Mathematical Biosciences and Engineering

MBE, 20(12): 21626–21642.
DOI: 10.3934/mbe.2023957
Received: 15 October 2023
Revised: 27 November 2023
Accepted: 30 November 2023
Published: 06 December 2023

http://www.aimspress.com/journal/MBE

Research article

Effect of color cross-correlated noise on the growth characteristics of tumor cells under immune surveillance

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Abstract: Based on the Michaelis-Menten reaction model with catalytic effects, a more comprehensive one-dimensional stochastic Langevin equation with immune surveillance for a tumor cell growth system is obtained by considering the fluctuations in growth rate and mortality rate. To explore the impact of environmental fluctuations on the growth of tumor cells, the analytical solution of the steady-state probability distribution function of the system is derived using the Liouville equation and Novikov theory, and the influence of noise intensity and correlation intensity on the steady-state probability distributional function are discussed. The results show that the three extreme values of the steady-state probability distribution function exhibit a structure of two peaks and one valley. Variations of the noise intensity, cross-correlation intensity and correlation time can modulate the probability distribution of the number of tumor cells, which provides theoretical guidance for determining treatment plans in clinical treatment. Furthermore, the increase of noise intensity will inhibit the growth of tumor cells when the number of tumor cells is relatively small, while the increase in noise intensity will further promote the growth of tumor cells when the number of tumor cells is relatively large. The color cross-correlated strength and cross-correlated time between noise also have a certain impact on tumor cell proliferation. The results help people understand the growth kinetics of tumor cells, which can a provide theoretical basis for clinical research on tumor cell growth.

Keywords: tumor cells; Langevin equation; noise; relevance; immune mechanism; Fokker-Planck equation

1. Introduction

Tumor cells are different from other cells in the body, as they are immortal, not controlled by the growth and proliferation of the body and invade the surrounding tissues. It is precisely because of these characteristics of tumor cells that people are helpless in the treatment and prevention of tumor cells [1]. Many scientists in these fields have tried to change this situation using their expertise, thus recording excellent research results, among which is the effects of noise and its connections to nonlinear systems [2]. The growth and change process of tumor cells is a complex nonlinear dynamic process; therefore, many researchers consider the influence of noise and its association on the growth process of tumor cells [3].

More and more facts have proven that tumors are considered a global disease that causes human death, mostly characterized by unnatural cell growth, rapid spread and obstruction of normal cell function [4]. With the increasing number of cancer patients, tumor research and treatment have become a major challenge in the fields of clinical medicine and biology. In recent decades, various mathematical and physical models for describing tumor growth have emerged, such as Logistic models, Gomperztion models, Self-limiting models and Eden models [5]. Due to the enormous pain and economic pressure that surgery, chemotherapy and radiation therapy can bring to patients and their families in clinical treatment, as well as their adverse effects on their lives, the understanding of immunotherapy has received considerable attention [6]. In practical situations, the growth process of tumor cells is always influenced by some random factors (such as temperature, radiation therapy, chemotherapy, drugs, etc.), and the appearance and growth of tumors are the result of a very complex interaction between the tumor and the immune system, which is likely to be nonlinear and time-varying [7].

Given the complexity of the growth process of tumor cells, a large number of researchers have begun to explore a more comprehensive and high-dimensional predator-prey model or a catalytic Michaelis Menten reaction model, striving to find a dynamic model that is closer to the growth process of tumor cells and has immune surveillance [8]. Zhong et al. [9] investigated the effect of multiplicative noise on tumor cell growth and confirmed that multiplicative noise has a differentiation effect on the growth law of tumor cells. Fiasconaro et al. [10] examined the effect of noise on the mobility of tumors under immune surveillance and the effect of noise on tumor apoptosis. Fang et al. [11] considered the correlation state between noise, and obtains that in some cases there is a certain form of correlation between noise, and this correlation state has a large influence on the state of the nonlinear system. After the results of these studies were brought up, Ai et al. [12] considered the influence of the association between additive white noise and multiplicative white noise on the growth state of tumor cells, which showed that the association strength of additive noise and multiplicative noise can affect the survival of tumor cells. Hua et al. [13] found the transition in a delayed tumor growth model with non-Gaussian colored noise. Alsakaji et al. [14] studied stochastic tumor-immune interaction model with external treatments and time delays, and they developed a stochastic optimality system to reduce tumor cells using some control variables. Rihan et al. [15] investigated the dynamics of a time-delay differential model for tumor-immune interactions with random noise, and they proved that in contrast to the deterministic model which shows no stable tumor-free state, the white noise can either lead to tumor dormancy or tumor elimination.

All these studies confirmed that noise and its association have a great impact on the complex nonlinear kinetic process of tumor cell proliferation and decay [16]. Although the immune system of the body cannot remove all the tumor cells in the body, the body will produce the corresponding tumor

immune mechanism against the abnormal proliferation of tumor cells [17]. As the tumor immune mechanism is nonlinear, and the temperature, humidity and nutrients in the body can promote the proliferation of tumor cells in some cases [18]. In the process of treatment, external factors such as radioactive substances, drug dose and intensity of radioactive substances can also promote the proliferation of tumor cells [19]. Under the simultaneous action of multiple different factors, whether tumor cells proliferate or die needs specific analysis [20].

