Mathematical Biosciences and Engineering

http://www.aimspress.com/journal/MBE

MBE, 17(5): 6045–6063.
DOI: 10.3934/mbe.2020321
Received: 05 July 2020
Accepted: 30 July 2020
Published: 14 September 2020

Research article

Soft pre-rough sets and its applications in decision making

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Abstract: Soft rough set model represents a different mathematical model to which many real-life data can be connected. In fact, this theory represents a link between soft set and rough set theories. The main goal of the present paper is to introduce a new approach to modify and generalize soft rough sets. We are discussing and exploring the basic properties for these approaches. In addition, we use the suggested approaches as a mathematical modeling for an uncertain data and deal with the ambiguity. Comparisons among the proposed methods and the previous one are obtained. Finally, a medical application of the suggested approximations in decision making of diagnosis of COVID-19 is illustrated. Moreover, we develop an algorithm following these concepts and apply it to a decision making problem to demonstrate the applicability of the proposed methods.

Keywords: soft sets; soft rough sets; soft pre-rough sets; decision making

1. Introduction

There are different methods of mathematical modeling for uncertainty and vagueness of data such as rough set theory [1], fuzzy set theory [2] and soft set theory [3]. Z. Pawlak [1] introduced the classical rough set models in the early of the eighties as a modern role for modeling the vagueness of data that collected from real-life problems. The core of this approach is an equivalence relation which is constructed from the data of an information system. But it seems that this relation restricts the applications, so different methods are introduced in many proposals to remove these restrictions such as similarity (reflexive and symmetric) relations [4], pre-order (reflexive and transitive) relations [5], reflexive relations [6], binary relations [7–9], topological approaches [10–13] and coverings [14–17]. Soft set represents a different mathematical model to deal with the uncertainty in data collected from real-life situations. This concept was introduced for the first time by

Molodtsov [3] in (1999) which is a free from any extra restrictions and then many different applications have been applied by soft set theory, such as "game theory, operations research, Riemann integration and theory of measurement. Recently, the theory of soft set becomes very widespread among scientists around the world and one of the most developing tools to handle uncertainty in various fields such as "information theory [18], computer sciences [19], engineering [20], medical sciences [21,22], and economy [19,23]". For more details about soft sets and its applications, we refer the reader to the references [24–36].

Undoubtedly the theory of rough sets differs than soft set theory, since Pawlak's approach requires an equivalence relation among the members of the set under investigation. But, in many daily life situations many real-life problems do not always involve crisp data, so such an equivalence relation is very difficult to find due to imprecise human knowledge. The limitations mentioned above which associated with these theories are due to lack of parameterization tools. In order to understand this different, let us consider the following information system (Table 1).

Table 1. An information system.

OB \ AT	a_1	a_2	$\cdots$	a_m
x_1	v_{11}	v_{12}	$\cdots$	v_{1m}
x_2	v_{21}	v_{22}	$\cdots$	v_{2m}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_n	v_{n1}	v_{n2}	$\cdots$	v_{nm}

Table 1 represents a simple representation for the structure (OB, AT, VAL, f) called "an information system", where

$OB = \{x_i : i = 1,2,\ldots,n\}$, is a finite set of objects,

$AT = \{a_j : j = 1,2,\ldots,m\}$, is a finite set of attributes,

$VAL = \{v_{ij} : i = 1,2,\ldots,n, j = 1,2,\ldots,m\}$is a finite set of values and the map $f: OB \times AT \to VAL$ is an information function such that: $f(x_i, a_j) = v_{ij}$.

1.1 Rough set approach

According to the information function f, we can classify the objects into disjoint subsets of OB such that the objects that have the same value belonging to the same category and then the mathematical models (categories) obtained by this classification forms a partition over the universe OB. Thus, these categories can be considered as the equivalence classes of an equivalence relations obtained by the following:

$$x_k R_{a_i} x_l \Leftrightarrow f(x_k, a_i) = f(x_l, a_i).$$

1.2 Soft set approach

A soft set induced by Table 1, is given by the pair (F, A) over OB, where A represents the set of attributes $AT$and defined by the set $A = \{e_1, e_2, e_3, \ldots, e_i\}$ and $F$is a mapping given by$F: A \to P(OB)$ such that $P(OB)$ is the power set of $OB$and for $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set(F, A). It is clear a soft set is free from any restrictions. So every soft set may be considered as an information system and accordingly we can generate Pawlak's rough sets from soft sets.

To solve many problems that acquired of intelligent systems identified by inadequate information, F. Feng et al. [37] introduced the soft rough sets theory. The used notions of the soft lower (resp. upper) approximations in this theory can be useful to fix the knowledge hidden in the information system and then expressed in the form of a decision making problem. Decision making performs a vital role in our daily life, and this process yields the best alternative among different choices. There are many applications for decision making such as ([38–45]). Decision making problems has been solved by Maji et al. [23]. The same authors also extended the classical soft sets to fuzzy soft sets [47,48]. In fact, F. Feng et al. replaced the equivalence classes by parameterized subsets of a set to provide the purpose of defining the lower and upper approximations of a subset. Although they introduced a new tool for approximating the sets, but these approaches did not satisfy the main properties of classical rough set theory. Accordingly, they were put some conditions to soft rough sets such as (soft set must be full soft set and intersection complete soft set) in order to satisfy the properties.

The main contribution in the present work is to introduce another model to soft rough sets without any restrictions and satisfy the properties of Pawlak's rough set theory. In fact, we suggest new tools to approximate the sets called "Soft pre-rough approximations". The properties of these approximations are studied and their relationships are examined with counter examples. We illustrate that the proposed approaches satisfy most properties of Pawlak's rough sets which are never held in F. Feng et al. [37]. Comparisons between the current method and the previous one are obtained. Several examples are given to evidence the connections between the soft rough sets and soft pre-rough sets. In addition, in section 4, two practical examples are introduced to illustrate the importance of the suggested approximations and to show the comparisons between the current method and the methods in [1] and [37]. Also, the second example shows that the proposed approach compatible with Pawlak's rough sets and more accurate than this approach, but Feng [37] does not compatible with Pawlak's rough sets.

Finally, we illustrate the importance of the proposed approach in medical science for application in decision making problems. In fact, a medical application in decision making for information system of medical diagnosis of COVID-19 (Corona Virus) disease is presented with algorithm. Overall we think this work supply a readable frame work to the respective areas with interesting applications such as COVID-19. In addition, we can say that the suggested algorithm represents an easy tool to find the optimal solution comparatively an easier and faster way than the existed algorithms.

2. Preliminaries

In this section some basic definitions and results that used in sequel are mentioned.

