in semi-Markov jump multi-agent systems. To effectively reduce communication costs and control updates, we propose an event-triggered protocol based on stochastic sampling. The stochastic sampling interval randomly switches between finite given values, while the event-triggered function depends on the stochastic sampled data from neighboring agents. Using the event-triggered strategy, we present sufficient conditions to ensure mean square consensus. Finally, we provide a numerical example demonstrating the effectiveness of the theoretical results.

Keywords: event-triggered; stochastic sampling; Lyapunov function; semi-Markovian switching; multi-agent systems; mean-square consensus

1. Introduction

It is universally acknowledged that multi-agent cooperative control has a lot of applications, such as unmanned air vehicles, traffic control, animal groups and automated highway systems [1–4]. The cooperative control consensus that has received a lot of attention can be roughly divided into leader-follower consensus and leaderless consensus [5,6]. Most of the research aims have been to design a consensus protocol to make agents exchange local information with their neighbors, so that a cluster of agents are capable of achieving the consistent state.

As a paradigmatic instance, a digital microprocessor is installed in each agent of the system, which is responsible for gathering information from its neighboring agents and updating the controller accordingly. The majority of research studies utilize continuous measurement signals, yet continuous communication in a constrained energy exchange network is not feasible. In order to avoid continuous communication, some studies have introduced sampling data control [7–9], and each agent transmits the corresponding data at the sampling instants. Control using periodic sampling data often requires estimation of an optimal sampling period since improper selection can result in either excessively frequent or infrequent sampling, both of which can be detrimental to system performance. Stochastic sampling can better solve this problem. Stochastic sampling has been gaining increasing attention due to its flexibility in terms of dynamically switching sampling periods between different values. In [10], the authors investigated the leaderless multi-agent consensus problem under stochastic sampling, where the sampling period was chosen randomly at a given value.

Compared to the conventional time-triggered mechanism, the utilization of an event-triggered mechanism presents a significant advantage in terms of improving the efficiency of communication resource allocation. Research on event-triggered mechanisms has been extensively conducted in various fields, including cyber-physical systems [11], cyber-control systems [12] and multi-agent systems [13–16]. To solve the high-order multi-agent consensus problem, in [16], Wu et al. presented by estimating the state of neighbor agents, a novel event-triggered protocol. For further research, many scholars put forward dynamic event-triggered protocol [17–21]. In [17], a dynamic event-triggered protocol was proposed for individual agents, which established a distributed adaptive consensus protocol. This protocol involves updating the coupling strength to achieve consensus among the agents. A dynamic event-triggered protocol was proposed in [20] for the investigation of consensus in multi-agent systems. This protocol involved internal dynamic variables, and the elimination of Zeno behavior played an important role. In [21], Du et al. studied the multi-agent problem of leader-follower consensus based on a dynamic event-triggered mechanism. However, since event-triggered control has its own limitation, there is a need for continuous event detection. To loosen that constraint, many researchers designed an event-triggered protocol based on sampling data [22–24]. The authors presented research on event-triggered consensus strategies for multi-agent systems in [25] and [26]. Specifically, Su et al. investigated sampled data-based leader-follower multi-agent systems with input delays, with a focus on making sampling-based mechanisms for event detection more realistic. Meanwhile, He et al. focused on mean-square leaderless consensus for networked non-linear multi-agent systems and presented an efficiently distributed event-triggered mechanism that reduces communication costs and controller updates for random sampling-based systems. In [27], Ruan et al. studied the consensus problem with bounded external disturbances under an event-triggered scheme based on two independent dynamic thresholds in the context of leader-follower dynamics. The authors presented a nonlinear dynamic event-triggered control strategy for achieving prescribed-time synchronization in networks of piecewise smooth systems in [28].

The above studies are mostly the ones based on the fixed topology of multiple agents. A large number of switching topologies, some of which are studied by establishing Markov models, can be observed in our real world [29–31]. In [29], Hu et al. investigated the multi-agent consensus of Markov jump systems based on event-triggered strategies. In [30, 31], the scholars studied consensus issues in multi-agent systems with a Markov network structure. Nevertheless, the application of time-varying topology based on Markov processes has certain limitations, primarily due to the exponential distribution of jump times in Markov chains. Some researchers have focused on semi-Markovian jump topologies [32–36]. This is because the sojourn time in the semi-Markovian exchange topology is a generic continuous random variable. Its probability distribution is general. The $\mathcal{H}_\infty$ consensus problem for multi-agent systems in a semi-Markov switching topology with incomplete known transmission rates was investigated in [35]. In a related research study by Xie et al. [36], the consensus problem of multi-agent systems was investigated under an attack scenario in which both the semi-Markov switching topology and network were susceptible to attacks, with the possibility of recovery. In [37], the authors presented a study on achieving cluster synchronization in finite/fixed time for semi-Markovian switching $T - S$ fuzzy complex dynamical networks with discontinuous dynamic nodes.

Building upon the aforementioned literature review, this research article is focused on the development and analysis of an event-triggered mechanism for the stochastic sampling of leader-follower multi-agent systems. The key contributions of this study can be summarized as follows:

(1). This paper proposes a novel event-triggered methodology by utilizing stochastic sampling, which is capable of significantly reducing the frequency of control updates and communication overhead among agents. Furthermore, the proposed mechanism ensures the avoidance of Zeno behavior.

(2). The dynamic system switching investigated in this paper is modeled by using the semi-Markov jump process. A sufficient condition for mean-square consensus is derived.

(3). The sampling period of stochastic sampling is randomly selected from a finite set. Stochastic sampling differs from the periodic sampling and stochastic sampling in the literature [26, 29].

The subsequent sections of this article are organized as follows. Section II presents the problem formulation and introductory suggestions. Section III reports the major findings. The numerical tests in Section IV validate the accuracy of the theoretical conclusions. Section V concludes with final remarks.

Notations: The n-dimensional identity matrix is denoted by I_n. A zero matrix of appropriate dimension is represented by O. A positive (negative) definite matrix A is denoted by $A > 0$ ($A < 0$). The element implied by the symmetry of a matrix is denoted by $*$. The function $C([a, b], R^n)$ maps the interval $[a, b]$ to a continuous vector-valued function R^n. The Euclidean norm of a vector is represented by $|\cdot|$. A superscript T and the symbol $\otimes$ indicate matrix transposition and the Kronecker product, respectively. Let $\mathbb{I}_N = \{1, 2, \ldots, N\}$ denote a finite index set.

2. Preliminaries and problem statement

2.1. Graph theorem

We consider a directed graph, denoted by $G = \{V, E, A\}$, where V represents a set of vertices $\{1, 2, \ldots, N\}$, E denotes a set of directed edges and A is a weighted adjacency matrix of size $N \times N$. The elements of A, denoted by a_{ij}, are positive if there exists a directed edge going from vertex i to vertex j, and zero otherwise. We can refer to the set of neighbors of vertex i as N_i, which is defined as $N_i = \{j \in V : (j, i) \in E\}$. The degree matrix D is defined as a diagonal matrix of size $N \times N$, where the diagonal entries d_i represent the weighted degree of vertex i. Specifically, $d_i = \sum_{j\in N_i} a_{ij}$. The Laplacian matrix of G is defined as $L = D - A = \left(l_{ij}\right)_{N\times N}$. The diagonal entries of L are given by $l_{ii} = -\sum_{j\in N_i} l_{ij}$, and the off-diagonal entries are defined as follows: $l_{ij} = -a_{ij}$ if (i, j) is an edge in G; otherwise, $l_{ij} = 0$.

