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Research article

Effects of congestion charging and subsidy policy on vehicle flow and revenue with user heterogeneity

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Abstract: Traffic congestion is a major issue in urban traffic networks. Both congestion charging and subsidy policy can solve traffic congestion to some extent, but which one is better? Based on this, this paper constructs a typical transit network consisting of three travel tools in four common travel modes. Travelers' values of time affect their choice of transportation in the congestion network, thus a stochastic user equilibrium model is established by considering travelers' heterogenous values of time to evaluate the effects of different combinations of congestion charging and subsidy policies on vehicle flow and revenue. Numerical results indicate that the effectiveness of congestion charging and subsidy policy in alleviating traffic congestion depends on the object of charging or subsidizing. Congestion charging for private cars can reduce traffic flow and alleviate traffic congestion, but charging for ridesharing cars does not reduce traffic flow and may even cause traffic congestion. Subsidizing public buses does not reduce traffic flow, but it can ease congestion by coordinating traffic flow on both edges of the dual-modal transport. The combination of no subsidy for public buses and charging for both private cars and ridesharing cars can obtain the greatest revenue, but it does not alleviate traffic congestion. Although the combination of charging for private cars and subsidizing public buses does not bring the most benefits, it can reduce traffic flow, and its revenue is also considerable. This study can provide quantitative decision support for the government to ease traffic congestion and improve government revenue.

Keywords: traffic congestion; private car; ridesharing car; public bus; transit subsidy; value of time

1. Introduction

Traffic congestion is one looming or urgent issue which most metropolitan areas around the world are facing [1–3]. According to a report from the Texas Transportation Institute, from 1982 to 2007, traffic congestion cost Americans $87.2 billion (in constant 2007 dollars) annually [4]. In 2010, traffic congestion costs for the United States reached $101 billion, including nearly two billion gallons of wasted fuel and 4.8 billion hours of travel delay [5]. The Texas Transportation Institute estimated that in 2011 urban Americans traveled about 5.5 billion additional hours and purchased an extra 2.9 billion gallons of fuel as a result of congestion [6]. In 2017, the estimated total economic costs due to congestion in Britain, Germany and the United States were $461 billion [7]. In addition, congestion has been worsening for many reasons, including the increasing demand for vehicles due to population growth and economic activity [8–10], physical capacity lagged behind demand growth [10,11] and low vehicle occupancy rate [12]. Therefore, searching for proper methods to alleviate traffic congestion to the greatest extent is a great challenge and is high on government agendas. In literature, two of the most popular ways to deal with congestion that have been suggested are congestion pricing and giving priority to public transportation [13].

Congestion charging is growing in popularity as one effective way to mitigate traffic congestion, which is to a certain extent advocated by economists [2,3,11,14]. The essential idea of congestion pricing or congestion charging is the use of a price mechanism to make users conscious of the costs that they impose upon one another when consuming during peak demand, and that they should pay for the additional congestion they create [15]. In practice, congestion charging has been successfully implemented in many metropolises, such as Singapore[1], London, Stockholm[2], Gothenburg and Milan [2,4,18–20]. However, congestion charging does not guarantee its efficiency for all types of cities as there need to be some specific attributes of the city to make better utilization of road charging [21].

Spurring the usage of the public transport system is another effective means to ease traffic congestion. Governments all over the world provide different forms of transport-related subsidies to encourage travelers to shift from private cars to public transport [6,22,23]. Ridesharing, as a complement to public transport [24], recently has emerged in many cities with the growth and acceptance of the sharing economy, the popularity of mobile internet technology as well as the application of innovative technologies. Emerging ridesharing platforms, e.g., Uber in the United States, Didi in China and Grab in Singapore, have facilitated the adoption of ridesharing by reducing the matching/meeting friction between drivers and riders [25]. Ridesharing, in the sense of carpooling, allows riders to travel with less expense by sharing a ride with peer passengers [26]. Ridesharing subsidies have the potential to improve social welfare and reduce congestion. However, providing too many subsidies to ridesharing users may increase congestion levels [27]. Thus, charging for it might relieve traffic congestion instead.

Public buses have a relatively higher occupancy rate than private and ridesharing cars. Many countries advocate charging for private and ridesharing cars and subsidizing buses. For example, 18%

[1] Singapore implements congestion pricing scheme [16].
[2] Stockholm implements urban congestion charge, after Singapore and London, and the second city using a time-differentiated scheme, after Singapore [17].

of the funds generated from congestion charging in the San Francisco Bay Area is used to subsidize bus transportation [6]. The city of Chicago introduced a tiered congestion surcharge on ride-sourcing trips to subsidize public transit [28].

There are many studies on congestion charging and public transportation in urban road networks, and some consider both congestion charging and public transportation subsidy [6,29–31]. The research on the impact of shared vehicles on traffic congestion is mainly based on ridesharing compensation [32–34]. Few studies focus on congestion charging for ridesharing cars, and those focusing on all the three elements simultaneously are fewer.

Therefore, this paper constructs a typical transit network with these three travel tools in four travel modes (only taking the private car, only taking the ridesharing car, transferring from the private car to the public bus, and transferring from the ridesharing car to the public bus). The congestion charging could be divided into four categories: no congestion charging, congestion charging for the private car, congestion charging for the ridesharing car, congestion charging for both; and subsidy could also be divided into four categories: no subsidy, subsidy for transferring from a private car to a public bus, subsidy for transferring from a ridesharing car to a public bus, subsidy for both. Thus, the four charging modes and four subsidy modes could be combined into 16 management policy combinations.

In addition, travelers' choice of travel modes depends on the trade-off between travel time and cost, since travelers may bear different costs on the same route if they choose different travel modes. Consequently, travelers' values of time (VOT) pose a significant influence on their decisions [35,36]. Commonly, travelers are heterogeneous in their VOT due to their socioeconomic characteristics and trip purpose [37]. Although some studies assume that all travelers are homogeneous [6,29,31], other studies have found that heterogeneity in travel mode and departure timing selection is crucial. Ignoring preference heterogeneity may cause a biased estimation of policy impacts [38]. Hence, preference heterogeneity is key to the travel mode.

Thus, a stochastic user equilibrium model is established by considering travelers' heterogenous values of time to evaluate the changes in network traffic flow and revenue caused by the implementation of different management policy combinations. This paper seeks to answer the following questions: 1) Does congestion charging or subsidy policy ease traffic congestion, and which one plays a dominant role? 2) Which kind of management policy combinations can bring more government revenue? 3) Should there be a congestion charge for private cars and/or ridesharing cars? 4) Should there be a subsidy for passengers who transfers private and/or ridesharing to public buses?

The rest of the paper is organized as follows: Section 2 reviews the literature on congestion charging and subsidy on the equilibrium network; Section 3 establishes a stochastic user equilibrium model to describe the transit problem; Section 4 conducts numerical experiments and quantitatively explores the effects of different management policy combinations on vehicle flow and revenue; The effects of three optimal management policy combinations on different travel modes are presented in Section 5; Section 6 concludes the paper and discusses future research.

