Mathematical Biosciences and Engineering

MBE, 19(6): 6379−6395.
DOI: 10.3934/mbe.2022299
Received: 20 November 2021
Revised: 26 February 2022
Accepted: 30 March 2022
Published: 22 April 2022

http://www.aimspress.com/journal/MBE

Research article

Quantitative analysis of facial proportions and facial attractiveness among Asians and Caucasians

Shikang Zheng[1], Kai Chen[1], Xinping Lin[1], Shiqian Liu[2], Jie Han[3] and Guomin Wu[1,*]

[1] Department of Oral, Plastic and Aesthetic Surgery, Hospital of Stomatology, Jilin University, Jilin 130021, China
[2] College of Mathematics, Jilin University, Jilin 130021, China
[3] Academy of Marxism, Jilin University, Jilin 130021, China

* **Correspondence:** Email: wugm@jlu.edu.cn; Tel: + 086043185291286.

Abstract: It has been proposed that the proportions of the human face are crucial for facial aesthetics. If this is the case, we should describe the relationship among proportions of face components quantitatively. This study aims to develop a mathematical model of facial proportions to provide a quantitative description of facial attractiveness. Furthermore, we expect that plastic surgeons can use models in clinical work to enhance communication efficiency between doctors and patients. Face alignment technique was used to analyse 5500 frontal faces with diverse properties (male/female, Asian/Caucasian, ages) to obtain the ratios among the nose length (N_L), the nasal base width (N), and the inner canthus width (E_I). A mathematical model ($N_L^2 = aE_I * N_L + bE_I * N + cN * N_L$) was developed to describe the relationship among these proportions. To validate the effectiveness of this approach, we simulated the post-operative photos using Adobe Photoshop. Our findings show that the ratio of nose length to nose width, the ratio of inner canthus width to nose length and the ratio of inner canthus to nose width play a significant role in determining facial attractiveness. These results provide a possible strategy to quantitatively describe the relationship among human face proportions.

Keywords: proportion; face; facial analysis; facial attractiveness; doctor-patient communication

Abbreviations: AF: Asian females; AM: Asian males; CF: Caucasian females; CM: Caucasian males

1. Introduction

The face is vital in determining human physical beauty and attractiveness. Facial attractiveness is related to mate selection strategies, health, self-esteem and survival of species [1,2]. Attractive people experience more positive outcomes, such as, receiving attention and being more socially prevalent [3−5]. It has been proposed that facial appearance is an indicator of an individual's physical health [6].

From ancient Greece to the Renaissance to modern society, people have been exploring facial attractiveness and the nature of beauty [7]. Many factors affect facial beauty and attractiveness, including: symmetry, youthfulness, skin texture, averageness, proportion, sexual dimorphism, good grooming, and even pregnancy [8−13].

Proportions have a critical impact on facial attractiveness [14]. Coincidentally, both in China and the West, people have adopted the method of dividing the face horizontally into thirds and vertically into fifths to analyse and test a person's face [15]. The ideal ratio is related to the Golden Ratio, which dates back to the ancient Greek period and is widespread. It is represented by the letter Φ (phi, $\Phi \approx 0.618$) [16,17]. Marquardt applied the sacred ratio to three-dimensional space and designed the golden ratio mask, which has wide application [18]. However, this has been questioned by critics who consider this mask too masculine for females [19]. One method people can use to achieve optimal facial attractiveness is by changing proportion through plastic surgery [20]. Proportions are, therefore, an important component of facial analysis and thus essential for optimizing plastic surgery [21,22].

Mateusz's et al. [23] results show that facial attractiveness increases as the uncovered eye surface increases and the nose and lip size decreases. However, only one variable was considered at a time, without examining the quantitative relationship between these variables. This raises the question of whether a mathematical formula can be established to describe the relationship among proportions of face components.

In this study, we focused on the factors that could significantly affect a person's physical beauty with the help of face alignment techniques which allowed us to obtain 68 facial feature points. Furthermore, we tried to describe the relationship among proportions of face components in the mathematical formulation, determining the intrinsic links among these proportions.

2 Material and methods

2.1. SCUT-FBP5500

SCUT-FBP5500 is a new diverse benchmark dataset proposed by South China University of Technology (SCUT). It contains 5500 frontal faces in total with diverse properties (male/female, Asian/Caucasian, ages). The dataset includes 2000 Asian females, 2000 Asian males, 750 Caucasian females and 750 Caucasian males, along with beauty scores [1,5] and beauty score distributions. Scores were obtained by 60 volunteers aged from 18−27 (average 21.6) rating facial attractiveness on a 1−5 scale (Dataset was open on the website: https://github.com/HCIILAB/SCUT-FBP5500-Database-Release) [24].

We selected clear photos of people without excessive make-up and occlusion from the dataset and, the face shown in the frontal aspect, with a neutral expression [25]. In total, 4740 photos were selected that met the criteria as the dataset for this experiment, including 1670 photos of Asia females (AF), 1778 photos of Asia males (AM), 627 photos of Caucasian females (CF), and 665 photos of Caucasian males (CM). The top 10% of scored photos make up the high attractiveness set, and the lowest-rated 10% of images make up the low attractiveness set.

2.2. Design and procedure

The photos were processed in Python using the "Dlib" and "OpenCV" Python packages.

