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Research article

Multi-airport system flight slot optimization method based on absolute fairness

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Abstract: With the rapid development of the civil aviation industry, the number of flights has increased rapidly. However, the availability of flight slot resources remains limited, and how to allocate flight slot resources effectively has been a hot research topic in recent years. A comprehensive flight slot optimization method can significantly enhance the rationality of the allocation results. The effective allocation of flight slot is the key to improving the operational efficiency of the multi-airport system. We will optimize the flight schedule of the entire multi-airport system considering the fairness of each airport in it. The optimization results will provide an important reference for the reasonable allocation of flight slot within the multi-airport system. Based on the operation characteristics of the multi-airport system, we have established a multi-objective flight slot allocation optimization model. In this model, we set the airport capacity limit, shared waypoint capacity limit and aircraft turnaround time limit as the constraints. The optimization goal of the model is to minimize total flight schedule displacement and the maximum deviation of fairness from the absolute fairness. Gurobi solver is used to solve the model. We have innovatively incorporated the rolling capacity constraint method into our model to ensure more accurate flight slot allocation results. The Beijing-Tianjin-Hebei regional multi-airport system is selected as an example to verify the above model, and the flight slot optimization results have successfully met the fairness goal. The comparative analysis has demonstrated that the rolling capacity constraint method significantly improves the accuracy of solution results, leading to more stable flight slot allocation. The results also prove that the flight slot allocation method of multi-airport system based on absolute fairness of peak demand can improve the fairness of the allocation results. To achieve a higher level of fairness, we have found that the peak-demand based fairness method requires a smaller slot displacement compared to the non-peak demand-based method. Through the optimization of flight slot of the multi-airport system, the coordination between airports can be significantly improved. It can provide a new solution for the efficient operation of the multi-

airport system.

Keywords: multi-airport system; flight slot optimization; absolute fairness; peak demand; rolling constraint

1. Introduction

With the continuous increase of aviation demand, the contradiction between limited airport and airspace resources and the growing flight slot demand is increasingly significant. Unreasonable flight slot allocation often results in a large number of flight delays. Especially in some large and busy international airports, serious delays have greatly reduced the efficiency of civil aviation transportation, caused inconvenience to passengers and brought economic losses to airlines. While airport expansions or air route additions can be used to solve this problem, these solutions are often costly and provide only temporary relief. As a result, flight slot optimization has become an increasingly important research topic.

Airport slot optimization involves optimizing the allocation of flight schedules supplied by airlines while meeting constraints, such as capacity and airplane turnaround time. The optimization purpose is to maximize the utilization of airport slot resources and improve airport operation efficiency. In this respect, both domestic and international researchers have conducted extensive research. For example, Zografos et al. [1] constructed a single airport flight slot optimization model for the first time. The model aimed to improve schedule efficiency through the minimization of the total schedule displacement. The author defined the slot displacement as the absolute difference between the allocated and requested slot time. So, the model took airport capacity and turnaround time as constraints, and the optimization objective was to minimize the total schedule displacement. Taking three airports in Greece as an example, the model was evaluated by programming in C++ and solving the linear problem using CLP library. Finally, the result show that the model greatly reduced the flight slot displacement and improved the flight schedule allocation efficiency. In order to get better optimization results, researchers continuously improve the flight schedule optimization model. For example, by increasing the number of objective functions, the original single objective optimization model has been transformed into a multi-objective optimization model. Jacquillat and Odoni [2] proposed a comprehensive method to alleviate airport congestion. He introduced a bi-objective slot allocation model which considers the minimization of the total displacement and the minimization of the maximum displacement of the requested slots. The minimization of the total displacement can improve the schedule efficiency and the minimization of the maximum displacement can ensure the fairness of the flights. To solve the model, they proposed an original iterative solution algorithm. The results verified that delays could be significantly reduced through limited adjustment of flight schedules. So, the model can achieve optimize airport slot strategically and improve airport capacity utilization tactically. Ribeiro et al. [3] proposed a multi-objective flight slot optimization model containing four objectives, which used the idea of priority to achieve the sum of multiple objectives by assigning corresponding weights to each objective to seek optimal results. The four objectives are as follows: minimization of total displacement, minimization of maximum displacement, minimization of unassigned slots and minimization of displaced slots. During the optimization process, the author also found that limited adjustment of WSG regulations could better meet the requirements of airlines and increase the efficiency and fairness of flight slot allocation. In fact, as the number of optimization objectives increases, the rationality and practicality of the model are also constantly improving. However, in addition to increasing the objective function, considering the scheduling preferences of more stakeholders can also increase the optimization effect of the model. For example, Katsigiannis and Zografos [4] considered the airline's schedule flexibility preference in the flight schedule optimization model, which was expressed by the Time Flexibility Index (TFI). Finally, the authors established a flight schedule optimization model considering flight schedule adjustment flexibility and airport total capacity constraints. The results showed that considering airlines' scheduling flexibility preference and inherent dynamic capacity limitation was beneficial to improving airport capacity utilization, improving flight schedule planning and reducing the total displacement. Slot allocation was generally carried out in a 6-month scheduling season, but it could cause congestion and reduce the airport capacity utilization. In order to alleviate the congestion problem, Fairbrother and Zografos [5] proposed two methods: changing the sequence threshold and scheduling the seasonal segmentation and proposed heuristic methods to shorten the solving time. The results showed that compared with changing the sequence threshold value, the new segmented scheduling method could effectively reduce the total displacement of the scheduling time. In summary, in terms of a single airport, the researchers have established a series of single objective or multi-objective airport slot optimization models. All these models can improve the airport slot utilization and airport operational efficiency to a certain extent and provide basic ideas for airport slot optimization.

With the proposed concepts of airport group and multi-airport system, it is more important to carry out collaborative optimization of the flight slot of the entire airport network within a multi-airport system. Some researchers began to extend the original single airport slot optimization model and proposed a flight slot optimization model for multi-airport system. Wu et al. [6] applied the transportation demand management theory to establish a flight schedule and frequency optimization model based on the multi-airport system. The model took the minimum travel loss time of passengers as the objective function, took the passenger attrition rate and airline load factor as constraints. The results showed that flight frequency optimization can reduce the time loss of passenger travel and reduce the number of flights needed to meet passenger demand. It could also provide an important reference for alleviating the congestion problem of hub airports. Pellegrini et al. [7] designed an integer linear programming model SOSTA to simulate the current slot allocation process in Europe with the goal of minimizing the total cost and constrained by capacity and aircraft turnaround time. The results proved that SOSTA could optimize the flight slot allocation process. Benlic [8] proposed a flight slot allocation model with the goal of minimizing unfulfilled requests and minimizing flight time displacement. The model also considered the constraints of existing rules and criteria of the IATA. Heuristic algorithms were adopted to solve the model, including a constructive heuristic method to generate feasible initial solutions and an iterative heuristic algorithm. The results showed that compared with the slot allocation of flights independently for each airport, the slot allocation at the network level will not bring significant degradation to the plan. However, due to the complexity of the model, its available scale was limited, and it was not able to properly allocate the flight slot of the whole quarter. Zhu et al. [9] established a flight slot optimization model which took the flight punctuality rate, airline market share, passenger loss time, maximum flight function positioning index as targets and took capacity limit as constraints. The author used the particle swarm optimization algorithm to solve the model and finally realized the allocation of flights with poor performance in the hub airport within the multi-airport system to other surrounding airports. The flight schedule allocation of each airport within the multi-airport system was more uniform and the operational efficiency of the multi-airport system was improved. Wang et al. [10] optimized the departure time of multi-airport flights in the terminal area. Based on consideration of the total delay time and total delay cost, they established a multi-airport collaborative release model of terminal area based on airport priority. The researchers used the improved multi-objective particle swarm algorithm to solve the model, and the solution results confirmed the effectiveness of the model. The model provided a theoretical basis for the multi-airport collaborative release mechanism. In summary, researchers have proposed many methods for optimizing flight schedules at multiple airports. For example, assigning flights from hub airports to other airports, optimizing flight frequency at airports or researching multi-airport collaborative release, etc. These methods can improve the operational efficiency and flight schedule utilization of airport clusters. However, few researchers have studied the flight schedule allocation of multi-airport system from the perspective of fairness in the use of public airspace.

Fairness refers to the reasonable distribution of the rights and obligations of social members. Reflected in the flight slot optimization level, social members refer to flight schedule applicants, rights refer to the number of flight slots applied by the applicants and obligations refer to the amount of flight schedule adjustments received by the applicants. If the allocation fairness is not considered and the total displacement is minimized on the premise of harming the interests of some applicants, it will inevitably cause dissatisfaction among some applicants and reduce the acceptability of the allocation results. Therefore, in the process of time allocation, it is necessary to comprehensively consider the interests of all parties to ensure that the amount of time adjustment allocated by each applicant should be coordinated with the amount of application to improve the fairness of allocation.

