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Research article

A study on aviation supply chain network controllability and control effect based on the topological structure

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Abstract: The present paper focuses on the controllability of the aviation supply chain network and establishes the judgment criterion for structural controllability of the aviation supply chain network. We determine the control effect by applying the control input to different nodes in the aviation supply chain network. These control nodes include the core enterprises of the aviation supply chain network, the upstream suppliers, and the downstream distributors. It is observed that the control effect is better when the control input is applied to the upstream suppliers of the aviation supply chain network than to the core enterprises of the aviation supply chain network. It is also more desirable to apply the control input to the core enterprises than to the distributors. That is, the control effect is the weakest when the control input is applied to the distributors, whereas the effect is best on application of the control to the upstream suppliers in the supply chain (that is, by choosing the upstream suppliers as the controlled nodes in the aviation supply chain network).

Keywords: network controllability; topological structure; control effect; supply chain

1. Introduction

The aviation supply chain network is a system of supply and demand relationships formed among aviation enterprises in the strategic partnership. In the market environment, the operation of the aviation supply chain network is characterized by high dynamics and complexity. The dynamic process characteristics of the aviation supply chain network need to be understood while studying the operation of the aviation supply chain network. At times, the aviation supply chain network can achieve the desired state and stability within a certain spatial and temporal scope by exerting an effective control over the supply chain. Therefore, studying the controllability and the stability of the aviation supply chain network is a frontier and important direction of the present investigation into the operation of the aviation supply chain network.

Disney and Towill applied the transfer function and spectral analysis to a single-level supply chain under a normal distribution and derived the analytical expression of the bullwhip effect, which refers to the scenario where the orders to the supplier tend to have larger fluctuations than sales to the buyer, and the distortion propagates upstream in an amplified form [1]. Disney obtained the order-demand transfer function using a causal loop diagram, block diagram and Z-transform, in order to analyze the stability and bullwhip effect of the supply chain [2]. The application of large-scale system methods to supply chain management was proposed by Cheng [3], who analyzed the process of information transfer within a typical large-scale supply chain system. These papers deeply studied the dynamic characteristics of the supply chain network, but they did not study the control of the supply chain network.

Lalwani et al. represented the state space model of a supply chain system under discrete time and analyzed the stability, controllability, and measurability of the system [4]. Laumanns and Lefeber treated the supply chain network as a process where materials flew dynamically, and each node was equivalent to a converter [5]. Materials passing through a certain node were simulated via first-order differential equation, while the supply chain optimization was realized by the robust optimal control. Although these studied the controllability of the supply chain, they did not pay more attention to the dynamic complexity of the supply chain network, which leads to a large defect in the practical application of their research results.

Liu studied the stability of the current level of the supply chain under non-returnable conditions by introducing a switched system and simulating its stability, with the analysis of each subsystem [6,7]. Using the decomposition-synthesis methodology, the generalized operator model for the coarse granularity was introduced. Then, the layer-by-layer decomposition was performed, and the generalized operator model for finer granularity was elaborated for each decision-making point or control step. Several scholars, including Helbing [8], applied the complex network theory to the supply chain network by treating it as a complex adaptive system with emergent, self-organizing, dynamic, non-linear and evolutionary features. Small variations at any link are likely to invoke changes in other links, which are closely associated with the topology and macroscopic property of the supply chain network. It was assumed that the topology of a complex supply chain network had a strong impact on the information amplification effect in the supply chain management. A reasonable supply chain structure can relieve the bullwhip effect while enhancing the robustness and anti-risk capacity [8]. These papers studied the complexity characteristics of supply chain networks, but did not conduct controllability research on the complex characteristics of supply chain networks.

H. Chen and X. M. Sun analyzed the supply chain control strategy considering the multi-level productivity of suppliers, which was modeled and optimized using the Arena simulation platform [9]. Orthogonal neural networks were applied to the stability analysis of dynamic supply chain models, and the stability of the collaborative operation in the time-delay supply chain system was studied [10]. The risks of the members of the collaborative supply chain that affect the profit distribution were studied, and a negotiation model for the residual profit distribution group was established considering the two key factors of risks and collaboration satisfaction [11]. Ivanov et al. explored various dimensions of the ripple effect and summarized recent developments in the field of supply chain disruption management from a multi-disciplinary perspective [12]. They further suggested that the ripple effect may be able to consolidate research in supply chain disruption management and recovery, similar to the bullwhip effect regarding demand and lead time fluctuation [12]. These fully considered the dynamic complexity of supply chain network operation, but did not study the complexity of supply chain network topology. In practice, the topology of the supply chain network has an impact on the complexity of supply chain network operation.