2. Literature review

The research content of this paper mainly involves three aspects: congestion charging, public transportation and ridesharing. Given the importance and complexity of congestion charging, many governments and scholars have been committed to maximally easing traffic congestion. The theoretical concept of congestion pricing was initiated by Pigou (1920), further developed by Nie and Liu [14] and Chen and Yang [15]. Many studies focus on the topics of step tolling, user heterogeneity, and tradable credit schemes. Some studies explore congestion pricing with the heterogeneity of travelers' value of time, etc. [39–42]. Research on public transportation focuses on optimizing public transportation pricing and services to alleviate traffic congestion. Yoshida [43] studied the influence of queuing rules at transit stops on mass-transit policies. Monchambert and de Palma [44] explored the two-way implication between punctuality level of public transport and customer public transport use via a bi-modal competitive system. To circumvent the Downs-Thomson paradox appearing in a competitive highway/transit system, Wang et al. [45] designed the transit subsidy policies from either government funding or road toll revenue. Yang and Tang [46] proposed a fare-reward scheme for easing rail transit peak-hour congestion with homogeneous commuters. As for heterogeneous commuters, Tang et al. [47] further proposed an incentive-based hybrid fare scheme. In recent years, more and more studies focus on the impact of ridesharing on traffic congestion. The impact of static ridesharing cars on traffic congestion was studied separately by Xu et al. [48] and Alisoltani et al. [49]. And some studies explored the potential of subsidizing ridesharing users, drivers, so as to reduce traffic congestion [27,32–34].

Based on the research above, different management policies (congestion charging policy and/or subsidy policy) were further studied on the equilibrium network with various travel modes. For example, Yang et al. [29] investigated the relationship among the auto toll, transit fare and subsidy scheme at a bi-modal network level, and linked road pricing and public transport provision via the use of congestion charge revenue to subsidize or improve transit services for an optimal modal split. Liu et al. [30] studied a Pareto-improving and revenue-neutral congestion pricing scheme on a simple two-mode network consisting of highway and transit with a different value of time distributions. Nie and Liu [15] considered a static congestion pricing model in which travelers choose a mode from driving on the highway or taking public transit, to minimize a combination of travel time, operating cost and toll and examine the effects of individuals' value of time on the policy of congestion pricing. Basso and Jara-Diaz [31] modeled and analyzed optimal prices and design of transport services in a bi-modal context, and optimized the congestion toll, the transit fare (i.e., the level of subsidies) and transit frequency. Chen and Nozick [6] developed a bi-level optimization model for identifying an optimal zonal pricing scheme to incentivize the expanded use of transit to stem congestion. Bagloee and Sarvi [50] employed a solution method to assign the toll or subsidy to each road, which is based on augmenting the travel time of roads up to the level where the traffic volumes do not exceed some target rates. Lucinda and Moita [51] combined a structural econometric model with a simulation algorithm to estimate an optimal congestion tax, and investigated commuters' willingness to switch to public transport. Sun and Szeto [52] proposed a logit-based multi-class ridesharing user equilibrium assignment framework that can incorporate different policy measures (e.g., car restrictions, cordon tolling and subsidization). Song et al. [27] proposed a simulation-based optimization framework to explore the potential of subsidizing ridesharing users, drivers and riders, which further could improve social welfare and reduce congestion. A detailed comparison of these literature in three perspectives (i.e., management policy, travel mode and user heterogeneity) is shown in Table 1.

Obviously, the impacts of congestion charging and public transportation subsidy simultaneously on traffic congestion are extensively explored, while congestion charging for ridesharing cars is sometimes ignored. And, research on ridesharing is increasing with the development of sharing economy, which ignores the roles of public bus.

Research on user heterogeneity has been increasing in recent years. Considering the preference heterogeneity of different travelers can better guide decision-making. Therefore, based on user heterogeneity, this paper considers congestion charging, ridesharing and subsidy strategies simultaneously, and explores the impact of different travel modes on traffic congestion.

Table 1. Comparison of literature about different management policies on the equilibrium network with various travel modes.

Literatures	Management policies		Travel tools			User heterogeneity
	Congestion charging	Subsidy	Private car	Public bus	Ride-sharing	
Yang et al. [29]	√	√	√	√	×	×
Liu et al. [30]	√	√	√	√	×	√
Nie and Liu [15]	√	×	√	√	×	√
Basso and Jara-Diaz [31]	√	√	√	√	×	×
Chen and Nozick [6]	√	√	√	√	×	×
Bagloee and Sarvi [50]	√	√	√	×	×	×
Lucinda and Moita [51]	√	√	√	√	×	√
Sun and Szeto [52]	√	√	√	×	√	√
Song et al. [27]	√	√	√	×	√	√
this paper	√	√	√	√	√	√

3. Modeling

3.1. Network representation

A transit network G = (V, E) consists of a node set V and an edge set E. The node set V includes the origin, destination and transit point. Each element in the edge set E connects two consecutive nodes. A travel mode is any path that a traveler follows on the transit network G from the origin to the destination by one or more travel tools, including the private car, the ride-sharing car and the public bus [53,54]. A detailed explanation is presented by a typical network with three nodes (origin, destination and transit point) and two edges (E_a, E_b), as shown in Figure 1. Travel tools on the edge E_a only include the private and ride-sharing car, and those on the edge E_b contain the private car, the ride-sharing car and the public bus. Four travel modes from the origin to the destination are as follows:

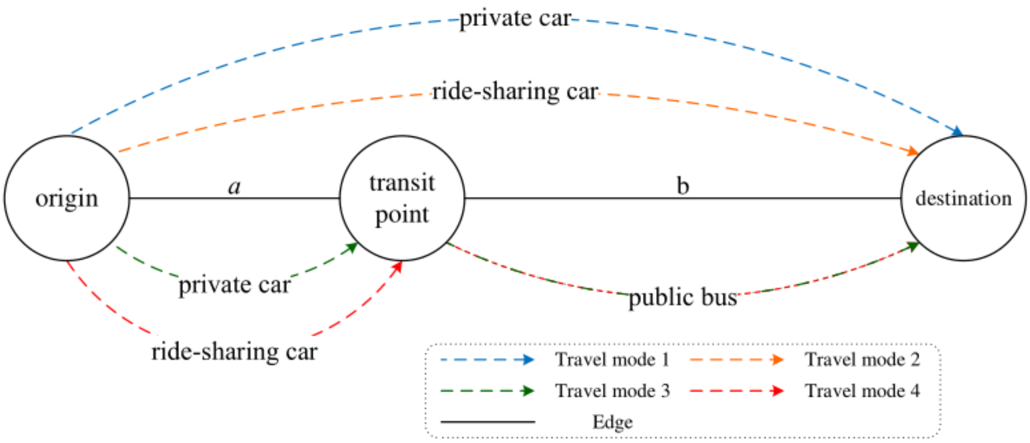

Figure 1. A network with four common travel modes.

Travel mode 1: a traveler travels straight by the private car from the origin to the destination.
Travel mode 2: a traveler travels straight by the ridesharing car from the origin to the destination.
Travel mode 3: a traveler travels along the edge E_a from the origin to the transit point by the private car, and then along the edge E_b from the transit point to the destination by the public bus.
Travel mode 4: a traveler travels along the edge E_a from the origin to the transit point by the ridesharing car, and then along the edge E_b to the destination by the public bus.

3.2. Travel time

A common quantitative measure of travel time in transportation literature and practice is the U.S. Bureau of Public Roads (BPR) formulation [55,56] which is continuously differentiable and strictly monotone [22]. Its mathematical expression is as follows:

$$t_j = t_j^0\left[\alpha + \beta\left(\frac{v_j}{H_j}\right)^{\phi}\right] \tag{1}$$

where t_j denotes travel time on the edge E_j, $j \in \{a,b\}$; t_j^0 is the free flow travel time on the edge E_j; H_j denotes capacity for the edge E_j; α, β and ϕ are parameters with no structural information[3]; v_j denotes vehicle flow on the edge E_j. Equation (1) describes the coupling of the assumption with the dependence between travel time and flow.

