

MATHEMATICAL MODELLING AND CONTROL OF ECHINOCOCCUS IN QINGHAI PROVINCE, CHINA

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ABSTRACT. In this paper, two mathematical models, the baseline model and the intervention model, are proposed to study the transmission dynamics of echinococcus. A global forward bifurcation completely characterizes the dynamical behavior of the baseline model. That is, when the basic reproductive number is less than one, the disease-free equilibrium is asymptotically globally stable; when the number is greater than one, the endemic equilibrium is asymptotically globally stable. For the intervention model, however, the basic reproduction number alone is not enough to describe the dynamics, particularly for the case where the basic reproductive number is less than one. The emergence of a backward bifurcation enriches the dynamical behavior of the model. Applying these mathematical models to Qinghai Province, China, we found that the infection of echinococcus is in an endemic state. Furthermore, the model appears to be supportive of human interventions in order to change the landscape of echinococcus infection in this region.

1. Introduction. Echinococcosis is a zoonotic parasitic disease. It is caused by infection with the larvae of echinococcus. The life of an echinococcus depends on two different hosts, a definitive host, such as dogs, wolves, foxes; and an intermediate host including sheep, deer and moose. In addition to these animals, humans are also important intermediate hosts to echinococcus granulosus. The reproduction process of echinococcus starts within a definitive host. The eggs released by adult echinococcus granulosus do not stay in the body of the definitive host. Instead, they are passed in the feces of the definitive host. An intermediate host becomes

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infected by accidentally ingesting an egg. As a result of the infection, the life of echinococcus continues within the body of the new host. The ingested egg then hatches and releases an oncosphere within the body of the intermediate host. Once the oncosphere invades the organs of the intermediate host, such as liver, brain and lungs, it develops into a cyst. The cyst slowly grows and, in the process, creates protoscolices. A definitive host will become infected after ingesting the cyst-containing organs of the infected intermediate host. The protoscolices ingested by the definitive hosts then develop into adult worms and the cycle starts all over again [1]. The life cycle of echinococcus granulosus is shown in Figure 1.

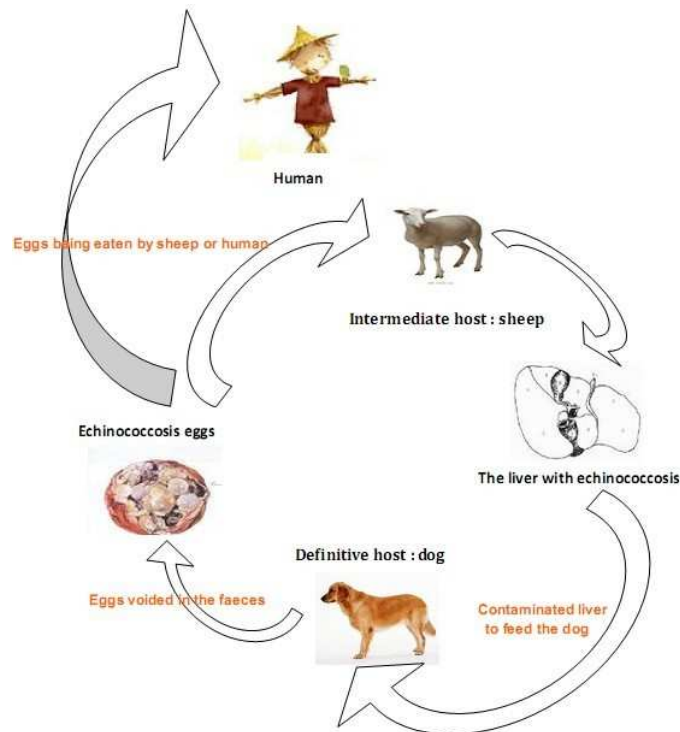


FIGURE 1. The life cycle of echinococcus granulosus.

Both cystic echinococcosis (CE) and alveolar echinococcosis (AE) are endemic in China, and they are one of the most challenging issues for public health and animal husbandry in the western region of China [2]. The canid intestinal tapeworms echinococcus granulosus and echinococcus multilocularis are the causative agents of infection of echinococcosis [3].

Human CE is associated with animal husbandry. In China, echinococcosis are found mainly in the western regions, including the Xinjiang Uygur autonomous region, Qinghai, Gansu and Sichuan provinces. A national survey of important parasitic diseases showed that the average prevalence of echinococcosis in these regions was about 1.08% in 2004 [4]. But the figures in some counties were way above the average. For example, the prevalence was as high as 9.74% from 1997 to 2001 in Chengduo county of Qinghai province; and it was 7.01% in 2006 [5]. Some measures, deworming domestic dogs, for instance, have been in practice against

the echinococcosis in China. To get rid of the worms from the dogs, praziquantel is given to each dog once a month. Usually the drug is mixed with the bait so that the dog can swallow it. Five days after the dog receives the drugs, the dog’s feces are collected and buried or burnt in order to prevent echinococcus eggs from spreading [6]. Other measures, such as hunting wild dogs regardless of the epidemiological status, also can be taken.

Over the last few decades, mathematical models have been formulated in the context of the dynamics of parasite disease[7], such as schistosomiasis and malaria [8, 9, 10, 11, 12]. Particularly interesting mathematic models in studying the spreading of echinococcosis can be seen in [13, 14], for example. A survey of Echinococcus granulosus, Taenia hydatigena and T. ovis for sheep and goats were undertaken in order to investigate the transmission dynamics of these parasites in northern Jordan [13]. It was found that Echinococcus granulosus was in an endemic steady state with no evidence of protective immunity in the intermediate host and the basic reproduction ratio was estimated between 1.5 and 1.8. In 2000, a study also in northern Jordan [14] suggested that the intensity of intermediate infection in donkeys increased with age in a linear fashion. The prevalence also increased with age approaching an asymptotic prevalence in the oldest animals. This implies that there was minimal regulation of the parasite population by intermediate host immunity.

To investigate the spread of echinococcus granulosus in China, we propose and study two epidemic models in this paper. We first construct a baseline model considering sheep, dogs and echinococcus granulosus’s eggs and study the global dynamics. We then discuss an intervention model which incorporates deworming dogs. We found backward bifurcation of this model, which suggests that the traditional requirement for the basic reproduction number to be below unity though necessary is not sufficient for disease control in this case.

The rest of this paper is organized as follows. In section 2, we develop and analyze the baseline mathematical model. We analyze the model with intervention in section 3. In section 4, uncertainty and sensitivity analysis are performed then we apply the model to some regions in China. A discussion section on the implications of the results completes the paper.

2. The baseline model. The baseline model is about the spreading of echinococcus granulosus in the cycle of dog-egg-sheep-dog. Both sheep and dogs are divided into the susceptible and the infected, denoted by S_d, I_d, S_s and I_s , respectively. The echinococcus granulosus’ eggs are denoted by E . Then the baseline model can be described by the following system of ordinary differential equations:

$$\left\{ \begin{array}{l} \frac{dS_d}{dt} = \Lambda_d - d_d S_d - (\alpha_s + d_s)\beta_{sd} S_d I_s, \\ \frac{dI_d}{dt} = (\alpha_s + d_s)\beta_{sd} S_d I_s - (\alpha_d + d_d) I_d, \\ \frac{dE}{dt} = \delta I_d - d_e E, \\ \frac{dS_s}{dt} = \Lambda_s - d_s S_s - \beta_{es} S_s E, \\ \frac{dI_s}{dt} = \beta_{es} S_s E - (\alpha_s + d_s) I_s. \end{array} \right. \tag{1}$$

In system (1), subscript d denotes dog and s denotes sheep. Λ_d describes the recruitment rate of the dog population; d_d is the natural death rate of dogs; α_d is the disease-induced death rate of infected dogs; A new infection of dogs comes from the food that the dogs ingest (fed with haslets of infected sheep by humans). β_{sd} describes the likelihood that a dog is infected given that the dog eats infected sheep haslets. Then the incidence rate of dogs is $(\alpha_s + d_s)\beta_{sd}S_dI_s$, which is proportional to the death rate of sheep. It is a quadratic form but it is the product of susceptible dogs and infected dead sheep. δ is the number of echinococcus granulosus's eggs produced by an infected dog. d_e is mortality rate of echinococcus granulosus's eggs. Parameters $\Lambda_s, d_s, \alpha_s, \beta_{es}$ have similar meanings. All parameters in system (1) are positive.