Pishvaee et al, and Jabbarzadeh et al proposed the robust control and optimization models for supply chain networks, respectively, based on actual supply chain networks [13,14]. Zhao Gang, Yang Ying-bao et al. studied the topology of supply chain networks of agricultural products based on complex network theory and calculated the main topology parameters of the supply chain networks [15]. Mizgier and his colleagues proposed a model for the quantification of risk in supply chain networks according to value at risk and expected shortfall and illustrated the mechanics of the model on complex network designs based on a Monte Carlo simulation [16]. Gang Zhao, Shu-li Gong et al. studied the dynamic mechanisms of risk propagation in complex supply chain networks are and topological evolutionary trends of complex supply chain networks [17]. Mousavi et al. applied the modified particle swarm optimization algorithm to the optimization control problem of a two-level supply chain network [18]. Bing Yang, Ming-hua Hu et al. analyzed in detail the state and structural controllability conditions of complex supply chain networks based on complex network theory [19]. An attempt made to describe the dynamic behaviours of each member company with an autonomous dynamic system to establish the cluster collaborative synchronisation dynamic model for dissipative coupling supply chain networks [20]. H. Li, Yang X and Wang S analyze the function perturbation impact on the finite-time stability and stabilization of the probabilistic Boolean networks [21]. Y. Li and H. Li investigate the periodic switching point controllability and stabilization of periodic switched Boolean control networks and apply the obtained results to the stabilization of deterministic asynchronous Boolean control networks [22]. These papers studied the topology characteristics, risk propagation and controllability conditions in complex supply chain networks and Boolean network controllability conditions and stability. However, in these studies, there no further research on the control effect of network system, and there were no controllability research combined with the topology of the supply chain network.

From the above analysis, it can be seen that limited work done on the control of the aviation supply chain network from the structural perspective of the supply chain network, and there are few research results on the control effect of the aviation supply chain network based on the controllability of the aviation supply chain network topology. The structural controllability of the aviation supply chain is a necessary condition for its state controllability. Studying the structural controllability of the aviation supply chain network is of significant importance for the operation and control of the aviation supply chain network. The present study focuses on the structural controllability and control effect of the aviation supply chain network from the point of view of the topological structure of the supply chain network. The present study has certain innovations in the field of supply chain network operation control. We apply the control input to different nodes in the aviation supply chain network, namely, the core enterprises, upstream suppliers, and downstream distributors and evaluate the control effect on the overall aviation supply chain network in each case.

2. Controllability of the topological structure of the aviation supply chain network and its criteria

Consider an aviation supply chain network consisting of N enterprise nodes and involving only forward logistics. Thus, we disregard the reverse logistics, which covers any operations related to the reuse of products and materials, including remanufacturing and refurbishing activities or any processes of moving goods from their typical final destination for capturing value or proper disposal. Given this, the aviation supply chain network under study is a directed network consisting of N nodes. The state equation for the enterprise nodes in this network has the following form:

$$\frac{dx_i}{dt} = \sum_{j=1}^{N} a_{ij} x_j + \sum_{j=1}^{M} b_{ij} u_j \quad . \tag{2.1}$$

The state equation for the aviation supply chain network can be written as follows:

$$\dot{x} = Ax + Bu, \quad x \in \Re^N, u \in \Re^M \tag{2.2}$$

where x is the state vector of the enterprise node, $x = (x_1, x_2, \cdots, x_N)^T$; $A = (a_{ij})_{N \times N}$ is the system matrix; $u = (u_1, u_2, \cdots, u_M)$ is the control input vector; $B = (b_{ij})_{N \times M}, (M \le N)$ is the control input matrix, while N enterprise nodes in this aviation supply chain network are called state nodes. The system parameter a_{ij} of the aviation supply chain network under control varies with different configurations of the state variables. For example, a_{ij} is apparently different when setting the state variable as the amount of working capital between the state nodes or as the logistics volume operating between the state nodes. The coordination and control of the aviation supply chain network usually implies the control of the volume of materials flow as the main target. So, the volume of materials flow is treated as the variable of the state nodes. For the controllability study of an assembly aviation supply chain network, variables of the state nodes may include volumes of producer goods, parts, and finished goods' flows at the enterprise nodes converted according to the bill of materials (BOM). For the local aviation supply chain network in the upstream of core enterprises, the parameter a_{ij} is related to the BOM of core enterprises and the number of suppliers of the same category of parts. For the local aviation supply chain network in the downstream of core enterprises, the parameter a_{ij} is related to the number of distributors, regional market sales, and historical sales performance of distributors. If the control inputs act on a specific state node of an enterprise, such node is referred to as a controlled enterprise node.

A controlled aviation supply chain network system is treated as state controllable if, for the initial $x(t_0) = x_0$ and final x_f states at any initial time t_0, there exist a finite time t_f and an unconstrained control input u, for which the equality $x(t_f) = x_f$ is valid.

The state controllability of an aviation supply chain network depends not only on the control input mechanism of the network and structural features of the controlled objects but also on parameters of the network system and control input matrix. The structural scheme design of a controlled aviation supply chain network and the structural link study of the controlled objects are usually focused on the aviation supply chain network topology. Therefore, noteworthy is the network structure effect on the structural controllability of the aviation supply chain network.

The structural controllability of the aviation supply chain network depends on the topology of network but not on the values of its parameters. Moreover, it applies not only to the linear approximation of the state equation of the network but also to the non-linearity and time variability of parameters of the state equation for the network, which are more typical for a real aviation supply chain network. If the structure of the aviation supply chain network can guarantee the existence of the input state control information, which can achieve the network controllability, this network is considered to be controllable in the structural sense. This condition can be formulated as follows.

If for the controlled aviation supply chain network system described by Eq (2.2), there exist arbitrary non-zero values of matrices A and B that make the system controllable, this network system is considered to possess structural controllability.

