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Modelling of a seasonally perturbed competitive three species impulsive system

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Abstract: The population of biological species in the ecosystem is known sensitive to the periodic fluctuations of seasonal change, food resources and climatic conditions. Research in the ecological management discipline conventionally models the behavior of such dynamic systems through specific impulsive response functions, but the results of such research are applicable only when the environments conform exactly to the conditions as defined by the specific response functions that have been implemented for specific scenarios. This means that the application of previous work may be somewhat limited. Moreover, the intra and inter competitions among species have been seldom studied for modelling the prey-predator ecosystem. To fill in the gaps this paper models the delicate balance of two-prey and one-predator system by addressing three main areas of: i) instead of using the specific impulse response this work models the ecosystem through a more general response function; ii) to include the effects due to the competition between species and iii) the system is subjected to the influences of seasonal factors. The seasonal factor has been implemented here in terms of periodic functions to represent the growth rates of predators. The sufficient condition for the local and global asymptotic stability of the prey-free periodic solution and the permanence of the system have been subsequently obtained by using the Comparison techniques and the Floquet theorems. Finally, the correctness of developed theories is verified by numerical simulation, and the corresponding biological explanation is given.

Keywords: seasonally succession; general functional response; two-prey and one-predator system; competition

1. Introduction

The modelling of the dynamics in the predator-prey ecosystem research has been by large concerning with two species system, without much consideration on the effects due to the intra and inter species competitions [1–4]. Other factors that may affect the delicate balance between the predator-prey and system, such as the species' population and the recycling of nutrients, have not been studied in great depth over the past couple decades of ecosystem research [5–8]. In the ecosystem of the real-world, it often involves the co-existence of three or more species competing each other even when short range (small) geographical surrounding is concerned. To model the rich dynamics of situations like this will need to include the competitions between species, so to understand the balance of the biological community better even for small local geographical areas of ecosystem. For example in the case of domestic farmlands where eagles prey on mice and snakes, eagles and foxes prey on rabbit; spiders and frogs prey on insects, frogs prey on spiders, and snakes prey on frogs and so on; a very complex food chain network is common even for a domestic farmland scenario. Thus this paper models the case of a three species system by including the intra- and inter-species prey-predator competitions. The present work reveals that the dynamic system of the model is more complex than the one without taking the intra- and inter-species competition into consideration. The more realistic of the present model to the real world ecosystem suggests that the result of this work may offer a better scope for practical application.

The change of environment due to natural disasters, climate and seasonal variations imposes direct impacts to the delicate balance of biological systems [9–12]. The environmental change affects the survival and development of biological species, their relationships such as their predator-prey habitats, mutual cooperation, competition and parasitism [13–15]. Due to the complex and diverse configurations in the ecosystems, the evolution of biological species in the natural environment has routinely been considered as an example of dynamic systems [16–19], which can then be modelled mathematically [20–23]. One objective of the present work is to extend previous research by including environmental factors through the modulation of the preys' intrinsic growth rate by periodically varying functions [24–27]. It is known that the seasonal effects vary periodically in time [28], thus one can include the periodical time function into the dynamic system for a more realistic modelling. In this paper, a periodic function $\sin(\omega t)$ is incorporated into the intrinsic growth rate to describe the change of the population of the system due to the seasonal variation, that is: $r + \lambda \sin(\omega t)$, in which $r > 0$ represents the prey's population intrinsic birth rate, $\lambda > 0$ denotes the magnitude of the fluctuation, $\frac{\lambda}{r}$ is the degree of the seasonality, $\omega > 0$ is the forcing term's angular frequency.

Furthermore, the inclusion of the environmental factors for modelling the evolution of two-prey and one-predator mutually competitive system by using a general functional response, has not been reported in the literature as far as the authors' knowledge [1, 29]. Traditionally, most researchers in the field have implemented specific functional responses for modelling the prey and predator densities and most of them only consider two species in their models, i.e. the one-prey and one-predator system [22–25]. Some of them have reported the study of three species system [26–29], including the two-prey one-predator, and also the one-prey and two-predator systems. However, all of them has adopted specific response functions, such as the Holling-type, Ivlev-type, Beddington-DeAngelies-type, Hassell-Varley-type, Watt-type, Square-Root-type, Monod-Haldance-type, ratio-dependent-type together with many others [13, 14, 26, 30], for modelling the time evolution of the predator and prey

biological species under specific environmental configurations. One critic in these studies has been the limitation to apply these modelling results for solving real world problem, because of the scenario specific response functions that have been adopted in the previous work. Only a handful of work have modelled the density of the prey in the prey-predator system through general response functions, i.e. in the forms of $g(u(t))$ which have been widely reported in the literature [1, 29]. The gaps in these preliminary studies are necessary to follow up in order to enhance the effectiveness of the ecosystem modelling: i) the reported work in Ref. [1] has adopted too many hypotheses thereby its practical usefulness for real world applications may be somewhat restricted. ii) The use of the general functional response only for the prey density alone (i.e. in the form of $g(u(t))$) may not be sufficient enough to model the evolution of the prey-predator system in the real environment. iii)The existing work only concerns with the two species system, i.e. the one-prey and one-predator system which may be still far away from being realistic to the situations in the real environment. Hence, a more general functional response which is specifically designed for a three species ecosystem has been chosen as the main core of the present work, i.e. the general functional response is in the form of : $g_i(u_1(t), u_2(t), v(t))$. The notion of using general functional responses for modelling the prey-predator system has been shown [1, 19, 29] to be an effective way for implementing the modelling results for practical applications. Previous work [19] has indicated that the periodic solutions of the modelling that utilized general functional response of $g(u(t), v(t))$, is capable to reproduce the results of those which employed specific response functions for the modelling [25–28]. This may indicate the validity of using generalized functional responses for modelling the evolution of the prey-predator system [19]. The use of the functional response in the generalized form may be regarded as one of the promising future directions of research in the area of ecosystem modelling.

Motivated by previous work in [1, 29] and [19], this paper attempts to establish the modelling of the three species ecosystem by deploying impulsive control strategies at different instances of time period through a generalize functional response of $g_i(u_1(t), u_2(t), v(t))$, and to compound the effects due to the periodic variation of the prey's intrinsic growth rate, here we mainly talk about the two-prey and one-predator ecosystem:

$$
\begin{cases}
\begin{rcases}
\frac{du_1(t)}{dt} = u_1(t)(r_1 - \frac{r_1}{K_1}u_1(t) + a_1 u_2(t) + \lambda_1 \sin(\omega_1 t) - g_1(u_1(t), u_2(t), v(t))v(t)) \\
\frac{du_2(t)}{dt} = u_2(t)(r_2 - \frac{r_2}{K_2}u_2(t) + a_2 u_1(t) + \lambda_2 \sin(\omega_2 t) - g_2(u_1(t), u_2(t), v(t))v(t)) \\
\frac{dv(t)}{dt} = v(t)(-D + k_1 g_1(u_1(t), u_2(t), v(t))u_1(t) + k_2 g_2(u_1(t), u_2(t), v(t))u_2(t))
\end{rcases} \\
t \neq (m + q - 1)T, t \neq mT \\
\begin{rcases}
\Delta u_1(t) = -l_1 u_1(t) \\
\Delta u_2(t) = -l_2 u_2(t) \\
\Delta v(t) = -l_3 v(t)
\end{rcases} t = (m + q - 1)T \\
\begin{rcases}
\Delta u_1(t) = 0 \\
\Delta u_2(t) = 0 \\
\Delta v(t) = \mu
\end{rcases} t = mT
\end{cases}
\tag{1.1}
$$