For system (1), it is easy to observe that the feasible region is

$$\Gamma = \left\{ (S_d, I_d, E, S_s, I_s) : 0 \leq S_d + I_d \leq \frac{\Lambda_d}{d_d}, 0 \leq E \leq \frac{\delta\Lambda_d}{d_d d_e}, 0 \leq S_s + I_s \leq \frac{\Lambda_s}{d_s} \right\}.$$

The closed set Γ is positively invariant to system (1).

2.1. The global stability of of the disease-free equilibrium. The disease-free equilibrium of system (1) is $U_0 = \left(\frac{\Lambda_d}{d_d}, 0, 0, \frac{\Lambda_s}{d_s}, 0 \right)$. Using the next-generation operator approach [15, 16], we obtain the basic reproductive number

$$\mathcal{R}_0 = \sqrt[3]{\frac{\delta\beta_{sd}\beta_{es}\Lambda_s\Lambda_d}{d_s d_d d_e (\alpha_d + d_d)}}. \quad (2)$$

Choosing $V = I_d + \frac{\alpha_d + d_d}{\delta} E + \frac{\beta_{sd}\Lambda_d}{d_d} I_s$ as a Lyapunov function, one can show that the disease-free equilibrium is asymptotically stable regardless of initial data, as stated in Theorem 2.1 below.

Theorem 2.1. *Consider system (1). The disease-free equilibrium U_0 is globally asymptotically stable whenever $\mathcal{R}_0 < 1$.*

The basic reproductive number in (2) is in a mathematically concise form. To better understand this number, we rewrite it as

$$\mathcal{R}_0 = \sqrt[3]{\left(\frac{\Lambda_d}{d_d} \frac{\beta_{sd}(\alpha_s + d_s)}{\alpha_d + d_d} \right) \left(\frac{\delta}{d_e} \right) \left(\frac{\Lambda_s}{d_s} \frac{\beta_{es}}{\alpha_s + d_s} \right)}. \quad (3)$$

This form now reflects the life cycle of echinococcus granulosus. The average number of eggs of echinococcus granulosus that could possibly be ingested by a typical sheep is measured by $\frac{\delta}{d_e}$; while $\frac{\Lambda_s}{d_s} \frac{\beta_{es}}{\alpha_s + d_s}$ accounts for the average number of infected sheep by a typical egg and $\frac{\Lambda_d}{d_d} \frac{\beta_{sd}(\alpha_s + d_s)}{\alpha_d + d_d}$ quantifies the average number of the infected dogs by a typical infected sheep. Overall, the basic reproduction number is the geometric mean of these fundament components.

2.2. The global stability of endemic equilibrium. In this subsection, we will show the existence of endemic equilibrium of system (1) and it's global stability.

Theorem 2.2. *When $\mathcal{R}_0 > 1$, there exists a unique endemic equilibrium U^* for system (1); and the endemic equilibrium U^* is globally asymptotically stable.*

Proof. Let $U^* = (S_d^*, I_d^*, E^*, S_s^*, I_s^*)$ be an endemic equilibrium for system (1). We first use E^* to express the remaining components of U^* .

$$S_d^* = \frac{\Lambda_d(d_s + \beta_{es}E^*)}{CE^* + d_d(d_s + \beta_{es}E^*)}, \tag{4}$$

$$I_d^* = \frac{BE^*}{\delta(CE^* + F + d_d\beta_{es}E^*)}, \tag{5}$$

$$S_s^* = \frac{\Lambda_s}{d_s + \beta_{es}E^*}, \tag{6}$$

$$I_s^* = \frac{\Lambda_s\beta_{es}E^*}{(d_s + \alpha_s)(d_s + \beta_{es}E^*)}, \tag{7}$$

where

$$B = \frac{\delta\beta_{sd}\beta_{es}\Lambda_s\Lambda_d}{\alpha_d + d_d}, \quad C = \beta_{sd}\beta_{es}\Lambda_s, \quad F = d_d d_s. \tag{8}$$

We then obtain $E^* = \frac{B-d_e F}{\delta(C+d_d\beta_{es})}$. Since $\mathcal{R}_0 = \sqrt[3]{\frac{B}{d_e F}}$, if $\mathcal{R}_0 > 1$, then $B > d_e F$. That is, $E^* > 0$ is equivalent to $\mathcal{R}_0 > 1$, there exists only one endemic equilibrium for system (1) when $\mathcal{R}_0 > 1$.

Obviously, the linearization matrix of system (1) around U^* is

$$J = \begin{pmatrix} -d_d - n\beta_{sd}I_s^* & 0 & 0 & 0 & -n\beta_{sd}S_d^* \\ n\beta_{sd}I_s^* & -m & 0 & 0 & n\beta_{sd}S_d^* \\ 0 & \delta & -d_e & 0 & 0 \\ 0 & 0 & -\beta_{es}S_s^* & -h & 0 \\ 0 & 0 & \beta_{es}S_s^* & \beta_{es}E^* & -n \end{pmatrix}. \tag{9}$$

The characteristic polynomial of the linearization matrix is

$$(\lambda + \alpha_d + d_d)(\lambda + d_e)(\lambda + \alpha_s + d_s)(\lambda + d_s + \beta_{es}E^*)(\lambda + \frac{CE^*}{d_s + \beta_{es}E^*} + d_d) - d_e(\alpha_d + d_d)(\alpha_s + d_s)(\lambda + d_d)(\lambda + d_s) = 0 \tag{10}$$

Rewrite the characteristic polynomial with simple form

$$\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 = 0$$

where

$$\begin{aligned}
a_1 &= d_e + h + m + n + l > 0, \\
a_2 &= h(d_e + m + n + l) + l(m + n) + d_e(m + n + l) + mn > 0, \\
a_3 &= hlm + hln + hmn + lmn + hld_e + hnd_e + lmd_e + hnd_e + lnd_e > 0, \\
a_4 &= hlmn + hlmd_e + hlnd_e + hmn d_e + lmn d_e - mnd_e(d_s + d_d) > 0, \\
a_5 &= hlmnd_e - mnd_e d_d d_s \\
&= mnd_e(CE^* + d_d \beta_{es} E^*) > 0, \\
h &= d_s + \beta_{es} E^*, \\
l &= \frac{CE^*}{d_s + \beta_{es} E^*} + d_d, \\
m &= \alpha_d + d_d, \\
n &= \alpha_s + d_s.
\end{aligned}$$

According to Routh-Hurwitz criteria, we can obtain

$$\begin{aligned}
H_1 &= a_1 > 0, & H_2 &= \begin{vmatrix} a_1 & a_3 \\ 1 & a_2 \end{vmatrix}, & H_3 &= \begin{vmatrix} a_1 & a_3 & a_5 \\ 1 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix}, \\
H_4 &= \begin{vmatrix} a_1 & a_3 & a_5 & 0 \\ 1 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & a_5 \\ 0 & 1 & a_2 & a_4 \end{vmatrix}, & H_5 &= \begin{vmatrix} a_1 & a_3 & a_5 & 0 & 0 \\ 1 & a_2 & a_4 & 0 & 0 \\ 0 & a_1 & a_3 & a_5 & 0 \\ 0 & 1 & a_2 & a_4 & 0 \\ 0 & 0 & a_1 & a_3 & a_5 \end{vmatrix}.
\end{aligned}$$