2.1. Pawlak rough set theory

In 1982, Pawlak [1] introduced the theory of rough set as a new mathematical methodology or easy tools in order to deal with the vagueness in knowledge-based systems, information systems and data dissection. This theory has many applications in many fields that are used to process control, economics, such as medical diagnosis, chemistry, psychology, finance, marketing, biochemistry, environmental science, intelligent agents, image analysis, biology, conflict analysis, telecommunication, and other fields.

Definition 2.1.1 [1] Let U be a finite set called universe, and R be an equivalence relation on$U$, we use $\frac{U}{R}$ to denote the family of all equivalence classes of R and $[x]_R$ to denote an equivalence class in R containing an element $x \in U$. Then, the pair (U, R) is called an approximation space and for any $X \subseteq U$, we can define the lower and upper approximation of X by $\underline{R}(X) = \{x \in U: [x]_R \subseteq X\}$ and $\overline{R}(X) = \{x \in U: [x]_R \cap X \neq \emptyset\}$, respectively.

According to Pawlak's definition, X is called an exact (resp. a rough) set if $\underline{R}(X) = \overline{R}(X)$ (resp. if $\underline{R}(X) \neq \overline{R}(X)$).

Definition 2.1.2 [1] Let (U, R) be an approximation space and$X \subseteq U$. Then, the "*boundary*", "*positive*" and "*negative*" regions and the "*accuracy*" of the approximations of $X \subseteq U$are defined respectively by:

$$BND_R(X) = \overline{R}(X) - \underline{R}(X),$$

$$POS_R(X) = \underline{R}(X),$$

$$NEG_R(X) = U - \overline{R}(X) \text{ and}$$

$$\mu_R(X) = \frac{|\underline{R}(X)|}{|\overline{R}(X)|} \text{ where } |\overline{R}(X)| \neq \emptyset.$$

Remark 2.1.1

(i) If the boundary region of X is empty ($BND_R(X) = \emptyset$), then $X$is crisp (or exact) with respect to$R$; in the opposite case, if $BND_R(X) \neq \emptyset$, then$X$is said to be rough (or inexact) with respect to R.

(ii) Note that sometimes the pair $(\overline{R}(X),\ \underline{R}(X))$ is also referred to as the rough set of $X$with respect to R.

(iii) If $X \subseteq U$ is defined by a predicate P and $x \in U$, we have the following interpretation:

- $x \in POS_R(X)$, means that x certainly has property P.
- $x \in BND_R(X)$, means that $x$possibly has property P.
- $x \in NEG_R(X)$, means that x definitely does not have property P.

Proposition 2.1.1 [1] Let $\emptyset$ be the empty set and X^c be the complement of $X \subseteq U$. Pawlak's rough set approximations have the following properties:

(L1) $\underline{R}(X) \subseteq X$.	(U1) $X \subseteq \overline{R}(X)$.
(L2) $\underline{R}(\emptyset) = \emptyset$.	(U2) $\overline{R}(\emptyset) = \emptyset$.
(L3) $\underline{R}(U) = U$.	(U3) $\overline{R}(U) = U$.
(L4)$\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$.	

(L5) If $X \subseteq Y$, then $\underline{R}(X) \subseteq \underline{R}(Y)$.	(U4) $\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$.
(L6)$\underline{R}(X) \cup \underline{R}(Y) \subseteq \underline{R}(X \cup Y)$.	(U5) If $X \subseteq Y$, then $\overline{R}(X) \subseteq \overline{R}(Y)$.
(L7) $\underline{R}(X^c) = (\overline{R}(X))^c$.	(U6) $\overline{R}(X) \cap \overline{R}(Y) \supseteq \overline{R}(X \cap Y)$.
(L8) $\underline{R}(\underline{R}(X) = \underline{R}(X)$.	(U7) $\overline{R}(X^c) = (\underline{R}(X))^c$.
(L9) If $X \in \frac{U}{R}$, then $\underline{R}(X) = X$.	(U8) $\overline{R}(\overline{R}(X)) = \overline{R}(X)$.
	(U9) If $X \in \frac{U}{R}$, then $\overline{R}(X) = X$.

2.2. Soft set theory and soft rough sets

Definition 2.2.1 [3] Let U be a non-empty set called "universe" and E the set of certain parameters in relation to the objects in U. A pair (F, A) is called a "*soft set*" over U, where $A \subseteq E$ and F is a mapping given by $F: A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A, F(a)$ may be considered as the set of a-approximate elements of the soft set (F, A).

Here note that for each $e \in E$, $F(e)$ is a crisp set. Thus the soft set (F, A) is called a standard soft set. In [23] Maji et al. defined a fuzzy soft set where, $F(e)$ is a fuzzy subset of U, for each parameter $e \in E$.

Definition 2.2.2 [37] Let $S = (F, A)$ be a soft set over U. Then the pair $A_s = (U, S)$ is called a soft approximation space. Based on the soft approximation space A_s, we define the "*soft A_s-lower and soft A_s-upper*" approximations of any subset $X \subseteq U$ respectively by the following two operations:

$$\underline{S}(X) = \{u \in U: \exists e \in A, [u \in F(e) \subseteq X]\}, \text{ and}$$

$$\overline{S}(X) = \{u \in U: \exists e \in A, [u \in F(e), F(e) \cap X \neq \emptyset]\}.$$

In general, we refer to $\underline{S}(X)$ and $\overline{S}(X)$ as soft rough approximations of $X \subseteq U$ with respect to A_s. Moreover, the sets

$$POS_{A_s}(X) = \underline{S}(X), NEG_{A_s}(X) = U - \overline{S}(X) = (\overline{S}(X))^c, \text{ and } BND_{A_s}(X) = \overline{S}(X) - \underline{S}(X).$$

are called the "*soft A_s-positive region, the soft A_s-negative region* and the *soft A_s-boundary*" region of $X \subseteq U$, respectively.

Clearly, if $\overline{S}(X) = \underline{S}(X), i.e. BND_{A_s}(X) = \emptyset$. Then $X \subseteq U$ is said to be "*soft A_s-definable*" or "*soft A_s-exact*" set; otherwise X is called a "*soft A_s-rough*" set.

Moreover, we can define the accuracy of the approximations as follows:

$$\mu_{A_s}(X) = \frac{|\underline{S}(X)|}{|\overline{S}(X)|}, \text{ where } \overline{S}(X) \neq \emptyset.$$

$\mu_{A_s}(X)$ is called the "*soft A_s-accuracy*" of $X \subseteq U$.

