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THE EXISTENCE OF POSITIVE PERIODIC SOLUTIONS OF A GENERALIZED N-SPECIES GILPIN-AYALA IMPULSIVE COMPETITION SYSTEM

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Abstract. In this paper, the existence of positive periodic solutions of a class of periodic n-species Gilpin-Ayala impulsive competition systems is studied. By using the continuation theorem of coincidence degree theory, a set of easily verifiable sufficient conditions is obtained. Our results are general enough to include some known results in this area.

In honor of Professor Tom Hallam's 70th birthday.

1. **Introduction.** The theory of impulsive differential equations is emerging as an important area of investigation since it is a lot richer than the corresponding theory of nonimpulsive differential equations. Many evolutionary processes in nature are characterized by the fact that at certain moments in time an abrupt change of state is experienced. That is the reason for the rapid development of the theory of impulsive differential equtions; see the monographs [5, 7].

The purpose of this paper is to study the existence of positive periodic solutions of a class of periodic n-species Gilpin-Ayala impulsive competition systems.

In [3], Gilpin and Ayala proposed the following competition model:

$$\frac{dN_i}{dt} = r_i N_i (1 - (\frac{N_i}{K_i})^{\theta_i} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \alpha_{ij} \frac{N_j}{K_j}), i = 1, 2, \cdots, n, \tag{1}$$

where N_i is the population density of the i^{th} species, r_i is the intrinsic exponential growth rate of the i^{th} species, K_i is the environmental carrying capacity of species i in the absence of competition, θ_i provides a nonlinear measure of intraspecific interference, and α_{ij} provides a measure of interspecific interference. Gilpin and Ayala's estimate of θ_i for *Drosophila* suggests that θ_i is typically less than one [4, 9].

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Recently, Fan and Wang [1] investigated the following generalized periodic n-species competition system:

$$y_i'(t) = y_i(t)[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)(y_j(t))^{\theta_{ij}}], y_i(0) > 0, \quad i = 1, 2, \cdots, n. \tag{2}$$

In this model, $\theta_{ij} > 0$, $a_{ij} \in C(R, [0, \infty)), r_i \in C(R, R), i = 1, \cdots, n$ are ω-periodic functions with $\int_0^\omega r_i(t)dt > 0$ and $\int_0^\omega a_{ii}(t)dt > 0$.

By using the method of coincidence degree, Fan and Wang deduced the following sufficient condition for the existence of positive periodic solution:

$$r_i > \sum_{\substack{j=1 \\ j\neq i}}^{n} \bar{a}_{ij}(\frac{\bar{r}_j}{\bar{a}_{jj}})^{\theta_{ij}/\theta_{jj}} e^{\theta_{ij}(\bar{r}_j + \bar{R}_j)\omega}. \tag{3}$$

We know that the birth in many species is not continuous but occurs at fixed time intervals (some wild animals have seasonal births). In the long run, the birth among these species can be considered as an impulse to the system. To describe this phenomenon exactly, we propose the following periodic impulsive system:

$$\begin{cases} y_i'(t) = y_i(t)[-d_i(t) - \sum\limits_{j=1}^{n} a_{ij}(t)(y_j(t))^{\theta_{ij}}], & t \geq 0, \ t \neq t_k, \\ y_i(t_k^+) - y_i(t_k) = b_{ik} y_i(t_k), & i = 1, 2, \cdots, n, \end{cases} \tag{4}$$

where $b_{ik} > 0$ is the birth rate of y_i in t_k. $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$ are fixed impulsive points with $\lim\limits_{k\to\infty} t_k = \infty$; $y_i(t)$ is continuous for $t \in [0, +\infty), t \neq t_k$; $y_i(t_k^+), y_i(t_k^-)$ exist and $y_i(t_k) = y_i(t_k^-)$ for $k = 1, 2, \cdots, \ i = 1, 2, \cdots, n$.