Based on the non-linear growth of tumor cells under the immune mechanism of the organism, the practical one-dimensional Langevin equation is obtained through the catalytic Michaelis-Menten reaction model [21,22]. In this paper, external and internal factors affecting the proliferation and decay of tumor cells were introduced into the model in the form of multiplicative and additive noise, respectively [23]. The effects of two kinds of noise on tumor cell proliferation and decay in the case of different color association strength and color association time were considered using the linearized approximation method and the fastest descent method [24]. Finally, the steady-state probability distribution function of the calculated system is derived [25]. By observing the changes in noise intensity, as well as the correlation strength and correlation time between noises, we aim to explore whether tumor cell therapy mainly manifests as proliferation or decay [26]. We hope to provide a more clinical theory for the treatment of tumor diseases by studying the proliferation and decay of tumor cells in such a complex environment. Based on this theory, clinical treatment can establish reasonable and effective treatment plans.

2. Model construction

Human immune mechanisms can recognize and remove a certain number of tumor cells in the human body [27]. In the past years, there were large number of related studies on tumor cell growth process under the effects of human immune mechanism, and some researchers proposed tumor cell immune surveillance model [28]. Considering the fact that the human immune mechanism is an indispensable influencing factor for the growth and proliferation of tumor cells, the growth and proliferation status of tumor cells under the influence of human immune mechanism is represented by enzymatic reactions as follows [29].

$$N\text{ormal cells} \xrightarrow{A} 2X \tag{1}$$

$$X \xrightarrow{\lambda} 2X \tag{2}$$

$$X + Y \xrightarrow{K_1} Z \xrightarrow{K_2} Y + P \tag{3}$$

where X in the above equation indicates the tumor cells, Y indicates the immune cells, Z indicates the compounds of tumor cells and immune cells, P indicates dead or unproductive cells, A shows the conversion rate of the normal cells, λ represents the replication rate of the tumor cells, K_1 indicates the binding rate of immune cells bound cancer cells and K_2 represents the decomposition rate of compound Z [30].

According to Michaelis-Menten theory, the above equations can be transformed into a univariate system, and the evolution equation of tumor cell over time can be simplified as the following Langevin

equation [31]. According to existing theories, this equation is more suitable for the actual growth and proliferation process of tumor cells [32]. It is expressed as follows.

$$\frac{dx}{dt} = \alpha + x(1-\theta x) - \beta \frac{x}{1-x} \tag{4}$$

where x represents the number of tumor cells, α and β denote variables related to tumor cell proliferation, in which β is the specific immune coefficient and satisfies $\beta \geq 0$ [33].

On the basis of the above equation, the expression of the definite theoretic potential function can be obtained as follows [34].

$$V(x) = -\alpha x - \frac{1}{2}x^2 + \frac{\theta}{3}x^3 + \beta x - \beta \ln(x+1) \tag{5}$$

The variables α in the above expression is related to the proliferation of the tumor cells. In the existing studies, $\alpha = 0$ was taken, that is, the tumor cell proliferation was not considered [35]. As can be seen from the $V(x)$ given in Figure 1, the value of α has an impact on the properties of the system. Therefore, in order to reduce the theoretical error, it is reassigned in the later study [36]. In other words, we used $\alpha \neq 0$ to modify the parameters related to tumor cell birth rate, making the model closer to the real situation [37].

It can be known that the process of tumor cell growth and proliferation are inevitably affected by external environmental factors, and the environmental factors that affect tumor cells exist in all aspects [38]. In this study, we focus on the effects of drugs, radioactive substances and drug dosage during treatment on the growth and proliferation of tumor cells [38,39].

As we can see, Equation (4) is a deterministic differential equation. Due to differences in individual genetics, behavior and abilities, or treatment methods such as surgery, chemotherapy and radiotherapy, the growth of tumor cells exhibits randomness, which affects the specific immune coefficient in the form of multiplicative noise [40]. Furthermore, considering the impact of environmental fluctuations on birth rate, such as temperature, oxygen and nutrient supply and host immune status, they belong to external noise and are added to the system in the form of additive noise [41]. For example, certain uncertain physical and chemical factors, the temperature of the body's survival, the supply of nutrients in the cell survival environment and fluctuations in the psychological state of patients with tumor diseases, sudden accidents leading to physiological discomfort, and the random effects of different treatment methods in different treatment plans, may lead to certain fluctuations in the growth rate of tumor cells and immune factors of the body [42]. These factors become additive noise terms and are added to the system.

From the perspective of mathematical statistics and signal processing, additive noise generally refers to thermal noise, shot noise, etc. [43]. Or in some cases, it is also referred to as external noise, which refers to the noise caused by external interference entering the system [44]. Their relationship with signals is additive, and noise exists regardless of whether there is a signal or not. In general communication, additive randomness is considered as the background noise of the system [45]. Multiplicative noise is generally caused by imperfect channels. Their relationship with the signal is multiplication, where the signal is present and the noise is present [46]. If the signal is not present, the noise will disappear. Multiplicative randomness is seen as a result of the time-varying or nonlinear

nature of the system. It is easy to know that in most cases, multiplicative noise actually exists in the form of internal noise in nonlinear systems, and it represents the noise caused by the internal components of the system, such as mutual interference between internal components. Multiplicative noise is widely present in real-world image applications [47].