Through the complex calculation, we obtain

$$\begin{aligned}
H_2 &= 2lmn + 2hmn + 2hnd_e + 2hmd_e + 2ld_en + 2lmd_e + 2hlm + 2hld_e + \\
&\quad 2hnl + 3mnd_e + (h + l + m + n)d_e^2 + (d_e + l + m + n)h^2 + (d_e + l \\
&\quad + h + n)m^2 + (d_e + l + m + h)n^2 + (d_e + h + m + n)l^2 > 0, \\
H_3 &> 2hld_e^3(m + n) + 2mn(hl^2 + hd_e + l^2d_e)(h + l) + 2hld_e(hm + nl)(h + \\
&\quad l + n + d_e) + (2hm^2ld_e + 2h^2nld_e + h^2m^2l)(h + m + d_e) + d_e(h^2m^2 \\
&\quad + h^2n^2 + l^2m^2)(m + l + n + h) + (4hlmnd_e + hl^2n^2)(m + l + n + d_e) \\
&\quad + n(l^2d_e^2 + l^2m^2 + h^2m^2)(m + l + h + d_e) + mnd_e(d_s + d_d)(h + l + \\
&\quad n + m + d_e)(m + l + n + d_e) + 2lmnh^2(m + h + n) + 2lmhn^2(l + n) \\
&\quad + 2lhn m^2(m + l + n) + 2hmd_e l^2(m + l + d_e) + 2hld_e n^2(d_e + h + n) \\
&\quad + 2ln^2 d_e^2(m + h + n + d_e) + a_1(mnd_e(mh + ml + nh + nl + d_e h + d_e l) \\
&\quad + m^2 d_e^2 h + h^2 n^2 l + h^2 l^2 m + l^2 m^2 h + h^2 d_e^2 m + l^2 d_e^2 h + n^2 d_e^2 h + \\
&\quad m^2 n_e^2 h + h^2 n^2 m + m^2 d_e^2 l + l^2 n^2 d_e + h^2 l^2 n + m^2 n^2 l + h^2 d_e^2 l + \\
&\quad l^2 d_e^2 m + l^2 n^2 m + h^2 d_e^2 n + h^2 l^2 d_e) > 0.
\end{aligned}$$

With the same method, we also have $H_4 > 0$. Since $H_5 = a_5 \cdot H_4$, then $H_5 > 0$. Therefore, we can confirm that U^* is locally asymptotically stable under the condition $\mathcal{R}_0 > 1$.

Now we will focus on the global stability of U^* . Consider a Lyapunov function[17]

$$\begin{aligned} V = & (S_d - S_d^*) - S_d^* \ln \frac{S_d}{S_d^*} + \left[(I_d - I_d^*) - I_d^* \ln \frac{I_d}{I_d^*} \right] + \mathfrak{B} \left[(E - E^*) - E^* \ln \frac{E}{E^*} \right] \\ & + \mathfrak{C} \left[(S_s - S_s^*) - S_s^* \ln \frac{S_s}{S_s^*} \right] + \mathfrak{D} \left[(I_s - I_s^*) - I_s^* \ln \frac{I_s}{I_s^*} \right] \end{aligned}$$

with $\mathfrak{B} = \frac{\alpha_d + d_d}{\delta}$, $\mathfrak{C} = \frac{d_e(\alpha_d + d_d)}{\delta\beta_{es}S_s^*}$, $\mathfrak{D} = \beta_{sd}S_d^*$. Computing the derivative of V along the trajectories of system (1), we obtain

$$\begin{aligned} \frac{dV}{dt} = & \Lambda_d - d_d S_d - (\alpha_s + d_s) \beta_{sd} S_d I_s - [\Lambda_d - d_d S_d - (\alpha_s + d_s) \beta_{sd} S_d I_s] \frac{S_d^*}{S_d} + \\ & \left[(\alpha_s + d_s) \beta_{sd} S_d I_s - (\alpha_d + d_d) I_d - [(\alpha_s + d_s) \beta_{sd} S_d I_s - (\alpha_d + d_d) I_d] \frac{I_d^*}{I_d} \right] \\ & + \mathfrak{B} \left[\delta I_d - d_e E - (\delta I_d - d_e E) \frac{E^*}{E} \right] \\ & + \mathfrak{C} \left[\Lambda_s - d_s S_s - \beta_{es} S_s E - (\Lambda_s - d_s S_s - \beta_{es} S_s E) \frac{S_s^*}{S_s} \right] \\ & + \mathfrak{D} \left[\beta_{es} S_s E - (\alpha_s + d_s) I_s - [\beta_{es} S_s E - (\alpha_s + d_s) I_s] \frac{I_s^*}{I_s} \right]. \end{aligned}$$

It follows from equations (4) and (6) that $\frac{d_e(\alpha_d + d_d)}{\delta\beta_{es}S_s^*} = \beta_{sd}S_d^*$ which further implies $\mathfrak{C} = \mathfrak{D}$. So, a direct calculation leads to

$$\begin{aligned} V' = & d_d S_d^* \left(2 - \frac{S_d^*}{S_d} - \frac{S_d}{S_d^*} \right) + \mathcal{R} \frac{d_s}{\beta_{es}} \left(2 - \frac{S_s^*}{S_s} - \frac{S_s}{S_s^*} \right) + 2\mathcal{R}E^* + \mathcal{P}S_d^*I_s^* \\ & + \mathcal{P}S_d^*I_s + \mathcal{Q}I_d^* - \mathcal{P} \frac{(S_d^*)^2 I_s^*}{S_d} - \mathcal{P} \frac{S_d I_s I_d^*}{I_d} - \mathcal{Q} \frac{E^*}{E} I_d - \mathcal{R} \frac{S_s^* E^*}{S_s} \\ & - \mathcal{P} \mathcal{R} \frac{I_s}{\beta_{sd} \beta_{es} S_s^*} - \mathcal{R} \frac{S_s I_s^* E}{S_s^* I_s} + \mathcal{P} \mathcal{R} \frac{I_s^*}{\beta_{sd} \beta_{es} S_s^*}, \end{aligned} \quad (11)$$

where $\mathcal{P} = (\alpha_s + d_s)\beta_{sd}$, $\mathcal{Q} = \alpha_d + d_d$, $\mathcal{R} = \frac{d_e(\alpha_d + d_d)}{\delta}$. Noticing the first two terms in (11) are negative and $\delta I_d^* = d_e E^*$, we drop them

$$\begin{aligned} V' = & 2\mathcal{R}E^* + \mathcal{P}S_d^*I_s^* + \mathcal{Q}I_d^* - \mathcal{P} \frac{(S_d^*)^2 I_s^*}{S_d} - \mathcal{P} \frac{S_d I_s I_d^*}{I_d} - \mathcal{P} \frac{S_d^* I_s^* I_d E^*}{I_d^* E} - \mathcal{R} \frac{S_s^* E^*}{S_s} \\ & - \mathcal{R} \frac{S_s I_s^* E}{S_s^* I_s} + \mathcal{P} \mathcal{R} \frac{I_s^*}{\beta_{sd} \beta_{es} S_s^*} \\ \leq & 3\mathcal{P}S_d^*I_s^* + 2\mathcal{R}E^* - \mathcal{P} \frac{(S_d^*)^2 I_s^*}{S_d} - \mathcal{P} \frac{S_d I_s I_d^*}{I_d} - \mathcal{P} \frac{S_d^* I_s^* I_d E^*}{I_d^* E} - \mathcal{R} \frac{S_s^* E^*}{S_s} \\ & - \mathcal{R} \frac{S_s I_s^* E}{S_s^* I_s} \end{aligned}$$

Applying the inequality of arithmetic and geometric means (for non-negative real numbers a_1, a_2, \dots, a_n , $\frac{a_1+a_2+\dots+a_n}{n} \geq (a_1a_2 \dots a_n)^{\frac{1}{n}}$ holds) to the last five negative terms, we obtain that

$$V' \leq -5\sqrt[5]{\mathcal{P}^3\mathcal{R}^2(S_d^*)^3(I_s^*)^3(E^*)^2} + 3\mathcal{P}S_d^*I_s^* + 2\mathcal{R}E^*.$$

Since $\mathcal{R}E^* = \mathcal{P}S_d^*I_s^*$, we have

$$V' \leq -5\mathcal{R}E^* + 5\mathcal{R}E^* = 0.$$

Obviously, the largest compact invariant set in $\{(S_d, I_d, E, S_s, I_s) \in \Gamma : V' = 0\}$ is the singleton $\{U^*\}$ when $\mathcal{R}_0 > 1$. Therefore, it follows from LaSalle’s Invariance Principle [18] that the endemic equilibrium U^* of system (1) is globally asymptotically stable whenever it comes to exist. This completes the proof. \square

Theorems 2.1 and 2.2 give a clear picture for the global dynamics of the baseline model. The bifurcation at $\mathcal{R}_0 = 1$ is of forward type and it is a global one. We, naturally, combine these two theorems into a whole theorem.

