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Research article

Event-triggered control of flexible manipulator constraint system modeled by PDE

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Abstract: The vibration suppression control of a flexible manipulator system modeled by partial differential equation (PDE) with state constraints is studied in this paper. On the basis of the backstepping recursive design framework, the problem of the constraint of joint angle and boundary vibration deflection is solved by using the Barrier Lyapunov function (BLF). Moreover, based on the relative threshold strategy, an event-triggered mechanism is proposed to save the communication workload between controller and actuator, which not only deals with the state constraints of the partial differential flexible manipulator system, but also effectively improves the system work efficiency. Good damping effect on vibration and the elevated system performance can be seen under the proposed control strategy. At the same time, the state can meet the constraints given in advance, and all system signals are bounded. The proposed scheme is effective, which is proven by simulation results.

Keywords: state constraints; partial differential equations; event-triggered control; Barrier Lyapunov function

1. Introduction

As computer technology and machinery manufacturing technology develops, people are expecting more and more from production automation. Since the last century, the manipulator system has gradually replaced human beings to complete the dangerous and repetitive work in various fields [1–4]. At the same time, the manipulator system can also significantly improve the production efficiency [5–7]. The flexible manipulator has better performance of high stability, high precision, high efficiency and low energy consumption than the traditional rigid manipulator [8–11]. Consequently, it is more adapted to the complex and changeable working environments in various fields. For example, for the sake of improving the automation level of agricultural production, the flexible manipulator system is adopted to pick fruit and vegetable crops in the field of agriculture, so as to further ensure the safety of agricultural products and improve the production efficiency in the process of processing and production. However, the flexible manipulator is characterized by its complex structure, low control accuracy, difficult control, etc. These defects may lead to vibration of the flexible manipulator system, which greatly affects the stability of the actual production. Thus, improving its stiffness and control accuracy, and suppressing the vibration of the flexible manipulator system have become the focus of current research.

By referring to the literature, we can know that the manipulator system with special flexible structure is a typical infinite dimensional distributed parameter system [12–14]. Most of the existing studies of the manipulators are based on the ordinary differential equation (ODE) dynamic models [15–18]. Nevertheless, these ODE models limit the system to a few key patterns, greatly affecting system performance [19]. For getting the accurate description of the flexible manipulator systems, the model cannot be constructed only through a single ODE; otherwise, there will be spillover instability [20]. Therefore, it is necessary to introduce partial differential equations (PDEs) in the flexible connection systems. At present, there are some research achievements on flexible systems described by PDEs. In [21], for the sake of the achievement of control goals, a boundary controller with input backlash is constructed based on the infinite-dimensional dynamic model. For the single flexible link manipulator system in [22], a sliding mode boundary controller is designed based on the adaptive radial basis function (RBF) neural network (NN) to drive the joint to the required position and quickly suppress the vibration on the beam. Then, an adaptive fault-tolerant control method is raised by using RBFNN and LaSalle's invariance principle to solve the failure problem of the actuator of the single-link flexible manipulator in [23].

Over the past two decades, systems modeled by PDEs have attracted more and more researchers because of their wide application in various fields, and numerous methods have been reported [24–28]. However, these results [21–28] all ignored the constraint problem. In fact, many real-world systems are limited by constraints in various ways [29]. It is possible that such constraints are due to physical restriction of systems, or caused by the requirements of safe operation [30]. Motivated by progress in constraints, lots of state constraint problems have been researched for ODE systems [31–33]. With the rise of the research on PDE systems, some scholars also put their attention to the problem of state constraints of PDE flexible mechanical systems. In [34], a class of flexible riser systems with backlash modeled by PDEs is considered. In order to solve its position and velocity constraints, logarithmic BLF is used. In [35], the state feedback control problem of moving vehicle-mounted manipulator modeled by PDE with output constraint is studied. Under the action of the designed control scheme, the position control and vibration suppression are effectively improved. For the uncertain PDE flexible manipulator system in [36], a NN fault-tolerant control scheme under state constraints is proposed. In the design process, the tangent BLF is utilized to handle the constraint problem, and get a good control.

In addition to the state constraint problem, in today's society, production resources are also tight. While meeting the quality of control, saving resources has become an important aspect that needs to be consider. In recent years, the event triggered control [37–40], as an effective method that can not only achieve control objectives, but also save resources, has raised the broad interest of all researchers. The event triggered control is a control mechanism of sampling on demand. System resources can only be used when necessary, and can meet the expected control performance indicators. In [41], a collaborative design scheme consisted of switching event triggering mechanism and mode dependent adaptive control law is proposed which solves the mismatch problem and avoids the Zeno behavior. In [42], for nonlinear uncertain systems, besides the design methods on the basis of fixed threshold strategy and relative threshold strategy, a new switching threshold strategy is proposed. However, the above results are only applicable to the system modeled by ordinary differential methods. When these methods are directly applied to the control system modeled by partial differential methods, it may lead to the failure of control strategy, and even bring huge losses to practical engineering. In addition, in the actual production and life, many control systems need to be modeled by partial differential method to achieve better control effect. Among them, the flexible manipulator system modeled by partial differential method is widely used in [43–45]. Therefore, in order to make efficient use of resources, the event-triggered control of flexible manipulator systems modeled by PDEs under state constraints is a significant topic of study that has inspired our own research.

It can be seen from the above analysis that although researchers have put forward many research results for flexible manipulator system, there are still some limitations. Therefore, the event-triggered control of a PDE flexible manipulator with constraints will be taken as the research object in this paper, and the control goal of saving communication resources will be achieved by designing event-triggered controllers. On the premise of achieving the stable performance of the system, good vibration suppression effect of the flexible manipulator will be maintained. On account of the above discussion, the innovation of this article is given below: when dealing with the state constraint of PDE flexible manipulator system, an event trigger control strategy is introduced.

In this paper, an event-triggered control design problem is studied for flexible manipulator systems with full state constraints. Under frameworks of adaptive backstepping control design technique, an event-triggered control scheme is proposed for flexible manipulator systems. The main contribution of the paper is summarized as follows:

1) In this paper, the design problem of event-triggered control is studied for flexible manipulator system with full state constraints and an event-triggered control method is proposed. Different from the constraint control scheme in [31,32], the event-triggered control strategy proposed in this paper can save unnecessary control signal transmission and improve the system performance.

