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Research article

Robust optimization of train scheduling with consideration of response actions to primary and secondary risks

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Abstract: Nowadays, with the rapid development of rail transportation systems, passenger demand and the possibility of the risks occurring in this industry have increased. These conditions cause uncertainty in passenger demand and the development of adverse impacts as a result of risks, which put the assurance of precise planning in jeopardy. To deal with uncertainty and lessen negative impacts, robust optimization of the train scheduling problem in the presence of risks is crucial. A two-stage mixed integer programming model is suggested in this study. In the first stage, the objective of the nominal train scheduling problem is to minimize the total travel time function and optimally determine the decision variables of the train timetables and the number of train stops. A robust optimization model is developed in the second stage with the aim of minimizing unsatisfied demand and reducing passenger dissatisfaction. Additionally, programming is carried out and the set of optimal risk response actions is identified in the proposed approach for the presence of primary and secondary risks in the train scheduling problem. A real-world example is provided to demonstrate the model's effectiveness and to compare the developed models. The results demonstrate that secondary risk plays a significant role in the process of optimal response actions selection. Furthermore, in the face of uncertainty, robust solutions can significantly and effectively minimize unsatisfied demand by a slightly rise in the travel time and the number of stops obtained from the nominal problem.

Keywords: nominal train scheduling; robust optimization; primary risk; secondary risk; response actions

1. Introduction

The railway transportation system is a highly complicated system that may be exposed to catastrophic risks. For instance, the most significant recent train accidents, which resulted in a total of 1142 fatalities, occurred in Great Britain in 1999, Australia in 2003, the United States in 2008 and Spain in 2013 [1]. Despite being relatively infrequent compared to other catastrophes in the transport industry, these catastrophic incidents cause enormous costs, injuries and delays. Traditional methods, however, are no longer able to handle the growing complexity of such contemporary social-technical systems. Therefore, instead of waiting for catastrophic events to occur, a control and risk reduction approach is necessary [2]. Due to its numerous commonalities in safety requirements, risk management may be very effective in the rail transport industry. Delays, incidents and other risks with diverse outcomes are frequent in the rail transport industry almost everywhere in the world. It is crucial to analyze contributory components and identify risk situations in order to avoid accidents. Numerous factors that put trains in danger and create delays can be avoided, or they can do less harm overall. Because the major objectives of the risk management approach are to decrease the chance of fatalities, reduce costs and improve passenger satisfaction, it is vital to pay attention to this approach [3]. To minimize risk to an acceptable level, risk management is an organized and systematic approach to risk identification and estimation. Additionally, it is highly useful in selecting the appropriate response actions to manage and control the identified risk variables and achieve the desired outcomes [4]. The majority of risk response action selection methods concentrate on reducing primary risks. However, it should be emphasized that secondary risks may also be produced through the use of risk response strategies, which might have destructive effects comparable to primary risks. Determining which primary risk response actions should not be performed to avoid secondary risks and what the secondary risk response actions should be in the event of secondary risks are therefore crucial [5].

On the other hand, the scheduling problem, which is an optimization problem in the rail transport industry, enables the selection of the best schedule for the arrival and departure of trains at the stations so that passengers and companies gain maximum productivity at the lowest cost possible. The development of train schedules is regarded as one of the most crucial and challenging tasks; this difficulty is brought on by the large and expanding range of real problems as well as the presence of various operational constraints. In addition to the challenges associated with developing rail transportation systems, customers now expect a higher quality of service. Therefore, the significance of the train scheduling issue has doubled as a result of the rise in passenger demand and the expansion of railway lines [6]. Optimization issues in the real world are frequently uncertain. There is seldom an area that is immune to measurement errors, implementation faults, system disruption flaws, or inadequate data at the moment, all of which can render definitive answers scientifically useless. Numerous strategies have been developed to cope with such uncertainty as a result. Recently, robust optimization has drawn a lot of interest since its major objective is to select solutions that can deal with uncertain data [7].

The literature to date reveals that researchers have not focused on the study of train scheduling robust optimization with consideration of response actions to primary and secondary risks. In order to reduce train travel time and choose the best course of action in the event of both primary and secondary risks, this article will construct a train scheduling model in the presence of risks. The rest of the paper is organized as follows. In Section 2, a review of the literature on risk in the field of the rail transportation industry, train scheduling and robust optimization is reviewed. Section 3 describes the problem. Furthermore, in this section, first, the nominal train scheduling problem in the presence of primary and secondary risks is formulated and then, a Robust optimization model is developed. Section 4 provides a real case study and clarifies the application of the proposed model. Finally, the conclusion is presented in Section 5.

2. Review of the literature

2.1. Risk and rail transportation industry

An increase in the number of fatalities and accidents in the transportation system has recently coincided with an expansion in supply in the rail transport industry, both for freight and passenger transport. Because of the variety of risks in the system, achieving a safety management system necessitates a systematic review of risks and planning to minimize their effects. All of these demands are satisfied by the risk management approach, which is the vanguard and essential component of safety management. Risk management is a dynamic system that addresses several challenges, including case identification, value estimation, planning, evaluating risk mitigation initiatives and risk control principles. Risk is one of the major factors causing time delays and monetary losses. It is recognized as the primary risk in the literature which must be identified for proper analysis [5].

By identifying the primary risk, it is possible to determine its likelihood of occurrence and the severity of it, allowing for the implementation of appropriate response actions and the effective control of the primary risk. The implementation of response actions to the primary risk generates another category of risk known as secondary risk, and to control this type of risk, appropriate response actions must be developed. The impact of the secondary risk should not be outweighed by the primary risks. Otherwise, the action to respond to the primary risk that caused the secondary risk should not be taken, or a different primary risk response action should be chosen.

The rail transport industry consists of several components, each of which can have an impact on other components and individuals. There are risks associated with each of these components, some of which are independent and others dependent. The management of rail transport must be aware of and control the relevant risk factors. It can be argued that this industry needs to implement a risk management approach due to its high sensitivities and irreparable costs because there is a lack of a comprehensive model for identifying risks and estimating the amount of risk in the transport industry, which is one of the most accident-prone industries, particularly in the rail transportation industry, which has so far had the least systemic view of safety [3].

Nowadays, one of the primary goals of international transportation companies is risk management knowledge and its application in the transportation and safety industry. In most advanced countries around the world, risk management is utilized as a dynamic and systematic process to evaluate the degree of safety in different sub-sectors and areas and check compliance with a standard framework to assure safety and reduce risks in the rail transport industry. Bubbico et al. [8] introduced an approach based on geographic information systems for the road and rail transportation of hazardous materials that enables quick risk assessment for multiple materials, trips and travel plans. Saat and Barkan [9] developed an optimization model based on the tank car safety design model that allows for the evaluation of all safety design improvement choices. Furthermore, its output reduces the risks of transporting hazardous materials via rail. Tian and Wang [10] investigated the preventive maintenance schedule of subway train components that optimized the perspective of failure risk using a game model.

