Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 11(7): 23210–23233.
DOI:10.3934/math.2026936
Received: 17 April 2026
Revised: 24 June 2026
Accepted: 17 July 2026
Published: 30 July 2026

Research article

State estimation for interval type-2 fuzzy singularly perturbed systems with important-data-based DoS attacks: the dual-scale channels case

Yue Hu[1,2], **Chaoqun Guo**[1,2], **Oh-Min Kwon**[3,*] **and Seung Hoon Lee**[4]

[1] Shandong Provincial Key Laboratory of New Power Distribution & Utilization Technology and Equipment, Shandong University of Technology, Zibo 255000, Shandong Province, China

[2] School of Electrical and Electronic Engineering, Shandong University of Technology, 266 Xincun West Road, Zibo 255000, Shandong, China

[3] School of Electrical Engineering, Chungbuk National University, Cheongju 28644, South Korea

[4] Division of Electronics and Electrical Information Engineering, National Korea Maritime and Ocean University, Busan 49112, South Korea

* **Correspondence:** Email: madwind@chungbuk.ac.kr.

Abstract: This article was dedicated to constructing an H_∞ state estimator for networked interval type-2 (IT2) fuzzy singularly perturbed systems (SPSs) subject to denial-of-service (DoS) attacks. The challenges lied in designing an asynchronous DoS attack model for SPSs with two-time-scale characteristics to attack the important data and providing a defense strategy to resist this attack. This work presented three major contributions: (1) An asynchronous important-data-based (IDB) DoS attack model was proposed to attack the critical data information of slow and fast sensors separately. (2) Then, an IT2 fuzzy composite resilient estimator was designed to resist the asynchronous IDB DoS attacks. (3) Finally, sufficient conditions were determined to ensure the error dynamics' asymptotic stability. A DC motor example was provided to highlight the applicability of our method.

Keywords: H_∞ state estimation; important-data-based DoS attacks; interval type-2 fuzzy approach; two-time-scale system
Mathematics Subject Classification: 93C42, 93C65, 93D20

1. Introduction

1.1. Research overview

In recent years, multi-time-scale phenomena caused by the coexistence of slow and fast dynamics in practical systems has attracted considerable attention [1–3]. Singularly perturbed systems (SPSs), as a mathematical tool, can effectively characterize this multi-time-scale behavior, under which the degree of time-scale separation between the slow and fast dynamics is quantified by the singular perturbation parameter (SPP) [4–7]. At present, many valuable achievements have been attained for SPSs. Specifically, due to the widespread occurrence of nonlinear dynamics in practical applications, significant focus has been committed to investigating nonlinear SPSs. The Takagi-Sugeno (T-S) fuzzy model, comprising several local linear subsystems blended through membership functions (MFs), has been demonstrated to be an effective method for representing nonlinear SPSs [8, 9].

In addition to nonlinearity, uncertainty often arises in many situations, such as parameter variations and measurement inaccuracies, which, if not adequately considered in the analysis, can degrade the system performance. Unfortunately, the traditional T-S (Type-1) fuzzy approach [10–12] often requires that the practical nonlinear physical plants do not contain parameter uncertainty. To confront this issue, a novel interval type-2 (IT2) fuzzy technique was devised, whose known lower and upper MFs effectively capture parameter uncertainties [13–15]. In light of this advantage, preliminary attention has been given to research on IT2 fuzzy SPSs [16–20]. For instance, in [18], an IT2 fuzzy technique was incorporated into the fuzzy quantized controller to account for parameter uncertainties for the nonlinear SPSs. In [19], the H_∞ filtering problem was tackled for a class of IT2 fuzzy SPSs with semi-Markov parameters. In [20], the authors discussed the control issue for fuzzy Markov SPSs and revealed that the IT2 fuzzy controller outperforms traditional Type-1 fuzzy controllers.

From another perspective, as computer networks are more extensively applied, the security communication problem has garnered significant attention in networked control systems (NCSs) [21–23]. Due to the high resource sharing of communication networks, NCSs tend to suffer from malicious cyber-attacks [24, 25]. Currently, two main classifications of network attacks exist in networked systems, namely, deception attacks [26] and denial-of-service (DoS) attacks [27]. The first type aims to undermine the integrity of data during transmission, whereas the second type disrupts the availability of network resources by interrupting the network channel. It is important to note that if not addressed appropriately, these cyber-attacks could severely compromise system performance and potentially lead to instability. As a result, security issues related to NCSs have garnered increasing attention from researchers.

1.2. Potential problems and motivation

Up to now, various cyber-attack issues have been explored for networked SPSs, and several preliminary studies have been conducted [28–33]. In [28], the authors analyzed the security problem for fuzzy SPSs with DoS attacks. In [29], the quantized control was researched for SPSs with deception attacks. In [30], the security control problem was studied for cyber-physical systems with dual time scales under DoS attacks. In [31], the resilient dissipative filtering problem was addressed for discrete-time Markov jump singularly perturbed systems with hybrid cyber-attacks. A periodic DoS attack model was adopted to depict the energy-restricted attacker in researching the security control problem for the networked SPSs [32]. When considering the time-scale difference in the network channels, a dual-scale DoS attack model was established for SPSs in [33]. Note that, different from deception attacks, DoS attacks demand minimal prior knowledge of the system and can cause significant destruction, making them easy to implement among adversaries. Therefore, researching the security control/state estimation problem for SPSs with DoS attacks has practical significance.

For the networked SPSs with DoS attacks, previous research work has always assumed that DoS attacks are launched indiscriminately. That is, the attacker ignores the information in the packets when carrying out the attacks. In fact, the transmitted data packets in NCSs contain different information, some of which are crucial for maintaining system performance when designing controllers or estimators, while others are not as important. In this regard, from the adversaries' perspective, an important-data-based (IDB) DoS attack model was given [34, 35], under which the importance degree of the data information can be evaluated and the destructiveness of the attacks can be increased. Nevertheless, when considering the significant time-scale differences between fast and slow dynamics, few results focus on security state estimation issues for the IT2 fuzzy SPSs subject to IDB DoS attacks, which motivates our current research.

From the above introduction, it is clear that several key issues (listed below) remain unresolved regarding the security estimation for IT2 fuzzy SPSs under asynchronous IDB DoS attacks.

Question 1: How can we construct an asynchronous IDB DoS attack model when the network channels exhibit two-time-scale characteristics?

Question 2: How can we construct an effective composite IT2 fuzzy estimator from the defender's perspective to resist asynchronous IDB DoS attacks?

Question 3: How can we tackle the numerical stiffness problem for the IT2 fuzzy SPSs with asynchronous IDB DoS attacks?

1.3. Key contributions and results

Inspired by Questions 1–3, we research the asynchronous IDB DoS attack model-based state estimation for IT2 fuzzy SPSs. The following summarizes the key innovations of our research.

(i) **Asynchronous IDB DoS attack model:** Compared with the conventional DoS attack model [32, 33], a new asynchronous IDB DoS attack model is designed to increase the destructiveness of attacks for networked IT2 fuzzy SPSs, in which attackers will launch attacks on important data packets in fast and slow channels based on different time scales.

(ii) **Composite estimator:** By considering the IT2 fuzzy method and the system's inherent two-time-scale characteristics, a composite IT2 fuzzy estimator is established, which can effectively counter asynchronous IDB DoS attacks.

(iii) **Security guarantee:** Constructing a suitable SPP-dependent Lyapunov functional prevents ill-conditioned problems and ensures that the error dynamics of the system are asymptotically stable.

2. System model and study objectives

2.1. System model

Consider the IT2 fuzzy SPSs model as follows:

Rule i: if $h_{\hat{\imath}}(x(k))$ is $\varrho^i_{\hat{\imath}}$, in which $\hat{\imath} \in \mathbb{G} \triangleq \{1, \cdots, \imath\}$, $i \in \mathbb{H} \triangleq \{1, \cdots, r\}$, $h_{\hat{\imath}}(x(k))$ is the premise variable, and $\varrho^i_{\hat{\imath}}$ indicates the fuzzy set corresponding to the ith rule, then

$$\begin{cases} x(k+1) = A_{\epsilon i}x(k) + D_i w(k), \\ y_1(k) = C_1 x(k) + F_1 w(k), \\ y_2(k) = C_2 x(k) + F_2 w(k), \\ z(k) = L_i x(k), \end{cases} \tag{2.1}$$

where $x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, $x_1(k) \in \mathbb{R}^{n_1}$ is the slow state vector, $x_2(k) \in \mathbb{R}^{n_2}$ is the fast state vector ($n = n_1 + n_2$). $y_1(k) \in \mathbb{R}^{n_1}$ and $y_2(k) \in \mathbb{R}^{n_2}$ denote the slow and fast measurement outputs, respectively. $z(k) \in \mathbb{R}^{n_z}$ is the output to be estimated. $w(k) \in \mathbb{R}^{n_w}$ is the external disturbance belonging to $l_2[0, +\infty)$. ϵ is the SPP with $0 < \epsilon \ll 1$. $C_1 = \begin{bmatrix} I_{n_1} & 0_{n_1 \times n_2} \end{bmatrix} \in \mathbb{R}^{n_1 \times n}$, $C_2 = \begin{bmatrix} 0_{n_2 \times n_1} & I_{n_2} \end{bmatrix} \in \mathbb{R}^{n_2 \times n}$, $F_1 \in \mathbb{R}^{n_1 \times n_w}$, $F_2 \in \mathbb{R}^{n_2 \times n_w}$ are known constant matrices. Morever, the matrices $A_{\epsilon i}$, D_i, and L_i, which are real constants with compatible dimensions, are listed below.

$$A_{\epsilon i} = \begin{bmatrix} A_i^{11} & \epsilon A_i^{12} \\ A_i^{21} & \epsilon A_i^{22} \end{bmatrix},\ D_i = \begin{bmatrix} D_i^1 \\ D_i^2 \end{bmatrix},\ L_i = \begin{bmatrix} L_i^1 & L_i^2 \end{bmatrix}.$$