2.2. Semi-Markov jump multi-agent systems

Consider a multi-agent system that has a leader and N followers. Label them respectively as 0 and $1, 2, 3, \ldots, N$. The dynamic equations for each agent are illustrated below:

$$\begin{cases} \dot{x}_i(t) = A(r(t))x_i(t) + B(r(t))u_i(t), & i \in \mathbb{I}_N, \\ \dot{x}_0(t) = A(r(t))x_0(t). \end{cases} \tag{2.1}$$

The control input and state for the ith state are denoted by $u_i(t) \in R^n$ and $x_i \in R^n$, respectively. A state of $x_0(t) \in R^n$ indicates the leader agent. The constant matrices $A(r(t))$ and $B(r(t))$ have the appropriate dimensions. The semi-Markov chain $\{r(t), t \geq 0\}$ is a right-continuous process defined on the complete probability space (Ω, F, P), where its values belong to the finite state space $D = \mathbb{I}_M$ and its generators are denoted by $\lambda = (\lambda mn(v))_{M\times M}$. The transition probabililty is

$$\Pr\{r(t+v) = n \mid r(t) = m\} = \begin{cases} \lambda_{mn}(v)v + o(v) & m \neq n, \\ 1 + \lambda_{mn}(v)v + o(v) & m = n. \end{cases} \tag{2.2}$$

v is the time interval, which stands for the amount of time that passes between two successive jumps. The transition rate from mode m at time t to mode n at time $t+v$ is denoted by $\lambda_{mn}(v)$, and it satisfies $\lambda_{mn}(v) \geq 0$. $o(v)$ can be defined as $\lim\limits_{v\to 0^+} \frac{o(v)}{v} = 0$. $\lambda_{mm}(v) = -\sum_{m\neq n} \lambda_{mn}(v)$.
Remark 1: The dwell time v is the time elapsed from the last jump of the system, and it is distinct from the time t. When the system jumps, v resets to 0 and the transition probability $\lambda_{mn}(v)$ only depends on v.

2.3. The event-triggered mean square consensus protocol based on stochastic sampled data

A consensus protocol for event-triggered consensus that is grounded in the stochastic sampling of data is presented. Assuming that the sampling time is $0 = t_0 < t_1 < t_2 < \cdots < t_s < \cdots$, the sampling period is $h = t_{s+1} - t_s$, in which h is selected from a random finite set $h_1, h_2, \cdots, h_l$. The probability is described as $\Pr\{h = h_s\} = \pi_s$, $s \in \mathbb{I}_l$, $\pi_s \in [0, 1]$, and $\sum_{s=1}^{l} \pi_s = 1$. For the sake of generality, we can set $0 = h_0 < h_1 < h_2 < \cdots < h_s < \cdots < h_l, l > 1$.

Assuming that the ith agent has a K-time event-triggered time of t_k^i, $\left\{t_k^i\right\}_{k=0}^{\infty}$ represents the event-triggered time sequence of the ith agent, $t_k^i \in \{t_s, s \in N\}$, $t_0^i = 0$. t_{k+1}^irepresents the next event-triggering time of the ith agent, which is determined by the following formula

$$t_{k+1}^i = \min_{t_s > t_k}\{t_s : (e_i(t_s))^T \Phi(e_i(t_s)) > \sigma_i (z_i(t_s))^T \Phi(z_i(t_s))\}. \tag{2.3}$$

The threshold parameter is represented in this case by $\sigma_i > 0$. A positive definite event-triggered matrix is the intended matrix of matrix Φ. $e_i(t_s) = x_i(t_k^i) - x_i(t_s)$ and $z_i(t_s) = \sum_{j=1}^{N} a_{ij}\left(x_i\left(t_k^i\right) - x_j\left(t_{k'}^j\right)\right) + b_i\left(x_i\left(t_k^i\right) - x_0(t_s)\right)$, where $t_s \in [t_k^i, t_{k+1}^i)$. $t_{k'}^i$ is the latest transmitted sampled data of its neighbors before t_k^i, that is $t_{k'}^j = \max\left\{t_k^j \mid t_k^j \leq t_k^i\right\}$, $k' = 0, 1, 2, \cdots$.
Remark 2: The stochastic sampling sequence is $0 = t_0 < t_1 < t_2 < \cdots < t_s < \cdots$. The sampling period is $h = t_{s+1} - t_s$, where h is selected from a random finite set $\{h_1, h_2, \cdots, h_l\}$, where $0 = h_0 < h_1 < h_2 < \cdots < h_l$. This stochastic sampling differs from the periodic sampling and stochastic sampling in the literature [26, 29]. They represent the special form of stochastic sampling when h is constant and l equals 2.
Remark 3: In accordance with the event-triggered condition (2.3), the ith agent broadcasts the most recent sampled data to its neighbors. The sampling sequence includes the event-triggered sequence because the sampling period $h = t_{s+1} - t_s$ is stochastic in the set of $\{h_1, h_2, \cdots, h_l\}, h > 0$. Zeno behavior is precisely precluded.
Remark 4: It should be noted that in an attempt to decrease unnecessary communication between agents, a stochastic sampling static event-triggered protocol is proposed.

The following consensus protocol should be taken into consideration in light of the discussion above:

$$u_i(t) = -K(r(t))\left[\sum_{j=1}^{N} a_{ij}\Big(x_i(t_k^i) - x_j(t_{k'}^j)\Big) + b_i\Big(x_i(t_k^i) - x_0(t_s)\Big)\right]. \tag{2.4}$$

Remark 5: The consensus protocol relies upon the stochastic sampling of event-triggered conditions as well as upon a semi-Markov switching system, where the feedback gain $K(r(t))$ depends on $r(t)$, which will be given by a theorem later. When the ith agent and its neighbor agents satisfy the trigger conditions, the controller will be updated. With a zero-order holder, $u_i(t)$ remains constant between two successive event instants.

Define

$$\begin{aligned} e_i(t_s) &= x_i\left(t_k^i\right) - x_i(t_s), \\ e_j(t_s) &= x_j\left(t_{k'}^j\right) - x_j(t_s), \\ \delta_i(t_s) &= x_i(t_s) - x_0(t_s). \end{aligned} \tag{2.5}$$

Submitting (2.5) into (2.4) , we can obtain

$$u_i(t) = -K(r(t))\left[\sum_{j=1}^{N} l_{ij}\Big(e_j(t_s) + \delta_j(t_s)\Big) + b_i\Big(e_i(t_s) + \delta_i(t_s)\Big)\right]. \tag{2.6}$$

Define $\tau(t) = t - t_s$, where $\tau(t)$ is a piecewise linear function with a slope of $\dot{\tau}(t) = 1$ for all $t \in [t_s, t_{s+1})$, except at time t_s. The control protocol (2.6) may be expressed as

$$u_i(t) = -K(r(t))\left[\sum_{j=1}^{N} l_{ij}\Big(e_j(t-\tau(t)) + \delta_j(t-\tau(t))\Big) + b_i\Big(e_i(t-\tau(t)) + \delta_i(t-\tau(t))\Big)\right]. \tag{2.7}$$

A new stochastic variable is introduced as follows:

$$\beta_s(t) = \begin{cases} 1 & t_{s-1} \le \tau(t) < t_s, \quad s = 1, 2, \cdots, l \\ 0 & \text{otherwise.} \end{cases} \tag{2.8}$$

In this way, we can obtain

$$\Pr\{\beta_s(t) = 1\} = \Pr\{t_{s-1} \le \tau(t) < t_s\} = \sum_{i=s}^{l} \pi_i \frac{h_i - h_{i-1}}{h_i} = \beta_s. \tag{2.9}$$

The Bernoulli distribution is satisfied by $\beta_s(t)$, given that $E\{\beta_s(t)\} = \beta_s$ and $E\left\{\beta_s(t) - \beta_s^2\right\} = \beta_s(1 - \beta_s)$ respectively. We can obtain the calculation of the following formula from the above study:

$$\begin{aligned} \dot{\delta}(t) =& (I_N \otimes A(r(t)))\delta(t) - \sum_{s=1}^{l} \beta_s(t)\Big(H \otimes B(r(t))K(r(t))\Big)e(t - \tau_s(t)) \\ & - \sum_{s=1}^{l} \beta_s(t)\Big(H \otimes B(r(t))K(r(t))\Big)\delta(t - \tau_s(t)), \end{aligned} \tag{2.10}$$

where $\delta = \operatorname{diag}\{\delta_1, \delta_2, \cdots, \delta_N\}$, $H = L + B_1$, $B_1 = \operatorname{diag}\{b_1, b_2, \cdots, b_N\}$.

