MATHEMATICAL BIOSCIENCES
AND ENGINEERING
Volume **4**, Number **1**, January **2007**

http://www.mbejournal.org/

pp. **101–112**

OPTIMAL CONTROL FOR MANAGEMENT OF AN INVASIVE PLANT SPECIES

Andrew J. Whittle

Department of Mathematics, University of Tennessee
Knoxville, Tennessee 37996-1300

Suzanne Lenhart

Department of Mathematics, University of Tennessee
Knoxville, Tennessee 37996-1300

Louis J. Gross

Department of Ecology and Evolutionary Biology, University of Tennessee
Knoxville, Tennessee 37996-1610

Abstract. Invasive plant populations typically consist of a large (main) focus and several smaller outlier populations. Management of the spread of invasives requires repeated control measures, constrained by limited funding and effort. Posing this as a control problem, we investigate whether it is best to apply control to the main focus, the outlier populations, or some combination of these. We first formulate and solve a discrete-time optimal control problem to determine where control is best applied over a finite time horizon. However, if limited funds are available for control, this optimal solution may not be feasible. In this case, we add an additional constraint to account for the fixed budget and solve the new optimality system. Our results have a variety of practical implications for invasive species management.

1. Introduction.

Modern transport has greatly expanded the rate of movement of species across the planet to regions in which they were not native. Although the majority of species propagules transported to new habitats are unable to survive and spread, a small fraction of such species produces successful invasives. Such invasives can cause a variety of detrimental impacts on natural systems, including reduction in biodiversity, habitat condition changes detrimental to native populations, and, in a number of cases, these invasives have had major economic impact [6]. A wide variety of programs exist around the world to limit the spread of harmful invasives, with different control measures (e.g., spraying herbicides, cutting, and controlled fires) applied depending upon local conditions and the treatments which are most effective at controlling the invasive. Control attempts have had mixed success in limiting invasive spread, and appropriate guidance for resource managers is needed to enhance the cost-effectiveness of control measures [10].

Invasion spread can occur in a variety of ways, depending upon the transport, dispersal, and growth properties of the particular species. Our emphasis in this paper is on invasive plants, though our techniques are generally applicable to animal

2000 *Mathematics Subject Classification.* 92D40.
Key words and phrases. optimal control, invasive species, discrete time, biological invasions, natural resources.

and disease spread as well. The spread of invasive plants typically begins from a focus of infection, with outlier populations developing through time as long-distance dispersal carries plant propagules to appropriate habitat locations distant from the primary initial infection location. This situation is the one we consider here, though there are examples of spread that are more appropriately described as a traveling wave front, or arise from continuous input from an outside source so that numerous invasive populations arise at disparate locations at the same time.

A wide variety of mathematical models have been applied to the spread of invasives (see Shigesada and Kawasaki [9]; Petrovskii and Li [5]). Very little of the modeling literature on invasives deals explicitly with the control problem, however. Our objective here is to analyze a general model that accounts for spread from a major focus and a collection of outlier populations. However, other control models have been developed for specific invasives, including the application of mathematical programming methods to investigate optimal spatial patterns of control [3].

Recent work by Cacho et al. [1] considered an invasion of a single focus with constant radial growth. Control was applied at discrete time intervals by treating the invasion so that its radius was reduced, and this treatment was repeated for several years to account for seed banks. They then maximized the uninfested area and minimized the cost of control over the time intervals, using a discount factor. However, they assumed that the cost of treating an area increases linearly. They found that one of three scenarios was optimal: eradication, containment (no increase or decrease in size), or no control depending on the initial size of the focus.

One of the most frequently cited papers on control of plant invasives is Moody and Mack [4], and we use this as a starting point for our investigation. This paper considered control of a biological invasion from an expanding point source that produced satellites. The authors' analysis compared two control scenarios: apply control to the main focus only or apply control to satellites only. They assume control was applied once at the initial point in time, and then the long-term (asymptotic) situation was considered to compare the effects of the alternative controls on growth of the invasives. Although this provides good insight on the control problem, the model is limited by the assumption of a single initial control and by the use of an asymptotic solution when the growth dynamics is more appropriately valid only over short periods, because of the eventual coalescence of the separate populations. Despite these drawbacks, the paper is often cited and advocated by natural resource managers. Expansion on the mathematical control scenarios investigated in Moody and Mack has been very limited, despite the fact that their scenario is quite common. We formulate a model similar to that investigated by Moody and Mack, allowing for a finite period, with control allowed to be applied once in each time step. This formulation is simple enough to readily allow comparisons of the results to those of Moody and Mack. We also consider the influence of total cost constraints on the application of the controls.