The vehicle flow v_j on the edge E_j is the sum of the flows of all travel modes going through the edge E_j for all travel modes. In particular, for the transit network in Figure 1, the vehicle flow v_a only includes the private car flow in travel mode 1 and 3, and the ridesharing car flow in travel mode 2 and 4; while the vehicle flow v_b contains the private car flow in travel mode 1, the ridesharing car flow in travel mode 2, and the bus flow in travel mode 3 and 4. Note that, when travelers on multiple travel modes select the public bus on the same edge, the bus flow is only recorded once. Consequently,

[3] Generally assume that $\alpha = 1, \beta = 0.15$ and $\phi = 4$, according to Liu and Meng [22], Zhao et al. [56], Manzo et al. [57], Almotahari and Yazici [58], Novak et al. [59].

the vehicle flow v_j can be expressed as

$$v_j = \begin{cases} \sum_i \frac{f_i}{\gamma_{j,i}} \cdot \delta_{j,i}, & j = a, i \in \{1,2,3,4\} \\ \sum_i \frac{f_i}{\gamma_{j,i}} \cdot \delta_{j,i} + v_{bus} \cdot \gamma_{bus}, & j = b, i \in \{1,2\} \end{cases} \tag{2}$$

where f_j denotes passenger flow of the *i*-th travel mode; $\delta_{j,i} = 1$ if the edge E_j is part of the *i*-th travel mode connecting the origin and destination, otherwise, $\delta_{j,i} = 0$; v_{bus} is the bus flow, which is a fixed value, only determined by frequency of public buses and is irrelevant to the passenger flow; γ_{bus} denotes the size ratio between public buses and cars[4]. In Eq (2), $\gamma_{j,i}$ is the average occupancy rate of the car used on the edge E_j of the *i*-th travel mode, which converts the flow-volume from passenger unit to corresponding vehicle unit,

$$\gamma_{j,i} = \begin{cases} \gamma_{priv}, & i \in \{1,3\}, \forall j \\ \gamma_{ride}, & i \in \{2,4\}, \forall j \end{cases} \tag{3}$$

where γ_{priv} and γ_{ride} are the average occupancy rates of the private and ridesharing cars, respectively. Therefore, the vehicle flow v_a and v_b in this paper can be written as follows:

$$\begin{aligned} v_a &= v_{a,1} + v_{a,2} + v_{a,3} + v_{a,4} = \frac{f_1}{\gamma_{priv}} + \frac{f_2}{\gamma_{ride}} + \frac{f_3}{\gamma_{priv}} + \frac{f_4}{\gamma_{ride}} \\ v_b &= v_{b,1} + v_{b,2} + v_{bus} \cdot \gamma_{bus} = \frac{f_1}{\gamma_{priv}} + \frac{f_2}{\gamma_{ride}} + v_{bus} \cdot \gamma_{bus} \end{aligned} \tag{4}$$

3.3. Travel cost

To construct travel cost, travelers' heterogenous values of time (VOT) is introduced firstly, i.e., different travelers value travel time differently, depending on their income levels or travel purposes. From an economic perspective, travel cost could be measured in the monetary unit and measurement is more appropriate when it comes to different time values [60]. Thus, the travel time can be converted from time-based value into uniform monetary value. Therefore, the travel cost of the *i*-th travel mode TC_i including time cost, monetary cost, congestion charge and subsidy, can be expressed as

$$TC_i(\tau) = \tau\left(\sum_j t_j \delta_{j,i} + w_i\right) + MC_i + CC_i - S_i \tag{5}$$

where MC_i, CC_i and S_i denote the monetary cost (e.g., extra fuel cost and faster depreciation of vehicles), the congestion charge for the private car and/or ridesharing car, and the subsidy for transferring from cars to public buses in the *i*-th travel mode, respectively; w_i denotes waiting time of the *i*-th travel mode, and

[4] Because the size of the public bus is larger than that of the private or ridesharing car, each bus occupies more flow than a car.

$$w_i = \begin{cases} 0, & i=1 \\ w_{ride}, & i=2 \\ w_{bus}, & i=3 \\ w_{ride}+w_{bus}, & i=4 \end{cases} \tag{6}$$

where w_{ride} and w_{bus} are the waiting time of the ridesharing car and the public bus, respectively; τ is the VOT, which is a stochastic variable. It is usually assumed to follow a log-normal distribution [15,61]. The probability density function of the stochastic variable τ is continuously differentiable and expressed as

$$f(\tau) = \frac{1}{(\tau-\theta)\sigma\sqrt{2\pi}} exp\left[-\frac{(\ln\frac{\tau-\theta}{m})^2}{2\sigma^2}\right], \qquad \tau > \theta, m > 0, \sigma > 0 \tag{7}$$

where m is the scale parameter, σ is the shape parameter and θ is the location parameter. Note that, ln m and σ are not the mean and variance of the random variable τ whose mean and variance are $e^{\ln m+\sigma^2/2}+\theta$ and $(e^{\sigma^2}-1)e^{2\ln m+\sigma^2}$, respectively. Its cumulative distribution function is

$$F(\tau) = \frac{1}{2}\left[1+erf\left(\frac{\ln\frac{\tau-\theta}{m}}{\sigma\sqrt{2}}\right)\right] \tag{8}$$

where $erf(\cdot)$ is the error function defined by

$$erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2} dt \tag{9}$$

3.4. Choice function

The choice probability that the travel cost of the *i*-th travel mode TC_i is less than that of any other travel mode (given τ) is

$$P_i = Pr(TC_i \le TC_l, \forall l \ne i \mid \tau) \tag{10}$$

The passenger flow of the *i*-th travel mode f_i can be written as

$$f_i = q \cdot P_i \tag{11}$$

where q is trip rate between the origin and the destination. Equation (11) characterizes the stochastic user equilibrium condition. After summing both sides of Eq (11), we get

$$\sum_i f_i = q \tag{12}$$

which satisfies the flow conservation constraint.

In summary, a stochastic user equilibrium model is composed of Eqs (1)–(12).

4. Numerical experiments

A series of numerical experiments were performed to explore how congestion charging and subsidy policies affect the vehicle flow on each edge and the revenue (the total congestion charge minus the total subsidy) when the network reached equilibrium. First, four congestion charging policies, four subsidy policies and parameters required in numerical experiments are listed. Then, results from the experiments are presented in detail. Finally, we analyze and summarize the results.

4.1. Management policies and parameters

To explore a more effective approach for alleviating traffic congestion, numerical experiments were carried out in two dimensions, i.e., congestion charging and subsidy. Congestion charging policies considered in this paper include: A) no congestion charging, B) congestion charging for the private car, C) congestion charging for the ridesharing car, D) congestion charging for both the private car and the ridesharing car. Subsidy policies include: I) no subsidy, II) subsidies for travelers transferring from the private car to the public bus, III) subsidies for travelers transferring from the ridesharing car to the public bus, IV) subsidies for travelers transferring from all cars to the public bus. Four charging modes and four subsidy modes are combined into 16 forms, abbreviated to A-I to D-IV. 16 different management policy combinations would be numerically analyzed.

Table 2. Common parameters under any management policy combination.

Parameter	Symbol	Value
Free flow travel time on the edge E_a	t_a^0	30
Free flow travel time on the edge E_b	t_b^0	90
Capacity for the edge E_a	H_a	100
Capacity for the edge E_b	H_b	80
	α	1
Parameters in the BPR function	β	0.15
	ϕ	4
Waiting time of the ridesharing car	w_{rc}	2
Waiting time of the public bus	w_{pb}	10
Average occupancy rate of the private car	γ_{pc}	1
Average occupancy rate of the ridesharing car	γ_{rc}	2.5
Bus flow	v_{pb}	1/6
Size ratio between bus and car	δ_{pb}	2
Scale parameter of the log-normal distribution	m	1.5
Shape parameter of the log-normal distribution	σ	1
Location parameter of the log-normal distribution	θ	0
Total number of travelers	C	100

Common parameters under any management policy combination are shown in Table 2. The monetary cost of the private car on the edge E_a and E_b were set to 25 and 75, respectively, and the monetary cost of the public bus on the edge E_b was set to 3. Suppose that the monetary cost of the ridesharing car was 80% of the private car. For the private or ridesharing car, the congestion charge was 10% of its monetary cost. When travelers transferring from the private or ridesharing car to the public bus were subsidized, the subsidy was set to 5; when travelers transferring from all cars to the public bus were subsidized, the subsidy was set to 2.5. Specific parameters under each management policy combination, including the monetary cost, the congestion charge and the subsidy in four travel modes, were calculated and are summarized in Table 3.

