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INTERNAL ERADICABILITY FOR AN EPIDEMIOLOGICAL MODEL WITH DIFFUSION

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Abstract. This work is concerned with the analysis of the possibility for eradicating a disease in an infected population. The epidemiological model under study is of SI type with diffusion. We assume the policy strategy acting on the infected individuals over a subset of the whole spatial territory. Using the framework of nonlinear reaction-diffusion equations, and spectral theory of linear differential operators, we give necessary conditions and sufficient conditions of eradicability.

1. **Introduction and main results.** We consider a nonlinear model describing the dynamics of an epidemiological system of the susceptible-infected type (SI) in a spatial domain $\Omega \subset \mathbb{R}^n$, $n \geq 1$, with a smooth boundary $\partial\Omega$. We denote by $S(t,x) \geq 0$ the density of the susceptible population and by $I(t,x) \geq 0$ the density of the infected population at time $t \geq 0$ and position $x \in \overline{\Omega}$. In an infected-free setting the susceptible population increases at a natural rate $r > 0$ and saturates at a level $K > 0$, known as the carrying capacity of the environment. The incubation period is assumed to be very short; thus the force of the infection is of the kind $pI(t,x)$, and the gain in the infective class is $pI(t,x)S(t,x)$, where $p > 0$ is the constant infection rate. The rate of removal of infectives is proportional to the number of infectives; that is, $aI(t,x)$, where $(a > 0)$ is a constant. In our model, we assume that the infected individuals do not contribute to the population renewal.

In our model, population fluxes obey a Fickian law and are proportional to their respective spatial gradients; this is $k_1\nabla S(t,x)$ for susceptible and $k_2\nabla I(t,x)$ for infected, k_1 and k_2 being positive numbers. These considerations yield a 2×2 system of reaction-diffusion equations in $(0,+\infty)\times\Omega$ for the SI dynamics; namely,

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$$\begin{cases} S_t(t,x) - k_1\Delta S(t,x) = r(1 - \dfrac{S(t,x)+I(t,x)}{K})S(t,x) - pS(t,x)I(t,x), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x \in \Omega,\ t>0, \\ I_t(t,x) - k_2\Delta I(t,x) = -aI(t,x) + pS(t,x)I(t,x), \qquad x\in\Omega,\ t>0, \\ \dfrac{\partial S}{\partial\nu}(t,x) = \dfrac{\partial I}{\partial\nu}(t,x) = 0, \qquad x\in\partial\Omega,\ t>0, \\ S(0,x) = S_0(x) \ge 0,\ I(0,x) = I_0(x) \ge 0, \qquad x\in\Omega, \end{cases} \tag{1}$$

where S_0 and I_0 are nonnegative and bounded initial data, assumed to be non-identically zero on Ω ($S_0, I_0 \in L^\infty(\Omega)$, $S_0(x), I_0(x) \ge 0$ a.e. $x \in \Omega$, $\|S_0\|_{L^\infty(\Omega)} > 0$, $\|I_0\|_{L^\infty(\Omega)} > 0$).
The no-flux boundary conditions on the boundary $\partial\Omega$ of Ω in (1) correspond to an isolated spatial environment. For biological significance of all the terms and parameters in (1), we refer to [6].

This paper concerns the internal eradicability (or zero-stabilizability) of the infected population. Let ω be a nonempty subdomain with a smooth boundary $\partial\omega$, such that $\bar\omega \subset \Omega$, and denote by m the characteristic function of $\bar\omega$, and by p_1 the constant $p + \frac{r}{K}$. The question we wish to answer is the following: is there any control $u \in L^\infty_{loc}([0,+\infty) \times \overline{\omega})$ such that the solution to the following system,