2) An event-triggered mechanism with relative threshold is designed, and the control signal update is event-driven under well-established event-triggered strategy. The proposed event-triggered control scheme effectively reduces the communication burden in the controller-to-the-actuator channel and still ensures the system stability, and it achieves the control objective.

The main contents of Sections 2 to 6 are as follows: Section 2 is the partial differential system model, and it gives the assumption and control objectives. The design procedure of the event trigger controllers based on Tan-BLF and backstepping technique is introduced in Section 3. Section 4 is the system stability analysis process. In Section 5, the effectiveness of the proposed method is further demonstrated with the help of a simulation example. Finally, the conclusion is given in Section 6.

Notations. To simplify and differentiate, notations $(\mathrm{A})_r = \partial(\mathrm{A})/\partial r$, $(\dot{\mathrm{A}}) = \partial(\mathrm{A})/\partial\tau$ throughout this paper. In the same way, $(\mathrm{A})_{rr}$ means $\partial^2(\mathrm{A})/\partial r^2$, $(\mathrm{A})_{rrr} = \partial^3(\mathrm{A})/\partial r^3$ and $(\mathrm{A})_{rrrr} = \partial^4(\mathrm{A})/\partial r^4$, $(\ddot{\mathrm{A}})$

$=\partial^2(\mathrm{A})/\partial\tau^2$. In addition, $(\mathrm{A})^T$ stands for transposition of (A).

2. System description and preliminaries

Based on the Hamiltonian principle [37], the dynamic model of the flexible manipulator system is solved as follows

$$\int_{t_1}^{t_2}\left(\epsilon E_k-\epsilon E_p+\epsilon W\right)dt=0 \tag{1}$$

where $\epsilon(\mathrm{A})$ means the variation of (A). The expressions of kinetic energy E_k, potential energy E_p and work W produced in the operation of the system are respectively listed as

$$E_k=\frac{1}{2}I_h\dot{\varrho}^2(\tau)+\frac{1}{2}\int_0^L \ell\dot{Y}^2(r,\tau)dr+\frac{1}{2}m\dot{Y}^2(X,\tau) \tag{2}$$

$$E_p=\frac{1}{2}\int_0^L EI\zeta_{rr}^2(r,\tau)dr \tag{3}$$

$$W=\Phi(\tau)\varrho(\tau)+O(\tau)Y(X,\tau) \tag{4}$$

where I_h stands for the hub inertia; $\varrho(\tau)$ represents the joint angle; ℓ and Y are the mass per unit length and the arc length at r of the flexible manipulator, respectively, where $Y(r,\tau)=r\varrho(\tau)+\zeta(r,\tau)$; the mass of the payload is m; the bending stiffness is denoted by EI; the manipulator length and the connecting rod vibration deflection at r are expressed by X and $\zeta(r,\tau)$; the torque input of the joint motor and the force input of the actuator are represented with $\Phi(\tau)$ and $O(\tau)$, respectively.

Combined with the Hamiltonian principle, through a series of derivations, the system PDE model can be written as follows:

$$\ell\ddot{Y}(r,\tau)=-EI\zeta_{rrrr}(r,\tau) \tag{5}$$

$$\Phi(\tau)=I_h\ddot{\varrho}(\tau)-EI\zeta_{rr}(0,\tau) \tag{6}$$

$$O(\tau)=m\ddot{Y}(X,\tau)-EI\zeta_{rrr}(X,\tau) \tag{7}$$

$$\zeta(0,\tau)=\zeta_r(0,\tau)=\zeta_{rr}(X,\tau)=0 \tag{8}$$

Furthermore, $\varrho(\tau)$ and $\zeta(X,\tau)$ are the system outputs, and they meet $\varrho(\tau)<k_{d_1}$ and $\zeta(X,\tau)<k_{d_2}$ with k_{d_1} and k_{d_2} being constants. There are two constants k_{c_1} and k_{c_2} such that following formulas hold:

$$\left|z_1(0)\right|=\left|\varrho(0)-\varrho_d\right|<k_{c_1} \tag{9}$$

$$\left|z_3(0)\right|=\left|\zeta(X,0)-\zeta_d(X,0)\right|<k_{c_2} \tag{10}$$

where $z_1 = \varrho(\tau) - \varrho_d$, ϱ_d is the ideal angle position, and ϱ_d is a constant, and $z_3 = \zeta(X,\tau) - \zeta_d(X,\tau)$, $\zeta_d(X,\tau)$ means the required vibration.

Assumption 1 [46]**.** Suppose that the parameters $\zeta_{rr}(0,\tau)$ and $\zeta_{rrr}(X,\tau)$ are attainable.

Control objective: The event-triggered controllers are designed to realize the following control objectives:

1) suppresses the vibration of the manipulator and stabilizes it in the desired position.
2) the joint angle $\varrho(\tau)$ and boundary vibration diversion $\zeta(X,\tau)$ are confined within the constraints.
3) the system signals are all bounded.
4) it can effectively avoid the occurrence of the Zeno behavior.

3. Design of the event-triggered boundary control

The following Eqs (11) and (12) are the system boundary errors:

$$z_1(\tau) = \varrho(\tau) - \varrho_d \tag{11}$$

$$z_2(\tau) = \dot{\varrho}(\tau) - \mu(\tau) \tag{12}$$

$$z_3(\tau) = \zeta(X,\tau) - \zeta_d(X,\tau) \tag{13}$$

$$z_4(\tau) = \dot{\zeta}(X,\tau) - \vartheta(\tau) \tag{14}$$

where $\mu(\tau) = -k_1 z_1(\tau)$ and $\vartheta(\tau) = -k_3 z_3(\tau)$ are virtual controls with $\dot{\varrho}_d = 0$ and $\dot{\zeta}_d = 0$, $k_1 > 0$ and $k_3 > 0$.

Taking the derivative of (11)–(14), and combining (5)–(8), one has

$$\dot{z}_1(\tau) = z_2(\tau) + \mu(\tau) - \dot{\varrho}_d = z_2(\tau) - k_1 z_1(\tau) \tag{15}$$

$$\dot{z}_2(\tau) = \left(\Phi(\tau) + EI\zeta_{rr}(0,\tau)\right) / I_h - \dot{\mu}(\tau) \tag{16}$$

$$\dot{z}_3(\tau) = z_4(\tau) + \vartheta(\tau) - \dot{\zeta}_d(X,\tau) = z_4(\tau) - k_3 z_3(\tau) \tag{17}$$

$$\dot{z}_4(\tau) = \left(O(\tau) + EI\zeta_{rrr}(X,\tau) - \rho X\left(\Phi(\tau) + EI\zeta_{rr}(0,\tau)\right)\right) / m - \dot{\vartheta}(\tau) \tag{18}$$

where $\rho = m/I_h$.