Theorem 2.3. *The global dynamics of the baseline model (1) is completely determined by the basic reproductive number \mathcal{R}_0 , regardless of initial values. When $\mathcal{R}_0 < 1$, the disease-free equilibrium is the global attractor, while if $\mathcal{R}_0 > 1$, the unique endemic equilibrium is the global attractor.*

3. Model with intervention. The exercises of deworming echinococcosis eggs and killing wild dogs are considered to construct a more practical mathematical model. The human population is introduced into the model to incorporate human interventions. Epidemiologically, the human population is also divided into susceptible and infected subpopulations, denoted by S_h and I_h respectively. $H = S_h + I_h$ is the total human population. Then the equations for the intervention model read

$$\left\{ \begin{array}{l} \frac{dS_d}{dt} = \Lambda_d - d_d S_d - (\alpha_s + d_s)\beta_{sd} S_d I_s, \\ \frac{dI_d}{dt} = (\alpha_s + d_s)\beta_{sd} S_d I_s - (\alpha_d + d_d) I_d, \\ \frac{dE}{dt} = \delta I_d - d_e E - c_h H E, \\ \frac{dS_s}{dt} = \Lambda_s - d_s S_s - \beta_{es} S_s E, \\ \frac{dI_s}{dt} = \beta_{es} S_s E - (\alpha_s + d_s) I_s, \\ \frac{dS_h}{dt} = \Lambda_h - \beta_{eh} S_h E - d_h S_h + \mu I_h, \\ \frac{dI_h}{dt} = \beta_{eh} S_h E - (\mu + d_h + \alpha_h) I_h. \end{array} \right. \tag{12}$$

In system (12), subscript h denotes human. Λ_h describes the birth rate of the human; d_h is the natural death rate of human; α_h is the disease-induced death rate; β_{eh} describes the transmission of echinococcosis of humans; μ is the recovery rate; c_h is the intervention coefficient. The rest of all other parameters have the same meanings as in model (1).

We assume that human efforts in deworming echinococcosis eggs and deep buried dog feces are proportional to the human population. Hence, $c_h H E$ is used to account

for losing rate of echinococcosis eggs because of the human intervention. Humans may get infected by accidentally inhaling echinococcosis eggs, which is modeled by the mass action law as consistently used for other incidence rates in this study.

We will explore model (12) in the following positively invariant region:

$$\Gamma = \left\{ (S_d, I_d, E, S_s, I_s, S_h, I_h) \in \mathbb{R}_+^7 : 0 \leq S_d + I_d \leq \frac{\Lambda_d}{d_d}, 0 \leq E \leq \frac{\delta \Lambda_d}{d_d d_e}, \right. \\ \left. 0 \leq S_s + I_s \leq \frac{\Lambda_s}{d_s}, 0 \leq S_h + I_h \leq \frac{\Lambda_h}{d_h} \right\}.$$

3.1. The disease-free equilibrium. The disease-free equilibrium of system (12) is $U_0^I = (S_d^0, 0, 0, S_s^0, 0, S_h^0, 0)$ with $S_d^0 = \frac{\Lambda_d}{d_d}$, $S_s^0 = \frac{\Lambda_s}{d_s}$ and $S_h^0 = \frac{\Lambda_h}{d_h}$ being the asymptotic carrying capacities for dog population, sheep and human, respectively. The intervention pressure of humans against echinococcosis eggs is measured through $c_h \frac{\Lambda_h}{d_h}$, conveniently denoted by k_h , which is the per-capita losing rate of the eggs due to the human’s intervention. Applying the next-generation operator approach to model (12) finds the basic reproduction number

$$\mathcal{R}_0^I = \sqrt[3]{\frac{\delta \beta_{sd} \beta_{es} \Lambda_s \Lambda_d}{d_s d_d (\alpha_d + d_d) (d_e + k_h)}}.$$

Theorem 3.1. *The disease-free equilibrium U_0^I of system (12) is locally asymptotically stable if $\mathcal{R}_0^I < 1$.*

The basic reproductive numbers for the baseline model and for the intervention model are closely related: the relationship is $\mathcal{R}_0^I = \mathcal{R}_0 \sqrt[3]{\frac{d_e}{d_e + k_h}}$. When $k_h = 0$ (no human intervention), \mathcal{R}_0^I and \mathcal{R}_0 agree. However, when human intervention comes to play, the basic reproductive number is reduced by a factor of $\sqrt[3]{\frac{d_e}{d_e + k_h}} < 1$. Since increasing intervention efforts results in decreasing the magnitude of echinococcus infection, human intervention is a practical approach to controlling echinococcus. Our simulations and sensitive analysis in later sections will further support this result.

3.2. The endemic equilibria. Now we turn to study the existence of endemic equilibrium of system (12).

Theorem 3.2. *When $\mathcal{R}_0^I > 1$, there exists a unique endemic equilibrium \hat{U} for system (12).*

Proof. Let $\hat{U} = (\hat{S}_d, \hat{I}_d, \hat{E}, \hat{S}_s, \hat{I}_s, \hat{S}_h, \hat{I}_h)$ be an endemic equilibrium for (12). We use \hat{E} to express the remaining components of \hat{U} .

$$\begin{aligned} \hat{S}_d &= \frac{\Lambda_d (d_s + \beta_{es} \hat{E})}{C \hat{E} + d_d (d_s + \beta_{es} \hat{E})}, & \hat{I}_d &= \frac{B \hat{E}}{\delta (C \hat{E} + F + d_d \beta_{es} \hat{E})}, \\ \hat{S}_s &= \frac{\Lambda_s}{d_s + \beta_{es} \hat{E}}, & \hat{I}_s &= \frac{\Lambda_s \beta_{es} \hat{E}}{(d_s + \alpha_s) (d_s + \beta_{es} \hat{E})}, \\ \hat{S}_h &= \frac{\Lambda_h (A + \mu)}{A (\beta_{eh} \hat{E} + d_h) + \mu d_h}, & \hat{I}_h &= \frac{\Lambda_h \beta_{eh} \hat{E}}{A (\beta_{eh} \hat{E} + d_h) + \mu d_h}, \end{aligned}$$

where the expressions for B, C, F are the same as in section 2 (see (8)), $A = d_h + \alpha_h$. From the expressions of \hat{I}_d, \hat{S}_h and \hat{I}_h , we get that

$$\delta \hat{I}_d = \frac{B \hat{E}}{C \hat{E} + F + d_d \beta_{es} \hat{E}} \quad (13)$$

$$c_h (\hat{S}_h + \hat{I}_h) \hat{E} = \frac{\Lambda_h (A + \mu + \beta_{eh} \hat{E})}{A (\beta_{eh} \hat{E} + d_h) + \mu d_h} \quad (14)$$