Table 3. Specific parameters under each management policy combination.

Management policy combination	Mode 1			Mode 2			Mode 3			Mode 4		
	Monetary cost	Congestion charge	Subsidy	Monetary cost	Congestion charge	Subsidy	Monetary cost	Congestion charge	Subsidy	Monetary cost	Congestion charge	Subsidy
A-I	100	0	0	80	0	0	28	0	0	23	0	0
B-I	100	10	0	80	0	0	28	2.5	0	23	0	0
C-I	100	0	0	80	8	0	28	0	0	23	2	0
D-I	100	10	0	80	8	0	28	2.5	0	23	2	0
A-II	100	0	0	80	0	0	28	0	5	23	0	0
B-II	100	10	0	80	0	0	28	2.5	5	23	0	0
C-II	100	0	0	80	8	0	28	0	5	23	2	0
D-II	100	10	0	80	8	0	28	2.5	5	23	2	0
A-III	100	0	0	80	0	0	28	0	0	23	0	5
B-III	100	10	0	80	0	0	28	2.5	0	23	0	5
C-III	100	0	0	80	8	0	28	0	0	23	2	5
D-III	100	10	0	80	8	0	28	2.5	0	23	2	5
A-IV	100	0	0	80	0	0	28	0	2.5	23	0	2.5
B-IV	100	10	0	80	0	0	28	2.5	2.5	23	0	2.5
C-IV	100	0	0	80	8	0	28	0	2.5	23	2	2.5
D-IV	100	10	0	80	8	0	28	2.5	2.5	23	2	2.5

4.2. Experiments

Grid search was implemented for each management policy combination. Only one group of equilibrium passenger flows was found, and any group of passenger flows converged to this equilibrium group over multiple iterations. Under 16 different management policy combinations, the passenger flow of four travel modes, the vehicle flow on each edge, the total congestion charge and subsidy, and the revenue of the equilibrium network are summarized in Table 4. Next, the results are analyzed and summarized from the perspective of the vehicle flow and the revenue.

Table 4. Experiments under different management policy combinations.

Policy combination	Passenger flow				Vehicle flow		Total congestion charge	Total subsidy	Revenue
	Mode 1	Mode 2	Mode 3	Mode4	E_a	E_b			
A-I	67	11	14	8	88.60	71.73	0	0	0
B-I	50	29	8	13	74.80	61.93	520	0	520
C-I	76	0	19	5	97.00	76.33	10	0	10
D-I	64	12	15	9	87.40	69.13	791.5	0	791.5
A-II	67	9	24	0	94.60	70.93	0	120	-120

Continued on next page

Policy combination	Passenger flow				Vehicle flow		Total congestion charge	Total subsidy	Revenue
	Mode 1	Mode 2	Mode 3	Mode4	E_a	E_b			
B-II	50	27	19	4	81.40	61.13	547.5	95	452.5
C-II	74	0	26	0	100.00	74.33	0	130	-130
D-II	64	10	25	1	93.40	68.33	784.5	125	659.5
A-III	67	11	5	17	83.20	71.73	0	85	-85
B-III	50	29	0	21	70.00	61.93	500	105	395
C-III	76	0	11	13	92.20	76.33	26	65	-39
D-III	64	12	6	18	82.00	69.13	787	90	697
A-IV	67	10	15	8	89.20	71.33	0	57.5	-57.5
B-IV	50	28	9	13	75.40	61.53	522.5	55	467.5
C-IV	75	0	20	5	97.00	75.33	10	62.5	-52.5
D-IV	64	11	16	9	88.00	68.73	786	62.5	723.5

4.3. Analysis of the vehicle flow

First of all, the vehicle flow on each edge was grouped by the congestion charging policy as shown in Figure 2, and the effects of subsidy policies on the vehicle flow were compared and analyzed. In Figure 2, light-colored bars represent the vehicle flow on the edge E_a; dark-colored bars represent the vehicle flow on the edge E_b; bars with the same color represent the vehicle flow under the same congestion charging policy; bars with the same hatching (pattern filled polygons) represent the vehicle flow under the same subsidy policy. In each group (bars with the same color), the difference in lengths of light-colored bars is obvious. By comparing the lengths of the light-colored bars in each group, we found that bar III is the shortest, followed by bar I, then bar IV, and lastly bar II. In other words, the priority order of the subsidy policy on controlling the vehicle flow of the edge E_a is III > I > IV > II. Consequently, the subsidy policy III (subsidy for travelers transferring from the ridesharing car to the public bus) is the most effective approach to control the traffic flow on the edge E_a. Not all subsidies can reduce the traffic flow on the edge E_a because the subsidy policy I outperforms both the subsidy policy II and IV. That is to say, no subsidy outperforms the subsidy for travelers transferring from the private car to the public bus or from all cars to the public bus. While the difference in lengths of dark-colored bars is slight compared with light-colored bars. Based on the comparison of the lengths of the dark-colored bars in each group, bar II is the shortest, followed by IV and lastly bar I and bar III which are the longest and isometric. Thus, the priority order of the subsidy policy on controlling the vehicle flow of the edge E_b is II > IV > I = III, which is exactly the opposite of that of the edge E_a.

Another interesting finding is that for any congestion charging policy, the vehicle flow on the edge E_b under both the subsidy policy I and III is equivalent. In order to explain the reason why the vehicle flows of these two subsidy policies are equal, their passenger flows are displayed in Table 5. From Table 5, it is clear that for any congestion charging policy the only difference between the subsidy policy I and III is the passenger flow choice between travel modes 3 and 4, and the decrease of the passenger flow choice of travel mode 3 is equal to the increase of that of travel mode 4. Therefore, when travelers transferring from the ridesharing car to the public bus are subsidized, some travelers will give up travel mode 3 and choose travel mode 4, but travelers who prefer travel mode 1 or 2 are not interested in this subsidy policy.