For an aviation supply chain network system, the matrix is $\Pi = (\pi_{ij}) \in \Re^{N\times N}$, which reflects the relational structure and coupling relationship between enterprises in the network. Hereinafter, Π is treated as a relational structure matrix of the enterprise nodes, which is a direct pathway of the control information transmission between state nodes of the enterprises in the network. For a directed aviation supply chain network with no reverse logistics, we assume that

$$\pi_{ij} = \begin{cases} 1 \\ 0 \end{cases}, \text{ when parameter } a_{ij} \begin{cases} \neq 0 \\ = 0 \end{cases}, \quad i \neq j .$$

Let $\Gamma = (\gamma_{ij}) \in \Re^{N\times M}$ be the relational structure matrix after removing the parameter information from the control input matrix of the aviation supply chain network. Then, we assume that the following condition holds:

$$\gamma_{ij} = \begin{cases} 1 \\ 0 \end{cases}, \text{ when } b_{ij} \begin{cases} \neq 0 \\ = 0 \end{cases}, \quad i \neq j, \quad \gamma_{ii} = 0 .$$

Next, we consider Γ as the structural matrix of the control input of the network and use symbols $\oplus$ and $\otimes$ as addition and multiplication operators, respectively, while $\vee$ and $\wedge$ correspond to the addition and multiplication operators in Boolean algebra.

For matrices $U = [u_{ij}]_{m\times n}$ and $Z = [z_{ij}]_{m\times n}$, their addition is applied as follows:

$$U \oplus Z = [u_{ij} \oplus z_{ij}]_{m\times n} = [u_{ij} \vee z_{ij}]_{m\times n} .$$

Similarly, their multiplication implies

$$U \otimes Z = [\overset{n}{\underset{k=1}{\oplus}} (u_{ik} \otimes z_{kj})]_{m\times s} = [(u_{i1} \wedge z_{1j}) \vee (u_{i2} \wedge z_{2j}) \vee \cdots \vee (u_{in} \wedge z_{nj})]_{m\times s} .$$

Let the reachability matrix of the relational structure of the state be C_A for aviation supply chain network, so there is

$$C_A = \Pi \oplus \Pi^2 \oplus \cdots \oplus \Pi^{N-1} = \overset{N-1}{\underset{m=1}{\oplus}} \Pi^m = \Pi \oplus \overset{N-1}{\underset{m=2}{\oplus}} \Pi^m \quad . \tag{2.3}$$

According to the above definition, Π is the direct pathway for control information transmission between state nodes. Accordingly, Π^m is the m-th logical power of Π. In the above formula, $\Pi^m(m = 2, 3\cdots, N-1)$ is the indirect pathway formed by series connections of m direct pathways between the state nodes.

Using the relational structure matrix Π and the input structural matrix Γ, one can calculate the structural reachability matrix of the control input information transmission in the network:

$$C_{AB} = \Gamma \oplus (C_A \otimes \Gamma) \quad . \tag{2.4}$$

As indicated by C_{AB} for the aviation supply chain network, the pathway that transmits the control information contained in the control input u to the state x of the enterprise node is structurally comprised of two parts, namely Γ and $C_A \otimes \Gamma$. Here, Γ is the direct control information transmission pathway for control input B, while $C_A \otimes \Gamma$ is the indirect control information transmission pathway formed by the series connection of control input B and the controlled aviation supply chain network system A via C_A. In Eq (2.3), the control information transmission pathway is composed of two parts, namely, the direct linking pathway Π between state nodes and indirect linking pathway $\overset{N-1}{\underset{m=2}{\oplus}} \Pi^m$ formed by multiple series connections of direct pathways.

For an aviation supply chain network system, if the state Eq (2.2) has non-zero values of matrices A and B that make the system controllable, such aviation supply chain network is considered to be structurally controllable. In other words, the extended system comprising the aviation supply chain network and input system provides at least one controllable information link. This satisfies the structural pre-condition that the state control input information can reach the state nodes, which is crucial for the network system to be structurally controllable. If the aviation supply chain system is structurally controllable, any variations of the non-zero parameter a_{ij} will not violate the system controllability. If matrix C_{AB} of the aviation supply chain network system has no rows where all elements are zero, the number of non-zero rows will be equal to the number of dimensions of the state vectors:

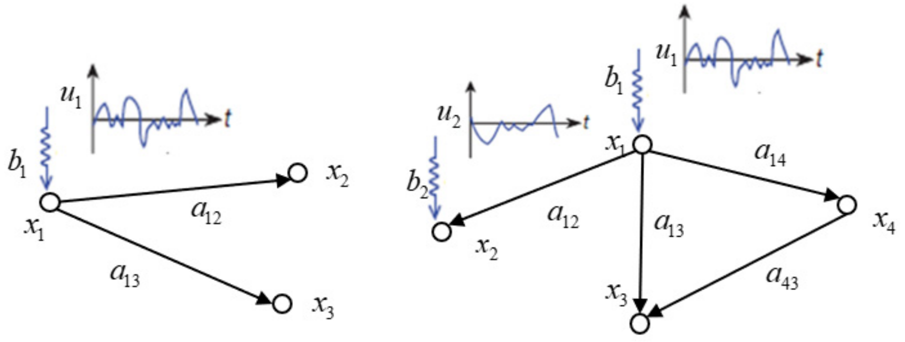
$$R(C_{AB}) = R[\Gamma \oplus (C_A \otimes \Gamma)] = R\left\{[\Gamma \oplus [(\Pi \oplus \Pi^2 \oplus \cdots \oplus \Pi^{N-1}) \otimes \Gamma]\right\} = N \tag{2.5}$$