To make the flight slot allocation results more reasonable and acceptable, some studies began to consider the fairness of flight slot applicants and established a flight slot optimization model based on fairness. For example, there are administrative capacity management methods, market capacity management methods and the combination of the two methods for flight schedule allocation. Cavusoglu and Macário [11] analyzed the problems existing in the current time allocation mechanism and investigated the market-based mechanism, with a view to establishing a more effective and fair time allocation method to better manage scarce airport capacity. From his work, we can learn that accomplishing more extensive integration by having all stakeholders participate at the decision-making level can increase the performance of the slot allocation. Castelli et al. [12] proposed a mechanism to solve the slot allocation problem of European airports, which considered the cost of the historical time priority principle and tried to realize equitable cost redistribution among airlines through monetary compensation. Zografos and Jiang [13] constructed a fairness index, that is, the proportion of flight slot displacement allocated by each airline to the total displacement was equal to the flight request proportion of the airline, so as to achieve a fair distribution of flight schedule displacement among airlines. Zografos and Jiang [14] used a combination of ε-constraint method and the roll generation algorithm to solve the bi-objective model considering fairness and efficiency. They also studied the efficiency-fairness trade-off effective boundary under the hierarchical and non-hierarchical slot scheduling mechanisms. Finally, the authors presented the results at the airport and airline level, which improved the transparency of slot allocation and provided a reference for decision-makers and airlines to choose fair and reasonable flight schedule allocation results. The study also showed the trade-off between fairness and efficiency. It proved that sacrificing little efficiency can significantly improve the fairness of flight slot allocation. Jiang and Zografos [15] constructed a bi-objective flight slot optimization model to measure the efficiency and fairness of slot allocation. The authors investigated three inter-airline fairness objectives: relative fairness objective, absolute fairness objective and fairness based on Gini index. The introduction of these fairness provides us with more ideas for solving the flight slot optimization problem of the multi-airport system. Based on the study of Zografos, Fairbrother et al. [16] first proposed the concept of peak demand-based fairness and established a two-stage flight slot allocation mechanism combining efficiency, fairness and airline preference. The first stage of this allocation mechanism was to establish a fairness-based reference schedule, and the second stage was for airlines to specify how to allocate the displacement they obtained in their requests. In this method the airline's preference is included in the allocation of the total offset. It can improve the transparency and acceptability of slot allocation. Katsigiannis et al. [17] modeled and solved the multi-objective and multi-level single airport slot allocation problem and established a multi-objective flight slot allocation model which considered the minimum total displacement, the minimum maximum displacement and the best demand-based fairness goals. The authors also considered the interaction between different levels of optimization objectives. This research showed that making small sacrifices to high-level targets can lead to more effective airport slot optimization results. Shui et al. [18] introduced the peak-demand based fairness into the flight slot allocation of the multi-airport system, constructed a multi-objective airport slot optimization model considering the minimum total displacement, minimum maximum slot displacement and minimum fairness target value. The model was programmed with python and solved by Gurobi solver. It was the first time to consider the peak-demand based fairness of the airport in the multi-objective airport slot optimization problem. This model can better balance fairness and efficiency at shared waypoints. However, the time interval considered by capacity constraints is too large, so the allocation results are not accurate enough in this research. Wu et al. [19] also used Gurobi solver to solve the flight slot optimization model of the multi-airport system and studied the changes of flight slot optimization and flight delay under three different optimization schemes with single airport, airport combination and multi-airport system as optimization objects respectively. His research demonstrates the usability and convenience of the Gurobi solver in solving the flight slot optimization problem of the multi-airport system. It also inspires us that the delay reduction effect was the best when we only consider the main airport combination in the multi-airport system. If all airports are optimized at the same time, in order to reduce the delay of small-traffic airports, the flight schedule optimization effect of high-traffic airports may be sacrificed and the overall delay may increase. It provides a reference for the application of our solution method and the selection of research objects. Zeng et al. [20] considered priorities in the flight slot optimization problem. Its fairness is mainly reflected in the process of determining priorities. He determined the flight priority according to the operational efficiency of the airport and found out the set of flights that needed to be adjusted. We can learn that prioritizing flights through actual operational situation can make the flight slot allocation result fairer and more reasonable.

Currently, the research on flight slot optimization based on fairness is not yet comprehensive. For example, the time interval considered by capacity constraints is too large, and the allocation results are not accurate enough. By considering capacity constraints at smaller intervals, a more balanced schedule allocation can be achieved, thereby reducing delays and improving resource utilization. A flight schedule optimization model based on the Gini coefficient has been established, but the applicability of other fairness models to the flight schedule optimization problem within a multi-airport system has not been explored. Increasing the analysis of other fairness models can provide more options for fairness studies within a multi-airport system.

Based on the shortcomings of previous studies, we improved the existing flight slot optimization model. On the one hand, this model focuses primarily on the fairness of flight slot allocation at the shared arrival waypoint within the multi-airport system. It proposes to use the concept of absolute fairness objective to measure the overall fairness level of the multi-airport system at the shared arrival waypoint. On the other hand, this model adds the 5-minute capacity constraint and adopts the rolling capacity constraint method for the 15-minute and 60-minute capacity constraints, resulting in more refined flight schedule optimization outcomes. We further verify the effectiveness of the model by utilizing operational data from three airports in Beijing and Tianjin. These findings provide valuable references for future research on cooperative operations and flight schedule optimization within multi-airport systems.

The rest of this article consists of four sections. The structure is as follows: Section 2 provides a brief overview of the flight slot optimization problem within multi-airport system and outlines the fundamental assumptions upon which the model is based; Section 3 introduces the flight slot optimization model for multi-airport system based on the concept of absolute fairness; Section 4 takes the Beijing-Tianjin-Hebei region multi-airport system as an example to verify the established model and analyzes the verification results; Section 5 summarizes the main conclusions of this paper.

2. Description of the multi-airport system flight slot optimization problem

The flight slot optimization model based on absolute fairness proposed in previous studies is mainly for a single airport, and the fairness goal is also proposed for airlines in airports. In this study, the concept of absolute fairness is extended to the scope of multi-airport system, and the fairness between airports is considered to achieve the best overall fairness of the multi-airport system. In a multi-airport system, the shared arrival and departure waypoints are important nodes that are related to each other. Due to limited energy of controllers and service capacity of facilities, the capacity of each arrival and departure waypoint within a given period is also limited. Therefore, how to reasonably allocate a large number of flight requests at each shared arrival and departure waypoint of each airport in the multi-airport system is also an important research content. The fair and reasonable allocation results are conducive to promoting the coordinated development of the multi-airport system, improving the efficiency of airspace utilization and the operational efficiency of the multi-airport system [21]. Therefore, we select a shared arrival and departure waypoint to explore the fairness of flight schedule allocation among airports in the multi-airport system at this waypoint.

To account for the unique flight operation characteristics of multi-airport systems, a multi-objective flight slot optimization model is proposed. This model incorporates airport capacity limits, shared arrival and departure waypoint capacity limits and turnaround time limits as constraints, while also considering the minimum total flight slot displacement and the maximum deviation of fairness from absolute fairness at shared arrival waypoints as objectives. To implement this model, the following assumptions are made:

a) The turnaround time of a flight refers to the time interval between the arrival of the preceding flight at the airport and the departure of the subsequent flight. The time interval between the arrival and departure slot should not be greater than the maximum turnaround time and should not be less than the minimum turnaround time.
b) The uncertainty of flight time is not considered, that is, the time for a flight to arrive at the shared approach and departure waypoint from the same airport is assumed to be a fixed value, which is the median of historical flight data in recent years.

c) Only the flights from airports within the multi-airport system involved in this study are considered, and the take-off, landing and turnaround times of flights at airports other than the multi-airport system are not considered.
d) The airline's current flight schedule is used as the initial flight request time.

3. Slot allocation model

3.1. Parameters

Table 1. Basic parameters.

parameters	
t	allocated slot time for airplane m
t_m	requested slot time for airplane m
T	a set of time intervals with a length of 5 minutes
m	the movement in the airport group
M	the set of movements requests in an airport group, $m\epsilon M$, $M = \{1,2,\cdots|M|\}$
a	the airport in the airport group
A	the set of all airports in an airport group, $a \in A$, $A = \{1,2,\cdots|A|\}$
u	key waypoints for fairness modeling in airport group
w	a waypoint in an airfield with both arrival and departure
W	all waypoints in an airfield with both arrival and departure, w$\in W, W = \{1,2,\cdots|W|\}$
i	arrival waypoint in the airport group
I	all arrival waypoints in the airport group, $i \in I, I = \{1,2,\cdots|I|\}$
d	departure waypoint in the airport group
D	All departure waypoints in the airport group, d$\in D, D = \{1,2,\cdots|D|\}$
$x_{t,m}^a$	$x_{t,m}^a$=1, if airplane m in airport a is allocated to slot t; otherwise, $x_{t,m}^a$=0

3.2. Decision variable

$$x_{t,m}^{a} = \begin{cases} 1, \text{airplane } m \text{ in airport } a \text{ is allocated to slot } t \\ 0, \text{otherwise} \end{cases} \tag{1}$$

$a \in A$, $A = \{1,2,\cdots|A|\}$. A represents the set of all airports in the multi-airport system. |A| is the total number of airports in the multi-airport system. $t\epsilon T$, $T = \{1,2,\cdots|T|\}$. T is the set of time intervals with an interval length of 5 minutes. $m\epsilon M$, $M = \{1,2,\cdots|M|\}$. M represents the set of all airplanes in the multi-airport system. |M| is the total number of airplanes in the multi-airport system.

3.3. Overall measure of absolute fairness of multi-airport system

The research on fairness was first proposed at the level of the single airport, and an airport fairness measurement formula based on the principle of proportionality was established. That is, the total displacement that should be allocated to an airline should be proportional to the slot requests that an airline has made. In order to improve the fairness level, the researchers considered flight demand in peak time and proposed an airport fairness index based on peak demand. In this study, the peak-demand based airport fairness index is extended to the multi-airport system level. The total displacement that should be allocated to an airport at a congested public waypoint should be proportional to the slot requests at the peak time that airport has made. The fairness index of each airport at the shared approach waypoint is listed as follows by reference [14]:

$$\mu_{a,u}=\begin{cases}\dfrac{\frac{S_{a,u}}{S_u}}{r_{a,u}},\ if\ r_{a,u}\neq 0\\ 1,\ if\ r_{a,u}=0\ and\ S_{a,u}=0\\ \infty,\ if\ r_{a,u=0}=0\ and\ S_{a,u}\neq 0\end{cases} \tag{2}$$

$\mu_{a,u}$ represents the peak-demand based fairness index of airport a at approach waypoint u. $\mu_{a,u}=1$ indicates airport a is treated fairly at waypoint u. $0\le\mu_{a,u}<1$ indicates that airport a is favored at waypoint u. $\mu_{a,u}>1$ indicates that airport a is disfavored at waypoint u. $S_{a,u}$ represents the displacement allocated to airport a at waypoint u. S_a represents the total displacement of all airports in the multi-airport system at the waypoint u. $r_{a,u}$ represents the proportion of flights requested by airport a at waypoint u during peak hours. The formulas can be expressed as:

$$S_{a,u}=\sum_{m\in M_{a,u}}\sum_{t\in T}|t-t_m|\,x_{t,m}^{a} \tag{3}$$

$$S_u=\sum_{a\in A}\sum_{m\in M_{a,u}}\sum_{t\in T}|t-t_m|\,x_{t,m}^{a}=\sum_{a\in A}S_{a,u} \tag{4}$$

$$r_{a,u}=\frac{N_{a,u}}{N_u}(N_u\neq 0) \tag{5}$$

$M_{a,u}$ represents the set of all flights in airport a that pass through the approach waypoint u. $N_{a,u}$ represents the number of peak hours flights in airport a that have requested pass through approach waypoint u. N_u represents the number of peak hours flights in the multi-airport system that have requested pass through approach waypoint u.