in which $u_i(t)$ denotes the densities of the two preys' species and $v(t)$ represents the densities of the predator species. $r_i > 0$ is the two preys population's intrinsic birth rate and $T > 0$ is the impulsive time interval. $K_i > 0$ denotes the environmental capacities of the prey population, $\frac{r_i}{K_i}$ is the effects of

intraspecific competition on species. $a_i > 0$ represents the effects of interspecific competition among different species. $\omega_i > 0$ represents the angular frequency of the perturbation caused by the environmental periodicity on species $u_i(t)$, $\lambda_i > 0$ denotes the magnitude of the fluctuation on species $u_i(t)$, $k_i > 0$ represents the rate of transforming the prey species $u_i(t)$ into the newborn predator species, $D > 0$ denotes the natural mortality of the predator species. $g_i(u_1(t), u_2(t), v(t))$ is the general functional response. $\Delta u_i(t) = u_i(t^+) - u_i(t), u_i(t^+) = \lim_{t \to t^+} u_i(t), \Delta v(t) = v(t^+) - v(t), v(t^+) = \lim_{t \to t^+} v(t)$. $\mu > 0$ is the density of the predator species which is released at time $t = mT, m \in Z_+$, and Z_+ is the set of all positive integers. l_1, l_2, l_3 is the fixed death rate of the prey population species $u_i(t)$ and the predator population at time $(m + q - 1)T$ due to the spraying pesticides, respectively, where q and all i appeared in the paper are expressed as $i = 1, 2$. All the above parameters are positive, $0 \le l_1, l_2, l_3 < 1$ and $0 \le q < 1$ and the range of these parameters are to be determined by practical significance, which conforms to the natural events that can be observed from the real ecosystem.

In model (1) the following conditions have been assumed: (i) All predators have the ability to hunt and they consume only the prey; (ii) All prey is assumed to be actively work around the ecosystem without refuging; (iii) In this work the impulsive control strategies is deployed at different instances of time period, so suppose T is the impulsive time interval and the natural enemies are released at time $t = mT$, then the pesticides are sprayed at time $(m + q - 1)T$, where $m = 1, 2, 3, 4 \cdots$; (iv)The general functional response $g_i(u_1(t), u_2(t), v(t))$ satisfies the monotonous decreasing about species $u_i(t)$ and $v(t)$, respectively, and $v(t) g_i(u_1(t), u_2(t), v(t))$ is monotonously increasing about species $v(t)$, which conforms to the natural events that can be observed from the actual ecosystem. The above assumptions are based on the following biological justifications: Both (i) and (ii) are the general conditions for modelling the interactions of prey-predator system and there is no special case has been considered. (iii) One contribution of this work is to consider the side effects of pesticides on natural enemies, hence the impulsive control is deployed at different instances of time period. (iv) The assumptions for modelling using the general functional response conforms to the complicated natural habitat of real ecosystem, which, cannot be fulfilled by using one specific functional response to model the activities of animals in such vast variety of environments.

The organization of this article is outlined as follows: The main lemmas and definitions are summarized in the section 2, which will be used in the later sections. In Section 3, the prey-free periodic solution's global asymptotically stability and local asymptotically stability are derived. Subsequently the sufficient conditions and the boundedness of their solutions for the ecosystem's permanence in equation (1.1) (thereafter abbreviated it as system (1.1) in the rest of the paper) are then investigated. Subsequently the theoretical results that obtained from section 4 are validated by numerical simulations and then the paper is concluded in Section 5 and the theoretical results are discussed from a biological viewpoint.

2. Preliminaries

Let $u_1(t) = u_2(t) = 0$, then the system (1.1) becomes:

$$\begin{cases} \frac{dv(t)}{dt} = -Dv(t) & t \neq (m+q-1)T, t \neq mT \\ v(t^+) = (1-l_3)v(t) & t = (m+q-1)T \\ v(t^+) = v(t) + \mu & t = mT \\ v_0 = v(0^+). \end{cases} \tag{2.1}$$

By using the fixed-point theorem and the stroboscopic mapping of the impulsive differential equations, we can get the following ecosystem (2.1)'s positive periodic solution:

$$\tilde{v}(t) = \begin{cases} \frac{\mu \exp\{-D[t-(m-1)T]\}}{1-(1-l_3)\exp(-DT)} & (m-1)T < t \leq (m+q-1)T \\ \frac{\mu(1-p_3)\exp\{-D[t-(m-1)T]\}}{1-(1-l_3)\exp(-DT)} & (m+q-1)T < t \leq mT \end{cases} \tag{2.2}$$

and $\tilde{v}(0^+) = \tilde{v}(mT^+) = \frac{\mu}{1-(1-l_3)\exp(-DT)}$, $\tilde{v}(qT^+) = \tilde{v}((m+q-1)T^+) = \frac{\mu(1-l_3)\exp(-DqT)}{1-(1-l_3)\exp(-DT)}$. When the initial value is $v_0 \geq 0$, through the expansion of the above we can obtain the solution of ecosystem (2.1):

$$v(t) = \begin{cases} (1-l_3)^{m-1}\left(v(0^+) - \frac{\mu}{1-(1-l_3)\exp(-DT)}\right)\exp(-Dt) + \tilde{v}(t), \\ (m-1)T < t \leq (m+q-1)T, \\ (1-l_3)^m\left(v(0^+) - \frac{\mu}{1-(1-l_3)\exp(-DT)}\right)\exp(-Dt) + \tilde{v}(t), \\ (m+q-1)T < t \leq mT. \end{cases} \tag{2.3}$$

Lemma 2.1. *Suppose $v(t)$ is the ecosystem (2.1)'s positive solution, and the initial value is $v_0 \geq 0$, when $t \to \infty$, then the following result is established: $|v(t) - \tilde{v}(t)| \to 0$.*

Thus we obtain the ecosystem (1.1)'s positive periodic solution about prey-free $(0, 0, \tilde{v}(t))$.

3. Main results

Theorem 3.1. *Provided that both of the following inequalities are established at the same time,*

$$\begin{cases} \ln(1-l_1) + (r_1+\lambda_1)T - \int_0^T g_1(0,0,\tilde{v}(s))\tilde{v}(s)\,ds < 0, \\ \ln(1-l_2) + (r_2+\lambda_2)T - \int_0^T g_2(0,0,\tilde{v}(s))\tilde{v}(s)\,ds < 0. \end{cases} \tag{3.1}$$

then we get the locally asymptotically stable periodic solution about prey-free $(0, 0, \tilde{v}(t))$.