2.2. Train scheduling and robust optimization

One of the most significant and affordable options for transporting individuals and goods is the usage of railways. Due to the significant expenses involved in extending the lines, existing infrastructure in the rail transport industry must be utilized as optimally as possible. One of the most crucial things that aid in maximizing the utilization of current infrastructure is optimizing the scheduling problem on railway lines. Scheduling specifies when each train arrives at each station when it departs, and at which stations it stops. The train scheduling problem has therefore captured the interest of scholars due to its great importance in exploiting railway systems. There are two approaches—nominal and robust approaches—are typically used in the literature review to address this particular set of problems. The robust optimization approach seeks to find solutions that are nearly insensitive to uncertainties, whereas the nominal approach helps to optimize objectives like decreasing travel time and maximizing passenger satisfaction. In other words, robust optimization allows for more precise planning. Additionally, numerous additional costs, time deviations and inconsistencies will all be considerably minimized.

Numerous studies in the area of nominal scheduling problems have been conducted recently. Burdett and Kozan [11] addressed a train scheduling problem by utilizing composite buffers to keep line occupancy levels at a high standard when trains are allowed to pass through intersections without making extra routing decisions. To develop a feasible train timetable with the flexibility to reschedule the train, Yalçnkaya and Bayhan [12] developed a simulation-based train scheduling scheme. To boost efficiency and reduce the net energy consumption of the railway line, Li and Lo [13] suggested a dynamic train scheduling and control system based on the Kuhn-Tucker approach. An integrated model that concurrently optimizes train scheduling and circulation plan by demand analysis was reported by Wang et al. [14] and simulation studies demonstrated that the optimized results outperform solutions designed by planners. To satisfy passenger demand, Mo et al. [15] suggested a flexible train scheduling model using a modified Tabu search solution algorithm and their objectives are to reduce energy costs and passenger waiting times. Rokhforoz and Fink [16] put forth train scheduling and predictive maintenance as a planning problem. Dual decomposition and mechanism design were used to build a hierarchical distributed learning algorithm to address this problem, and the effectiveness of the model in analyzing a rail network was confirmed. Considering train timetabling and coupling, Feng et al. [6] introduced an integrated optimization approach to operationally manage daily fluctuating demand.

Furthermore, the robust optimization approach to train scheduling has garnered a lot of interest in recent years. One of the latest methods in mathematical programming, robust optimization aims to select solutions with the capacity to handle uncertain data. This method assumes that the uncertain data are limited and unknown, and that the uncertainty space is often thought of as convex. Additionally, robust optimization differs from conventional programming in that it does not require knowledge of the probability distribution of uncertain data [17]. The operational level deviations in train scheduling lead to an infeasible nominal timetable. However, compared to the basic plan, using robust optimization can reduce the severity of deviations. To boost robustness, Kroon et al. [18] developed a stochastic optimization model that deals with the allocation of buffer time in the train timetable. For high-speed trains, Li et al. [19] developed a robust sampled-data cruise control scheduling that analyzes the stability of time-varying delay systems and also ensures the trains' optimal speed and safety. Jamili and Pourseyed-Aghaee [20] introduced a non-linear robust model to determine the optimal stop pattern in urban rail transit systems to manage uncertain passenger arrival and departure demand. Jovanović et al. [21] allocated the optimal buffer times based on the sensitivity of their delay and the impact of the delay on all other events to improve the robustness of the train schedule. This strategy was developed based on the well-known knapsack problem.

A multi-objective model for Robust Skip-Stop Scheduling with consideration of Earliness and Tardiness Penalties was put forth by Rajabighamchi et al. [22] and it could effectively reduce passengers' overall travel time while stabilizing train timetables. To reduce the number of people waiting in subway systems, Zhou et al. [23] expanded a robust scheduling optimization approach based on two heuristic algorithms. Cacchiani et al. [24] developed robust optimization models that simultaneously minimize the uncertainty in demand and limit the worsening of the objective function values of the nominal problem.

Finally, to clarify the implementation of the study of the robust optimization of train scheduling with consideration of response actions to primary and secondary risks, Figure 1 provides the development roadmap to illustrate the summarized steps of the approach.

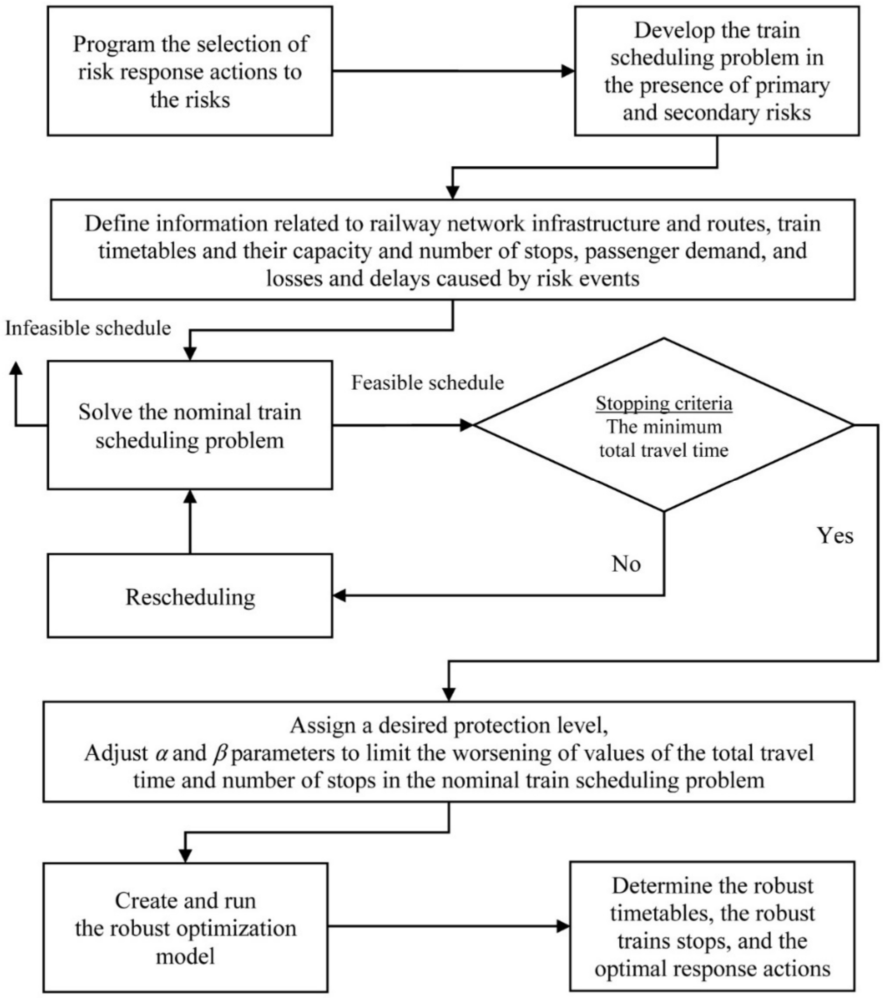

Figure 1. Development roadmap of the robust optimization of train scheduling with consideration of response actions to risks.