The peak time in this study refers to: at a time interval of 5 minutes, if the number of flights at a key approach waypoint at that time is greater than or equal to the maximum number of flights that can be accommodated by the waypoint in 5 minutes, that moment is a peak time. The flights allocated to this waypoint within this time are called peak time flights. We introduce the variable $A_{t,m}^{a,u}$ to describe whether a moment is a peak time. $A_{t,m}^{a,u}=1$ means that the requested time t of flight m at airport a is the peak time of the approach waypoint u, otherwise $A_{t,m}^{a,u}=0$. Thus $N_{a,u}$ and N_u can be expressed as:

$$N_{a,u}=\sum_{m\in M_{a,u}}\sum_{t\in T}A_{t,m}^{a,u} \tag{6}$$

$$N_u = \sum_{a\in A}\sum_{m\in M_{a,u}}\sum_{t\in T} A_{t,m}^{a,u} = \sum_{a\in A} N_{a,u} \tag{7}$$

The fairness indicator $\mu_{a,u}$ evaluates the fairness of each airport at approach waypoint u relative to the multi-airport system. However, the fairness of the whole multi-airport system at the approach waypoint u cannot be evaluated intuitively and accurately by the fairness index of a single airport. Therefore, we introduce the concept of absolute fairness objective (MMA), which is the maximum value of the absolute difference between the fairness value of a given airport and the absolute fairness value (Jiang and Zografos [15]). The mathematical expression of MMA is provided by Eq (8).

$$\min r_{MMA} = \min \max_{a\in A} |\mu_{a,u} - 1| \tag{8}$$

Where, r_{MMA} represents the absolute fairness target value of multi-airport system, which is used to measure the absolute fairness level of the multi-airport system. $\mu_{a,u} = 1$ indicates airport a is treated fairly at waypoint u. The value of the absolute difference between the fairness value of a given airport and the absolute fairness value indicates how favorably or dis-favorably an airport has been treated. The maximum value of this difference is associated with the airport that has been most or least favorably treated. Therefore, the minimization of the maximum value of this difference leads to a slot allocation outcome that avoids extreme unfairness conditions and improve the overall fairness level of multi-airport system.

3.4. Objective function

1). Minimize the total schedule displacement

Schedule displacement is defined as the difference between the allocated and requested slot. The objective function adopted in this paper is the minimum total schedule displacement of all airports in the multi-airport system. The mathematical expression is provided by Eq (9).

$$\min \sum_{a\in A}\sum_{m\in M}\sum_{t\in T} |t - t_m| x_{t,m}^{a} \tag{9}$$

$|t - t_m|$ is the slot displacement of airport m.

2). Minimize the maximum deviation of fairness from the absolute fairness

$$\min r_{MMA} = \min \max_{a\in A} |\mu_{a,u} - 1| \tag{10}$$

3.5. Constraint condition

1). Airport capacity constraint

The airport capacity constraint requiring that the declared capacity of the airport should not be violated. The airport declared capacity is the number of airplanes that the airport can operate per unit time. Studies [18] have made 15-minute and 60-minute capacity constraints for airports. Based on this, we add a 5-minute capacity constraint for each airport. The rolling capacity constraint method was used when considering 15-minute and 60-minute capacity constraints.

We consider three flight operation types: approach, departure and total flights. The capacity constraint requires that the declared capacity of each operation type at the airport should not be violated. For example, as for the approach capacity constraint, the number of approach flights of each airport per time interval should be less than or equal to the maximum number of approach flights that the airport can operate during this time interval. We mostly consider the 5-minute, 15-minute and 60-minute approach capacity constraints of airport a. The formulas can be expressed as:

$$\sum_{m\in M_a^k}\sum_{t\in T}x_{t,m}^a \le C_a^k, \forall a\in A, k\in K \tag{11}$$

$$\sum_{m\in M_a^k}\sum_{t}^{t+2}x_{t,m}^a \le CQ_a^k, \forall t\in T, \forall a\in A, k\in K \tag{12}$$

$$\sum_{m\in M_a^k}\sum_{t}^{t+11}x_{t,m}^a \le CH_a^k, \forall t\in T, \forall a\in A, k\in K \tag{13}$$

Where, Eq (11) represents the 5-minute capacity constraint of airport a. Eq (12) represents the 15-minute rolling capacity constraint of airport a. Eq (13) represents the 60-minute rolling capacity constraint of airport a. $T = \{1,2,3,\cdots|T|\}$ represents the set of time periods with a time interval length of 5 minutes. $K = \{Arr, Dep, Total\}$ represents the set of flight operation types. M_a^k represents a collection of all flights of operation type k at airport a. For example, M_a^{Arr} represents a collection of all approach flights at airport a. C_a^k, CQ_a^k, CH_a^k represent the 5-minute, 15-minute and 60-minute airport capacity of flight operation type k at Airport a. For example, C_a^{Arr} represents the 5-minute approach capacity of airport a. CQ_a^{Arr} represents the 15-minute approach capacity of airport a.

2). Waypoint capacity constraints

(Shui et al., 2023) have made 15-minute and 60-minute capacity constraints for shared waypoints. Based on this, we add a 5-minute capacity constraint for each shared waypoint. For 15-minute and 60-minute capacity constraints, we use a rolling capacity constraints method.

i. Approach waypoints

5-minute capacity constraints for approach waypoint:

$$\sum_{a\in A}\sum_{m\in M_i^{Arr}}\sum_{t-l_{a,i}^{Arr}} x_{t,m}^a \le C_i, \forall t\in T, \forall a\in A, i\in I \tag{14}$$

Where, M_i^{Arr} represents the set of all flights passing through approach waypoint i. C_i represents the maximum number of flights that can pass through the approach waypoint i in 5 minutes. $l_{a,i}^{Arr}$ represents the flight time from approach waypoint i to airport a. $I = \{1,2,3\cdots|I|\}$ represents all shared approach waypoints in the multi-airport system.

15-minute and 60-minute rolling capacity constraints for approach waypoint:

$$\sum_{a\in A}\sum_{m\in M_i^{Arr}}\sum_{t-l_{a,i}^{Arr}}^{t-l_{a,i}^{Arr}+2} x_{t,m}^a \le CQ_i, \forall t\in T, \forall a\in A, i\in I \tag{15}$$

$$\sum_{a\in A}\sum_{m\in M_i^{Arr}}\sum_{t-l_{a,i}^{Arr}}^{t-l_{a,i}^{Arr}+11} x_{t,m}^a \le CH_i, \forall t\in T, \forall a\in A, i\in I \tag{16}$$

Where, CQ_i represents the maximum number of flights that can pass through the approach waypoint i in 15 minutes. CH_i represents the maximum number of flights that can pass through the approach waypoint i in 60 minutes.

ii. Departure waypoints

5-minute capacity constraints for departure waypoint:

$$\sum_{a\in A}\sum_{m\in M_d^{Dep}}\sum_{t+l_{a,d}^{Dep}} x_{t,m}^{a} \le C_d, \forall t\in T, \forall a\in A, d\in D \tag{17}$$

Where, M_d^{Dep} represents the collection of all airplanes passing through the departure waypoint d. C_d represents the maximum number of flights that can pass through the departure waypoint d in 5 minutes. $l_{a,d}^{Dep}$ represents the flight time from airport a to departure waypoint d. $D = \{1,2,3 \cdots |D|\}$ represents all shared departure waypoints in the multi-airport system.

15-minute and 60-minute rolling capacity constraints for departure waypoint:

$$\sum_{a\in A}\sum_{m\in M_d^{Dep}}\sum_{t+l_{a,d}^{Dep}}^{t+l_{a,d}^{Dep}+2} x_{t,m}^{a} \le CQ_d, \forall t\in T, \forall a\in A, d\in D \tag{18}$$

$$\sum_{a\in A}\sum_{m\in M_d^{Dep}}\sum_{t+l_{a,d}^{Dep}}^{t+l_{a,d}^{Dep}+11} x_{t,m}^{a} \le CH_d, \forall t\in T, \forall a\in A, d\in D \tag{19}$$

Where, CQ_d represents the maximum number of flights that can pass through the departure waypoint d in 15 minutes. CH_d represents the maximum number of flights that can pass through the departure waypoint d in 60 minutes.

iii. Waypoints for arrival and departure flights

5 minutes capacity constraints:

$$\sum_{a\in A}\sum_{m\in M_w^{Arr}}\sum_{t-l_{a,w}^{Arr}} x_{t,m}^{a} + \sum_{a\in A}\sum_{m\in M_w^{Dep}}\sum_{t+l_{a,w}^{Dep}} x_{t,m}^{a} \le C_w, \forall t\in T, \forall a\in A, w\le W \tag{20}$$

Where, C_w represents the maximum number of flights that can pass in 5 minutes at the waypoint where there are both arrival and departure flights. $W = \{1,2,3 \cdots |W|\}$ represents the collection of waypoints in the multi-airport system that both have arrival and departure flights.

15-minute and 60-minute rolling capacity constraint:

$$\sum_{a\in A}\sum_{m\in M_w^{Arr}}\sum_{t-l_{a,w}^{Arr}}^{t-l_{a,w}^{Arr}+2} x_{t,m}^{a} + \sum_{a\in A}\sum_{m\in M_w^{Dep}}\sum_{t+l_{a,w}^{Dep}}^{t+l_{a,w}^{Dep}+2} x_{t,m}^{a} \le CQ_w, \forall t\in T, \forall a\in A, w\in W \tag{21}$$

$$\sum_{a\in A}\sum_{m\in M_w^{Arr}}\sum_{t-l_{a,w}^{Arr}}^{t-l_{a,w}^{Arr}+11} x_{t,m}^{a} + \sum_{a\in A}\sum_{m\in M_w^{Dep}}\sum_{t+l_{a,w}^{Dep}}^{t+l_{a,w}^{Dep}+11} x_{t,m}^{a} \le CH_w^{Total}, \forall t\in T, \forall a\in A, w\in W \tag{22}$$

Where, CQ_w represents the maximum number of flights that can pass in 15 minutes at a waypoint where there are both arrival and departure flights. CH_w represents the maximum number of flights that can pass in 60 minutes at a waypoint where there are both arrival and departure flights.