In the next section, we describe our discrete time model for the growth of foci of infection and explain the control format and the objective functional. In Section 3, we explain the methods used to solve the optimal control problem and explicitly give the adjoint system and the characterization of the optimal control. We apply an extension of the Pontryagin Maximum Principle to discrete time systems [2, 7, 8]. In the last section, we present our results from numerical solutions of the optimality system, which is the state system coupled with the adjoint system along with the control characterization. We also discuss our conclusions and the limitations and possible extensions of this work.

2. **Mathematical model.**

Moody and Mack [4] initially considered a finite number of satellites with a main focus and either allowed all satellites to be eradicated or an amount of the main focus to be removed, that amount being equivalent to the total area of all satellites (assuming the focus and the satellites were disks). Their model is phrased in terms of the radii of the main focus and the satellite populations. They then allowed the invasive populations to expand and found that the total area invaded was always greater, after a sufficiently long period of time, for the second scenario (removing area from the main focus). This is not surprising, as after an initial transient period, all disks become so large that asymptotically the area becomes the same for each foci. Thus having two or more satellites remaining will give greater area than just one remaining. During the transient period, the main focus, due to its size, is larger in area than the satellites since all the disks grow at the same rate. However, the satellites catch up, and only the number of infestations matters.

Using a discrete time model, we consider a finite time horizon and allow control at each time step, for a similar situation. The equations for the radius of the invasion over time are given by

$$r_{j,t+1} = \left(r_{j,t} + \frac{r_{j,t}k}{\varepsilon + r_{j,t}} \right)(1 - \beta_{j,t})$$

for $t = 0, 1...T-1$, where $r_{j,t}$ represents the radius of the focus j at time t. The initial size of the foci will determine which is the main focus and will be specified in the numerical simulations. The spread rate is given by k and is scaled by $\frac{r}{\varepsilon+r}$ for ε small. The scaling ensures that if a focus is eradicated, $r_{j,t} = 0$, it remains eradicated and does not grow back. The control coefficient $\beta_{j,t}$ is the amount of radius decrease due to control at time t for the focus j. We form an objective functional comprising the costs to be minimized. Our objective will therefore be to minimize the area covered by the invasive species at the end of the time period. Coupled with this is the cost of the control over the entire time period, which we take to be quadratic for simplicity of the control analysis. Hence with controls $\beta = (\beta_1, \beta_2, ..., \beta_n)$ where $\beta_j = (\beta_{j,0}, \beta_{j,1}, ..., \beta_{j,T-1})$, our cost functional or objective functional is given by

$$J(\beta) = J_1(\beta) + J_2(\beta)$$

where

$$\begin{aligned} J_1(\beta) &= \sum_{j=1}^{n} r_{j,T}^2 \\ J_2(\beta) &= \sum_{j=1}^{n} \sum_{t=0}^{T-1} B\beta_{j,t}^2. \end{aligned}$$

The coefficient B is a positive-valued balancing constant that adjusts the cost of control relative to the economic cost of the invasive. We seek to minimize $J(\beta)$ over controls with components $0 \le \beta_{j,t} \le 1$.

3. **Methods.** The optimal control can be determined by solving the state system and its corresponding adjoint system. The adjoint system is found from the formation of a Hamiltonian. Applying an extension of Pontryagin's Maximum Principle [2] to discrete systems, we find a characterization for the optimal control for the system. Thus we obtain the following theorem.

THEOREM 3.1. *Given an optimal control β_j^* for $j = 1...n$, and corresponding state solutions r_j, there exist adjoint variables λ_j such that the adjoint variables satisfy the system*

$$\lambda_{j,t-1} = \lambda_{j,t}(1 - \beta^*_{j,t})\left(1 + \frac{\varepsilon k}{(\varepsilon + r_{j,t})^2}\right) \tag{1}$$

for $t = 1, 2...T - 1$ with the transversality conditions

$$\lambda_{j,T-1} = 2r_{j,T}. \tag{2}$$

Furthermore the optimal control is given by

$$\beta^*_{j,t} = \max\left(0, \min\left(\frac{\lambda_{j,t}}{2B}\left(r_{j,t} + \frac{r_{j,t}k}{\varepsilon + r_{j,t}}\right), 1\right)\right). \tag{3}$$