If $b_{ik} \equiv 0$, then (4) has a form similar to (2). So, (4) is the generalization of (2). In this paper, we will study (4) with the following assumptions:

(H_1) $d_i(t)$ is the death rate of y_i at time $t, d_i \in C([0, \infty), [0, \infty)), \ d_i(t + \omega) = d_i(t), i = 1, 2, \cdots, n$;

(H_2) $a_{ij} \in C([0, \infty), [0, \infty)), i, j = 1, 2, \cdots, n, a_{ij}(t+\omega) = a_{ij}(t)$ with $\int_0^\omega a_{ij}(t)dt > 0$; and

(H_3) there exists $q > 0$ such that $t_{k+q} = t_k + \omega, \ b_{i(k+q)} = b_{ik}, k = 1, 2, \cdots, \ i = 1, 2, \cdots, n$.

Without loss of generality, we assume $t_k \neq 0$ and $[0, \omega] \cap \{t_k\} = \{t_1, t_2, \cdots, t_m\}$, so $q = m$.

By the definition of y_i, we have $y_i(0) > 0$. In view of

$$y_i(t) = y_i(0) \exp\{\int_0^t [-d_i(s) - \sum_{j=1}^{n} a_{ij}(s)(y_j(s))^{\theta_{ij}}]ds\}, \quad t \in [0, t_1],$$

$$y_i(t) = y_i(t_k^+) \exp\{\int_{t_k}^t [-d_i(s) - \sum_{j=1}^{n} a_{ij}(s)(y_j(s))^{\theta_{ij}}]ds\}, \quad t \in (t_k, t_{k+1}],$$

$$y_i(t_k^+) = (1 + b_{ik}) y_i(t_k), \ k = 1, 2, \cdots, \ i = 1, 2, \cdots, n,$$

the solution of (4) is positive.

Let $x_i(t) = \ln y_i(t), i = 1, 2, \cdots, n$, then equation (4) is transformed into

$$\begin{cases} x_i'(t) = -d_i(t) - \sum\limits_{j=1}^{n} a_{ij}(t) e^{\theta_{ij} x_j(t)}, & t \geq 0, \ t \neq t_k, \\ x_i(t_k^+) - x_i(t_k) = \ln(1 + b_{ik}), & i = 1, 2, \cdots, n. \end{cases} \tag{5}$$

Hence, the existence of periodic solution of (4) is equivalent to that of (5).

Definition 1.1. A function $x = (x_1, x_2, \cdots, x_n)^T \in ([0, \infty), R^n)$ is said to be a solution of (5) on $[0, \infty)$, if the following conditions are satisfied.

(i) $x(t)$ is continuous on each interval $(t_{k-1}, t_k), k = 1, 2, \cdots$;

(ii) for any $t_k, k = 1, 2, \cdots, x(t_k^+), x(t_k^-)$ exist and $x(t_k^-) = x(t_k)$; and

(iii) $x(t)$ satisfies (5) almost everywhere in $[0, \infty)$ and at impulsive points t_k situated in $(0, \infty)$, may have a discontinuity of the first kind.

2. **Existence of positive periodic solutions.** In this section, we will investigate the existence of positive periodic solutions of (4) based on the coincidence degree theory. For convenience, we first introduce some concepts and results on coincidence degree theory.

Let X, Z be normed vector spaces, $L : \text{dom } L \subset X \to Z$ be a linear mapping, and $N : X \to Z$ be a continuous mapping.

L is said to be a Fredholm mapping of index zero, if dimKer$L$=codimIm$L < +\infty$ and ImL is closed in Z.

If L is a Fredhlom mapping of index zero, then there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$, such that ImP = Ker$L$, Ker$Q$ = ImL = Im$(I - Q)$. It follows that $L \mid$ dom$L\cap$ Ker$P : (I - P)X \to$ ImL is invertible. We denote the inverse of that map by K_P.

The mapping N is said to be L-compact on $\bar{\Omega}$, if Ω is an open bounded subset of X, $QN(\bar{\Omega})$ is bounded and $K_P(I - Q)N : \bar{\Omega} \to X$ is compact.

Since ImQ is isomorphic to Ker$L$, there exists isomorphism J: Im$Q \to$ Ker$L$.

Lemma 2.1. [2]. *Let L be a Fredholm mapping*

(a) for each $\lambda \in (0, 1)$, every solution x of $Lx = \lambda Nx$ satisfies $x \notin \partial\Omega$, and

(b) for each $x \in KerL \cap \partial\Omega$, $QNx \neq 0$ and $deg_B\{JQN, \Omega \cap KerL, 0\} \neq 0$, where deg_B denotes the Brouwer degree, then the equation $Lx = Nx$ has at least one solution in $domL \cap \bar{\Omega}$.