Proposition 2.2.1 [37] Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ a soft approximation space. Then, for each $X \subseteq U$:

$$\underline{S}(X) = \bigcup_{e\in A}\{F(e): F(e) \subseteq X\} \text{and } \overline{S}(X) = \bigcup_{e\in A}\{F(e): F(e) \cap X \neq \emptyset\}.$$

Proposition 2.2.2 [37] Let $S = (F, A)$ be a soft set over $U$and $A_s = (U, S)$ a soft approximation space. Then, the soft A_s-lower and A_s-upper approximations of $X \subseteq U$ satisfy the following properties:

(i)	$\underline{S}(\emptyset) = \overline{S}(\emptyset) = \emptyset$.	(v)	$\overline{S}(X \cup Y) = \overline{S}(X) \cup \overline{S}(Y)$.
(ii)	$\underline{S}(U) = \overline{S}(U) = \bigcup_{e\in A} f(e)$.	(vi)	$\overline{S}(X \cap Y) \subseteq \overline{S}(X) \cap \overline{S}(Y)$.
(iii)	$\underline{S}(X \cap Y) \subseteq \underline{S}(X) \cap \underline{S}(Y)$.	(vii)	If $X \subseteq Y$, then $\underline{S}(X) \subseteq \underline{S}(Y)$.
(iv)	$\underline{S}(X \cup Y) \supseteq \underline{S}(X) \cup \underline{S}(Y)$.	(viii)	If $X \subseteq Y$, then $\overline{S}(X) \subseteq \overline{S}(Y)$.

Proposition 2.2.3 [37] Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ a soft approximation space. Then, for each $X \subseteq Y$:

(i)	$\underline{S}(\underline{S}(X)) = \underline{S}(X)$.	(iii)	$\underline{S}(X) \subseteq \overline{S}(\underline{S}(X))$.
(ii)	$\overline{S}(\overline{S}(X)) \supseteq \overline{S}(X)$.	(iv)	$\underline{S}(\overline{S}(X)) = \overline{S}(X)$.

Definition 2.2.3 [37] Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ a soft approximation space. Then, $S$is said to be a "*full soft set*" if $U = \bigcup_{e\in A} F(e)$.

It is clear that if S is a full soft set, then $\forall x \in U, \exists e \in A$ such that $x \in F(e)$.

Proposition 2.2.4 [37] Let $S = (F, A)$ be a full soft set over U and $A_s = (U, S)$ a soft approximation space. Then, the following conditions are true:

(i) $\underline{S}(U) = \overline{S}(U) = U$.

(ii) $X \subseteq \overline{S}(X), \forall X \subseteq U$.

(iii) $\overline{S}(\{x\}) \neq \emptyset, \forall x \in U$.

Definition 2.2.4 Let $S = (F, A)$ be a full soft set over U, $A_s = (U, S)$ a soft approximation space and $X \subseteq U$. Then, we define the following four basic types of soft rough sets:

- X is roughly soft A_s-definable if $\underline{S}(X) \neq \emptyset$ and $\overline{S}(X) \neq U$.
- X is internally soft A_s-indefinable if $\underline{S}(X) = \emptyset$ and $\overline{S}(X) \neq U$.
- X is externally soft A_s-indefinable if $\underline{S}(X) \neq \emptyset$ and $\overline{S}(X) = U$.
- X is totally soft A_s-indefinable if $\underline{S}(X) = \emptyset$ and $\overline{S}(X) = U$.

3. Generalized soft rough approximations

In this section, we define new generalized soft rough approximations so-called "*Soft pre-rough approximations*". The properties of suggested approaches are superimposed. Relationships among the current approaches and previous one in F. Feng, et al. [37] are obtained. Many examples and counter examples are introduced. We will prove that the proposed method is a generalization and more accurate than [37].

Definition 3.1 Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ a soft approximation space. Then, the soft "*pre-lower*" and "*pre-upper*" approximations of any subset $X \subseteq U$ are defined respectively, by:

$$\underline{S}_p(X) = X \cap \underline{S}(\overline{S}(X)) \text{ and } \overline{S}_p(X) = X \cup \overline{S}(\underline{S}(X)).$$

In general, we refer to $(\underline{S}_p(X), \overline{S}_p(X))$ as "soft pre-rough approximations" with respect to A_s.

Definition 3.2 Let $A_s = (U, S)$ be a soft approximation space and $X \subseteq U$. Then, the soft "*pre-positive*, *pre-negative*, *pre-boundary*" regions and the "*pre-accuracy*" of the soft pre-approximations are defined respectively by:

$$POS_p(X) = \underline{S}_p(X), NEG_p(X) = U - \overline{S}_p(X), BND_p(X) = \overline{S}_p(X) - \underline{S}_p(X), \text{ and}$$

$$\mu_p(X) = \frac{|\underline{S}_p(X)|}{|\overline{S}_p(X)|}, \text{ where } \overline{S}_p(X) \neq \emptyset.$$

Clearly, if $\overline{S}(X) = \underline{S}(X), i.e. BND_{A_s}(X) = \emptyset$ and $\mu_p(X) = 1$. Then $X \subseteq U$ is said to be "*soft pre-definable*" or "*soft pre-exact*" set; otherwise X is called a "*soft pre*-rough" set.

The main goal of the following results is to introduce and superimposed the basic properties of soft pre-rough approximations $\underline{S}_p$ and $\overline{S}_p$.

Proposition 3.1 Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ a soft approximation space. Then, the soft pre-lower and pre-upper approximations of $X \subseteq U$ satisfy the following properties:

(i) $\underline{S}_p(\emptyset) = \overline{S}_p(\emptyset) = \emptyset$.	(v) $\underline{S}_p(X \cap Y) \subseteq \underline{S}_p(X) \cap \underline{S}_p(Y)$.
(ii) $\underline{S}_p(U) = \bigcup_{e \in A} f(e)$ and $\overline{S}_p(U) = U$.	(vi) $\underline{S}_p(X \cup Y) \supseteq \underline{S}_p(X) \cup \underline{S}_p(Y)$.
(iii) If $X \subseteq Y$, then $\underline{S}_p(X) \subseteq \underline{S}_p(Y)$.	(vii) $\overline{S}_p(X \cap Y) \subseteq \overline{S}_p(X) \cap \overline{S}_p(Y)$.
(iv) If $X \subseteq Y$, then $\overline{S}_p(X) \subseteq \overline{S}_p(Y)$.	(viii) $\overline{S}_p(X \cup Y) = \overline{S}_p(X) \cup \overline{S}_p(Y)$.