To reduce the arithmetic intensity, OpenCV was used to convert the images to grayscale (Figure 1b). The law of the gamma transformation, $g(x,y) = (f(x,y))^{gamma}$, was used to remove non-uniform illumination of the images (Figure 1−c). Gamma values ranged from 0 to 1 and 0.3 was selected. Sixty-eight facial feature points of a face were extracted by the "Dlib" Python package (Figure 1d).

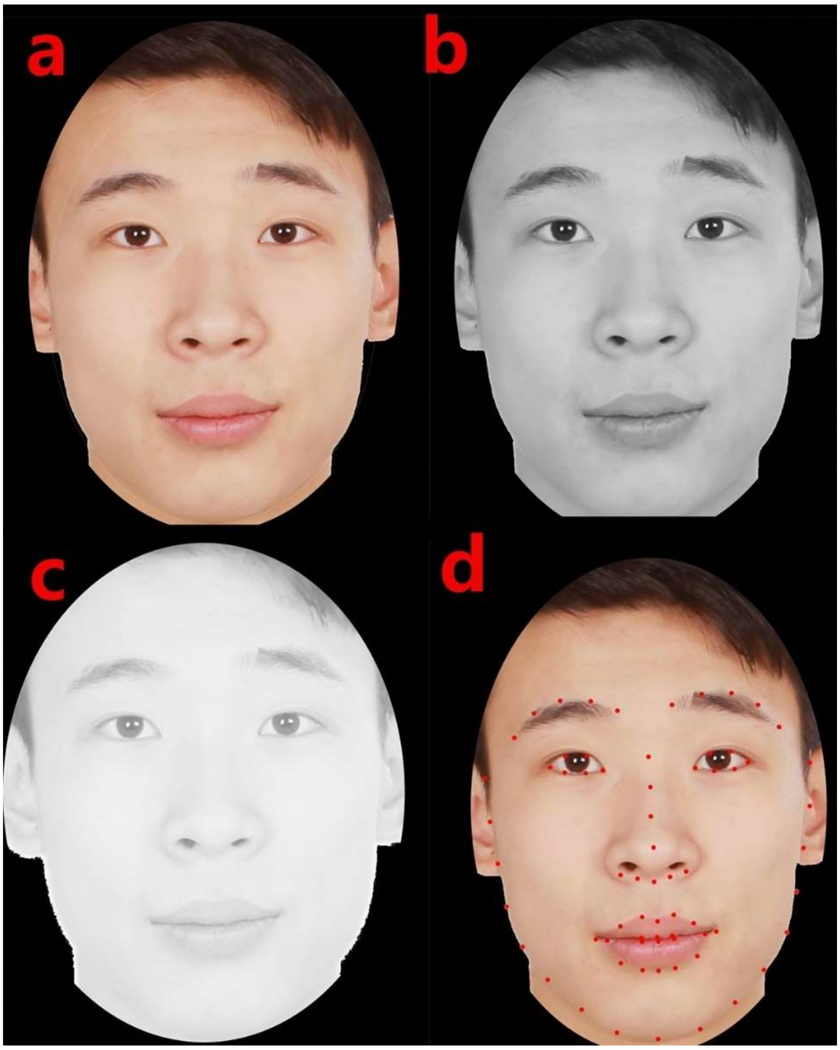

Figure 1. Image processing process.

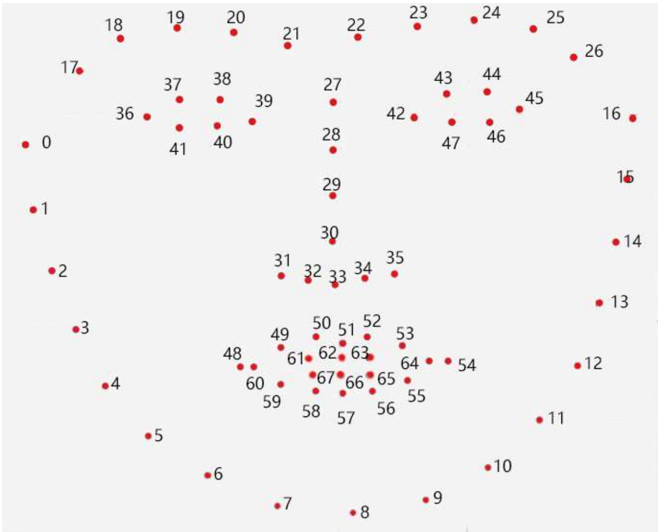

Figure 2. The distribution of 68 feature points model.

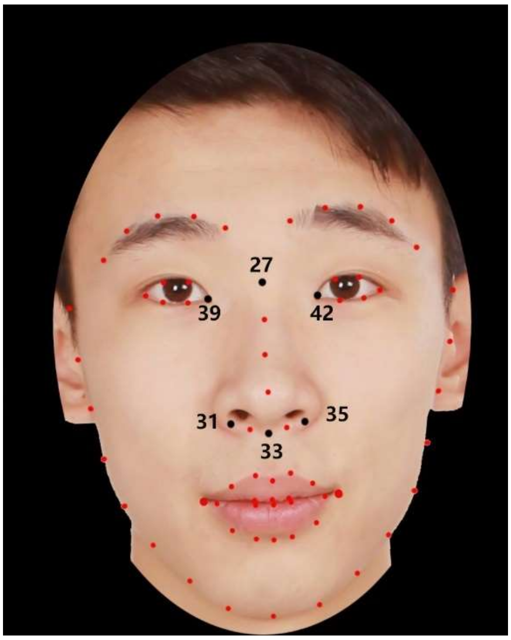

Figure 3. The quantities measured in this study.