Taking these factors into account, the impact of multiplicative noise $\xi(t)$ is introduced into the tumor cell proliferation system and it acts on the specificity coefficient β; thus, one can obtain $\beta \to \beta + \xi(t)$. Similarly, the additive noise $\eta(t)$ is introduced into Eq (4) to describe the fluctuations in a parameter related to tumor cell birth rate and mortality rate α, hence it can be rewritten as $\alpha \to \alpha + \eta(t)$ [48]. After bringing the above influencing factors into the equation, the Langevin equation is more consistent with the change of tumor cell growth [49]. It can be expressed as follows

$$\frac{dx}{dt} = \alpha + x(1-\theta x) - \beta \frac{x}{1+x}\xi(t) + \eta(t) \tag{6}$$

where $\xi(t)$ and $\eta(t)$ represent multiplicative Gaussian white noise and additive white Gaussian noise with a mean of zero, and they satisfy the following statistical properties as follows [50,51].

$$\xi(t) = \eta(t) = 0 \tag{7}$$

$$\left\langle \xi(t)\xi(t') \right\rangle = 2D\delta(t-t') \tag{8}$$

$$\left\langle \eta(t)\eta(t') \right\rangle = 2Q\delta(t-t') \tag{9}$$

$$\left\langle \xi(t)\eta(t') \right\rangle = \left\langle \eta(t)\xi(t') \right\rangle = \frac{\lambda\sqrt{DQ}}{\tau} \exp\left\{ \frac{-|t-t'|}{\tau} \right\} \tag{10}$$

where the magnitude of the D and Q represents the noise intensity, λ represents the color cross-correlated strength between multiplicative Gaussian white noise and additive white Gaussian noise, t and t' denote time and τ represents the color correlation time between the two kinds of noise [51].

3. Fokke-Planck equation and its steady-state solutions

According to the method of reference, make an equivalent transformation to Eq (6) and rewrite it as follows [52].

$$\frac{dx}{dt} = f(x) + g_1(x)\xi(t) + g_2(x)\eta(t) \tag{11}$$

$$f(x) = \alpha + x(1-\theta x) - \beta \frac{x}{1+x} \tag{12}$$

$$g_1(x) = -\frac{1}{1+x}, g_2(x) = 1 \tag{13}$$

According to the Liouville equation, the following results can be obtained.

$$\frac{\partial \rho(x,t)}{\partial t} = -\frac{\partial}{\partial x}[f(x) + g_1(x)\xi(t) + g_2(x)\eta(t)]\rho(x,t) \tag{14}$$

In terms of the theory of nonlinear dynamics, the Liouville equation and the VAN Kampen Lemma, it can be inferred that the ensemble average is as follows.

$$P(x,t) = < \delta(x(t) - x) > \tag{15}$$

and finally the following equation can be obtained.

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x} f(x)P(x,t) - \frac{\partial}{\partial x} g_1(x) < \xi(t)\delta(x(t) - x) > -\frac{\partial}{\partial x} g_2(x) < \mu(t)\delta(x(t) - x) > \tag{16}$$

According to Novikov theorem [53],

$$< \zeta_k \Phi[\zeta_1, \zeta_2] > = \int_0^t dt' \gamma_{kl}(t,t') < \frac{\delta(\delta(x(t) - x))}{\delta \zeta_l(t')} >, \ (k,l = 1,2) \tag{17}$$

the following equation can be obtained.

$$\begin{aligned} \frac{\partial P(x,t)}{\partial t} &= -\frac{\partial}{\partial x}\{f(x) + Dg_1(x)g_1'(x) + \lambda\sqrt{DQ}[g_1(x)g_2'(x) + g_1'(x)g_2(x)] + Qg_2(x)g_2'(x)\}P(x,t) \\ &+ \frac{\partial^2}{\partial x^2}[Dg_1^2(x) + 2\lambda\sqrt{DQ}g_1(x)g_2(x) + Qg_2^2(x)]P(x,t) \end{aligned} \tag{18}$$

By solving the above equation, the corresponding Fokker-Planck equation is obtained and it can be expressed as follows [54].

$$\partial P(x,t) = -\frac{\partial}{\partial(x)} A(x)P(x,t) + \frac{\partial^2}{\partial x^2} B(x)P(x,t) \tag{19}$$

where $P(x, t)$ in the above equation represents the steady-state probability distribution function in the model, and $A(x)$ and $B(x)$ can be indicated as follows [55].

$$A(x) = f(x) + Dg_1(x)g_1'(x) + \frac{\lambda\sqrt{DQ}}{1 - \tau f'(x_s)}(g_1(x)g_2'(x)) \tag{20}$$

$$B(x) = Dg_1^2(x) + \frac{2\lambda\sqrt{DQ}}{1 - \tau f'(x_s)} g_1(x)g_2(x) + \theta g_2^2(x) \tag{21}$$

Since the number of tumor cells cannot be negative, it can be determined that $x \geq 0$ in Eq (19). According to the reflection boundary conditions, the steady-state solution of the Fokker-Planck equation can be obtained as follows [56].

$$P_{st}(x) = \frac{N}{\sqrt{B(x)}} \exp\{\int \frac{f(x)}{B(x)} dx\}. \tag{22}$$