Proof. Firstly, let's consider the following impulsive different equations:

$$
\begin{cases}
\frac{du_{11}(t)}{dt} = u_{11}(t)(r_1 - \frac{r_1}{K_1}u_{11}(t) + a_1 u_{12}(t) + \lambda_1 - g_1(u_{11}(t), u_{12}(t), v_1(t))v_1(t)) \\
\frac{du_{12}(t)}{dt} = u_{12}(t)(r_2 - \frac{r_2}{K_2}u_{12}(t) + a_2 u_{11}(t) + \lambda_2 - g_2(u_{11}(t), u_{12}(t), v_1(t))v_1(t)) \\
\frac{dv_1(t)}{dt} = v_1(t)(-D + k_1 g_1(u_{11}(t), u_{12}(t), v_1(t))u_{11}(t) + k_2 g_2(u_{11}(t), u_{12}(t), v_1(t))u_{12}(t)) \\
t \neq (m+q-1)T, t \neq mT \\
\left.\begin{array}{l} \Delta u_{11}(t) = -l_1 u_{11}(t) \\ \Delta u_{12}(t) = -l_2 u_{12}(t) \\ \Delta v_1(t) = -l_3 v_1(t) \end{array}\right\} t=(m+q-1)T \\
\left.\begin{array}{l} \Delta u_{11}(t) = 0 \\ \Delta u_{12}(t) = 0 \\ \Delta v_1(t) = \mu \end{array}\right\} t=mT
\end{cases}
\tag{3.2}
$$

Since the system (1.1) and system (3.2) have the same prey-free periodic solution $(0, 0, \tilde{v}(t))$ and $u_{11}(t) \geq u_1(t)$, $u_{12}(t) \geq u_2(t)$, $v_1(t) \geq v(t)$, thus the system (3.2)'s prey-free periodic solution is needed to be proved that it is locally asymptotically stable.

Let us denote $z_1(t) = u_{11}(t)$, $z_2(t) = u_{12}(t)$, $w(t) = v_1(t) - \tilde{v}(t)$. By taking the linear part of the Taylor expansion and the form of the system (3.2) can be written as:

$$
\begin{cases}
\frac{dz_1(t)}{dt} = z_1(t)(r_1 + \lambda_1 - g_1(0, 0, \tilde{v}(t))\tilde{v}(t)) \\
\frac{dz_2(t)}{dt} = z_2(t)(r_2 + \lambda_2 - g_2(0, 0, \tilde{v}(t))\tilde{v}(t)) \\
\frac{dw(t)}{dt} = -Dw(t) + k_1 g_1(0, 0, \tilde{v}(t))\tilde{v}(t)z_1(t) + k_2 g_2(0, 0, \tilde{v}(t))\tilde{v}(t)z_2(t) \\
t \neq (m+q-1)T, t \neq mT \\
\left.\begin{array}{l} z_1((m+q-1)T^+) = (1-l_1)z_1((m+q-1)T) \\ z_2((m+q-1)T^+) = (1-l_2)z_2((m+q-1)T) \\ w((m+q-1)T^+) = (1-l_3)w((m+q-1)T) \end{array}\right\} t=(m+q-1)T \\
\left.\begin{array}{l} z_1(mT^+) = z_1(mT) \\ z_2(mT^+) = z_2(mT) \\ w(mT^+) = w(mT) \end{array}\right\} t=mT
\end{cases}
\tag{3.3}
$$

Through the simple calculation it can be shown that the fundamental solution matrix:

$$
\varphi(t) = \begin{pmatrix} A_1(t) & 0 & 0 \\ 0 & A_2(t) & 0 \\ * & * & \exp(-Dt) \end{pmatrix},
$$

in which $A_i(t) = \exp\left(\int_0^t (r_i + \lambda_1 - g_i(0, 0, \tilde{v}(t))\tilde{v}(t))\,ds\right) (i = 1, 2)$ and $\varphi(t)$ satisfies:

$$
\frac{d\varphi(t)}{dt} = \begin{pmatrix} r_1 + \lambda_1 - g_1(0, 0, \tilde{v}(t))\tilde{v}(t) & 0 & 0 \\ 0 & r_2 + \lambda_2 - g_2(0, 0, \tilde{v}(t))\tilde{v}(t) & 0 \\ k_1 g_1(0, 0, \tilde{v}(t))\tilde{v}(t) & k_2 g_2(0, 0, \tilde{v}(t))\tilde{v}(t) & -D \end{pmatrix}\varphi(t).
$$

The ecosystem (7)'s resetting impulsive conditions can then be written as:

$$
\begin{pmatrix} z_1\left((m+q-1)T^+\right) \\ z_2\left((m+q-1)T^+\right) \\ w\left((m+q-1)T^+\right) \end{pmatrix} = \begin{pmatrix} 1-l_1 & 0 & 0 \\ 0 & 1-l_2 & 0 \\ 0 & 0 & 1-l_3 \end{pmatrix} \begin{pmatrix} z_1\left((m+q-1)T\right) \\ z_2\left((m+q-1)T\right) \\ w\left((m+q-1)T\right) \end{pmatrix},
$$

$$
\begin{pmatrix} z_1\left(mT^+\right) \\ z_2\left(mT^+\right) \\ w\left(mT^+\right) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z_1\left(mT\right) \\ z_2\left(mT\right) \\ w\left(mT\right) \end{pmatrix}.
$$

Let λ_1, λ_2 be the monodromy matrix's eigenvalues

$$
M = \begin{pmatrix} 1-l_1 & 0 & 0 \\ 0 & 1-l_2 & 0 \\ 0 & 0 & 1-l_3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \varphi(T),
$$

where

$$
\lambda_i = (1-l_i)\exp\left(\int_0^T r_i + \lambda_i - g_i\left(0, 0, \tilde{v}(t)\right)\tilde{v}(t)ds\right)(i=1,2),
$$
$$
\lambda_3 = (1-l_3)\exp(-DT) < 1.
$$

As $|\lambda_3| < 1$, and by applying the Floquent theory on the impulsive different equation, and when $|\lambda_i| < 1(i=1,2)$, we can obtain the following inequality which holds at the same time:

$$
(1-l_i)\exp\left(\int_0^T r_i + \lambda_i - g_i\left(0, 0, \tilde{v}(t)\right)\tilde{v}(t)ds\right) < 0(i=1,2).
$$

By taking logarithms on both sides at the same time, then the inequality (5) is established. □

Corollary 3.2. *When the general functional response becomes $u(t)g(u(t))$, then we can get the similar result as Theorem 3.1, namely the following inequality is established:*

$$
\ln(1-l_1) + (r+\lambda)T - g(0)\int_0^T \tilde{v}(s)\,ds < 0, \tag{3.4}
$$

which is the theorem 3.1 in the reference [15].

Theorem 3.3. *Provided that both of the following inequalities are established at the same time,*

$$
\begin{cases} \ln(1-l_1) + (r_1+\lambda_1)T - \int_0^T g_1\left(K_1, K_2, \tilde{v}(s)\right)\tilde{v}(s)\,ds < 0, \\ \ln(1-l_2) + (r_2+\lambda_2)T - \int_0^T g_2\left(K_1, K_2, \tilde{v}(s)\right)\tilde{v}(s)\,ds < 0. \end{cases} \tag{3.5}
$$

then we get the globally asymptotically stable periodic solution about prey-free $(0, 0, \tilde{v}(t))$.