According to the initial conditions of Equation (2.10), let $\delta(t) = \phi(t), -h_l \le t \le 0$, $\phi(t) = \left[\phi_1^T(t), \phi_2^T(t), \cdots, \phi_N^T(t)\right]$ and $\phi_i(t) \in C([-h_l, 0], R^n)$.

Definition 1 [34]. Under semi-Markov switching topologies, the leader-follower consensus of multi-agent system (2.1) with the consensus protocol is said to be achieved if $\lim_{t\to\infty} E\|x_i(t) - x_0(t)\| = 0$, $i \in \mathbb{I}_N$ holds for any initial distribution $r_0 \in D$ and any initial condition $\phi(t), \forall t \in [-h_l, 0]$.

Assumption 1. The directed spanning tree in the network graph G has the leader's root.

Lemma 1 [34]. For symmetric matrices $R > 0$ and X and any scalar μ, the following inequality holds:

$$-XR^{-1}X \leq \mu^2 R - 2\mu X.$$

3. Results

Theorem 1. Under Assumption 1 and utilizing the protocol given by (2.7), the constants provided are $0 = h_0 < h_1 < \cdots < h_s < \cdots < h_l$ and $\pi_i \in [0, 1]$, $\sigma_i > 0$, $i \in \mathbb{I}_N$. Consensus can be achieved in the mean-square sense for the multi-agent system (2.10) by employing the stochastic sampling event-triggered strategy given by (2.3), provided that constant metrics are present, namely $P(m) > 0, Q_s > 0, R_s > 0, W_s > 0, m \in Dands \in \mathbb{I}_l$. In such a way, the inequality below holds:

$$\begin{pmatrix} \Xi & F^T(m) & \Sigma(m,v) \\ * & \Psi & 0 \\ * & * & -\mathrm{X}_2(m) \end{pmatrix} < 0, \tag{3.1}$$

where

$$\Xi = \aleph_1 + \aleph_2 + \aleph_3 + \aleph_4 + \aleph_5 + \aleph_6.$$

$$\aleph_1 = F^T(m)(I_N \otimes P(m))\varepsilon_1 + \varepsilon_1^T(I_N \otimes P(m))F(m) + \sum_{n=1}^{M} \lambda_{mn}(v)\left(\varepsilon_1^T(I_N \otimes P(n))\varepsilon_1\right).$$

$$\begin{aligned}\aleph_2 =& \beta_1\varepsilon_1^T(I_N \otimes Q_s)\varepsilon_1 - \beta_1\varepsilon_{l+1}^T(I_N \otimes Q_s)\varepsilon_{i+1} + \sum_{s=2}^{l}\beta_s(\varepsilon_{l+s}^T(I_N \otimes Q_s)\varepsilon_{l+s} \\ &- \varepsilon_{l+s+1}^T(I_N \otimes Q_s)\varepsilon_{l+s+1}).\end{aligned}$$

$$\aleph_3 = -\sum_{s=1}^{l}\beta_s\frac{1}{h_s - h_{s-1}}((\varepsilon_{l+s}^T - \varepsilon_{l+s+1}^T) \times (I_N \otimes (R_s + W_s))(\varepsilon_{l+s} - \varepsilon_{l+s+1})).$$

$$\aleph_4 = \varepsilon_1^T(I_N \otimes P(m))\varepsilon_1.$$

$$\aleph_5 = -\textstyle\sum_{s=1}^{l}\beta_s\varepsilon_{s+1}^T\Phi\varepsilon_{s+1}.$$

$$\aleph_6 = (\varepsilon_{s+1} + \varepsilon_{2l+s+1})^T\left(H^T\Lambda H \otimes \Phi\right)(\varepsilon_{s+1} + \varepsilon_{2l+s+1}).$$

$$F(m) = (I_N \otimes A(m))\varepsilon_1 - \sum_{s=1}^{l}\beta_s(H \otimes B(m)K(m))\varepsilon_{2l+s+1} - \sum_{s=1}^{l}\beta_s(H \otimes B(m)K(m))\varepsilon_{s+1}.$$

$$\Phi = \mathrm{diag}\{\Phi_1, \Phi_2, \cdots, \Phi_N\}, \Sigma(m,v) = \lambda(m,v)\mathrm{X}_1(m).$$

$$\Psi = -\left(\textstyle\sum_{s=1}^{l}\beta_s(h_s - h_{s-1})(I_N \otimes (R_s + W_s))\right)^{-1}.$$

$$\lambda(m,v) = \left(\sqrt{\lambda_{m1}(v)}, \sqrt{\lambda_{m2}(v)}, \cdots, \sqrt{\lambda_{mm-1}(v)}, \sqrt{\lambda_{mm+1}(v)}, \cdots, \sqrt{\lambda_{mM}(v)}\right).$$

$$\mathrm{X}_1(m) = \mathrm{diag}\{I_N \otimes P(m), \cdots, I_N \otimes P(m)\}_{M-1}.$$

$$\mathrm{X}_2(m) = \mathrm{diag}\{I_N \otimes P(1), I_N \otimes P(2), \cdots, I_N \otimes P(m-1), I_N \otimes P(m+1), \cdots, I_N \otimes P(M)\}_{M-1}.$$

$A(m) = A(r(t) = m), P(m) = P(r(t) = m)$ and β_s is defined the same way as in (2.10). Define ε_s as a block matrix consisting of $3l + 1$ block elements. The s-th block element is an $Nn \times Nn$ identity matrix, denoted by I_{Nn}, while all other block elements are zero matrices. Therefore, ε_s can be expressed as $\varepsilon_s = [0, 0, \cdots, I_{Nn}, 0, 0, \cdots, 0] \in \mathbb{R}^{Nn\times(3l+1)Nn}$ for $s = 1, 2, \cdots, 3l+1$.

Proof of Theorem 1. Think about the Lyapunov-Krasovskii functional presented as follows:

$$V\left(t, \delta_t, \dot{\delta}_t, r(t)\right) = \sum_{i=1}^{3} V_i\left(t, \delta_t, \dot{\delta}_t, r(t)\right), t \in [t_s, t_{s+1}], \tag{3.2}$$

where

$$V_1\left(t, \delta_t, \dot{\delta}_t, r(t)\right) = \delta^T(t)\left(I_N \otimes P(r(t))\right)\delta(t), \tag{3.3}$$

$$V_2\left(t, \delta_t, \dot{\delta}_t\right) = \sum_{s=1}^{l} \beta_s \int_{t-h_s}^{t-h_{s-1}} \delta^T(\mu) \times (I_N \otimes Q_s)\,\delta(\mu)d\mu, \tag{3.4}$$

$$V_3(t, \delta_t, \dot{\delta}_t) = \sum_{s=1}^{l} \beta_s \int_{-h_s}^{-h_{s-1}} \int_{t+v}^{t} \dot{\delta}^T(\mu)(I_N \otimes (R_s + W_s))\dot{\delta}(\mu)d\mu dv. \tag{3.5}$$

Consider the weak infinitesimal generator

$$\Im V(t, z_t) = \lim_{\Delta\to 0^+} \frac{1}{\Delta}\{E\{V(t+\Delta, \delta_{t+\Delta}, \dot{\delta}_{t+\Delta}, r(t+\Delta)) \mid \delta_t, r(t) = m\} - V(t, \delta_t, \dot{\delta}_t, r(t))\}. \tag{3.6}$$