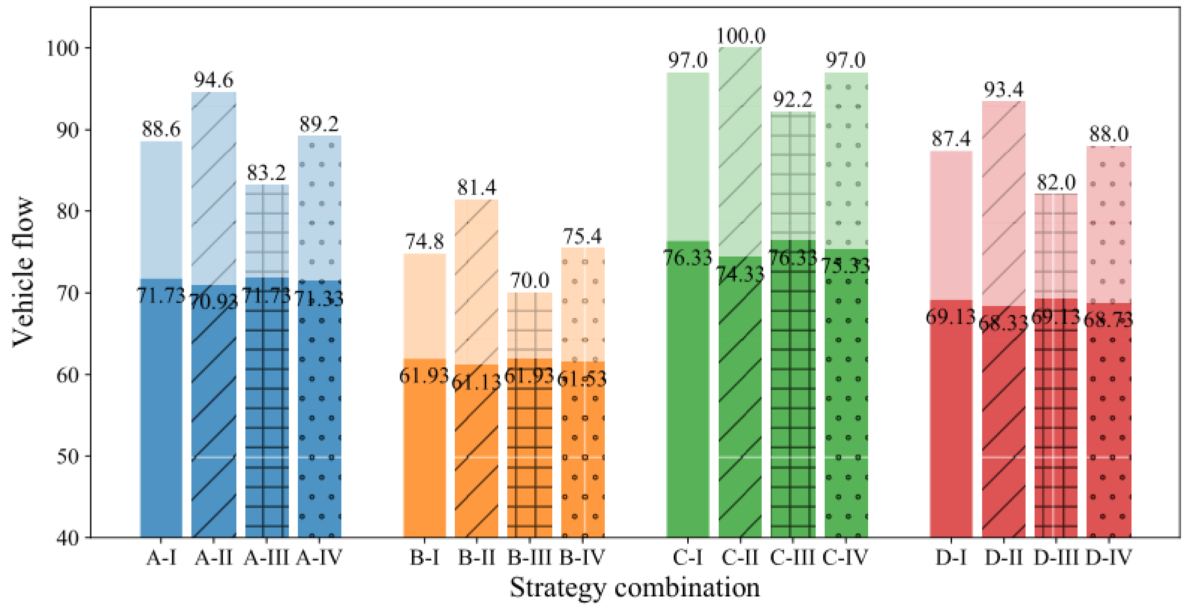

Figure 2. Comparison of the vehicle flows grouped by the congestion charging policy.

Table 5. Comparison of the subsidy policy I and III.

Policy combination	Passenger flow				Vehicle flow on E_b
	Mode 1	Mode 2	Mode 3	Mode 4	
A-I	67	11	14	8	71.73
A-III	67	11	5	17	71.73
B-I	50	29	8	13	61.93
B-III	50	29	0	21	61.93
C-I	76	0	19	5	76.33
C-III	76	0	11	13	76.33
D-I	64	12	15	9	69.13
D-III	64	12	6	18	69.13

The vehicle flow on each edge was grouped by the subsidy policy as shown in Figure 3, and the effects of congestion charging policies on the vehicle flow were compared and analyzed. For either light-colored bars or dark-colored bars, in each group with the same hatching, bar B is the shortest, followed by bar D then bar A and lastly bar C. In other words, the priority orders of the congestion charging policy on controlling the vehicle flow on both two edges (E_a and E_b) are B > D > A > C. Therefore, the congestion charging policy B (congestion charging for the private car) is the most effective approach to control the traffic flow. Besides, not all congestion charging policies are better than no congestion charging, such as C < A.

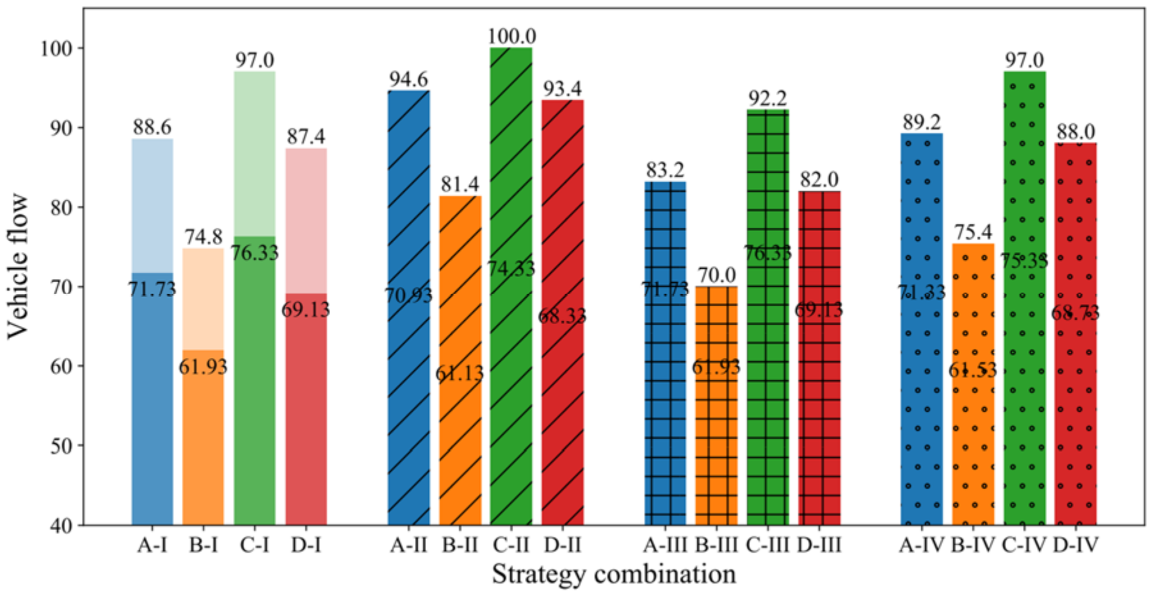

Figure 3. Comparison of the vehicle flows grouped by the subsidy policy.

In summary, 1) the optimal management policy combination to control the traffic flow on the edge E_a is B-III, i.e., congestion charging for the private car and subsidies for travelers transferring from the ridesharing car to the public bus; 2) the optimal management policy combination to control the traffic flow on the edge E_b is B-II, i.e., congestion charging for the private car and subsidies for travelers transferring from the private car to the public bus.

4.4. Analysis for the revenue

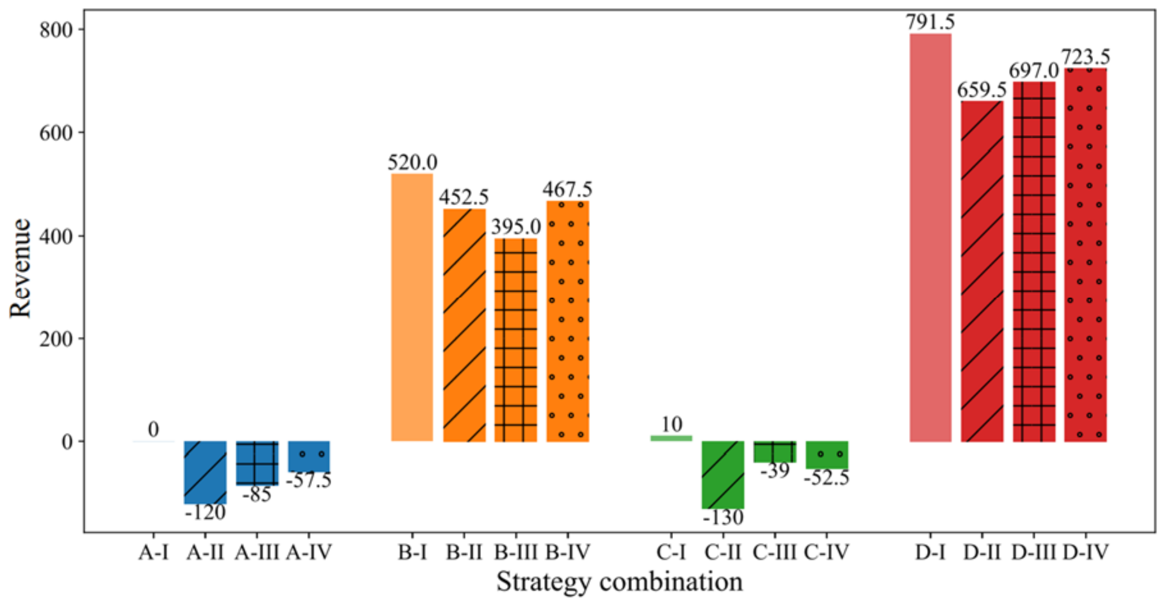

Figure 4. Comparison of the revenues grouped by the congestion charging policy.