Proof. The Hamiltonian is given by

$$H = \sum_{j=1}^{n}\left[r^2_{j,T} + \sum_{t=0}^{T-1} B\beta^2_{j,t} + \lambda_{j,t}\left[\left(r_{j,t} + \frac{r_{j,t}k}{\varepsilon + r_{j,t}}\right)(1 - \beta_{j,t})\right]\right].$$

Applying the extension of Pontryagin's Maximum Principle, we obtain

$$\lambda_{j,t-1} = \frac{\partial H}{\partial r_{j,t}}$$

for $t = 1...T$, evaluated at the optimal control and corresponding states, which gives system (1) and (2). Using the optimality conditions,

$$\frac{\partial H}{\partial \beta_{j,t}} = 2B\beta^*_{j,t} - \lambda_{j,t}\left(r_{j,t} + \frac{r_{j,t}k}{\varepsilon + r_{j,t}}\right) = 0$$

on the set $\{(j,t)|0 < \beta^*_{j,t} < 1\}$ and solving for $\beta^*_{j,t}$ subject to the constraints, the characterization is derived. □

When controlling an invasion, the amount of control (or at least of funds used for control) may be limited. For example, an agency may give a pool of money only for controlling an invasive for a finite period of time. Thus, the optimal solution found previously may require more money than is realistically possible. Therefore, we create an extra constraint to insure that the sum of the cost of the controls is fixed,

$$\begin{aligned} m &= J_2(\beta) \\ &= \sum_{j=1}^{n}\sum_{t=0}^{T-1} B\beta^2_{j,t}, \end{aligned}$$

where m is the fixed allocation of money available for the entire time period. Adding this constraint gives a smaller class of feasible controls than those for the unconstrained problem. Also, as the cost of the control is fixed, the objective functional becomes $J(\beta) = J_1(\beta)$ over the class of controls with the sum constraint and the same upper and lower bounds on components. We therefore have the following theorem.

THEOREM 3.2. *Assume $m < Bn$. For objective functional J_1, given an optimal control β_j^* for $j = 1...n$ where $m = J_2(\beta)$ and corresponding state solution r_j, there exist adjoint variables λ_j and a constant μ such that the adjoint variables satisfy the system*

$$\lambda_{j,t-1} = \lambda_{j,t}(1-\beta_{j,t}^*)\left(1+\frac{\varepsilon k}{(\varepsilon + r_{j,t})^2}\right)$$

for $t = 0, 1...T-1$ with the transversality conditions

$$\lambda_{j,T-1} = 2r_{j,T}.$$

Furthermore the optimal control is given by

$$\beta_{j,t}^* = \max\left(0, \min\left(\frac{\lambda_{j,t}}{2B\mu}\left(r_{j,t}+\frac{r_{j,t}k}{\varepsilon + r_{j,t}}\right), 1\right)\right).$$

Proof. We introduce a new state variable M_t to track the amount of control used up to year t. Therefore

$$M_{t+1} = M_t + \sum_{j=1}^{n} B\beta_{j,t}^2$$

for $t = 0...T-1$ with initial condition $M_0 = 0$ and terminal condition $M_T = m$. As this is another state, we have another adjoint variable. Thus the Hamiltonian is given by

$$\begin{aligned} H &= \sum_{j=1}^{n}\left[r_{j,T}^2 + \sum_{t=0}^{T-1}\lambda_{j,t}\left[\left(r_{j,t}+\frac{r_{j,t}k}{\varepsilon + r_{j,t}}\right)(1-\beta_{j,t})\right]\right] \\ &\quad + \sum_{t=0}^{T-1}\mu_t\left[M_t + \sum_{j=1}^{n}B\beta_{j,t}^2\right]. \end{aligned}$$

Applying the extension of Pontryagin's Maximum Principle, the adjoint system becomes

$$\begin{aligned} \lambda_{j,t-1} &= \tfrac{\partial H}{\partial r_{j,t}} \\ \mu_{t-1} &= \tfrac{\partial H}{\partial M_t} = \mu_t \end{aligned}$$

for $t = 1...T$ evaluated at the optimal control and corresponding states with final time condition $\lambda_{j,T-1} = 2r_{j,T}$. Note μ_t is constant for all t; i.e., $\mu_t = \mu$ for $t = 0, 1, ...T-1$. One can show if $\mu = 0$, then for all $j = 1, 2..., n$, $\beta_{j,t} = 1$ for some $t\epsilon\{0, 1..., T-1\}$, which contradicts $m < Bn$. Hence we can assume $\mu \neq 0$. Now by considering the optimality condition,

$$\frac{\partial H}{\partial \beta_{j,t}} = 2B\mu\beta_{j,t}^* + \lambda_{j,t}\left(r_{j,t}+\frac{r_{j,t}k}{\varepsilon + r_{j,t}}\right) = 0,$$

on the set $\{(j,t)|0 < \beta_{j,t}^* < 1\}$ and solving for $\beta_{j,t}^*$ subject to the bounds, the characterization is derived. □