This implies that the aviation supply chain network is structurally controllable. In the above formula, $R(C_{AB})$ is the number of non-zero rows in matrix C_{AB}, while N is the number of dimensions of the state vectors in the aviation supply chain network, i.e., the number of enterprises. In Eq (2.5), each state node of the enterprise has at least one control information transmission pathway. Thus, it is ensured structurally that the flow of control input information reaches every state node and that the control input B matches with aviation supply chain network A. Equation (2.5) can be used as the state controllability criterion for the aviation supply chain network. The structural controllability of the aviation supply chain network is a structural feature of the state controllability. Noteworthy is that an aviation supply chain network with no state controllability may still possess the structural controllability. Therefore, the state controllability of an aviation supply chain network is a sufficient but not necessary condition of its structural controllability.

3. Correlation between the structural controllability and the state controllability of the aviation supply chain network system

The structural controllability of an aviation supply chain network system is one structural feature of its state controllability which usually indicates the inevitable structural controllability. However, it does not necessarily indicate the state controllability. The state controllability may be achieved under specific operational parameters and the control input. The structural uncontrollability of the aviation supply chain network system always indicates state uncontrollability.

To illustrate, we describe a simple case in Figure 1 showing two local aviation supply chain network systems under different control inputs. Figure 1(a) has 3 enterprise state nodes and 1 control input, the latter is applied to one state node, whereas Figure 1(b) has 4 enterprise state nodes and 2 control inputs, and the latter are applied to the two state nodes.



(a) (b)

Figure 1. A local aviation supply chain network system under different control inputs.

At first, we analyze the state controllability by representing the state equation of the aviation supply chain network system in Figure 1(a) as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ a_{12} & 0 & 0 \\ a_{13} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} u .$$

The judgment matrix of the state controllability of the aviation supply chain network system in Figure 1(a) is written as:

$$M_C = \left[B, AB, A^2B\right] = b\begin{pmatrix} 1 & 0 & 0 \\ 0 & a_{12} & 0 \\ 0 & a_{13} & 0 \end{pmatrix}.$$

Apparently, there is $Rank(M_C) = 2 < N = 3$. Thus, the aviation supply chain network system in Figure 1(a) can be said to have the state uncontrollability.

We analyze the structural controllability of the aviation supply chain network system shown in Figure 1(a) according to the judgment criterion for the structural controllability of the aviation supply chain network system by the following equations:

$$C_A = \bigoplus_{m=1}^{N-1} \Pi^m = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$C_{AB} = \Gamma \oplus (C_A \otimes \Gamma) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \oplus \left(\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},$$

$$R(C_{AB}) = R[\Gamma \oplus (C_A \otimes \Gamma)] = R\left\{[\Gamma \oplus [(\Pi \oplus \Pi^2 \oplus \cdots \oplus \Pi^{N-1}) \otimes \Gamma]\right\} = 3.$$

As seen from the above, the state uncontrollability of the aviation supply chain network does not necessarily indicate structural controllability.

Below, we analyze the state controllability of the aviation supply chain network system in Figure 1(b). The state equation of the aviation supply chain network system in Figure 1(b) is written as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{12} & 0 & 0 & 0 \\ a_{13} & 0 & 0 & a_{43} \\ a_{14} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix},$$

$$M_C = \left[B, AB, A^2B, A^3B\right] = \begin{pmatrix} b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2 & a_{12}b_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{13}b_1 & 0 & a_{43}a_{14}b_1 & 0 & 0 & 0 \\ 0 & 0 & a_{14}b_1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

In this matrix, irrespective of any non-zero value assigned to the parameter of the aviation supply chain network system, the following always holds true: $Rank(M_C) = 4 = N$. Hence, the aviation supply chain network system in Figure 1(b) can be said to have the state controllability.

The structural controllability of the aviation supply chain network system in Figure 1(b) is analyzed below:

$$C_A = \overset{N-1}{\underset{m=1}{\oplus}} \Pi^m = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