3. Problem description and mathematical models

3.1. Problem description

A set of stations *S* along the railway lines and a set of trains *K* moving in the same direction are thought of as a railway network. According to each train $k \in K$, the set of stations is divided into several subset of stations S_k ($S_k \subseteq S$). For each train $k \in K$, we define a subset S_k of stations that the train visits, which include its fixed origin station O_k and its fixed destination station D_k. Passengers with demands Q_{ij} intends to travel between stations i and j $(i, j \in S, i \neq j)$. The number of train stops, as well as the train departure and arrival times at each visited station, must be properly determined to satisfy the demand. Therefore, a decision must be made as to whether or not the train would stop at station *s*. When the train stops in a small subset of stations, the quality of service decreases while passengers' satisfaction with the likelihood of reaching the destination faster increases. To improve service quality and satisfy passengers with travel time, the maximum number of stops N_k that a train can have throughout its journey is determined in advance for each train *k*, and the minimum number of trains that should stop at the station num_i is established in advance for each station *i*. Another way to increase quality is to have the same number of seats as passengers. As a result, the capacity of the train C_k should be considered at the time of satisfying passenger demand.

The train *k* departs from its origin station O_k at the time T_k, and to correct the train departure time, a maximum allowable delay time $\Delta T_k$is determined. After train *k* departs, the travel time t_{ki}^{tr} from station *i* to station *i* + 1 is determined. If train *k* arrives at station *i*, it stays there for time t_{ki}^{dwell}. Only at stops may trains be overtaken. In order to avoid trains from colliding at a station, the minimum departure headway time h_{dep} and the minimum arrival headway time h_{arr} must also be defined.

There are a number of risks *R* in the railway network. Where a subset of risks R_i $(R^i \subseteq R)$ is allocated based on the main risk set for each station *i*. Each primary risk *r* $(r \in R^i)$ that occurs at station *i* imposes expected time delays D_{ir}^{time} and expected monetary lossL_{ir}^{co} to the respective station, the values of which are determined by experts based on the significance of station *i* and the effect of risk *r*. Risk response actions that are sustainable should be implemented to mitigate the negative effects of risks. As a result, a set of risk *A* response measures is identified. Based on the set of original risk response actions, a subset of primary risk response actions A_i $(A^i \subseteq A)$ is allocated to each station *i*. The implementation of response action *a* to reduce the effect of the primary risk *r* at station *i* incurs a cost c_{ira}^{act}. The estimated number of days and the estimated cost that are reduced after acting and are represented by e_{ira}^{time} and e_{ira}^{co}, respectively.

By selecting primary risk response actions, a subset of potential secondary risks stemming from taking the actions is created and identified. It is assumed that the station i would experience the expected time delays D_{ira}^{stime} and monetary losses L as a result of the secondary risk r caused by the implementation of action *a* at station. In addition, actions A_i $(A^i \subseteq A)$ have been developed in advance to respond to secondary risks. Analogous to the primary action, estimated number of days e_{ira}^{stime} and estimated cost e_{ira}^{sco} can be diminished by implementing the secondary action at the cost of c_{ira}^{sact}. In addition, for each station i established values of the maximum permitted delays t_i^* and cost B_i have been taken into consideration in order to manage the time delays and the risk-related costs as efficiently as possible.

According to this statement of the problem, the nominal problem's objective is to satisfy passenger demand while reducing the overall travel time of the T_T train when both primary and secondary risks are present. This is considered to one of the most crucial factors for passengers and one of the most important objectives in the matter of train schedules. To satisfy passenger demand, timetables and the number of train stops, as well as the choice of risk response strategies, should be made in a way that ensures the minimum travel time. However, if passenger demand exceeds the level of service offered, it may result in poor service or an infeasible solution to the nominal problem. Therefore, in addition to obtaining the minimum travel time, handling the additional unexpected passenger demand should be taken into consideration when selecting the timetables and the number of train stops.

The desired protection level may be considered for the number of extra passengers who want to travel from station i to station j by expanding the nominal problem into a robust problem. The slack variable is specified as an integer that reflects the number of people that cannot travel between station i and station j in order to keep the problem in the feasible condition when the protection level is not reached. Therefore, the objective of the robust problem is to minimize the number of these passengers. Adding more passengers will result in longer travel times and more stops. As a result, we have a robust solution with low efficiency. To resolve this issue, the nominal problem objective needs to be put in the robust problem to provide a robust and efficient solution. To this end, constraints on the total travel time of the train in the nominal problem and the total number of stops in the nominal problem are applied and added to the robust optimization model to avoid their values worsening.

3.2. Mathematical models

First, in Subsection 3.2.1, the nominal train scheduling problem is formulated in the presence of primary and secondary risks, and then in Subsection 3.2.2, the robust optimization model is developed.

3.2.1. Nominal train scheduling problem

The model of Qi et al. [25] is used in this study to develop and formulate the nominal train scheduling problem and to propose the problem of train schedule in the presence of primary and secondary risks. The decision variables in this problem fall into five categories: the times of departure from and arrival at the station, the order of the trains to prevent accidents, the number of stations at which the train stops, the number of passengers in each train and the number of actions taken in response to primary and secondary risks. To describe the nominal scheduling problem, it is assumed that the variables are non-negative integers t_{ki}^{dep} $(k \in K, i \in S_k \backslash \{D_k\})$ and t_{ki}^{arr} $(k \in K, i \in S_k \backslash \{O_k\})$, which represent the departure time of train k from station i and the arrival time of train k at station i respectively. The number of passengers who want to travel by train k from station i to station j is represented by a non-negative integer variable q_{ij}^{k}. The binary variable y_{kli} reflects the order of trains between consecutive stations and also has a value of 1 if train k departs from station i earlier than train l or reaches station $i + 1$ earlier than train l, otherwise, it has a value of 0 $(k, l \in K, k < l, i \in S_k \backslash \{D_k\} \cap S_l \backslash \{D_l\})$. The binary variable x_{ki} also indicates the number of train stops, with a value of 1 if train k stops at station i and a value of 0 otherwise. Finally, z_{ira} and z'_{ira} are binary variables that denote the number of primary and secondary risk response actions, respectively. If the primary risk response action a is chosen to reduce risk r at station i the value of z_{ira} is 1, otherwise, it is 0. Also, if the secondary risk response action a is chosen to reduce risk r at station i the value of z'_{ira} is 1, otherwise, it is 0. The following is the mathematical model for the nominal train scheduling problem:

$$MInT_T = \sum_{k\in K}(t_{kD_k}^{arr} - t_{kO_k}^{dep}) \tag{1}$$

Subject to:

$$x_{kO_k} = x_{kD_k} = 1, k \in K \tag{2}$$

$$\sum_{k\in K:i,j\in S_k} q_{ij}^k = Q_{ij}, i,j \in S, i < j \tag{3}$$

$$\sum_{j\in S_k, i<j} q_{ij}^k \leq \sum_{j\in S_k, i<j} Q_{ij}\, x_{ki}\ ;\ k \in K, i \in S_k \backslash \{D_k\} \tag{4}$$

$$\sum_{j\in S_k, i>j} q_{ji}^k \leq \sum_{j\in S_k, i>j} Q_{ji}\, x_{ki}\ ;\ k \in K, i \in S_k \backslash \{D_k\} \tag{5}$$

$$\sum_{i'\in S_k, i'\leq i} \sum_{j\in S_k, i<j} q_{i'j}^k \leq C_k, k \in K, i \in S_k \backslash \{D_k\} \tag{6}$$

$$\sum_{i\in S_k} x_{ki} \leq N_k, k \in K \tag{7}$$

$$\sum_{k\in k:i\in S_k} x_{ki} \geq num_i, i \in S \tag{8}$$

$$\begin{gathered}\sum_{r\in R^i} L_{ir}^{co} - \sum_{r\in R^i}\sum_{a\in A^i} e_{ira}^{co} z_{ira} + \sum_{r\in R^i}\sum_{a\in A^i} c_{ira}^{act} z_{ira} + \sum_{r\in R^i}\sum_{a\in A^i} L_{ira}^{sco} z_{ira} \\ - \sum_{r\in R^i}\sum_{a\in A^i} e_{ira}^{sco} z_{ira}^{'} + \sum_{r\in R^i}\sum_{a\in A^i} c_{ira}^{sact} z_{ira}^{'} \leq B_i, i \in S\end{gathered} \tag{9}$$

$$\begin{gathered}\sum_{l\in R^i} L_{ir}^{co} - \sum_{r\in R^i}\sum_{a\in A^i} e_{ira}^{co} z_{ira} + \sum_{r\in R^i}\sum_{a\in A^i} c_{ira}^{act} z_{ira} \geq \sum_{r\in R^i}\sum_{a\in A^i} L_{ira}^{sco} z_{ira} \\ \sum_{r\in R^i}\sum_{a\in A^i} e_{ira}^{sco} z_{ira}^{'} + \sum_{r\in R^i}\sum_{a\in A^i} c_{ira}^{sact} z_{ira}^{'}, i \in S\end{gathered} \tag{10}$$

$$\begin{gathered}\sum_{r\in R^i} D_{ir}^{time} - \sum_{r\in R^i}\sum_{a\in A^i} e_{ira}^{time} z_{ira} \geq \\ \sum_{r\in R^i}\sum_{a\in A^i} D_{ira}^{stime} z_{ira} - \sum_{r\in R^i}\sum_{a\in A^i} e_{ira}^{stime} z_{ira}^{'}, i \in S\end{gathered} \tag{11}$$

$$\sum_{r\in R^i} D_{ir}^{time} - \sum_{r\in R^i}\sum_{a\in A^i} e_{ira}^{time} z_{ira} + \sum_{r\in R^i}\sum_{a\in A^i} D_{ira}^{stime} z_{ira} - \sum_{r\in R^i}\sum_{a\in A^i} e_{ira}^{stime} z_{ira}^{'} = t_i^{risk}, i \in S \tag{12}$$

$$0 \le t_i^{risk} \le t_i^{*}, i \in S \tag{13}$$

$$z_{ira} \ge z_{ira}^{'}, \forall i \in S, \forall r \in R^i, \forall a \in A^i \tag{14}$$

$$t_{ki+1}^{arr} - t_{ki}^{dep} = t_{ki}^{tr} + t_i^{risk}, k \in K, i \in S_k \backslash \{D_k\} \tag{15}$$

$$T_k \le t_{kO_k}^{dep} \le T_k + \Delta T_k, k \in K \tag{16}$$

$$t_{ki}^{dep} - t_{ki}^{arr} \ge t_{ki}^{dwell} x_{ki}, k \in K, i \in S_k \backslash \{O_k, D_k\} \tag{17}$$

$$t_{ki}^{dep} + h_{dep} \le t_{li}^{dep} + M(1 - y_{kli}), i \in (S_k \backslash \{D_k\}) \cap (S_l \backslash \{D_l\}), k, l \in K, k < l \tag{18}$$

$$t_{li}^{dep} + h_{dep} \le t_{ki}^{dep} + M y_{kli}, i \in (S_k \backslash \{D_k\}) \cap (S_l \backslash \{D_l\}), k, l \in K, k < l \tag{19}$$

$$t_{ki+1}^{arr} + h_{arr} \le t_{li+1}^{arr} + M(1 - y_{kli}), i \in (S_k \backslash \{D_k\}) \cap (S_l \backslash \{D_l\}), k, l \in K, k < l \tag{20}$$

$$t_{li+1}^{arr} + h_{arr} \le t_{ki+1}^{arr} + M y_{kli}, i \in (S_k \backslash \{D_k\}) \cap (S_l \backslash \{D_l\}), k, l \in K, k < l \tag{21}$$

$$t_{ki}^{dep} \ge 0,\text{integer, k} \in \text{K, i} \in S_k \backslash \{D_k\} \tag{22}$$

$$t_{ki}^{arr} \ge 0,\text{integer, k} \in \text{K, i} \in S_k \backslash \{O_k\} \tag{23}$$

$$q_{ij}^{k} \ge 0,\text{integer, k} \in \text{K, i,j} \in S_k, i < j \tag{24}$$

$$y_{kli} \in \{0,1\}, i \in (S_k \backslash \{D_k\}) \cap (S_l \backslash \{D_l\}), k, l \in K, k < l \tag{25}$$

$$x_{ki} \in \{0,1\}, k \in K, i \in S_k \tag{26}$$

$$z_{ira}, z_{ira}^{'} \in \{0,1\}, \forall i \in S, \forall r \in R^i, \forall a \in A^i. \tag{27}$$

The objective function (1) in the nominal train scheduling problem seeks to minimize the total travel time of trains, which is denoted as T_T. Constraint (2) guarantees that each train departs at its origin station and stops at its destination station following its operating area. Constraint (3) is used to assure the movement of full passenger demand between stations i and j. Constraint (4) specifies that if train k does not stop at station i no passengers will be transferred; if train k does stop at station i it will travel to station j at most based-on passenger demand. The requirements for the arrival of train k and the transfer of passengers to station i are considered in constraint (5), with definitions similar to those in constraint (4). Constraint (6) was used to meet the capacity of each train k at station i between $i'(i' \leq i)$ and j $(i < j)$, and it should be maximum C_k.. This constraint takes into consideration passengers who boarded at or before station i and exited after station i. Constraint (7) limits the maximum number of stops throughout the trip to N_k for train k, whereas constraint (8) limits the minimum number of train stops at station i to num_i.