Suppose $J \subset R$ be any interval. Define

$PC[J, R^n] = \{x : J \to R^n, x(t)$ is continuous for $t \in J,\ t \neq t_k$, and $x(t_k^+), x(t_k^-)$ exist and $x(t_k) = x(t_k^-)\}$;

$PC^1[J, R^n] = \{x \in PC[J, R^n], x(t)$ is continuously differentiable, for $t \in J, t \neq t_k$, and $x'(t_k^+), x'(t_k^-)$ exist and $x'(t_k) = x'(t_k^-)\}$.

Obviously, $PC[J, R^n]$ is a Banach space with the norm $\|x\|_{PC} = \sup_{t\in J} \|x(t)\|$, and $PC^1[J, R^n]$ is also a Banach space with the norm $\|x\|_{PC^1} = \max\{\|x\|_{PC}, \|x'\|_{PC}\}$, where $\|\cdot\|$ is any norm of R^n.

The following result follows from an application of Arzela-Ascoli's Theorem.

Lemma 2.2. *$H \subset PC[J, R^n]$ is relatively compact if and only if the functions in H are uniformly bounded on J and equicontinuous on $(t_{k-1}, t_k], k = 1, 2, \cdots, K$, for any fixed $K > 1$.*

For convenience, let

$$\overline{d}_i = \frac{1}{\omega} \int_0^\omega d_i(t)dt$$

$$\gamma_i \stackrel{def}{=} \frac{\sum\limits_{k=1}^{m} \ln(1+b_{ik})}{\omega} - \overline{d}_i.$$

In the following, we will give the main result of this paper.

Theorem 2.3. *Assume $(H_1)-(H_3)$ hold and the system of algebraic equations*

$$g(u) = (\gamma_i - \sum_{j=1}^{n} \bar{a}_{ij} u_j^{\theta_{ij}})_{n\times 1} = 0 \tag{6}$$

has a finite solution $u^* = (u_1^*, \cdots, u_n^*)^T \in R_+^n$ with $u_i^* > 0$ and $\sum_{u^*} sgnJ_g(u^*) \neq 0$. In addition, if $\gamma_i > 0$, and

$$\gamma_i > \sum_{\substack{j=1\\ j\neq i}}^{n} \bar{a}_{ij} \{\frac{\gamma_j}{\bar{a}_{jj}}\}^{\theta_{ij}/\theta_{jj}} [\prod_{k=1}^{m}(1+b_{jk})]^{\theta_{ij}},$$

then (4) *has at least one positive ω-periodic solution, where $\overline{a}_{ij} = \frac{1}{\omega}\int_0^\omega a_{ij}(t)dt,\ i,j = 1,2,\cdots,n$.*

Proof. As stated in Section 1, we need only to prove that (5) has at least one ω-periodic solution.

Let

$$X = \{x \in PC(R,R^n) | x(t+\omega) = x(t), \forall\ t \in R\},$$
$$Z = X \times (R^n)^m.$$

For $x \in X$, take $\| x \|_{PC} = \sup\limits_{t\in[0,\omega]} \|x(t)\|$, where $\|\cdot\|$ is any convenient norm on R^n, and for $z \in Z$, take $\|z\|_Z = \|x\|_{PC} + \|u\|$, where $x \in X, u \in R^{nm}$, and $\|\cdot\|$ is any convenient norm on R^{nm}, then X, Z are both Banach Spaces with the norm $\|\cdot\|_{PC}$ and $\|\cdot\|_Z$, respectively.

Let

$$\text{dom}L = X \cap PC^1(R,R^n),$$
$$L : \text{dom}L \to Z, x \to (x', \triangle x(t_1), \cdots, \triangle x(t_m)),$$
$$N : X \to Z,$$

$$Nx = \left(\begin{pmatrix} h_1 \\ \vdots \\ h_n \end{pmatrix}, \begin{pmatrix} \ln(1+b_{11}) \\ \vdots \\ \ln(1+b_{n1}) \end{pmatrix}, \cdots, \begin{pmatrix} \ln(1+b_{1m}) \\ \vdots \\ \ln(1+b_{nm}) \end{pmatrix} \right),$$

where

$$h_i(t) \stackrel{def}{=} -d_i(t) - \sum_{j=1}^{n} a_{ij}(t) e^{\theta_{ij} x_j(t)},\ i = 1,2,\cdots,n.$$

It is clear that

$$\ker L = \{x : x \in X, x(t) = h, h \in R^n\},$$
$$\text{Im}L = \{z : z = (f, C_1, \cdots, C_m) \in Z, \int_0^\omega f(s)ds + \sum_{k=1}^{m} C_k = 0\}.$$

So, ImL is closed in Z, and dimKer$L = n =$ codimImL. Hence, L is a Fredholm mapping of index zero.