The steady-state probability distributional function $P_{st}(x)$ is the solution to Eq (19) obtained under steady-state conditions, which is used to describe the probability of the number of tumor cells appearing in a certain number of distributions in the dynamic stochastic differential equation given by Eq (19) under steady-state conditions. N indicates the normalization constant, and the value of N is determined by the normalization conditions as follows.

$$\int_0^{+\infty} P_{st}(x)dx = 1. \tag{23}$$

In the exponential part of Eq (22), its calculation result is defined as a modified potential function and represented by $U(x)$, and its specific form is as follows.

$$U(x) = \int \frac{f(x)}{B(x)} dx. \tag{24}$$

The generalized potential function is a generalized concept used to discuss the relationship between the potential function and the generalized velocity. The modified potential function $U(x)$ of the system is also known as the generalized potential function. The changes in the modified potential function are shown in Figure 2.

4. Discussion

In the previous theoretical analysis, the factors that affect the study results have been considered, and the relevant variables have been integrated together. By keeping other variables constant and only changing the value of noise intensity D, numerical simulation is performed on the steady-state probability distribution function $P_{st}(x)$, and the results are shown in Figure 3.

The steady-state distribution of a system refers to the probability distribution of each state in the system remaining unchanged after a series of state transitions. If we simply consider from a physical perspective, when there is a phase particle with an initial position of x in the phase space, the steady-state probability density distribution function of the system shows a maximum value, which means that the cumulative probability of the particle appearing at position x is the highest. Figure 3(a) shows the overall variation of the steady-state probability distribution function, Figure 3(b) shows the third steady-state point in the steady-state probability distribution function image, which is the magnified image of the most obvious peak. Figure 3(c) shows the image near the first steady-state point after changing the noise intensity value D. In the subsequent figures, similar processing is performed. From a mathematical perspective, the steady-state probability distribution function has three extreme points, with two maxima and one minimum, representing three states in the tumor system, i.e., tumor outbreak, disappearance and being in an intermediate state.

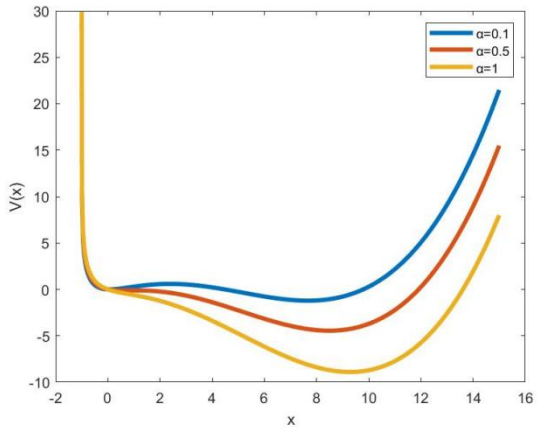

Figure 1. Potential function of the tumor cells. α = 0.5, β = 2.8, λ = 0.5. The above deterministic potential function equation has one unstable solution and two stable solutions.

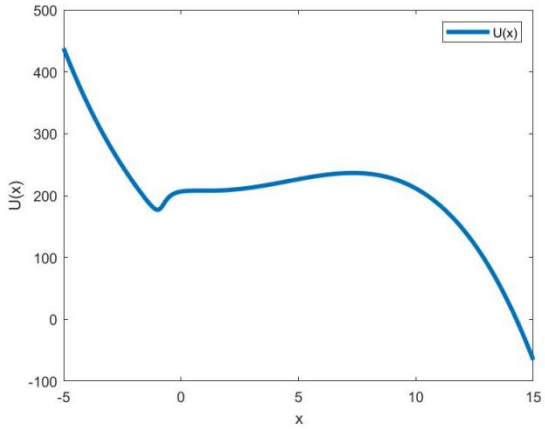

Figure 2. Modified potential function of the tumor cells. α = 0.5, β = 2.8, λ = 0.5. The modified potential function of the system is also known as the generalized potential function. The generalized potential function is a generalized concept used to discuss the relationship between the potential function and the generalized velocity.

According to Figure 3, it can be seen that the steady-state probability distribution function shows a trend of two peaks and one valley. It is obvious that when the value of x is around 7, the extreme value of the steady-state probability distribution function $P_{st}(x)$ reaches its maximum, indicating the maximum probability of tumor cell growth and proliferation at this time. It can be concluded that reducing the intensity of the multiplicative noise increases the value of the steady-state probability distribution function $P_{st}(x)$ before it reaches the second peak. However, when the value of x is greater than 7, the value of the steady-state probability distribution function increases with the multiplicative noise intensity. When the state variable x is at an unstable point, it indicates that the number of tumor cells can transition to two states, i.e., cell death or tumor eruption.

Preliminary analysis shows that when the number of tumor cells in the body is small, the multiplicative noise intensity has less effect on tumor cell proliferation. When the number of tumor cells reaches a certain value, the multiplicative noise intensity has a large influence on the tumor cell proliferation. As previously described, in treatment methods such as surgery, chemotherapy and

radiation therapy, the growth of tumor cells exhibits randomness. These nonlinear factors will affect the specific immune coefficient in the form of multiplicative noise and further affect the growth status of tumor cells. When the number of tumor cells is relatively small, the increase of multiplicative noise intensity will inhibit the growth of tumor cells, thereby preventing their spread. However, when the number of tumor cells is relatively large, the increase in multiplicative noise intensity will further increase the probability of growth of tumor cells.