Constraint (9) evaluates the costs of identifying primary and secondary risks, as well as the costs of conducting response actions, to a maximum of B_i to successfully implement the risk assessment process at station i. Constraint (10) requires that the secondary risk costs at station i be less than the primary risk costs. Furthermore, constraint (11) indicates that secondary risk time delays should be less than primary risk time delays. If constraints (10) and (11) are rejected, no actions should be taken to address the primary risk to avoid the emergence of secondary risks. Constraint (12) shows the remaining time delays after implementing the risk response actions at station i in which the value t_i^{risk} is added to the train's travel time in the problem. Constraint (13) provides a desired predetermined time delay value for each station i to control the time delays calculated in constraint (12). Constraint (14) states that secondary risk response actions are meaningful when primary risk response actions are taken because secondary risks come from primary risk response actions.

Constraints (15)–(17) are concerned with train time information. Constraint (15) represents the train k's travel time from station i to station $i + 1$ by the sum of t_{ki}^{tr} and t_i^{risk}. Constraint (16) shows the departure time from the origin station can be moved by at most ΔT_k. Constraint (17) states that if train k stops at station i, its dwell time is at least equal to t_{ki}^{dwell}. Constraints (18)–(21) are in place to avoid train accidents. It considers the departure time of trains k and l from station i with the minimum h_{dep} time relative to each other by activating just one of the limitations (18) and (19). Similarly, by activating only one of the constraints (20) and (21), it takes into consideration the arrival times of trains k and l to station $i + 1$ with the minimum time h_{arr} relative to each other. Constraints (22) to (27) define the range of decision variables at the end of the nominal train scheduling problem.

3.2.2. Robust optimization model

The programming model introduced in Section 4.1 is developed into a robust optimization model in this subsection. The Light Robustness technique introduced by Fischetti and Monaci [26] is used for this purpose, which is based on inserting the desired protection level against uncertainty and using slack variables when the protection level cannot be guaranteed.

The goal of this technique is to achieve maximum robustness against uncertainty by minimizing the sum of slack variables, while limiting, by an additional constraint, the worsening of the objective function value of the nominal problem. In this paper, a desired protection level Δ_{ij} for the number of additional passengers wanting to travel from station i to station j is considered. When the desired protection level is not fully satisfied, a slack variable of integers is defined as γ_{ij} which represents the number of passengers who are unable to travel between stations i and j, or the unsatisfied demand. As a result, the robust optimization model's objective is to minimize the number of passengers whose demand is not satisfied. In addition, to offer efficient and robust solutions, two constraints on the worsening of the value of the total train travel time in the nominal problem T_T^* and the value of the total number of stops are applied in the nominal problem N_S^* are applied. It should be noted that α and β are percentages chosen to control the worsening of the above values. The proposed robust optimization model is shown as follows:

$$Min \sum_{i,j\in S,i<j} \gamma_{ij} \tag{28}$$

Subject to:

The constraints represented in Eqs (2) and (4)–(27) are valid here again.

$$\sum_{k\in K:i,j\in S_k} q_{ij}^k \geq Q_{ij}, i,j \in S, i<j \tag{29}$$

$$\sum_{k\in K:i,j\in S_k} q_{ij}^k + \gamma_{ij} = Q_{ij} + \Delta_{ij}, i,j \in S, i<j \tag{30}$$

$$\sum_{k\in K}(t_{kD_k}^{arr} - t_{kO_k}^{dep}) \leq (1+\alpha)T_T^* \tag{31}$$

$$\sum_{k\in K}\sum_{i\in S_k} x_{ki} \leq (1+\beta)N_S^* \tag{32}$$

$$\gamma_{ij} \geq 0,\text{integer, k} \in \text{K, i,j} \in S, i<j. \tag{33}$$

The objective function (28) seeks to minimize the number of passengers who cannot travel between stations i and j, that is, the sum of the γ_{ij} variables that are activated by constraints (30). Constraints (2), (4) and (27) are transferred from the nominal train scheduling problem to this model. Constraint (29) replaces constraint (3), which considers the potential of additional passenger movement in addition to satisfying the nominal passenger demand. Constraint (30) establishes a desired protection level Δ_{ij} for the number of additional passengers who want to go from station i to station j in addition to Q_{ij}. in this constraint, when the protection level is not satisfied, decision variables γ_{ij} are employed. Constraints (31) and (32) are applied to ensure the efficiency of the robust solutions. Constraint (31) is established to control the maximum worsening of the total train travel time in the nominal problem and constraint (32) is to control the maximum worsening of the total number of stops in the nominal problem. Finally, constraint (32) defines the domain of the slack variable.

4. Case study for a light rail transit line

In this section, the light rail transit (LRT) line of Kermanshah, located in the west of Iran, is considered as a practical example from the real world to demonstrate the effectiveness and efficiency of the proposed models. This line includes 13 stations, 12 routes, A and B operating areas and 6 LRT trains. Operating region A contains the stations from Taqebstan to Ferdowsi, where LRT1 to LRT4 trains run at a speed of 80 km/h, while operating region B includes the stations from Nowbahar to Ferdowsi, where LRT5 and LRT6 trains run at a speed of 100 km/h. Figure 2 shows the light rail transit line of Kermanshah.