Choose the following Lyapunov function:

$$V_1(\tau) = \frac{1}{2}\log\left(\frac{k_{c_1}^2}{k_{c_1}^2 - z_1^2(\tau)}\right) \tag{19}$$

Then, taking the derivative of $V_1(\tau)$ based on $\dot{z}_1(\tau)$, one gets

$$\dot{V}_1(\tau) = -k_1 \frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} + \frac{z_1(\tau) z_2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} \tag{20}$$

In order to eliminate the $z_1(\tau) z_2(\tau)$ in (20), the Lyapunov function $V_2(\tau)$ is selected in the following form:

$$V_2(\tau) = V_1(\tau) + \frac{1}{2} I_h z_2^2(\tau) \tag{21}$$

The derivative of $V_2(\tau)$ along time is

$$\dot{V}_2(\tau) = -k_1 \frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} + \frac{z_1(\tau) z_2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} + z_2(\tau)\big(\Phi(\tau) + EI\zeta_{rr}(0,\tau) - I_h \dot{\mu}(\tau)\big) \tag{22}$$

The boundary controller is designed as

$$\Phi(\tau) = -EI\zeta_{rr}(0,\tau) + I_h \dot{\mu}(\tau) - \frac{z_1(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2(\tau) \tag{23}$$

where $k_2 > 0$ is a constant. Substituting (23) into (22) yields

$$\dot{V}_2(\tau) = -k_1 \frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2^2(\tau) \tag{24}$$

Construct the following Lyapunov function, $V_3(\tau)$, as

$$V_3(\tau) = V_2(\tau) + \frac{1}{2}\log\left(\frac{k_{c_2}^2}{k_{c_2}^2 - z_3^2(\tau)}\right) \tag{25}$$

Then, from (17), the $\dot{V}_3(\tau)$ can be obtained as

$$\begin{aligned}\dot{V}_3(\tau) &= \dot{V}_2(\tau) - k_3 \frac{z_3^2(\tau)}{k_{c_2}^2 - z_3^2(\tau)} + \frac{z_3(\tau) z_4(\tau)}{k_{c_2}^2 - z_3^2(\tau)} \\ &= -k_1 \frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2^2(\tau) - k_3 \frac{z_3^2(\tau)}{k_{c_2}^2 - z_3^2(\tau)} + \frac{z_3(\tau) z_4(\tau)}{k_{c_2}^2 - z_3^2(\tau)}\end{aligned} \tag{26}$$

Select $V_4(\tau)$ as

$$V_4(\tau) = V_3(\tau) + \frac{1}{2} m z_4^2(\tau) \tag{27}$$

Then, the differential coefficient of $V_4(\tau)$ is

$$\begin{aligned}\dot{V}_4(\tau) &= \dot{V}_3(\tau) + m z_4(\tau)\dot{z}_4(\tau) \\ &= -k_1 \frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2^2(\tau) - k_3 \frac{z_3^2(\tau)}{k_{c_2}^2 - z_3^2(\tau)} + \frac{z_3(\tau) z_4(\tau)}{k_{c_2}^2 - z_3^2(\tau)} \\ &\quad + z_4(\tau)\Big(O(\tau) + EI\zeta_{rrr}(X,\tau) - \rho X\big(\Phi(\tau) + EI\zeta_{rr}(0,\tau)\big) - m\dot{\vartheta}(\tau)\Big)\end{aligned} \tag{28}$$

The desired boundary controller is designed as

$$O(\tau) = -EI\zeta_{rrr}(X,\tau) + \rho X\big(\Phi(\tau) + EI\zeta_{rr}(0,\tau)\big) + m\dot{\vartheta}(\tau) - \frac{z_3(\tau)}{k_{c_2}^2 - z_3^2(\tau)} - k_4 z_4(\tau) \tag{29}$$

where $k_4 > 0$ is a constant.

From (29) and (28), one has

$$\dot{V}_4(\tau) = -k_1 \frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2^2(\tau) - k_3 \frac{z_3^2(\tau)}{k_{c_2}^2 - z_3^2(\tau)} - k_2 z_4^2(\tau) \tag{30}$$

The event trigger mechanism is proposed so that the communication resources are commendably reduced.

Under the event-triggering mechanism, the boundary control strategy is designed as follows

$$\Phi_0(\tau) = \hbar_1(\tau_k),\ \forall \tau \in [\tau_k, \tau_{k+1}) \tag{31}$$

$$\tau_{k+1} = \inf\left\{\tau \in \mathbb{R} \,\|e_1(\tau)| \ge \delta_1 |\Phi_0(\tau)| + \kappa_1\right\} \tag{32}$$

$$O_0(\tau) = \hbar_2(\tau_s),\ \forall \tau \in [\tau_s, \tau_{s+1}) \tag{33}$$

$$\tau_{s+1} = \inf\left\{\tau \in \mathbb{R} \,\|e_2(\tau)| \ge \delta_2 |O_0(\tau)| + \kappa_2\right\} \tag{34}$$

where $e_1(\tau) = \hbar_1(\tau) - \Phi_0(\tau)$, $e_2(\tau) = \hbar_2(\tau) - O_0(\tau)$, κ_1, κ_2, $0 < \delta_1 < 1$, $0 < \delta_2 < 1$ are all positive design parameters. $\tau_k, k \in \mathbb{Z}^+$, and $\tau_s, s \in \mathbb{Z}^+$ are the moments when the event is triggered. The times will be respectively marked as τ_{k+1} and τ_{s+1} whenever (32) and (34) are triggered, and the control values $\Phi_0(\tau_{k+1})$ and $O_0(\tau_{s+1})$ will be applied to the system.