Proof. From the system (3.2) we found that:

$$
\begin{cases} \dfrac{du_{1i}(t)}{dt} \le u_{1i}(t)\left(r_i + \lambda_i - \dfrac{r_i}{K_i}u_{1i}(t)\right), & t \ne (m+q-1)T, t \ne mT, \\ u_{1i}(t^+) = (1-l_i)u_{1i}(t) \le u_{1i}(t), & t = (m+q-1)T, t = mT. \end{cases}
$$

By employing the Comparison theorem of the impulsive different equation, $u_{1i}(t) \leq \widetilde{u}_i(t)\,(i = 1, 2)$ can be obtained. Then

$$\widetilde{u}_i(t) = \frac{K_i x_0}{(K_i - x_0)\,e^{-(r_i + \lambda_i)t} + x_0} \to K_i\,(t \to \infty), \tag{3.6}$$

where $\widetilde{u}_i(t)$ satisfies the following equation:

$$\begin{cases} \frac{\mathrm{d}\widetilde{u}_i(t)}{\mathrm{d}t} = \widetilde{u}_i(t)\left(r_i + \lambda_i - \frac{r_i}{K_i}\widetilde{u}_i(t)\right), \\ \widetilde{u}_i(0^+) = x_0. \end{cases}$$

Thus for all sufficiently large t and for any $\varepsilon_i > 0$, we can obtain $u_{1i}(t) \leq K_i + \varepsilon_i$.

By Choosing $\delta_i > 0$, it satisfies

$$\eta_i = (1 - l_i)\exp\left(\int_0^T r_i + \lambda_i - g_i(K_1 + \varepsilon_1, K_2 + \varepsilon_2, \tilde{v}(s) - \delta_i)(\tilde{v}(s) - \delta_i)\,ds\right) \in (0, 1).$$

where $i = 1, 2$. In the same way, we can see that $\frac{\mathrm{d}v_1(t)}{\mathrm{d}t} \geq -Dv_1(t)$, therefore $v_1(t) \geq \tilde{v}(t)$. By employing the Lemma 2.1, $v_1(t) \to \tilde{v}(t)$ can be obtained when $t \to \infty$, then

$$v_1(t) \geq \tilde{v}(t) > \tilde{v}(t) - \delta \tag{3.7}$$

holds when all t is large enough. Without loss of generality, we can assume (3.7) holds for all $t > 0$. Notice that:

$$\frac{\mathrm{d}u_{1i}}{\mathrm{d}t} \leq u_{1i}(r_i + \lambda_i - g_i(K_1 + \varepsilon_1, K_2 + \varepsilon_2, \tilde{v}(s) - \delta_i)(\tilde{v}(s) - \delta_i)).$$

It is easy to figure out:

$$\begin{aligned} u_{1i}((m + q)T) &\leq u_{1i}((m + q - 1)T^+)A_i \\ &= u_{1i}((m + q - 1)T)(1 - l_i)A_i \\ &= u_{1i}((m + q - 1)T)\eta_i, \end{aligned}$$

where $A_i = \exp\left(\int_{(m+q-1)T}^{(m+q)T} r_i + \lambda_i - g_i(K_1 + \varepsilon_1, K_2 + \varepsilon_2, \tilde{v}(s) - \delta_i)(\tilde{v}(s) - \delta_i)\,ds\right)$. Therefore we have:

$$u_{1i}((m + q)T) \leq u_{1i}(qT)\eta_i^m \to 0\,(m \to \infty).$$

As

$$0 \leq u_{1i}(t) \leq u_{1i}((m + q - 1)T)(1 - l_i)\,e^{rT}$$

holds for $t \in [(m + q - 1)T, (m + q)T]$, thus $u_{1i}(t) \to 0\,(i = 1, 2)$ as $t \to \infty$.

Next we will prove the claim that when $\lim\limits_{t \to \infty} u_{1i}(t) = 0$, where $i = 1, 2$, then $\lim\limits_{t \to \infty} v_1(t) = \tilde{v}(t)$. For any positive and small enough number $\varepsilon_3, \varepsilon_4 > 0$, by using the monotonicity of the general functional response function, we can obtain:

$$-Dv_1(t) \leq \frac{\mathrm{d}v_1(t)}{\mathrm{d}t} \leq v_1(t)(-D + k_1\varepsilon_3 g_1(0, 0, \tilde{v}(s) - \delta_1) + k_2\varepsilon_4 g_2(0, 0, \tilde{v}(s) - \delta_2)),$$

and

$$\tilde{v}_1(t) \leq v_1(t) \leq \tilde{v}_2(t), \tag{3.8}$$

in which $\tilde{v}_1(t)$ is the positive periodic solutions of the system (2.1) and $\tilde{v}_2(t)$ is the positive periodic solutions of the system (2.1)with D changes into $-D + k_1\varepsilon_3 g_1(0, 0, \tilde{v}(s) - \delta_1) + k_2\varepsilon_4 g_2(0, 0, \tilde{v}(s) - \delta_2)$. By employing the Lemma 2.1, for all sufficiently large t, we can get

$$\tilde{v}_1(t) \to \tilde{v}(t), \tilde{v}_2(t) \to \tilde{v}(t).$$

From inequality (3.8) we can get that $\lim_{t\to\infty} v_1(t) \to \tilde{v}(t)$. □

Corollary 3.4. *When the general response function becomes $u(t)g(u(t))$, then we can get the similar result as Theorem 3.3, namely the following inequalities holds:*

$$\ln(1 - l_1) + (r + \lambda)T - K * \int_0^T \tilde{v}(s)\, ds < 0, \tag{3.9}$$

where $K = g(K_1), u(t) \le K_1$, as $g(u(t))$ only depends on the prey density $u(t)$, which is the theorem 3.2 in the reference [15].

Theorem 3.5. *Provided that both of the following inequalities hold at the same time,*

$$\begin{cases} \ln(1 - l_1) + (r_1 - \lambda_1)T - \int_0^T g_1(0, 0, \tilde{v}(s))\, \tilde{v}(s)\, ds > 0, \\ \ln(1 - l_2) + (r_2 - \lambda_2)T - \int_0^T g_2(0, 0, \tilde{v}(s))\, \tilde{v}(s)\, ds > 0, \end{cases} \tag{3.10}$$

then the system (1.1) is permanent.

Proof. There exists a positive constant number M, we can get that $u_1(t) \le M, u_2(t) \le M, v(t) \le M$ as $t \to \infty$, which can be found in the literature [31], and it holds for all $t > 0$ as according to theorem 3.3. Let $\tilde{m} = \tilde{v} - \varepsilon > 0$, and according to the Lemma 2.1, $v(t) > \tilde{m}$ can be obtained. In the following, for all large enough t, we only need to find $m_0 > 0$, such that $u(t) > m_0$.