The firing strength range for the ith plant rule can be expressed as

$$\mathcal{W}_i = \left[\underline{\mu}_i(x(k)), \overline{\mu}_i(x(k)) \right], \tag{2.2}$$

in which

$$\underline{\mu}_i(x(k)) = \Pi_{\hat{\imath}=1}^{\iota} \underline{\beta}_{\varrho_{\hat{\imath}}^i}(h_{\hat{\imath}}(x(k))) > 0,\ \overline{\mu}_i(x(k)) = \Pi_{\hat{\imath}=1}^{\iota} \overline{\beta}_{\varrho_{\hat{\imath}}^i}(h_{\hat{\imath}}(x(k))) > 0.$$

$\underline{\beta}_{\varrho_{\hat{\imath}}^i}(h_{\hat{\imath}}(x(k)))$ and $\overline{\beta}_{\varrho_{\hat{\imath}}^i}(h_{\hat{\imath}}(x(k)))$ satisfy

$$0 \le \underline{\beta}_{\varrho_{\hat{\imath}}^i}(h_{\hat{\imath}}(x(k))) \le \overline{\beta}_{\varrho_{\hat{\imath}}^i}(h_{\hat{\imath}}(x(k))) \le 1.$$

Consequently, the global IT2 fuzzy SPSs are formulated as

$$\begin{cases} x(k+1) = \sum_{i=1}^{r} \mu_i(x(k))[A_{\epsilon i} x(k) + D_i w(k)], \\ y_1(k) = C_1 x(k) + F_1 w(k), \\ y_2(k) = C_2 x(k) + F_2 w(k), \\ z(k) = \sum_{i=1}^{r} \mu_i(x(k)) L_i x(k), \end{cases} \tag{2.3}$$

where

$$\mu_i(x(k)) = \frac{\hat{\mu}_i(x(k))}{\sum_{i=1}^{r} \hat{\mu}_i(x(k))},\ \sum_{i=1}^{r} \mu_i(x(k)) = 1,\ \hat{\mu}_i(x(k)) = \underline{\mu}_i(x(k)) \underline{a}_i(x(k)) + \overline{\mu}_i(x(k)) \overline{a}_i(x(k)),$$
$$\underline{a}_i(x(k)) + \overline{a}_i(x(k)) = 1.$$

In this case, $\underline{a}_i(x(k))$, $\overline{a}_i(x(k)) \in [0, 1]$ are adopted to represent the nonlinear weight factors in system (2.3).

2.2. Study objectives

To address the H_∞ state estimation issue, we will define the following essential terms and present related lemmas.

Definition 1. *Let a scalar $\gamma > 0$ be given, for any nonzero $w(k) \in l_2(0, \infty]$, if system* (2.3) *under the zero initial condition satisfies*

$$\sum_{k=0}^{\infty} \|\tilde{z}(k)\|^2 \leq \gamma^2 \sum_{k=0}^{\infty} \|w(k)\|^2. \tag{2.4}$$

Then the system realizes the required H_∞ performance γ.

Lemma 1. *[17] For a given scalar $\bar{\epsilon} > 0$ and proper symmetric matrices W_1, W_2, we can obtain*

$$W_1 + \epsilon W_2 < 0, \forall \epsilon \in (0, \bar{\epsilon}],$$

holds if and only if $W_1 < 0$ and $W_1 + \bar{\epsilon} W_2 < 0$.

Objectives: This paper focuses on handling the aforementioned Questions 1–3 and obtaining the following outcomes.

1) Establish an asynchronous IDB DoS attack model for the system (2.3).
2) Design a composite IT2 fuzzy resilient estimator through the utilization of the IT2 fuzzy framework.
3) Address the H_∞ state estimation problem of system (2.3) in the context of a fixed SPP upper bound.

3. Design of asynchronous IDB DoS attacks and the composite estimator

In this part, the asynchronous IDB DoS attack model and IT2 fuzzy composite estimator will be investigated.

3.1. Asynchronous IDB DoS attack model

As discussed in [34], the "important" data packets impact the system performance, and the attackers can disrupt the system performance by attacking "important" data packets. Hence, inspired by references [34] and [35], to describe this crucial phenomenon for SPSs with two-time-scale chracteristics, an asynchronous IDB DoS attack model with the following three features is urgently established.

1) The asynchronous IDB DoS attacks can obtain the slow and fast measurement outputs $y_1(k)$ and $y_2(k)$ at current time k, and record the latest attack-free packets $y_1(t_k)$ and $y_2(s_k)$, where t_k, s_k are unattacked moments of slow and fast outputs, respectively.
2) This asynchronous IDB DoS attack mechanism can evaluate the important data packets that affect SPSs' performance.
3) If a data packet is evaluated as the "important" data packet, an attack will be initiated by the network invader. Accordingly, this attacked data will not be further transmitted to the estimator.

Moreover, the structure diagram of networked IT2 fuzzy SPSs with asynchronous IDB DoS attacks is presented in Figure 1.

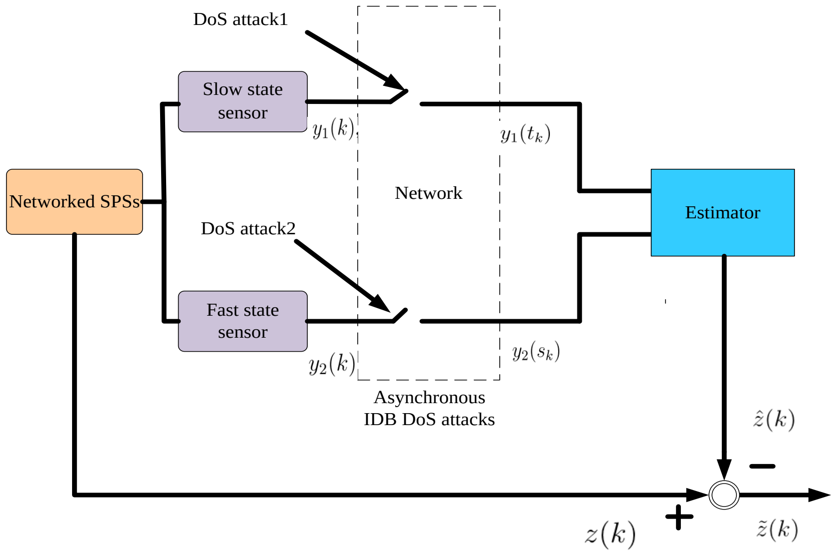

Figure 1. Structure diagram of networked IT2 fuzzy SPSs under asynchronous IDB DoS attacks.

Unlike the current DoS attack models [32, 33], the asynchronous IDB DoS attack model will be designed based on an idea opposite to event-triggered thinking. First, an asynchronous "important information" recognition mechanism is constructed as:

$$\psi_1(\cdot) = (y_1(t_k) - y_1(k))^{\mathrm{T}}(y_1(t_k) - y_1(k)) - \sigma_1 y_1^{\mathrm{T}}(t_k) y_1(t_k), \tag{3.1}$$

$$\psi_2(\cdot) = (y_2(s_k) - y_2(k))^{\mathrm{T}}(y_2(s_k) - y_2(k)) - \sigma_2 y_2^{\mathrm{T}}(s_k) y_2(s_k), \tag{3.2}$$

with scalars $\sigma_1 > 0$, $\sigma_2 > 0$. The sequences $\{t_k\}(k = 1, 2, ...)$ and $\{s_k\}(k = 1, 2, ...)$ represent the time instants of attack-free packets in the slow and fast transmission channels, respectively. Moreover, $y_1(k)$ and $y_2(k)$ denote the current slow and fast measurement output, respectively, and $y_1(t_k)$ and $y_2(s_k)$ are the latest attack-free measurements received through the slow and fast channels, respectively.

Then, based on the important data recognition mechanisms $\psi_1(\cdot)$ and $\psi_2(\cdot)$, attackers selectively launch attacks, and the attack model is established as follows:

$$\lambda_1(k) = \begin{cases} 1, & \text{if } \psi_1(\cdot) > 0; \\ 0, & \text{otherwise}. \end{cases} \tag{3.3}$$

$$\lambda_2(k) = \begin{cases} 1, & \text{if } \psi_2(\cdot) > 0; \\ 0, & \text{otherwise}. \end{cases} \tag{3.4}$$

If $\lambda_1(k) = 1$, the current slow measurement output $y_1(k)$ will be attacked at time k; if $\lambda_1(k) = 0$, the slow data signal $y_1(t_k) = y_1(k)$ will be saved and no attack will occur at time k. Similarly, when $\lambda_2(k) = 1$, the fast transmission channel is subject to jamming at time k, which results in the corruption of the current fast measurement $y_2(k)$. In contrast, when $\lambda_2(k) = 0$, no attack occurs at time k, and the fast measurement is successfully transmitted, i.e., $y_2(s_k) = y_2(k)$.

Remark 1. *In [32, 33], the proposed network attack models only adopt some predesigned attack sequences to attack the network channels, which neglects the influence of packet importance on control performance. In fact, the importance degree of each packet on system performance is not identical. In spired by [34, 35], an asynchronous important data recognition mechanism is constructed by adopting the inverse idea of an event-triggered scheme (ETS), under which the system will select "important" packets and discard "unimportant" packets via the* ψ_i *conditions,* $i = 1, 2$.

Remark 2. *For the SPSs, the presence of SPP inevitably results in a huge distinction between the fast and slow dynamics. Slow states evolve gradually, whereas fast states change quickly. Different from references [34, 35], the asynchronous IDB DoS attack model is established to describe the attack behavior of networked SPSs channels in this paper, under which the attacker can launch attacks on fast and slow network channels according to the system dynamic characteristics. Meanwhile the attackers monitor the measured output of IT2 fuzzy SPSs and selectively attack "important" data packets.*

3.2. Composite IT2 fuzzy resilient estimator design

Considering the networked SPSs under the IDB DoS attacks, a composite IT2 fuzzy estimator will be designed in this part to resist this asynchronous IDB DoS attack model.