Design $\hbar_1(\tau)$ and $\hbar_2(\tau)$ as follows:

$$\hbar_1(\tau) = -(1+\delta_1)\left[\phi_1(\tau)\tanh\left(\frac{z_2(\tau)\phi_1(\tau)}{a_1}\right) + \bar{\kappa}_1 \tanh\left(\frac{z_2(\tau)\bar{\kappa}_1}{a_1}\right)\right] \tag{35}$$

$$\hbar_2(\tau) = -(1+\delta_2)\left[\phi_2(\tau)\tanh\left(\frac{z_4(\tau)\phi_2(\tau)}{a_2}\right) + \bar{\kappa}_2 \tanh\left(\frac{z_4(\tau)\bar{\kappa}_2}{a_2}\right)\right] \tag{36}$$

where

$$\phi_1(\tau) = -EI\zeta_{rr}(0,\tau) + I_h\dot{\mu}(\tau) - \frac{z_1(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2(\tau)$$

$$\phi_2(\tau) = -EI\zeta_{rrr}(X,\tau) + \rho X\left(I_h\dot{\mu}(\tau) - \frac{z_1(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2(\tau)\right) + m\dot{\vartheta}(\tau) - \frac{z_3(\tau)}{k_{c_2}^2 - z_3^2(\tau)} - k_4 z_4(\tau)$$

$a_i > 0, i = 1,2$, and $\bar{\kappa}_i > \kappa_i/(1-\delta_i), i = 1,2$.

According to (32) and (34), it holds that $|\hbar_1(\tau) - \Phi_0(\tau)| < \delta_1|\Phi_0(\tau)| + \kappa_1$ and $|\hbar_2(\tau) - O_0(\tau)| < \delta_2|O_0(\tau)| + \kappa_2$, for $\forall\tau\in[\tau_k,\tau_{k+1})$ and $\forall\tau\in[\tau_s,\tau_{s+1})$. Then, there are parameters $\lambda_1(\tau)\le 1$ $\lambda_2(\tau)\le 1, \eta_1(\tau)\le 1$ and $\eta_2(\tau)\le 1$, such that

$$\hbar_1(\tau) = (1+\lambda_1(\tau)\delta_1)\Phi_0(\tau) + \lambda_2(\tau)\kappa_1 \tag{37}$$

$$\hbar_2(\tau) = (1+\eta_1(\tau)\delta_2)O_0(\tau) + \eta_2(\tau)\kappa_2 \tag{38}$$

Then, we get

$$\Phi_0(\tau) = \frac{\hbar_1(\tau)}{1+\lambda_1(\tau)\delta_1} - \frac{\lambda_2(\tau)\kappa_1}{1+\lambda_1(\tau)\delta_1} \tag{39}$$

$$O_0(\tau) = \frac{\hbar_2(\tau)}{1+\eta_1(\tau)\delta_2} - \frac{\eta_2(\tau)\kappa_2}{1+\eta_1(\tau)\delta_2} \tag{40}$$

Further, $\dot{V}_2(\tau)$ and $\dot{V}_4(\tau)$ can be rewritten as

$$\begin{aligned}\dot{V}_2(\tau) = &-k_1\frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} + \frac{z_1(\tau)z_2(\tau)}{k_{c_1}^2 - z_1^2(\tau)}\\ &+z_2(\tau)(\frac{\hbar_1(\tau)}{1+\lambda_1(\tau)\delta_1} - \frac{\lambda_2(\tau)\kappa_1}{1+\lambda_1(\tau)\delta_1}\\ &+EI\zeta_{rr}(0,\tau) - I_h\dot{\mu}(\tau))\end{aligned} \tag{41}$$

Note that, for $\forall n\in R$ and $a_i > 0, i=1,2$, one has $-n\tanh(n/a_i)\le 0$. Then, based on (35), one has

$$z_2(\tau)\hbar_1(\tau)\le 0 \tag{42}$$

In addition, due to $\lambda_1(\tau)\le 1$ and $\lambda_2(\tau)\le 1$, (41) and (42) hold

$$\frac{z_2(\tau)\hbar_1(\tau)}{1+\lambda_1(\tau)\delta_1} \le \frac{z_2(\tau)\hbar_1(\tau)}{1+\delta_1} \tag{43}$$

$$\left|\frac{\lambda_2(\tau)\kappa_1}{1+\lambda_1(\tau)\delta_1}\right| \le \frac{\kappa_1}{1-\delta_1} \tag{44}$$

Thus, according to (35), one gets

$$\frac{z_2(\tau)\hbar_1(\tau)}{1+\lambda_1(\tau)\delta_1} \le -z_2(\tau)\hbar_1(\tau)\tanh\left(\frac{z_2(\tau)\hbar_1(\tau)}{a_1}\right) - z_2(\tau)\bar{\kappa}_1\tanh\left(\frac{z_2(\tau)\bar{\kappa}_1}{a_1}\right) \tag{45}$$

Both adding and subtracting $|z_2(\tau)\bar{\kappa}_1|$ and $z_2(\tau)\hbar_1(\tau)$, one obtains

$$\begin{aligned}\frac{z_2(\tau)\hbar_1(\tau)}{1+\lambda_1(\tau)\delta_1} \le {} & |z_2(\tau)\hbar_1(\tau)| - z_2(\tau)\hbar_1(\tau)\tanh\left(\frac{z_2(\tau)\hbar_1(\tau)}{a_1}\right) + |z_2(\tau)\bar{\kappa}_1| \\ & - z_2(\tau)\bar{\kappa}_1\tanh\left(\frac{z_2(\tau)\bar{\kappa}_1}{a_1}\right) - |z_2(\tau)\bar{\kappa}_1| + z_2(\tau)\hbar_1(\tau)\end{aligned} \tag{46}$$

Consider the property of $\tanh(\bullet)$ that

$$0 \le |D| - D\tanh\left(\frac{D}{\gamma}\right) \le 0.2785\gamma \tag{47}$$

with $D \in R$ and $\gamma > 0$. Therefore, one has

$$\frac{z_2(\tau)\hbar_1(\tau)}{1+\lambda_1(\tau)\delta_1} \le 0.557a_1 - |z_2(\tau)\bar{\kappa}_1| + z_2(\tau)\hbar_1(\tau) \tag{48}$$

Substituting (48) into (47), one has

$$\begin{aligned}\dot{V}_2(\tau) \le {} & -k_1\frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2^2(\tau) + 0.557a_1 \\ & - |z_2(\tau)\bar{\kappa}_1| - z_2(\tau)\frac{\lambda_2(\tau)\kappa_1}{1+\lambda_1(\tau)\delta_1}\end{aligned} \tag{49}$$