Similarly, let's also consider the following impulsive differential equations:

$$\begin{cases} \frac{du_{21}(t)}{dt} = u_{21}(t)(r_1 - \frac{r_1}{K_1}u_{21}(t) + a_1 u_{22}(t) - \lambda_1 - g_1(u_{21}(t), u_{22}(t), v_2(t))v_2(t)) \\ \frac{du_{22}(t)}{dt} = u_{22}(t)(r_2 - \frac{r_2}{K_2}u_{22}(t) + a_2 u_{21}(t) - \lambda_2 - g_2(u_{21}(t), u_{22}(t), v_2(t))v_2(t)) \\ \frac{dv_2(t)}{dt} = v_2(t)(-D + k_1 g_1(u_{21}(t), u_{22}(t), v_2(t))u_{21}(t) + k_2 g_2(u_{21}(t), u_{22}(t), v_2(t))u_{22}(t)) \\ \quad t \ne (m+q-1)T, t \ne mT \\ \left.\begin{array}{l} \Delta u_{21}(t) = -l_1 u_{21}(t) \\ \Delta u_{22}(t) = -l_2 u_{22}(t) \\ \Delta v_2(t) = -l_3 v_2(t) \end{array}\right\} t=(m+q-1)T \\ \left.\begin{array}{l} \Delta u_{21}(t) = 0 \\ \Delta u_{22}(t) = 0 \\ \Delta v_2(t) = \mu \end{array}\right\} t=mT \end{cases} \tag{3.11}$$

Since $u_{21}(t) \le u_1(t), u_{22}(t) \le u_2(t)$ and $v_2(t) \le v(t)$ and they hold for any $t > 0$, thus we only need to prove that $u_{2i}(t) > m_{i0}(i = 1, 2)$ as $t \to \infty$. Next we will only need to prove that $u_{21}(t) > m_{10}$ because we can obtain that $u_{22}(t) > m_{20}$ in the same way. So the following process is divided into two steps:

1. Let m_1 to be small enough such that

$$0 < m_1 < \frac{D - k_2 M g_2\,(0,0,\tilde{m})}{k_1 g_1\,(0,0,\tilde{m})},$$

$$\sigma = \ln\,(1 - l_1) + \left(r - \lambda - \frac{r}{K}m_1\right)T - \int_0^T g\,(0,\tilde{\omega}\,(t) + \varepsilon_1)\,(\tilde{\omega}\,(t) + \varepsilon_1)\,dt > 0.$$

Now we can claim that $u_{21}\,(t_1) > m_1$ and it holds for some $t_1 > 0$. Otherwise, for any $t > 0$ we can obtain that $u_{21}\,(t_1) \leq m_1$. It can be seen from system (3.11) that

$$\frac{dv_2\,(t)}{dt} \leq v_2\,(t)\,(-D + k_1 m_1 g_1\,(0,0,\tilde{m}) + k_2 M g_2\,(0,0,\tilde{m}))\,.$$

It is easy to deduce from the comparison theorem that $v_2\,(t) \leq \omega\,(t)$, $\omega\,(t) \to \tilde{\omega}\,(t)$, where $\omega\,(t)$ is the positive solution of the following system:

$$\begin{cases} \frac{d\omega(t)}{dt} = (-D + k_1 m_1 g_1\,(0,0,\tilde{m}) + k_2 M g_2\,(0,0,\tilde{m}))\,\omega\,(t), \\ t \neq (m + q - 1)\,T, t \neq mT \\ \omega\,(t^+) = (1 - l_3)\,\omega\,(t), \qquad t = (m + q - 1)\,T \\ \omega\,(t^+) = \omega\,(t) + \mu, \qquad t = mT \\ \omega_0 = v\,(0^+)\,. \end{cases} \tag{3.12}$$

and we can also get the positive periodic solution:

$$\tilde{\omega}\,(t) = \begin{cases} \dfrac{\mu \exp\{(-D + k_1 m_1 g_1(0,0,\tilde{m}) + k_2 M g_2(0,0,\tilde{m}))[t - (m-1)T]\}}{1 - (1 - l_2)\exp\left[(-D + k_1 m_1 g_1(0,0,\tilde{m}) + k_2 M g_2(0,0,\tilde{m}))T\right]}, \\ (m - 1)\,T < t \leq (m + q - 1)\,T, \\ \dfrac{\mu(1 - l_2) \exp\{(-D + k_1 m_1 g_1(0,0,\tilde{m}) + k_2 M g_2(0,0,\tilde{m}))[t - (m-1)T]\}}{1 - (1 - l_2)\exp\left[(-D + k_1 m_1 g_1(0,0,\tilde{m}) + k_2 M g_2(0,0,\tilde{m}))T\right]}, \\ (m + q - 1)\,T < t \leq mT. \end{cases}$$

So we can get that there exists $T_1 > 0$ such that $v_2\,(t) \leq \omega\,(t) \leq \tilde{\omega}\,(t) + \varepsilon_1$, and

$$\frac{du_{21}\,(t)}{dt} \geq u_{21}\,(t)\left(r_1 - \lambda_1 - \frac{r_1}{K_1}m_1 - g_1\,(0,0,\tilde{\omega}\,(t) + \varepsilon_1)\,(\tilde{\omega}\,(t) + \varepsilon_1)\right). \tag{3.13}$$

By integrating the inequality (3.13) over $[(m + q - 1)\,T,\ (m + q)\,T]$, we can obtain:

$$\begin{aligned} u_{21}\,((m + q)\,T) &\geq u_{21}\,((m + q - 1)\,T^+)\,B \\ &= u_{21}\,((m + q - 1)\,T)\,(1 - l_1)\,B \\ &= u_{21}\,((m + q - 1)\,T)\exp\,(\sigma), \end{aligned}$$

where $B = \exp\left(\int_{(m+q-1)T}^{(m+q)T} r_1 - \lambda_1 - \frac{r_1}{K_1}m_1 - g_1\,(0,0,\tilde{\omega}\,(t) + \varepsilon_1)\,(\tilde{\omega}\,(t) + \varepsilon_1)\,ds\right)$. Therefor

$$u_{21}\,((m + q)\,T) \geq u_{21}\,(qT)\exp\,(m\sigma) \to \infty\,(m \to \infty)\,,$$

which contradicts to $u_{21}\,(t) \leq M$.

2. Next, we will prove the claim that when $t > t_1$, then $u_{21}(t_1) > m_1$ holds. Otherwise, for some $t > t_1$, we can obtain that $u_{21}(t) \le m_1$. Define $\widetilde{t} = \inf_{t \ge t_1}\{u_{21}(t) < m_1\}$, we can get $u_{21}(t_1) \ge m_1$ for $t \in \left[t_1, \widetilde{t}\right)$ and $\widetilde{t} \in [p_1 T, (p_1 + 1)T]$, $p_1 \in N$. It is easy to deduce from the continuity of the $u_{21}(t)$ that $u_{21}(\widetilde{t}) = m_1$. For $p_2, p_3 \in N$, such that

$$p_2 T > \frac{1}{(-D + k_1 m_1 g_1(0,0,\tilde{m}) + k_2 M g_2(0,0,\tilde{m}))} \ln \frac{\varepsilon_1}{M + \mu},$$

$$\exp\left(\delta\left(p_2 + 1\right)T\right)\exp\left(p_3\sigma\right) > 1,$$

where $\delta \overset{\Delta}{=} r_1 - \lambda_1 - \frac{r_1}{K_1}m_1 - g_1(0,0,M)M < 0$.