Note that the specific parameters σ_1 and σ_2 in (3.3) and (3.4) are unknown by the defender. In view of the defender does not knowing the attack parameters σ_1 and σ_2, the corresponding parameters $\underline{\sigma}_1 > 0$ and $\underline{\sigma}_2 > 0$ will be designed to represent them, respectively. Meanwhile, define $\phi_1(k) = y_1(t_k) - y_1(k)$, $\phi_2(k) = y_2(s_k) - y_2(k)$. Then, the attack strategies adopted by the defender are described by

$$\tilde{\lambda}_1(k) = \begin{cases} 1, \text{ if } \tilde{\psi}_1(\cdot) > 0; \\ 0, \text{ otherwise.} \end{cases} \tag{3.5}$$

$$\tilde{\lambda}_2(k) = \begin{cases} 1, \text{ if } \tilde{\psi}_2(\cdot) > 0; \\ 0, \text{ otherwise.} \end{cases} \tag{3.6}$$

The functions $\psi_1(\cdot)$ and $\psi_2(\cdot)$ are denoted as

$$\tilde{\psi}_1(\cdot) = \phi_1^{\mathrm{T}}(k)\phi_1(k) - \underline{\sigma}_1(\phi_1(k) + y_1(k))^{\mathrm{T}}(\phi_1(k) + y_1(k)), \tag{3.7}$$

$$\tilde{\psi}_2(\cdot) = \phi_2^{\mathrm{T}}(k)\phi_2(k) - \underline{\sigma}_2(\phi_2(k) + y_2(k))^{\mathrm{T}}(\phi_2(k) + y_2(k)), \tag{3.8}$$

where the values of $\underline{\sigma}_1$ and $\underline{\sigma}_2$ are computed in Algorithm 1 according to the following point: All packets arriving at the estimator must fail to satisfy the conditions $\tilde{\psi}_1(\cdot) > 0$ and $\tilde{\psi}_2(\cdot) > 0$ in (3.7) and (3.8). Namely, if we define

$$\sigma_1(k) = \frac{||y_1(t_{k-1}) - y_1(t_k)||^2}{||y_1(t_{k-1})||^2}, \ \sigma_2(k) = \frac{||y_2(s_{k-1}) - y_2(s_k)||^2}{||y_2(s_{k-1})||^2},$$

then, it can be obtained that $\sigma_1(k) < \sigma_1$, $\sigma_2(k) < \sigma_2$. Accordingly, if we choose $\underline{\sigma}_1 = \max\{\sigma_1(k)\}$, $\underline{\sigma}_2 = \max\{\sigma_2(k)\}$ and k is large enough, it can be implied that $\underline{\sigma}_1$ and $\underline{\sigma}_2$ approximate σ_1 and σ_2 from the left side, respectively, and if k is large enough, both $\underline{\sigma}_1 - \sigma_1$ and $\underline{\sigma}_2 - \sigma_2$ will approach 0.

In these attack strategies adopted by the defender, $\tilde{\lambda}_1(k) = 1$ implies that the defender is convinced the attacker initiated the attack in slow channels at time k. $\tilde{\lambda}_2(k) = 1$ implies that the defender is convinced the attacker initiated the attack in fast channels at time k. $\tilde{\lambda}_1(k) = 0$ and $\tilde{\lambda}_2(k) = 0$ indicate that the slow and fast channels, respectively, are not subject to attacks at time k.

Algorithm 1 Calculation of $\underline{\sigma}_1$, $\underline{\sigma}_2$

Step 1: Choose $\underline{\sigma}_1 = 0$, $\underline{\sigma}_2 = 0$, $k = 1, 2, ..., M$, where M is a sufficiently large positive integer.
Step 2: Calculate variable parameters $\sigma_1(k)$, $\sigma_2(k)$: $\sigma_1(k) = \|y_1(t_{k-1}) - y_1(t_k)\|^2/\|y_1(t_{k-1})\|^2$, $\sigma_2(k) = \|y_2(s_{k-1}) - y_2(s_k)\|^2/\|y_2(s_{k-1})\|^2$,
Step 3: Update $\underline{\sigma}_1$, $\underline{\sigma}_2$: If $\sigma_1(k) \geq \underline{\sigma}_1$, $\sigma_2(k) \geq \underline{\sigma}_2$, then $\underline{\sigma}_1 = \sigma_1(k)$, $\underline{\sigma}_2 = \sigma_2(k)$.
Step 4: If $k = M$, then jump to **Step 5**; else $k = k + 1$ and jump to **Step 2**.
Step 5: Calculate $\underline{\sigma}_1 = \underline{\sigma}_1 + \kappa_1$, $\underline{\sigma}_2 = \underline{\sigma}_2 + \kappa_2$, where κ_1 and κ_2 are constants small enough.
Step 6: Output $\underline{\sigma}_1$, $\underline{\sigma}_2$.

Remark 3. *It should be noted that under the parameters $\underline{\sigma}_1$, $\underline{\sigma}_2$ in* (3.5) *and* (3.6)*, the attacked packet number is not lower than that under the parameters σ_1, σ_2 in* (3.3), (3.4)*. That is, if the system's performance is maintained under conditions* (3.5) *and* (3.6)*, then the system's dynamic performance can also be preserved under the IDB DoS attack conditions* (3.3) *and* (3.4).

Then, according to (3.5) and (3.6), the resilient estimator is constructed by:

1) **Slow and fast estimators:** Rule j: if $h_1(\hat{x}(k))$ is ϱ_1^j, $\cdots$, and $h_q(\hat{x}(k))$ is ϱ_q^j, then

$$\begin{cases} \hat{x}(k+1) = A_{\epsilon j}\hat{x}(k) + K_{1j}(y_1(t_k) - C_1\hat{x}(k)) + \epsilon K_{2j}(y_2(s_k) - C_2\hat{x}(k)), \\ \quad \hat{z}(k) = L_j\hat{x}(k), \end{cases} \tag{3.9}$$

where $\hat{x}(k) = \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix}$, $\hat{z}(k)$ is the corresponding estimate of $z(k)$, $h_{\hat{j}}(\hat{x}(k))$ denotes the premise variable, and K_{1j} and K_{2j} are slow and fast estimator gains, respectively, that need to be designed. For any $j \in \mathbb{I} \triangleq \{1, 2, \cdots, s\}$, $\hat{j} \in \mathbb{J} \triangleq \{1, 2, \cdots, \jmath\}$, $\varrho_{\hat{j}}^j$ is the fuzzy set of $h_{\hat{j}}(\hat{x}(k))$. The lower and upper MFs are $\underline{\beta}_{\varrho_{\hat{j}}^j}(h_{\hat{j}}(\hat{x}(k)))$ and $\overline{\beta}_{\tilde{\varrho}_{\hat{j}}^j}(h_{\hat{j}}(\hat{x}(k)))$, respectively. Afterward, the firing strength range for the jth rule can be expressed as

$$\mathcal{M}_j(\hat{x}(k)) = [\underline{\mu}_j(\hat{x}(k)), \overline{\mu}_j(\hat{x}(k))], \tag{3.10}$$

where

$$\underline{\mu}_j(\hat{x}(k)) = \Pi_{\hat{j}=1}^{\jmath}\underline{\beta}_{\varrho_{\hat{j}}^j}(h_{\hat{j}}(\hat{x}(k))) > 0,\ \overline{\mu}_j(\hat{x}(k)) = \Pi_{\hat{j}=1}^{\jmath}\overline{\beta}_{\varrho_{\hat{j}}^j}(h_{\hat{j}}(\hat{x}(k))) > 0,$$
$$0 \leq \underline{\beta}_{\varrho_{\hat{j}}^j}(h_{\hat{j}}(\hat{x}(k))) \leq \overline{\beta}_{\varrho_{\hat{j}}^j}(h_{\hat{j}}(\hat{x}(k))) \leq 1.$$

2) **Composite fuzzy resilient estimator:** Then, the overall IT2 fuzzy estimator is described by

$$\begin{cases} \hat{x}(k+1) = \sum_{j=1}^{s} v_j(\hat{x}(k))[A_{\epsilon j}\hat{x}(k) + K_{1j}(y_1(t_k) - C_1\hat{x}(k)) + \epsilon K_{2j}(y_2(s_k) - C_2\hat{x}(k))], \\ \quad \hat{z}(k) = \sum_{j=1}^{s} v_j(\hat{x}(k))L_j\hat{x}(k), \end{cases} \tag{3.11}$$

where

$$v_j(\hat{x}(k)) = \frac{\hat{v}_j(\hat{x}(k))}{\sum_{j=1}^{s} \hat{v}_j(\hat{x}(k))}, \ \sum_{j=1}^{s} v_j(\hat{x}(k)) = 1, \ \hat{v}_j(\hat{x}(k)) = \underline{\mu}_j(\hat{x}(k))\underline{b}_j(\hat{x}(k)) + \overline{\mu}_j(\hat{x}(k))\overline{b}_j(\hat{x}(k)),$$

$\underline{b}_j(\hat{x}(k)), \overline{b}_j(\hat{x}(k)) \in [0, 1], \underline{b}_j(\hat{x}(k)) + \overline{b}_j(\hat{x}(k)) = 1.$

In this case, $\underline{b}_j(\hat{x}(k))$ and $\overline{b}_j(\hat{x}(k))$ are used to describe the nonlinear weight factors in the resilient estimator construction process.