The 68 facial feature points contain essential information about a face, including the contour of the face, eyes, nose, mouth, etc. (Figure 2). The input to the procedure is a 2D image, and the output is a set of (x, y) coordinates of the facial landmark points. In this study, the inner canthus width (E_I), nose length (N_L), and nasal base width (N) were selected for facial analysis (Figure 3). The definition of the landmark [26] used in the analysis is shown in Table 1. The Euclidean distances of the above four facial feature quantities were calculated using:

$$\text{Inner canthus width: } E_I = \sqrt{(x_{39} - x_{42})^2 + (y_{39} - y_{42})^2} \tag{1}$$

$$\text{Length of Nose: } N_L = \sqrt{(x_{27} - x_{33})^2 + (y_{27} - y_{33})^2} \tag{2}$$

$$\text{Nasal base width: N} = \sqrt{(x_{31} - x_{35})^2 + (y_{31} - y_{35})^2} \tag{3}$$

The ratios were then calculated between the quantities. R_1 is the ratio of E_I to N. R_2 is the ratio of N_L to N. R_3 is the ratio of E_I to N_L.

Table 1. Definition of landmark used in analysis.

27	Soft-tissue nasion	33	soft-tissue subnasal
42	Left eye media canthus	39	Right eye media canthus
35	Left Nasal ala	31	Right Nasal ala

2.3. Statistical analysis

Independent samples t-test was used to analyse the differences in facial proportions(R_1, R_2 and R_3) across gender and race. We compared the difference in facial proportions(R_1, R_2 and R_3) between those with high attractiveness and those with low attractiveness. The normal distribution and variance homogeneity were tested by the P-P plot and Leven's test. In addition, a stepwise linear regression model was constructed to quantitively analyse the relationship among R_1, R_2 and R_3.

The data were processed using IBM SPSS Statistics 26 (IBM Corporation, Armonk, NY, USA) and $P < 0.05$ and $P < 0.01$ were acknowledged as statistically significant.

2.4. Verification

A total of 20 images were selected from the low attractiveness set, including 5 images each from AF, AM, CF, CM. The nose length, inner canthus width, and nose width were obtained. An ideal inner canthus width was calculated by putting the nose length and the nose width into the formula derived from the high attractiveness set. The ideal nose width was obtained by the same method. Photoshop (Adobe Photoshop CC 2018) was used to separately change the inner canthus and the nasal base width in the photos to the ideal width. To vary the width of the inner canthus width and nose width, Filter-Liquify in Photoshop was used, allowing us to change the facial proportions quantitatively. Thus, three photographs were obtained for each image: the original, one with the nasal base width changed to the ideal, and one with the inner canthus width changed to the ideal. The photos were rated on a 1–5 scale by 48 volunteers ages 18−25 (average age 20.9, 24 females, 24 males), where 1 represented "extremely unattractive" and 5 represented "very attractive". Independent samples t-test was used to analyse the difference in attractiveness scores before and after the changes. $P < 0.05$ and $P < 0.01$ were acknowledged as statistically significant.

2.5. Ethics approval of research

The study was approved by the Ethics Committee of the Hospital of Stomatology, Jilin University (2020–38). Participants provided prior informed written consent.

All procedures performed in studies involving human participants were in accordance with the ethical standards of the hospital research committee and with the 1964 Helsinki Declaration and its later amendments or comparable ethical standards.

3. Results

The description of facial proportions is presented in Table 2.

Significant differences exist in facial proportions between race and gender. Independent samples t-test results showed that R_1 was significantly higher in Asian females than in Asian males ($t = 12.666$, $p < 0.01$), Caucasian females ($t = 7.685$, $p < 0.01$) and Caucasian males ($t = 25.471$, $p < 0.01$). R_2 was significantly higher in Asian females than in Asian males ($t = 11.881$, $p < 0.01$), Caucasian females ($t = 5.865$, $p < 0.01$) and Caucasian males ($t = 13.204$, $p < 0.01$). R_3 was significantly lower in Caucasian males than in Asian females ($t = 12.856$, $p < 0.01$) and Asian males ($t = 12.650$, $p < 0.01$). Differences in facial proportions between race and gender are showed in Figure 4.

Significant differences exist in facial proportions between the high attractiveness set and low attractiveness set. Independent samples t-test results showed that images with high attractiveness scores were significantly higher in R_1 than images with low attractiveness scores among Asian females ($t = 2.901$, $p < 0.01$), Caucasian females ($t = 11.814$, $p < 0.01$) and Caucasian males ($t = 5.336$, $p < 0.01$). Furthermore, images with high attractiveness scores were significantly higher in R_2 than images with low attractiveness scores among Asian females ($t = 11.719$, $p < 0.01$), Asian males ($t = 12.507$, $p < 0.01$), Caucasian females ($t = 13.245$, $p < 0.01$) and Caucasian males ($t = 10.610$, $p < 0.01$). Additionally, images with high attractiveness scores were significantly lower in R_3 than images with low attractiveness scores among Asian females ($t = -10.579$, $p < 0.01$), Asian males ($t = -14.011$, $p < 0.01$) Caucasian females ($t = -2.477$, $p = 0.015$) and Caucasian males ($t = -6.545$, $p < 0.001$). Differences of facial proportions between high attractiveness set and low attractiveness set are showed in Figure 5.