3). Turnaround time constraint

The aircraft turnaround time constraint requiring that the time interval between the arrival and departure slot should be greater than or equal to the minimum aircraft turn-around time and not greater than the maximum turnaround time. The formula is as follows:

$$TF_{\min} \le \sum_{a\in A}\sum_{t\in T}(t * x^{a}_{t,m^{z}_{dep}}) - \sum_{a\in A}\sum_{t\in T}(t * x^{a}_{t,m^{z}_{arr}}) \le TF_{\max} \tag{23}$$

TF_{min} represents the minimum turnaround time of connecting flight z. TF_{max} represents the maximum turnaround time of connecting flight z. $(m^z_{arr},\ m^z_{dep})$ represents a movement pair for connecting flight z, m^z_{dep} is the departure and m^z_{arr} is the arrival movement of connecting flight z. The minimum turnaround time is set as 6 and the maximum turnaround time as 36 according to the Normal Statistics Method of Civil Aviation Flights.

4). Each flight request must be allocated at one airport in the multi-airport system and every flight must be allocated to one and only one time interval:

$$\sum_{t\in T} x^{a}_{t,m} = 1, \forall a \in A, m \in M \tag{24}$$

3.6. Model solving

The model proposed in this study is a nonlinear bi-objective model. In order to solve the model, we need to transform it into a linear single objective model. So, the ε-constraint is adopted to transform the fairness objective into the fairness constraint (Ehrgott [22]). The specific transformation process is as follows:

$$r_u^{MMA} = |\mu_{a,u} - 1| \le \varepsilon \tag{25}$$

$$\left| \frac{\dfrac{\sum_{m\in M_{a,u}}\sum_{t\in T}|t-t_m|x^{a}_{t,m}}{\sum_{a\in A}\sum_{m\in M_{a,u}}\sum_{t\in T}|t-t_m|x^{a}_{t,m}}}{\dfrac{\sum_{m\in M_{a,u}}\sum_{t\in T}A^{a,u}_{t,m}}{\sum_{a\in A}\sum_{m\in M_{a,u}}\sum_{t\in T}A^{a,u}_{t,m}}} - 1 \right| \le \varepsilon \tag{26}$$

$$\left| \frac{\sum_{m\in M_{a,u}}\sum_{t\in T}|t-t_m|x^{a}_{t,m}}{\dfrac{\sum_{m\in M_{a,u}}\sum_{t\in T}A^{a,u}_{t,m}}{\sum_{a\in A}\sum_{m\in M_{a,u}}\sum_{t\in T}A^{a,u}_{t,m}}} - \sum_{a\in A}\sum_{m\in M_{a,u}}\sum_{t\in T}|t-t_m|x^{a}_{t,m} \right| \le \varepsilon * \sum_{a\in A}\sum_{m\in M_{a,u}}\sum_{t\in T}|t-t_m|x^{a}_{t,m} \tag{27}$$

$\mu_{a,u}$ represents peak-demand based fairness index of airport a at approach waypoint u. $M_{a,u}$

represents the set of all flights in airport a that pass through the approach waypoint u. t_m is the request flight time for aircraft m and t is the actual allocated time of the flight. $A_{t,m}^{a,u}$ represents that the requested time t of flight m at airport a is the peak time of the approach waypoint u.

r_u^{MMA} is the absolute fairness target value of the multi-airport system at the shared approach waypoint u. In this paper, the value is used to measure the overall fairness level of the multi-airport system at the approach waypoint u at peak time. A smaller value of r_u^{MMA} indicates that the waypoint is fairer at peak time. So, it can also be referred to as the absolute fairness target value of airport based on peak demand. The value of ε represents the maximum allowable difference between the fairness value of a given airport and the absolute fairness value. If the value of ε is too high, the fairness cannot be well reflected and the results would be same as those resulting from a model that does not consider fairness. If it is too small, it cannot find a feasible solution. The value of ε can be selected according to the need.

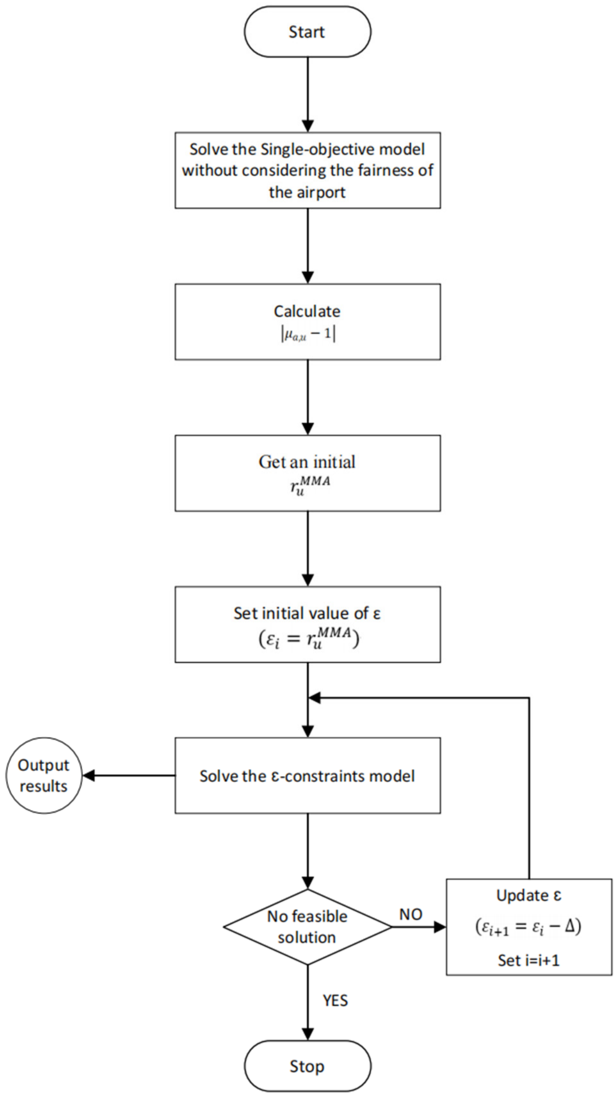


Figure 1. Flowchart of the solution algorithm.

Selection of parameter ε and specific calculation process: The first step is to solve the flight slot optimization problem without considering the fairness of the airport. The result of the solution is substituted into $|\mu_{a,u} - 1|$ and an initial r_u^{MMA} value is found. In the second step, the calculated initial r_u^{MMA} value is substituted as the initial ε value into the Eq (26) to solve the flight schedule optimization problem considering fairness. In the third step, gradually decrease the value of ε by a certain step, solve the ε-constraint model and calculate the optimal solution under each ε. The algorithm terminates when no feasible solution can be found for an ε. We draw the flowchart of the solution algorithm (Feng et al. [23,24]). The process is shown in Figure 1.

The obtained flight schedule displacement is different when the value of ε is different, and this article will analyze the trade-off between fairness and total displacement using two methods:

Method 1: The drawing method: Through the above series of calculation results, draw a graph with absolute fairness as the horizontal coordinate and total flight slot displacement as the vertical coordinate. The graph can clearly show the tradeoff between fairness and total displacement.

Method 2: The formula method: This tradeoff relationship can be seen directly through the graph. In order to more accurately analyze the trade-off between the overall fairness of the multi-airport system and the total displacement of airport slot, we introduce the concept of fairness cost. The formula is shown as follows:

$$F(\varepsilon) = \frac{TD(\varepsilon) - TD^*}{TD^*} \tag{28}$$

$TD(\varepsilon)$ is the total schedule displacement at fairness level ε. TD^* is the minimum displacement value. Therefore, the fairness cost $F(\varepsilon)$ reflects the additional flight schedule displacement that is required to achieve a given level of fairness compared to the minimum displacement.

Compared with the study [18], we apply the cost of fairness in the result analysis at the multi-airport system level for the first time. On the one hand, the calculation of fairness cost can help us to analyze the trade-off between fairness and displacement more accurately from the perspective of data. On the other hand, the introduction of the fairness cost can provide some help for the selection of ε value and the decision-making of decision-makers. Although all airports want to achieve a more equitable distribution, but the total flight slot displacement they can accept is limited. So, fairness cost can be used to evaluate the additional flight slot displacement required to achieve different absolute fairness target values. Decision makers can weigh the demand for fairness and the acceptance degree of displacement to choose a more appropriate fairness target value, and then get the best flight slot allocation results.

3.7. Model comparison

In order to further verify the rationality and effectiveness of the flight schedule optimization model established in this study, several comparison models are also designed.

3.7.1. Non-rolling capacity constraint method

The model established in this paper considers rolling capacity constraints of 15 minutes and 60 minutes. Taking 15-minute capacity constraint as an example, if there are 6 time intervals with 5-minute units (1, 2, 3, 4, 5, 6) and the maximum 15-minute airport capacity for departure is 10, then the rolling capacity constraints should ensure that the requests for departure that will be allocated to intervals {1, 2, 3}, {2, 3, 4}, {3, 4, 5} and {4, 5, 6} will not be more than the maximum departure capacity 10. Non-rolling capacity constraints only need to ensure that the requests for departure within the intervals {1, 2, 3} and {4, 5, 6} does not exceed the maximum departure capacity of the airport in 15 minutes. The specific form is shown in Figure 2.

Previous studies only considered the 15-minute or 60-minute capacity constraints, which may result in the situation that there are no flights in the first 10 minutes and all flights are concentrated in the last 5 minutes. Therefore, a 5-minute capacity constraint is added, and a 15-minute rolling capacity constraint is set to ensure that the capacity saturation state will not last for a long time in a day. It can help to reduce the workload of controllers, improve airport operation efficiency and make the results of flight slot allocation more balanced and accurate.