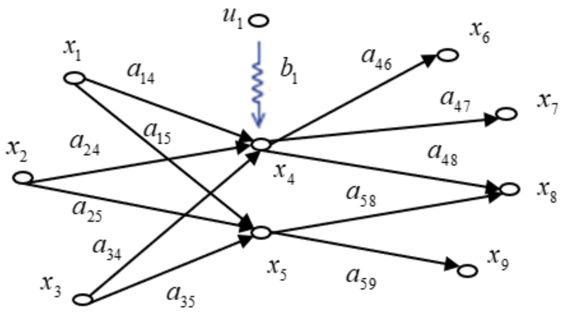
$$C_{AB} = \Gamma \oplus (C_A \otimes \Gamma) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \oplus \left(\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \right) = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix},$$

$$R(C_{AB}) = R[\Gamma \oplus (C_A \otimes \Gamma)] = N = 4.$$

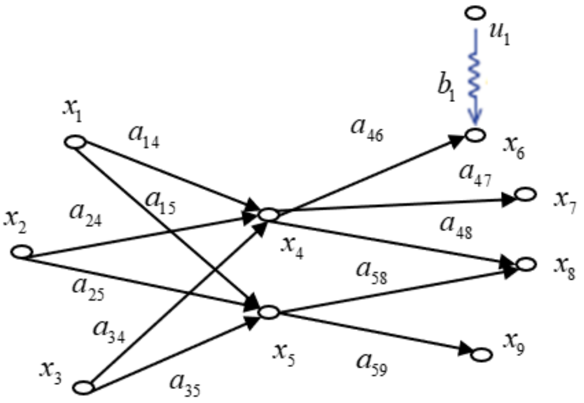
Apparently, the aviation supply chain network system in Figure 1(b) is structurally controllable.

4. The control effect of applying the control input to different nodes

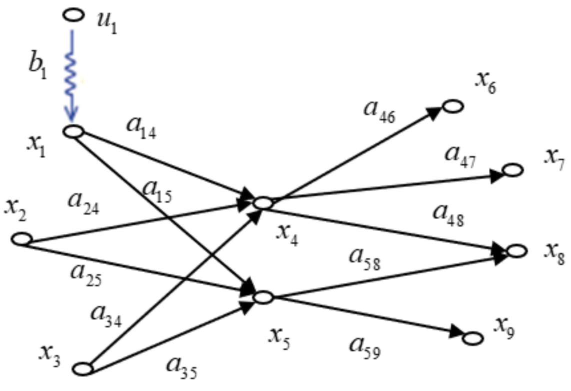
To further investigate the controllability of the aviation supply chain network system, we analyze the control effect by applying the control input to different nodes. An aviation supply chain network system under control is built with 2 core enterprises, 3 suppliers, and 4 regional distributors. First, one control input is introduced to this aviation supply chain network. This control input is applied to one core enterprise, one distributor, and one supplier, respectively, as shown in Figure 2.



(a) The controlled node is a state node of a core enterprise.



(b) The controlled node is a distributor state node.



(c) The controlled node is a supplier state node.

Figure 2. An aviation supply chain network system into which one control input is introduced.

The judgment criterion in Eq (5) for structural controllability of the aviation supply chain network system is used to analyze the structural controllability of the aviation supply chain network system as shown in Figure 2(a).

The state equation of the aviation supply chain network system in Figure 2(a) is written as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{14} & a_{24} & a_{34} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{15} & a_{25} & a_{35} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{46} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{47} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{48} & a_{58} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{59} & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ b_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} u_1 .$$

The structural controllability of the aviation supply chain network system shown in Figure 2(a) is analyzed by using Eq (5):

$$C_A = \Pi \oplus \Pi^2 \oplus \cdots \oplus \Pi^{N-1} = \bigoplus_{m=1}^{N-1} \Pi^m = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C_{AB}=\Gamma\oplus(C_A\otimes\Gamma)=\begin{pmatrix}0\\0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix}\oplus\left(\begin{pmatrix}0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\1&1&1&0&0&0&0&0&0\\1&1&1&0&0&0&0&0&0\\1&1&1&1&0&0&0&0&0\\1&1&1&1&0&0&0&0&0\\1&1&1&1&1&0&0&0&0\\1&1&1&0&1&0&0&0&0\end{pmatrix}\otimes\begin{pmatrix}0\\0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix}\right)=\begin{pmatrix}0\\0\\0\\1\\0\\1\\1\\1\\0\end{pmatrix},$$

$$R(C_{AB})=R[\Gamma\oplus(C_A\otimes\Gamma)]=R\left\{[\Gamma\oplus[(\Pi\oplus\Pi^2\oplus\cdots\oplus\Pi^{N-1})\otimes\Gamma]\right\}=5<9.$$

The aviation supply chain network system with only one control input applied to the core enterprise state node x_4 in Figure 2(a) is structurally uncontrollable. Thus, in this aviation supply chain network system under control, the system matrix A never matches the control input matrix B. Whatever be the value, the system operation parameter a_{ij} will never enable the state controllability of the aviation supply chain network system by adjusting the input control information.

Then, the control input is applied to one distributor state node, as shown in Figure 2(b). $C_A=\overset{N-1}{\underset{m=1}{\oplus}}\prod^m$ and $C_{AB}=\Gamma\oplus(C_A\otimes\Gamma)$ are calculated to derive

$$R(C_{AB})=R[\Gamma\oplus(C_A\otimes\Gamma)]=1.$$

When the control input is applied to one distributor state node x_6, the aviation supply chain network system will be structurally uncontrollable as shown in Figure 2(b). Besides, whatever value is assigned to the operational parameter of the aviation supply chain network, the control input always has a worse control effect on the system's state as compared to the situation in Figure 2(a).