Introduce
$y(t) = (\delta^T(t), \delta^T(t-\tau_1(t)), \cdots, \delta^T(t-\tau_l(t)), \delta^T(t-h_1), \cdots, \delta^T(t-h_l), e^T(t-\tau_1(t)), \cdots,$
$e^T(t-\tau_l(t))), y(t) \in R^{(3l+1)Nn}, A(m) = A(r(t) = m), P(m) = P(r(t) = m), \forall m \in D$.
Thus, we obtain

$$\begin{aligned}
&\mathrm{E}[\Im V_1(t, \delta_t, \dot{\delta}_t, r(t))] \\
&= E[\lim_{\Delta\to 0^+} \frac{1}{\Delta}\{E\{V_1(t+\Delta, \delta_{t+\Delta}, \dot{\delta}_{t+\Delta}, r(t+\Delta) \mid \delta_t, r(t) = m)\} - V_1\left(t, \delta_i, \dot{\delta}_i, r(t)\right)\}] \\
&= E[y^T(t)(F^T(m)(I_N \otimes P(m))\varepsilon_1 + \varepsilon_1^{\tau}(I_N \otimes P(m))F(m) + \sum_{n=1}^{M} \lambda_{mn}(v)\varepsilon_1^{\tau}(I_N \otimes P(n))\varepsilon_1)y(t)],
\end{aligned} \tag{3.7}$$

where

$$F(m) = (I_N \otimes A(m))\varepsilon_1 - \sum_{s=1}^{l} \beta_s\Big(H \otimes B(m)K(m)\Big)\varepsilon_{2l+s+1} - \sum_{s=1}^{l} \beta_s\Big(H \otimes B(m)K(m)\Big)\varepsilon_{s+1}.$$

$$\begin{aligned}
&E\left[\Im V_2\left(t, \delta_i, \dot{\delta}_t\right)\right] \\
&= E\{y^T(t)(\beta_1\delta^T(t-h_0)(I_N \otimes Q_1)\delta(t-h_0) - \beta_1\delta^T(t-h_1)(I_N \otimes Q_1)\delta(t-h_1) \\
&\quad + \sum_{s=2}^{l} \beta_s(\delta^T(t-h_{s-1})(I_N \otimes Q_s)\delta(t-h_{s-1}) - \delta^T(t-h_s)(I_N \otimes Q_s)\delta(t-h_s)))y(t)\},
\end{aligned} \tag{3.8}$$

and

$$E[\Im V_3\left(t,\delta_t,\dot{\delta}_t\right)] = E\Big[\sum_{s=1}^{l}\beta_s(h_s-h_{s-1})\dot{\delta}^T(t)(I_N\otimes(R_s+W_s))\dot{\delta}(t) - \sum_{s=1}^{l}\beta_s\int_{t-h_s}^{t-h_{s-1}}\dot{\delta}^T(v)(I_N\otimes(R_s+W_s))\dot{\delta}(v)dv\Big]. \tag{3.9}$$

According to Jensen's inequality, it can be obtained that

$$\begin{aligned} &-\sum_{s=1}^{l}\beta_s\int_{t-h_s}^{t-h_{s-1}}\dot{\delta}^T(v)(I_N\otimes(R_s+W_s))\dot{\delta}(v)dv \\ &\leq -\sum_{s=1}^{l}\beta_s\frac{1}{h_s-h_{s-1}}\int_{t-h_s}^{t-h_{s-1}}\dot{\delta}^T(v)dv\times(I_N\otimes(R_s+W_s))\int_{t-h_s}^{t-h_{s-1}}\dot{\delta}(v)dv. \end{aligned} \tag{3.10}$$

Submit (2.10) and (3.10) into (3.9), and we can obtain

$$\begin{aligned} &E\left[\Im V_3\left(t,\delta_t,\dot{\delta}_t,r(t)\right)\right] \\ &\leq y^T(t)\left[\sum_{s=1}^{l}\beta_s\left(h_s-h_{s-1}\right)\times F^T(m)\left(I_N\otimes(R_s+W_s)\right)F(m)\right. \\ &\left.-\sum_{s=1}^{l}\beta_s\frac{1}{h_s-h_{s-1}}\left(\varepsilon_{l+s}^T-\varepsilon_{l+s+1}^T\right)\times(I_N\otimes(R_s+W_s))\left(\varepsilon_{l+s}^T-\varepsilon_{l+s+1}^T\right)\right]y(t). \end{aligned} \tag{3.11}$$

From (2.3), we can obtain

$$\begin{aligned} &e^T\left(t-\tau_s\right)\Phi e\left(t-\tau_s\right)\leq z^T\left(t-\tau_s\right)(\Lambda\otimes\Phi)z\left(t-\tau_s\right) \\ &=\left(\delta\left(t-\tau_s\right)+e\left(t-\tau_s\right)\right)^T(H^T\Lambda H\otimes\Phi)\times\left(\delta\left(t-\tau_s\right)+e\left(t-\tau_s\right)\right), \end{aligned} \tag{3.12}$$

where $e\left(t-\tau_s\right)=col\{e_1\left(t-\tau_s\right),e_2\left(t-\tau_s\right),\cdots,e_N\left(t-\tau_s\right)\}$, $\Lambda=\operatorname{diag}\{\sigma_1,\sigma_2,\cdots,\sigma_N\}$. Additionally, $z\left(t-\tau_s\right)=col\{z_1\left(t-\tau_s\right),z_2\left(t-\tau_s\right),\cdots,z_N\left(t-\tau_s\right)\}$.
Thus, combine (3.8–3.10) with (3.11), and we can obtain

$$\begin{aligned} &\mathrm{E}\left[\Im V\left(t,\delta_t,\dot{\delta}_t,r(t)\right)\right] \\ &\leq E[y^T(t)\sum_{s=1}^{l}\beta_s\left(\Xi-\mathcal{E}_{s+1}^T\Phi\mathcal{E}_{s+1}+(\mathcal{E}_{s+1}+\mathcal{E}_{2l+s+1})^T(H^T\Lambda H\otimes\Phi)(\mathcal{E}_{s+1}+\mathcal{E}_{2l+s+1})\right]y(t), \end{aligned} \tag{3.13}$$

and

$$\begin{aligned} &\mathrm{E}\left[\Im V\left(t,\delta_t,\dot{\delta}_t,r(t)\right)\right] \\ &\leq E[y^T(t)\sum_{s=1}^{l}\beta_s\left(\Xi-\mathcal{E}_{s+1}^T\Phi\mathcal{E}_{s+1}+(\mathcal{E}_{s+1}+\mathcal{E}_{2l+s+1})^T(H^T\Lambda H\otimes\Phi)(\mathcal{E}_{s+1}+\mathcal{E}_{2l+s+1})\right]y(t). \end{aligned} \tag{3.14}$$

Applying the Schur complement and (3.1) leads to the conclusion that

$$\mathrm{E}\left[\Im V\left(t,\delta_t,\dot{\delta}_t,r(t)\right)\right]<0, \tag{3.15}$$

where

$$\Xi = \aleph_1 + \aleph_2 + \aleph_3 + \aleph_4 + \aleph_5 + \aleph_6.$$

Hence, the consensus (3.1) of the multi-agent system can be attained in a mean squared sense under the event-triggered method given by (2.3). □

Keeping Theorem 1's results in mind, this method provides an efficient approach to design consensus controller gains.

Theorem 2. Under Assumption 1, and by utilizing the protocol given by (2.7), we have the following constants $0 = h_0 < h_1 < \cdots < h_s < \cdots < h_l, \pi_i \in [0, 1], \sigma_i > 0, i \in \mathbb{I}_N$ and $\mu > 0$. The multi-agent system consensus (2.10) can be achieved in the mean-square sense with the stochastic sampled even-triggered strategy (2.3) if there exist the matrices $\hat{P}(m) > 0, \hat{Q}_s > 0, \hat{R}_s > 0, \hat{W}_s > 0$ and $\hat{\Phi} > 0$ and the matrices $\hat{K}(m), m \in D$ and $s \in \mathbb{I}_l$ satisfy the following inequality:

$$\begin{pmatrix} \hat{\Xi} & (h_m - h_{(m-1)})\hat{F}^T(m) & \hat{\Sigma}(m, v) \\ * & \hat{\Psi} & O \\ * & * & -\hat{X}_2(m) \end{pmatrix} < 0, \tag{3.16}$$

where $\hat{\Xi} = \hat{\aleph}_1 + \hat{\aleph}_2 + \hat{\aleph}_3 + \hat{\aleph}_4 + \hat{\aleph}_5 + \hat{\aleph}_6$.

