MATHEMATICAL BIOSCIENCES
AND ENGINEERING
Volume **4**, Number **4**, October **2007**

http://www.mbejournal.org/

pp. **573–594**

EVALUATING GROWTH MEASURES IN AN IMMIGRATION PROCESS SUBJECT TO BINOMIAL AND GEOMETRIC CATASTROPHES

J.R. Artalejo

Faculty of Mathematics, Complutense University of Madrid
Madrid 28040, Spain

A. Economou

Department of Mathematics, University of Athens
Panepistemiopolis, Athens 15784, Greece

M.J. Lopez-Herrero

School of Statistics, Complutense University of Madrid
Madrid 28040, Spain

(Communicated by Yang Kuang)

Abstract. Populations are often subject to the effect of catastrophic events that cause mass removal. In particular, metapopulation models, epidemics, and migratory flows provide practical examples of populations subject to disasters (e.g., habitat destruction, environmental catastrophes). Many stochastic models have been developed to explain the behavior of these populations. Most of the reported results concern the measures of the risk of extinction and the distribution of the population size in the case of total catastrophes where all individuals in the population are removed simultaneously. In this paper, we investigate the basic immigration process subject to binomial and geometric catastrophes; that is, the population size is reduced according to a binomial or a geometric law. We carry out an extensive analysis including first extinction time, number of individuals removed, survival time of a tagged individual, and maximum population size reached between two consecutive extinctions. Many explicit expressions are derived for these system descriptors, and some emphasis is put to show that some of them deserve extra attention.

1. **Introduction.** Stochastic models for the growth of populations subject to the effect of random catastrophes have been extensively studied in the literature (see, e.g., Adler and Nüernberger, 1994; Bartoszynski et al., 1989; Brockwell et al., 1982; Casagrandi and Gatto, 2002; Economou, 2004; Pakes, 1987; Pollet, 2001). Often, the mathematical models are treated in the framework provided by continuous-time Markov processes. More specifically, the most widely studied models use birth-and-death processes and their generalizations.

The catastrophe mechanism, in its simplest version, instantaneously removes the whole population (total catastrophe case) whenever a catastrophic event occurs.

2000 *Mathematics Subject Classification.* Primary: 92D25; Secondary: 60J99.

Key words and phrases. Disasters, environmental catastrophes, extinction time, immigration process, Markov chain, maximum population size, metapopulation dynamics, persistence time, survival time.