Proof:

(i) Since $\underline{S}(\emptyset) = \overline{S}(\emptyset) = \emptyset$. Then $\underline{S}_p(\emptyset) = \emptyset \cap \underline{S}(\overline{S}(\emptyset)) = \emptyset$ and $\overline{S}_p(\emptyset) = \emptyset \cup \overline{S}(\underline{S}(\emptyset)) = \emptyset$.

(ii) Since $\underline{S}(U) = \overline{S}(U) = \bigcup_{e \in A} f(e)$, then $\underline{S}_p(U) = U \cap \underline{S}(\overline{S}(U)) = U \cap \underline{S}(\bigcup_{e \in A} f(e)) = \bigcup_{e \in A} f(e)$ and $\overline{S}_p(U) = U \cup \overline{S}(\underline{S}(U)) = U \cup \overline{S}(\bigcup_{e \in A} f(e)) = U$.

(iii) Since $\underline{S}(X) \subseteq \underline{S}(Y)$ and $\overline{S}(X) \subseteq \overline{S}(Y)$ for each $X \subseteq Y$. Then, for each $X \subseteq Y$:

$\underline{S}_p(X) = X \cap \underline{S}(\overline{S}(X)) \subseteq Y \cap \underline{S}(\overline{S}(Y)) = \underline{S}_p(Y)$.

(iv) By similar way as (iii).
(v) Since $X \cap Y \subseteq X$ and $X \cap Y \subseteq Y$. Then, by (iii), $\underline{S}_p(X \cap Y) \subseteq \underline{S}_p(X) \cap \underline{S}_p(Y)$.
(vi) Since $X \subseteq X \cup Y$ and $Y \subseteq X \cup Y$. Then, by (iv), $\underline{S}_p(X \cup Y) \supseteq \underline{S}_p(X) \cup \underline{S}_p(Y)$.
(vii) By similar way as (v).
(viii) By using (v)-(vii), the proof is obvious.

Proposition 3.2 Let $S = (F, A)$ be a soft set over $U$and $A_s = (U, S)$ a soft approximation space. Then, for each $X \subseteq Y$:

(i) $\underline{S}_p(\underline{S}_p(X)) = \underline{S}_p(X)$.	(iii) $\underline{S}_p(X) \subseteq \overline{S}_p(\underline{S}_p(X))$.
(ii) $\overline{S}_p(X) \subseteq \overline{S}_p(\overline{S}_p(X))$.	(iv) $\underline{S}_p(\overline{S}_p(X)) \subseteq \overline{S}_p(X)$.

Proof: Straightforward.

Remark 3.1 Note that the inclusion relations in Proposition 3.2 may be strict, as shown in Example 3.1 and Example 3.2.

Example 3.1 Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ a soft approximation space, where, $U = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2, e_3, \dots, e_6\}$ and $A = \{e_1, e_2, e_3\} \subseteq E$ such that $(F, A) = \{(e_1, \{x_1, x_4\}), (e_2, \{x_3\}), (e_3, \{x_2, x_3, x_4\})\}$. Now, let $X = \{x_3, x_4\}$. Then, we get $\overline{S}_p(X) = \{x_2, x_3, x_4\}$ which implies $\overline{S}_p(\overline{S}_p(X)) = U$. Hence, $\overline{S}_p(X) \neq \overline{S}_p(\overline{S}_p(X))$.

Example 3.2 Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ be a soft approximation space, where $U = \{x_1, x_2, x_3, \dots, x_6\}$, $E = \{e_1, e_2, e_3, \dots, e_6\}$and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$ such that $(F, A) = \{(e_1, \{x_1, x_6\}), (e_2, \{x_3\}), (e_3, \emptyset), (e_4, \{x_1, x_2, x_5\})\}$. Now let $X = \{x_1, x_6\}$ and $Y = \{x_3, x_4, x_5\}$. Then, we get $\underline{S}_p(X) = \{x_1, x_6\}$ which implies $\overline{S}_p(\underline{S}_p(X)) = \{x_1, x_2, x_5, x_6\}$. Hence, $\underline{S}_p(X) \neq \overline{S}_p(\underline{S}_p(X))$. Also, $\overline{S}_p(Y) = \{x_3, x_4, x_5\}$ and this means that $\underline{S}_p(\overline{S}_p(Y)) = \{x_3\}$. Hence, $\underline{S}_p(\overline{S}_p(Y)) \neq \overline{S}_p(Y)$.

Proposition 3.3 Let $S = (F, A)$ be a full soft set and $A_s = (U, S)$ be a soft approximation space. Then:

(i) $\underline{S}_p(U) = U$.	(ii) $\underline{S}_p(\overline{S}_p(X)) = \overline{S}_p(X), \forall X \subseteq U$.

Proof:

(i) Let $S = (F, A)$ be a full soft set, then by Proposition 2.2.4, we get:

$$\underline{S}_p(U) = U \cap \underline{S}(\overline{S}(U)) = U \cap \underline{S}(U) = U \cap U = U.$$

(ii) Firstly, by Proposition 3.2, we get: $\underline{S}_p(\overline{S}_p(X)) \subseteq \overline{S}_p(X), \forall X \subseteq U$. Thus, we must prove the inverse relation $\overline{S}_p(X) \subseteq \underline{S}_p(\overline{S}_p(X))$ as follows:

Let $S = (F, A)$ be a full soft set, then by Proposition 2.2.4, we get:

$X \subseteq \overline{S}(X), \forall X \subseteq U$and by Proposition 2.2.3, we get: $\underline{S}(\overline{S}(X)) = \overline{S}(X), \forall X \subseteq U$.

Thus, $X \subseteq \underline{S}(\overline{S}(X)), \forall X \subseteq U$ and since $\underline{S}(X) \subseteq X, \forall X \subseteq U$. Then $\overline{S}(\underline{S}(X)) \subseteq \overline{S}(X)$ and this implies $\overline{S}_p(X) = [X \cup \overline{S}(\underline{S}(X))] \subseteq \underline{S}(\overline{S}(X))$. Accordingly, $\overline{S}_p(X) \subseteq \underline{S}(\overline{S}(\overline{S}_p(X)))$ and this means that$\overline{S}_p(X) \subseteq \underline{S}_p(\overline{S}_p(X)), \forall X \subseteq U$.

Remark 3.2 Propositions 3.1, 3.2 and 3.3 represent one of the deviations between the proposed method and F. Feng, et al. [37] approach. In fact, according to these propositions, the suggested approximations satisfied most properties of Pawlak's rough sets and then Table 2 summarize these properties which represents a first comparison among soft pre-approximation and soft rough approximations [37].

Table 2. Properties of soft rough and soft pre-rough approximations.