Set

$$Px = \frac{1}{\omega}\int_0^\omega x(t)dt,$$
$$Qz = Q(f, C_1, \cdots, C_m) = (\frac{1}{\omega}[\int_0^\omega f(s)ds + \sum_{k=1}^{m} C_k], 0, 0, \cdots, 0).$$

It is easy to show that P and Q are continuous projectors, such that

$$\mathrm{Im}P = \mathrm{Ker}L, \mathrm{Im}L = \mathrm{Ker}Q = \mathrm{Im}(I - Q).$$

Furthermore, the generalized inverse (to L) K_P: Im $L \to \mathrm{Ker}P \cap \mathrm{Dom}L$ exists.

Set $z = (f, C_1, \cdots, C_m) \in$Im L, then there exists $x \in \mathrm{Ker}P \cap \mathrm{Dom}L$ satisfying

$$\begin{cases} x'(t) = f(t), & t \neq t_k, \ k = 1, 2, \cdots, \\ x(t_k^+) - x(t_k) = C_k, & \end{cases}$$

that is,

$$x(t) = \int_0^t f(s)ds + \sum_{t>t_k} C_k + x(0). \tag{7}$$

Because of $x(t) \in \mathrm{Ker}P$, we have $\int_0^\omega x(t)dt = 0$. So, from (7),

$$\int_0^\omega \int_0^t f(s)dsdt + \int_0^\omega \sum_{t>t_k} C_k dt + \omega x(0) = 0.$$

Then, from the last equation and (7),

$$x(t) = \int_0^t f(s)ds + \sum_{t>t_k} C_k - \frac{1}{\omega}\int_0^\omega \int_0^t f(s)dsdt - \frac{1}{\omega}\sum_{k=1}^{m}(\omega - t_k)C_k. \tag{8}$$

That is,

$$K_P z = \int_0^t f(s)ds + \sum_{t>t_k} C_k - \frac{1}{\omega}\int_0^\omega \int_0^t f(s)dsdt - \frac{1}{\omega}\sum_{k=1}^{m}(\omega - t_k)C_k. \tag{9}$$

$$QNx = \left(\begin{pmatrix} \frac{1}{\omega}\{\int_0^\omega h_1(t)dt + \sum_{k=1}^{m} \ln(1 + b_{1k})\} \\ \vdots \\ \frac{1}{\omega}\{\int_0^\omega h_n(t)dt + \sum_{k=1}^{m} \ln(1 + b_{nk})\} \end{pmatrix}, 0, \cdots, 0 \right),$$

$$K_P(I-Q)Nx=\begin{pmatrix}\int_0^t h_1(s)ds+\sum\limits_{t>t_k} ln(1+b_{1k})\\ \vdots \\ \int_0^t h_n(s)ds+\sum\limits_{t>t_k}\ln(1+b_{nk})\end{pmatrix}$$
$$-\begin{pmatrix}\frac{1}{\omega}\int_0^\omega\int_0^t h_1(s)dsdt+\frac{1}{\omega}\sum\limits_{k=1}^m(\omega-t_k)\ln(1+b_{1k})\\ \vdots \\ \frac{1}{\omega}\int_0^\omega\int_0^t h_n(s)dsdt+\frac{1}{\omega}\sum\limits_{k=1}^m(\omega-t_k)\ln(1+b_{nk})\end{pmatrix}$$
$$-\begin{pmatrix}(\frac{t}{\omega}-\frac{1}{2})\{\int_0^\omega h_1(s)ds+\sum\limits_{k=1}^m\ln(1+b_{1k})\}\\ \vdots \\ (\frac{t}{\omega}-\frac{1}{2})\{\int_0^\omega h_n(s)ds+\sum\limits_{k=1}^m\ln(1+b_{nk})\}\end{pmatrix}.$$

Clearly, QN and $K_P(I-Q)N$ are continuous. By Lemma 2.2, we can easily show that $K_P(I-Q)N(\bar{\Omega})$ is relatively compact for any open bounded set $\Omega\subset X$. Moreover, $QN(\bar{\Omega})$ is bounded. Thus, N is $L-$compact on $\bar{\Omega}$ for any open bounded set $\Omega\subset X$.