Let $T' = (p_2 + p_3)T$, we can obtain the claim that there exists $t_2 \in \left[(p_1 + 1)T, (p_1 + 1)T + T'\right]$ which implies that $u_{21}(t_1) \ge m_1$. Otherwise $u_{21}(t_1) < m_1$ and as according to system (3.11) when $\omega\left((p_1 + 1)T^+\right) = v\left((p_1 + 1)T^+\right)$ we can get:

$$\omega(t) = \begin{cases} (1 - l_3)^{m-1}\left(\omega\left((p_1 + 1)T^+\right) - \frac{\mu}{1 - (1-l_3)\exp\left[(-D + k_1 m_1 g_1(0,0,\tilde{m}) + k_2 M g_2(0,0,\tilde{m}))T\right]}\right) * \\ \exp\left[(-D + k_1 m_1 g_1(0,0,\tilde{m}) + k_2 M g_2(0,0,\tilde{m}))(t - (p_1 + 1)T)\right] + \tilde{\omega}(t), \\ (1 - l_3)^{m}\left(\omega\left((p_1 + 1)T^+\right) - \frac{\mu}{1 - (1-l_3)\exp\left[(-D + k_1 m_1 g_1(0,0,\tilde{m}) + k_2 M g_2(0,0,\tilde{m}))T\right]}\right) * \\ \exp\left[(-D + k_1 m_1 g_1(0,0,\tilde{m}) + k_2 M g_2(0,0,\tilde{m}))(t - (p_1 + 1)T)\right] + \tilde{\omega}(t). \end{cases}$$

Thus

$$|\omega(t) - \tilde{\omega}(t)| < (M + \mu)\exp\left[(-D + k_1 m_1 g_1(0,0,\tilde{m}) + k_2 M g_2(0,0,\tilde{m}))(t - (p_1 + 1)T)\right] < \varepsilon_1,$$

and when t satisfies the condition of $(p_1 + p_2 + 1)T \le t \le (p_1 + 1)T + T'$, we can obtain:

$$v(t) \le \omega(t) \le \tilde{\omega}(t) + \varepsilon_1.$$

So when t satisfies the condition of $(p_1 + p_2 + 1)T \le t \le (p_1 + 1)T + T'$, in the same way, we can also obtain the inequality (3.13), then

$$u_{21}\left((p_1 + p_2 + p_3 + 1)T\right) \ge u_{21}\left((p_1 + p_2 + 1)T\right)\exp\left(p_3\sigma\right). \tag{3.14}$$

It is observed that when $t \in \left(\widetilde{t}, (p_1 + 1)T\right]$ there are two possible cases .

(Case 1) If $u_{21}(t) \le m_1$ for $t \in \left(\widetilde{t}, (p_1 + 1)T\right]$, then $u_{21}(t) \le m_1$ for all $t \in \left(\widetilde{t}, (p_1 + p_2 + 1)T\right]$. From system (3.11) we can observe that:

$$\frac{\mathrm{d}u_{21}(t)}{\mathrm{d}t} \ge u_{21}(t)\left(r_1 - \lambda_1 - \frac{r_1}{K_1}m_1 - g_1(0,0,M)M\right) \overset{\Delta}{=} \delta u_{21}(t). \tag{3.15}$$

We can get the following inequality by integrating the inequality (3.15) over $\left(\widetilde{t}, (p_1 + p_2 + 1)T\right]$:

$$u_{21}\left((p_1 + p_2 + 1)T\right) \ge m_1 \exp\left(\delta\left(p_2 + 1\right)T\right). \tag{3.16}$$

Through the simple calculation of inequalities (3.14) and (3.16), we can obtain:

$$u_{21}\left((p_1 + p_2 + p_3 + 1)T\right) \ge m_1 \exp\left(\delta\left(p_2 + 1\right)T\right)\exp\left(p_3\sigma\right) > m_1,$$

which is in contradiction to $u_{21}(t) \leq m_1$ for all t. Let's define $t^* = \inf_{t \geq \tilde{t}} \{u_{21}(t) \geq m_1\}$, then $u_{21}(t^*) = m_1$. The inequality (3.15) holds for $t \in [\tilde{t}, t^*)$ and the integration over $t \in [\tilde{t}, t^*)$ to get

$$u_{21}(t) \geq u_{21}(\tilde{t}) \exp\left(\delta\left(t - \tilde{t}\right)\right) \geq m_1 \exp\left(\delta\left(p_2 + p_3 + 1\right)T\right) \overset{\Delta}{=} m_0.$$

As $u_{21}(t^*) \geq m_1$, the same argument applies to $t > t^*$. Hence $u_{21}(t) \geq m_0$ holds for all $t > t_1$.

(Case 2) There exists $t' \in (\tilde{t}, (n_1 + 1)T]$ which implies that $u_{21}(t') > m_1$. By setting $\hat{t} = \inf_{t \geq \tilde{t}} \{u_{21}(t) \geq m_1\}$, then $u_{21}(t) < m_1$ holds for $t \in [\tilde{t}, \hat{t})$ and $u_{21}(\hat{t}) = m_1$. Then the inequality (3.15) holds for $t \in [\tilde{t}, \hat{t})$ and thus we can also observe that:

$$u_{21}(t) \geq u_{21}(\tilde{t}) \exp\left(\delta\left(t - \tilde{t}\right)\right) \geq m_1 \exp\left(\delta T\right) \geq m_{10}.$$

By the same argument, we can proof that $u_{21}(t) \geq m_{10}$ for all $t \geq t_1$. By using the same method, we also obtain that $u_{22}(t) \geq m_{20}$.

Since $u_{21}(t) \leq u_1(t), u_{22}(t) \leq u_2(t)$, therefore in both cases, we can deduce that $u_1(t) \geq m_{10}$, $u_2(t) \geq m_{20}$ and it holds for all $t \geq t_1$ in both cases. □

Corollary 3.6. *When the general functional response becomes $u(t)g(u(t))$, then we can get the similar result as Theorem 3.5, namely the following inequalities holds:*

$$\ln(1 - l_1) + (r - \lambda)T - g(0)\int_0^T \tilde{v}(s)\,ds > 0, \tag{3.17}$$

which is exactly identical to the theorem 3.3 in [15].

4. Numerical simulations

This section is dedicated to the validation of the theoretical results derived in section 3 by substituting the generalized function with specific functional response and to compare the output with previously results that have been reported in [25–28].