By combining (3.7)–(3.11), the overall IT2 fuzzy resilient estimator can be rewritten as

$$\begin{cases} \hat{x}(k+1) = \sum_{j=1}^{s} v_j(\hat{x}(k))[A_{\epsilon j}\hat{x}(k) + K_{1j}(C_1 x(k) - C_1\hat{x}(k) + F_1 w(k) + \phi_1(k)) + \epsilon K_{2j}(C_2 x(k) \\ \qquad - C_2\hat{x}(k) + F_2 w(k) + \phi_2(k))], \\ \hat{z}(k) = \sum_{j=1}^{s} v_j(\hat{x}(k)) L_j \hat{x}(k). \end{cases} \tag{3.12}$$

Remark 4. *According to asynchronous IDB DoS attack conditions* (3.3) *and* (3.4)*, the attacker implements the DoS attack in the slow transmission channel when a significant difference exists between the latest transmitted signal* $y_1(t_k)$ *and the current signal* $y_1(k)$*. Similarly, the attacker implements the DoS attack in the fast transmission channel when a significant difference exists between the latest transmitted signal* $y_2(t_k)$ *and the current signal* $y_2(k)$*. Compared with the exist DoS attack model [30, 32, 33], this asynchronous IDB DoS attack model will cause more severe damage to system performance. In particular, the proposed asynchronous IDB DoS attacks can cause an even greater divergence between the updated signal and current signal, which brings more frequent DoS attacks. To deal with this problem, we design the composite fuzzy resilient estimator* (3.12) *on the defense side. As depicted in Remark 3, the proposed resilient estimator* (3.12) *under conditions* (3.5) *and* (3.6) *can counter the asynchronous IDB DoS attacks.*

Denote $e(k) = x(k) - \hat{x}(k)$, $\tilde{z}(k) = z(k) - \hat{z}(k)$. The IT2 fuzzy error dynamics system model can be depicted as

$$\begin{cases} e(k+1) = \sum_{i=1}^{r} \mu_i(x(k)) \sum_{j=1}^{s} v_j(\hat{x}(k))[(A_{\epsilon i} - A_{\epsilon j})x(k) + (A_{\epsilon j} - K_{1j}C_1 - \epsilon K_{2j}C_2)e(k) - K_{1j}\phi_1(k) \\ \qquad - \epsilon K_{2j}\phi_2(k) + (D_i - K_{1j}F_1 - \epsilon K_{2j}F_2)w(k)], \\ \tilde{z}(k) = \sum_{i=1}^{r} \mu_i(x(k)) \sum_{j=1}^{s} v_j(\hat{x}(k))[(L_i - L_j)x(k) + L_j e(k)]. \end{cases} \tag{3.13}$$

Define $\xi(k) = \begin{bmatrix} x^{\mathrm{T}}(k) & e^{\mathrm{T}}(k) \end{bmatrix}^{\mathrm{T}}$. By combining (3.12) and (3.13), the corresponding augmented system model can be written as

$$\begin{cases} \xi(k+1) = \sum_{i=1}^{r} \sum_{j=1}^{s} \mu_{ij}(k)[\mathcal{A}_{\epsilon ij}\xi(k) - \bar{K}_{1j}\phi_1(k) - \bar{K}_{2j}\phi_2(k) + \bar{D}_{ij}w(k)], \\ \tilde{z}(k) = \sum_{i=1}^{r} \sum_{j=1}^{s} \mu_{ij}(k)\bar{L}_{ij}\xi(k), \end{cases} \tag{3.14}$$

where

$\mu_{ij}(k) = \mu_i(x(k))v_j(\hat{x}(k)),$

$\mathcal{A}_{\epsilon ij} = \begin{bmatrix} A_{\epsilon i} & 0 \\ A_{\epsilon i} - A_{\epsilon j} & A_{\epsilon j} - K_{1j}C_1 - \epsilon K_{2j}C_2 \end{bmatrix},\ \bar{K}_{1j} = \begin{bmatrix} 0 \\ K_{1j} \end{bmatrix},\ \bar{K}_{2j} = \begin{bmatrix} 0 \\ \epsilon K_{2j} \end{bmatrix},\ \bar{D}_{ij} = \begin{bmatrix} D_i \\ D_i - K_{1j}F_1 - \epsilon K_{2j}F_2 \end{bmatrix},$

$\bar{L}_{ij} = \begin{bmatrix} L_i - L_j & L_j \end{bmatrix}.$

4. Main results

Next, this section focuses on the H_∞ state estimation of the augmented system (3.14).

Theorem 1. Let scalars $\underline{\sigma}_1$, $\underline{\sigma}_2$, $\gamma > 0$, the SPP $\epsilon > 0$, and the estimator gains K_{1j}, K_{2j} be given. If there exist suitable matrices $\mathcal{P}_\epsilon > 0$, $\Xi_{iju} \geq 0$ and positive scalars α_1, α_2, m_{iju} ($i \in \mathbb{H}$, $j \in \mathbb{I}$) such that

$$\Pi_{ij} - \Xi_{iju} < 0, \tag{4.1}$$

$$\underline{\mu}_{iju}\Pi_{ij} + m_{iju}\Xi_{iju} < 0, \tag{4.2}$$

where

$\Pi_{ij} = \begin{bmatrix} \Pi_{ij}^{11} & \Pi_{ij}^{12} & \Pi_{ij}^{13} & \Pi_{ij}^{14} \\ * & \Pi_{ij}^{22} & \Pi_{ij}^{23} & \Pi_{ij}^{24} \\ * & * & \Pi_{ij}^{33} & \Pi_{ij}^{34} \\ * & * & * & \Pi_{ij}^{44} \end{bmatrix},\ \mathcal{I} = \begin{bmatrix} I & 0 \end{bmatrix},\ \Pi_{ij}^{44} = \bar{D}_{ij}^{\mathrm{T}}\mathcal{P}_\epsilon\bar{D}_{ij} - \alpha_1\underline{\sigma}_1F_1^{\mathrm{T}}F_1 - \alpha_2\underline{\sigma}_2F_2^{\mathrm{T}}F_2 - \gamma^2 I,$

$\Pi_{ij}^{11} = \mathcal{A}_{\epsilon ij}^{\mathrm{T}}\mathcal{P}_\epsilon\mathcal{A}_{\epsilon ij} - \mathcal{P}_\epsilon - \alpha_1\underline{\sigma}_1\mathcal{I}^{\mathrm{T}}C_1^{\mathrm{T}}C_1\mathcal{I} + \bar{L}_{ij}^{\mathrm{T}}\bar{L}_{ij} - \alpha_2\underline{\sigma}_2\mathcal{I}^{\mathrm{T}}C_2^{\mathrm{T}}C_2\mathcal{I},\ \Pi_{ij}^{12} = -\mathcal{A}_{\epsilon ij}^{\mathrm{T}}\mathcal{P}_\epsilon\bar{K}_{1j} - \alpha_1\sigma_1\mathcal{I}^{\mathrm{T}}C_1^{\mathrm{T}},$
$\Pi_{ij}^{22} = \bar{K}_{1j}^{\mathrm{T}}\mathcal{P}_\epsilon\bar{K}_{1j} + \alpha_1(1-\sigma_1)I,\ \mathcal{P}_\epsilon = \mathrm{diag}\{P_\epsilon, P_\epsilon\},\ \Pi_{ij}^{13} = -\mathcal{A}_{\epsilon ij}^{\mathrm{T}}\mathcal{P}_\epsilon\bar{K}_{1j} - \alpha_2\sigma_2\mathcal{I}^{\mathrm{T}}C_2^{\mathrm{T}},$
$\Pi_{ij}^{23} = \bar{K}_{1j}^{\mathrm{T}}\mathcal{P}_\epsilon\bar{K}_{2j},\ \Pi_{ij}^{33} = \bar{K}_{2j}^{\mathrm{T}}\mathcal{P}_\epsilon\bar{K}_{2j} + \alpha_2(1-\sigma_2)I,\ \Pi_{ij}^{14} = \mathcal{A}_{\epsilon ij}^{\mathrm{T}}\mathcal{P}_\epsilon\bar{D}_{ij} - \alpha_1\underline{\sigma}_1\mathcal{I}^{\mathrm{T}}C_1^{\mathrm{T}}F_1 - \alpha_2\underline{\sigma}_2\mathcal{I}^{\mathrm{T}}C_2^{\mathrm{T}}F_2,$
$\Pi_{ij}^{24} = \bar{K}_{1j}^{\mathrm{T}}\mathcal{P}_\epsilon\bar{D}_{ij} - \alpha_1\underline{\sigma}_1F_1,\ \Pi_{ij}^{34} = \bar{K}_{2j}^{\mathrm{T}}\mathcal{P}_\epsilon\bar{D}_{ij} - \alpha_2\underline{\sigma}_2F_2,$

then augmented system (3.14) is asymptotically stable and satisfies the H_∞ performance level (2.4).

Proof. Now, employing the following Lyapunov functional candidate to analyze the SPSs (3.9):

$$V(k) = \xi^{\mathrm{T}}(k)\mathcal{P}_\epsilon\xi(k). \tag{4.3}$$

By evaluating the difference of $V(k)$, we obtain

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) = \xi^{\mathrm{T}}(k+1)\mathcal{P}_\epsilon\xi(k+1) - \xi^{\mathrm{T}}(k)\mathcal{P}_\epsilon\xi(k) \\ &\leq \sum_{i=1}^{r}\sum_{j=1}^{s}\mu_{ij}(k)\{[\mathcal{A}_{\epsilon ij}\xi(k) - \bar{K}_{1j}\phi_1(k) - \bar{K}_{2j}\phi_2(k) + \bar{D}_{ij}w(k)]^{\mathrm{T}}\mathcal{P}_\epsilon[(\mathcal{A}_{\epsilon ij}\xi(k) - \bar{K}_{1j}\phi_1(k) \\ &\quad - \bar{K}_{2j}\phi_2(k) + \bar{D}_{ij}w(k)] - \xi^{\mathrm{T}}(k)\mathcal{P}_\epsilon\xi(k)\}. \end{aligned} \tag{4.4}$$

Inspired by [38], by taking (3.5)–(3.8) and (4.4) into consideration, we infer that

$$\begin{aligned} \Delta V(k) \leq &\sum_{i=1}^{r}\sum_{j=1}^{s}\mu_{ij}(k)\{[\mathcal{A}_{\epsilon ij}\xi(k) - \bar{K}_{1j}\phi_1(k) - \bar{K}_{2j}\phi_2(k) + \bar{D}_{ij}w(k)]^{\mathrm{T}}\mathcal{P}_\epsilon[(\mathcal{A}_{\epsilon ij}\xi(k) - \bar{K}_{1j}\phi_1(k) - \bar{K}_{2j}\phi_2(k) \\ &+ \bar{D}_{ij}w(k)] - \xi^{\mathrm{T}}(k)\mathcal{P}_\epsilon\xi(k)\} - \alpha_1(\underline{\sigma}_1(\phi_1(k) + y_1(k))^{\mathrm{T}}(\phi_1(k) + y_1(k)) - \phi_1^{\mathrm{T}}(k)\phi_1(k)) \\ &- \alpha_2(\underline{\sigma}_2(\phi_2(k) + y_2(k))^{\mathrm{T}}(\phi_2(k) + y_2(k)) - \phi_2^{\mathrm{T}}(k)\phi_2(k)). \end{aligned} \tag{4.5}$$