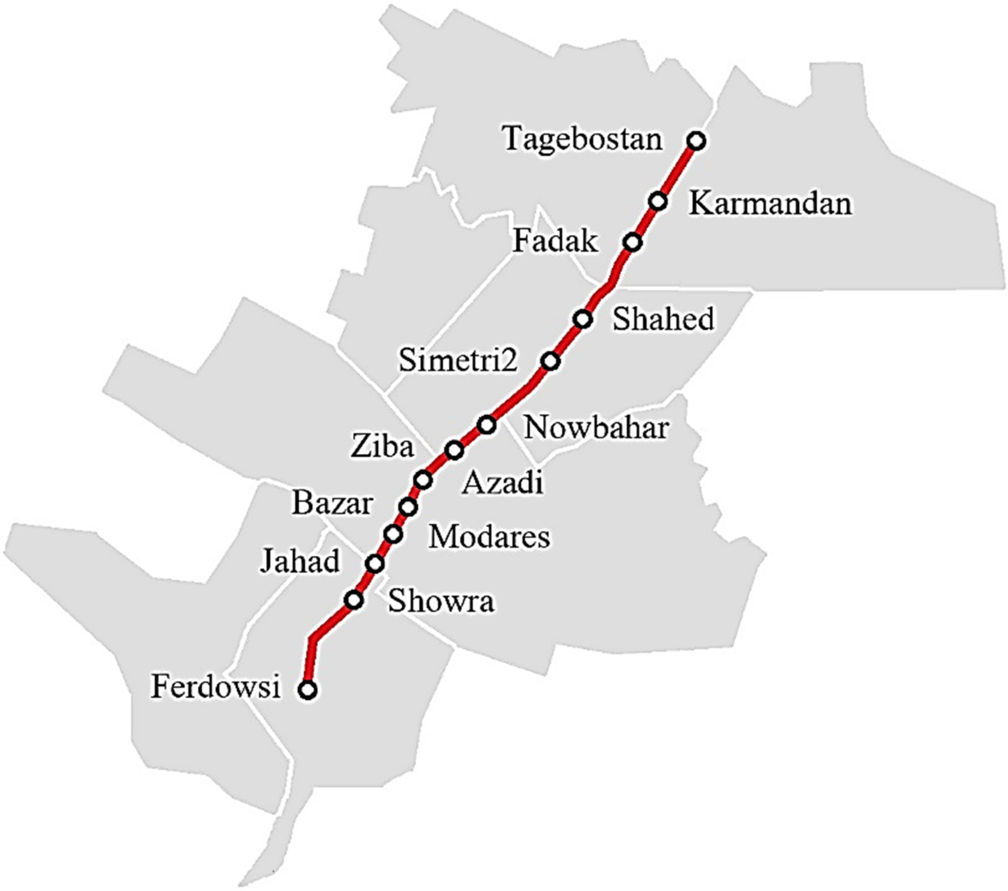

Figure 2. Layout of the Kermanshah light rail transit line.

Table 1. Travel time for each LRT on each route.

Route	LRT1, LRT2, LRT3 and LRT4	LRT5 and LRT6
Taqebostan–Karmandan	9	-
Karmandan–Fadak	6	-
Fadak–Shahed	10	-
Shahed–Simetri2	7	-
Simetri2–Nowbahar	5	-
Nowbahar–Ziba	7	5
Ziba–Azadi	8	6
Azadi–Bazar	8	6
Bazar–Modares	10	7
Modares–Jahad	8	6
Jahad–Showra	7	5
Showra–Ferdowsi	7	5

Some basic input data should initially be defined to apply the case study. The expected departure times for LRT1, LRT2, LRT3, LRT4, LRT5 and LRT6 from their respective origin stations are 10, 20, 30, 40, 25 and 35 minutes, respectively, and the 10-minute maximum delay from the scheduled departure time for each train is taken into consideration. Table 1 shows the travel time of each train along each route of the light rail transit line along the direction of the destination.

Table 2. Passenger demand for each pair of selected origin and destination stations.

Origin/Destination	Taqebostam	Karmandan	Fadak	Shahed	Simetri2	Nowbahar	Ziba	Azadi	Bazar	Modares	Jahad	Showra	Ferdowsi
Taqebostam	-	105	138	120	57	61	70	18	84	34	117	73	381
Karmandan	-	-	109	91	129	63	8	52	80	12	62	28	137
Fadak	-	-	-	145	146	176	56	60	11	55	111	107	38
Shahed	-	-	-	-	72	44	29	54	19	17	75	78	141
Simetri2	-	-	-	-	-	117	16	60	45	88	46	160	238
Nowbahar	-	-	-	-	-	-	324	161	300	300	30	146	345
Ziba	-	-	-	-	-	-	-	141	121	176	122	98	232
Azadi	-	-	-	-	-	-	-	-	88	11	115	247	214
Bazar	-	-	-	-	-	-	-	-	-	343	64	279	233
Modares	-	-	-	-	-	-	-	-	-	-	76	300	243
Jahad	-	-	-	-	-	-	-	-	-	-	-	172	294
Showra	-	-	-	-	-	-	-	-	-	-	-	-	120
Ferdowsi	-	-	-	-	-	-	-	-	-	-	-	-	-

Trains with a capacity of 850 people have been selected to successfully transfer passengers from the origin to the destination, and the amount of passenger demand between each of the stations along the exit direction is indicated in Table 2. To improve service quality and satisfy passengers, each train in operating region A can make a maximum of 10 stops and each train in operating region B can make a maximum of 5 stops, and at least 1 train is allowed to stop at each station. In addition, the minimum departure headway time and the minimum arrival headway time are both set to 3 to prevent trains from clashing at the same stop. Loading and unloading passengers, changing crews and other activities at scheduled stops need a minimum dwelling time of 4 minutes for each train.

The experts were then invited to use the checklist to determine the primary risks at each station and prepare effective response actions for each of them. Secondary risks that may be formed as a result of the selection of primary risk response actions were also discovered, and all of this information is provided in Table 3.

Table 3. List of primary risks, primary response actions and secondary risks.

Primary risks (PR)	Primary response actions (PA)	Secondary risks (SR)
Damage to antiquities (PR_1)	Use of alternative routes around antiquities sites (PA_1)	-
Disturbing residents (PR_2)	Minimizing the effects of crowd, pollution and noise (PA_2)	-
The illogicality of the construction organizational plan (PR_3)	Employing experts to redesign the scheme (PA_3)	-
Toxic gas leak (PR_4)	Dispatch of specialist forces to seal the leak (PA_4)	Fall (SR_1)
Pipeline explosion (PR_5)	Dispatch of specialist forces to seal the leak (PA_4)	Fall (SR_1)
High level of underground water (PR_6)	Place a series of wells or use a pump (PA_5)	land subsidence (SR_2)
Fall (PR_7)	Removing damaged parts with the oscillatory drilling and cutting machines (PA_6)	Loss of land (SR_3)
Instability of the supporting structure (PR_8)	Using advanced strutting system (PA_7), Triple grouting technology (PA_8)	- Significant problems in the strength of foundation reinforcement (SR_4)
Changing the shape of the retaining wall (PR_9)	Demolition of deformed wall and reconstruction with better quality cement (PA_9)	landslide (SR_5)
Failure to strengthen the foundation (PR_{10})	Triple grouting technology (PA_8)	Significant problems in the strength of foundation reinforcement (SR_4)
Mechanical failure (PR_{11})	Mechanical maintenance (PA_{10})	-

After identifying the primary risks, the probability and impact of each risk in terms of time delays and monetary losses were evaluated, and the expected time delays and monetary losses imposed at each station were determined by multiplying these two factors together. Furthermore, to mitigate the negative consequences of risks, the appropriate actions to respond to the primary risk were determined, which include the cost of application and result in a specified level of reduction in expected time delays and monetary losses in each station. Table 4 displays the data for these parameters.

Because secondary risks might occur during the implementation of some primary risk response actions, secondary risk information, like primary risk information, should be provided; these data are given in Table 5. Furthermore, to optimally manage time delays and risk-related costs, the predetermined values of t_i^* equaling 10 minutes and B_i equaling 65 billion Rials are considered for each station.

Table 4. Primary risks data and primary response actions data.