$\hat{\aleph}_1 = \hat{F}^T(m)\varepsilon_1 + \varepsilon_1^T \hat{F}(m) + \varepsilon_1^T \lambda_{mm}(v)\left(I_N \otimes \hat{P}(m)\right)\varepsilon_1.$

$$\hat{\aleph}_2 = \beta_1\varepsilon_1^T\left(I_N \otimes \hat{Q}_1\right)\varepsilon_1 - \beta_1\varepsilon_{l+1}^T\left(I_N \otimes \hat{Q}_1\right)\varepsilon_{l+1} + \sum_{s=2}^{l}\beta_s\left(\varepsilon_{l+s}^T\left(I_N \otimes \hat{Q}_s\right)\varepsilon_{l+s}\right.$$
$$\left. - \varepsilon_{l+s+1}^T\left(I_N \otimes \hat{Q}_s\right)\varepsilon_{l+s+1}\right).$$

$$\hat{\aleph}_3 = -\sum_{s=1}^{l}\beta_s\frac{1}{h_s - h_{s-1}}\left(\left(\varepsilon_{l+s}^T - \varepsilon_{l+s+1}^T\right)\left(I_N \otimes \left(\hat{R}_s + \hat{W}_s\right)\right)\left(\varepsilon_{l+s} - \varepsilon_{l+s+1}\right)\right).$$

$\hat{\aleph}_4 = \varepsilon_1^T(I_N \otimes \hat{P}(m))\varepsilon_1.$

$\hat{\aleph}_5 = -\sum_{s=1}^{l}\beta_s\varepsilon_{s+1}^T\left(\hat{\Phi}\right)\varepsilon_{s+1}.$

$\hat{\aleph}_6 = (\varepsilon_{s+1} + \varepsilon_{2l+s+1})^T\left(H^T\Lambda H \otimes \hat{\Phi}\right)(\varepsilon_{s+1} + \varepsilon_{2l+s+1}).$

$$\hat{F}(m) = \left(I_N \otimes A(m)\hat{P}(m)\right)\varepsilon_1 - \sum_{s=1}^{l}\beta_s(H \otimes B(m)\hat{K}(m))\varepsilon_{2l+s+1} - \sum_{s=1}^{l}\beta_s(H \otimes B(m)\hat{K}(m))\varepsilon_{s+1}.$$

$\hat{\Phi} = \mathrm{diag}\left\{\hat{\Phi}_1, \hat{\Phi}_2, \cdots, \hat{\Phi}_N\right\}, \hat{\Phi}_i = \hat{P}(m)\Phi_i\hat{P}(m), \hat{P}(m) = P^{-1}(m), \hat{\Phi}_s = \hat{P}(m)\Phi_s\hat{P}(m).$

$$\hat{\Psi} = \sum_{s=1}^{l}\left(\beta_s\left(h_s - h_{s-1}\right)\right)^{-1}\left(\mu^2\left(I_N \otimes (R_s + W_s)\right) - 2\mu\left(I_N \otimes \hat{P}(m)\right)\right).$$

$\hat{R}_s = \hat{P}(m)R_s\hat{P}(m), \hat{W}_s = \hat{P}(m)W_s\hat{P}(m), \hat{\Sigma}(m, v) = \lambda(m, v)\hat{X}_1(m).$

$\lambda(m, v) = \left(\sqrt{\lambda_{m1}(v)}, \sqrt{\lambda_{m2}(v)}, \cdots, \sqrt{\lambda_{mm-1}(v)}, \sqrt{\lambda_{mm+1}(v)}, \cdots, \sqrt{\lambda_{mM}(v)}, 0, \cdots, 0\right).$

$\hat{X}_1(m) = \mathrm{diag}\left\{I_N \otimes \hat{P}(m), \cdots, I_N \otimes \hat{P}(m), O, \cdots, O\right\}.$

$\hat{X}_2(m) = \mathrm{diag}\left\{I_N \otimes \hat{P}(1), I_N \otimes \hat{P}(2), \cdots, I_N \otimes \hat{P}(m-1), I_N \otimes \hat{P}(m+1), \cdots, I_N \otimes \hat{P}(M), O, \cdots, O\right\}.$

In addition, the feedback gain is supplied by $K(m)=\hat{K}(m)\hat{P}^{-1}(m)$ and the event-triggered parameter matrix is given by $\Phi(m) = \hat{P}^{-1}(m)\hat{\Phi}(m)\hat{P}^{-1}(m)$.

Proof of Theorem 2. Here we present the definitions of matrix variables $K(m) = \hat{K}(m)\hat{P}^{-1}(m), \hat{P}(m) = P^{-1}(m)$ and $\hat{\Phi}(m) = \hat{P}(m)\Phi(m)\hat{P}(m)$. We pre- and post-multiply both sides of (3.3) by the

matrix diag$\{I_N \otimes P^{-1}(m), I_N \otimes P^{-1}(m), I_N \otimes P^{-1}(m), I_{nN}\}$, and both sides of (3.2) by the matrix diag$\{I_N \otimes P^{-1}(m), I_N \otimes P^{-1}(m)\}$, respectively.
Lemma 1 enables one to derive the subsequent inequality.
$-\left(I_N \otimes \hat{P}(m)\right)\left(\sum_{s=1}^{l} \beta\left(h_s - h_{s-1}\right)\left(I_N \otimes (R_s + W_s)\right)\right)^{-1} \times \left(I_N \otimes \hat{P}(m)\right)$
$\leq \mu^2 \left(\sum_{s=1}^{l} \beta\left(h_s - h_{s-1}\right)\left(I_N \otimes (R_s + W_s)\right)\right) - 2\mu\left(I_N \otimes \hat{P}(m)\right);$
we can get

$$\begin{pmatrix} \hat{\Xi} & (h_m - h_{(m-1)})\hat{F}^T(m) & \hat{\Sigma}(m, v) \\ * & \hat{\Psi} & O \\ * & * & -\hat{X}_2(m) \end{pmatrix} < 0, \tag{3.17}$$

where $\hat{\Xi} = \hat{\aleph}_1 + \hat{\aleph}_2 + \hat{\aleph}_3 + \hat{\aleph}_4 + \hat{\aleph}_5 + \hat{\aleph}_6$,

$$\hat{\aleph}_1 = \hat{F}^T(m)\varepsilon_1 + \varepsilon_1^T \hat{F}(m) + \varepsilon_1^T \lambda_{mm}(v)\left(I_N \otimes \hat{P}(m)\right)\varepsilon_1,$$

$$\begin{aligned}\hat{\aleph}_2 =& \beta_1 \varepsilon_1^T \left(I_N \otimes \hat{Q}_1\right)\varepsilon_1 - \beta_1 \varepsilon_{l+1}^T \left(I_N \otimes \hat{Q}_1\right)\varepsilon_{l+1} + \sum_{s=2}^{l} \beta_s \left(\varepsilon_{l+s}^T \left(I_N \otimes \hat{Q}_s\right)\varepsilon_{l+s}\right. \\ &\left. -\varepsilon_{l+s+1}^T \left(I_N \otimes \hat{Q}_s\right)\varepsilon_{l+s+1}\right),\end{aligned}$$

$$\hat{\aleph}_3 = -\sum_{s=1}^{l} \beta_s \frac{1}{h_s - h_{s-1}} \left(\left(\varepsilon_{l+s}^T - \varepsilon_{l+s+1}^T\right)\left(I_N \otimes \left(\hat{R}_s + \hat{W}_s\right)\right)\left(\varepsilon_{l+s} - \varepsilon_{l+s+1}\right)\right),$$

$$\hat{\aleph}_4 = \varepsilon_1^T (I_N \otimes \hat{P}(m))\varepsilon_1,$$

$$\hat{\aleph}_5 = -\sum_{s=1}^{l} \beta_s \varepsilon_{s+1}^T \left(\hat{\Phi}\right)\varepsilon_{s+1},$$

$$\hat{\aleph}_6 = \left(\varepsilon_{s+1} + \varepsilon_{2l+s+1}\right)^T \left(H^T \Lambda H \otimes \hat{\Phi}\right)\left(\varepsilon_{s+1} + \varepsilon_{2l+s+1}\right),$$