Define $\check{\eta}(k) = \begin{bmatrix} \xi^{\mathrm{T}}(k) & \phi_1^{\mathrm{T}}(k) & \phi_2^{\mathrm{T}}(k) \end{bmatrix}^{\mathrm{T}}$. When $w(k) = 0$, Eq (4.4) indicates that

$$\Delta V(k) \leq \check{\eta}^{\mathrm{T}}(k) \sum_{i=1}^{r} \sum_{j=1}^{s} \mu_{ij}(k) \check{\Pi}_{ij} \check{\eta}(k), \tag{4.6}$$

where

$$\check{\Pi}_{ij} = \begin{bmatrix} \check{\Pi}_{ij}^{11} & \check{\Pi}_{ij}^{12} & \check{\Pi}_{ij}^{13} \\ * & \check{\Pi}_{ij}^{22} & \check{\Pi}_{ij}^{23} \\ * & * & \check{\Pi}_{ij}^{33} \end{bmatrix}, \tag{4.7}$$

$$\begin{aligned} \check{\Pi}_{ij}^{11} &= \mathcal{A}_{\epsilon ij}^{\mathrm{T}} \mathcal{P}_{\epsilon} \mathcal{A}_{\epsilon ij} - \mathcal{P}_{\epsilon} - \alpha_1 \underline{\sigma}_1 \mathcal{I}^{\mathrm{T}} C_1^{\mathrm{T}} C_1 \mathcal{I} - \alpha_2 \underline{\sigma}_2 \mathcal{I}^{\mathrm{T}} C_2^{\mathrm{T}} C_2 \mathcal{I},\ \check{\Pi}_{ij}^{12} = -\mathcal{A}_{\epsilon ij}^{\mathrm{T}} \mathcal{P}_{\epsilon} \bar{K}_{1j} \\ &\quad - \alpha_1 \sigma_1 \mathcal{I}^{\mathrm{T}} C_1^{\mathrm{T}}, \check{\Pi}_{ij}^{22} = \bar{K}_{1j}^{\mathrm{T}} \mathcal{P}_{\epsilon} \bar{K}_{1j} + \alpha_1 (1 - \sigma_1) I, \\ \check{\Pi}_{ij}^{13} &= -\mathcal{A}_{\epsilon ij}^{\mathrm{T}} \mathcal{P}_{\epsilon} \bar{K}_{1j} - \alpha_2 \sigma_2 \mathcal{I}^{\mathrm{T}} C_2^{\mathrm{T}},\ \check{\Pi}_{ij}^{23} = \bar{K}_{1j}^{\mathrm{T}} \mathcal{P}_{\epsilon} \bar{K}_{2j},\ \check{\Pi}_{ij}^{33} = \bar{K}_{2j}^{\mathrm{T}} \mathcal{P}_{\epsilon} \bar{K}_{2j} + \alpha_2 (1 - \sigma_2) I. \end{aligned}$$

To utilize the useful information of MFs and obtain the sufficient conditions of stability criteria, we partition the state space of interest $\mathbb{U}$ into substate spaces u. Then, in each subspace, the lower and upper bounds of the local MFs are represented as $\underline{\mu}_{iju}$ and $\bar{\mu}_{iju}$, respectively. Meanwhile, the slack matrices Ξ_{iju} with the constraints that $\Xi_{iju} > 0$ and $\Xi_{iju} > \Pi_{ij}$ are introduced to obtain the following conditions:

$$\begin{aligned} \sum_{i=1}^{r} \sum_{j=1}^{s} \mu_{iju}(k) \Pi_{ij} &= \sum_{i=1}^{r} \sum_{j=1}^{s} (\mu_{iju}(k) + \underline{\mu}_{iju} - \underline{\mu}_{iju}) \Pi_{ij} \\ &\leq \sum_{i=1}^{r} \sum_{j=1}^{s} \underline{\mu}_{iju} \Pi_{ij} + \sum_{i=1}^{r} \sum_{j=1}^{s} (\mu_{iju}(k) - \underline{\mu}_{iju}) \Pi_{ij} \leq \sum_{i=1}^{r} \sum_{j=1}^{s} (\underline{\mu}_{iju} \Pi_{ij} + m_{iju} \Xi_{iju}), \end{aligned} \tag{4.8}$$

in which the constants m_{iju} are defined as the differences between the upper and lower bounds of the IT2 fuzzy membership functions, i.e., $m_{iju} = \bar{\mu}_{iju} - \underline{\mu}_{iju}$.

Then, when $w(k) = 0$, it can be obtained from (4.1) and (4.2) that $\sum_{i=1}^{r} \sum_{j=1}^{s} (\underline{\mu}_{iju} \check{\Pi}_{ij} + m_{iju} \Xi_{iju}) < 0$. Accordingly, by considering Theorem 1 and (4.6), it can be concluded that $\Delta V(k) < 0$ when external disturbance $w(k) = 0$. Therefore, it follows that the augmented system (3.14) achieves asymptotic stability.

Next, let us analyze the H_∞ disturbance attenuation level of system (3.14). Adding $z^{\mathrm{T}}(k) z(k) - \gamma^2 w^{\mathrm{T}}(k) w(k)$ to (4.5) on both sides, one has

$$\begin{aligned} &\Delta V(k) + z^{\mathrm{T}}(k) z(k) - \gamma^2 w^{\mathrm{T}}(k) w(k) \\ &\quad \leq \sum_{i=1}^{r} \sum_{j=1}^{s} \mu_{ij}(k) \{ [\mathcal{A}_{\epsilon ij} \xi(k) - \bar{K}_{1j} \phi_1(k) - \bar{K}_{2j} \phi_2(k) + \bar{D}_{ij} w(k)]^{\mathrm{T}} \mathcal{P}_{\epsilon} [(\mathcal{A}_{\epsilon ij} \xi(k) - \bar{K}_{1j} \phi_1(k) - \bar{K}_{2j} \phi_2(k) \\ &\quad\quad + \bar{D}_{ij} w(k)] - \xi^{\mathrm{T}}(k) \mathcal{P}_{\epsilon} \xi(k) \} - \alpha_1 (\underline{\sigma}_1 (\phi_1(k) + y_1(k))^{\mathrm{T}} (\phi_1(k) + y_1(k)) - \phi_1^{\mathrm{T}}(k) \phi_1(k)) \\ &\quad\quad - \alpha_2 (\underline{\sigma}_2 (\phi_2(k) + y_2(k))^{\mathrm{T}} (\phi_2(k) + y_2(k)) - \phi_2^{\mathrm{T}}(k) \phi_2(k)) + z^{\mathrm{T}}(k) z(k) - \gamma^2 w^{\mathrm{T}}(k) w(k) \\ &\quad \leq \eta^{\mathrm{T}}(k) \sum_{i=1}^{r} \sum_{j=1}^{s} \mu_{ij}(k) \Pi_{ij} \eta(k), \end{aligned} \tag{4.9}$$

where

$$\eta(k) = \begin{bmatrix} \xi^{\mathrm{T}}(k) & \phi_1^{\mathrm{T}}(k) & \phi_2^{\mathrm{T}}(k) & w^{\mathrm{T}}(k) \end{bmatrix}^{\mathrm{T}},$$

$$\Pi_{ij} = \begin{bmatrix} \Pi_{ij}^{11} & \Pi_{ij}^{12} & \Pi_{ij}^{13} & \Pi_{ij}^{14} \\ * & \Pi_{ij}^{22} & \Pi_{ij}^{23} & \Pi_{ij}^{24} \\ * & * & \Pi_{ij}^{33} & \Pi_{ij}^{34} \\ * & * & * & \Pi_{ij}^{44} \end{bmatrix},$$

$$\begin{aligned}
\Pi_{ij}^{11} &= \mathcal{A}_{\epsilon ij}^{\mathrm{T}} \mathcal{P}_\epsilon \mathcal{A}_{\epsilon ij} - \mathcal{P}_\epsilon - \alpha_1 \underline{\sigma}_1 \mathcal{I}^{\mathrm{T}} C_1^{\mathrm{T}} C_1 \mathcal{I} - \alpha_2 \underline{\sigma}_2 \mathcal{I}^{\mathrm{T}} C_2^{\mathrm{T}} C_2 \mathcal{I} + \bar{L}_{ij}^{\mathrm{T}} \bar{L}_{ij}, \\
\Pi_{ij}^{12} &= -\mathcal{A}_{\epsilon ij}^{\mathrm{T}} \mathcal{P}_\epsilon \bar{K}_{1j} - \alpha_1 \sigma_1 \mathcal{I}^{\mathrm{T}} C_1^{\mathrm{T}},\ \Pi_{ij}^{22} = \bar{K}_{1j}^{\mathrm{T}} \mathcal{P}_\epsilon \bar{K}_{1j} + \alpha_1 (1 - \sigma_1) I, \\
\Pi_{ij}^{13} &= -\mathcal{A}_{\epsilon ij}^{\mathrm{T}} \mathcal{P}_\epsilon \bar{K}_{1j} - \alpha_2 \sigma_2 \mathcal{I}^{\mathrm{T}} C_2^{\mathrm{T}},\ \Pi_{ij}^{33} = \bar{K}_{2j}^{\mathrm{T}} \mathcal{P}_\epsilon \bar{K}_{2j} + \alpha_2 (1 - \sigma_2) I, \\
\Pi_{ij}^{14} &= \mathcal{A}_{\epsilon ij}^{\mathrm{T}} \mathcal{P}_\epsilon \bar{D}_{ij} - \alpha_1 \underline{\sigma}_1 \mathcal{I}^{\mathrm{T}} C_1^{\mathrm{T}} F_1 - \alpha_2 \underline{\sigma}_2 \mathcal{I}^{\mathrm{T}} C_2^{\mathrm{T}} F_2,\ \Pi_{ij}^{24} = \bar{K}_{1j}^{\mathrm{T}} \mathcal{P}_\epsilon \bar{D}_{ij} - \alpha_1 \underline{\sigma}_1 F_1, \\
\Pi_{ij}^{34} &= \bar{K}_{2j}^{\mathrm{T}} \mathcal{P}_\epsilon \bar{D}_{ij} - \alpha_2 \underline{\sigma}_2 F_2,\ \Pi_{ij}^{23} = \bar{K}_{1j}^{\mathrm{T}} \mathcal{P}_\epsilon \bar{K}_{2j},\ \Pi_{ij}^{44} = \bar{D}_{ij}^{\mathrm{T}} \mathcal{P}_\epsilon \bar{D}_{ij} - \alpha_1 \underline{\sigma}_1 F_1^{\mathrm{T}} F_1 \\
&\quad - \alpha_2 \underline{\sigma}_2 F_2^{\mathrm{T}} F_2 - \gamma^2 I,\ \mathcal{I} = \begin{bmatrix} I & 0 \end{bmatrix}.
\end{aligned}$$