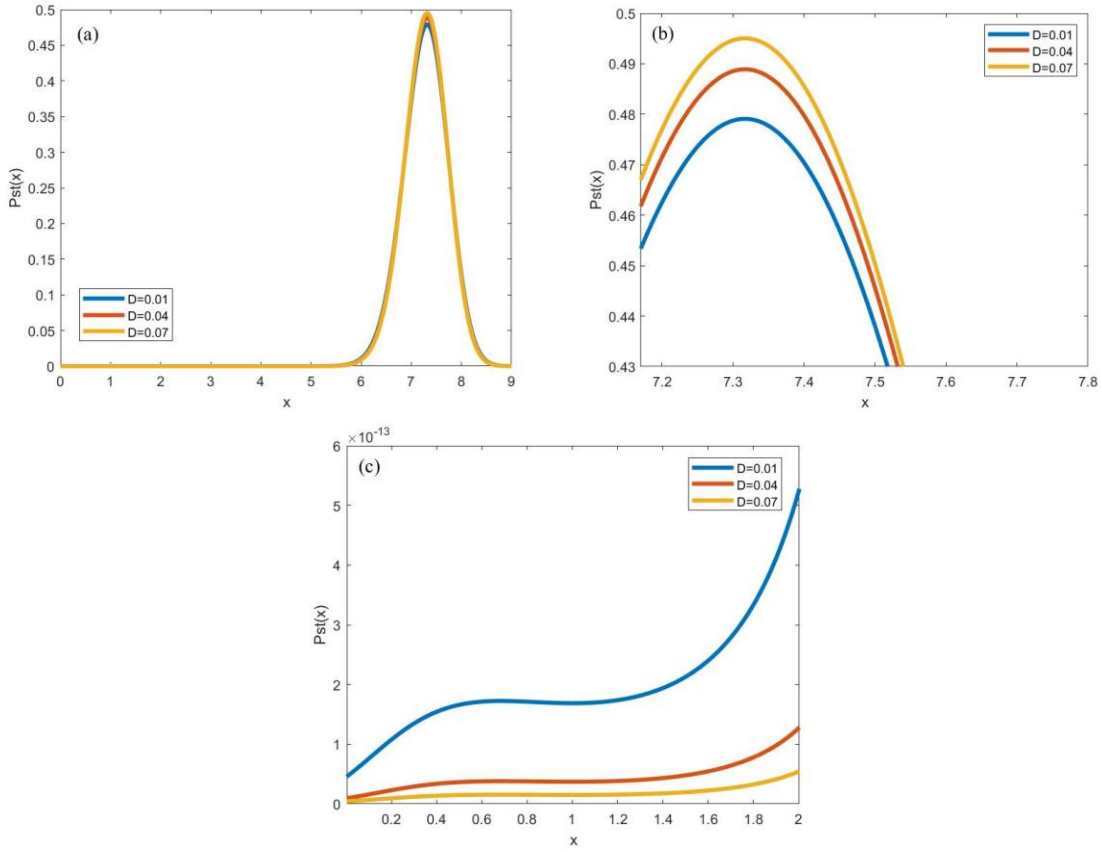

Figure 3. Effect of the multiplicative noise intensity D on the steady-state probability distribution function. (a) $\alpha = 0.5$, $\beta = 2.8$, $\theta = 0.1$, $\lambda = 0.5$, $Q = 0.09$, $\tau = 0.1$; (b) $\alpha = 0.5$, $\beta = 2.8$, $\theta = 0.1$, $\lambda = 0.5$, $Q = 0.09$, $\tau = 0.1$. The variation of multiplicative noise intensity has a certain impact on the proliferation process of tumor cells. When the number of tumor cells is relatively small, the increase of multiplicative noise intensity will inhibit the growth of tumor cells, thereby preventing their spread. However, when the number of tumor cells is relatively large, the increase in multiplicative noise intensity will further increase the probability of tumor cell growth.

However, long-term treatment can lead to tumor cells establishing tolerance to treatment methods and drugs. In addition, radiation therapy and the use of chemicals during the treatment process can cause normal cells to transform into cancer cells, leading to an increase in the probability of tumor cell growth and proliferation in subsequent treatment. That is to say, the multiplicative noise intensity D cannot be fixed throughout the treatment process. Reasonably changing the magnitude of multiplicative noise intensity D during the treatment process is more conducive to promoting the decay of tumor cells and achieving satisfactory treatment results.