The effects of congestion charging and subsidy policies on the revenue were analyzed. Results of the revenue grouped by the congestion charging policy are shown in Figure 4 and results of the revenue grouped by the subsidy policy are shown in Figure 5. In Figure 4, in each group with the same hatching, bar D is the longest, followed by bar B, but the remaining two bars are disordered. The revenue under the congestion charging policy D is the highest. Although the revenue under the congestion charging policy B is second, and it is also impressive. In Figure 5, in each group with the same color, bar I is the longest, but the other three bars are disordered. No subsidy can obtain more revenue than the other three subsidy policies, which is consistent with common sense.

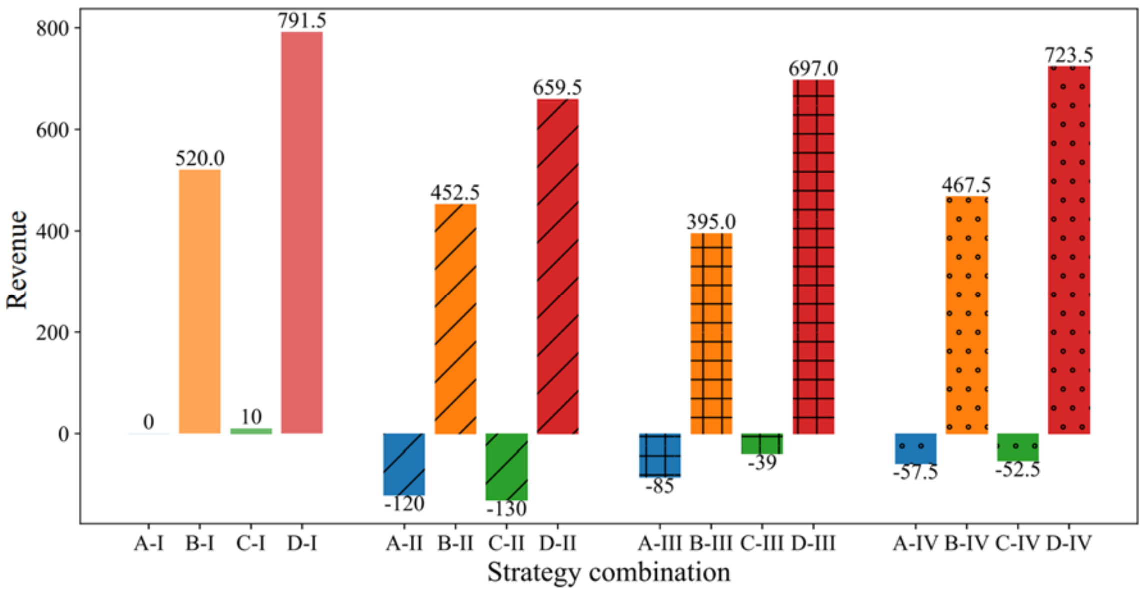

Figure 5. Comparison of the revenues grouped by the subsidy policy.

Note that most revenues under the congestion charging policy C were negative, i.e., congestion charging for the ridesharing car may not necessarily lead to benefits. To explore this interesting finding, the passenger flow, total congestion charge, total subsidy and revenue under the congestion charging policy C were compared and the results are shown in Table 6. All passenger flows of travel mode 2 were zero, and all passenger flows of travel mode 4 were very small, thus the total congestion charge was very limited. In addition, some travelers need to be paid certain subsidies, and consequently the revenues under the management policy combinations C-II, C-III and C-IV were negative. Therefore, when the travelers by the ridesharing car were charged for congestion fees, they would give up travel mode 2, and only when a few travelers chose travel mode 4, the revenue was consequently little or even negative.

Table 6. Partial results under the congestion charging policy C.

Policy combination	Passenger flow				Total congestion Charge	Total subsidy	Revenue
	Mode 1	Mode 2	Mode 3	Mode 4			
C-I	76	0	19	5	10	0	10
C-II	74	0	26	0	0	130	-130
C-III	76	0	11	13	26	65	-39
C-IV	75	0	20	5	10	62.5	-52.5

Thus, the optimal management policy combination to maximize the revenue is D-I, i.e., congestion charging for both the private car and the ridesharing car, and no subsidy.

4.5. Summary

In terms of vehicle flow control, the priority order of the congestion charging policy on the edge E_a and E_b is B > D > A > C; the priority order of the subsidy policy on the edge E_a is III > I > IV > II; the priority order of the subsidy policy on the edge E_b is II > IV > I = III.

In terms of the revenue, congestion charging policies may bring benefits, but may also cause loss. Therefore, the choice of appropriate subsidy goal is essential, and the congestion charging policy D is the best, followed by B. Subsidy policies can only reduce the revenue, and no subsidy is the best choice. When the goal is to minimize the vehicle flow on the edge E_a, the optimal management policy combination is B-III, which can effectively alleviate traffic congestion of urban roads without public bus transit lines; when the goal is to minimize the vehicle flow on the edge E_b, the optimal management policy combination is B-II, which is a useful way to relieve stress in traffic congestion for urban roads with public bus transit lines; when the goal is to maximize the revenue, the optimal management policy combination is D-I, which can improve government revenue that might be allocated to develop transport infrastructures.

5. Effect on choice of travel modes

In the previous section, the optimal management policy combinations in three scenarios were obtained, i.e., the management policy combination B-III for reducing the vehicle flow on the edge E_a, B-II for reducing the vehicle flow on the edge E_b and D-I for increasing the revenue. The combination of no congestion charging and no subsidy policy (i.e., A-I) was used as a reference combination, and the effect of these three optimal management policy combinations on four travel modes was analyzed. According to the model construction in Section 3, the probability density function (pdf of lognormal) of travelers' values of time (VOT) τ is plotted, and the passenger flow is used to analyze the impact of congestion charging and subsidy strategy on the travel mode choices of passengers with different reflections on the value of time.

First, the transformation caused by the management policy combination B-III to travelers was discussed. The difference of the passenger flow between the management policy combinations B-III and A-I is shown in Figure 6. A total of 31% of travelers would re-select their travel modes. In particular, 13% of travelers gave up travel mode 3 and chose travel mode 4; 1% of travelers changed from travel mode 3 to 2; 17% of travelers changed from travel mode 1 to 2. The management policy combination B-III increased the proportion of travelers who chose travel modes 2 and 4, and reduced proportion of travelers who chose travel modes 1 and 3.

To explore whether the changes of these travelers were mainly affected by the congestion charging policy B or the subsidy policy III, this paper only analyzed the impact of the congestion charging policy first (i.e., the passenger flows between the management policy combinations A-I and B-I were compared), and further analyzed the impact of the subsidy policy (i.e., the passenger flows between the management policy combinations B-I and B-III were compared). The results are shown in Figure 7. From the management policy combinations A-I to B-I, 5% of travelers changed from travel mode 3 to 4, 1% of travelers varied from travel mode 3 to 2, and 17% of travelers changed from travel mode 1 to 2; from the management policy combinations B-I to B-III, only 8% of travelers varied from travel mode 3 to 4. Therefore, most traveler changes in travel modes were caused by congestion charging for the private car (the congestion charging policy B), and the changes of only 8% of travelers were attributed to subsidies for travelers transferring from the ridesharing car to the public bus (i.e., the subsidy policy III).