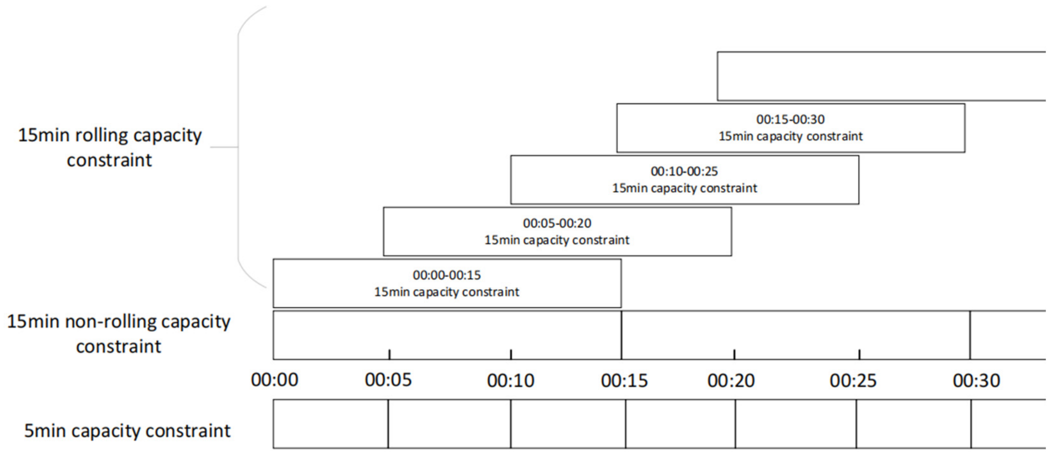


Figure 2. Capacity constraint method.

To verify the effectiveness, rationality and accuracy of the rolling capacity constraint method, this paper will design a set of models with non-rolling capacity constraints for comparison. The meanings of parameters such as M_a^{Arr}, C_a^{Arr}, CQ_a^{Arr} and CH_a^{Arr} in the non-rolling constraint formula are consistent with those in the rolling constraint model. However, t in the 15-minute rolling capacity constraint formulas no longer takes all $T = \{1,2,3,\cdots|T|\}$, but $t = 3t_f - 2, \forall t_f \in T_f$. $T_f = \{1,2,3,\cdots|T_f|\}$ represents a set of time periods with a time interval length of 15 minutes. The value of t in the 60-minute rolling capacity constraint is set to $t = 12t_s - 11, \forall t_s \in T_s$. $T_s = \{1,2,3,\cdots|T_s|\}$ represents a set of time periods with a time interval length of 60 minutes. The specific capacity constraint formulas are shown as follows:

1). Airport capacity constraints

5-minute,15-minute and 60-minute non-rolling capacity constraints at airport a.

$$\sum_{m\in M_a^k}\sum_{t\in T} x_{t,m}^a \le C_a^k, \forall a\in A, k\in K \tag{29}$$

$$\sum_{m\in M_a^k}\sum_{t}^{t+2} x_{t,m}^a \le CQ_a^k, t=3t_f-2, \forall t_f \in T_f, \forall a\in A, k\in K \tag{30}$$

$$\sum_{m\in M_a^k}\sum_{t}^{t+11} x_{t,m}^a \le CH_a^k, t=12t_s-11, \forall t_s \in T_s, \forall a\in A, k\in K \tag{31}$$

2). Waypoint capacity constraints

i. Arrival waypoints

5-minute, 15-minute and 60-minute non-rolling capacity constraints at arrival waypoint:

$$\sum_{a\in A}\sum_{m\in M_i^{Arr}}\sum_{t-l_{a,i}^{Arr}} x_{t,m}^a \le C_i, \forall t\in T, \forall a\in A, i\in I \tag{32}$$

$$\sum_{a\in A}\sum_{m\in M_i^{Arr}}\sum_{t-l_{a,i}^{Arr}}^{t-l_{a,i}^{Arr}+2} x_{t,m}^a \le CQ_i, t=3t_f-2, \forall t_f \in T_f, \forall a\in A, i\in I \tag{33}$$

$$\sum_{a\in A}\sum_{m\in M_i^{Arr}}\sum_{t-l_{a,i}^{Arr}}^{t-l_{a,i}^{Arr}+11} x_{t,m}^a \le CH_i, t=12t_s-11, \forall t_s \in T_s, \forall a\in A, i\in I \tag{34}$$

ii. Departure waypoints

5-minute, 15-minute and 60-minute non-rolling capacity constraints at departure waypoint:

$$\sum_{a\in A}\sum_{m\in M_d^{Dep}}\sum_{t+l_{a,d}^{Dep}} x_{t,m}^a \le C_d, \forall t\in T, \forall a\in A, d\in D \tag{35}$$

$$\sum_{a\in A}\sum_{m\in M_d^{Dep}}\sum_{t+l_{a,d}^{Dep}}^{t+l_{a,d}^{Dep}+2} x_{t,m}^a \le CQ_d, t=3t_f-2, \forall t_f \in T_f, \forall a\in A, d\in D \tag{36}$$

$$\sum_{a\in A}\sum_{m\in M_d^{Dep}}\sum_{t+l_{a,d}^{Dep}}^{t+l_{a,d}^{Dep}+11} x_{t,m}^a \le CH_d, t=12t_s-11, \forall t_s \in T_s, \forall a\in A, d\in D \tag{37}$$

iii. Waypoints for arrival and departure flights

5-minute, 15-minute and 60-minute non-rolling capacity constraints at the waypoints for both arrival and departure flights:

$$\sum_{a\in A}\sum_{m\in M_w^{Arr}}\sum_{t-l_{a,w}^{Arr}} x_{t,m}^a + \sum_{a\in A}\sum_{m\in M_w^{Dep}}\sum_{t+l_{a,w}^{Dep}} x_{t,m}^a \le C_w, \forall t\in T, \forall a\in A, w\in W \tag{38}$$

$$\sum_{a\in A}\sum_{m\in M_w^{Arr}}\sum_{t-l_{a,w}^{Arr}}^{t-l_{a,w}^{Arr}+2} x_{t,m}^a + \sum_{a\in A}\sum_{m\in M_w^{Dep}}\sum_{t+l_{a,w}^{Dep}}^{t+l_{a,w}^{Dep}+2} x_{t,m}^a \le CQ_w, t=3t_f-2, \forall t_f \in T_f, \forall a\in A, w\in W \tag{39}$$

$$\sum_{a\in A}\sum_{m\in M_w^{Arr}}\sum_{t-l_{a,w}^{Arr}}^{t-l_{a,w}^{Arr}+11} x_{t,m}^a + \sum_{a\in A}\sum_{m\in M_w^{Dep}}\sum_{t+l_{a,w}^{Dep}}^{t+l_{a,w}^{Dep}+11} x_{t,m}^a \le CH_w, t=12t_s-11, \forall t_s \in T_s, \forall a\in A, w\in W \tag{40}$$

3.7.2. Non-peak demand-based fairness method

When establishing the absolute fairness constraint of waypoints, the model adopted in this study considers the absolute fairness of airports at shared waypoints under peak demand, that is, the flight slot displacement is allocated according to the number of flights requested by each airport at shared waypoints during peak hours. We believe that considering the peak-demand based fairness will further enhance the fairness of flight slot allocation. To verify this conjecture, the absolute fairness method not based on peak demand is designed for comparison. The specific formulas are as follows:

$$\mu_{a,u}^{*}=\begin{cases}\dfrac{\dfrac{S_{a,u}}{S_u}}{r_{a,u}^{*}}, if\ r_{a,u}\neq 0\\ 1, if\ r_{a,u}=0\ and\ S_{a,u}=0\\ \infty, if\ r_{a,u}=0\ and\ S_{a,u}\neq 0\end{cases} \tag{41}$$

$$r_{a,u}^{*}=\frac{N_{a,u}^{*}}{N_u^{*}}(N_u^{*}\neq 0) \tag{42}$$

$$\min r_{MMA}^{*}=\min\max_{a\in A}\left|\mu_{a,u}^{*}-1\right| \tag{43}$$

Where, $\mu_{a,u}^{*}$ represents the non-peak-demand based fairness index of airport a at waypoint u. $S_{a,u}$ represents the flight schedule displacement allocated to airport a at waypoint u. S_u represents the total flight schedule displacement of all airports in the multi-airport system at the waypoint u. $r_{a,u}^{*}$ represents the proportion of flights requested by airport a at waypoint u. $N_{a,u}^{*}$ represents the number of flights within airport a that have requested pass through arrival waypoint u. N_u^{*} represents the number of flights within the multi-airport system that have requested to pass through the waypoint u.

It can be seen from the Eq (42), unlike the absolute fairness method based on peak demand, the calculation method of $r_{a,u}^{*}$ in the absolute fairness method that is not based on peak demand has changed. The value is equal to the ratio of the number of flights requested by an airport at a shared waypoint to the total number of flights requested by all airports at that waypoint. Therefore, airports with a larger number of flights requested at that waypoint will eventually have a relatively larger displacement in their flight schedules. However, the peak time is the focus of flight slot optimization. In theory, the flight slot optimization considering peak demand is fairer. In the following chapter, we carry out simulation verification through examples to prove the correctness of the above conjecture.

In this paper, Python 3.10.7 was used for programming under Windows 10 operating system, and Gurobi 10.0.0 solver was used to solve the model.

4. Model application and results

4.1. Model verification

4.1.1. Airport selection

There are nine airports in the Beijing-Tianjin-Hebei region multi-airport system. The important airports are Capital International Airport, Daxing International Airport, Tianjin Binhai International Airport and Shijiazhuang Zhengding International Airport. Taking the Capital Airport as the benchmark, the linear distance between Capital Airport and Daxing Airport is about 67 kilometers, Tianjin Binhai Airport is about 125 kilometers, and Shijiazhuang Airport is about 260 kilometers. These airports are relatively close, so they are bound to have mutual constraints and interdependence during operation. The coordinated development strategy of Beijing-Tianjin-Hebei region multi-airport system is put forward to alleviate the unreasonable structure of air route network and the shortage of airspace resources caused by the unbalanced and uncoordinated development of civil aviation in North China. In 2019, the passenger throughput of Beijing-Tianjin-Hebei region multi-airport system reached 1466.6556 million, an increase of 1.1% over the previous year. With the recovery of the epidemic and the rapid development of the civil aviation industry, this figure is bound to increase further, and the competition among airports in the shared airspace resources such as shared waypoints, air routes and corridors will also further intensify. How to allocate the limited airspace resources more scientifically and reasonably is the key to achieve the coordinated development of the Beijing-Tianjin-Hebei region multi-airport system. Therefore, in order to improve the operation efficiency and cooperative development level of the Beijing-Tianjin-Hebei region multi-airport system, it is necessary to optimize the airport slot of the multi-airport system.