To apply Lemma 2.1, we have to obtain an appropriate open bounded subset Ω. Corresponding to the operator equation $Lx=\lambda Nx$ with $\lambda\in(0,1)$, we have

$$\begin{cases} x_i'(t)=\lambda[-d_i(t)-\sum_{j=1}^n a_{ij}(t)e^{\theta_{ij}x_j(t)}], & t\neq t_k,\ k=1,2,\cdots,\\ x_i(t_k^+)-x_i(t_k)=\lambda\ln(1+b_{ik}), & i=1,2,\cdots,n,\\ x_i(0)=x_i(\omega). \end{cases}\tag{10}$$

Integrating (10) from 0 to ω, we have

$$\int_0^\omega[-d_i(s)-\sum_{j=1}^n a_{ij}(s)e^{\theta_{ij}x_j(s)}]ds+\sum_{k=1}^m\ln(1+b_{ik})=0,\ i,j=1,2,\cdots,n,$$

that is,

$$\sum_{j=1}^n\int_0^\omega a_{ij}(t)e^{\theta_{ij}x_j(t)}dt=\gamma_i\omega.\tag{11}$$

From (10) and (11), it follows that

$$\int_0^\omega|x_i'(t)|dt\le\bar{d}_i\omega+\sum_{j=1}^n\int_0^\omega a_{ij}(t)e^{\theta_{ij}x_j(t)}dt=\sum_{k=1}^m\ln(1+b_{ik}).\tag{12}$$

Since $x(t)\in X$, there exists $\xi_i\in[0,\omega]$ such that

$$x_i(\xi_i)=\min_{t\in[0,\omega]}x_i(t),\quad i=1,2,\cdots,n.\tag{13}$$

From (11) and (13), we have

$$\omega\bar{a}_{ii}e^{\theta_{ii}x_i(\xi_i)}\le\gamma_i\omega,\ i=1,2,\cdots,n.$$

Moreover,

$$x_i(\xi_i)\le\frac{1}{\theta_{ii}}\ln\{\frac{\gamma_i}{\bar{a}_{ii}}\},\ i=1,2,\cdots,n.\tag{14}$$

Then

$$x_i(t) \le x_i(\xi_i) + \int_0^\omega |x_i'(t)|dt \le \frac{1}{\theta_{ii}} \ln\{\frac{\gamma_i}{\bar{a}_{ii}}\} + \sum_{k=1}^m \ln(1+b_{ik}) \overset{def}{=} M_i^+. \tag{15}$$

On the other hand, since $\sup\limits_{t\in[0,\omega]} x_i(t)$ exists, there exists $\eta_i \in [0,\omega]$ satisfying

$$x_i(\eta_i^+) = \sup_{t\in[0,\omega]} x_i(t), \ i = 1, 2, \cdots, n. \tag{16}$$

From (16), if $\eta_i \neq t_k$, then $x_i(\eta_i^+) = x_i(\eta_i)$; if $\eta_i = t_k$, then $x_i(\eta_i^+) = x_i(t_k^+)$. From (11) and (16), it follows that

$$\bar{a}_{ii} e^{\theta_{ii} x_i(\eta_i^+)} \ge \gamma_i - \sum_{\substack{j=1\\ j\neq i}}^n \bar{a}_{ij} e^{\theta_{ij} x_j(\eta_j^+)}.$$

That is

$$x_i(\eta_i^+) \ge \frac{1}{\theta_{ii}} \ln \frac{\gamma_i - \sum\limits_{\substack{j=1\\ j\neq i}}^n \bar{a}_{ij} (\frac{\gamma_j}{\bar{a}_{jj}})^{\theta_{ij}/\theta_{jj}} [\prod\limits_{k=1}^m (1+b_{jk})]^{\theta ij}}{\bar{a}_{ii}} \overset{def}{=} M_i. \tag{17}$$