Station	Possible Primary Risks	L_{ir}^{co} (Billion Rials)	D_{ir}^{time} (Minute)	c_{ira}^{act} (Billion Rials)	e_{ira}^{co} (Billion Rials)	e_{ira}^{time} (Minute)
Taqebostan	PR_1	14.63	35	2.4	14.01	31
Karmandan	PR_2, PR_3	2.45	14	0.09	2.05	12
Fadak	-	-	-	-	-	-
Shahed	PR_4, PR_5	35.66	40	2.7	32.16	36
Simetri2	PR_6	0.13	4	0.05	0.10	2
Nowbahar	PR_7, PR_8	11.82	26	2.8	9.48	22
Ziba	PR_7, PR_8	11.82	26	2.19	9.98	22
Azadi	PR_6, PR_8	9.92	25	2.02	8.64	23
Bazar	PR_9	0.05	5	0.02	0.03	3
Modares	PR_9, PR_{10}, PR_{11}	15.11	45	2.7	14.32	41
Jahad	PR_9, PR_{10}	11.05	20	1.75	10.28	18
Showra	PR_6, PR_9	11.8	22	3	10.28	18
Ferdowsi	-	-	-	-	-	-

Table 5. Secondary risks data and secondary response actions data.

Station	Possible Secondary Risks (SR)	L_{ira}^{sco} (Billion Rials)	D_{ira}^{stime} (Minute)	c_{ira}^{sact} (Billion Rials)	e_{ira}^{sco} (Billion Rials)	e_{ira}^{stime} (Minute)
Taqebostan	-	-	-	-	-	-
Karmandan	-	-	-	-	-	-
Fadak	-	-	-	-	-	-
Shahed	SR_1	13.64	27	2.8	13	26
Simetri2	SR_2	4.5	10	2.4	3.9	7
Nowbahar	SR_3, SR_4	8.5	13	3.6	8.3	12
Ziba	SR_3, SR_4	8.4	13	3.5	8.3	11
Azadi	SR_2, SR_4	4.3	17	2.08	4	17
Bazar	SR_5	3.8	13	1.6	3.5	10
Modares	SR_2, SR_3, SR_5	9.15	16	3.22	9.02	14
Jahad	SR_5	7.22	14	2.38	7.09	13
Showra	SR_2, SR_5	7.81	14	4.11	7.62	14
Ferdowsi	-	-	-	-	-	-

After collecting the initial data, the nominal train scheduling problem was developed in GAMS optimization software and solved by the CPLEX solver to determine train timetables, train stops and optimal response actions on the risks Kermanshah light rail transit line, with the goal of minimizing total train travel time. Two final conditions were proposed for this purpose: stopping the solver when the optimality gap is smaller than 5% or when the computing time surpasses two hours. Furthermore, OPTCR was set to 0.05 as an index to evaluate the final solution's quality to produce an optimal solution with high quality.

The value of the objective function was found by solving the nominal problem to be 806 minutes, which corresponds to the minimum total travel time of trains. In this case, trains stop at 40 stations to transport 9528 people. Then, a robust optimization model with the same characteristics and conditions as the nominal problem was developed with GAMS software to deal with the uncertainty in passenger demand. For this purpose, the desired protection level Δ_{ij} for the additional passengers was calculated to be 5% of the value of Q_{ij}, so that the nominal problem's passenger demand increases by 5%. Furthermore, both α and β parameters were adjusted to 5% to create a robust and efficient solution. The results indicated that the overall travel time of the trains increased to 846 minutes in the robust optimization model, during which 9887 people were transferred by stopping the trains at 42 stops.

In terms of robustness against uncertain passenger demand, comparisons of the robust optimization model with the nominal problem of train scheduling reveal that the robust solution performs significantly better than the nominal problem in coping with additional passenger demand. In the nominal problem, there are 359 unsatisfied demands, but in the robust model, there are only 82. This indicates that the robust solution reduces unsatisfied passenger demands by about 4.5 times. Table 6 ashows the timetable and train stop plan pattern based on the robust optimization model. The train does not stop at stations when the train arrival time and train departure time are the same, as illustrated by the train stop plan pattern. The filled black dots in the train stop plan pattern show where the trains stop (Figure 3).

Table 6. Robust timetable of the Kermanshah light rail transit line.

Station	LRT1		LRT2		LRT3		LRT4		LRT5		LRT6	
	t^{arr}	t^{dep}	t^{arr}	t^{dep}	t^{arr}	t^{dep}	t^{arr}	t^{dep}	t^{arr}	t^{dep}	t^{arr}	t^{dep}
Taqebostan	-	13	-	20	-	30	-	40	-	-	-	-
Karmandan	26	30	33	33	43	47	53	53	-	-	-	-
Fadak	38	44	41	41	55	55	61	65	-	-	-	-
Shahed	54	58	51	51	65	69	75	79	-	-	-	-
Simetri2	70	74	63	67	81	81	91	95	-	-	-	-
Nowbahar	83	83	76	76	90	94	104	108	-	25	-	45
Ziba	95	95	88	88	106	110	118	122	35	39	55	83
Azadi	109	113	102	102	124	128	136	136	51	51	95	99
Bazar	123	123	112	112	138	142	146	150	59	59	107	109
Modares	138	142	127	127	157	157	165	169	71	75	121	123
Jahad	156	160	141	141	171	175	183	183	87	91	135	135
Showra	170	174	151	155	185	185	193	197	99	99	143	147
Ferdowsi	185	-	166	-	196	-	208	-	108	-	156	-

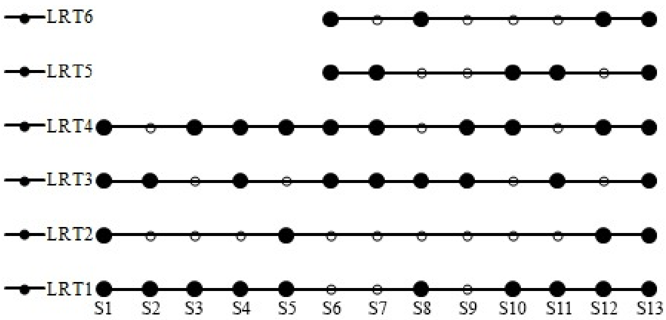

Figure 3. Train stop design template of the Kermanshah light rail transit line.

Apart from the uncertainty issue, the selection of a set of appropriate risk response actions to cope with possible primary and secondary risks in the light rail transit line is an essential issue in the successful implementation of the train scheduling problem. Equations (9) to (14) were defined for this reason. In this way, the set of optimal risk response actions was selected while keeping in mind the constraints of costs and time delays caused by primary and secondary risks. Based on this and the robust optimization model outputs, Table 7 reports the optimal costs and time delays produced by the risks in each of the stations.