REMARK 3.1. *Although the upper bound on $J_2(\beta)$ is nBT, the realistic upper bound, taking the dynamics into account, is nB. If $\beta_{j,s}^* = 1$ for some j, s then $r_{j,t} = 0$ for all $t > s$, which implies $\beta_{j,t}^* = 0$ for all $t > s$. Once a focus is eliminated, there is no need to apply control to it afterwards. Therefore, for each j, one would only choose $\beta_{j,s}^* = 1$ for at most one time s. Consequently if $m \geq nB$, then the optimal strategy would be to eliminate every focus.*

4. **Numerical results.** We solved the optimality systems by an iterative method with forward-solving of the state system followed by backward-solving of the adjoint system using MATLAB. We start with an initial guess for the control at the first iteration and then before each following iteration, we update the control by using the characterization. We continued until convergence of successive iterates was achieved.

When there is a fixed cost for the total amount of control we solve the optimality system in a similar way. We let M_T be *free* and then find the unknown adjoint variable μ by exhaustive search method, such that $M_T = m$.

To illustrate our results, we choose the initial radii to be $r_{1,0} = 10$, $r_{2,0} = 0.5$, $r_{3,0} = 1.0$, $r_{4,0} = 1.5$, $r_{5,0} = 2.0$ and $r_{6,0} = 2.5$. We designate the first focus with the largest radius as the main focus and the others are considered to be satellites. Figures 1 and 2 show optimal control solutions for different cost coefficients, B, with $k = 1$ and $\varepsilon = 0.01$.

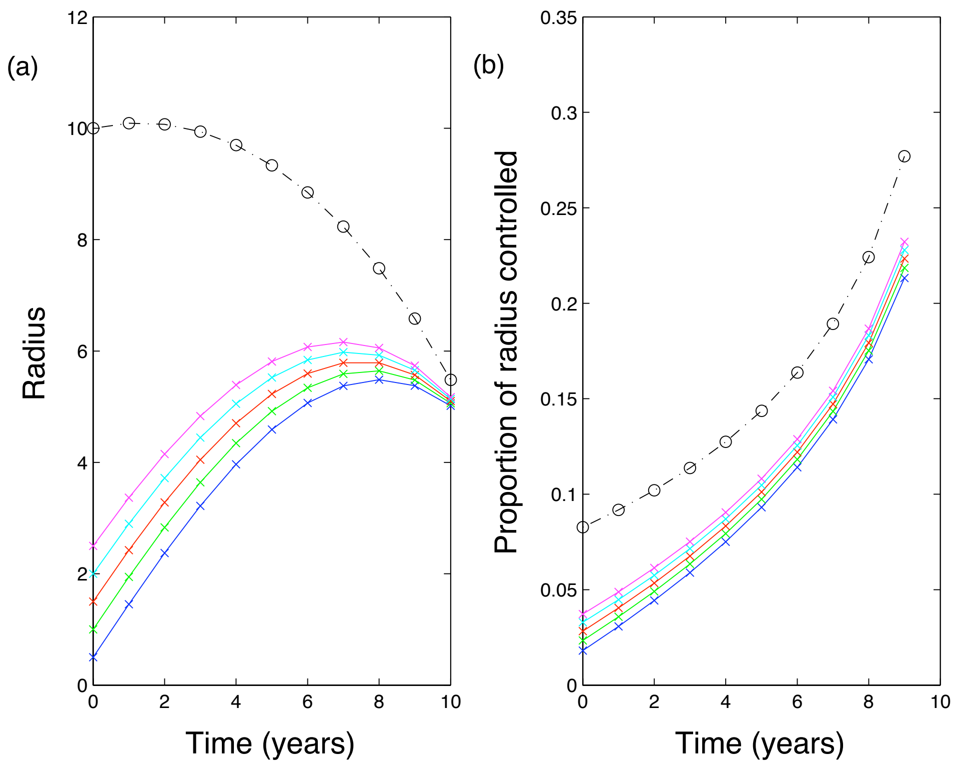

FIGURE 1. The optimal control solution for $B = 150$. Figure (a) shows the radii of 6 populations. Figure (b) shows the portion of radius removed from each focus.