	$\underline{S}$	$\underline{S}_p$		$\overline{S}$	$\overline{S}_p$
L1	*1*	*1*	***U1***	*1*	*1*
L2	*1*	*1*	***U2***	*1*	*1*
L3	*0*	*0*	***U3***	*0*	*1*
L4	*0*	*0*	***U4***	*1*	*1*
L5	*1*	*1*	***U5***	*1*	*1*
L6	*1*	*1*	***U6***	*1*	*1*
L7	*1*	*1*	***U7***	*1*	*1*
L8	*1*	*1*	***U8***	*1*	*1*
L9	*1*	*1*	***U9***	*1*	*1*

Note that: The number "1" denotes "yes" and "0" denotes "no".

The main goal of the following results is to illustrate the relationship between soft rough approximations [37] and soft pre-rough approximations (current methods).

Theorem 3.1 Let $A_s = (U, S)$ be a soft approximation space and $X \subseteq U$. Then:

(i) $\underline{S}(X) \subseteq \underline{S}_p(X)$.

(ii) $\overline{S}_p(X) \subseteq \overline{S}(X)$.

Proof: We will prove the first statement and the other similarly.

Let$x \notin \underline{S}_p(X)$. Then, either $x \notin X$or $x \notin \underline{S}(\overline{S}(X))$ and this implies:

Case (1): $x \notin X \Rightarrow x \notin \underline{S}(X)$.

Case (2): $x \notin \underline{S}(\overline{S}(X)) \Rightarrow \exists e \in E$, such that $x \in F(e)$ and$F(e) \not\subset \overline{S}(X)$. Thus, $x \in F(e)$ and $F(e) \not\subset \underline{S}(X)$ which means that $x \notin \underline{S}(X)$. Accordingly, $\underline{S}(X) \subseteq \underline{S}_p(X), \forall X \subseteq U$.

Corollary 3.1 Let $A_s = (U, S)$ be a soft approximation space and $X \subseteq U$. Then,

(i) $BND_p(X) \subseteq BND_{A_s}(X)$.

(ii) $\mu_{A_s}(X) \leq \mu_p(X)$.

Corollary 3.2 Let $A_s = (U,S)$ be a soft approximation space and $X \subseteq U$. If X is a soft exact set, then it is a soft pre-exact set.

Remark 3.3 The converse of the above results is not true in general as Example 3.3 illustrated.

Example 3.3 Consider Example 3.2, and let $X = \{x_3, x_4, x_5\}$ and $Y = \{x_3, x_6\}$. Thus $\underline{S}(X) = \{x_3\}$ and $\overline{S}(X) = \{x_1, x_2, x_3, x_5\}$. But $\underline{S_p}(X) = \{x_3, x_5\}$ and $\overline{S}_p(X) = \{x_3, x_4, x_5\}$. It is clear that $\underline{S}(X) \subseteq \underline{S_p}(X)$ and $\overline{S}_p(X) \subseteq \overline{S}(X)$. Moreover, $X \not\subset \overline{S}(X)$. But, $\underline{S_p}(X) \subseteq X \subseteq \overline{S}_p(X)$. Similarly, $\underline{S}(Y) = \{x_3\}$, $\overline{S}(Y) = \{x_1, x_3, x_6\}$ and then $BND_{A_s}(Y) = \{x_1, x_6\}$ and $\mu_{A_s}(Y) = \frac{1}{3}$. But $\underline{S_p}(Y) = \overline{S}_p(Y) = Y$ and then $BND_p(Y) = \emptyset$ and $\mu_p(Y) = 1$. It is clear that Y is a soft pre-exact in our approach although it is a soft rough with respect to [37].

According to Theorem 3.1, we define the following important definition:

Definition 3.4 Let $S = (F, A)$ be a full soft set over U, $A_s = (U,S)$ a soft approximation space and $X \subseteq U$. Then, we define the following four basic types of soft pre-rough sets:

- X is roughly soft pre-definable if $\underline{S_p}(X) \neq \emptyset$ and $\overline{S}_p(X) \neq U$.
- X is internally soft pre-indefinable if $\underline{S_p}(X) = \emptyset$ and $\overline{S}_p(X) \neq U$.
- X is externally soft pre-indefinable if $\underline{S_p}(X) \neq \emptyset$ and $\overline{S}_p(X) = U$.
- X is totally soft pre-indefinable if $\underline{S_p}(X) = \emptyset$ and $\overline{S}_p(X) = U$.

The intuitive meaning of this classification is as follows:

- If X is roughly soft pre-definable, this means that we are able to decide for some elements of U that they belong to X, and meanwhile for some elements of U, we are able to decide that they belong to X^c, by using available knowledge from a soft approximation space A_s.
- If X is internally soft pre-indefinable, this means that we are able to decide about some elements of U that they belong to X^c, but we are unable to decide for any element of U that it belongs to X, by employing A_s.
- If X is externally soft pre-indefinable, this means that we are able to decide for some elements of U that they belong to X, but we are unable to decide, for any element of U that it belongs to X^c, by employing A_s.
- If X is totally soft pre-indefinable, we are unable to decide for any element of U whether it belongs to X or X^c, by employing A_s.

Theorem 3.2 Let $A_s = (U,S)$ be a soft approximation space and $X \subseteq U$. Then:

(i) If X is roughly soft pre-definable, then X is roughly soft A_s-definable.
(ii) If X is internally soft pre-definable, then X is internally soft A_s-indefinable.
(iii) If X is externally soft pre-definable, then X is externally soft A_s-indefinable.
(iv) If X is totally soft pre-indefinable, then X is totally soft A_s-indefinable.

Proof: By Theorem 3.1, the proof is obvious.

Remark 3.4

(i) Theorem 3.2 represents another difference between soft rough approximations [37] and soft pre-rough approximations (current methods). Moreover, this theorem shows the importance of our approaches in defining the sets, for example: if X is totally soft A_s-indefinable, then $\underline{S}(X) = \emptyset$ and $\overline{S}(X) = U$ and this means that we are unable to decide for any element of U whether it belongs to X or X^c. But, by using soft pre-rough approximations, $\underline{S_p}(X) \neq \emptyset$ and $\overline{S}_p(X) \neq U$ and then X can be roughly soft pre-definable which means that we are able to decide for some elements of U that they belong to X, and meanwhile for some elements of U, we are able to decide that they belong to X^c, by using available knowledge from the soft approximation space A_s (Example 3.2 and 4.1 illustrate this fact).

(ii) The converse of Theorem 3.3 is not true in general as Example 4.1 illustrated.