Then, based on (44), this leads to

$$\begin{aligned}\dot{V}_2(\tau) \le {} & -k_1\frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2^2(\tau) + 0.557a_1 \\ & - |z_2(\tau)\bar{\kappa}_1| + \left|\frac{z_2(\tau)\kappa_1}{1-\delta_1}\right|\end{aligned} \tag{50}$$

Further consider $\bar{\kappa}_1 > \kappa_1/(1-\delta_1)$, this leads to

$$-|z_2(\tau)\bar{\kappa}_1| + \left|\frac{z_2(\tau)\kappa_1}{1-\delta_1}\right| \le 0 \tag{51}$$

Then, $\dot{V}_2(\tau)$ is further expressed as

$$\dot{V}_2(\tau) \le -k_1\frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2^2(\tau) + 0.557a_1 \tag{52}$$

and $\dot{V}_3(\tau)$ can be rewritten as

$$\dot{V}_3(\tau) \le -k_1 \frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2^2 + 0.557 a_1 - k_3 \frac{z_3^2(\tau)}{k_{c_2}^2 - z_3^2(\tau)} + \frac{z_3(\tau) z_4(\tau)}{k_{c_2}^2 - z_3^2(\tau)} \tag{53}$$

In the same way, consider (40), and $\dot{V}_4(\tau)$ can be rewritten as

$$\begin{aligned} \dot{V}_4(\tau) &= \dot{V}_3(\tau) + m z_4(\tau) \dot{z}_4(\tau) \\ &\le -k_1 \frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2^2(\tau) - k_3 \frac{z_3^2(\tau)}{k_{c_2}^2 - z_3^2(\tau)} + \frac{z_3(\tau) z_4(\tau)}{k_{c_2}^2 - z_3^2(\tau)} \\ &\quad + 0.557 a_1 + z_4(\tau) \left(\frac{\hbar_2(\tau)}{1 + \eta_1(\tau)\delta_2} - \rho X \left(\Phi(\tau) + EI \zeta_{rr}(0,\tau) \right) \right. \\ &\quad \left. + EI \zeta_{rrr}(X,\tau) - m \dot{\vartheta}(\tau) - \frac{\eta_2(\tau)\kappa_2}{1 + \eta_1(\tau)\delta_2} \right) \end{aligned} \tag{54}$$

Note that when $a_2 > 0$ for $\forall n \in R$, $-n \tanh(n/a_2) \le 0$ is always true. Thus, from (36), it can be sure that

$$z_4(\tau) \hbar_2(\tau) \le 0 \tag{55}$$

Since $|\eta_i| \le 1, i = 1, 2$, it can be seen that

$$\frac{z_4(\tau)\hbar_2(\tau)}{1 + \eta_1(\tau)\delta_2} \le \frac{z_4(\tau)\hbar_2(\tau)}{1 + \delta_2} \tag{56}$$

$$\left| \frac{\eta_2(\tau)\kappa_2}{1 + \eta_1(\tau)\delta_2} \right| \le \frac{\kappa_2}{1 - \delta_2} \tag{57}$$

Further, according to (36), one has

$$\frac{z_4(\tau)\hbar_2(\tau)}{1 + \eta_1(\tau)\delta_2} \le -z_4(\tau)\hbar_2(\tau) \tanh\left(\frac{z_4(\tau)\hbar_2(\tau)}{a_2} \right) - z_4(\tau)\bar{\kappa}_2 \tanh\left(\frac{z_4(\tau)\bar{\kappa}_2}{a_2} \right) \tag{58}$$

Then, both adding and subtracting $|z_4(\tau)\bar{\kappa}_2|$ and $z_4(\tau)\hbar_2(\tau)$ on the right side of (58), it holds that

$$\begin{aligned} \frac{z_4(\tau)\hbar_2(\tau)}{1 + \eta_1(\tau)\delta_2} &\le |z_4(\tau)\hbar_2(\tau)| - z_4(\tau)\hbar_2(\tau) \tanh\left(\frac{z_4(\tau)\hbar_2(\tau)}{a_2} \right) + |z_4(\tau)\bar{\kappa}_2| \\ &\quad - z_4(\tau)\bar{\kappa}_2 \tanh\left(\frac{z_4(\tau)\bar{\kappa}_2}{a_2} \right) - |z_4(\tau)\bar{\kappa}_2| + z_4(\tau)\hbar_2(\tau) \end{aligned} \tag{59}$$

Because of the property in (47), it is further known that

$$\frac{z_4(\tau)\hbar_2(\tau)}{1+\eta_1(\tau)\delta_2} \le 0.557a_2 - \left|z_4(\tau)\bar{\kappa}_2\right| + z_4(\tau)\hbar_2(\tau) \tag{60}$$

Substituting (60) into (54), one has

$$\begin{aligned}\dot{V}_4(\tau) \le & -k_1\frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2^2(\tau) - k_3\frac{z_3^2(\tau)}{k_{c_2}^2 - z_3^2(\tau)} - k_4 z_4^2(\tau) \\ & + 0.557a_1 + 0.557a_2 - \left|z_4(\tau)\bar{\kappa}_2\right| - z_4(\tau)\frac{\eta_2(\tau)\kappa_2}{1+\eta_1(\tau)\delta_2}\end{aligned} \tag{61}$$

Then, based on (57), this leads to

$$\begin{aligned}\dot{V}_4(\tau) \le & -k_1\frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2^2(\tau) - k_3\frac{z_3^2(\tau)}{k_{c_2}^2 - z_3^2(\tau)} - k_4 z_4^2(\tau) \\ & + 0.557a_1 + 0.557a_2 - \left|z_4(\tau)\bar{\kappa}_2\right| + \left|\frac{z_4(\tau)\kappa_2}{1-\delta_2}\right|\end{aligned} \tag{62}$$

Further consider $\bar{\kappa}_2 > \kappa_2/(1-\delta_2)$, and one has

$$-\left|z_4(\tau)\bar{\kappa}_2\right| + \left|\frac{z_4(\tau)\kappa_2}{1-\delta_2}\right| \le 0 \tag{63}$$

Then, $\dot{V}_4(\tau)$ is further expressed as

$$\begin{aligned}\dot{V}_4(\tau) \le & -k_1\frac{z_1^2(\tau)}{k_{c_1}^2 - z_1^2(\tau)} - k_2 z_2^2(\tau) - k_3\frac{z_3^2(\tau)}{k_{c_2}^2 - z_3^2(\tau)} \\ & - k_4 z_4^2(\tau) + 0.557a_1 + 0.557a_2\end{aligned} \tag{64}$$

4. Stability analysis

We can get the theorem result as below according to the above analysis process.