In this study, the Beijing-Tianjin-Hebei region multi-airport system is taken as an example to verify the above model. According to the statistics, the arrival and departure waypoints of the four major airports in the Beijing-Tianjin-Hebei region multi-airport system are shown in Table 2. It can be seen from Table 2 that the four airports don’t have shared arrival or departure waypoints, while the three airports of PEK, PKX and TSN have three shared departure waypoints and two shared approach waypoints. Therefore, Beijing Capital International Airport (PEK), Beijing Daxing International Airport (PKX) and Tianjin Binhai International Airport (TSN) are selected as examples to verify the model. The shared arrival waypoints AVBOX and DUMAP and the shared departure waypoints IDKEX, MUGLO and ELKUR are selected as important waypoints. The capacity constraints of shared waypoints are modeled based on these waypoints.

Table 2. Statistics of Beijing-Tianjin-Hebei Airport arrival and departure waypoints.

	Arrival waypoints	Departure waypoints
PEK	GUVBA, OSUBA, DUMAP, AVBOX, DUGEB	IDKEX, DOTRA, LULTA, MUGLO, IGMOR, ELKUR, BOTPU, RUSDO
PKX	BUMDU, DUMAP, ABVOX, ELAPU, BELAX	IDKEX, DOTRA, MUGLO, ELKUR, OMDEK, PEGSO
TSN	DUMAP, AVBOX, GUVBA	IDKEX, MUGLO, IGMOR, ELKUR, OMDEK, PEGSO, BOTPU
SJW	VADKA, PEGSO, TONOV, AVLIS, LIKTI	BELAX, UKMIS, VADKA, ADBES

4.1.2. Parameter setting

According to statistics, there were 2335 arrival and departure flights at the three airports of Beijing and Tianjin on January 16, 2023. The data scale is very large, but the flight volume is not large in some periods of a day, so there is no need to optimize the flight time of those periods. Therefore, this paper will select a busy period of that day to verify and analyze the model. After successful verification, the model can be applied to optimize the airport slot in a longer period.

We select 722 flights from 8:00 to 14:00 on January 16, 2023, to verify this model. There included 307 flights from Beijing Capital International Airport, 262 flights from Daxing International Airport and 153 flights from Tianjin Binhai International Airport. The following describes how to obtain the relevant information and how to set the parameters.

a) Flight information: Collect the flight schedule of the day from the official website of each airport, including the flight number, airport of departure and landing, planned takeoff and landing time, actual takeoff and landing time etc.
b) Airport capacity information: Based on the official information released by the Civil Aviation Administration of China, air traffic control units, airports, controllers' work experience and airport historical operation data, the airport capacity standards of each airport for 5 minutes, 15 minutes and 60 minutes are obtained, as shown in Table 3.

Table 3. Airport capacity information.

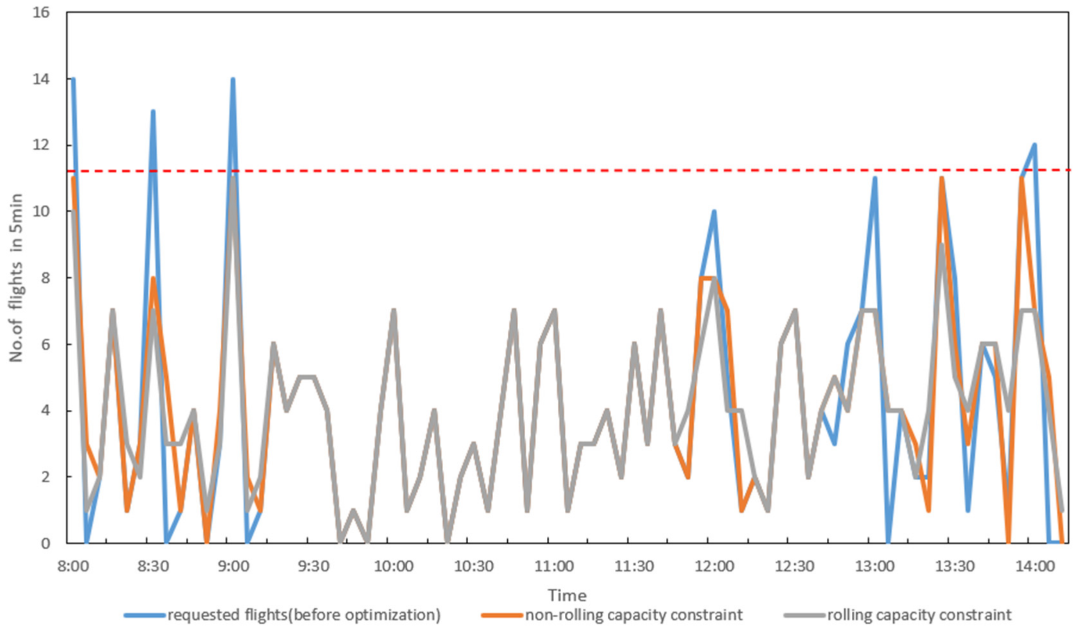
Airport	5 minutes capacity (No. of flights)	15 minutes departure capacity (No. of flights)	15 minutes arrival capacity (No. of flights)	15 minutes total movement capacity (No. of flights)	60 minutes total movement capacity (No. of flights)
PEK	11	12	12	18	70
PKX	9	11	9	17	62
TSN	3	8	7	12	32

c) Waypoint capacity information: Set the maximum capacity of each shared waypoint for 5 minutes, 15 minutes and 60 minutes according to the controller's work experience and historical operation data. For example, set the maximum capacity of IDKEX point for each time interval as: 5, 9 and 26. The maximum capacity of each time interval at ELKUR is set as: 6, 13, 32. The maximum capacity of each time interval at MUGLO is set as: 5, 9, 30. The maximum capacity of each time interval at MUGLO is set as: 5, 10, 31. The maximum capacity of each time interval at DUMAP is set to 4, 8 and 25.
d) In this paper, the arrival waypoint AVBOX is selected as the key research object, and the absolute fairness constraint is established at that point.

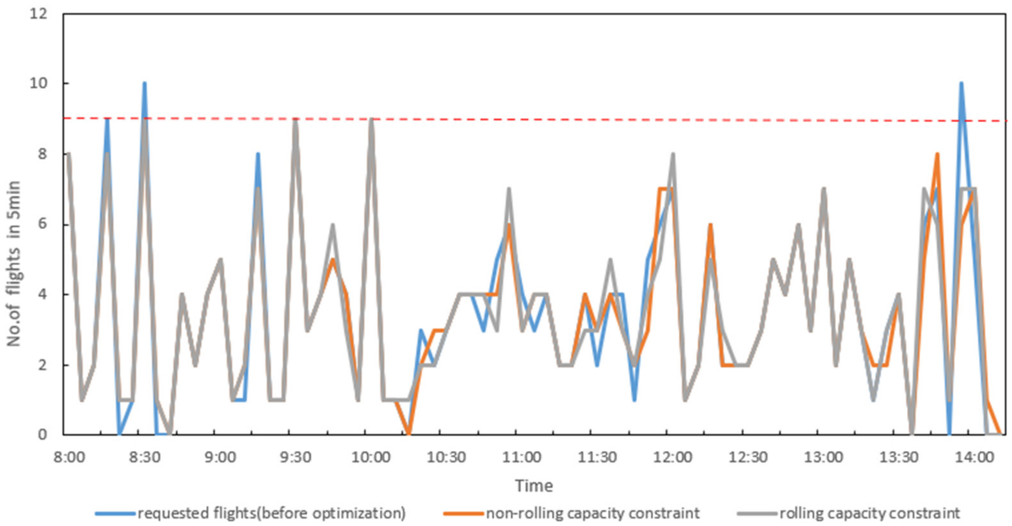
4.2. Result analysis

4.2.1. Effect of rolling capacity constraint method on optimization results

Figure 3 shows the changes in flight schedules at the three airports before and after the optimization. The red dotted line represents the maximum allowable flight volume of each airport within a 5-minute interval. In other words, the maximum allowable flight volume is the airport's available capacity of the 5-min interval. The line chart shows that the optimized schedule has a decrease in peak values. On the premise that the capacity limit is not exceeded, the low flight volume times share part of the adjacent peak time flights, so that the number of flights in each time interval is within the maximum capacity limit of the airport.



(a) The number of flights per 5 min before and after optimization of PEK



(b) The number of flights per 5 min before and after optimization of PKX

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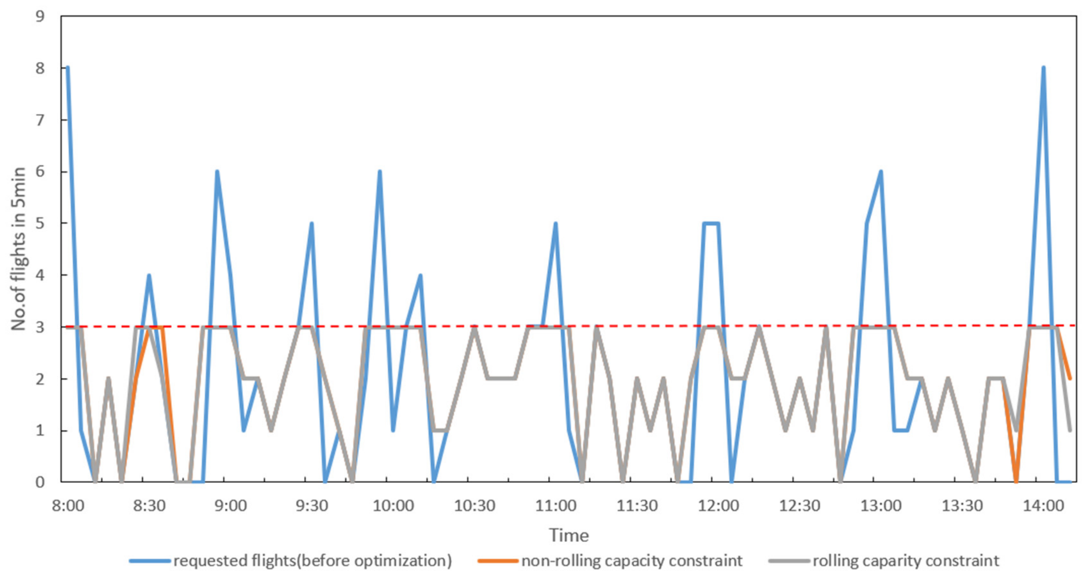


(c) The number of flights per 5 min before and after optimization of TSN

Figure 3. The number of flights per 5 min before and after optimization of the three airports.