$$\hat{F}(m) = \left(I_N \otimes A(m)\hat{P}(m)\right)\varepsilon_1 - \sum_{s=1}^{l} \beta_s (H \otimes B(m)\hat{K}(m))\varepsilon_{2l+s+1} - \sum_{s=1}^{l} \beta_s (H \otimes B(m)\hat{K}(m))\varepsilon_{s+1},$$

$$\hat{\Phi} = \text{diag}\left\{\hat{\Phi}_1, \hat{\Phi}_2, \cdots, \hat{\Phi}_N\right\},\ \hat{\Phi}_i = \hat{P}(m)\Phi_i \hat{P}(m),\ \hat{P}(m) = P^{-1}(m),\ \hat{\Phi}_s = \hat{P}(m)\Phi_s \hat{P}(m),$$

$$\hat{\Psi} = \sum_{s=1}^{l} \left(\beta_s \left(h_s - h_{s-1}\right)\right)^{-1} \left(\mu^2 \left(I_N \otimes (R_s + W_s)\right) - 2\mu\left(I_N \otimes \hat{P}(m)\right)\right),$$

$$\hat{R}_s = \hat{P}(m)R_s \hat{P}(m), \hat{W}_s = \hat{P}(m)W_s \hat{P}(m),\ \hat{\Sigma}(m, v) = \lambda(m, v)\hat{X}_1(m),$$

$$\lambda(m, v) = \left(\sqrt{\lambda_{m1}(v)},\ \sqrt{\lambda_{m2}(v)}, \cdots,\ \sqrt{\lambda_{mm-1}(v)},\ \ \sqrt{\lambda_{mm+1}(v)}, \cdots,\ \sqrt{\lambda_{mM}(v)}, 0, \cdots, 0\right),$$

$$\hat{X}_1(m) = \text{diag}\left\{I_N \otimes \hat{P}(m), \cdots, I_N \otimes \hat{P}(m), O, \cdots, O\right\},$$

$$\begin{aligned}\hat{X}_2(m) = \text{diag}&\left\{I_N \otimes \hat{P}(1), I_N \otimes \hat{P}(2), \cdots, I_N \otimes \hat{P}(m-1),\right. \\ &\left. I_N \otimes \hat{P}(m+1), \cdots, I_N \otimes \hat{P}(M), O, \cdots, O\right\}.\end{aligned}$$

The proof is therefore complete. □

In Theorems 1 and 2, we establish sufficient conditions for achieving consensus in event-triggered semi-Markov jump multi-agent systems through stochastic sampling. But the sufficient conditions do not satisfy linear matrix inequality (LMI) conditions , because $\lambda(m, v)$ is time-varying. As a result, the problems cannot be directly solved by using the LMI toolbox in MATLAB. Nevertheless, we can establish lower and upper bounds for the transition rate and apply the theorem presented below to overcome this issue.

Theorem 3. Under Assumption 1, and by utilizing the protocol given by (2.7), we have the following constants $0 = h_0 < h_1 < \cdots < h_s < \cdots < h_l, \pi_i \in [0,1], \sigma_i > 0, i \in \mathbb{I}_N$ and $\mu > 0$. By utilizing the stochastic sampled event-triggered strategy (2.3) and assuming the existence of positive matrices $\hat{P}(m), \hat{Q}s, \hat{R}s, \hat{W}s, \hat{\Phi}$, consensus of the multi-agent system (2.10) can be achieved in a mean square sense. This is subject to the condition that the matrices $\hat{K}(m), m \in D$ and $s \in \mathbb{I}_l$ satisfy the following inequality:

$$\begin{pmatrix} \underline{\hat{\Xi}} & (h_m - h_{(m-1)})\hat{F}^T(m) & \underline{\hat{\Sigma}}(m) \\ * & \hat{\Psi} & O \\ * & * & -\hat{X}_2 \end{pmatrix} < 0, \tag{3.18}$$

$$\begin{pmatrix} \overline{\hat{\Xi}} & (h_m - h_{(m-1)})\hat{F}^T(m) & \overline{\hat{\Sigma}}(m) \\ * & \hat{\Psi} & O \\ * & * & -\hat{X}_2 \end{pmatrix} < 0, \tag{3.19}$$

where $\underline{\hat{\Xi}} = \underline{\hat{\aleph}}_1 + \hat{\aleph}_2 + \hat{\aleph}_3 + \hat{\aleph}_4 + \hat{\aleph}_5 + \hat{\aleph}_6$,
$\underline{\hat{\aleph}}_1 = \hat{F}^T(m)\varepsilon_1 + \varepsilon_1^T \hat{F}(m) + \varepsilon_1^T \underline{\lambda}_{mm} \left(I_N \otimes \hat{P}(m)\right)\varepsilon_1$,
$\overline{\hat{\Xi}} = \overline{\hat{\aleph}}_1 + \hat{\aleph}_2 + \hat{\aleph}_3 + \hat{\aleph}_4 + \hat{\aleph}_5 + \hat{\aleph}_6$,
$\overline{\hat{\aleph}}_1 = \hat{F}^T(m)\varepsilon_1 + \varepsilon_1^T \hat{F}(m) + \varepsilon_1^T \overline{\lambda}_{mm} \left(I_N \otimes \hat{P}(m)\right)\varepsilon_1$,
$\underline{\hat{\Sigma}}(m) = \underline{\lambda}(m)\hat{X}_1(m), \overline{\hat{\Sigma}}(m) = \overline{\lambda}(m)\hat{X}_1(m)$,
$\underline{\lambda}(m) = \left(\sqrt{\underline{\lambda}_{m1}}, \sqrt{\underline{\lambda}_{m2}}, \cdots, \sqrt{\underline{\lambda}_{mm-1}}, \sqrt{\underline{\lambda}_{mm+1}}, \cdots, \sqrt{\underline{\lambda}_{mM}}, 0, \cdots, 0\right)$,
$\overline{\lambda}(m) = \left(\sqrt{\overline{\lambda}_{m1}}, \sqrt{\overline{\lambda}_{m2}}, \cdots, \sqrt{\overline{\lambda}_{mm-1}}, \sqrt{\overline{\lambda}_{mm+1}}, \cdots, \sqrt{\overline{\lambda}_{mM}}, 0, \cdots, 0\right)$.

The definitions of Theorem 2 are applicable to the remaining terms in the inequalities. By using the same strategy for proof as Theorem 2 of [35], the theorem may be simply constructed. Therefore, it is omitted here.

Remark 6. It is worth mentioning that Theorem 3's conclusion is relatively conservative. To decrease conservativeness, the sojourn-time division method is used by dividing the sojourn time υ by J and denoting the pth segment as $\overline{\lambda}_{nm,p}$ and $\underline{\lambda}_{nm,p}$ to represent the upper and lower bounds on the transmission probability, respectively. The conclusions drawn are relatively lenient.

Corollary 1. Under Assumption 1 and the protocol (2.7), where $0 = h_0 < h_1 < \cdots < h_s < \cdots < h_l$, $\pi_i \in [0,1]$, $\sigma_i > 0, i \in \mathbb{I}_N$ and $\mu > 0$, the multi-agent system consensus (2.10) can be achieved in a mean-square sense with the stochastic sampled event-triggered strategy (2.3). This can be achieved if there exists a positive matrix $\hat{P}(m)$, and positive matrices $\hat{Q}_s, \hat{R}_s, \hat{W}_s, \hat{\Phi}$, along with the matrices $\hat{K}(m), m \in D$, $s \in \mathbb{I}_l$, which satisfy the following LMI:

$$\begin{pmatrix} \underline{\hat{\Xi}}(m,p) & (h_m - h_{(m-1)})\hat{F}^T(m,p) & \underline{\hat{\Sigma}}(m,p) \\ * & \hat{\Psi}(m,p) & O \\ * & * & -\hat{X}_2(m,p) \end{pmatrix} < 0,$$

$$\begin{pmatrix} \overline{\hat{\Xi}}(m,p) & (h_m - h_{(m-1)})\hat{F}^T(m,p) & \overline{\hat{\Sigma}}(m,p) \\ * & \hat{\Psi}(m,p) & O \\ * & * & -\hat{X}_2(m,p) \end{pmatrix} < 0,$$