4. A decision making for information system

In the present section, we introduce two practical examples as applications of the current approaches in decision making for information system. First example represents a set valued information system of food nutrients for students in a one of school. In the second example, we used the interesting example that given in [37] to illustrate the importance of the suggested methods and thus we obtain a comparison between our approaches and the previous one such as Pawlak approach [1] and F. Feng et al. [37] approach.

Example 4.1 A set valued information system is presented in Table 4, where $U = \{S_1, S_2, S_3, \ldots, S_6\}$ of students, $E = \{e_1 =$ Food contains preservatives, $e_2 =$ Carbohydrates, $e_3 =$Protein, $e_4 =$Vitamins, $e_5 =$Fats, $e_6 =$Minerals, $e_7 =$Junk food, $e_8 =$Ice-cream$\}$ be a set of parameters which illustrate food nutrients for students.

Consider the soft set (F, E) which describes the attractiveness of the students given by $(F, E) = \{$Students eat food containing "Food containing preservatives"$= \emptyset$, Students eat food containing "Carbohydrate" $= U$, Students eat food containing "Protein"$= \{S_1, S_2, S_3, S_4, S_6\}$, Students eat food containing "Vitamins" $= U$, Students eat food containing "Fat" $= \{S_1, S_3, S_6\}$, Students eat food containing "Minerals"$= \{S_1, S_2, S_6\}$, Students eat "Junk food" $= \{S_2, S_4, S_5\}$, Students eat "Ice-cream"$= \{S_1, S_3, S_6\}$. Suppose that, Mr. X is interested to buy food on the basis of his choice parameters $\{e_1, e_3, e_6, \ldots, e_8\}$ which constitute the subset $A = \{e_2, e_3, e_4, e_5, e_6\}$ of the set E. That means, out of available food in U, he is to select that food which qualifies with all (or with maximum number of) parameters of the soft set. The problem is to select the food which is a suitable choice of parameters set by Mr. X. Let us first make a tabular representation of the problem. Consider the soft set (F, A) where A is the choice parameter of Mr. X as in Table 3. Here (F, A) is a soft subset of (F, E) such that:

$$(F, E) = \{(e_2, U), (e_3, \{S_1, S_2, S_3, S_4, S_6\}), (e_4, U), (e_5, \{S_1, S_3, S_6\}), (e_6, \{S_1, S_2, S_6\})\}.$$

Thus, we can represent the soft set by tabular representation as shown in Table 3.

Now, we will calculate the approximations of some subsets using F. Feng method [37] and our method as shown in Table 4.

Table 3. Food Information System.

Students	e_2	e_3	e_4	e_5	e_6
S_1	1	1	1	1	1
S_2	1	1	1	0	1
S_3	1	1	1	1	0
S_4	1	1	1	0	0
S_5	1	0	1	0	0
S_6	1	1	1	1	1

Table 4. Comparisons among the soft rough approximations [37] and soft pre-rough approximations (Current methods).

X	**Feng method in Definition 2.2.4**				**Current method in Definition 3.1**			
	$\underline{S}(X)$	$\overline{S}(X)$	$BND_{A_s}(X)$	$\mu_{A_s}(X)$	$\underline{S}_p(X)$	$\overline{S}_p(X)$	$BND_p(X)$	$\mu_p(X)$
$\{S_2, S_3\}$	$\emptyset$	U	U	0	$\{S_2, S_3\}$	$\{S_2, S_3\}$	$\emptyset$	1
$\{S_1, S_3, S_6\}$	$\emptyset$	U	U	0	$\{S_1, S_3, S_6\}$	$\{S_1, S_3, S_6\}$	$\emptyset$	1
$\{S_1, S_2, S_3, S_6\}$	$\emptyset$	U	U	0	$\{S_1, S_2, S_3, S_6\}$	$\{S_1, S_2, S_3, S_6\}$	$\emptyset$	1
$\{S_2, S_3, S_4, S_5\}$	$\emptyset$	U	U	0	$\{S_2, S_3, S_4, S_5\}$	$\{S_2, S_3, S_4, S_5\}$	$\emptyset$	1

Remark 4.1 From Table 4, we can notice the following:

(i) There are many subsets which are soft rough (totally soft rough indefinable), but they are soft pre-exact (totally soft pre-definable). The suggested approaches "soft pre-rough approximations" represent the best tools for approximating the sets since by using them, the boundary regions decreased (or canceled). Moreover, the accuracy of the soft pre-approximations is more accurate than F. Feng et al. approach [37].

(ii) According to the results in Section 3 (Theorem 3.1 and Corollary 3.1 & 3.2), we can say that the proposed method is more accurate than [37] in decision making and accordingly these method is very useful in real-life applications.

Thus, we conclude that this approach represents a best tool to identify the best feeding system for students. Accordingly, using the soft pre-rough approximations, we can study the properties of the feeding system of students and give the accuracy of decision making.

For example, assume that Mr. X wants to choice a best feeding system that contains some of the basic elements in food and then according to the soft set (F, E) make their initial decision. Let Mr. X points out that $(F, B) = \{F(e_2), F(e_3), F(e_4), F(e_5)\}$ is the best choice for feeding system that contains the main components $\{e_2, e_3, e_4, e_5\}$. Then, according to Table 3, the set of students $Y = \{S_1, S_3, S_6\}$ satisfy this feeding system (F, B) by choice of Mr. X. Thus, we can make the following computations (Table 5) to compare among [37] method and our method in decision making.

From the above results of Table 5, we conclude that: Using soft pre-rough approximation (Current method), Mr. X is most likely (or must) to buy the food in $POS_p(Y) = Y$ and then he is sure for his choice for these students. In addition, he also is sure that his choice food $(F, B) = \{F(e_2), F(e_3), F(e_4), F(e_5)\}$ will not be taken outside the range of Y since to the students $NEG_p(Y) = \{S_2, S_4, S_5\}$ which represents the students are lie out of his choice. Thus, Mr. X's initial demand for food is analyzed and delineated by using the concept of soft pre-rough tools and related soft computing techniques. On the other hand, if we use the soft rough approximations [37], Mr. X

can not to decide the best food or the food never be taken exactly (Because $\underline{S}(Y) = \emptyset$ and $\overline{S}(Y) = U$, that is Yhere is totally soft rough set. In addition, the soft positive $POS_{A_s}(Y) = NEG_{A_s}(Y) = \emptyset$ and the soft accuracy measure is $\mu_{A_s}(Y) = 0$.

Table 5. Comparison between soft approximations of Y using [37] and current method.