$$\begin{cases} S_t(t,x) - k_1\Delta S(t,x) = r(1 - \dfrac{S(t,x)}{K})S(t,x) - p_1\, S(t,x)I(t,x), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x \in \Omega,\ t>0, \\ I_t(t,x) - k_2\Delta I(t,x) = -aI(t,x) + pS(t,x)I(t,x) + m(x)u(t,x), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x\in\Omega,\ t>0, \\ \dfrac{\partial S}{\partial\nu}(t,x) = \dfrac{\partial I}{\partial\nu}(t,x) = 0, \qquad x\in\partial\Omega,\ t>0, \\ S(0,x) = S_0(x) \ge 0,\ I(0,x) = I_0(x) \ge 0, \qquad x\in\Omega, \end{cases} \tag{2}$$

satisfies both

$$S(t,x) \ge 0, \qquad I(t,x) \ge 0, \quad \forall t>0,\ a.e.\ x \in \Omega$$

and

$$\lim_{t\to+\infty} I(t) = 0 \quad \text{ in } L^\infty(\Omega)?$$

REMARK 1. *The nonnegativity of S and I is a natural requirement, because S and I represent densities of population. The existence and uniqueness of a solution (S, I) to the nonlinear system (2) (for any $S_0, I_0 \in L^\infty(\Omega)$, $S_0(x), I_0(x) \ge 0$ a.e. $x \in \Omega$, and for any $u \in L^\infty_{loc}([0,+\infty)\times\overline{\omega})$) follows by way of the Banach fixed-point theorem and using comparison results for the solutions to parabolic equations. If in addition $u := 0$, then the solution to (2) (the solution to (1)) is nonnegative.*

DEFINITION 1.1. *We say that the infected population is internally eradicable (or zero-stabilizable) if for any $S_0, I_0 \in L^\infty(\Omega)$, $S_0(x), I_0(x) \ge 0$ a.e. $x \in \Omega$, $\|S_0\|_{L^\infty(\Omega)} > 0$, $\|I_0\|_{L^\infty(\Omega)} > 0$, the answer to the above-mentioned question is affirmative.*

We say that the infected population is internally null-controllable if for any $S_0, I_0 \in L^\infty(\Omega)$, $S_0(x), I_0(x) \ge 0$ a.e. $x \in \Omega$, $\|S_0\|_{L^\infty(\Omega)} > 0$, $\|I_0\|_{L^\infty(\Omega)} > 0$, and for any $T > 0$, there exists $u \in L^\infty((0,T)\times\omega)$ such that the solution (S, I) to (2) satisfies $I(T, x)$ a.e. $x \in \Omega$.

For basic results of controllability, we refer to [7].

REMARK 2. *Note that one can easily prove that when $Kp < a$, the infected population is stabilized to zero naturally (without external control); indeed, using the maximum principle and comparison theorems one obtains $S(t,x) \leq y(t)$, where y is the solution of $y_t = r(1 - \frac{y}{K})y$, $y(0) = \|S_0\|_{L^\infty(\Omega)}$. Thus for each $\varepsilon > 0$, one can find $\tilde{T}$ s.t. $K - \varepsilon < y(t) < K + \varepsilon$, for each $t > \tilde{T}$. Going back to the equation describing the dynamics of the infected population, integrating by parts on Ω and choosing ε appropriately (less than $\frac{a-Kp}{p}$), one obtains via Gronwall's lemma that $I(t,\cdot) \to 0$ in $L^1(\Omega)$ as $t \to +\infty$ and via the parabolic regularity that $I(t,\cdot) \to 0$ in $L^\infty(\Omega)$ as $t \to +\infty$.*

When $Kp > a$, an endemic situation occurs for the free SI system, and an external control is needed to stabilize to zero the infective population.

So, our main problem is to treat the case when $Kp > a$.

We denote the principal eigenvalue of the following problem,

$$\begin{cases} -\Delta\varphi = \lambda\varphi & \text{in } \Omega \setminus \overline{\omega}, \\ \varphi = 0 & \text{in } \partial\omega, \\ \dfrac{\partial\varphi}{\partial\nu} = 0 & \text{in } \partial\Omega, \end{cases} \tag{3}$$

by λ_1^ω.

The main result of our paper is as follows.