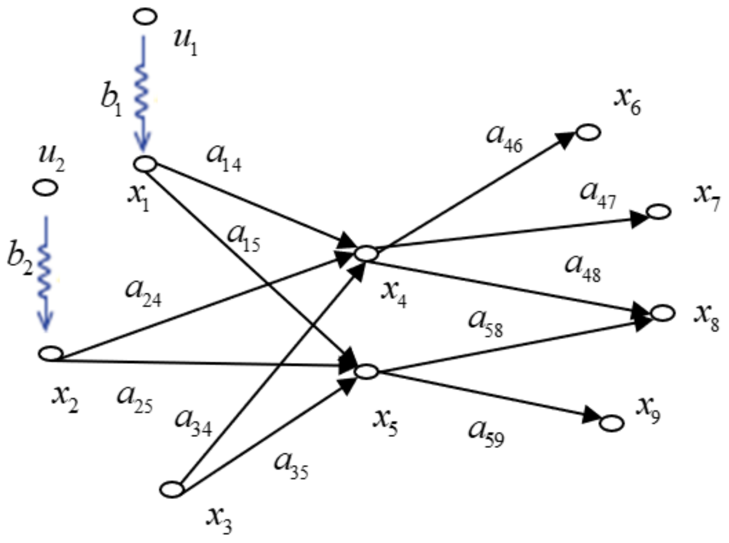
Next, the control input is applied to one supplier state node x_1, as shown in Figure 2(c). $C_A=\overset{N-1}{\underset{m=1}{\oplus}}\prod^m$ and $C_{AB}=\Gamma\oplus(C_A\otimes\Gamma)$ are calculated to derive:

$$R(C_{AB})=R[\Gamma\oplus(C_A\otimes\Gamma)]=7<9.$$

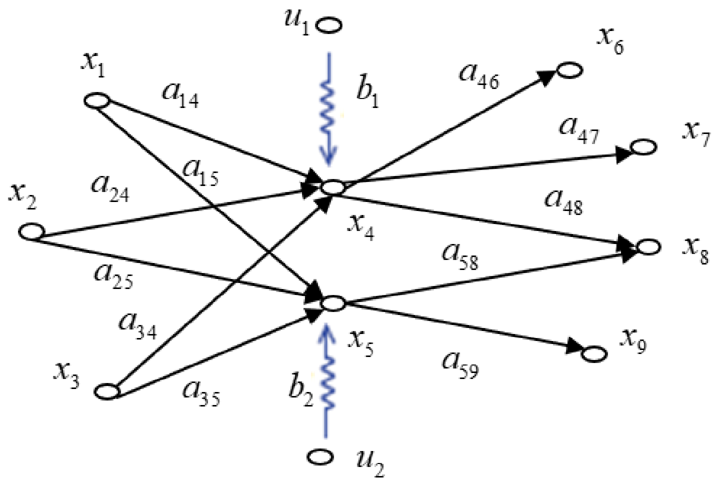
When the control input is applied to one supplier state node x_1, this aviation supply chain network system still remains uncontrollable as shown in Figure 2(c). Nevertheless, the control effect is improved as compared to the situation shown in Figure 2(b),(c).

Further, the calculation is done for all the enterprise state nodes. The entire aviation supply chain network system will never be controllable whatever be the chosen controlled node (namely, one supplier state node, one core enterprises state node, or one distributor state node). Moreover, the control effect is better, if one control input is applied to the suppliers than to the core enterprises. It is even more preferable to control the core enterprises than the distributors. The control effect is worst when the control input is applied to the distributor nodes.

In addition, two control inputs are applied to two different nodes, respectively, as shown in Figure 3.



(a) The controlled nodes are two suppliers.



(b) The controlled node are two core enterprises.

Figure 3. An aviation supply chain network system with two different control inputs.

The state equation of the aviation supply chain network system in Figure 3(a) is written as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{14} & a_{24} & a_{34} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{15} & a_{25} & a_{35} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{46} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{47} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{48} & a_{58} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{59} & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{pmatrix} + \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} .$$

The controllability of the system in Figure 3(a) is analyzed by using Eq (5):

$$C_A = \bigoplus_{m=1}^{N-1} \Pi^m = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C_{AB} = \Gamma \oplus (C_A \otimes \Gamma) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \oplus \left(\begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 1&1&1&0&0&0&0&0&0 \\ 1&1&1&0&0&0&0&0&0 \\ 1&1&1&1&0&0&0&0&0 \\ 1&1&1&1&0&0&0&0&0 \\ 1&1&1&1&1&0&0&0&0 \\ 1&1&1&0&1&0&0&0&0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \right) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix},$$

$$R(C_{AB}) = R[\Gamma \oplus (C_A \otimes \Gamma)] = R\left\{[\Gamma \oplus [(\Pi \oplus \Pi^2 \oplus \cdots \oplus \Pi^{N-1}) \otimes \Gamma]\right\} = 8 < 9 .$$

Apparently, the aviation supply chain network system in Figure 3(a) is uncontrollable. In this system, no state controllability of the system can be achieved under any operational parameter or control input.

Next, the control inputs are applied, respectively, to the two state nodes of the core enterprises, namely, x_4 and x_5, as shown in Figure 3(b) to calculate $C_A = \overset{N-1}{\underset{m=1}{\oplus}} \prod^m$ and $C_{AB} = \Gamma \oplus (C_A \otimes \Gamma)$. Thus, $R(C_{AB}) = R[\Gamma \oplus (C_A \otimes \Gamma)] = 6 < 9$.