	F. Feng et al. [37]	**Soft pre-rough (Current method)**
Soft lower	$\emptyset$	$\{S_1, S_3, S_6\}$
Soft upper	U	$\{S_1, S_3, S_6\}$
Soft boundary region	U	$\emptyset$
Soft positive region	$\emptyset$	$\{S_1, S_3, S_6\}$
Soft negative region	$\emptyset$	$\{S_2, S_4, S_5\}$
Soft accuracy	0	1

Example 4.2 [37] Let us consider the following soft set $S = (F, E)$ which describes ‘‘life expectancy’’. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ consists of six persons and $E = \{e_1, e_2, e_3, e_4\}$ is a set of decision parameters. The e_i $(i = 1,2,3,4)$ stands for ‘‘under stress’’, ‘‘young’’, ‘‘drug addict’’ and ‘‘healthy’’. Set $F(e_1) = \{u_5\}, F(e_2) = \{u_1, u_2\}, F(e_3) = \emptyset$ and $F(e_4) = \{u_1, u_2, u_3, u_6\}$. Thus the soft set (F, E) is given by:

$(F, E) = \{(\text{under stress}, \{u_5\}), (\text{young}, \{u_1, u_2\}), (\text{drug addict}, \emptyset), (\text{healthy}, \{u_1, u_2, u_3, u_6\})\}$.

Table 6. An information table.

	u_1	u_2	u_3	u_4	u_5	u_6
Sex	Woman	Woman	Man	Man	Man	Man
Age category	Young	Young	Mature age	Old	Mature age	Baby
Living area	City	City	City	Village	City	Village
Habits	NSND	NSND	Smoke	SD	Smoke	NSND

By other words, "life expectancy" topic can be presented using rough sets as follows: The estimate will be done in codes of attributes: "sex", "age category", "living area", "habits", stands by the value sets "{man, woman}","{baby, young, mature age, old}", "{village, city}" and "{smoke, drinking, smoke and drinking, no smoke and no drinking}". We denote "smoke and drinking" by SD and "no smoke and no drinking" by NSND. Table 4, gives the information such that the rows represent the attributes and the table entries are the different values of attributes for every person.

Therefore, we get the following equivalence classes that generated by these attributes: $[u_1]_R = [u_2]_R = \{u_1, u_2\}, [u_3]_R = [u_5]_R = \{u_3, u_5\}, [u_4]_R = \{u_4\}, [u_6]_R = \{u_6\}$.

Thus, we calculate the approximations of some subsets of U by using “Pawlak method [1]”, “F. Feng et al. method [37]” and “Current method” as Table 7 illustrated.

Remark 4.2 From Table 6, we can notice the following:

Table 6. Comparison among the rough approximations (Pawlak [1]), the soft rough approximations (F. Feng et al [37]) and soft pre-rough approximations (Current method).

X	**Pawlak Method**			**Feng Method**		
	$\underline{R}(X)$	$\overline{R}(X)$	$\mu_R(X)$	$\underline{S}(X)$	$\overline{S}(X)$	$\mu_{A_s}(X)$
$\{u_5\}$	$\emptyset$	$\{u_2, u_5\}$	0	$\{u_5\}$	$\{u_5\}$	1
$\{u_6\}$	$\{u_6\}$	$\{u_6\}$	1	$\emptyset$	$\{u_1, u_2, u_3, u_6\}$	0
$\{u_1, u_3\}$	$\emptyset$	$\{u_1, u_2, u_3, u_5\}$	0	$\emptyset$	$\{u_1, u_2, u_3, u_6\}$	0
$\{u_2, u_3\}$	$\emptyset$	$\{u_1, u_2, u_3, u_5\}$	0	$\emptyset$	$\{u_1, u_2, u_3, u_6\}$	0
$\{u_2, u_5\}$	$\{u_2, u_5\}$	$\{u_2, u_5\}$	1	$\{u_5\}$	$\{u_1, u_2, u_3, u_5, u_6\}$	$\frac{1}{5}$
$\{u_4, u_5\}$	$\{u_4\}$	$\{u_3, u_4, u_5\}$	$\frac{1}{3}$	$\{u_5\}$	$\{u_5\}$	1
$\{u_1, u_5, u_6\}$	$\{u_1, u_5, u_6\}$	$\{u_1, u_5, u_6\}$	1	$\{u_5\}$	$\{u_1, u_2, u_3, u_5, u_6\}$	$\frac{1}{5}$
$\{u_1, u_2, u_3, u_6\}$	$\{u_1, u_2, u_6\}$	$\{u_1, u_2, u_3, u_5, u_6\}$	$\frac{3}{5}$	$\{u_1, u_2, u_3, u_6\}$	$\{u_1, u_2, u_3, u_6\}$	1
U	U	U	1	$\{u_1, u_2, u_3, u_5, u_6\}$	$\{u_1, u_2, u_3, u_5, u_6\}$	1

X	Current Method		
	$\underline{S}_p(X)$	$\overline{S}_p(X)$	$\mu_p(X)$
$\{u_5\}$	$\{u_5\}$	$\{u_5\}$	1
$\{u_6\}$	$\{u_6\}$	$\{u_6\}$	1
$\{u_1, u_3\}$	$\{u_1, u_3\}$	$\{u_1, u_3\}$	1
$\{u_2, u_3\}$	$\{u_2, u_3\}$	$\{u_2, u_3\}$	1
$\{u_2, u_5\}$	$\{u_2, u_5\}$	$\{u_2, u_5\}$	1
$\{u_4, u_5\}$	$\{u_4, u_5\}$	$\{u_4, u_5\}$	1
$\{u_1, u_5, u_6\}$	$\{u_1, u_5, u_6\}$	$\{u_1, u_5, u_6\}$	1
$\{u_1, u_2, u_3, u_6\}$	$\{u_1, u_2, u_3, u_6\}$	$\{u_1, u_2, u_3, u_6\}$	1
U	U	U	1

Exact (definable) set (shaded cells)

(i) There are different methods to approximate the sets. The best of them is "soft pre-rough approximations since, by using them, the boundary regions decreased (or canceled) by increasing the lower approximation and decreasing the upper approximation. Moreover, the accuracy of the soft pre-approximations is more accurate than the other accuracy measures such as [1] and [37].

(ii) There are many sets which are rough according to [1] and [37], but they are soft pre-exact according to the presented method. Moreover, there are some subsets which are soft rough according to [37] although they are exact in Pawlak, but they are soft pre-exact in our approach. So, we can say that the proposed approaches are useful in removing the vagueness of rough sets. Hence, these approaches will be useful in decision making for extracting the information and help in removing the vagueness of the data in real-life problems.