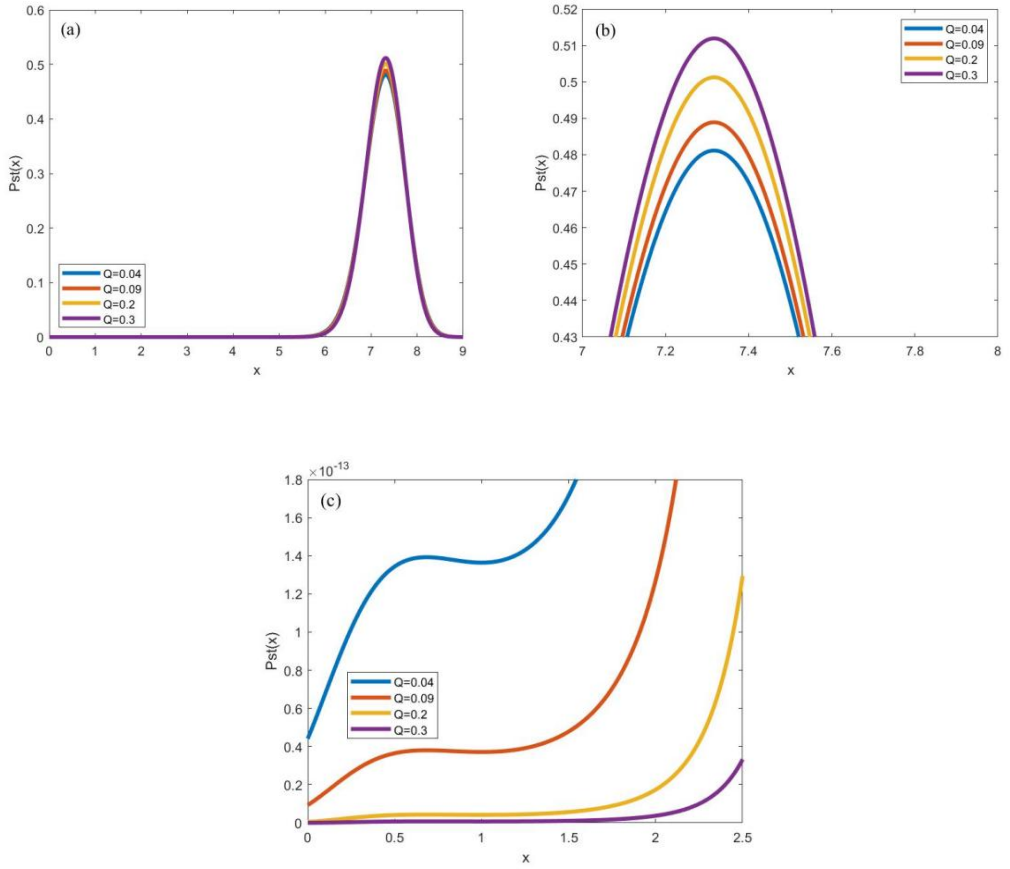

Figure 4. Effect of the additive noise intensity Q on the steady-state probability distribution function. (a) $\alpha = 0.5$, $\beta = 2.8$, $\theta = 0.1$, $\lambda = 0.5$, $D = 0.04$, $\tau = 0.1$; (b) $\alpha = 0.5$, $\beta = 2.8$, $\theta = 0.1$, $\lambda = 0.5$, $D = 0.04$, $\tau = 0.1$. The increase of additive noise intensity will inhibit the growth of tumor cells when the number of tumor cells is relatively small, while the increase in additive noise intensity will further promote the growth of tumor cells when the number of tumor cells is relatively large.

Next, the effect of the additive noise intensity on the value of tumor cell growth is discussed when the multiplicative noise intensity is fixed, as shown in Figure 4(a). Figure 4(b) shows the image where the maximum peak of the $P_{st}(x)$ function varies with different additive noise intensity Q. Figure 4(c) shows a magnified image of the steady-state probability distribution function $P_{st}(x)$ when changing the additive noise intensity Q at the relatively insignificant peak of the change.

As can be seen from Figure 4, the structure of the steady-state probability distribution function in this case is similar to that when changing the value of the multiplicative noise intensity D. However, at the relatively small x value, the effect of changing the multiplicative noise intensity Q is greater than changing the additive noise intensity D before the second peak of the steady-state probability distribution function appears. In addition, the value of the magnitude of the additive noise intensity Q does not change significantly when the value of x is around 7. When the noise intensity changes, it can be seen from the image of the corresponding steady-state probability distribution function that if the number of tumor cells in the body is small, the impact of changing the intensity of the two types of noise on the probability of tumor cell proliferation is not significant.