We find that for the cost coefficient $B = 150$, the main focus and the satellites are reduced by an increasing amount each year (see Figure 1(b)). However, initially the reduction in radii of the satellites is not as great as their growth, so the satellites increase in size (Figure 1(a)). Yet toward the end of the period, the reduction in area becomes greater than the satellite's growth, reducing the satellite's size. For the radius of the main focus, there is a slight initial increase in size for the first year, followed by a decrease each subsequent year. By the end of the period, the main focus has been reduced to about half its original radius. The total amount of control used each year increases, with more control applied to the main focus compared to the satellites. This has the effect of reducing the main focus size to a size comparable to the satellites'. For larger values of B, it is more costly to control the invasion, and therefore less control is used on both the main focus and the satellites. However, the proportion of control applied to the main focus in comparison to the satellites increases, as B increases.

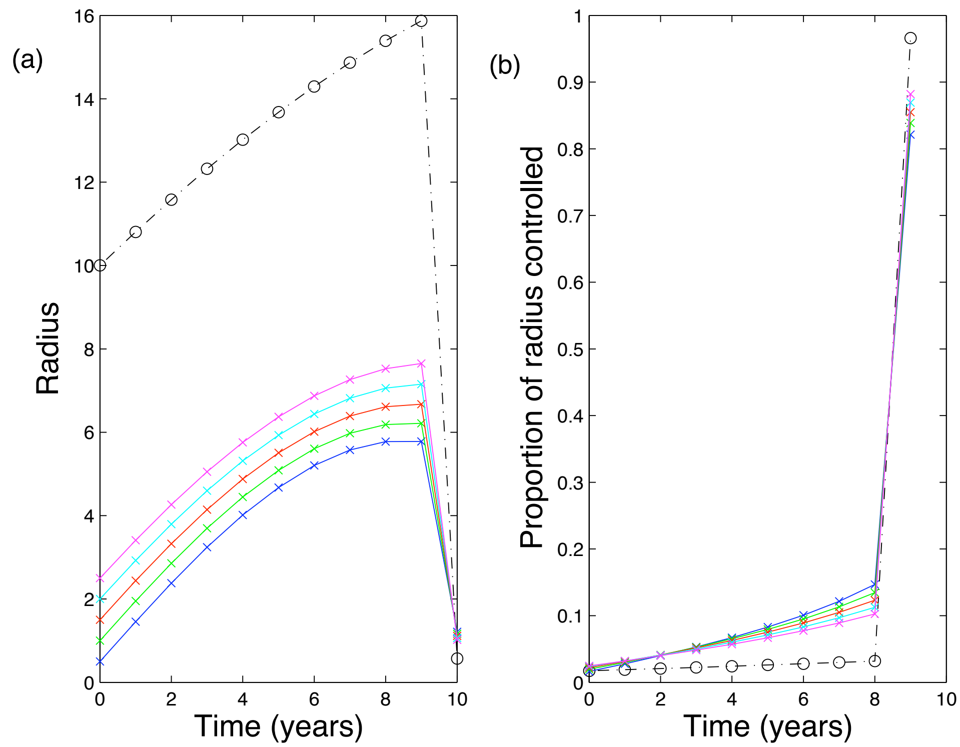

FIGURE 2. The optimal control solution for $B = 10$. Figure (a) shows the radii of 6 populations. Figure (b) shows the portion of radius removed from each focus.

In Figure 2(b) we see that for a lower cost coefficient, $B = 10$, the optimal solution is to apply little control to both the main focus and the satellites until the penultimate time step. Then large amounts of control are used to reduce the invasion to a small amount (Figure 2(a)). In this case, it has become less important to control the main focus and satellites throughout most of the time period. This is due to the low cost coefficient, B, and also because, in this model, the removal of a proportion of a disk with a large radius incurs the same cost as removing the same proportion of the radius of a small disk. We also note that throughout the majority of the period (before the penultimate time step), a greater proportion of radius is removed from the satellites than from the main focus. So the optimal control strategies can be very different depending on the cost coefficient, B.