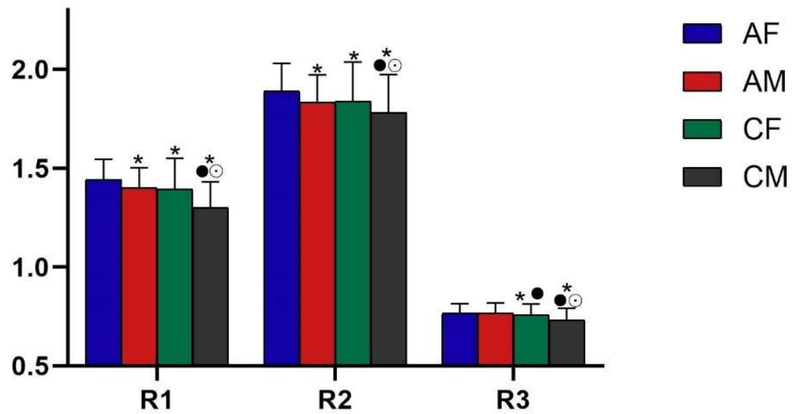

Figure 4. Differences of facial proportions between race and genders (*p<0.05 vs AF, ● p < 0.05 vs AM Group, ⊙P < 0.05 vs CF Group).

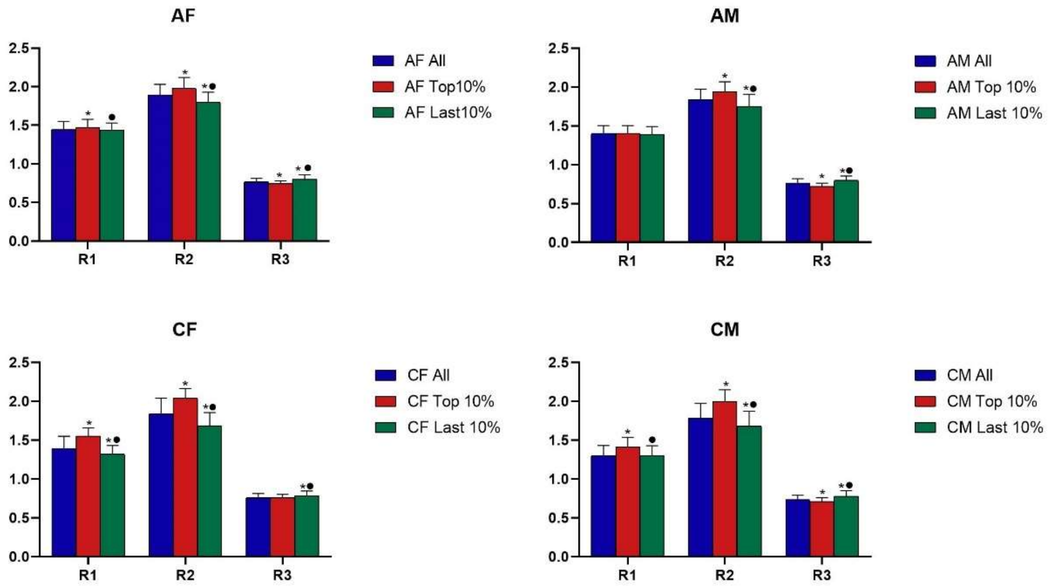

Figure 5. Differences of facial proportions between high attractiveness set and low attractiveness (*p < 0.05 vs All Group, ●p < 0.05 vs Top 10% Group).

The mathematical model of R_1, R_2 and R_3 among Asian females, Asian males, Caucasian females and Caucasian males was obtained after stepwise linear regression analysis: $R_2 = \mathrm{a}R_1 + \mathrm{b}R_3 + \mathrm{c}$. The value of Durbin Watson was within the acceptable range, suggesting the independence of these variables. Furthermore, these variables presented no collinearity (VIF < 1.5). After a simple algebraic simplification, a polynomial equation with three variables was obtained: $N_L^2 = \mathrm{a}E_I * N_L + \mathrm{b}E_I * \mathrm{N} + \mathrm{cN} * N_L$. The result of stepwise linear regression analysis among AF, AM, CF and CM and the result of stepwise linear regression analysis among high attractiveness and low attractiveness sets are presented in Table 3. The relationship between R_1, R_2 and R_3 among AF, AM, CF and CM is presented in Figure 6.