From (12) and (17),

$$\begin{aligned} x_i(t) \quad &\ge x_i(\eta_i^+) - \textstyle\int_0^\omega |x_i'(t)|dt \\ &\ge M_i - \sum_{k=1}^m \ln(1+b_{ik}) \overset{def}{=} M_i^-. \end{aligned} \tag{18}$$

By (15) and (18), we have

$$\sup_{t\in[0,\omega]} |x_i(t)| < \max\{|M_i^+|, |M_i^-|\} \overset{def}{=} H_i. \tag{19}$$

It is evident that, H_i is independent of the choice of λ.

Let $H = \|(H_1, H_2, \cdots, H_n)^T\| + C$, where C is large enough so that the unique solution of (6) satisfies $\|(\ln\{u_1^*\}, \ln\{u_2^*\}, \cdots, \ln\{u_n^*\})^T\| < C$, then $\|x\|_{PC} < H$.

Let $\Omega = \{x \in X : \|x\|_{PC} < H\}$. It is clear that Ω satisfies Condition (a) in Lemma 2.1. When $x \in KerL \cap \partial\Omega = R^n \cap \partial\Omega$, x is a constant vector in R^n with $\|x\| = H$. Then

$$\begin{aligned} QNx \quad &= \left(\begin{pmatrix} -\bar{d}_1 - \sum\limits_{j=1}^n \bar{a}_{1j} e^{\theta_{1j} x_j} + \frac{1}{\omega} \sum\limits_{k=1}^m \ln(1+b_{1k}) \\ \vdots \\ -\bar{d}_n - \sum\limits_{j=1}^n \bar{a}_{nj} e^{\theta_{nj} x_j} + \frac{1}{\omega} \sum\limits_{k=1}^m \ln(1+b_{nk}) \end{pmatrix}, 0, \cdots, 0 \right) \\ &= \left(\begin{pmatrix} \gamma_1 - \sum\limits_{j=1}^n \bar{a}_{1j} e^{\theta_{1j} x_j} \\ \vdots \\ \gamma_n - \sum\limits_{j=1}^n \bar{a}_{nj} e^{\theta_{nj} x_j} \end{pmatrix}, 0, \cdots, 0 \right) \neq 0. \end{aligned}$$

Take J: Im$Q \to$ Ker$L$, $(d, 0, \cdots, 0) \to d$; then if $x \in KerL \cap \Omega$, we have

$$JQNx = \begin{pmatrix} \gamma_1 - \sum\limits_{j=1}^{n} \bar{a}_{1j} e^{\theta_{1j} x_j} \\ \vdots \\ \gamma_n - \sum\limits_{j=1}^{n} \bar{a}_{nj} e^{\theta_{nj} x_j} \end{pmatrix}.$$

Furthermore, in view of the assumptions in Theorem 2.3, it is easy to prove that

$$deg\{JQNx, \Omega \cap KerL, 0\} \neq 0.$$

Now we have proved that Ω satisfies all the conditions in Lemma 2.1. Hence by Lemma 2.1, (5) has at least one ω-periodic solution $x^*(t)$ in $\bar{\Omega}$. So, $y^*(t) = (y_1^*(t), \cdots, y_n^*(t))^T$ with $y_i^*(t) = e^{x_i^*(t)}$ is a positive ω-periodic solution of (4).

□

Corollary 1. *Assume* $(H_1) - (H_3)$ *hold and the system of algebraic equations*

$$\sum_{j=1}^{n} \bar{a}_{ij} u_j^{\theta_{ij}} = \gamma_i$$

has a unique solution $u^* = (u_1^*, \cdots, u_n^*)^T \in R_+^n$ *with* $u_i^* > 0$, *the* $n \times n$ *matrix* $(p_{ij})_{n\times n}$ *with* $p_{ij} = \theta_{ij} \bar{a}_{ij} u_j^{*\theta_{ij}-1}$ *is nonsingular and* $\gamma_i > 0$,

$$\gamma_i > \sum_{\substack{j=1 \\ j\neq i}}^{n} \bar{a}_{ij} \{\frac{\gamma_j}{\bar{a}_{jj}}\}^{\theta_{ij}/\theta_{jj}} [\prod_{k=1}^{m} (1 + b_{jk})]^{\theta_{ij}};$$

then (4) has at least one ω*-periodic solution with strictly positive components.*