As shown in Figure 3(b), when the control inputs are applied to the two state nodes of the core enterprises, namely, x_4 and x_5, the aviation supply chain network system becomes uncontrollable. The control effect is worse than when the two control inputs are applied to the two state nodes of the suppliers.

Similarly, the two state nodes of the distributors are chosen as the controlled nodes. Under this situation, the control effect is weaker than by choosing the two core enterprise nodes as the controlled nodes. Based on the analysis of the aviation supply chain network in extension system of Figures 2 and 3, more control inputs are introduced and applied to the enterprise nodes in the aviation supply chain network. The best control effect is obtained when the control is exerted on the upstream suppliers in the aviation supply chain. It is better to exert control on the suppliers than on the core enterprises. Similarly, it is also better to exert control on the core enterprises than on the distributors. The control effect is the worst when the control input is applied to the distributors. That is, the control effect is best when the control input is applied to the upstream suppliers.

Based on the aforesaid analyses, the three state nodes of the suppliers are chosen as the controlled nodes to verify whether the aviation supply chain network system is controllable shown in Figure 4.

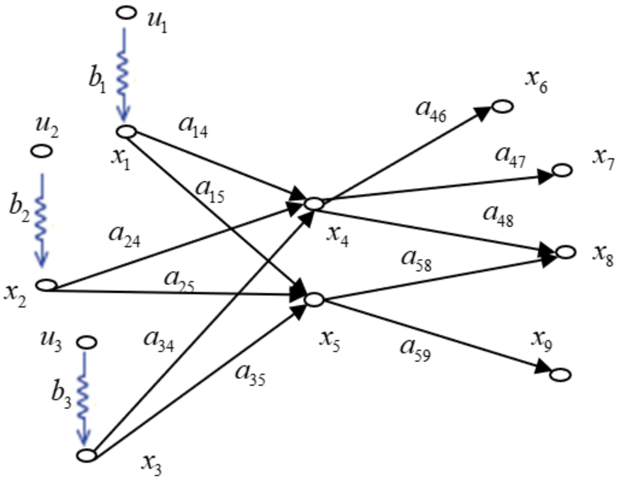


Figure 4. An aviation supply chain network system where three control inputs are introduced.

The state equation of the aviation supply chain network system in Figure 4(a) is written as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{14} & a_{24} & a_{34} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{15} & a_{25} & a_{35} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{46} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{47} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{48} & a_{58} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{59} & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{pmatrix} + \begin{pmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}.$$

The controllability of the system in Figure 4 is analyzed according to Eq (5):

$$C_A = \overset{N-1}{\underset{m=1}{\oplus}} \Pi^m = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C_{AB} = \Gamma \oplus (C_A \otimes \Gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \oplus \left(\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

Therefore, $R(C_{AB}) = R[\Gamma \oplus (C_A \otimes \Gamma)] = 9$

As seen from the aforesaid discussion, when the control inputs are applied to the three suppliers, the aviation supply chain network system is structurally controllable as seen in Figure 4. The state controllability can be achieved for this system under a specific value of operational parameter a_{ij} and the control input b_{ij} .

5. Conclusions

The structural controllability of the aviation supply chain network is the basis for operation state control of the aviation supply chain network. At present, in the field of supply chain network research, there is a lack of research on the controllability and control effect of the aviation supply chain network based on topology. An attempt made in this paper to analyze the structural controllability of the aviation supply chain network system and the control effect. The judgment criterion for the structural controllability of the aviation supply chain network system was established by applying and assessing the control effect of the control inputs to the upstream suppliers, the core enterprises, and downstream distributors separately. The study demonstrated that the judgment criterion for the structural controllability of the aviation supply chain network system was eligible for determining the structural controllability of the aviation supply chain network system. In the field of aviation supply chain networks, it is believed that when the core enterprises of the aviation supply chain network are controlled, the control effect of the aviation supply chain network is the best. However, the present study shows that the control effect is better when the control inputs act on the upstream suppliers of the aviation supply chain network than the core enterprises of the aviation supply chain network, and it is also better to apply the control inputs to the core enterprises than to the distributors. On the other hand, the control effect is the weakest when the control inputs act on the distributors. The control effect is best when the control to the upstream suppliers in the aviation supply chain was applied (that is, by choosing the upstream suppliers as the controlled nodes in the aviation supply chain network). The research conclusion certain innovative contribution in the field of supply chain network operation control. Based on this work, our future course of research will be to investigate the collaboration behavior of the aviation supply chain network system and an in-depth study on the dynamics of the synchronous collaboration behavior of the aviation supply chain network.

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Conflict of interest

The authors declare there is no conflict of interest.