Substituting these two expressions into the third equation of system (12), we arrive at an equation about \hat{E} alone

$$\frac{B}{C \hat{E} + F + d_d \beta_{es} \hat{E}} = \frac{c_h \Lambda_h (A + \mu + \beta_{eh} \hat{E})}{A (\beta_{eh} \hat{E} + d_h) + \mu d_h} + d_e. \quad (15)$$

Equation (15) can be simplified into

$$\mathfrak{H}(\hat{E})^2 + \mathfrak{K} \hat{E} + \mathfrak{L} = 0, \quad (16)$$

where

$$\begin{aligned} \mathfrak{H} &= AC d_e \beta_{eh} + A d_d d_e \beta_{es} \beta_{eh} + C \Lambda_h c_h \beta_{eh} + \Lambda_h c_h d_d \beta_{es} \beta_{eh} > 0, \\ \mathfrak{K} &= F \beta_{eh} (D + \alpha_h d_e) + D(A + \mu)(C + d_d \beta_{es}) - \frac{ADF(\mathcal{R}_0^I)^3}{d_h} \beta_{eh}, \\ \mathfrak{L} &= DF(A + \mu)(1 - (\mathcal{R}_0^I)^3), \\ D &= d_h d_e + c_h \Lambda_h. \end{aligned}$$

The number of endemic equilibrium is determined by the number of positive solutions of the quadratic equation (16). Since $\mathfrak{L} < 0$ holds if and only if $\mathcal{R}_0^I > 1$, (16) has a unique positive root when $\mathcal{R}_0^I > 1$. Consequently, there exists a unique endemic equilibrium for system (12) whenever $\mathcal{R}_0^I > 1$. \square

When $\mathcal{R}_0^I < 1$, quadratic equation (16) might have a pair of positive solutions. This provides an opportunity for an emergence of multiple endemic equilibria.

Theorem 3.3. *If $\mathcal{R}_c < (\mathcal{R}_0^I)^3 < 1$, system (12) have two positive equilibria. where*

$$\begin{aligned} \mathcal{R}_c &= \max\{\mathcal{R}_a, \mathcal{R}_b\}, \\ \mathcal{R}_a &= \frac{d_h [\beta_{eh} F (D + \alpha_h d_e) + (\mu + A)(C + \beta_{es} d_d) D]}{AFD \beta_{eh}}, \\ \mathcal{R}_b &= \frac{2d_h [AF \beta_{eh} (D + \alpha_h d_e) + (\mu + A)(C + \beta_{es} d_d)(AD - 2d_h(D + \alpha_h d_e))]}{A^2 DF \beta_{eh}}. \end{aligned}$$

Proof. Because equilibrium of system (12) must satisfy (16), system (12) has two endemic equilibria if and only if

$$\mathfrak{L} > 0, \mathfrak{K} < 0 \text{ and } \mathfrak{K}^2 - 4\mathfrak{H}\mathfrak{L} > 0.$$

First, $\mathfrak{L} > 0$ is guaranteed because of $\mathcal{R}_0^I < 1$. Then, $\mathfrak{K} < 0$ if and only if

$$F \beta_{eh} (D + \alpha_h d_e) + D(A + \mu)(C + d_d \beta_{es}) < \frac{ADF \beta_{eh}}{d_h} (\mathcal{R}_0^I)^3,$$

which is equivalent to $\mathcal{R}_a < (\mathcal{R}_0^I)^3$. Finally, let us check $\mathfrak{K}^2 - 4\mathfrak{H}\mathfrak{L} > 0$.

$$\begin{aligned} \mathfrak{K}^2 - 4\mathfrak{H}\mathfrak{L} &= [D(\mu + A)(C + \beta_{es}d_d) - F\beta_{eh}(D + \alpha_h d_e)]^2 + \left(\frac{ADF\beta_{eh}}{d_h}(\mathcal{R}_0^I)^3\right)^2 \\ &\quad - \frac{2ADF\beta_{eh}}{d_h}[D(\mu + A)(C + \beta_{es}d_d) + F\beta_{eh}(D + \alpha_h d_e)](\mathcal{R}_0^I)^3 \\ &\quad + 4[DF\beta_{eh}(\mu + A)(C + \beta_{es}d_d)(D + \alpha_h d_e)](\mathcal{R}_0^I)^3. \end{aligned}$$

Hence, $\mathfrak{K}^2 - 4\mathfrak{H}\mathfrak{L} > 0$ is equivalent to

$$\frac{2d_h (AF\beta_{eh}(D + \alpha_h d_e) + (\mu + A)(C + \beta_{es}d_d)(AD - 2d_h(D + \alpha_h d_e)))}{A^2DF\beta_{eh}} = \mathcal{R}_b,$$

and

$$\mathcal{R}_b < (\mathcal{R}_0^I)^3$$

Therefore, system (12) has two positive equilibria when $\mathcal{R}_c < (\mathcal{R}_0^I)^3 < 1$. □

Theorem 3.3 has established the existence of multiple equilibria without knowing any information about their stability. Next we use the bifurcation approach to settle down this concern.

The following lemma provides a criteria for determining if a bifurcation is backward or forward at $\mathcal{R}_0^I = 1$. Its general form and proof can be found in [19, 16, 20].

Lemma 3.4. [20] *Consider a system of ordinary differential equations*

$$\frac{dx}{dt} = f(x, \phi), f : \mathbb{R}^n \rightarrow \mathbb{R}^n \text{ and } f \in \mathcal{C}^2(\mathbb{R}^n \times \mathbb{R}) \tag{17}$$

Assume that

1. $D_x f(0, 0) = (\frac{\partial f_i}{\partial x_i}(0, 0))$ is the linearization matrix of system (17) around the equilibrium 0 with ϕ evaluated at 0. Zero is a simple eigenvalue of $D_x f(0, 0)$ and all other eigenvalues have negative real parts;
2. Matrix $D_x f(0, 0)$ has a right null vector (a right eigenvector $D_x f(0, 0)$ associated with the zero eigenvalue) ω and a left null vector ν . associated with the zero eigenvalue

Let f_k be the k th component of f and

$$\begin{aligned} a &= \sum_{k,i,j=1}^n v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}(0, 0), \\ b &= \sum_{k,i=1}^n v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \phi}(0, 0). \end{aligned}$$

If both a and b are positive, equilibrium 0 is asymptotical stable and there exist unstable positive equilibria when $\phi < 0$ and $|\phi| \ll 1$. That is, the direction of the bifurcation at $\phi = 0$ is backward. On the other hand, if $a < 0$ and $b > 0$, equilibrium 0 is asymptotical stable and there exist stable positive equilibria when $\phi > 0$ and $|\phi| \ll 1$. That is, the direction of the bifurcation at $\phi = 0$ is forward.

Applying Lemma 3.4 to our model (12), we are able to determine direction of the bifurcation at $\mathcal{R}_0^I = 1$.

Theorem 3.5. System (12) undergoes a backward bifurcation at $\mathcal{R}_0^I = 1$ if

$$\mathcal{R}_d = \left(\frac{d_s}{\beta_{es}(d_h d_e + c_h \Lambda_h)} \right) \left(\frac{d_d}{\beta_{sd} \Lambda_s + d_d} \right) \left(\frac{k_h \beta_{eh} \alpha_h}{\mu + \alpha_h + d_h} \right) > 1. \quad (18)$$

If $\mathcal{R}_d < 1$, the bifurcation at $\mathcal{R}_0^I = 1$ is forward.