in which $\underline{\hat{\Xi}}(m,p), \overline{\hat{\Xi}}(m,p), \underline{\hat{\Sigma}}(m,p), \overline{\hat{\Sigma}}(m,p), \hat{F}(m,p), \hat{P}(m,p), \hat{X}_2(m,p), \hat{\Psi}(m,p), \hat{\Upsilon}_1(m,p)$ and $\hat{\Upsilon}_2(m,p)$ are similarly defined as in Theorem 3, with the exception that (m) is substituted by (m,p). Furthermore, the feedback gain is given by $K(m,p) = \hat{K}(m,p)\hat{P}^{-1}(m,p)$. Moreover, the expression for the feedback gain is defined as $K(m,p) = \hat{K}(m,p)\hat{P}^{-1}(m,p)$. The parameter matrix for the event-triggered strategy is denoted as $\Phi(m,p) = \hat{P}^{-1}(m,p)\hat{\Phi}(m,p)\hat{P}^{-1}(m,p)$, where $\hat{\Phi}(m,p)$ is an estimated parameter and m is an element in the set D while p is an element in the index set $\mathbb{I}_J$.

4. Numerical example

Within this part, we will provide a numerical illustration to showcase the efficacy of the suggested design methodology. For consideration of a semi-Markov jump multi-agent system, which contains a leader and five followers, we assume that the model is described in formula (2.1). The coefficient matrices of the system equation are $Ar, Br, Cr, r = 1,2,3$. $A(1) = \begin{pmatrix} -14 & -18 \\ -11 & -28 \end{pmatrix}$, $A(2) = \begin{pmatrix} -16 & -16 \\ -15 & -23 \end{pmatrix}$, $A(3) = \begin{pmatrix} -13 & -18 \\ -11 & -20 \end{pmatrix}$, $B(1) = \begin{pmatrix} 17 \\ 2 \end{pmatrix}$, $B(2) = \begin{pmatrix} 6 \\ 12 \end{pmatrix}$, $B(3) = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$. The topology of the network is shown in Figure 1. The corresponding Laplacian matrix L and the leader adjacency matrix B can be derived in the manner shown as follows:

$$L = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}, B = \mathrm{diag}(1,1,0,0,0).$$

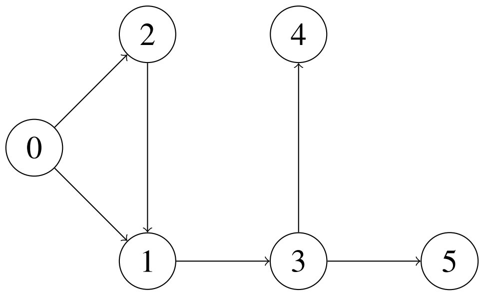

Figure 1. The leader and five followers topology.

Let the event-trigged parameters be $\sigma_1 = 6.353, \sigma_2 = 7.163, \sigma_3 = 6.093, \sigma_4 = 7.533$ and $\sigma_5 = 6.312$. The stochastic sampling period h takes values from the set $\{h_1, h_2\} = \{0.1s, 0.2s\}$ with probabilities of occurrence $\pi_1 = Pr\{h = h_1\} = 0.2$ and $\theta_2 = Pr\{h = h_2\} = 0.8$. With these values, we can obtain that $\rho_1 = 0.6, \rho_2 = 0.4$. By utilizing MATLAB's LMI toolbox, we can verify the feasibility of solutions to LMIs (3.18, 3.19) for $\mu = 4$ in Theorem 3. The event-triggered parameter metrics are derived as follows: $\Phi_1 = \begin{pmatrix} 3.1159 & 0.0189 \\ 0.0189 & 3.1294 \end{pmatrix}$, $\Phi_2 = \begin{pmatrix} 3.1153 & 0.0195 \\ 0.0196 & 3.1279 \end{pmatrix}$, and $\Phi_3 = \begin{pmatrix} 3.1149 & 0.0195 \\ 0.0195 & 3.1279 \end{pmatrix}$. The consensus feedback matrices are as follows: $K(1) = \begin{pmatrix} 0.0058 & -0.0038 \end{pmatrix}$, $K(2) = \begin{pmatrix} 0.0027 & -0.0011 \end{pmatrix}$, $K(3) = \begin{pmatrix} 0.0048 & -0.0023 \end{pmatrix}$. The initial states of the leader and followers were selected as follows: $x_0(0) = \begin{pmatrix} 1 & 0 \end{pmatrix}$, $x_1(0) = \begin{pmatrix} 3.547 & 9.553 \end{pmatrix}$, $x_2(0) = \begin{pmatrix} -6.154 & 5.902 \end{pmatrix}$, $x_3(0) = \begin{pmatrix} 3.594 & -7.611 \end{pmatrix}$, $x_4(0) =$

$\begin{pmatrix} 9.341 & 7.841 \end{pmatrix}$, $x_5(0) = \begin{pmatrix} 2.301 & 12.24 \end{pmatrix}$. The tracking errors between the leader and the followers are shown in Figures 2 and 3.

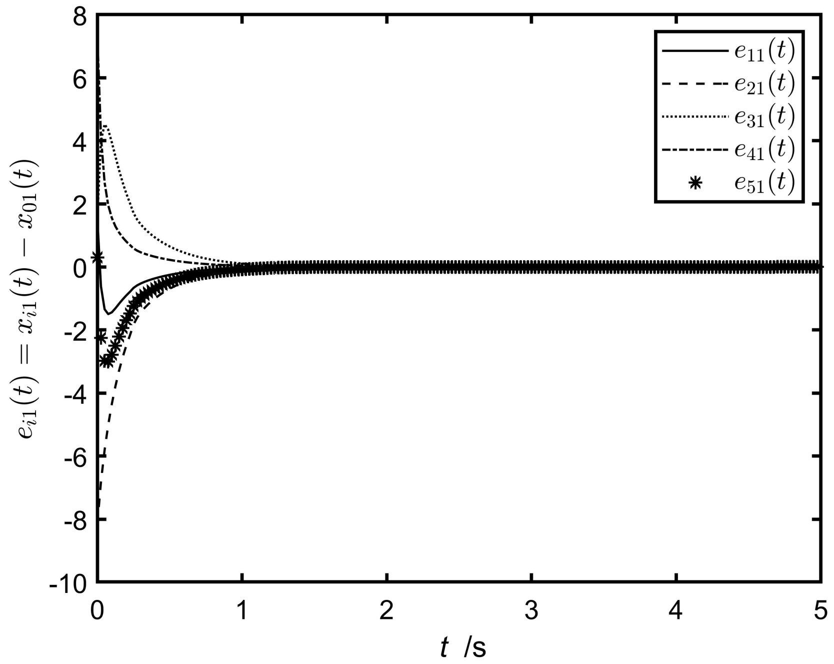

Figure 2. The system state e_1.

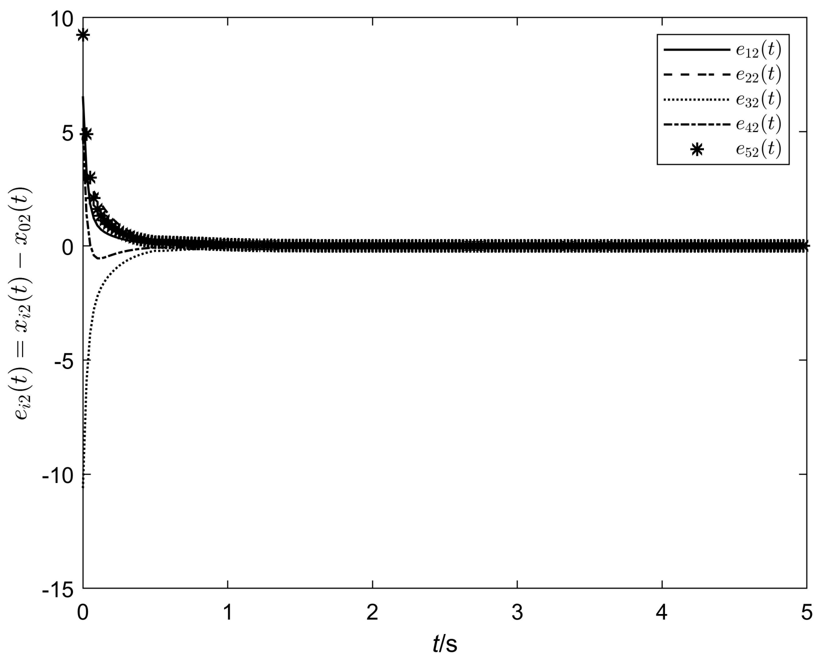

Figure 3. The system state e_2.