References

1. S. M. Disney, D. R. Towill, On the bullwhip and inventory variance produced by an ordering policy, *Omega*, **31** (2003), 157–167. https://doi.org/10.1016/S0305-0483(03)00028-8
2. S. M. Disney, D. R. Towill, A discrete transfer function model to determine the dynamic stability of a vendor managed inventory supply chain, *Int. J. Prod. Res.*, **40** (2002), 179–204. https://doi.org/10.1080/00207540110072975
3. Y. S. Cheng, Application of large system method in supply chain management, *Ind. Eng. J.*, **10** (2007), 34–37. https://doi.org/10.1016/j.inoche.2006.08.015
4. C. S. Lalwani, S. M. Disney, D. R. Towill, Controllable, observable and state space representations of a generalized order-up-to policy, *Int. J. Prod. Econ.*, **101** (2006), 172–184. https://doi.org/10.1016/j.ijpe.2005.05.014
5. M. Laumanns, E. Lefeber, Robust optimal control of material flows in demand-driven supply networks, *Phys. A*, **363** (2006), 24–31. https://doi.org/10.1016/j.physa.2006.01.045

6. H. X. Liu, Complex behavior of a simple supply chain, *Comput. Integr. Manuf. Syst.*, **13** (2013), 585–607. https://doi.org/10.3969/j.issn.1006-5911.2007.03.028
7. H. X. Liu, H. W. Wang, Z. G. Wang, Modeling the dynamics of a two-stage supply chain: A switch system theory approach, *J. Huazhong Univ. Sci. Technol.*, **33** (2005), 92–95. https://doi.org/10.1159/000198556
8. D. Helbing, Information and material flows in complex networks, *Phys. A*, **363** (2006), 11–16. https://doi.org/10.1016/j.physa.2006.01.042
9. H. Chen, X. M. Sun, Supply chain system control strategy simulation with supplier's multi-productivity, *Comput. Eng. Appl.*, **50** (2014), 262–266. http://cea.ceaj.org/EN/Y2014/V50/I13/262
10. X. L. Zhang, Dynamic supply chains with time delay characteristic modeling and stability analysis, *J. Ind. Eng. Eng. Manage.*, **29** (2015), 95–101. http://en.cnki.com.cn/Article_en/CJFDTOTAL-GLGU201504011.htm
11. N. T. Jiang, Research on the coordination supply chain surplus profits allocation based on the collaborative risk and the cooperation satisfaction under the background of the supply-front reform, *Logist. Sci. Tech.*, **39** (2016), 112–118. http://en.cnki.com.cn/Article_en/CJFDTotal-LTKJ201609033.htm
12. D. Ivanov, B. Sokolov, A. Dolgui, The ripple effect in supply chains: trade-off "efficiency flexibility-resilience" in disruption management, *Int. J. Prod. Res.*, **52** (2014), 2154–2172. https://doi.org/10.1080/00207543.2013.858836
13. M. S. Pishvaee, J. Razmi, S. A. Torabi, Robust possibilistic programming for socially responsible supply chain network design: A new approach, *Fuzzy Sets Syst.*, **206** (2012), 1–20. https://doi.org/10.1016/j.fss.2012.04.010
14. A. Jabbarzadeh, B. B. Fahimnia, S. Seuring, Dynamic supply chain network design for the supply of blood in disasters: A robust model with real world application, *Transp. Res. Part E.*, **70** (2014), 225–244. https://doi.org/10.1016/j.tre.2014.06.003
15. G. Zhao, Y. Yang, X. Bao, Q. Peng, On the topological properties of urban complex supply chain network of agricultural products in mainland China, *Transp. Lett.: Int. J.Trans. Res.*, **7** (2015), 188–195. https://doi.org/10.1179/1942787515Y.0000000007
16. K. J. Mizgier, S. M. Wagner, M. P. Jüttner, Disentangling diversification in supply chain networks, *Int. J. Prod. Econ.*, **162** (2015), 115–124. https://doi.org/10.1016/j.ijpe.2015.01.007
17. G. Zhao, S. Gong, Y. Yang, M. Lin, Model and dynamic behavior of risk propagation in complex agricultural supply chain networks in China and their topological evolution, *J. Int. Technol.*, **17** (2016), 483–493. https://doi.org/10.6138/JIT.2016.17.3.20160225a
18. S. M. Mousavi, A. Bahreinineja, S. N. Musa, F. Yusof, A modified particle swarm optimization for solving the integrated location and inventory control problems in a two-echelon supply chain network, *J. Intell. Manuf.*, **28** (2017), 191–206. https://doi.org/10.1007/s10845-014-0970-z
19. B. Yang, M. Hu, G. Zhao, Y. Yang, Structural and state controllability study of the supply chain network based on the complex network theory, *J. Int. Technol.*, **20** (2019), 214–223. https://doi.org/10.3966/160792642019102006018
20. Y. B. Yang, C. P. Liu, B. Yang, G. Zhao, M. H. Hu, A cluster collaborative synchronisation dynamic model of the dissipative coupling supply chain network, *Int. J. Secur. Networks*, **15** (2020), 1879–1887. https://doi.org/10.1504/IJSN.2020.10033127

21. H. Li, X. Yang, S. Wang, Perturbation analysis for finite-time stability and stabilization of probabilistic Boolean networks, *IEEE Trans. Cybern.*, **51** (2021), 4623–4633. https://doi.org/10.1109/TCYB.2020.3003055
22. Y. Li, H. Li, Controllability and stabilization of periodic switched Boolean control networks with application to asynchronous updating, *Nonlinear Anal. Hybrid Syst.*, **41** (2021), 101054. https://doi.org/10.1016/j.nahs.2021.101054