The models obtained by stepwise linear regression analysis for each group are as follows.

$$\text{AF}: N_L^2 = 1.300E_I * N_L - 2.445E_I * \mathrm{N} + 1.886\mathrm{N} * N_L\ (r^2 = 0.991) \tag{4}$$

$$\text{Top 10\% AF}: N_L^2 = 1.325 * N_L - 2.645E_I * \mathrm{N} + 1.999\mathrm{N} * N_L\ (r^2 = 0.992) \tag{5}$$

$$\text{Last 10\% AF}: N_L^2 = 1.230E_I * N_L - 2.255E_I * \mathrm{N} + 1.838\mathrm{N} * N_L\ (r^2 = 0.990) \tag{6}$$

$$\text{AM}: N_L^2 = 1.292E_I * N_L - 2.349E_I * \mathrm{N} + 1.823\mathrm{N} * N_L\ (r^2 = 0.990) \tag{7}$$

$$\text{Top 10\% AM}: N_L^2 = 1.385E_I * N_L - 2.661E_I * \mathrm{N} + 1.922\mathrm{N} * N_L\ (r^2 = 0.992) \tag{8}$$

$$\text{Last 10\% AM}: N_L^2 = 1.242E_I * N_L - 2.184E_I * \mathrm{N} + 1.765\mathrm{N} * N_L\ (r^2 = 0.992) \tag{9}$$

$$\text{CF}: N_L^2 = 1.307E_I * N_L - 2.391E_I * \mathrm{N} + 1.833\mathrm{N} * N_L\ (r^2 = 0.992) \tag{10}$$

$$\text{Top 10\% CF}: N_L^2 = 1.313E_I * N_L - 2.731E_I * \mathrm{N} + 2.082\mathrm{N} * N_L\ (r^2 = 0.992) \tag{11}$$

$$\text{Last 10\% CF}: N_L^2 = 1.334E_I * N_L - 2.211E_I * \mathrm{N} + 1.665\mathrm{N} * N_L\ (r^2 = 0.991) \tag{12}$$

$$\text{CM}: N_L^2 = 1.367E_I * N_L - 2.405E_I * \text{N} + 1.767\text{N} * N_L \; (r^2 = 0.991) \tag{13}$$

$$\text{Top 10\% CM}: N_L^2 = 1.396E_I * N_L - 2.696E_I * \text{N} + 1.935\text{N} * N_L \; (r^2 = 0.990) \tag{14}$$

$$\text{Last 10\% CM}: N_L^2 = 1.314E_I * N_L - 2.241E_I * \text{N} + 1.716\text{N} * N_L \; (r^2 = 0.990) \tag{15}$$

Table 2. Description of the facial proportions among AF、AM、CF and CM.

Groups		N	Mean	Std. Error	Variance	95%CI	Median
AF	R1	1670	1.4447	0.00249	0.010	1.4398~1.4496	1.4400
	R2	1670	1.8908	0.00342	0.020	1.8841~1.8975	1.8900
	R3	1670	0.7656	0.00120	0.002	0.7633~0.7680	0.7600
AF	R1	167	1.4682	0.00839	0.012	1.4516~1.4848	1.4700
Top 10%	R2	167	1.9766	0.01115	0.021	1.9546~1.9987	1.9700
	R3	167	0.7439	0.00281	0.001	0.7383~0.7494	0.7400
AF	R1	167	1.4359	0.00733	0.009	1.4214~1.4503	1.4357
Last 10%	R2	167	1.7998	0.01017	0.017	1.7797~1.8199	1.8100
	R3	167	0.8001	0.00451	0.003	0.7912~0.8090	0.8000
AM	R1	1778	1.4007	0.00242	0.010	1.3959~1.4054	1.4000
	R2	1778	1.8345	0.00328	0.019	1.8281~1.8410	1.8400
	R3	1778	0.7656	0.00128	0.003	0.7631~0.7681	0.7600
AM	R1	178	1.4008	0.00765	0.010	1.3857~1.4159	1.4100
Top 10%	R2	178	1.9395	0.00960	0.016	1.9206~1.9584	1.9300
	R3	178	0.7228	0.00302	0.002	0.7168~0.7287	0.7200
AM	R1	178	1.3912	0.00751	0.010	1.3764~1.4061	1.3900
Last 10%	R2	178	1.7521	0.01151	0.024	1.7294~1.7748	1.7500
	R3	178	0.7969	0.00435	0.003	0.7883~0.8055	0.8000
CF	R1	627	1.3927	0.00628	0.025	1.3804~1.4051	1.3800
	R2	627	1.8404	0.00789	0.039	1.8249~1.8559	1.8300
	R3	627	0.7583	0.00220	0.003	0.7540~0.7627	0.7500
CF	R1	63	1.5510	0.01358	0.012	1.5238~1.5781	1.5500
Top 10%	R2	63	2.0386	0.01589	0.016	2.0068~2.0703	2.0400
	R3	63	0.7616	0.00528	0.002	0.7510~0.7721	0.7600
CF	R1	63	1.3170	0.01441	0.013	1.2882~1.3458	1.3200
Last 10%	R2	63	1.6867	0.02129	0.029	1.6441~1.7292	1.6600
	R3	63	0.7848	0.00772	0.004	0.7693~0.8002	0.7800
CM	R1	665	1.2988	0.00511	0.017	1.2898~1.3098	1.2900
	R2	665	1.7835	0.00737	0.036	1.7690~1.7980	1.7800
	R3	665	0.7318	0.00234	0.004	0.7272~0.7364	0.7300
CM	R1	67	1.4145	0.1503	0.015	1.3845~1.4445	1.4200
Top 10%	R2	67	1.9987	0.1844	0.023	1.9618~2.0355	1.9700
	R3	67	0.7091	0.00609	0.002	0.6970~0.7213	0.7000
CM	R1	67	1.2981	0.01581	0.017	1.2665~1.3296	1.2800
Last 10%	R2	67	1.6778	0.02397	0.039	1.6299~1.7256	1.6500
	R3	67	0.7782	0.00863	0.005	0.7610~0.7954	0.7700

Table 3. Results of stepwise linear regression analysis.