Proof. We choose β_{es} as the bifurcation parameter. Then $\mathcal{R}_0^I = 1$ corresponds to

$$\beta_{es} = \frac{d_s d_d (\alpha_d + d_d) (d_e + k_h)}{\delta \beta_{sd} \Lambda_s \Lambda_d}. \quad (19)$$

The linearization matrix of system (12) around U_0^I is

$$J = \begin{pmatrix} -d_d & 0 & 0 & 0 & -a_1 & 0 & 0 \\ 0 & -(\alpha_d + d_d) & 0 & 0 & a_1 & 0 & 0 \\ 0 & \delta & -a_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta_{es} S_s^0 & -d_s & 0 & 0 & 0 \\ 0 & 0 & \beta_{es} S_s^0 & 0 & -(\alpha_s + d_s) & 0 & 0 \\ 0 & 0 & -\beta_{eh} S_h^0 & 0 & 0 & -d_h & \mu \\ 0 & 0 & \beta_{eh} S_h^0 & 0 & 0 & 0 & -a_3 \end{pmatrix}, \quad (20)$$

where $a_1 = (\alpha_s + d_s) \beta_{sd} S_d^0$, $a_2 = d_e + k_h$, $a_3 = \mu + \alpha_h + d_h$.

The characteristic polynomial of the linearization matrix is

$$\begin{aligned} & \lambda(\lambda + d_d)(\lambda + d_s)(\lambda + d_h)(\lambda + \mu + \alpha_h + d_h) \\ & \cdot [\lambda^2 + (\alpha_d + d_d + d_e + k_h + \alpha_s + d_s)\lambda + [(\alpha_d + d_d)(d_e + k_h) \\ & + (\alpha_d + d_d)(\alpha_s + d_s) + (d_e + k_h)(\alpha_s + d_s)]] = 0 \end{aligned} \quad (21)$$

Explicitly, there are 5 known eigenvalues

$$\lambda_1 = 0, \quad \lambda_2 = -d_s, \quad \lambda_3 = -d_h, \quad \lambda_4 = -(\mu + \alpha_h + d_h), \quad \lambda_5 = -d_d.$$

The other two eigenvalues λ_6, λ_7 are governed by the following quadratic equation

$$\begin{aligned} & \lambda^2 + (\alpha_d + d_d + d_e + k_h + \alpha_s + d_s)\lambda \\ & + [(\alpha_d + d_d)(d_e + k_h) + (\alpha_d + d_d)(\alpha_s + d_s) + (d_e + k_h)(\alpha_s + d_s)] = 0 \end{aligned}$$

which guarantees $\Re\{\lambda_6\} < 0$ and $\Re\{\lambda_7\} < 0$. Hence, $\lambda_1 = 0$ is a simple eigenvalue and all other eigenvalues have negative real parts, which meet the hypothesis of Lemma 3.4. A right eigenvector associated with the simple eigenvalue $\lambda = 0$ is

$$\omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7),$$

$$\text{where } \omega_1 = -\frac{(\alpha_d + d_d)(d_e + k_h)}{\delta d_d}, \quad \omega_2 = \frac{(d_e + k_h)}{\delta}, \quad \omega_3 = 1, \quad \omega_4 = -\frac{\beta_{es} \Lambda_s}{d_s^2},$$

$$\omega_5 = \frac{\beta_{es} \Lambda_s}{d_s(\alpha_s + d_s)}, \quad \omega_6 = -\frac{(\alpha_h + d_h)}{d_h} \frac{\beta_{eh} \frac{\Lambda_h}{d_h}}{\mu + \alpha_h + d_h}, \quad \omega_7 = \frac{\beta_{eh} \frac{\Lambda_h}{d_h}}{\mu + \alpha_h + d_h}.$$

Similarly, a left eigenvector associated with $\lambda = 0$ is

$$\nu = \left(0, \frac{d_d}{\beta_{sd} \Lambda_d}, \frac{d_d(\alpha_d + d_d)}{\delta \beta_{sd} \Lambda_d}, 0, 1, 0, 0 \right).$$

The application of Lemma 3.4 requires the computation of two quantities a and b using ω , ν , and the second derivatives of the vector field. The results are

$$\begin{aligned}
 a &= 2\nu_2\omega_1\omega_5 \frac{\partial^2 f_2}{\partial S_d \partial I_s} + 2\nu_3\omega_3\omega_6 \frac{\partial^2 f_3}{\partial S_h \partial E} + 2\nu_3\omega_3\omega_7 \frac{\partial^2 f_3}{\partial I_h \partial E} + 2\nu_5\omega_3\omega_4 \frac{\partial^2 f_5}{\partial S_s \partial E} \\
 &= 2\nu_2\omega_1\omega_5(\alpha_s + d_s)\beta_{sd} + 2\omega_3\omega_4\beta_{es} - 2c_h\nu_3\omega_3(\omega_6 + \omega_7) \\
 &= -2 \frac{d_d}{\beta_{sd}\Lambda_d} \frac{(\alpha_d + d_d)(d_e + c_h \frac{\Lambda_h}{d_h})}{\delta d_d} \frac{\beta_{es}\Lambda_s}{d_s(\alpha_s + d_s)} (\alpha_s + d_s)\beta_{sd} - 2 \frac{\beta_{es}^2 \Lambda_s}{d_s^2} \\
 &\quad + 2c_h \frac{d_d(\alpha_d + d_d)}{\delta \beta_{sd}\Lambda_d} \frac{\alpha_h}{d_h} \frac{\beta_{eh} \frac{\Lambda_h}{d_h}}{\mu + \alpha_h + d_h} \\
 &= -2 \frac{(\alpha_d + d_d)(d_e + k_h)\beta_{es}\Lambda_s}{\delta d_s \Lambda_d} - 2 \frac{\beta_{es}^2 \Lambda_s}{d_s^2} + 2c_h \frac{d_d(\alpha_d + d_d)}{\delta \beta_{sd}\Lambda_d} \frac{\alpha_h}{d_h} \frac{\beta_{eh} \frac{\Lambda_h}{d_h}}{\mu + \alpha_h + d_h} \\
 &= -2 \frac{\beta_{es}\Lambda_s}{d_s} \left(\frac{(\alpha_d + d_d)(d_e + k_h)}{\delta \Lambda_d} + \frac{\beta_{es}}{d_s} \right) + 2c_h \frac{d_d(\alpha_d + d_d)}{\delta \beta_{sd}\Lambda_d} \frac{\alpha_h}{d_h} \frac{\beta_{eh} \frac{\Lambda_h}{d_h}}{\mu + \alpha_h + d_h} \\
 &= -2 \frac{\beta_{es}^2 \Lambda_s}{d_s^2} \left(\frac{\beta_{sd}\Lambda_s}{d_d} + 1 \right) + 2 \frac{d_d(\alpha_d + d_d)}{\delta \beta_{sd}\Lambda_d} \frac{\alpha_h}{d_h} \frac{\beta_{eh} k_h}{\mu + \alpha_h + d_h} \\
 b &= \omega_4 \frac{\partial^2 f_5}{\partial S_s \partial \beta_{es}} + \omega_3 \frac{\partial^2 f_5}{\partial E \partial \beta_{es}} \\
 &= S_s^0 > 0.
 \end{aligned}$$

We rearrange the inequality $a > 0$ as

$$\mathcal{R}_d = \left(\frac{d_s}{\beta_{es}(d_h d_e + c_h \Lambda_h)} \right) \left(\frac{d_d}{\beta_{sd}\Lambda_s + d_d} \right) \left(\frac{k_h \beta_{eh} \alpha_h}{\mu + \alpha_h + d_h} \right) > 1.$$

That is, $a > 0$ holds if and only if $\mathcal{R}_d > 1$. Therefore, system (12) undergoes a backward bifurcation at $\mathcal{R}_0^I = 1$ if $\mathcal{R}_d > 1$. When $\mathcal{R}_d < 1$ the bifurcation at $\mathcal{R}_0^I = 1$ is forward. \square

Figure 2 illustrates the appearance of backward bifurcation. As can be seen that when $\mathcal{R}_0^I < 1$, the stable disease-free equilibrium and a stable endemic equilibrium co-exist, thus creating a bi-stability situation. This figure suggests that the classical requirement for the basic reproduction number \mathcal{R}_0^I to be below unity is not sufficient for disease control when a backward bifurcation happens. For instance, we estimate that $\mathcal{R}_c = 0.458$ from the data of Chengduo county of Qinghai province, China. So, the basic reproductive number \mathcal{R}_0^I should be smaller than 0.458 to eliminate echinococcosis from this region regardless of the initial epidemic status.