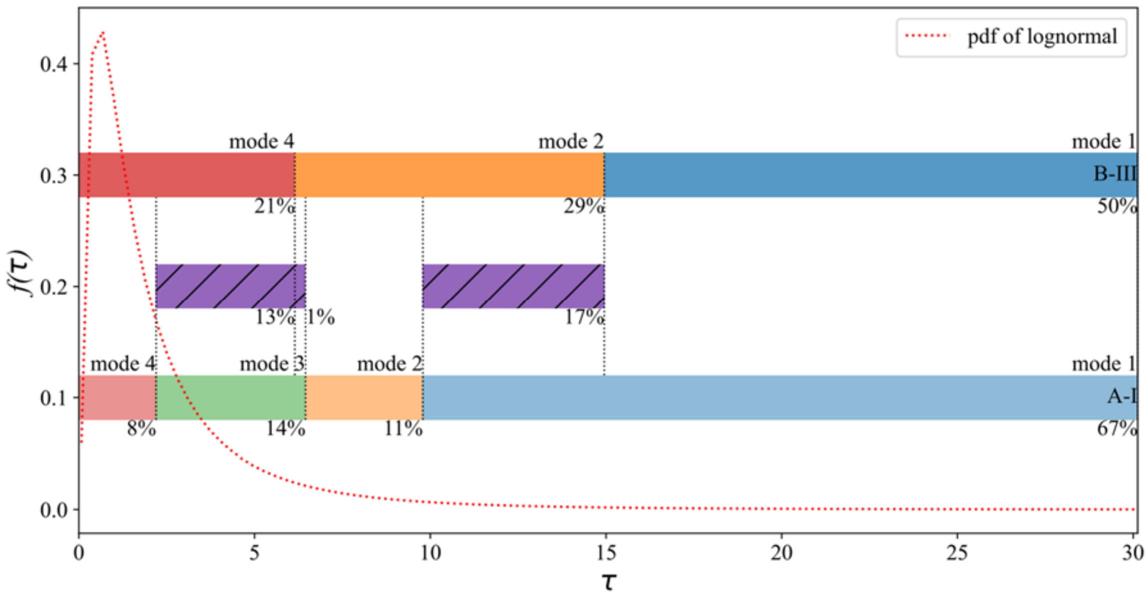

Figure 6. B-III vs A-I.

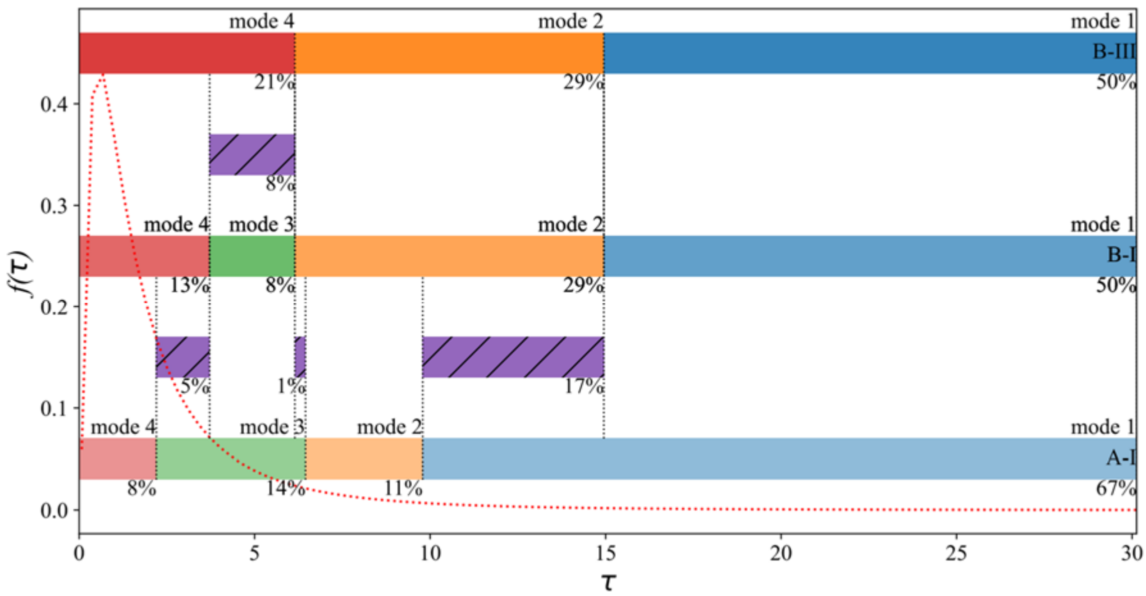

Figure 7. B-III vs B-I and B-I vs A-I.

Second, the changes which the management policy combination B-II brought were analyzed in a similar way and the results are shown in Figures 8 and 9. According to these two figures, 17% of travelers varied from travel mode 1 to 2 due to congestion charging for the private car (i.e., the congestion charging policy B); 4% of travelers changed from travel mode 4 to 3, 1% of travelers gave up travel mode 2 and chose travel mode 3, and these changes were caused by subsidies for travelers transferring from the private car to the public bus (i.e., the subsidy policy II). Overall, the management policy combination B-II increased the proportion of travelers who chose travel modes 2 and 3, and reduced proportion of travelers who chose travel modes 1 and 4.

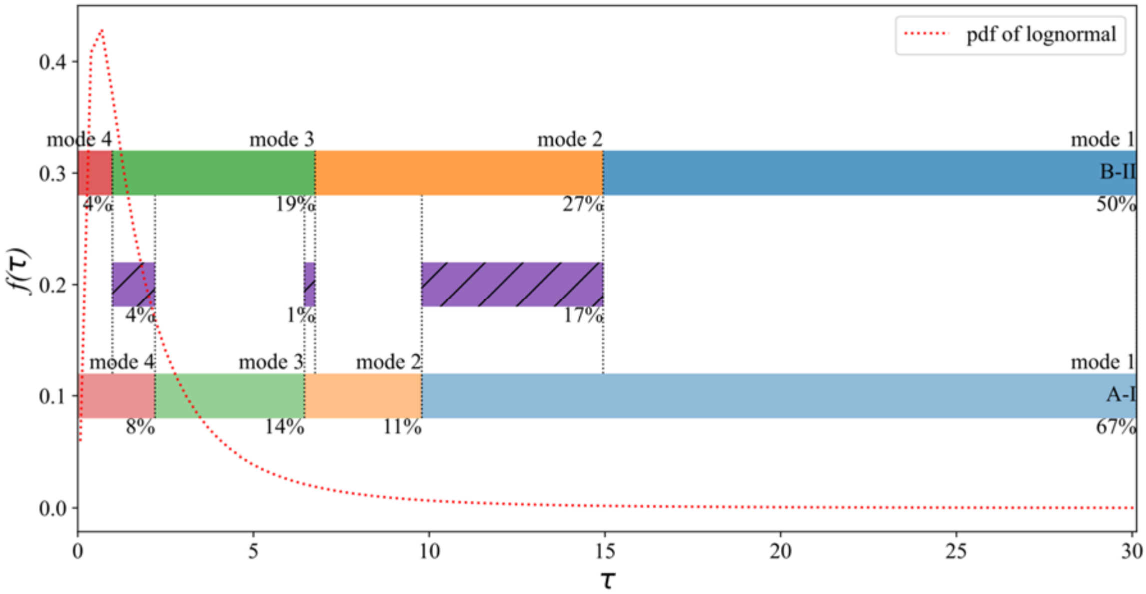

Figure 8. B-II vs A-I.