Figure 4 illustrates the point in time at which an event is triggered. It indicates that the triggering of the event occurs at a lower frequency than the sampling rate.

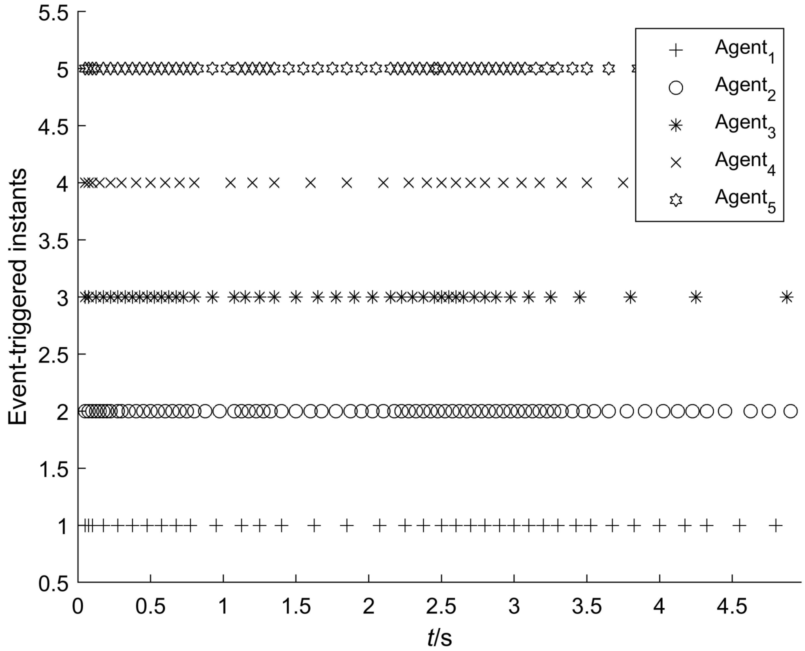

Figure 4. Event-triggered instants for each agent.

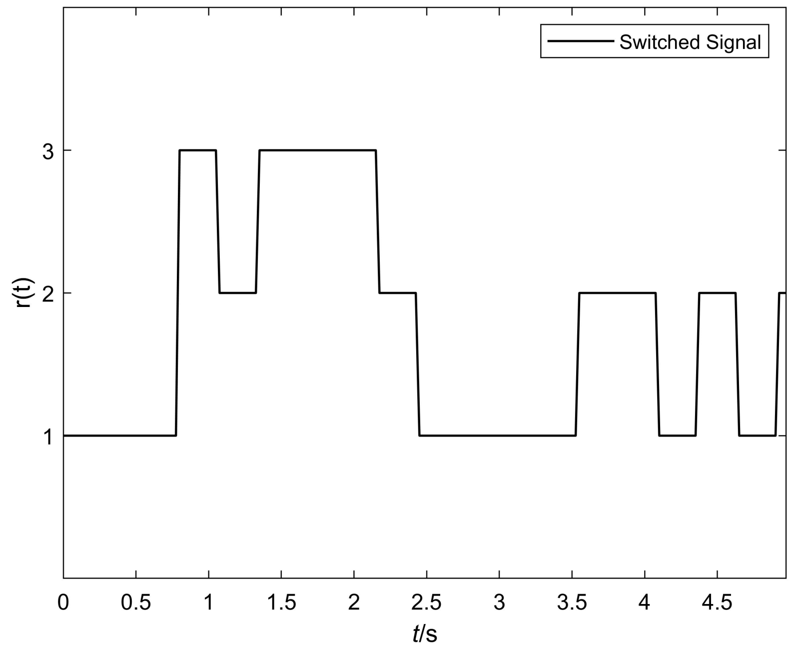

Figure 5. The semi-Markov switching system state trajectory under the event-triggered protocol.

Furthermore, the stochastic sampling period h is shown in Figure 6.

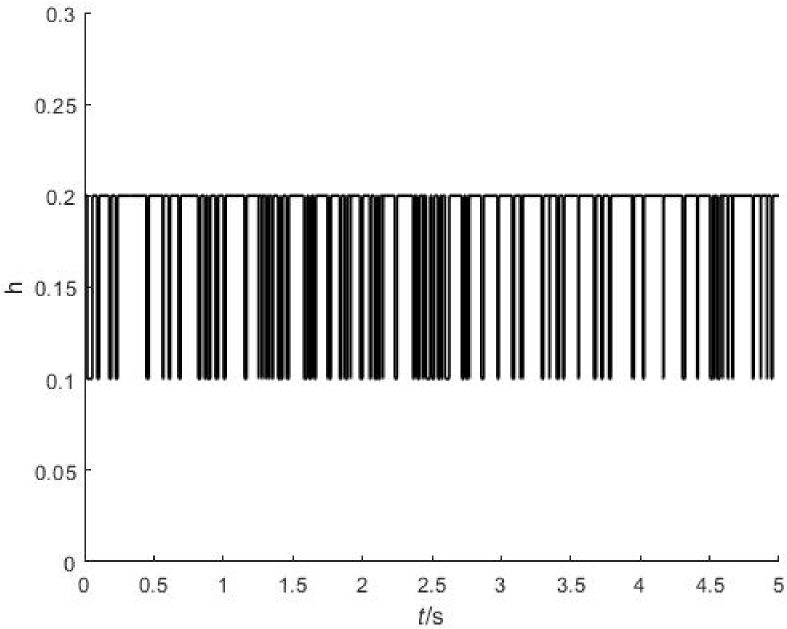

Figure 6. Stochastic sampling period *h*.

We can compare our simulation example with that in [34]. Both our article and [34] share the same state equations. However, [34] adopted a static event-triggered protocol, while we added a process of stochastic sampling to its event-triggered protocol. Compared with the figure in [34], the multi-agent system controlled by stochastic sampling event-triggered control has a faster convergence rate and smaller steady-state error. Thus, this example validates the validity of Theorem 3.

5. Discussion

The paper presented a study on the mean-square consensus of a semi-Markov jump multi-agent system based on event-triggered stochastic sampling. We have proposed a novel approach to improve the efficiency of multi-agent systems for consensus control.

The results of the study showed that the proposed approach was effective in achieving mean-square consensus in multi-agent systems. The use of event-triggering via stochastic sampling reduced the communication frequency and improved the computational efficiency of the system. The semi-Markov jump model provided a more accurate representation of the state transitions in the system.

However, there are some limitations to this study. The numerical examples presented in the paper were relatively small, and it is unclear how the proposed approach would scale to larger multi-agent systems. Additionally, the study assumed perfect knowledge of the system parameters, which may not be the case in real-world scenarios.

Future research can further investigate the robustness of the proposed approach against uncertainties and disturbances in the system. The scalability of the approach can also be explored in more details, and the approach can be tested on more complex multi-agent systems.

6. Conclusions

The study presented in this article focuses on the use of multi-agent systems for the leader-follower consensus control topic. We have proposed a novel event-triggered stochastic sampling approach and investigated the use of a semi-Markov switching system architecture. We have also developed appropriate measures for mean-square consensus in multi-agent systems.

The results of the numerical example presented in this study demonstrate the accuracy of the theoretical computations. The proposed approach has the potential to improve the efficiency of multi-agent systems for consensus control in various applications.

Use of AI tools declaration

The authors declare that they have not used artificial intelligence tools in the creation of this article.

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Conflict of interest

The authors declare that there is no conflict of interest.

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