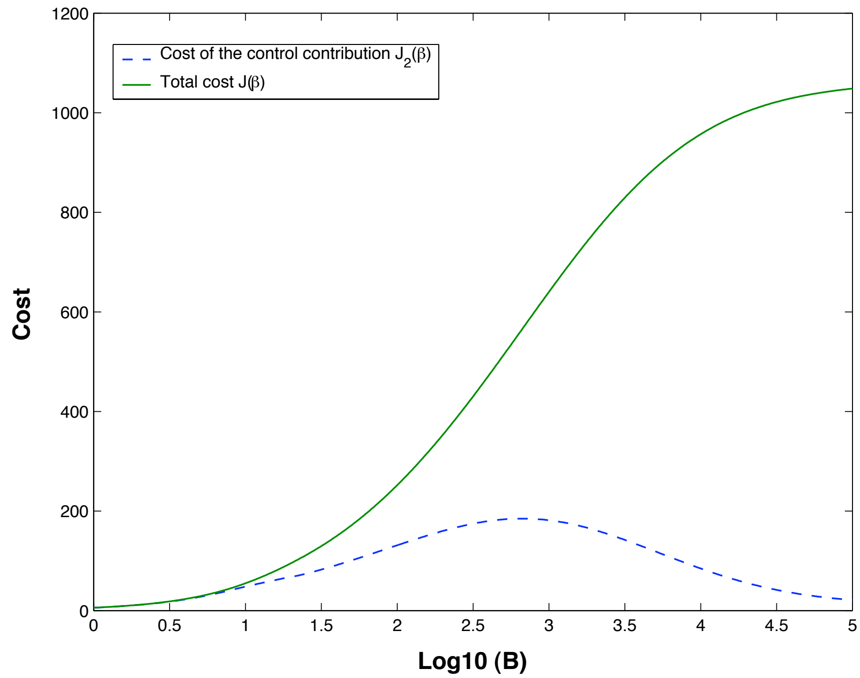

FIGURE 3. Costs $J(\beta)$ and $J_2(\beta)$ against cost per unit area of control, B, for the optimal solution.

Figure 3 shows how the total cost, $J(\beta)$, and the cost of controlling the invasion, $J_2(\beta)$, changes against the cost coefficient, B. The total cost (solid line) of the optimal strategy increases monotonically as the cost coefficient increases. We see that the total cost increases significantly in the interval [1,4] for $\log_{10}(B)$. For low values of B, we see that the contribution from the cost of controlling the invasion to the total cost is very high (solid and dashed lines are very close together); thus, the area of the invasion is small at the final time. As B increases, the cost of controlling the invasion also increases, however, the contribution to the total cost decreases (dashed line increases slower than solid line) and the area of the invasion is larger at the final time. Further increases in B cause the cost of the amount of control to peak, and for larger values of B, the amount of control is reduced.

The above situations did not constrain the amount of control, so we next consider the problem when limited resources or funding allow only a finite amount of control. We assume the constraint on the total cost of the control is, $m = 38.6$ for cost coefficient $B = 20$. Figure 4 shows the difference in optimal solutions with and without the constraint on the total amount spent on the control.