Comparing Theorem 2.3 with Theorem 3.5, we find that the human intervention is the driver behind the occurrence of the backward bifurcation. The condition for the appearance of backward bifurcation in equation (18) is rather complicated, but we still can look into it for simple cases. If we consider $\mathcal{R}_d(c_h)$ as a function of c_h (all other parameters are fixed), we can find $\mathcal{R}_d(c_h)$ is monotonic increasing with

$$\lim_{c_h \rightarrow \infty} \mathcal{R}_d(c_h) = \frac{d_d d_s \beta_{eh} \alpha_h}{\beta_{es} d_h (\beta_{sd} \Lambda_s + d_d) (\mu + \alpha_h + d_h)}.$$

This is the absolute maximum value for \mathcal{R}_d if all other parameters are fixed. If $\frac{d_d d_s \beta_{eh}}{\beta_{es} d_h (\beta_{sd} \Lambda_s + d_d)} \frac{\alpha_h}{(\mu + \alpha_h + d_h)} < 1$, then for whatever the human effort is in fighting against echinococcosis, the bifurcation at $\mathcal{R}_0^I = 1$ is always forward. This is a condition to avoid the occurrence of backward bifurcation.

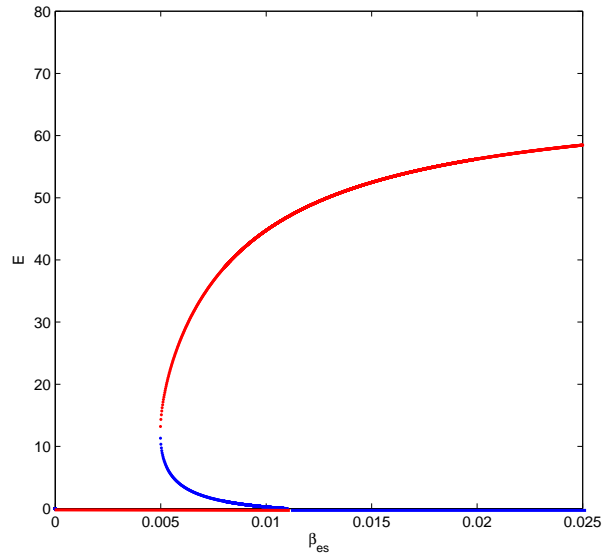


FIGURE 2. Illustration of a backward bifurcation. Plot of E at equilibrium versus the bifurcation parameter β_{es} . The blue line presents the unstable equilibria, and the red line presents the stable equilibria. $\beta_{eh} = 0.011$ is fixed and other parameter values are given in Table 1.

4. Numerical simulations.

4.1. Mean values of parameters and initial values. In the following, we shall present some simulations for model (12) using the data from Chengduo county, Qinghai province of China. The data concerning echinococcosis are obtained mainly from Chinese Center for Disease Control and Prevention [5]. However, these involving the number of dogs and sheep cannot be acquired directly. We have to rely on other resources or estimation. The mean values of estimated parameters are listed in Table 1. The numbers of dogs, sheep, human were estimated according to the China Yearbook [21] and Chinese Center for Disease Control and Prevention [5]. The initial values are given in Table 2.

4.2. Uncertainty and sensitivity analysis to \mathcal{R}_0^I . Our analysis in previous sections has clearly demonstrated that the quantity of \mathcal{R}_0^I plays a crucial role in determining the dynamic behavior of our models. We, therefore, need more specific information on this re-parameterized quantity. Table 1 roughly estimates the mean value for each parameter. Variations of these parameters in our deterministic model lead to uncertainty to model predictions since the basic reproductive number varies with parameters. The variation of the basic reproductive number is studied by looking into the distribution of \mathcal{R}_0^I using Latin Hypercube Sampling.

TABLE 1. Definitions, distributions and values for the parameters

Para	Description	Value	Source
Λ_d	annual crop of newborn puppies	4000	estimated
Λ_s	annual crop of newborn lambs	5000	[21]
Λ_h	human annual birth population	2247	[21]
α_d	dog disease-related death rate	0.53	estimated
d_d	dog natural mortality rate	0.21	estimated
α_s	sheep disease-related death rate	0.0093	estimated
d_s	sheep natural mortality rate	0.00525	[21]
α_h	human disease-related death rate	0.04	[21]
d_h	human natural mortality rate	0.014	[21]
μ	human recovery rate	0.02	estimated
δ	number of eggs produced by per dog	200	estimated
β_{es}	eggs-to-sheep transmission rate	0.015	estimated
β_{sd}	eggs-to-human transmission rate	0.002	estimated
β_{eh}	dog-to-human transmission rate	0.0011	estimated
c_h	intervention coefficient	0.15	estimated
d_e	eggs mortality rate	1	[24]

TABLE 2. Initial Conditions

Variable	Description	Initial Value
S_d	susceptible dog	6000
I_d	infected dog	2000
E	echinococcosis egg	50
S_s	susceptible sheep	78680
I_s	infected sheep	51320
S_h	susceptible human	54264
I_h	infected human	1736

Each parameter is treated as a random variable with its mean value listed in Table 1. We simply assume that all these parameters have uniform distributions. Figure 3 draws the histograms of the parameters from running Latin hypercube sampling 1,000 times. With these 1,000 runs of Latin hypercube sampling, the derived sampling distribution of \mathcal{R}_0^I is shown in Figure 4. This sampling concludes that the mean of \mathcal{R}_0^I is 1.83 and the standard deviation is 1.563. Hence, statistically we are very confidential that echinococcosis is in an endemic state since $\mathcal{R}_0^I > 1$ unless further action is taken to change \mathcal{R}_0^I . The Latin hypercube sampling also generates $P(\mathcal{R}_0^I > \mathcal{R}_c) = 0.927$, that is, the probability that \mathcal{R}_0^I is bigger than \mathcal{R}_c is 0.927. One can see there is little chance to bring \mathcal{R}_0^I less than \mathcal{R}_c . Thus, more likely we will observe endemic cases.

Next we use sensitivity analysis to analyze the influence of each parameter on the basic reproductive number. Partial rank correlation coefficients (PRCC) between the basic reproductive number and each parameter were derived from the previous 1,000 runs of Latin hypercube sampling. The ordering of these PRCCs corresponds to the level of statistical influence that the parameter has on the variability of the \mathcal{R}_0^I [22]. The larger PRCCs in absolute value, the more important the parameter

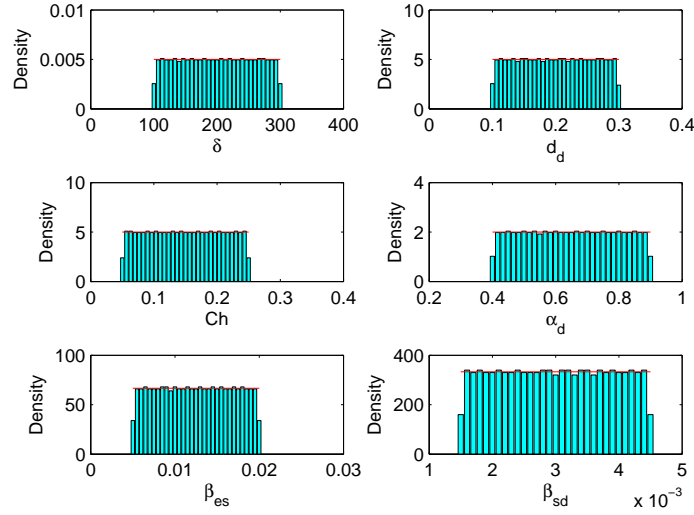


FIGURE 3. Frequencies of parameters obtained from Latin Hypercube Sampling of 1,000 runs.