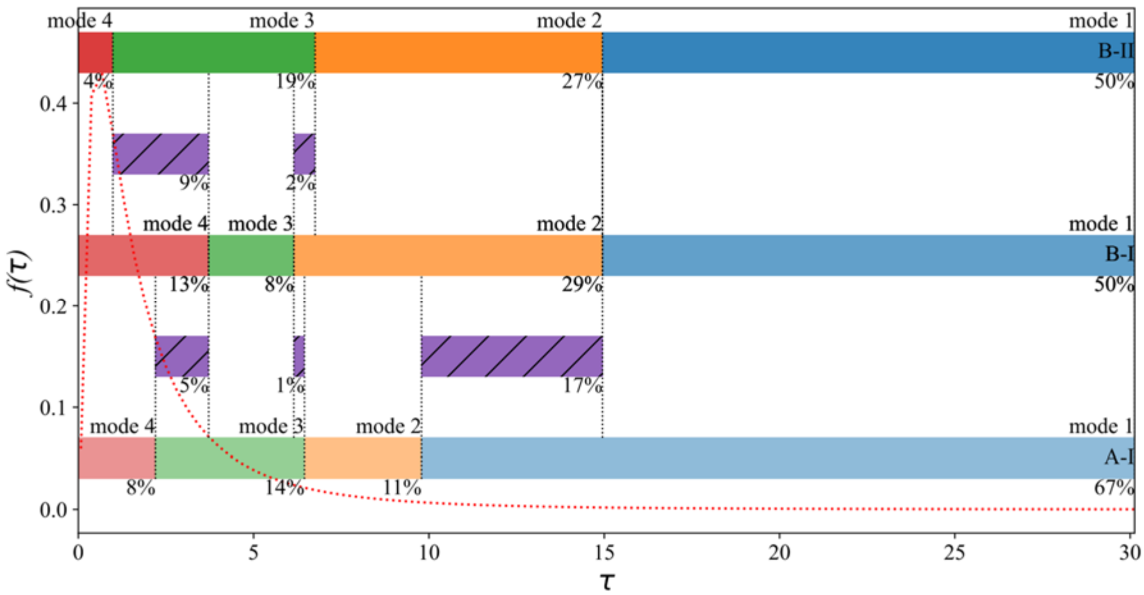

Figure 9. B-II vs B-I and B-I vs A-I.

Finally, we analyzed the transformation of travel modes caused by the management policy combination D-I, and the results are shown in Figure 10. Based on the comparison of the management policy combinations D-I and A-I, 1% of travelers with lower VOT changed from travel mode 3 to 4; 2% of travelers gave up travel mode 2 and chose travel mode 3; 3% of travelers with higher VOT varied from travel mode 1 to 2. These changes in travel modes were caused by congestion charging for all cars (i.e., the congestion charging policy D). Although only 6% of travelers changed their travel modes, the management policy combination D-I can improve the revenue to a large extent.

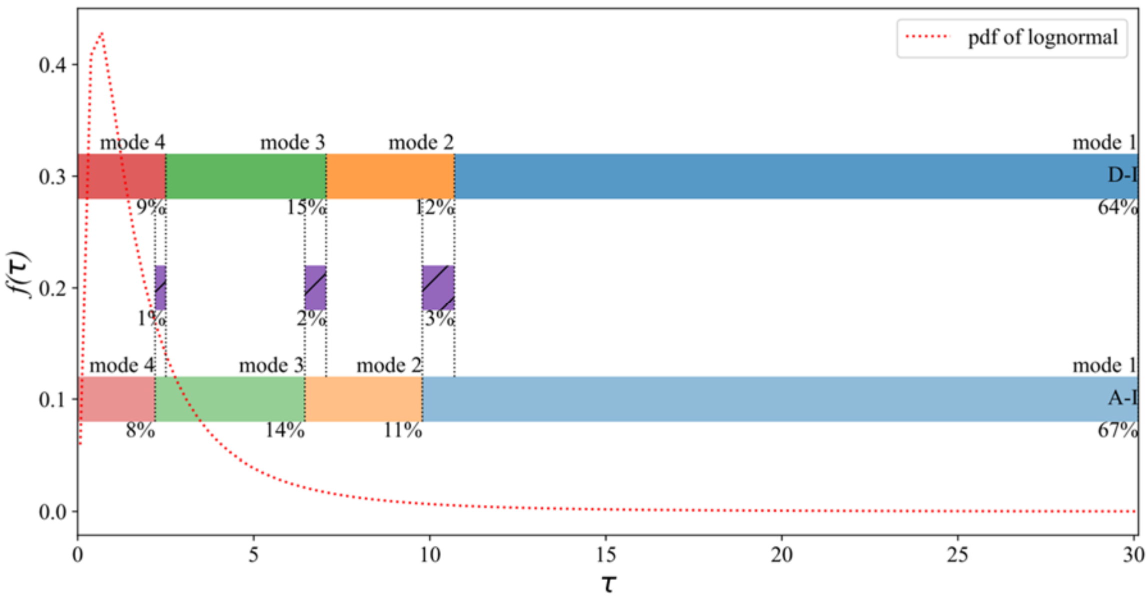

Figure 10. D-I vs A-I.

6. Conclusions

This paper establishes a stochastic user equilibrium model considering travelers' heterogenous values of time. Four charging modes and four subsidy modes are combined into 16 management policy combinations. The effects of different policy combinations on the traffic flow and revenue are explored by numerical experiments. Our main conclusions are as follows:

1) Charging for private cars can reduce traffic flow and alleviate traffic congestion, but charging for ridesharing cars cannot reduce traffic flow, which might even cause traffic congestion; subsidizing public buses cannot reduce traffic flow, but it can alleviate traffic congestion by coordinating the traffic flow on both edges of the bimodal transport.

2) Obviously, charging for all cars and no subsidy is the best combination to improve government revenue. But this is not the best way to relieve traffic congestion. The combination of congestion charging for the private car and subsidies for travelers transferring from the ridesharing car to the public bus can minimize the vehicle flow on urban roads without public bus transit lines; the combination of congestion charging for the private car and subsidies for travelers transferring from the private car to the public bus can minimize the vehicle flow on urban roads with public bus transit lines. Although the two management policy combinations do not bring the most revenue, their revenues are also considerable.

3) Charging for private cars makes more travelers change their travel modes from private cars to

ridesharing cars, which would reduce traffic flow and alleviate traffic congestion. The effect of charging for ridesharing cars on reducing traffic flow is worse than that of no charging. Therefore, in this paper, private cars should be charged, and ridesharing cars should not be charged.

4) Subsidizing passengers who transfer from private and/or ridesharing car to public buses leads some travelers to change their travel modes and reduces traffic flow. The effect is not as good as charging for private cars, but better than charging for ridesharing cars. Therefore, subsidizing public transportation should be considered.

Proper management policy combination not only can promote efficient use of urban transportation systems, mitigate urban traffic congestion, save a large amount of travel time and reduce CO_2 emissions, but also has the advantage that it does not require costly new transportation infrastructure construction and generates revenues which are used for investments in essential transportation infrastructure, such as expanding the road capacity, providing better maintenance and improving public transport. In addition, it reveals more transparently how part of the revenues obtained from congestion pricing are distributed to subsidize travelers, improve the equity in the traffic system [29] and reduce the motoring public's opposition to congestion charge [62].

Therefore, this paper can provide quantitative decision support for the government (such as the Ministry of Transport) and help them choose the appropriate policy combination according to urban development planning or current demand. For instance, at the early stages of urban transport development, which requires enormous capital to develop road infrastructure, the combination of congestion charging for all cars and no subsidy (policy D-I) may be more appropriate; when congestion often happens on roads without public bus transit lines, the combination of congestion charging for the private car and subsidies for travelers transferring from the ridesharing car to the public bus (policy B-III) is preferred; when congestion occurs on roads with public bus transit lines, the combination of congestion charging for the private car and subsidies for travelers transferring from the private car to the public bus (policy B-II) is most worth considering.

In the future, travel modes could be enriched and more combinations could be generated, which would take into consideration public buses, private cars-ridesharing cars and make further complex comparison. Additionally, the exploration of CO_2 emissions could be incorporated in different modes, which might contribute to low carbon development. Lastly, more realistic data (e.g., urban road network and population density) could be collected to perform empirical research.

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Conflict of interest

The authors declare there is no conflict of interest.

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