From (4.1), (4.2), (4.8), and (4.9), it follows that

$$\Delta V(k) + z^{\mathrm{T}}(k) z(k) - \gamma^2 w^{\mathrm{T}}(k) w(k) \le 0. \tag{4.10}$$

Summing up (4.11) from 0 to ∞ on both sides, it follows that

$$\sum_{k=0}^{\infty} z^{\mathrm{T}}(k) z(k) - \gamma^2 \sum_{k=0}^{\infty} w^{\mathrm{T}}(k) w(k) \le V(0) - V(\infty). \tag{4.11}$$

Accordingly, under the zero initial condition $x(0) = 0$, one has $V(0) = 0$. Hence,

$$\sum_{k=0}^{\infty} z^{\mathrm{T}}(k) z(k) - \gamma^2 \sum_{k=0}^{\infty} w^{\mathrm{T}}(k) w(k) \le 0. \tag{4.12}$$

Therefore, the H_∞ disturbance attenuation level of system (3.14) can be achieved. This proof is complete. □

Now, a sufficient criterion for solving the resilient estimator problem is proposed in this section.

Theorem 2. Let scalars $\underline{\sigma}_1, \underline{\sigma}_2, \bar{\epsilon} > 0, \gamma > 0$ be given. For any $\epsilon \in (0, \bar{\epsilon}]$, the closed loop system (3.14) is asymptotically stable and satisfies H_∞ performance level (2.4), if there exist appropriate dimensional matrices $\bar{\mathcal{P}} > 0$, $\tilde{\mathcal{P}}$, $X = \mathrm{diag}\{X_1, X_2\}$, $\Xi_{iju} \ge 0$, $\mathcal{K}_{1j}$, $\mathcal{K}_{2j}$ and scalars $\alpha_1 > 0$, $\alpha_2 > 0$, m_{iju} ($i \in \mathbb{H}$, $j \in \mathbb{I}$) such that

$$\bar{\mathcal{P}} > 0, \tag{4.13}$$

$$\bar{\mathcal{P}} + \bar{\epsilon} \tilde{\mathcal{P}} > 0, \tag{4.14}$$

$$\begin{bmatrix} \bar{\Upsilon}_{ij} - \Xi_{iju} & \bar{\Gamma}_{ij} \\ * & \bar{\mathbb{F}}_{ij} \end{bmatrix} < 0, \tag{4.15}$$

$$\begin{bmatrix} \tilde{\Upsilon}_{ij} - \Xi_{iju} & \tilde{\Gamma}_{ij} \\ * & \tilde{\mathbb{F}}_{ij} \end{bmatrix} < 0, \tag{4.16}$$

$$\begin{bmatrix} \underline{\mu}_{iju} \bar{\Upsilon}_{ij} + m_{iju}\Xi_{iju} & \sqrt{\underline{\mu}_{iju}}\bar{\Gamma}_{ij} \\ * & \bar{\mathbb{F}}_{ij} \end{bmatrix} < 0, \tag{4.17}$$

$$\begin{bmatrix} \underline{\mu}_{iju} \tilde{\Upsilon}_{ij} + m_{iju}\Xi_{iju} & \sqrt{\underline{\mu}_{iju}}\tilde{\Gamma}_{ij} \\ * & \tilde{\mathbb{F}}_{ij} \end{bmatrix} < 0, \tag{4.18}$$

where

$$\bar{\Upsilon}_{ij} = \begin{bmatrix} \bar{\Upsilon}_{ij}^{11} & -\alpha_1\underline{\sigma}_1\mathcal{I}^{\mathrm{T}}C_1^{\mathrm{T}} & -\alpha_2\underline{\sigma}_2\mathcal{I}^{\mathrm{T}}C_2^{\mathrm{T}} & -\alpha_1\underline{\sigma}_1\mathcal{I}^{\mathrm{T}}C_1^{\mathrm{T}}F_1 - \alpha_2\underline{\sigma}_2\mathcal{I}^{\mathrm{T}}C_2^{\mathrm{T}}F_2 \\ * & \alpha_1(1-\underline{\sigma}_1)I & 0 & -\alpha_1\underline{\sigma}_1F_1 \\ * & * & \alpha_2(1-\underline{\sigma}_2)I & -\alpha_1\underline{\sigma}_2F_2 \\ * & * & * & -\alpha_1\underline{\sigma}_1F_1^{\mathrm{T}}F_1 - \alpha_2\underline{\sigma}_2F_2^{\mathrm{T}}F_2 - \gamma^2 I \end{bmatrix},$$

$$\tilde{\Upsilon}_{ij} = \begin{bmatrix} \tilde{\Upsilon}_{ij}^{11} & -\alpha_1\underline{\sigma}_1\mathcal{I}^{\mathrm{T}}C_1^{\mathrm{T}} & -\alpha_2\underline{\sigma}_2\mathcal{I}^{\mathrm{T}}C_2^{\mathrm{T}} & -\alpha_1\underline{\sigma}_1\mathcal{I}^{\mathrm{T}}C_1^{\mathrm{T}}F_1 - \alpha_2\underline{\sigma}_2\mathcal{I}^{\mathrm{T}}C_2^{\mathrm{T}}F_2 \\ * & \alpha_1(1-\underline{\sigma}_1)I & 0 & -\alpha_1\underline{\sigma}_1F_1 \\ * & * & \alpha_2(1-\underline{\sigma}_2)I & -\alpha_1\underline{\sigma}_2F_2 \\ * & * & * & -\alpha_1\underline{\sigma}_1F_1^{\mathrm{T}}F_1 - \alpha_2\underline{\sigma}_2F_2^{\mathrm{T}}F_2 - \gamma^2 I \end{bmatrix},$$

$$\tilde{\Gamma}_{ij} = \begin{bmatrix} \mathcal{A}_{\bar{\epsilon}ij}^{\mathrm{T}}X^{\mathrm{T}} \\ \bar{\mathcal{K}}_{1j} \\ \bar{\epsilon}\bar{\mathcal{K}}_{2j} \\ \tilde{D}_{ij}^{\mathrm{T}}X^{\mathrm{T}} \end{bmatrix},\ \bar{\Gamma}_{ij} = \begin{bmatrix} \mathcal{A}_{0ij}^{\mathrm{T}}X^{\mathrm{T}} \\ \bar{\mathcal{K}}_{1j} \\ 0 \\ \hat{D}_{ij}^{\mathrm{T}}X^{\mathrm{T}} \end{bmatrix},\ \bar{\mathbb{F}}_{ij} = \bar{\mathcal{P}} - X - X^{\mathrm{T}},\ \tilde{\mathbb{F}}_{ij} = \bar{\mathcal{P}} + \bar{\epsilon}\tilde{\mathcal{P}} - X - X^{\mathrm{T}},$$

$$\mathcal{A}_{0ij} = \begin{bmatrix} A_{0ij} & 0 \\ 0 & A_{0ij} - K_{1j}C_1 \end{bmatrix},\ \mathcal{A}_{\bar{\epsilon}ij} = \begin{bmatrix} A_{\bar{\epsilon}ij} & 0 \\ 0 & A_{\bar{\epsilon}ij} - K_{1j}C_1 - \bar{\epsilon}K_{2j}C_2 \end{bmatrix},\ A_{0ij} = \begin{bmatrix} A_{ij}^{11} & 0 \\ A_{ij}^{21} & 0 \end{bmatrix},$$

$$A_{\bar{\epsilon}ij} = \begin{bmatrix} A_{ij}^{11} & \bar{\epsilon}A_{ij}^{12} \\ A_{ij}^{21} & \bar{\epsilon}A_{ij}^{22} \end{bmatrix},\ \bar{\mathcal{K}}_{1j} = \begin{bmatrix} 0 \\ \mathcal{K}_{1j} \end{bmatrix}, \mathcal{A}_{0ij}^{\mathrm{T}}X^{\mathrm{T}} = \begin{bmatrix} A_{0ij}^{\mathrm{T}}X_1^{\mathrm{T}} & 0 \\ 0 & A_{0ij}^{\mathrm{T}}X_2^{\mathrm{T}} - C_1^{\mathrm{T}}\mathcal{K}_{1j} \end{bmatrix},\ \tilde{\mathcal{P}} = \begin{bmatrix} \tilde{P} & 0 \\ 0 & \tilde{P} \end{bmatrix},$$

$$\mathcal{A}_{\bar{\epsilon}ij}^{\mathrm{T}}X^{\mathrm{T}} = \begin{bmatrix} A_{\bar{\epsilon}ij}^{\mathrm{T}}X_1^{\mathrm{T}} & 0 \\ 0 & A_{\bar{\epsilon}ij}^{\mathrm{T}}X_2^{\mathrm{T}} - C_1^{\mathrm{T}}\mathcal{K}_{1j} - \bar{\epsilon}C_2^{\mathrm{T}}\mathcal{K}_{2j} \end{bmatrix},\ \bar{\mathcal{P}} = \begin{bmatrix} \bar{P} & 0 \\ 0 & \bar{P} \end{bmatrix},\ \bar{\mathcal{K}}_{2j} = \begin{bmatrix} 0 \\ \mathcal{K}_{2j} \end{bmatrix},$$