Compared the solving results of rolling capacity constraint method and non-rolling capacity constraint method, it is found that the solving result of rolling capacity constraint method further reduces the number of peak flights, and the flights at peak time are dispersed to adjacent time, which makes the distribution of schedule flights more uniform. For example, it can be reflected in the 8:15−8:45 and 13:15−14:00 periods at PEK and the 10:00−10:30 and 12:15−12:30 periods at PKX. Figure 4 further illustrates this change.

Figure 4. Number of flights in 5 minutes and 15 minutes under different methods.

Figure 4 shows the changes in the number of flights in 5 minutes and 15 minutes before and after the flight slot optimization of PEK airport using different capacity constraint methods. The red dotted line in the figure points to the vertical axis value of 18, indicating that the 15-minute airport capacity of PEK is 18. The green dotted line in the figure points to the vertical axis value of 11, indicating that the 5-minute airport capacity of PEK is 11. As can be seen from Figure 4, when using the non-rolling capacity constraint method, only the constrained 15 minutes will meet the capacity limit. Unconstrained 15 minutes are still possible to exceed the maximum capacity, such as the 13:50−14:05 period and the 13:20−13:35 period. The rolling capacity constraint method can ensure that any 15 minutes in a day does not exceed the maximum capacity limit.

Comparing the six lines in Figure 4, it can be seen that the line obtained by the rolling capacity constraint method is smoother and the fluctuation range is smaller. The sudden increase in the number of flights in the adjacent 15-minute period is further reduced, and the distribution of flights throughout the day is also even. However, although the flight slot adjusted by the non-rolling capacity constraint method also meets the capacity limit, there is still an uneven distribution of flight slots. There may be no flights in the first 10 minutes of a 15-minute interval, and all flights are concentrated in the last 5 minutes. For example, after adjusting for the non-rolling capacity constraint method, the number of flights at 13:20 is only 1, while the number of flights at 13:25 reaches the airport's 5-minute capacity limit of 11. These times when the number of flights is extremely high is the moment to potentially break through the capacity constraints. In the actual operation, the capacity constraints of these time may be broken at any time as unexpected situations arise. The rolling capacity constraint method can further adjust these times with a large number of flights and allocate some peak time flights to adjacent idle times. So that the entire flight distribution curve is smoother. For example, the flight slot allocation result obtained by the rolling capacity constraint method adjusts the number of flights at 13:20 to 4, and the number of flights at 13:25 to 9. In fact, in addition to making the flight distribution more uniform and improving the operation efficiency of the airport, this time adjustment obtained by using the rolling capacity constraint method also enhances the flight coordination ability in the face of emergencies during peak hours.

Table 4. The flight distribution of PEK under different methods.

Time	The number of requested flights	The number of flights (non-rolling constraint method)	The number of flights (rolling constraint method)
13:45	5	6	6
13:50	1	0	4
13:55	11	11	7
14:00	12	7	7
14:05	0	5	4
14:10	0	0	1

Taking the flight distribution of Beijing Capital International Airport before and after optimization from 13:45 to 13:55 as an example. As shown in Table 4, after adjusting the original schedule with the non-rolling capacity constraint method, the flight volume of 13:45 is 6, the flight volume of 13:50 is 0 and the flight volume of 13:55 is 11. After adjusting the original schedule with rolling capacity constraint method, the number of flights in the 13:45 period is 6, the number of flights in the 13:50

period is 4 and the number of flights in the 13:55 period is 7. Therefore, after adopting the rolling capacity constraint method, the flight pressure in the 13:55 period is shared by the 13:50 period and the flight distribution is more uniform.

In summary, the rolling capacity constraint method can further adjust the demand during peak hours. Under the premise of fully satisfying the capacity constraints, the smooth adjustment of the model to some potential capacity conflict periods further reduces the possibility of breaking through the capacity constraints during potential conflict periods. The result is a smoother distribution of flight slot. A more uniform distribution of flight slot is beneficial to improve airspace utilization and airport operation efficiency and reducing delays. At the same time, the effective control of the number of flights in each time period can reduce the workload of controllers and ensure the safety of airport operation.

4.2.2. The tradeoff between total schedule displacement of the flight schedule of the multi-airport system and absolute fairness of the multi-airport

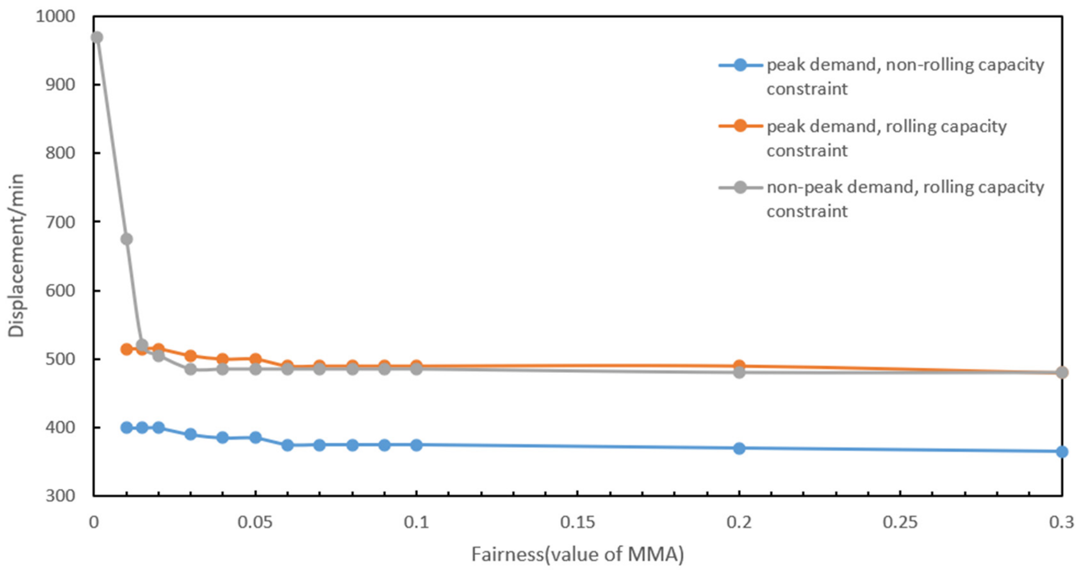


Figure 5. The trade-off between flight displacements and MMA airport fairness.

The results presented in Figure 5 are consistent with those obtained using the Gini coefficient-based fairness method in the literature [19]. This shows that the absolute fairness method used in this paper is also applicable within the scope of multi-airport system.

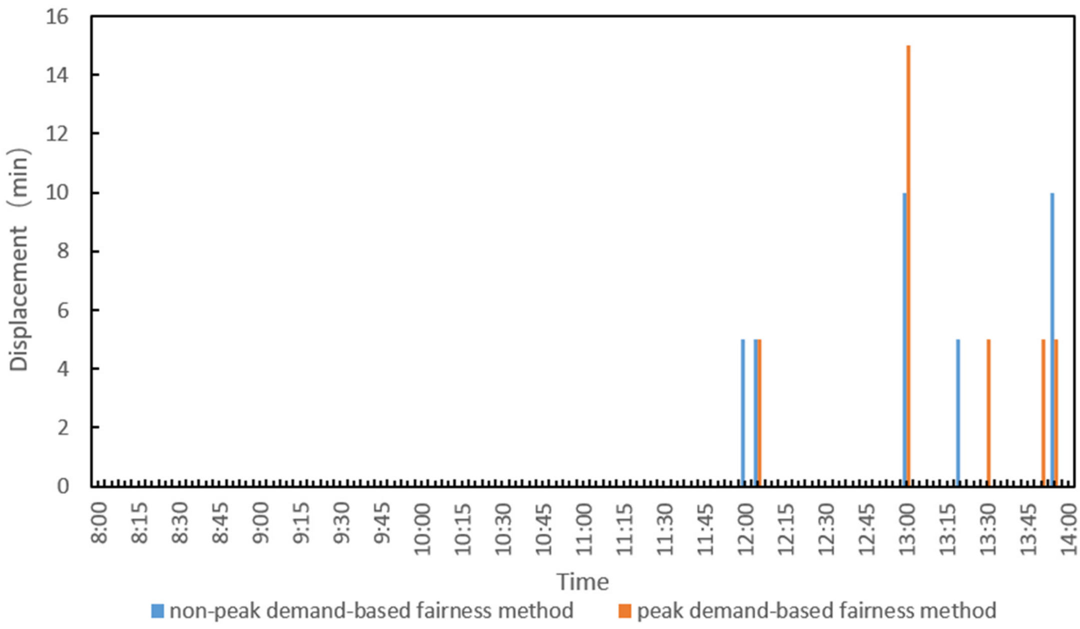
It can be seen from Figure 5 that the smaller the absolute fairness index of the airport, the larger the total flight schedule displacement. The smaller the absolute fairness objective of airports, the fairer the treatment of each airport, the better the overall fairness of the multi-airport system. In order to achieve this fairness, the greater the amount of flight slot adjustment for each airport. This problem can also be illustrated by calculating the fairness cost. For example, when the peak-demand based method and rolling constraint method are adopted, the fairness cost is 0.073 when the absolute fairness objective of the airport is 0.02. It means that 7.3% of the optimal displacement should be increased to achieve the absolute fairness objective of 0.02. Similarly, to achieve the absolute fairness goal of 0.05, the optimal displacement should be increased by 4.2%. To achieve absolute fairness objective of 0.08, we need to increase the optimal displacement by 2.1%. Compared with the absolute fairness value changing from 0.08 to 0.05, when the absolute fairness value changing from 0.05 to 0.02, the optimal displacement needs to increase more. It further indicates that when the absolute fairness requirement is higher, the increase of flight slot displacement will be larger. Decision makers can weigh the demand for fairness and the acceptance degree of displacement to choose a more appropriate fairness target value, and then get the best flight slot allocation results.

As shown in Figure 5, when the absolute fairness objective is in the range of [0, 0.016], the flight slot displacement based on non-peak demand fairness method is greater than the flight slot displacement based on peak-demand fairness method. Moreover, within this range, the total flight slot displacement calculated by non-peak demand-based fairness method increases greatly. When the absolute fairness value of airport is 0.015, the fairness cost is 0.073 and when the absolute fairness value of airport is 0.001, the fairness cost is as high as 1.02. This indicates that non-peak demand-based optimization algorithms are more sensitive to the change of fairness objective value when fairness value is in the range of [0, 0.016]. Therefore, when a higher level of fairness is needed, the slot optimization using peak-based fairness method requires a smaller total displacement and the optimization effect is better. In the range [0.016, 0.3], the flight schedule displacement obtained by the peak demand-based fairness method is larger than the flight schedule displacement obtained by the non-peak demand-based fairness method. However, the values are similar.