Corollary 2. *Assume* $(H_1) - (H_3)$ *hold and the system of algebraic equations*

$$\bar{a}_{ii}(u_i)^{\theta_{ii}} + \sum_{\substack{j=1 \\ j\neq i}}^{n} \bar{a}_{ij} u_j = \gamma_i$$

has a unique solution $u^* = (u_1^*, \cdots, u_n^*)^T \in R_+^n$ *with* $u_i^* > 0$, *the* $n \times n$ *matrix* $(p_{ij})_{n\times n}$ *with* $p_{ii} = \theta_{ii} \bar{a}_{ii} u_i^{*\theta_{ii}-1}$ *and* $p_{ij} = \bar{a}_{ij}$ *for* $j \neq i$ *is nonsingular and* $\gamma_i > 0$,

$$\gamma_i > \sum_{\substack{j=1 \\ j\neq i}}^{n} \bar{a}_{ij} \{\frac{\gamma_j}{\bar{a}_{jj}}\}^{1/\theta_{jj}} [\prod_{k=1}^{m} (1 + b_{jk})].$$

Then (4) with $\theta_{ij} \equiv 1 (i \neq j)$ *has at least one* ω*-periodic solution with strictly positive components.*

Lemma 2.4. *Assume* $(H_1) - (H_3)$ *hold and* $\gamma_i > 0$,

$$\gamma_i > \sum_{\substack{j=1 \\ j\neq i}}^{n} \bar{a}_{ij} \frac{\gamma_j}{\bar{a}_{jj}};$$

then the system of algebraic equations

$$\sum_{j=1}^{n} \bar{a}_{ij} u_j = \gamma_i$$

has a unique solution $u^* = (u_1^*, \cdots, u_n^*)^T \in R_+^n$ *with* $u_i^* > 0$.

Proof. The proof is similar to that of Lemma 4.1.1 in [6]; we omit it here. □

Corollary 3. *Assume* $(H_1)-(H_3)$ *hold and* $\theta_{ij}\equiv\theta_j$ *with* $\theta_j>0$*; then if* $\gamma_i>0$ *and*

$$\gamma_i>\sum_{\substack{j=1\\ j\neq i}}^{n}\bar{a}_{ij}\frac{\gamma_j}{\bar{a}_{jj}}[\prod_{k=1}^{m}(1+b_{jk})]^{\theta_j},$$

(4) *has at least one* ω*-periodic solution with strictly positive components.*

Proof. Note that

$$\gamma_i>\sum_{\substack{j=1\\ j\neq i}}^{n}\bar{a}_{ij}\frac{\gamma_j}{\bar{a}_{jj}}[\prod_{k=1}^{m}(1+b_{jk})]^{\theta_j}>\bar{a}_{ij}\frac{\gamma_j}{\bar{a}_{jj}};$$

then from Lemma 2.4, it follows that the system of algebraic equation $\sum_{j=1}^{n}\bar{a}_{ij}(u_j)^{\theta_j}=\gamma_i$ has a unique solution $(u_1^*,\cdots,u_n^*)^T\in R_+^n$ with $u_i^*>0$. The conclusion follows immediately from Theorem 2.3.

□

Corollary 4. *Assume* $(H_1)-(H_3)$ *hold,* $\gamma_i>0$ *and*

$$\gamma_i>\sum_{\substack{j=1\\ j\neq i}}^{n}\bar{a}_{ij}\frac{\gamma_j}{\bar{a}_{jj}}[\prod_{k=1}^{m}(1+b_{jk})].$$

Then the Lotka-Volterra n*-species competition system with impulses, namely* (4) *with* $\theta_{ij}\equiv 1$*, has at least one* ω*-periodic solution with strictly positive components.*

Remark 1. Obviously, if $\theta_{ij}\equiv 1$ and $n=2$, Corollary 4 is reduced to Theorem 2.1 in [8] on Lotka-Volterra two species competition system. Therefore, our results are new and extend those in [8].

Remark 2. The results in this paper indicate that under certain perturbations the generalized n-species periodic Gilpin-Ayala competition system preserves the original periodicity of nonimpulsive system (2).

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