Firstly, the Holling II functional response and the ratio-dependent functional response is selected for the substitution into the generalized response, namely, $g_1(u_1(t), u_2(t), v(t)) = \frac{r}{a + bu_1(t)}$, $g_2(u_1(t), u_2(t), v(t)) = \frac{c}{my(t) + u_2(t)}$. Define:

$$f_1(T) = \ln(1 - l_1) + (r_1 + \lambda_1)T - \frac{rB}{a},$$

$$f_2(T) = \ln(1 - l_2) + \left(r_2 + \lambda_2 - \frac{c}{m}\right)T,$$

$$f_3(T) = \ln(1 - l_1) + (r_1 - \lambda_1)T - \frac{rB}{a},$$

$$f_4(T) = \ln(1 - l_2) + \left(r_2 - \lambda_2 - \frac{c}{m}\right)T,$$

where $B \overset{\Delta}{=} \int_0^T \tilde{v}(s)ds = \frac{\mu[1-l_3\exp(-DqT)-(1-l_3)\exp(-DT)]}{D[1-(1-l_3)\exp(-DT)]}$. As $\lim_{T\to 0} B(T) = 0$, so $f_i(0) = \ln(1-l_i) < 0 (i = 1, 2, 3, 4)$. And due to $\lim_{T\to\infty} B(T) = \frac{\mu}{D}$, so $\lim_{T\to\infty} f_i(T) = \infty (i = 1, 2, 3, 4)$. And since $f_i(T)(i = 1, 2, 3, 4)$ is monotonous increasing about T, so it can be seen that $f_i(T) = 0(i = 1, 2, 3, 4)$ has unique positive solutions which can be denoted by T_1^*, T_2^*, T_3^* and T_4^*, respectively. It is known that $(0, 0, \tilde{v}(t))$ satisfies the theorem 3.1 when $T < \min(T_1^*, T_2^*)$ and when $T > \max(T_3^*, T_4^*)$, which satisfies the theorem 3.5. It is quite obvious that $T_1^* < T_3^*$ and $T_2^* < T_4^*$. From the theorem 3.1, it is obtained that only the prey (pest) density $u_1(t)$ is extinct when $T \in [\min(T_1^*, T_2^*), \max(T_1^*, T_2^*)]$.

Next, it is assumed that $r_1 = 2, K_1 = 10, r_2 = 3, K_2 = 10, a_1 = 0.5, a_2 = 0.8, \lambda_1 = 0.1, \omega_1 = \pi, \lambda_2 = 0.1, \omega_2 = 2\pi, a = 5, b = 3, r = 3, c = 4.4, m = 2, k_1 = 0.4, k_2 = 0.4, D = 0.5, l_1 = 0.9, l_2 = 0.85, l_3 = 0.9, u = 10, q = 0.2, u_1(0) = 1, u_2(0) = 3.5, v(0) = 10$. A briefly calculation may suggest that $T_1^* \approx 2.79, T_2^* \approx 2.11, T_3^* \approx 2.96$ and $T_4^* \approx 2.71$. It is easy to see that the prey (pest) density $u_1(x)$ and $u_2(x)$ are rapidly decreasing to zero while the predator (natural enemy) population $v(t)$ oscillates in a cycle which tends to stabilize when $T = 2 < \min(T_1^*, T_2^*)$, then $(0, 0, \tilde{v}(t))$ is locally asymptotically stable as according to the theorem 3.1(see Figure 1(a) and 1(b)). And when $T = 2.5 \in [T_2^*, T_1^*]$ only the prey (pest) density $u_1(t)$ is extinct, the predator population $v(t)$ and the prey population $u_2(t)$ oscillate in a cycle which tends to stabilize(see Figure 2(a), 2(b), 2(c) and 2(d)). Similarly, when $T = 3 > \max(T_3^*, T_4^*)$ the system (1.1) is permanent according to the theorem 3.5, then the predator population $v(t)$, the prey population $u_1(t)$ and $u_2(t)$ can coexist in a stable limited cycle, as it is shown in Figure 3(a) and 3(b).
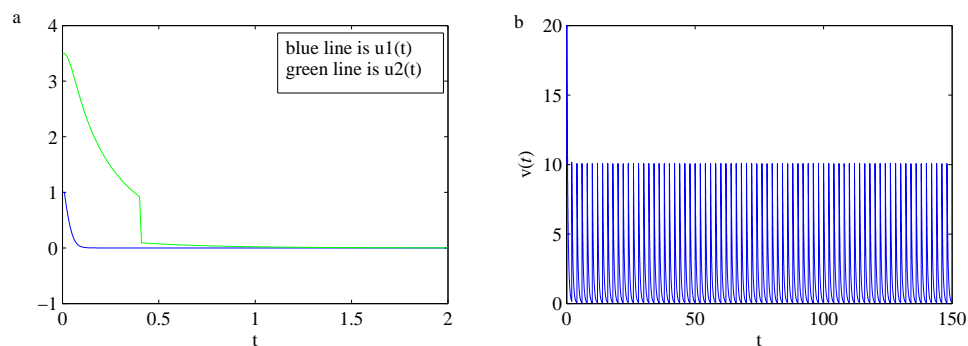

Figure 1. Dynamical behavior of system (1.1) with $T = 2$.(a)Time sequence diagram of prey population $u_1(t)$ and $u_2(t)$.(b)Time sequence diagram of predator population $v(t)$.
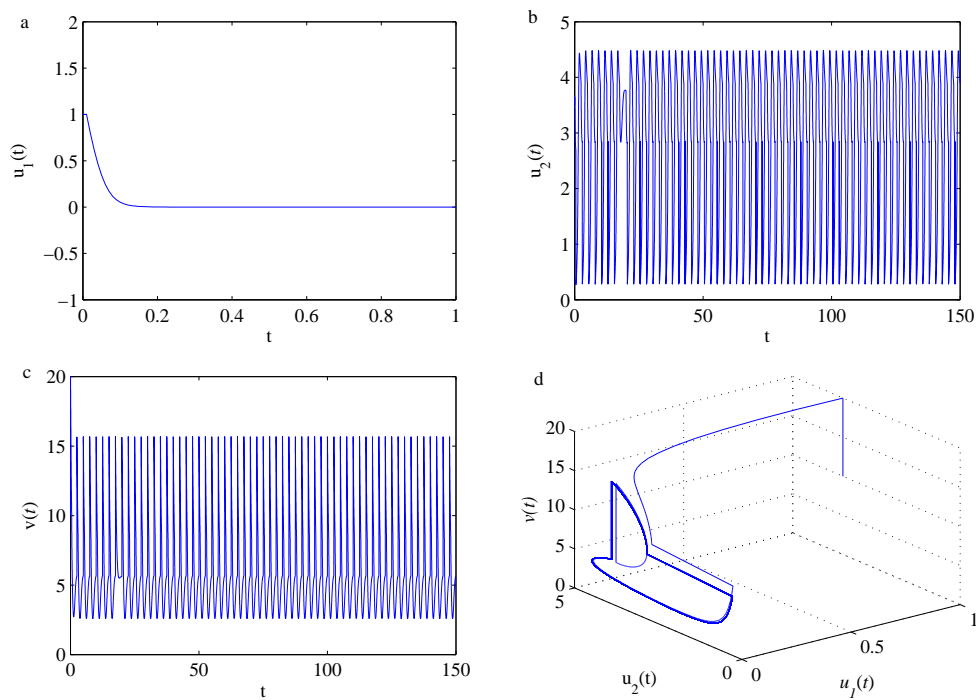
Figure 2. Dynamical behavior of system (1.1) with $T = 2.5$.(a)Time sequence diagram of prey population $u_1(t)$.(b)Time sequence diagram of predator population $u_2(t)$.(c)Time sequence diagram of prey population $v(t)$.(d)The phase diagram of $u_1(t), u_2(t)$ and $v(t)$.
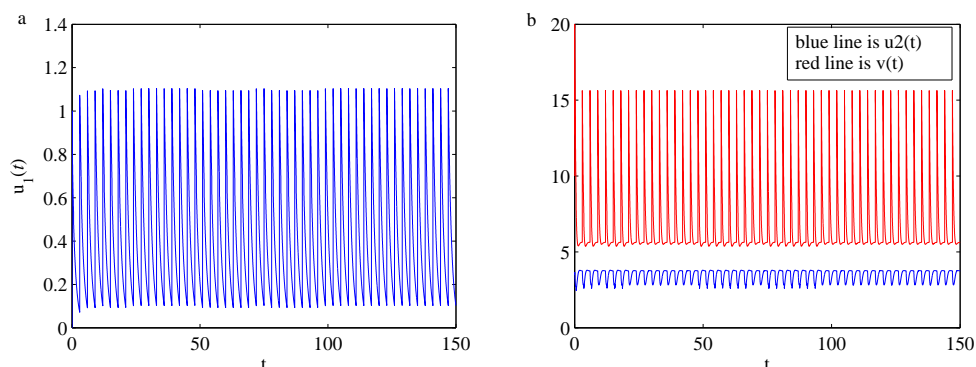