THEOREM 1.1. *If the infected population is eradicable, then $k_2\lambda_1^\omega + a - pK \geq 0$. Moreover, if $k_2\lambda_1^\omega + a - pK > 0$, then there is a feedback control that stabilizes the infected population to zero.*

REMARK 3. *It is important to notice that this is a stabilizability problem with state constraints; this is because the solution (S, I) to (2) must satisfy*

$$S(t,x) \geq 0, \quad I(t,x) \geq 0, \qquad \forall t \geq 0, \text{ a.e. } x \in \Omega.$$

If $k_2\lambda_1^\omega + a - pK > 0$, then we shall also provide a feedback control that stabilizes the infected-population density to zero.

The control of the infected population means that we destroy (eliminate) or separate (quarantine) the infected individuals.

Another question that we will discuss is related to the null controllability of the infected population. In fact, we shall prove that for any $I_0 \in L^\infty(\Omega)$, $I_0(x) \geq 0$ a.e. $x \in \Omega$ such that $\|I_0\|_{L^\infty(\Omega\setminus\overline{\omega})} > 0$ and that for any $T > 0$, there is no control $u \in L^\infty((0,T) \times \omega)$ such that the solution (S, I) of (2) satisfies

$$I(T,x) = 0, \quad \text{a.e. } x \in \Omega$$

and

$$S(t,x) \geq 0, \quad I(t,x) \geq 0, \quad \forall t \in (0,T), \text{ a.e. } x \in \Omega.$$

Related stabilizability results for nonnegative solutions to some parabolic equations and to the age-dependent population dynamics have been established in [3, 4, 1]. The detailed description of the SI systems can be found in [6, 9]. Stabilizability results for another epidemic model and, using a different technique, can be found in [5]. For basic results concerning the parabolic boundary value problems in L^k frame, we refer to [8].

Our paper is organized as follows: in section 2 we remind some auxilliary results concerning the principal eigenvalue of $-\Delta$ and give the proof of Theorem 1.1. In section 3 we give some remarks concerning the shape of the domain Ω, to get the fastest stabilization of the infected population.

2. **Proof of the main results.** For any arbitrary but fixed real numbers ε and γ, we consider the eigenvalue problem

$$\begin{cases} -\Delta\varphi = \varepsilon\varphi - m(x)\gamma\varphi + \lambda\varphi & \text{in } \Omega, \\ \dfrac{\partial\varphi}{\partial\nu} = 0 & \text{in } \partial\Omega, \end{cases} \tag{4}$$

and we denote the principal eigenvalue of (4) by $\lambda_{1,\varepsilon}^{\gamma}$.

The following two lemmas have been proved by the authors in [2].

Lemma 2.1. *If $\varepsilon = 0$, then*

$$\lim_{\gamma\to+\infty} \lambda_{1,0}^{\gamma} \to \lambda_1^{\omega},$$

where λ_1^{ω} is the principal eigenvalue for (3).

Lemma 2.2. *For any $\gamma > 0$, we have*

$$\lim_{\varepsilon\searrow 0} \lambda_{1,\varepsilon}^{\gamma} = \lambda_{1,0}^{\gamma}.$$

The proofs of these two lemmas are mainly based on Rayleigh's principle.

Let us now state and prove two useful lemmas concerning the asymptotic behavior of the solution to (1).

Lemma 2.3. *Assume that $S_0, I_0 \in L^\infty(\Omega)$, $S_0(x), I_0(x) \geq 0$ a.e. $x \in \Omega$, $\|S_0\|_{L^\infty(\Omega)} > 0$, $\|I_0\|_{L^\infty(\Omega)} > 0$ and $u \in L^\infty_{loc}([0,+\infty) \times \overline{\omega})$. Let (S, I) be a nonnegative solution to* (2). *For each $\varepsilon > 0$, there exists $T > 0$ such that*

$$0 \leq S(t,x) \leq K + \varepsilon, \quad \text{for any } t > T \ \text{ and a.e. } x \in \Omega.$$