$$\tilde{\Upsilon}_{ij}^{11} = -(\bar{\mathcal{P}} + \bar{\epsilon}\tilde{\mathcal{P}}) + \bar{L}_{ij}^{\mathrm{T}}\bar{L}_{ij} - \alpha_1\underline{\sigma}_1\mathcal{I}^{\mathrm{T}}C_1^{\mathrm{T}}C_1\mathcal{I} - \alpha_2\underline{\sigma}_2\mathcal{I}^{\mathrm{T}}C_2^{\mathrm{T}}C_2\mathcal{I},\ \hat{D}_{ij}^{\mathrm{T}}X^{\mathrm{T}} = \begin{bmatrix} D_i^{\mathrm{T}}X_1^{\mathrm{T}} & D_iX_2^{\mathrm{T}} - F_{1j}^{\mathrm{T}}\mathcal{K}_{1j} \end{bmatrix},$$

$$\tilde{D}_{ij}^{\mathrm{T}}X^{\mathrm{T}} = \begin{bmatrix} D_i^{\mathrm{T}}X_1^{\mathrm{T}} & D_iX_2^{\mathrm{T}} - F_1^{\mathrm{T}}\mathcal{K}_{1j} - \bar{\epsilon}F_2^{\mathrm{T}}\mathcal{K}_{2j} \end{bmatrix},\ \bar{\Upsilon}_{ij}^{11} = -\bar{\mathcal{P}} + \bar{L}_{ij}^{\mathrm{T}}\bar{L}_{ij} - \alpha_1\underline{\sigma}_1\mathcal{I}^{\mathrm{T}}C_1^{\mathrm{T}}C_1\mathcal{I} - \alpha_2\underline{\sigma}_2\mathcal{I}^{\mathrm{T}}C_2^{\mathrm{T}}C_2\mathcal{I}.$$

In this situation, the desired estimator gains are calculated as

$$K_{1j} = X_2^{-1}\mathcal{K}_{1j}^{\mathrm{T}},\ K_{2j} = X_2^{-1}\mathcal{K}_{2j}^{\mathrm{T}}. \tag{4.19}$$

Proof. Denoting $\mathcal{P}_\epsilon \triangleq \bar{\mathcal{P}} + \epsilon\tilde{\mathcal{P}} > 0$, for $\epsilon \in (0, \bar{\epsilon}]$, it follows from (4.13), (4.14), and Lemma 1 that $\mathcal{P}_\epsilon > 0$. Besides, by combining (4.15)–(4.18) and Lemma 1, it can be obtained that

$$\begin{bmatrix} \Upsilon_{ij} - \Xi_{iju} & \Gamma_{ij} \\ * & \mathbb{F}_{ij} \end{bmatrix} < 0, \tag{4.20}$$

$$\begin{bmatrix} \underline{\mu}_{iju}\Upsilon_{ij} + m_{iju}\Xi_{iju} & \sqrt{\underline{\mu}_{iju}}\Gamma_{ij} \\ * & \mathbb{F}_{ij} \end{bmatrix} < 0, \tag{4.21}$$

where

$$\Upsilon_{ij} = \begin{bmatrix} \Upsilon_{ij}^{11} & -\alpha_1\underline{\sigma}_1\mathcal{I}^{\mathrm{T}}C_1^{\mathrm{T}} & -\alpha_2\underline{\sigma}_2\mathcal{I}^{\mathrm{T}}C_2^{\mathrm{T}} & -\alpha_1\underline{\sigma}_1\mathcal{I}^{\mathrm{T}}C_1^{\mathrm{T}}F_1 - \alpha_2\underline{\sigma}_2\mathcal{I}^{\mathrm{T}}C_2^{\mathrm{T}}F_2 \\ * & \alpha_1(1-\underline{\sigma}_1)I & 0 & -\alpha_1\underline{\sigma}_1F_1 \\ * & * & \alpha_2(1-\underline{\sigma}_2)I & -\alpha_2\underline{\sigma}_2F_2 \\ * & * & * & -\alpha_1\underline{\sigma}_1F_1^{\mathrm{T}}F_1 - \alpha_2\underline{\sigma}_2F_2^{\mathrm{T}}F_2 - \gamma^2 I \end{bmatrix} < 0,$$

$$\Gamma_{ij} = \begin{bmatrix} \mathcal{A}_{\epsilon ij}^{\mathrm{T}}X^{\mathrm{T}} \\ \bar{\mathcal{K}}_{1j}^{\mathrm{T}} \\ \epsilon\bar{\mathcal{K}}_{2j}^{\mathrm{T}} \\ \bar{D}_{ij}^{\mathrm{T}}X^{\mathrm{T}} \end{bmatrix}, \ \mathbb{F}_{ij} = \mathcal{P}_\epsilon - X - X^{\mathrm{T}},$$

$$\Upsilon_{ij}^{11} = -\mathcal{P}_\epsilon + \bar{L}_{ij}^{\mathrm{T}}\bar{L}_{ij} - \alpha_1\underline{\sigma}_1\mathcal{I}^{\mathrm{T}}C_1^{\mathrm{T}}C_1\mathcal{I}^{\mathrm{T}} - \alpha_2\underline{\sigma}_2\mathcal{I}^{\mathrm{T}}C_2^{\mathrm{T}}C_2\mathcal{I}^{\mathrm{T}},\ \bar{D}_{ij}^{\mathrm{T}}X^{\mathrm{T}} = \begin{bmatrix} D_i^{\mathrm{T}}X_1^{\mathrm{T}} & D_i^{\mathrm{T}}X_2^{\mathrm{T}} - F_1^{\mathrm{T}}\mathcal{K}_{1j} - \epsilon F_2^{\mathrm{T}}\mathcal{K}_{2j} \end{bmatrix}.$$

From reference [17], it follows that

$$X\mathcal{P}_\epsilon^{-1}X^{\mathrm{T}} \le \mathcal{P}_\epsilon - X - X^{\mathrm{T}}. \tag{4.22}$$

Accordingly, by applying (4.22) to (4.21), it yields that

$$\begin{bmatrix} \underline{\mu}_{iju}\Upsilon_{ij} + m_{iju}\Xi_{iju} & \sqrt{\underline{\mu}_{iju}}\Gamma_{ij} \\ * & \hat{\mathbb{F}}_{ij} \end{bmatrix} < 0, \tag{4.23}$$

where $\hat{\mathbb{F}}_{ij} = X\mathcal{P}_\epsilon^{-1}X^{\mathrm{T}}$.

Then, by taking (4.19) into consideration, one has

$$\begin{bmatrix} \underline{\mu}_{iju}\Upsilon_{ij} + m_{iju}\Xi_{iju} & \sqrt{\underline{\mu}_{iju}}\hat{\Gamma}_{ij} \\ * & \hat{\mathbb{F}}_{ij} \end{bmatrix} < 0, \tag{4.24}$$

where

$$\hat{\Gamma}_{ij} = \begin{bmatrix} X\mathcal{A}_{\epsilon ij} & X\bar{K}_{1j} & X\epsilon\bar{K}_{2j} & X\bar{D}_{ij} \end{bmatrix}^{\mathrm{T}}.$$

The left-hand side of (4.24) is pre- and post-multiplied by diag$\{I, I, I, I, X^{-1}\}$ and its transpose, respectively, and it can be obtained that

$$\begin{bmatrix} \underline{\mu}_{iju}\Upsilon_{ij} + m_{iju}\Xi_{iju} & \sqrt{\underline{\mu}_{iju}}\hat{\hat{\Gamma}}_{ij} \\ * & -\mathcal{P}_\epsilon^{-1} \end{bmatrix} < 0, \tag{4.25}$$

where $\hat{\hat{\Gamma}}_{ij} = \begin{bmatrix} \mathcal{A}_{\epsilon ij} & \bar{K}_{1j} & \epsilon\bar{K}_{2j} & \bar{D}_{ij} \end{bmatrix}^{\mathrm{T}}$.

Then, applying the Schur complement lemma yields the equivalence between inequality (4.2) and conditions (4.17) and (4.18), which guarantee the asymptotic stability of the augmented system (3.14) and the fulfillment of the H_∞ performance index (2.4). This proof is complete.

Remark 5. *The existence of the SPP often leads to significant time scale differences between the slow and fast dynamics in the SPSs. In order to achieve the maximum damage to the dynamics of the SPSs, inspired by the ETS, an asynchronous IDB DoS attack model is designed, under which attackers will launch attacks on important data packets in fast and slow channels based on different time scales. Meanwhile, affected by the estimated state, the IT2 fuzzy SPSs model always includes the mismatched MFs between the plant and the estimator design. Meanwhile, the introduction of the SPP unavoidably gives rise to numerical stiffness in the stability analysis process of the SPSs. More importantly, the coexistence of IDB asynchronous DoS attacks and mismatched MFs makes the augmented system* (3.14) *more complex. To deal with this problem, Lemma 1, the information of the local MFs, and the IDB DoS attack conditions* (3.7) *and* (3.8) *are considered in Theorem 2. By designing the resilient estimator gains* $K_{1j} = X_2^{-1}\mathcal{K}_{1j}^{\mathrm{T}}$, $K_{2j} = X_2^{-1}\mathcal{K}_{2j}^{\mathrm{T}}$, *the stability condition can be obtained by* (4.13)–(4.18).