Model		B	t	Sig	Tolerance	VIF	R^2	Durbin-Watson
AF	Constant	1.886	322.14	0.000			0.991	1.943
	R1	1.300	372.29	0.000	0.842	1.188		
	R3	−2.445	−335.87	0.000	0.842	1.188		
AF	Constant	1.999	92.81	0.000			0.992	1.735
Top 10%	R1	1.325	137.64	0.000	0.883	1.132		
	R3	−2.645	−90.44	0.000	0.883	1.132		
AF	Constant	1.838	103.75	0.000			0.990	2.116
Last 10%	R1	1.230	−113.17	0.000	0.796	1.256		
	R3	−2.255	99.76	0.000	0.796	1.256		
AM	Constant	1.823	337.82	0.000			0.990	1.916
	R1	1.292	364.33	0.000	0.813	1.231		
	R3	−2.349	−349.37	0.000	0.813	1.231		
AM	Constant	1.922	117.22	0.000			0.992	2.011
Top 10%	R1	1.385	140.08	0.000	0.748	1.337		
	R3	−2.661	−106.15	0.000	0.748	1.337		
AM	Constant	1.765	99.10	0.000			0.992	2.101
Last 10%	R1	1.242	117.76	0.000	0.934	1.071		
	R3	−2.184	−119.93	0.000	0.934	1.071		
CF	Constant	1.833	179.04	0.000			0.992	2.058
	R1	1.307	265.13	0.000	0.851	1.176		
	R3	−2.391	−170.00	0.000	0.851	1.176		
CF	Constant	2.082	76.22	0.000			0.992	1.493
Top 10%	R1	1.313	83.17	0.000	0.713	1.403		
	R3	−2.731	−67.29	0.000	0.713	1.403		
CF	Constant	1.665	55.01	0.000			0.991	2.373
Last 10%	R1	1.334	71.71	0.000	0.896	1.116		
	R3	−2.211	−63.69	0.000	0.896	1.116		
CM	Constant	1.767	188.48	0.000			0.991	1.922
	R1	1.367	246.15	0.000	0.885	1.129		
	R3	−2.405	−198.51	0.000	0.885	1.129		
CM	Constant	1.935	69.767	0.000			0.990	1.727
Top 10%	R1	1.396	77.91	0.000	0.707	1.415		
	R3	−2.696	−60.90	0.000	0.707	1.415		
CM	Constant	1.716	53.877	0.000			0.990	1.918
Last 10%	R1	1.314	67.082	0.000	0.913	1.095		
	R3	−2.241	−62.429	0.000	0.913	1.095		

Dependent Variable: R2; Predictors: (Constant), R1, R3

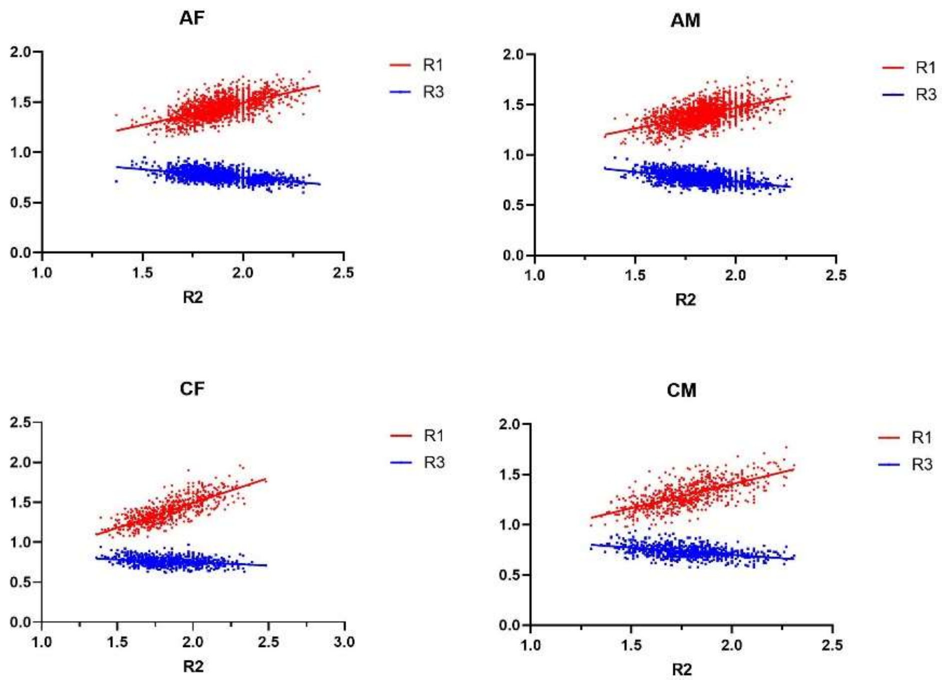

Figure 6. The relationship between R_1, R_2 and R_3 among AF, AM, CF and CM.