Proof. Let us denote by $\bar{S}$ the solution to

$$\begin{cases} \bar{S}_t(t) = r(1 - \frac{\bar{S}(t)}{K})\bar{S}(t), & t > 0, \\ \bar{S}(0) = \|S_0\|_{L^\infty(\Omega)}. \end{cases} \tag{5}$$

Thus $0 \leq S(t,x) \leq \bar{S}(t)$, $\forall t \geq 0$, a.e. $x \in \Omega$, and $\lim_{t\to\infty} \bar{S}(t) = K$. Then for each $\varepsilon > 0$, there exists $T > 0$ such that

$$0 \leq S(t,x) \leq K + \varepsilon, \quad \forall t > T, \text{ a.e. } x \in \Omega.$$

Lemma 2.4. *Assume that $S_0, I_0 \in L^\infty(\Omega)$, $S_0(x), I_0(x) \geq 0$ a.e. $x \in \Omega$, $\|S_0\|_{L^\infty(\Omega)} > 0$, $\|I_0\|_{L^\infty(\Omega)} > 0$ and $u \in L^\infty_{loc}([0,+\infty) \times \overline{\omega})$. Let (S, I) be a nonnegative solution to* (2). *If we assume*

$$I(t) \to 0 \quad \text{in } L^\infty(\Omega),$$

as $t \to +\infty$, then

$$S(t) \to K \quad \text{in } L^\infty(\Omega),$$

as $t \to +\infty$.

Proof. Let $\varepsilon > 0$ be a fixed and small enough number. Then there exists $T > 0$ such that for $t > T$ one has $\|I(t)\|_{L^\infty(\Omega)} < \varepsilon$. Let us denote by $\tilde{S}$ the solution of

$$\begin{cases} \tilde{S}_t(t,x) - k_1\Delta\tilde{S}(t,x) = r(1 - \frac{\tilde{S}(t,x)}{K})\tilde{S}(t,x) - p_1\varepsilon\tilde{S}(t,x), & x \in \Omega,\ t > T, \\ \dfrac{\partial \tilde{S}}{\partial \nu}(t,x) = 0, & x \in \partial\Omega,\ t > T, \\ \tilde{S}(T,x) = S(T,x), & x \in \Omega\ . \end{cases}$$

Using the comparison result for the solutions of parabolic equations, we get that $\tilde{S}(t,x) \le S(t,x) \le \bar{S}(t,x)$, $\forall t > T$, a.e. $x \in \Omega$ ($\bar{S}$ is the solution to (5)). Now using arguments similar to those in [2], we obtain that

$$\lim_{t\to\infty} \tilde{S}(t) = \frac{K(r - p_1\varepsilon)}{r}$$

in $L^\infty(\Omega)$. Passing to the limit $\varepsilon \to 0$, we get the conclusion.

Proof of Theorem 1.1. Let us assume that the infected population is eradicable. Let u be the control that stabilizes to zero the infected population and denote by (S, I) the solution of (2) corresponding to u, where $S_0, I_0 \in L^\infty(\Omega)$, $S_0(x), I_0(x) \ge 0$ a.e. $x \in \Omega$, $\|S_0\|_{L^\infty(\Omega)} > 0$, $\|I_0\|_{L^\infty(\Omega)} > 0$. Let $K > \varepsilon > 0$ be arbitrary but fixed. There exists $T > 0$ such that for any $t > T$,

$$\|S(t) - K\|_{L^\infty(\Omega)} < \varepsilon.$$

Consider the solution $\tilde{I}$ to

$$\begin{cases} \tilde{I}_t(t,x) - k_2\Delta\tilde{I}(t,x) = -a\tilde{I}(t,x) + p(K-\varepsilon)\tilde{I}(t,x), & x \in \Omega\setminus\bar{\omega},\ t > T, \\ \dfrac{\partial \tilde{I}}{\partial \nu}(t,x) = 0, & x \in \partial\Omega,\ t > T, \\ \tilde{I}(t,x) = 0, & x \in \partial\omega,\ t > T, \\ \tilde{I}(T,x) = I(T,x), & x \in \Omega\setminus\bar{\omega}\ . \end{cases}$$

Then $0 \le \tilde{I}(t,x) \le I(t,x)$, $\forall t > T$, a.e. $x \in \Omega\setminus\bar{\omega}$. Thus

$$\lim_{t\to\infty} \tilde{I}(t) = 0 \tag{6}$$

in $L^\infty(\Omega\setminus\bar{\omega})$.