According to Table 7, it is evident that the implementation of risk response actions does not cause any additional risks in Taqebostan and Karmandan stations. Moreover, they perform optimally within the cost and delay limits that were previously specified. However, in other stations, the implementation of risk response actions causes secondary risks, and secondary risk response actions should also be implemented. At each station, the cost and time delays caused by the primary risk should be compared to the cost and time delays induced by the secondary risk at the same station. For example, the cost and time delay caused by primary risks at Simetri 2 station are 0.08 billion Rials and 2 minutes, respectively, whereas similar figures for secondary risks are 3 billion Rials and 3 minutes, respectively.

Table 7. Optimal cost and delay due to primary and secondary risks.

Station	Primary risk		Secondary risk	
	Risk cost (Billion Rials)	Risk delay (Minute)	Risk cost (Billion Rials)	Risk delay (Minute)
Taqebostan	3.02	4	-	-
Karmandan	0.49	2	-	-
Fadak	-	-	-	-
Shahed	6.20	4	3.44	1
Simetri2	0.08	2	3.00	3
Nowbahar	5.14	4	3.80	1
Ziba	4.03	4	3.60	2
Azadi	3.30	2	2.38	0
Bazar	0.04	2	1.90	3
Modares	3.49	4	3.35	2
Jahad	2.52	2	2.51	1
Showra	4.52	4	4.30	0
Ferdowsi	-	-	-	-

Table 8. Optimal set of response actions of the Kermanshah light rail transit line.

Station	Primary risk		Secondary risk	
	Mitigated risks	Selected response actions	Mitigated risk	Response actions
Taqebostan	PR_1	PA_1	-	-
Karmandan	PR_2, PR_3	PA_2, PA_3	-	-
Shahed	PR_4, PR_5	PA_4	SR_1	*Yes*
Nowbahar	PR_7, PR_8	PA_6, PA_7, PA_8	SR_3, PSR_4	*Yes*
Ziba	PR_7, PR_8	PA_6, PA_7, PA_8	SR_3, SR_4	*Yes*
Azadi	PR_6, PR_8	PA_5, PA_7, PA_8	SR_2, SR_4	*Yes*
Modares	PR_9, PR_{10}, PR_{11}	PA_8, PA_9, PA_{10}	SR_2, SR_3, SR_5	*Yes*
Jahad	PR_9, PR_{10}	PA_9, PA_{10}	SR_2	*Yes*
Showra	PR_6, PR_9	PA_5, PA_9	SR_2, SR_5	*Yes*

As a result, because the possible negative consequences of secondary risks are greater than those of primary risks, primary risk response actions should not be taken to prevent such imposed negative effects. Similarly, the possible greater negative effects of secondary risks highlight the Bazar station's lack of response actions to the primary risk. In addition, the impacts of secondary risks are crucial in determining the optimal response actions to the primary risk. Table 8 presents the list of reduced risks and appropriate risk-response actions based on the impact of secondary risks.

Sensitivity analysis is performed to investigate the impact of changing some parameters on the optimal solutions of the proposed robust optimization model. First, the effect of changing parameters α and β is examined, which were considered to control the increase in overall train travel time and the total number of train stops, respectively. Table 9 displays the results obtained when two parameters were changed from 1% to 25%.

Table 9. Sensitivity analysis on a maximum worsening controlled by parameters α and β.

	Value	$\sum_{i,j\in S,i<j}\gamma_{ij}$	Total travel time	Number of stops	Gap %	CPU time
α	01%	82	814.06	42	0	00:05.058
	05%	82	846.30	42	0	00:08.701
	25%	82	914.00	42	0	00:02.276
β	01%	82	846.30	40	0	10:06.143
	05%	82	846.30	42	0	00:08.929
	25%	82	846.30	50	0	00:01.510

It can be observed that the robust objective function has the same value for each variant. As a result, increasing the travel time and the number of stops will not increase the robustness. The reason for this is that the trains are nearly full to satisfy the nominal passenger demand. It is worth noting that by increasing the parameters by 1%, the same level of robustness may be obtained, i.e., the value of the robust objective function equals 82. However, reducing the effect of the parameters, particularly the effect of the parameter β increases the computational time required to find a robust solution. As a result, imposing constraints (31) and (32) is critical to ensuring the robust solution's efficiency. The desired protection level Δ_{ij} is another parameter that is analyzed in the range of 1% to 25%, with the results shown in Table 10.

Table 10. Sensitivity analysis on a desired protection level by parameter Δ_{ij}.

Value	Unsatisfied demand		Total travel time	Number of stops	Gap %	CPU time
	Robust solution	Nominal solution				
01%	0000	0061	846.30	42	0	00:12.457
05%	0082	0359	846.30	42	0	00:08.123
25%	1077	1281	846.30	42	0	00:11.647

The nominal train scheduling problem and its robust optimization model were investigated in this study by taking into consideration the response actions to the primary and secondary risks. To demonstrate the relevance and effectiveness of the proposed models, an actual case study was implemented on a light rail transit line. The results demonstrated that the robust optimization model performs significantly better in terms of robustness when confronted with uncertain passenger demand because a small increase in the value of travel time and the number of stops in the nominal problem decreases unsatisfied passenger demand by 4.5 times. Furthermore, the risk-related programming demonstrated that the presence of secondary risks has a considerable impact on the choice of response actions to the primary risks. It indicates that these primary risk response actions are selected and implemented only when the negative effects of secondary risks are less severe than those of the primary type. Otherwise, primary response actions are ignored, or other primary response actions must be used. To sum up, the proposed robust optimization model can be effectively applied in environments with uncertain passenger demand and under the influence of primary and secondary risks. The multi-objective design of the train scheduling problem, which takes into consideration more indicators of evaluation and more factors of uncertainty such as the passenger demand of each train and the travel time of each route, is an interesting subject worthy of continuing study efforts. On this basis, advanced algorithms may be designed to find robust solutions for more complicated and larger instances.

5. Conclusions

The nominal train scheduling problem and its robust optimization model were investigated in this study by taking into consideration the response actions to the primary and secondary risks. To demonstrate the relevance and effectiveness of the proposed models, an actual case study was implemented on a light rail transit line. The results demonstrated that the robust optimization model performs significantly better in terms of robustness when confronted with uncertain passenger demand because a small increase in the value of travel time and the number of stops in the nominal problem decreases unsatisfied passenger demand by 4.5 times. Furthermore, the risk-related programming demonstrated that the presence of secondary risks has a considerable impact on the choice of response actions to the primary risks. It indicates that these primary risk response actions are selected and implemented only when the negative effects of secondary risks are less severe than those of the primary type. Otherwise, primary response actions are ignored, or other primary response actions must be used. To sum up, the proposed robust optimization model can be effectively applied in environments with uncertain passenger demand and under the influence of primary and secondary risks. The multi-objective design of the train scheduling problem, which takes into consideration more indicators of evaluation and more factors of uncertainty such as the passenger demand of each train and the travel time of each route, is an interesting subject worthy of continuing study efforts. On this basis, advanced algorithms may be designed to find robust solutions for more complicated and larger instances.

Conflict of interest

The authors declare there is no conflict of interest.

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