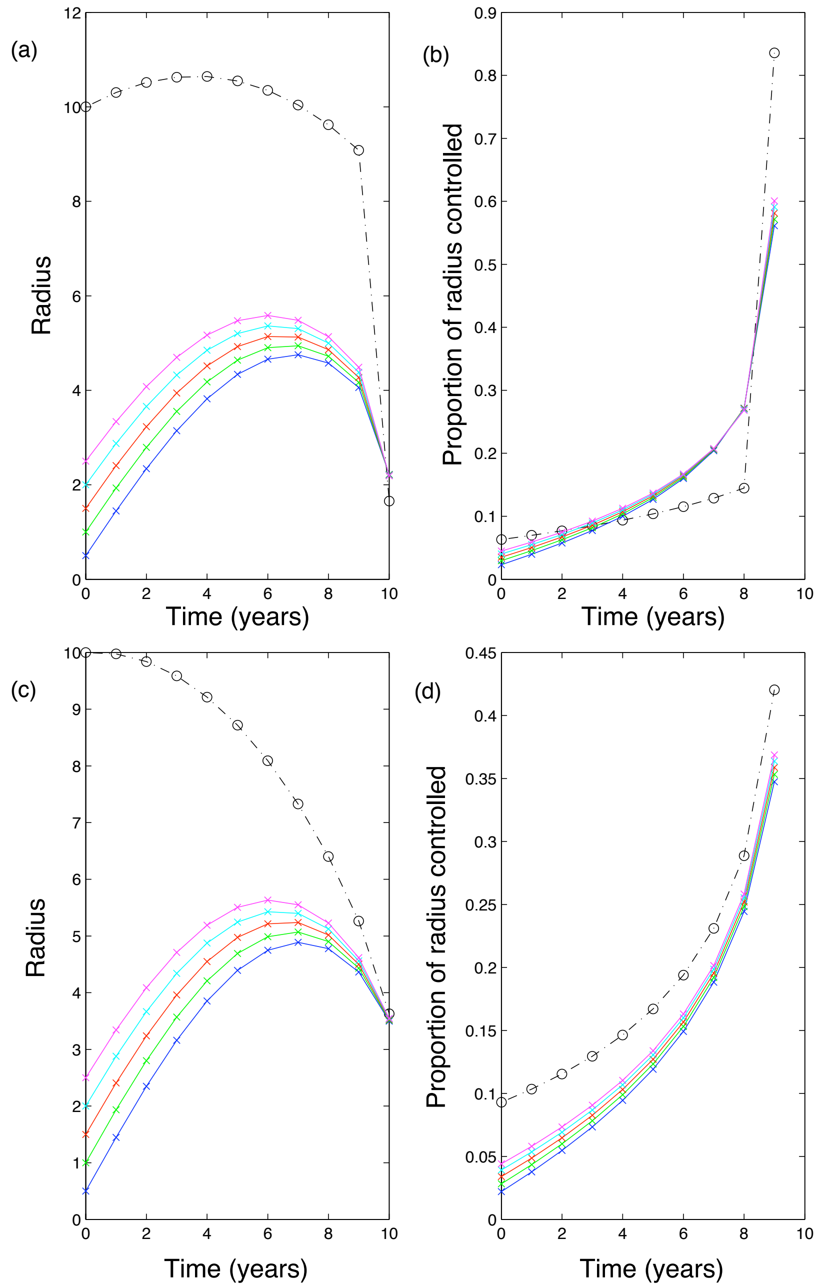

FIGURE 4. Comparison between constrained and unconstrained optimal solutions for $B = 20$. Figures (a) and (b) show the unconstrained optimal solution where $J(\beta) = 95.4$ of which 68.3 is the cost of the control. Figures (c) and (d) show the constrained optimal solution for fixed cost, $m = 38.6$. Here $J_1(\beta) = 113.8$.

There is a clear difference between the constrained optimal solution and the unconstrained optimal solution (Figure 4). The resulting total cost is much higher in the constrained problem due to the restriction on the amount of control used. Strikingly, we find that the control strategy changes considerably because of the restriction on cost. In the restrictive case, more control is used earlier in the control period (as in Figure 1). However, in the unconstrained case, less control is used until toward the end of the period, when a large amount of control is used (as in Figure 2).

In Table 1 we compare the total cost of the objective functional for four different strategies over a ten-year period. We let the initial radii be $r_{1,0} = 10$, $r_{2,0} = 0.5$, $r_{3,0} = 1.0$, $r_{4,0} = 1.5$, $r_{5,0} = 2.0$ and $r_{6,0} = 2.5$ and let $k = 1$, $\varepsilon = 0.01$ with $B = 20$. The optimal solutions for the constrained and unconstrained problems

Table 1. Comparison of total costs between both Moody and Mack scenarios and the constrained and unconstrained optimal solutions.

Control method	Total cost of the objective functional (cost of the control)
Satellite elimination (Moody and Mack)	499.7 (100.0)
Main focus reduction (Moody and Mack)	1042.6 (10.2)
Unconstrained optimal solution	95.4 (68.3)
Constrained optimal solution	279.8 (10.2)

compare favorably with the solutions of the two scenarios from Moody and Mack. By constraining the total cost of the control allowed during the period to the same cost (10.2) as in the Moody and Mack scenario in which part of the radius of the main focus is removed, there is a considerable gain in the total cost. Therefore, allowing the same amount of control to be applied at discrete intervals to a combination of the main focus and the satellites is far more cost effective. Removing the constraint allows use of more control, causing a further decrease in the total cost. For our model, the cost of eliminating the satellites at the start is also high compared to our time-dependent control solutions.

5. **Discussion.** Our results provide several conclusions that differ from those of the original model by Moody and Mack. This is hardly surprising given the different assumptions our approach takes, but we attempted to provide a method that is sufficiently similar to theirs that comparisons could be reasonably made. Our reasoning is that since the Moody and Mack results are widely cited as appropriate for invasion management, it is important to determine whether use of their conclusions for invasive management is reasonable. We believe that, in general, their conclusion (focus efforts on eradicating the satellite populations) is far too limiting to be appropriate guidance for resource managers. This is in part due to our results showing that allocation of control efforts on both a central focus and satellites is optimal, but also because realistic invasive control is not done at a single time, but at multiple times. Our model allows for this possibility in addition to providing other realistic extensions of the original Moody and Mack model.