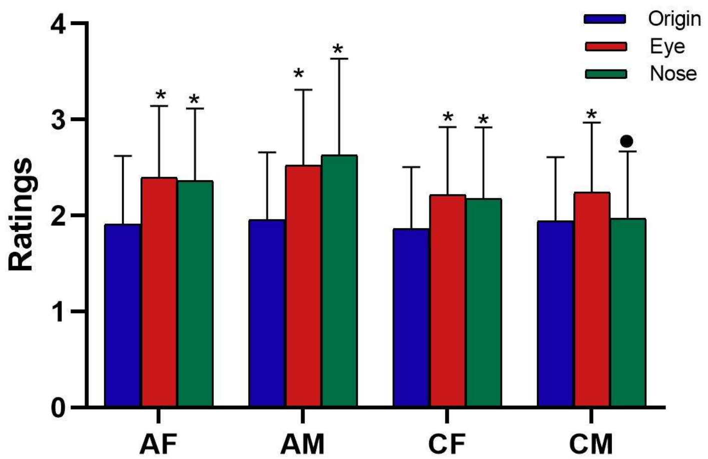

Figure 7. Result of the verification (*p < 0.05 vs Origin Group, ●p < 0.05 vs Eye Group).

The mean of the original image scores for the five low attractiveness Asian females was 1.91. The mean of the scores after changing the width of the inner canthus was 2.40, significantly higher than the original image score ($t = -7.306$, $p < 0.01$). The mean of the scores after changing the width of the nose was 2.36, which was also significantly higher than the original image score ($t = 6.779$, $p < 0.01$). The mean of the original image scores for the five low attractiveness Asian males was 1.95. The mean of the scores after changing the width of the inner canthus was 2.53, significantly higher than the original image score (t = −8.424, $p < 0.01$). The mean of the scores after changing the width of the nose was 2.63, significantly higher than the original image score (t = 8.501, $p < 0.01$). The mean of the original image scores for the five low attractiveness Caucasian females was 1.86. The mean of the scores after changing the width of the inner canthus was 2.22, significantly higher than the original image score (t = −5.814, $p < 0.01$). The mean of the scores after changing the width of the nose was 2.18, significantly higher than the original image score (t = 4.982, $p < 0.01$). The mean of the original image scores for the five Caucasian males was 1.94. The mean score after changing the width of the inner canthus was 2.24, significantly higher than the original image score (t = −4.713, $p < 0.01$). The mean of the scores after changing the width of the nose was 1.97. The results of the independent samples t-test are shown in Figure 7.

4. Discussion

In this study, we explored the relationship between facial proportions and attractiveness. We found that the ratio of inner canthus width to nose length, the ratio of nose length to nose width and the ratio of inner canthus width to nose length play a significant role in determining facial attractiveness in Asian females and Caucasian males. The ratio of nose length to nose width and the ratio of inner canthus width to nose length play a significant role in determining facial attractiveness in Asian males, whereas the ratio of inner canthus width to nose width and the ratio of nose length to nose width play an important role in determining facial beauty in Caucasian females. Furthermore, we developed a model to describe the relationship between the proportions of face components quantitatively.

In contrast to Mateusz's et al. [23] findings, where the facial attractiveness increased as the nose size reduced, we found that the facial attractiveness increased as the ratio of N_L to N increased among AF, AM, CF and CM. Additionally, the facial attractiveness increased as the ratio of E_I to N_L decreased among AF, AM and CM. Thus, a person's facial attractiveness increases when the length of the nose is longer, and the width of the nose is shorter. The size of the nose can be described in terms of length and width, and our experiment investigated the effect of these two quantities on facial attractiveness separately. Rather than discuss the effect of a particular proportion on facial attractiveness in isolation, we emphasized the relationship between facial proportions.

No one aesthetic standard can represent universal beauty because people's perception of beauty is influenced by racial and cultural differences [15,27]. A common inaccuracy, which has existed since the 15th century, is that beauty can be defined and evaluated by proportions and linear parameters, dismissing racial and cultural differences [27−31]. For example, according to Da Vinci, the width of the mouth ideally equals the distance between the lips and the edge of the jaw [27,32]. Other findings suggest in a well-proportioned face the nasal base width should equal the inner canthus width [15]. In this experiment, ethnic and cultural differences in beauty perception were not considered, but it would be crucial to relate facial proportions to ethnic and cultural differences [27].

The mathematical model ($N_L^2 = aE_I * N_L + bE_I * N + cN * N_L$) was obtained after stepwise linear regression analysis. From different sets of facial pictures, we can obtain different parameters (a, b and c) allowing a person to select photos of faces based on their aesthetic preferences. Based on this set of photos, we can build a mathematical model about facial proportions. This method can thus consider the cultural-ethnic differences in facial analysis by taking into account the subjects' facial preferences.

This mathematical model may provide a strategy for plastic surgical decision-making. In this mathematical model, three facial variables are involved: nose length, inner canthus width, and nasal base width. The length of the nose is usually difficult to alter by plastic surgery, whereas the surgery of the inner canthus and the nasal base width is relatively well established. Therefore, the patient's nose length can be taken as a fixed value. According to the patient's willingness and the plastic surgeon's evaluation, the inner canthus width or the nasal base width could be altered to achieve ideal facial proportions through plastic surgery. For example, if a patient wants to change the inner canthus width, we can measure the nasal base width as m and measure the nose length as n. The optimal inner canthus width can be calculated by simply bringing m and n into the formula. In this way, we can get the optimal inner canthus width: $E_I = \frac{n^2 - c*m*n}{a*n+b*m}$. Models can be built based on the patient's aesthetic preferences and used as a reference for plastic surgery. This ensures that plastic surgery outcomes are individualized, instead of one standard being applied to all.