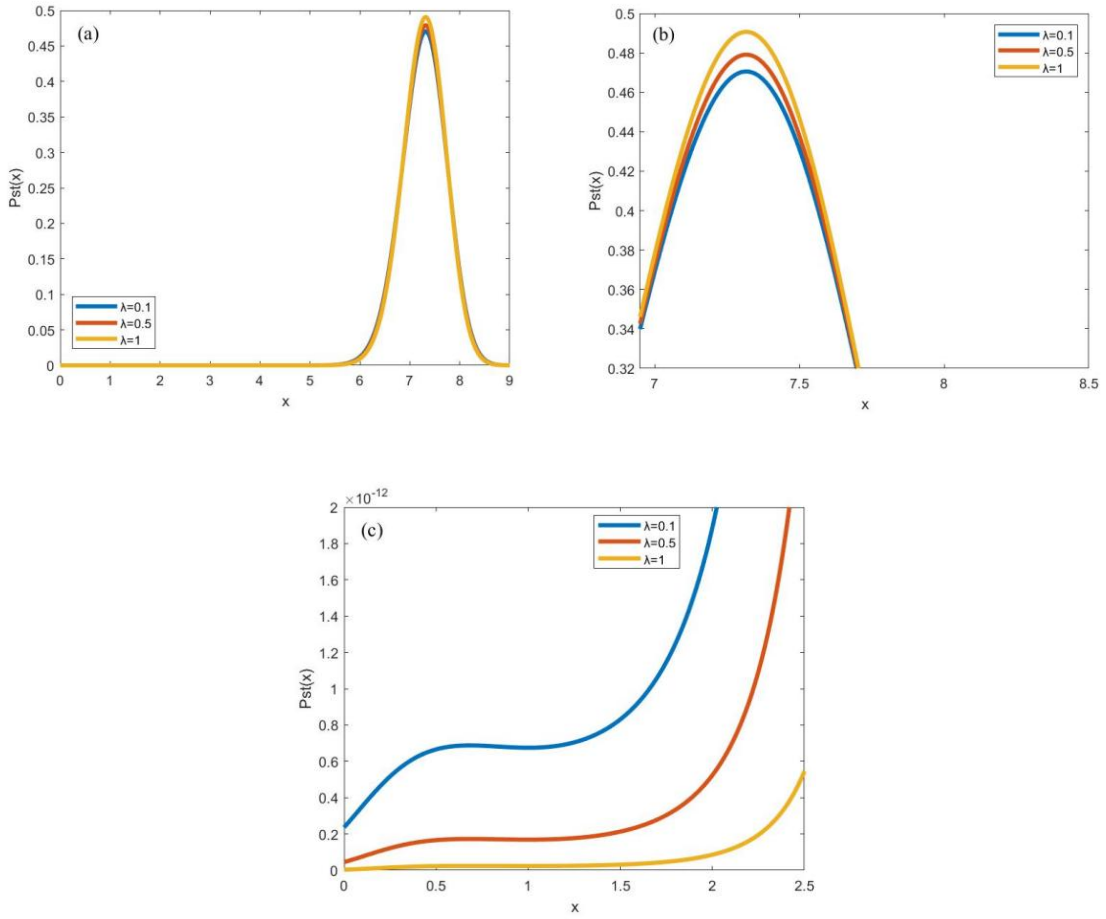

Figure 5. Effect of the correlation strength λ on the steady-state probability distribution function. (a) $\alpha = 0.5$, $\beta = 2.8$, $\theta = 0.1$, $D = 0.01$, $Q = 0.09$, $\tau = 0.1$; (b) $\alpha = 0.5$, $\beta = 2.8$, $\theta = 0.1$, $D = 0.01$, $Q = 0.09$, $\tau = 0.1$. For a small number of tumor cells, the strong cross correlation between noise can maintain a low probability level of tumor cell numbers. For a larger number of tumor cells, the stronger the cross correlation, the higher the corresponding probability, indicating a higher probability of the occurrence of a large number of tumor cells.

It is known from the above analysis that additive and multiplicative noise intensity have similar but different effects on tumor cell growth and proliferation. However, we cannot ignore the fact that the treatment of tumor diseases is done under the combined action of two kinds of noise, so the effect of two kinds of noise on the growth and proliferation of tumor cells is studied in the following. Figure 5(b) shows a locally enlarged image of the steady-state probability distribution function $P_{st}(x)$ as a function of the cross-correlated intensity of the noise.

As can be seen in Figure 5, when the value of x is relatively small, the extreme value of the steady-state probability distribution function shifts to the left as the additive noise intensity Q increases. Overall, the value of the steady-state probability distribution function changes very little, and the overall value of the image is approaching 0 indefinitely, indicating that the size of the correlation strength between noise has little effect on the steady-state probability distribution function. However, when the value of x is about 7, the steady-state probability distribution function appears as the most obvious peak. Additionally, the greater the strength of correlation between noise, the greater the peak

of the steady-state probability distribution function. When the value of x is close to this value, reduce the strength of the color association between the noise can promote the decay of tumor cells.