Our main result is that, rather than focusing all control effort on just the main focus of infestation or just the satellites (these were the limited options investigated by Moody and Mack), we find that the optimal solution is always to apply control to both the main focus and the satellites. The proportion of control applied to the main focus in comparison to the satellites changes depending on the balancing constant, B, which governs the relative costs of management to the costs of the infestation. Larger B values lead to higher optimal control on the main focus, while as B decreases, the optimal solution is to apply more control to the satellites.

The formulation of the optimal control problem gives a different view of how several applications of control equally spaced over a set period affect optimal management for invasives. Additionally, our approach provides a comparison of the explicit time course of optimal control, separately for different sizes of satellites, and provides estimates of the differential application of control to these satellite infestations. Thus, given an explicit set of information about sizes of infestations, our model provides estimates of the relative amount of control, either in an unconstrained cost model or a constrained cost model, which should be applied to each infestation. This offers the potential to provide explicit guidance to managers in a considerably more realistic manner than applying a rule of thumb based on the earlier Moody and Mack results.

Moody and Mack included a second model formulation (allowing for initiation of new foci of infection) that we have not investigated here. Our results are additionally limited because of the way the model is formulated, assuming the same cost for a proportional reduction of radius for all infestations independent of their size.

Many other extensions of our approach could be investigated, and we expect that if implemented these could change the details of our results, though we do not expect them to bring about changes to our main conclusion. Our model does not include discount factors associated with the costs, in part because we view the time horizon for practical application of this approach to be sufficiently short that such discounting would have small effect at current interest and discount rates. If long-term (over several decades) applications were to be considered, then discounting would be more appropriately included. Our model has no explicit spatial components and ignores the potential impact of distance factors in affecting the control costs. We have focused as well only on a final-time infestation area in the objective, but it would be relatively easy to incorporate the costs of infestation throughout the time period rather than only at final time. It would be reasonable to investigate how specific optimal control patterns determined from our model might change as the spatial allocation of total area is modified—easily done by allocating a fixed area of outliers among more or less satellites with the same total area.

In conclusion, we have developed a relatively simple methodology for developing plans for management of invasives that might reasonably be applied to realistic measurements of a real extent of infestations. While the biology associated with this model is extremely naive, it may well be appropriate for situations in which only coarse estimates of the radial spread rate of the infestation are available. At the least, we hope that our results encourage resource managers to consider alternatives to the simple rules based upon the earlier models, which we argue are likely too constrained to be used for cost-effective management.

Acknowledgments. This paper was supported by the National Science Foundation through awards DMS-0110920 and IIS-0427471 to the University of Tennessee. The authors thank Dr. Michael Fuller for discussions regarding invasives and Dr. Dan Simberloff for first suggesting that we investigate the model of Moody and

Mack. We would also like to thank Julius Oyier for preliminary work and Rachael Miller and Dr. Vladimir Protopopescu for useful discussions.

REFERENCES

[1] O. J. Cacho, R. M. Wise, S. M. Hester, and J. A. Sinden. Invasive species in aquatic ecosystems: Economics and matrix population models. *Working paper series in Agricultural and Resource Economics, University of New England, Australia*, 2005.

[2] C. Clark. *Mathematical Bioeconomics: The Optimal Management of Renewable Resources.* Wiley, 2nd edition, 1990.

[3] J. Hof and M. M. Bevers. *Spatial Optimization in Ecological Applications.* Columbia Univ. Press, 2002.

[4] M. E. Moody and R. N. Mack. Controlling the spread of plant invasions the importance of nascent foci. *Journal of Applied Ecology*, 25:1009–1021, 1988.

[5] S. V. Petrovskii and B. Li. *Exactly Solvable Models of Biological Invasion.* CRC Press, 2005.

[6] D. Pimentel. *Biological Invasions: Economic and Environmental Costs of Alien Plant, Animal, and Microbe Species.* CRC Press, 2002.

[7] L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelize, and E. F. Mishchenko. *The MAthematical Theory of Optimal Processes.* Wiley, 1962.

[8] S. P. Sethi and G. L. Thompson. *Optimal Control Theory: Application to Management Science and Economics.* Kluwer, 2000.

[9] N. Shigesada and K. Kawasaki. *Biological Invasions: Theory and Practice.* Oxford Univ. Press, 2005.

[10] D. Simberloff, I. M. Parker, and P. N. Windle. Introduced species policy, management, and future research needs. *Frontiers in Ecology and the Environment*, 3:12–20, 2005.

Received on January 25, 2006. Revised on July 30, 2006.

E-mail address: whittle@math.utk.edu
E-mail address: lenhart@math.utk.edu
E-mail address: gross@tiem.utk.edu