Good communication skills are essential for surgeons and for effective clinical practice [33,34]. Disputes between doctors and patients in plastic surgery are often reported [35]. This involves the issue of communication and harmonization of aesthetic standards between the patient and the physician in cosmetic surgery. Additionally, the modern biopsychosocial model of medicine requires a doctor to be a skilled operator of medical techniques and a person integrated with relevant knowledge and skills, which is more evident in aesthetic surgery [36–38]. Communication between surgeons and patients can be streamlined by allowing patients to select photos of faces based on his or her aesthetic preferences. A mathematical model can be produced based on this data, improving interactions between the practitioner and the patient, and allowing the patient's aesthetic preferences to be fully expressed.

In recent years, various cutting-edge computer technologies, such as deep learning, have been used in facial analysis and other medical fields [39,40]. Through leveraging the marked attractive faces, Zhang et al. proposed a geometric beauty score function to evaluate attractiveness quantitively, modeled by the proposed semi-supervised HSSL learning method [41]. Zhang et al. quantitatively analysed the effect of facial geometric features on facial beauty, finding that the face can become more beautiful by making its geometric features closer to the average face shape [42]. In this experiment, we built a mathematical model from the SCUT-FBP5500, which reflects the average nature of facial features in the population. Verification tests have also demonstrated that the face can become more attractive as facial features are becoming closer to the average face shape. Xu et al. proposed a new network framework, classification and regression network (CRNet), to predict facial beauty [43]. Hong et al. presented a novel framework for automatically assessing facial attractiveness that considered four feature ratio sets as objective elements of facial attractiveness based on deep learning [44]. Tong's et al. findings show that a deep neural network (DNN) model can learn putative ratios from face images based only on categorical annotation when no annotated facial features for attractiveness are explicitly given [45]. He's et al. study shows that DiscoStyle can determine users' facial preferences reasoning and recommend preferred facial styles in different genders and races [46]. With the mathematical model obtained in this paper, we can further infer the aesthetic preferences of the users. Users can build a set of images based on their preferences that could then be used to create a mathematical model that could represent the user's preferences.

Deep learning can predict facial attractiveness by establishing a nonlinear relationship among multiple features of the face. By using deep learning, the accuracy and efficiency of predicting facial attractiveness have been improved compared to traditional methods. Additionally, deep learning can consider a larger number of facial features.

In this experiment, we focused on three facial features (E_I, N and N_L) that are frequently used in clinical practice. We developed linear regression models ($N_L^2 = aE_I * N_L + bE_I * N + cN * N_L$) based on populations of different ethnicities and genders. Although we did not use deep learning in our experiments, the linear regression models all showed R^2 values higher than 0.990, VIF < 1.5 and the value of Durbin Watson was within the acceptable range, indicating that the model establishes the relationship among the three facial features properly. Through validation tests, we also confirmed that facial attractiveness can be improved by changing the facial features of the participants with this model.

There are four limitations to our study. First of all, in this experiment, we used 2D images, while the human brain processes 3D images [42]. Compared to 3D analysis, 2D images can result in the loss of facial information and lead to systematic errors. For example, the nose length (N_L) in this experiment is not the actual length of the nose but the projected length of the actual nose length in the vertical plane where the face is located. Therefore, in future studies, it is recommended to use 3D images to study facial features. Secondly, the perception of attractiveness is affected by multiple other factors such as age [47] and female hormone status [48]. For example, female raters in different hormonal states will give different ratings to the same face and younger people have different aesthetic preferences from older people. Future studies should investigate the influence of these factors on mathematical models. Additionally, the perception of attractiveness is influenced by factors besides proportions, including symmetry, averageness, skin texture, and even filters [13,49−51]. Therefore, we encourage future studies to consider these factors. Thirdly, we did not try nonlinear modelling, such as neural networks. Only three facial feature quantities were selected in this study; other facial features should be investigated in the future. Last but not least, due to the aesthetic preferences of the East versus that of the West, the scores of 1500 Caucasians in the dataset rated by Chinese volunteers may not be representative of how Caucasians would rate the pictures.

5. Conclusions

Our results show that the ratio of nose length to nose width, the ratio of inner canthus width to nose length and the ratio of inner canthus to nose width play a significant role in facial attractiveness among Asian females, Asian males, Caucasian females, and Caucasian males. We established a mathematical model ($N_L^2 = aE_I * N_L + bE_I * N + cN * N_L$) to describe the relationship between a person's facial features. This model may provide some reference for plastic surgeons and facilitate better communication between doctors and patients. More facial feature quantities should be investigated in the future.

Acknowledgments

This study was funded by Fund of Health Technology Innovation Project of Jilin Provincial Department of Finance (No.2019J031); Fund Project of Science and Technology Project of Jilin Provincial Department of Finance (No. JCSZ2019378-15); Fund Project of Science and Technology Project of Jilin Provincial Department of Finance (No. JCSZ2020304-10)

Thanks to Jason Foster and Yang Guang for proofreading the article. We also thank He Yueyang and Meng Linghao of their recommendation for this mathematical model.

Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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