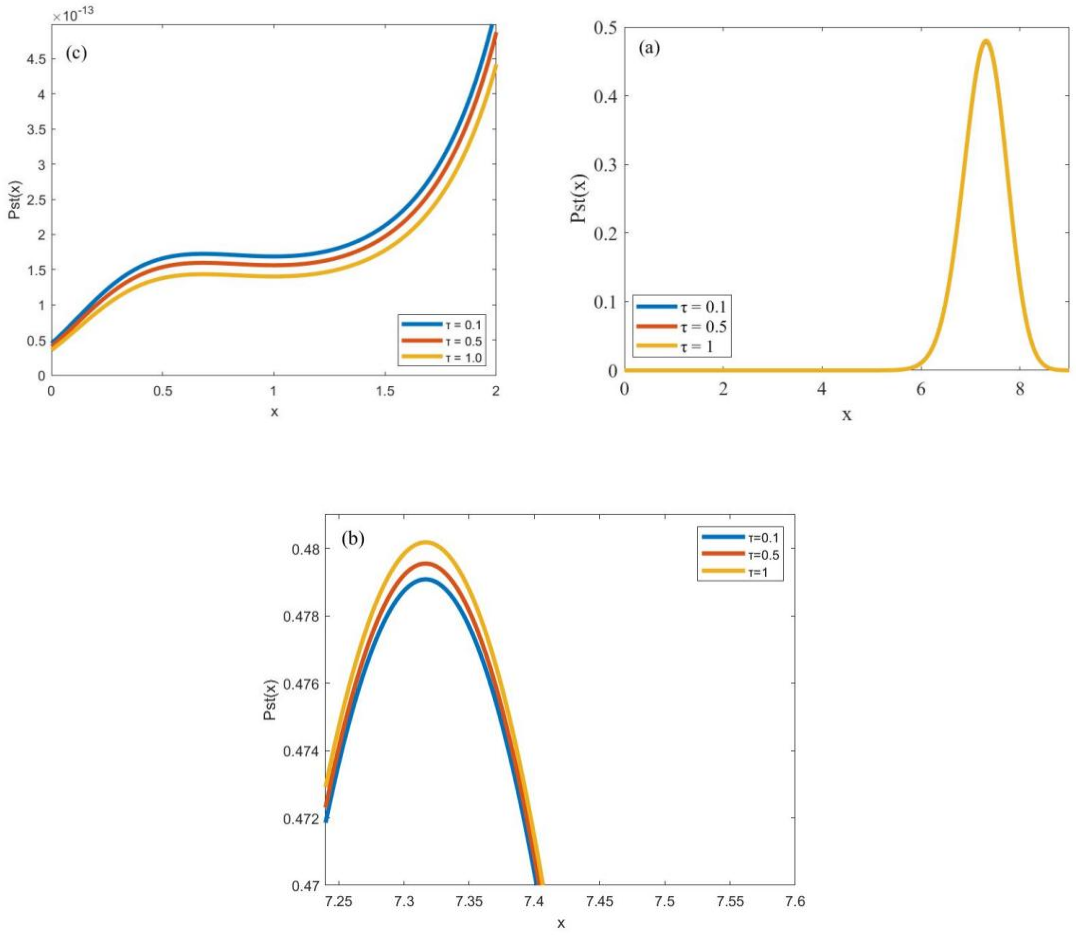

Figure 6. Effect of the cross-correlated time τ on the steady-state probability distribution function. (a) $\alpha = 0.5$, $\beta = 2.8$, $\theta = 0.1$, $D = 0.01$, $\lambda = 0.5$, $Q = 0.09$; (b) $\alpha = 0.5$, $\beta = 2.8$, $\theta = 0.1$, $D = 0.01$, $\lambda = 0.5$, $Q = 0.09$. The steady-state probability distribution function generally exhibits a structure of two maxima and one minimum. The probability distribution function decreases with the increase of correlation time when the number of tumor cells is relatively small, while the probability distribution function increases with the increase of correlation time when the number of tumor cells is relatively large.

Figure 6 shows the effect of the cross-correlated time between the noise on tumor cell proliferation and death when other conditions are fixed. It can be seen that the maximum and minimum values of the steady-state probability distribution function are the same as those shown in the figure above, and one can conclude that the cross-correlated time between noise affects tumor cell proliferation and decay more strongly than the association strength. When the value of x is relatively small, reducing the cross-correlated time between noise has a similar effect to prolonging the cross-correlated time between noise when the x value is relatively large, which can inhibit the proliferation of tumor cells, but the degree of inhibition is different.

5. Conclusions

Considering factors such as the body's immunity and external therapeutic environment, a stochastic Langevin equation is obtained to describe the proliferation and death of tumor cells based on the catalytic Michaelis-Menten reaction model. Throughout the analysis process, these internal and external nonlinear influencing factors were introduced into the tumor cell proliferation system in the form of noise. Subsequently, a series of algorithms such as Liouville equation, VAN Kampen Lemma and Novikov theory, are used to obtain the Fokker-Planck equation. Under steady-state conditions, an analytical solution to the steady-state probability distribution function is obtained, and the impact of changes in noise parameters on the steady-state probability distribution function is analyzed to characterize the clinical efficacy of different treatment methods for tumor cell therapy.

Research results show that the multiplicative noise intensity, the additive noise intensity, the color cross-correlated intensity and the color cross-correlated time can promote the growth or death of tumor cells, but the patterns of effects are not the same. From a mathematical perspective, the steady-state probability distribution function has three extreme points, with two maxima and one minimum, representing three scenarios in the tumor system, i.e., tumor outbreak, disappearance and being in an intermediate state. When the number of tumor cells is relatively small, the increase of noise intensity will inhibit the growth of tumor cells, thereby preventing their spread. However, when the number of tumor cells is relatively large, the increase in noise intensity will further increase the probability of growth of tumor cells. If the cross-correlated time and cross-correlated intensity between noises are changed, it can be observed that the steady-state probability distribution function will also change.

Compared with previous studies, the proliferation of tumor cells in the original model was not taken into account. However, the parameters related to tumor cell birth rate are modified to make the model closer to the real situation in our study. Moreover, the form of color cross-correlated between multiplicative and additive white noise has not been discussed before. The above results indicate that different treatment plans should be given according to different stages of treatment to achieve relatively satisfactory treatment results. This conclusion provides some theoretical support for the application of treatment means and treatment intensity.

Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work is supported by Jiangxi Provincial Natural Science Foundation of China (No. 20232BAB201048), the Natural Science Foundation of Jiangxi Provincial Department of Education (No. GJJ2202903) and Innovation and Entrepreneurship Project of Yuzhang Normal University College Students (YZCXCY2022037).

Conflict of interest

The authors declare there are no conflicts of interest.

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