Figure 3. Dynamical behavior of system (1.1) with $T = 3$.(a)Time sequence diagram of prey population $u_1(t)$.(b)Time sequence diagram of predator population $u_2(t)$ and $v(t)$.

Finally, the profound influence of environmental factors (the magnitudes $\lambda_i(i = 1, 2)$ and the frequencies $\omega_i(i = 1, 2)$) on the complexity of the system (1.1) is considered. let $\lambda_1 = 0.4$ and it can be figured out that $T_1^* \approx 2.36$, when $T = 2 < T_1^*$ and as according to Figure 4(a), the prey (pest) population is found approaching to zero, more slowly than that of in Figure 4(b). It is easy to understand that when λ is increased which is equivalent to the increase of the intrinsic birth rate of the prey, a shorter cycle T_1^* will be needed for applying pesticides and to release natural enemies in order to make the pest to extinct. Note that when the larger is the λ, the longer time that it will take for the pests to become

extinct. Although from the results derived in section 3, look as if the frequencies $\omega_i(i = 1, 2)$ do not affect the dynamics of the system (1.1), but they are significant ingredients of the environment(see Figure 5(a) and 5(b)), which can cause various and complex dynamic behavior.
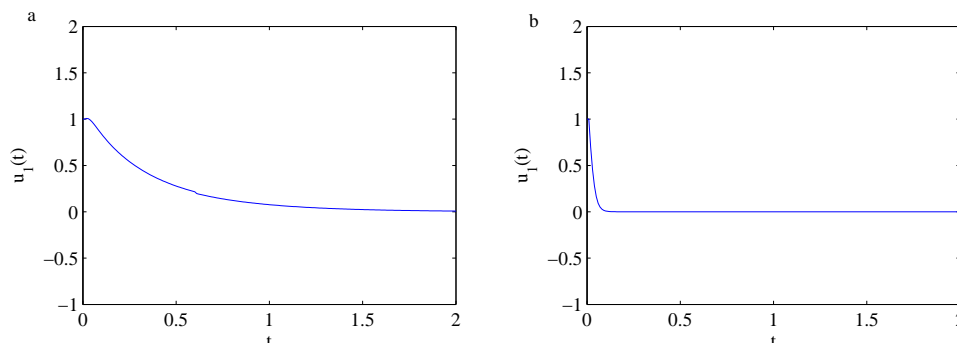

Figure 4. (a)Time sequence diagram of prey population $u_1(t)$ when $T = 2, \lambda_1 = 0.4$.(b)Time sequence diagram of predator population $u_1(t)$ when $T = 2, \lambda_1 = 0.1$.
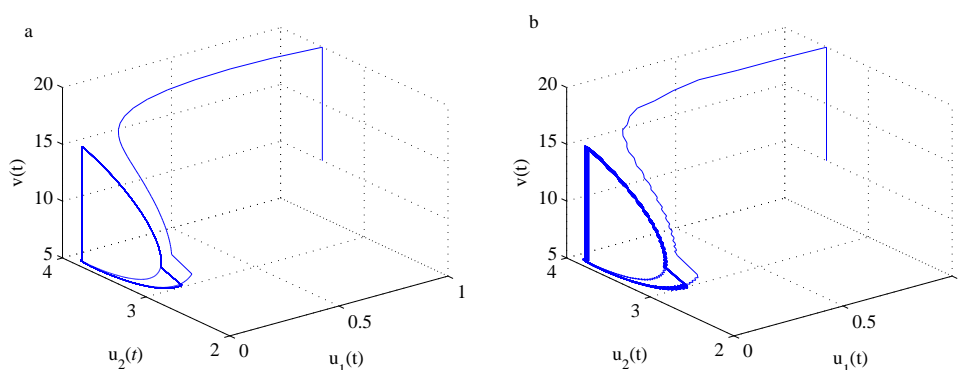

Figure 5. Dynamical behavior of system (1.1) with $T = 2.5$.(a)The phase diagram of $u_1(t), u_2(t)$ and $v(t)$ when $\omega_1 = 2\pi$.(b)The phase diagram of $u_1(t), u_2(t)$ and $v(t)$ when $\omega_2 = 2$.

5. Conclusions and biological significance

Considering the impact of environmental factors on the prey-predator system, the paper extends previous work [15–18] to establish the two-prey and one-predator system with a more generalized functional response including the intra- and inter-species prey-predator competitions. The sufficient condition for the local and global asymptotic stability of the prey-free periodic solution and the permanence of the system have been subsequently derived according to the Theorem 3.1, Theorem 3.2 and Theorem 3.5 in the section 3, which generalize the existing conclusions that have been reported in the literature [25–28].

These results and simulations show that dynamical properties of (1) are very complex and depend on impulsive period T, the magnitudes $\lambda_i(i = 1, 2)$, the releasing amount of predator μ, the two preys population's intrinsic birth rate $r_i > 0(i = 1, 2)$, due to the spraying pesticides the fixed death rate of the prey population species $\leq l_1, l_2$ and more generalized functional response, look as if the rest of

the parameters do not affect the dynamics of the system (1.1) from the results, but from simulations we can clearly see that the profound influence of frequencies $\omega_i(i = 1, 2)$ on the complexity of the system (1.1). Therefore, we just obtain the sufficient condition for the local and global asymptotic stability of the prey-free periodic solution and the permanence of the system, and the next step is to get the necessary and sufficient condition bout them. As both the environmental noise [32–35]and regime switching [36, 37]are important factors which affect the balancing of the prey-predator populations in the ecosystem [38–40], subsequently we will consider these factors in our next stage of research work.

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Conflict of interest

The authors declare there is no conflict of interest.

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