Note that there exists $I_0 \in L^\infty(\Omega)$, $I_0(x) \ge 0$ a.e. $x \in \Omega$ such that $I(T,x)$ is not identically zero. Indeed, if we suppose that $I(T,x) = 0$ a.e. $x \in \Omega$, for any $I_0 \in L^\infty(\Omega)$, $I_0(x) \ge 0$ a.e. $x \in \Omega$, then the solution to the following problem,

$$\begin{cases} \bar{I}_t(t,x) - k_2\Delta\bar{I}(t,x) = -a\bar{I}(t,x), & x \in \Omega\setminus\bar{\omega},\ t > 0, \\ \dfrac{\partial \bar{I}}{\partial \nu}(t,x) = 0, & x \in \partial\Omega,\ t > 0, \\ \bar{I}(t,x) = 0, & x \in \partial\omega,\ t > 0, \\ \bar{I}(0,x) = I_0(x), & x \in \Omega\setminus\bar{\omega}, \end{cases}$$

satisfies $0 \le \bar{I}(t,x) \le I(t,x)$ for any $t \in (0,T)$ and a.e. $x \in \Omega\setminus\bar{\omega}$ (we have used again the comparison result for the solutions of parabolic equations). Using the backward uniqueness theorem, we may infer that $\bar{I}(0,x) = I_0(x) = 0$ a.e. $x \in \Omega\setminus\bar{\omega}$; this is absurd if we choose I_0 such that $\|I_0\|_{L^\infty(\Omega\setminus\bar{\omega})} > 0$. Using now (6), we may infer that the principal eigenvalue $\lambda_1^\varepsilon = k_2\lambda_1^\omega + a - p(K-\varepsilon)$ of the following eigenfunction problem,

$$\begin{cases} -k_2\Delta\varphi(x) = -a\varphi(x) + p(K-\varepsilon)\varphi(x) + \lambda\varphi(x), & x \in \Omega \setminus \bar{\omega}, \\ \dfrac{\partial\varphi}{\partial\nu}(x) = 0, & x \in \partial\Omega, \\ \varphi(x) = 0, & x \in \partial\omega, \end{cases}$$

is positive. Passing to the limit $\varepsilon \to 0$, we get that $k_2\lambda_1^\omega + a - pK \geq 0$.

Let us prove now the second assertion of our theorem. Assume that the principal eigenvalue of the following eigenvalue problem,

$$\begin{cases} -k_2\Delta\varphi(x) = -a\varphi(x) + pK\varphi(x) + \lambda\varphi(x), & x \in \Omega \setminus \bar{\omega}, \\ \dfrac{\partial\varphi}{\partial\nu}(x) = 0, & x \in \partial\Omega, \\ \varphi(x) = 0, & x \in \partial\omega, \end{cases}$$

is positive (this is equivalent to the fact that $k_2\lambda_1^\omega + a - pK > 0$), and let $S_0, I_0 \in L^\infty(\Omega)$, $S_0(x), I_0(x) \geq 0$ a.e. $x \in \Omega$, $\|S_0\|_{L^\infty(\Omega)} > 0$, $\|I_0\|_{L^\infty(\Omega)} > 0$.

Consider the solution to

$$\begin{cases} \tilde{S}_t(t,x) - k_1\Delta\tilde{S}(t,x) = r(1 - \frac{\tilde{S}(t,x)}{K})\tilde{S}(t,x), & x \in \Omega,\ t > 0, \\ \dfrac{\partial\tilde{S}}{\partial\nu}(t,x) = 0, & x \in \partial\Omega,\ t > 0, \\ \tilde{S}(0,x) = S_0(x), & x \in \Omega. \end{cases}$$

For any $\varepsilon > 0$ arbitrary but fixed, there exists $T > 0$ such that $K - \varepsilon \leq \tilde{S}(t,x) \leq K + \varepsilon$, for any $t \geq T$ and a.e. $x \in \Omega$.