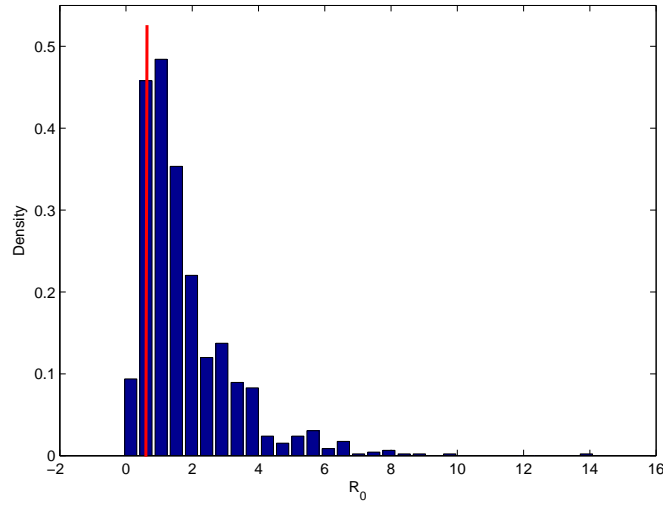


FIGURE 4. Sampling distribution of \mathcal{R}_0^I from 1,000 runs of Latin hypercube sampling. The mean of \mathcal{R}_0^I is 1.83; the standard deviation is 1.563; and probability that the value of \mathcal{R}_0^I bigger than \mathcal{R}_c is 0.927.

in responding to the change in \mathcal{R}_0^I . Plus sign or minus sign means the influence is positive or negative respectively. For example, when the PPCC to a parameter value is positive, then bigger value of this parameter results in bigger \mathcal{R}_0^I . Parameters in Table 3 are arranged in ascending according to the corresponding absolute value of the PRCC, so that the top one has the greatest impact on \mathcal{R}_0^I . Table 3 shows that intervention coefficient c_h has the greatest impact on \mathcal{R}_0^I , followed by the transmission rate from echinococcosis eggs to sheep β_{es} , then the transmission rate from sheep and dogs β_{sd} , then δ the number of echinococcosis eggs produced by an infected dog. Hence, our dynamical analysis together with sensitivity analysis consistently conclude that the most effective approach to reduce the infection is to increase the human intervention, which is within our reach because c_h is controllable.

TABLE 3. PRCC between \mathcal{R}_0^I and each parameter

Parameters	Correlation Coefficients
c_h	-0.9218
β_{es}	0.9065
β_{sd}	0.8696
δ	0.8671
d_d	-0.735
α_d	-0.6756

4.3. Model predictions for Chengduo county. Using the parameter values in Table 1 and initial values in Table 2, we numerically solve the intervention model. Figure 5 illustrates the infected human cases in the next 30 years. In the first few years, the infection cases increase rapidly and reach the peak (about 6770), then decrease steadily in the next 7 or 8 years and finally level off. Ignoring the transition dynamics for the first few years, we can see that the infection cases are eventually stabilized.

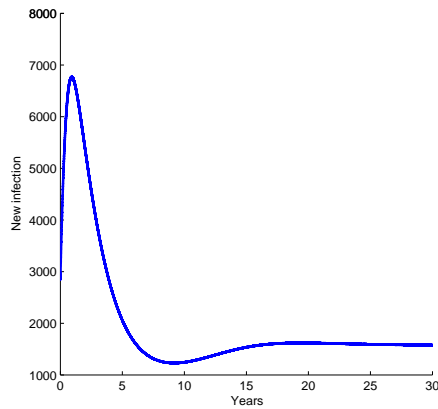


FIGURE 5. The newly increased number of human cases I_h in the next 30 years for Chengduo county.

5. Discussion. Bi-stability could happen in the intervention model, but this is very theoretical. There is little chance one can observe it. Simply looking at the expression of \mathcal{R}_d which indicates if a backward bifurcation can occur, we find that when $\alpha_h = 0$, a backward bifurcation cannot happen. We recall that α_h is the echinococcosis induced-death rate for humans. This rate is very tiny because human cases of echinococcosis can be successfully treated [23]. We are almost sure the dynamical results for the intervention model is regardless of the initial values.

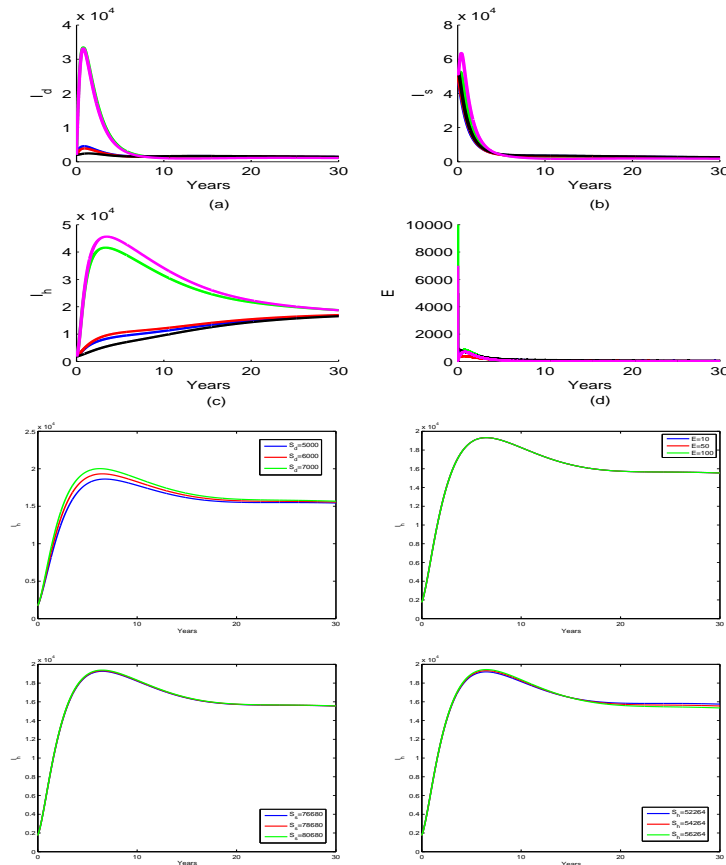


FIGURE 6. Numerical test for the stability of the endemic equilibrium when $\mathcal{R}_0^I > 1$.

For our intervention model, we showed that there exists a unique equilibrium when $\mathcal{R}_0^I > 1$. The stability of this equilibrium has not been fully studied. Theorem 3.5 only implies the endemic equilibrium is locally asymptotically stable when $\mathcal{R}_0^I > 1$, but very close to 1. When \mathcal{R}_0^I is not close to 1, we test the stability of the endemic equilibrium by numerical approach. Figure 6 verifies that, indeed, the endemic equilibrium is locally asymptotically stable.

Chengduo county has eight townships, four of which live on pure animal husbandry and the other four have both agriculture and animal husbandry. The population is 97.8% Tibetan. With their common belief in Buddhism, they avoid killing

wild dogs. Herdsman usually raise dogs, with each household raising 2-4 dogs on average. There are also many stray dogs in Chengduo, which makes echinococcosis more serious [5]. Facing up to the epidemic situation in Chengduo county, both central and local governments have sought forceful methods to reduce echinococcosis transmission. In this article, in order to explore effective control and prevention measures, we proposed two models to study the transmission dynamics of echinococcosis in Chengduo County. From the sensitivity analysis, we find that the human intervention coefficient (c_h), the number of echinococcosis eggs produced per infected dog (δ) and the probability of infected sheep eaten by dogs (β_{sd}) are very important factors among others. Weak human intervention and the high probability of infected sheep eaten by dog are two serious hidden troubles for echinococcosis in Chengduo County.

Therefore, in order to prevent the spread of echinococcosis, deworming dogs and reducing the chances that infected sheep are being eaten by dogs are crucial. Also since dog feces is the source of parasite, its careful disposal can also be a very effective method to control echinococcosis spreading.

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