(iii) The importance of the current approximations is not only that it is reducing or deleting the boundary regions, but also it is satisfying most properties of Pawlak's rough sets without any restrictions. Note that the subset $X = \{u_4, u_5\}$ is not soft definable (according to [37]) although

$\underline{S}(X) = \overline{S}(X)$ and $\mu_{A_s}(X) = 1$since$X \not\subset \overline{S}(X)$. Similarly, the universe set $U$is not soft definable (with respect to [37]) although $\underline{S}(U) = \overline{S}(U)$ and $\mu_{A_s}(U) = 1$, since$\underline{S}(U) \neq U$ and $\overline{S}(U) \neq U$. But these sets are soft pre-exact (definable) in our approach.

(iv) Theorem 3.1 and Corollary 3.1 & 3.2 show that soft pre-rough set approximation is a worth considering alternative to the rough set approximation. Soft pre-rough sets could provide a better approximation than soft rough or rough sets do, depending on the structure of the equivalence classes and of the subsets$F(e)$, where$e \in A$.

5. A medical application for "COVID-19"

Currently, the emergence of a novel human coronavirus, (SARS-CoV-2) or (COVID-19), has become a global health concern causing severe respiratory tract infections in humans. Human-to-human transmissions have been described with incubation times between 2-10 days, facilitating its spread via droplets, contaminated hands or surfaces. According to paper [48], human coronaviruses can remain infectious on inanimate surfaces for up to 9 days. A novel coronavirus (SARS-CoV-2) or (COVID-19) has recently emerged from China with a total of 45171 confirmed cases of pneumonia (as of February 12, 2020) [48]. Most two impact factors for infections transmission namely "Contact with infected surfaces" and "Interactions with infected people" of the virus. In the present section, we introduce an application to show how our approaches are the best tools in decision making about the infection of COVID-19.

Example 5.1 Suppose that the universe $U = \{p_1, p_2, p_3, \dots, p_{10}\}$ consists of ten persons and $A = \{e_1, e_2, e_3, e_4\}$ is a set of attributes parameters, where e_1 stands for ''stay at home'', e_2 stands for ''go out the home and contact with infected people'', e_3 stands for ''work at hospital'', e_4 stands for ''study at home'' and e_5 stands for ''study out the home''. Let (F, A) be a soft set over given by Table 7.

Table 7. Tabular representation for a soft set (F, A).

$\frac{U}{A}$	e_1	e_2	e_3	e_4	e_5	***Infected with*** **COVID-19**
p_1	1	0	0	1	1	0
p_2	0	1	1	0	1	0
p_3	0	1	1	0	1	1
p_4	1	0	1	0	0	1
p_5	1	0	0	1	1	0
p_6	0	0	0	1	1	0
p_7	0	1	1	0	0	1
p_8	1	0	0	1	0	0
p_9	0	1	1	0	1	1
p_{10}	1	0	0	0	1	0

Here 1 and 0 denote "yes" and "no" respectively.

From Table 7, we get the set of infected patient with COVID-19 is $X = \{p_3, p_4, p_7, p_9\}$. Thus:

5.1. According to F. Feng et al. [37] approach

$\underline{S}(X) = \emptyset$ and $\overline{S}(X) = U$ which implies $BND_{\mathcal{A}_S}(X) = U$ and $\mu_{\mathcal{A}_S}(X) = 0$. Thus, X is rough (totally soft $\mathcal{A}_S$-indefinable) set which means that no elements in X or no patients having COVID-19 which contradicts Table 5.1. Accordingly, by using [8], we are unable to decide for any element of U whether it belongs to X or X^c.

5.2. According to current approach

$\underline{S}_p(X) = \overline{S}_p(X) = X$ which implies $BND_p(X) = \emptyset$ and $\mu_p(X) = 1$. Thus, X is exact (totally soft pre-definable) set which means that, by using an available knowledge, we can decide the infected patients with more accurate than the other methods. On the other hand, we can decide whether patients are not infected. Hence, we can say that the suggested approaches are useful in removing the vagueness of the rough sets. Hence, these approaches will be useful in decision making for extracting the information and help in removing the vagueness of the data in real-life problems.

At the end of the paper, we give an algorithm which can be used to have a decision making for information system in terms of the soft pre-approximations.

Algorithm 5.1	**A decision making via Soft pre-rough approximations.**
Step 1:	Input the soft set (F, E).
Step 2:	Input the set A of choice parameters of Mr. X which is a subset of E.
Step 3:	Investigate the soft pre-upper approximation, say, $\overline{S}_p(X)$ and soft pre-lower approximation, say, $\underline{S}_p(X)$, for every $X \subseteq U$. According to **Definition 3.1**.
Step 4:	Determine a boundary region, say, $BND_p(X)$ from **Step 2**, for every$X \subseteq U$. According to **Definition 3.1**.
Step 5:	Calculate the accuracy of the approximation, say, $\mu_p(X)$ by **Step 2**, for every$X \subseteq U$. According to **Definition 3.1**.
Step 6:	Decide, exactly, rough sets and exact sets. Using **Definition 3.1**.

6. Conclusion

In this paper, we have presented some solutions to improve the approach of soft rough sets [33]. The suggested method can be considered as a modification to soft rough set models (given by F. Feng et al. [37]). In fact, new approximations called "Soft pre-rough approximations" have been introduced and their properties have been studied. Comparisons among these approaches and the other works in [1] and [37] have been provided. Moreover, according to our results "Theorem 3.1 and its corollaries", the proposed methods are more accurate than [37] in decision making and accordingly these methods are very useful in real-life applications. The importance of the current paper is not only that it introduces a new type of generalized soft rough set approximations, but also the suggested approaches satisfied most properties of Pawlak's rough sets, without any extra restrictions, that never held in [37].

Finally, we have introduced two applications examples in decision making to illustrate the importance of current methods and also to compare between this method and the previous one in [1] and [37]. Moreover, we have introduced a medical application about Corona virus "COVID-19" to illustrate the importance of the proposed approximations in decision making. In addition, we obtain an algorithm for the proposed methods to be useful in decision making of any future real-life problem. More importantly the present paper not only provides a complete new range of approximation spaces but also increase the accuracy of approximations of the subsets of a set.

Acknowledgements

The Authors would like to express their Gratitude's to the ministry of education and the deanship of scientific research-Najran University-Kingdom of Saudi Arabia for their financial and Technical support under code number [NU/ESCI/17/022].

Funding

This research work supported by the Deanship of the Scientific Research at Najran University, Najran, Saudi Arabia [NU/ESCI/17/022].

Conflict of interest

The authors declare no conflicts of interest in this paper.

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