Taking in (2), $u := 0$ for $t \in (0,T)$ and $u := -\gamma I$ (with $\gamma > 0$) for $t \geq T$, one has that there exists a unique solution (S, I) to (2), which is nonnegative and satisfies

$$S(t,x) \leq K + \varepsilon, \quad \forall t > T, \text{ a.e. } x \in \Omega$$

(because $S(t,x) \leq \tilde{S}(t,x)$, $\forall t > T$, a.e. $x \in \Omega$) and $0 \leq I(t,x) \leq I^*(t,x)$, for any $t > T$, a.e. $x \in \Omega$, where I^* is the solution to

$$\begin{cases} I_t^*(t,x) - k_2\Delta I^*(t,x) = -aI^*(t,x) + p(K+\varepsilon)I^*(t,x) - m(x)\gamma I^*(t,x), & \\ & x \in \Omega,\ t > T, \\ \dfrac{\partial I^*}{\partial\nu}(t,x) = 0, & x \in \partial\Omega,\ t > T, \\ I^*(T,x) = I(T,x), & x \in \Omega\ . \end{cases}$$

Using Lemmas 2.1 and 2.2, we conclude that there exist $\varepsilon > 0$ (small enough) and $\gamma > 0$ (large enough) such that the principal eigenvalue of the following eigenvalue problem,

$$\begin{cases} -k_2\Delta\varphi(x) = -a\varphi(x) + p(K+\varepsilon)\varphi(x) - m(x)\gamma\varphi(x) + \lambda\varphi(x), & x \in \Omega, \\ \dfrac{\partial\varphi}{\partial\nu}(x) = 0, & x \in \partial\Omega, \end{cases}$$

is positive. In conclusion, $0 \leq \lim_{t\to\infty} I(t) \leq \lim_{t\to\infty} I^*(t) = 0$ in $L^2(\Omega)$, and consequently by the regularizing effect of the parabolic operator, we get that

$$\lim_{t\to\infty} I(t) = \lim_{t\to\infty} I^*(t) = 0$$

in $L^\infty(\Omega)$.

3. **Final remarks.** The argument used in the first part of the proof of Theorem 1.1 allows us to conclude that for any $S_0, I_0 \in L^\infty(\Omega)$, $S_0(x), I_0(x) \geq 0$ a.e. $x \in \Omega$ satisfying $\|I_0\|_{L^\infty(\Omega\setminus\overline{\omega})} > 0$ and for any $T > 0$, there is no control $u \in L^\infty((0,T)\times\omega)$ such that

$$I(T, x) = 0, \quad \text{a.e. } x \in \Omega$$

and

$$S(t, x) \geq 0, \quad I(t, x) \geq 0, \quad \forall t \in (0, T), \text{ a.e. } x \in \Omega.$$

It means that the infected population is not exactly null controllable at a finite moment T.

If the infected population is zero-stabilizable, then, at infinity it behaves (in $L^2(\Omega)$) as the exponential

$$e^{(-k_2\lambda_1^\omega - a + pK)t}.$$

Assume now that $\omega \subset \mathbb{R}^n$ is a ball of radius $\rho > 0$. We wish to find the domain $\Omega \subset \mathbb{R}^n$ of class C^1, of a given measure $L > meas(\omega)$ such that $\bar{\omega} \subset \Omega$ and that the principal eigenvalue of problem (3) is maximal. The answer to this question has been given by the authors in [2]. We remind it:

THEOREM 3.1. *Let $\omega \subset \mathbb{R}^n$ be a ball of radius $\rho > 0$. The maximal value for the principal eigenvalue corresponding to* (3) *in the class of all domains $\Omega \subset \mathbb{R}^n$ of class C^1 with measure $L > meas(\omega)$, which contain ω, is realized when Ω is the ball of measure L with the same center as the given ball ω.*

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