Theorem 1: Consider the flexible manipulator system as shown in (5)–(8), under Assumption 1, and design the event-triggered controllers in (35) and (36). Then the presented approach guarantees that 1) the vibration of the manipulator is effectively restrained and stabilized, 2) all the signals displaying in the closed-loop system are bounded, 3) the joint angle $\varrho(\tau)$ and the boundary vibration diversion $\zeta(X,\tau)$ fulfill the constraint conditions $\varrho(\tau) < k_{d_1}$ and $\zeta(X,\tau) < k_{d_2}$, respectively, and 4) the system can effectively avoid the occurrence of the Zeno behavior.

Proof:

The Barrier Lyapunov function is considered as

$$V(\tau)=V_4(\tau)=\frac{1}{2}\log\left(\frac{k_{c_1}^2}{k_{c_1}^2-z_1^2(\tau)}\right)+\frac{1}{2}I_h z_2^2(\tau)$$
$$+\frac{1}{2}\log\left(\frac{k_{c_2}^2}{k_{c_2}^2-z_3^2(\tau)}\right)+\frac{1}{2}mz_4^2(\tau) \tag{65}$$

On the basis of the above analysis, one obtains

$$\dot{V}(\tau)\le -k_1\frac{z_1^2(\tau)}{k_{c_1}^2-z_1^2(\tau)}-k_2 z_2^2(\tau)-k_3\frac{z_3^2(\tau)}{k_{c_2}^2-z_3^2(\tau)}$$
$$-k_4 z_4^2(\tau)+0.557a_1+0.557a_2 \tag{66}$$
$$\le -cV(\tau)+d$$

where $c=\min(2k_1,2k_2/I_h,2k_3,2k_4/m)$, and $d=0.557a_1+0.557a_2$.

Multiplying $e^{c\tau}$ on both sides of (66), and integrating (66) over $[0,\tau]$, one can obtained that

$$V(\tau)\le\left(V(0)-\frac{d}{c}\right)e^{-c\tau}+\frac{d}{c} \tag{67}$$

According to (66) and (67), the boundedness of errors $z_i, i=1,2,3,4$ is known. Meanwhile, since ϱ_d, μ, $\zeta_d(X,\tau)$ and ϑ are bounded, according to (11)–(14), one gets that $\varrho(\tau)$, $\dot{\varrho}(\tau)$, $\zeta(X,\tau)$ and $\dot{\zeta}(X,\tau)$ are also bounded. Similarly, according to (35) and (36), the boundedness of $\hbar_1(\tau)$ and $\hbar_2(\tau)$ are obviously proved. Considering $e_1=\hbar(\tau)-\nu_d$ and $e_3=\rho(X,\tau)-\rho_d(X,\tau)$, we can get $|\varrho(\tau)|=|z_1+\varrho_d|\le|z_1|+|\varrho_d|$ and $|\zeta(X,\tau)|=|z_3+\zeta_d(X,\tau)|\le|z_3|+|\zeta_d(X,\tau)|$. According to (9) and (10), it holds that $|\varrho(\tau)|\le k_{c_1}+|\varrho_d|<k_{d_1}$ and $|\zeta(X,\tau)|\le k_{c_2}+|\zeta_d(X,\tau)|<k_{d_2}$, which means that system states are within their constraint bounds.

Recall the definition of $e_i(\tau), i=1,2$, i.e., $e_1(\tau)=\hbar_1(\tau)-\Phi_0(\tau)$, $e_2(\tau)=\hbar_2(\tau)-U_0(\tau)$, where $\Phi_0(\tau)=\hbar_1(\tau_k)$ for $\forall\tau\in[\tau_k,\tau_{k+1})$, and $O_0(\tau)=\hbar_2(s_k)$ for $\forall\tau\in[\tau_s,\tau_{s+1})$. Then, one has

$$\dot{e}_1(\tau)=\dot{\hbar}_1(\tau)-\dot{\hbar}_1(\tau_k)$$
$$\dot{e}_2(\tau)=\dot{\hbar}_2(\tau)-\dot{\hbar}_2(\tau_s)$$

Here $\dot{\hbar}_1(\tau_k)$ and $\dot{\hbar}_2(\tau_s)$ can be regarded as constants at the time interval $[\tau_k,\tau_{k+1})$ and $[\tau_s,\tau_{s+1})$, which means that $\dot{\hbar}_1(\tau_k)=0$ and $\dot{\hbar}_2(\tau_s)=0$. Then, we have

$$\frac{d}{d\tau}|e_1|=\text{sign}(e_1)\dot{e}_1\le|\dot{\hbar}_1| \tag{68}$$

$$\frac{d}{d\tau}|e_2|=\text{sign}(e_2)\dot{e}_2\le|\dot{\hbar}_2| \tag{69}$$

According to the definition of $\hbar_1(\tau)$ and $\hbar_2(\tau)$ in (35) and (36), we know that $\dot{\hbar}_1(\tau)$ and

$\dot{\hbar}_2(\tau)$ are the functions of $z_i, i = 1,2,3,4$. From the result before, all the system signals are bounded, so $\dot{\hbar}_1(\tau)$ and $\dot{\hbar}_2(\tau)$ are bounded. Then, we assume that $\dot{\hbar}_1(\tau) \le \bar{\hbar}_1$, $\dot{\hbar}_2(\tau) \le \bar{\hbar}_2$ with $\bar{\hbar}_1$ and $\bar{\hbar}_2$ being constants. In addition, $e_1(\tau_k) = 0$, $e_2(\tau_s) = 0$ and $\lim_{\tau \to \tau_{k+1}} e_1(\tau_{k+1}) = \delta_1 |\Phi_0(\tau)| + \kappa_1$, $\lim_{\tau \to \tau_{s+1}} e_2(\tau_{s+1}) = \delta_2 |O_0(\tau)| + \kappa_2$. By integrating (68) and (69) on their both sides, one gets $\tau_{k+1} - \tau_k \ge T = \left(\delta_1 |\Phi_0(\tau)| + \kappa_1\right) / \bar{\hbar}_1$ and $\tau_{s+1} - \tau_s \ge T = \left(\delta_2 |O_0(\tau)| + \kappa_2\right) / \bar{\hbar}_2$. Thus, the Zeno behavior can be avoided.