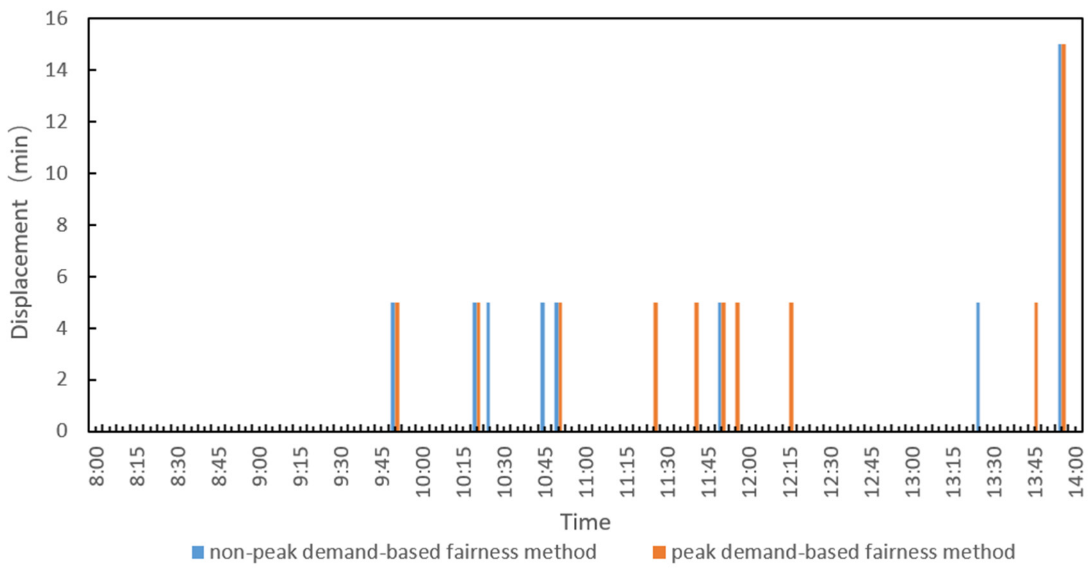
By comparing the solution results of the model with and without the consideration of rolling constraints, it can be seen that the flight slot displacement obtained by the method considering the rolling constraints is larger. It shows that the rolling-constraints method can enhance the constraints on the model, increase the amount of flight slot adjustment and make the adjustment results more accurate.

4.2.3. The effect of peak-based fairness approach on flight slot displacement of airports at key waypoints

Figure 6 shows the change in flight schedule displacement of each airport at AVBOX after the flight schedule optimization. According to statistics, the peak periods of AVBOX on that day are 10:40 to 10:50 and 13:45 to 14:00. The flight application amount of each airport at ABVOX point and the flight slot displacement after optimization of each airport are shown in Table 5. It can be seen from Table 5 that TSN has the highest total flight applications volume at ABVOX, and the ratio of total flight applications volume of PKX and TSN is close to 1:1. However, in peak hours, the number of flight applications of PKX is the highest and the ratio of application volume of PKX and TSN is close to 3:2.

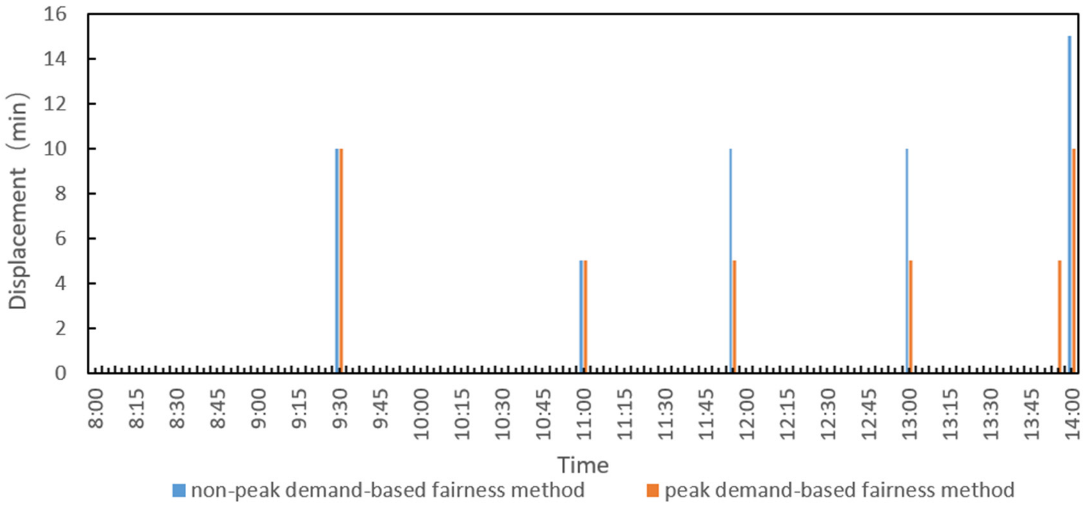


(a) The schedule displacement of PEK at AVBOX per 5 min under the peak demand-based fairness method and the non-peak demand-based fairness method



(b) The schedule displacement of PKX at AVBOX per 5 min under the peak demand-based fairness method and the non-peak demand-based fairness method

Continued on next page



(c) The schedule displacement of TSN at AVBOX per 5 min under the peak demand-based fairness method and the non-peak demand-based fairness method

Figure 6. The flight slot displacement at AVBOX of each airport under the peak demand-based fairness method and the non-peak demand-based fairness method.

When the non-peak-based approach is adopted, the total flight slot displacement allocated to TSN airport is higher due to the high total flight application volume. It can be seen intuitively from Figure 6(c) that TSN will allocate the higher total displacement it allocates by increasing the displacement of off-peak flights. However, with a peak demand-based method, TSN does not allocate the highest flight schedule displacement, and its off-peak flight displacement also decreases.

Peak period is the key point to reduce transportation efficiency. Airports with a higher number of flight applications in peak period should receive more "punishment", that is, allocate more flight slot displacement. Therefore, it is fairer and more reasonable to adopt the peak-demand based flight time allocation method.

Table 5. Flight information at AVBOX points at each airport.

	PEK	PKX	TSN
Total number of flight requests (No. of movements)	36	52	53
Number of applications during peak hours (No. of movements)	7	12	8
Schedule displacement solved by non-peak demand-based fairness methods (min)	35	50	50
Schedule displacement solved by peak demand based fairness methods (min)	35	60	40

5. Conclusions

Based on the operation characteristics of the multi-airport system, we establish a multi-objective flight slot allocation optimization model. The model is constrained by the airport capacity limit, shared waypoints capacity limit and aircraft turnaround time limit. The objective of the optimization is to minimize the total schedule displacement and the maximum deviation of fairness from the absolute fairness. The solution results of the model satisfy all constraints while minimizing the flight slot displacement. In this study, we collect the operation data of Beijing-Tianjin-Hebei multi-airport system and utilized Python programming combined with Gurobi solver to verify and solve the model. Additionally, we established the non-fairness demand-based model and the non-rolling constraint model for comparison. The conclusions of this study are as follows:

We add 5-minute capacity constraint when establishing the capacity limit of airports and waypoints to improve the accuracy of flight slot allocation. By doing so, the flight volume during each 5-minute interval remains within the acceptable range of airports and controllers. Regarding the 15-minute and 60-minute capacity constraints, our results show that the rolling constraint method leads to a more uniform distribution of flight slot compared to the non-rolling constraint method. It can reduce the occurrence of extremely high flight volume at one time and very little flight volume at the adjacent time. Thus, improving the accuracy of allocation. Additionally, it can alleviate the workload of controllers and increase the efficiency of airspace usage.

In this paper, the tradeoff curves between the total time displacement and the absolute fairness objective value are depicted under different conditions. The results indicate that, when the absolute fairness objective is the same, the rolling capacity constraint method produces a larger flight slot displacement compared to the non-rolling capacity constraint method. This suggests that the rolling capacity constraint method imposes stronger constraints on the model and facilitates more adjustments. Moreover, when fairness demand is high, the non-peak demand-based method generates more flight slot displacement than the peak demand-based method.

The absolute fairness target is established based on peak demand in this model. To evaluate the validity of considering peak demand, we also establish a non-peak demand-based fairness model for comparison. Our findings demonstrate that, with the peak-demand fairness method, the ratio of flight slot displacement allocated by each airport at a waypoint is consistent with the ratio of flight requests of each airport at that waypoint during peak times. So, the airport with more flight applications at peak time will be allocated more flight slot displacement. In the non-peak demand-based fairness model, the ratio of flight slot displacement assigned by each airport is close to the ratio of flight volume requested at the waypoint. According to statistics, airports with a large number of total flight applications do not necessarily apply for the largest number of flights at peak hours. Peak hours have a significant impact on how efficiency airspace is used. Airports with a greater number of flight applications at peak hours should receive a higher "penalty" in the form of more allocated flight slot displacement. Hence, the peak demand-based method enhances the fairness of flight slot optimization and reduces the penalty imposed on off-peak flights.

In summary, using rolling capacity constraint method can further improve the uniformity of flight slot allocation results, and adding 5-minute capacity constraint can make slot allocation more accurate. The peak-demand based absolute fairness constraint method is used to further improve the fairness of flight slot allocation at shared waypoints. Both these methods can be considered and applied in future flight slot optimization studies to improve the rationality and acceptability of optimization results.

There are still some limitations in this paper. For instance, we optimize flight slot for only a certain period of a day. The flight slot can be optimized for a day, a week or even a season in the future. In this paper, all the requested flights at the airport are treated with equal importance and optimized simultaneously, without considering the principle of priority. Future research could consider dividing the flights at the airport into three categories based on their slot request priority: historic, new entrant and others. This prioritization principle can be applied to optimize flight slots more effectively. we only consider fairness at a shared arrival waypoint. However, future research could explore the possibility of incorporating fairness constraints at multiple waypoints to ensure fairness throughout the entire flight trajectory. Another possible extension of our research is considering the cost of flight slot adjustment. By incorporating the minimum total adjustment cost into the optimization objective, the acceptability of flight slot optimization results can be enhanced.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

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