Theorem 1 is demonstrated integrally.

5. Simulation

In order to verify the effectiveness of the control strategy designed in this paper, a system simulation based on (5)–(8) is considered. The system parameters are selected as follows: $EI = 10\,Nm^2$, $X = 1\,\mathrm{m}$, $\ell = 0.5\,kgm^{-1}$, $m = 2\,kg$, and $I_h = 1\,kgm^2$. The other related parameters are chosen as $\varrho_d = 0.03$, $\zeta_d = 0$, $k_1 = 10$, $k_2 = 10$, $k_3 = 10$, $k_4 = 10$, $\kappa_1 = 1$, $\kappa_2 = 1$, $\delta_1 = 0.15$, $\delta_2 = 0.15$, $\bar{\kappa}_1 = 2$, $\bar{\kappa}_2 = 2$, $a_1 = 0.4$, $a_2 = 0.4$. In order to further compare with other control methods, the proportional differential (PD) control $\hbar_1(\tau) = -2.5\zeta_{rr}(0,\tau) - 1.5\varrho(\tau) - 1.5\dot{\varrho}(\tau)$, $\hbar_2(\tau) = -2.5\zeta_{rrr}(X,\tau) - 1.5\zeta(X,\tau) - 1.5\dot{\zeta}(X,\tau)$ is proposed in this paper. The simulation results are given as Figures 1–9.

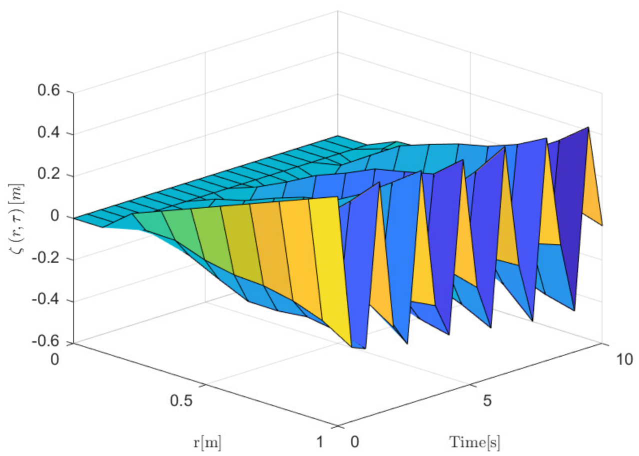

Figure 1. Displacement $\zeta(r,\tau)$ of the system without control.

Figure 1 shows the system vibration deflection without any control. It is obvious that the manipulator moves freely with large amplitude. Figure 2 shows the system vibration deflection under the action of the event trigger controller. It can be seen from the figure that the amplitude of the manipulator becomes gentle within a short time, forming an obvious contrast with Figure 1. Figure 3 is the trajectory of boundary vibration deflection $\zeta(X,\tau)$ of the system with (curve) or without (dotted line) control. It can be seen that when the system does not apply any control, $\zeta(X,\tau)$

changes periodically, and the amplitude is changed greatly, which will damage the flexible manipulator system and reduce the working accuracy. In the case of control, the boundary vibration deflection $\zeta(X,\tau)$ gradually tends to be stable, which greatly reduces the system loss. Figures 4 and 5 respectively indicate the trajectories of state junction angle $\varrho(\tau)$ and state boundary deflection $\zeta(X,\tau)$. From the figures we can know, that $\varrho(\tau)$ and $\zeta(X,\tau)$ remain within the constraint boundaries. In the meantime, the tracking performances of $\varrho(\tau)$ and $\zeta(X,\tau)$ are good. It can be clearly seen that based on the adopted control solution; they are adjusted to the expected value. Figure 6 is the trajectories of event trigger controllers $\hbar_1(\tau)$ and $\hbar_2(\tau)$. Figure 7 is the time interval for triggering events, which indicates that the Zeno phenomenon is successfully avoided. The displacement and the boundary output changes under state constraints with PD control are shown in Figures 8 and 9. Compared with the simulation results in the previous case, it is obvious that the control strategy proposed in this paper is smoother and more effective than PD control. Obviously, they are bounded. From all the analysis so far, we conclude the following result. The control objectives of this paper can be realized under the action of the proposed control strategy.

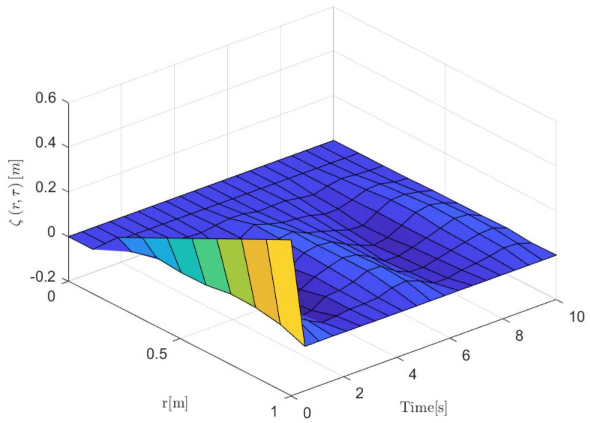

Figure 2. Displacement $\zeta(r,\tau)$ of the system with event trigger control.

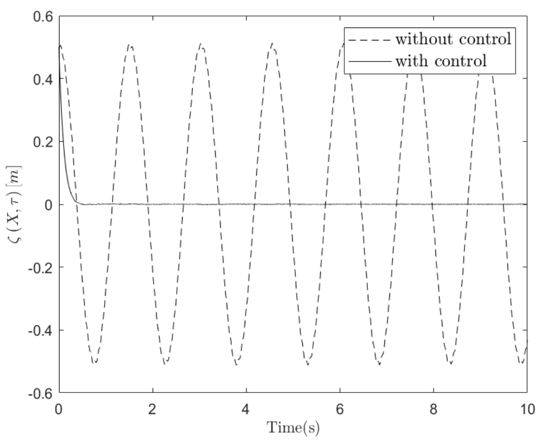

Figure 3. Trajectories of $\zeta(X,\tau)$ under control (solid line) and without control (dotted line).

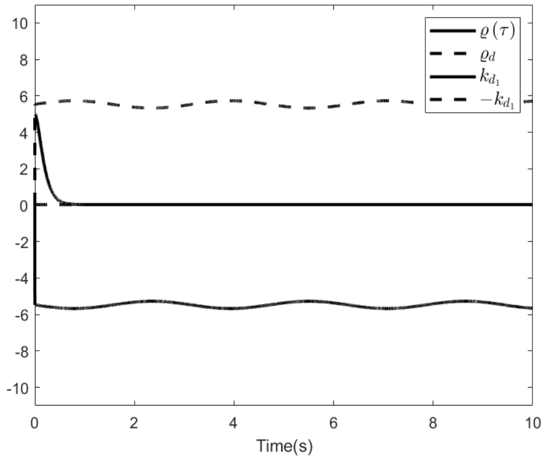

Figure 4. Trajectories of $\varrho(\tau)$, ϱ_d, k_{d_1} and $-k_{d_1}$.

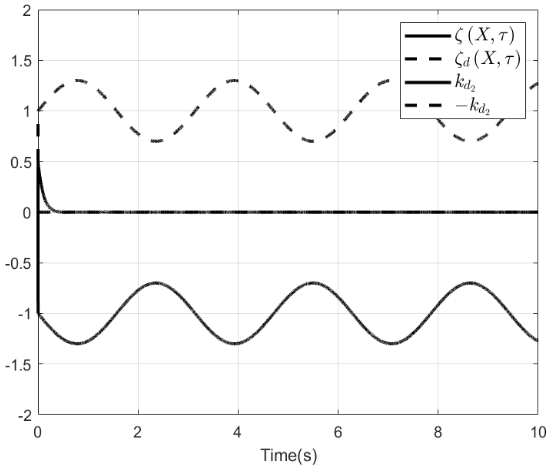

Figure 5. Trajectories of $\zeta(X,\tau)$, c, k_{d_2} and $-k_{d_2}$.

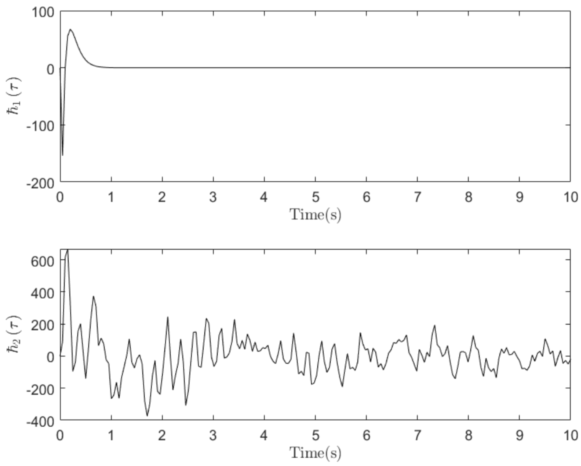

Figure 6. Trajectories of $\hbar_1(\tau)$ and $\hbar_2(\tau)$.

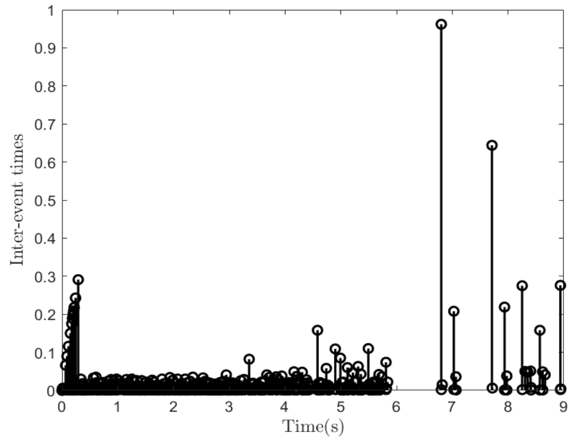

Figure 7. The time intervals of the event-triggered manipulator.

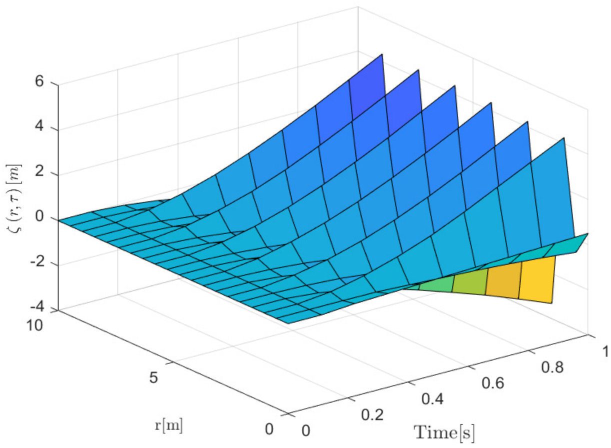

Figure 8. Displacement $\zeta(r,\tau)$ of the system with PD control.

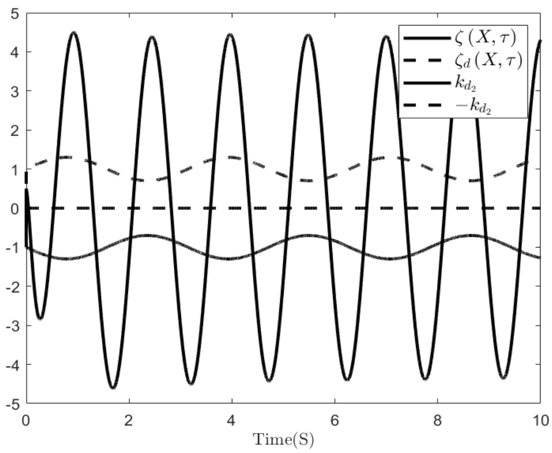

Figure 9. Trajectories of $\zeta(X,\tau)$, c, k_{d_2} and $-k_{d_2}$ under PD control.

6. Conclusions

A vibration suppression control algorithm with event-triggered mechanism is proposed for manipulator system described by PDEs with state constraints. State constraints and event-triggered problems are considered simultaneously in the course controller design. Based on the BLF and relative threshold strategy, the method we developed reduces the cost of information transmission and guarantees that the system signals of the under consideration are all bounded. The elastic vibration of flexible manipulator is well suppressed, and the constrained states do not break the constraint bounds. In addition, the Zeno phenomenon is successfully avoided. Simulation results show the validity of the proposed algorithm. At a future date, the proposed scheme can be further studied and spread in other partial differential practical systems with DoS attacks like in [47].

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Conflict of interest

The authors declare that they have no conflict of interest.

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