Remark 6. *In this article, the inequalities involving* $\tilde{\psi}_1(\cdot)$ *and* $\tilde{\psi}_2(\cdot)$ *are derived from the inverse ETS used to model important-data-based DoS attacks. In the stability analysis, the considered condition corresponds to the case where the data are successfully delivered (i.e., the attack condition is not activated). Therefore, the derivation of* (4.5) *is performed under the admissible transmission condition associated with the proposed attack model, rather than requiring* $\tilde{\psi}_1(\cdot) \geq 0$ *and* $\tilde{\psi}_2(\cdot) \geq 0$ *to hold for all time instants. Then, by using Lyapunov stability theory, the asymptotic stability of augmented system* (3.14) *can be achieved.*

Remark 7. *Unlike existing studies that separately investigate SPSs, interval type-2 fuzzy systems, or conventional DoS attacks, this paper considers their coexistence within a unified state-estimation framework. The main challenge originates from the coupling among the singular perturbation dynamics, fuzzy uncertainties, and asynchronous IDB DoS attacks. Specifically, the attack decisions depend on the transmitted data importance, while the slow and fast subsystems are subject to different attack processes. Consequently, existing estimation conditions developed for synchronous attacks or single-channel transmission systems cannot be directly applied. To address this issue, a composite fuzzy resilient estimator and corresponding attack-dependent linear matrix inequality conditions are developed.*

□

5. Illustrative example

This part uses a DC motor model from [36] to demonstrate the effectiveness of our works.

First, consider the DC motor system model as follows:

$$\begin{cases} \acute{\mathbf{J}}\frac{d\zeta(t)}{dt} = -\mathfrak{F}\zeta(t) + \mathfrak{K}\mathfrak{L}_r i^2(t), \\ \acute{L}\frac{di(t)}{dt} = -\mathfrak{K}\mathfrak{L}_r\zeta(t)i(t) - \tilde{R}i(t) + \varpi(t), \end{cases}$$

where $\zeta(t)$ is the error of angular speed, $i(t)$ stands for the error of armature current, and $\varpi(t)$ means the external disturbance. $\acute{\mathbf{J}} = 1\mathrm{kg}\cdot\mathrm{m}^2$ and $\acute{L} = \epsilon$ are the moment of the inertia and inductance, respectively. $\mathfrak{F} = 0.08\mathrm{N}\cdot\mathrm{m/rad/s}$ is the viscous friction coefficient, $\mathfrak{K}$ is the torque/backemf invariable, $\mathfrak{L}_r$ represents the winding inductance, and $\tilde{R} = 5\Omega$ is the resistance.

Define $x_1(k)$ and $x_2(k)$ as $x_1(k) = \zeta(k)$, $x_2(k) = i(k)$, respectively. Let $x(k) = \begin{bmatrix} x_1^{\mathrm{T}}(k) & x_2^{\mathrm{T}}(k) \end{bmatrix}^{\mathrm{T}}$. Given that the DC motor model contains both SPP and nonlinear terms, similar to the treatment in [16], the sampling period is selected as $T = 0.05$ s. By applying the slow-sampling approach [39] to discretize the continuous-time singularly perturbed system and subsequently employing the IT2 fuzzy modeling technique to approximate the nonlinear dynamics, the following discrete-time IT2 fuzzy singularly perturbed system is obtained:

$$x(k+1) = \sum_{i=1}^{2} \mu_i(x(k))[A_{\epsilon i}x(k) + D_i w(k)],$$

where

$$A_{\epsilon 1} = \begin{bmatrix} 0.3549 & -2.2023\epsilon \\ -0.0088 & -0.0547\epsilon \end{bmatrix}, D_1 = \begin{bmatrix} -0.1834 \\ 0.1957 \end{bmatrix}, \ A_{\epsilon 2} = \begin{bmatrix} 0.3549 & 2.2023\epsilon \\ -0.0088 & -0.0547\epsilon \end{bmatrix}, D_2 = \begin{bmatrix} 0.1834 \\ 0.1957 \end{bmatrix}.$$

In this case, matrices $A_{\epsilon 1}$, D_1, $A_{\epsilon 2}$, and D_2 correspond to the local subsystem matrices associated with the two fuzzy rules generated from the nonlinear term of the DC motor model. Besides, the system measurement output can be depicted as

$$\begin{cases} y_1(k) = C_1 x(k) + F_1 w(k), \\ y_2(k) = C_2 x(k) + F_2 w(k), \end{cases}$$

where

$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \ F_1 = 0.3, \ F_2 = 0.1.$$

The controlled output is given by

$$z(k) = \sum_{i=1}^{2} L_i x(k),$$

where

$$L_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \ L_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Denote $f(k) = \Re\mathfrak{L}_r x_2(k)$ with $f(k) \in [-3, \ 3]$. In addition, $\Re\mathfrak{L}_r$ belongs to $[0, 1]$. Inspired by [37], the grades of membership in the plant model are chosen as

$$\begin{aligned} \underline{\mu}_1(x_2(k)) &= 0.3(1 - \frac{f(k)}{3}), \ \overline{\mu}_1(x_2(k)) = \frac{1}{2}(1 - \frac{f(k) - 0.1}{3 + 0.1}), \\ \overline{\mu}_2(x_2(k)) &= 0.3(1 + \frac{f(k)}{3}), \ \underline{\mu}_2(x_2(k)) = \frac{1}{2}(1 + \frac{f(k) + 0.1}{3 + 0.1}). \end{aligned}$$

Accordingly, in the system model, the nonlinear weighting coefficients are presented as $\underline{a}_i(\hat{x}(k)) = 0.2$, $\overline{a}_i(\hat{x}(k)) = 0.8$ $(i = 1, 2)$.

Next, the grades of membership in the estimator design are selected as

$$\underline{\nu}_1(\hat{x}_2(k)) = e^{\frac{-\hat{x}_2^2(k)}{0.5}}, \ \overline{\nu}_1(\hat{x}_2(k)) = e^{-\hat{x}_2^2(k)}, \ \underline{\nu}_2(\hat{x}_2(k)) = 1 - e^{-\hat{x}_2^2(k)}, \ \overline{\nu}_2(\hat{x}_2(k)) = 1 - e^{\frac{-\hat{x}_2^2(k)}{0.5}}.$$

Likewise, set $\underline{b}_j(\hat{x}(k)) = \cos^2(\hat{x}_2(k))$, $\overline{b}_j(\hat{x}(k)) = 1 - \underline{b}_j(\hat{x}(k))$ $(j = 1, 2)$.

The state space $\mathbb{U}$ is divided into five uniform subdomains. The operating domain $x_2(k)$ is broken into 5 uniform subdomains, each of which is described by $[-(21/5) + (6/5)u, -3 + (6/5)u]$, $u = 1, 2, ..., 5$. Based on the above relationship, we can calculate the constant parameters $\bar{\mu}_{iju}$ and $\underline{\mu}_{iju}$.

In this simulation example, we set the threshold parameters $\sigma_1 = 1.6$, $\sigma_2 = 1.1$ and select $w(k) = \sin(k)e^{-0.1k}$.

Validation.

According to Theorem 2, we choose the H_∞ performance index $\gamma = 3.5$ and the SPP $\epsilon = 0.01$, and calculate the thresholds $\underline{\sigma}_1 = 1.5284$, $\underline{\sigma}_2 = 1.0083$. Then, by using the MATLAB toolbox, the resilient estimator gains can be solved as

$$K_{11} = \begin{bmatrix} 0.1670 \\ -0.0112 \end{bmatrix},\ K_{21} = \begin{bmatrix} -0.0042 \\ 6.6732 \end{bmatrix},\ K_{12} = \begin{bmatrix} 0.1670 \\ -0.0090 \end{bmatrix},\ K_{22} = \begin{bmatrix} -0.0042 \\ 4.6960 \end{bmatrix}.$$

Furthermore, by choosing $x(0) = [-0.5 \quad 1.5]^{\mathrm{T}}$ and $\hat{x}(0) = [-1 \quad 1]^{\mathrm{T}}$, the slow and fast state responses of the networked SPSs are illustrated in Figures 2 and 3, respectively. Meanwhile, the slow and fast DoS attack sequences are shown in Figures 4 and 5, respectively. From Figure 4, we see that the attacker launched 14 attacks in the slow transmission channel. From Figure 5, we conclude that the attacker launched 44 attacks in the fast transmission channel. From Figures 2 and 3, we conclude that the obtained estimator gains can effectively estimate the system's states when the system is attacked. Besides, the controlled output of the networked SPSs can be shown in Figure 6.

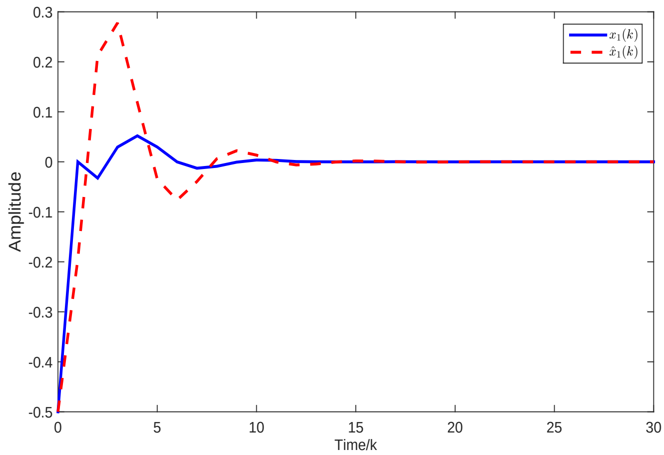

Figure 2. The slow state response and its estimation.

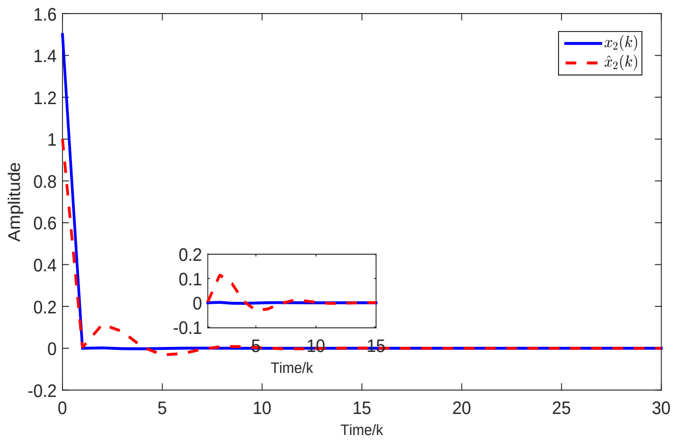

Figure 3. The fast state response and its estimation.

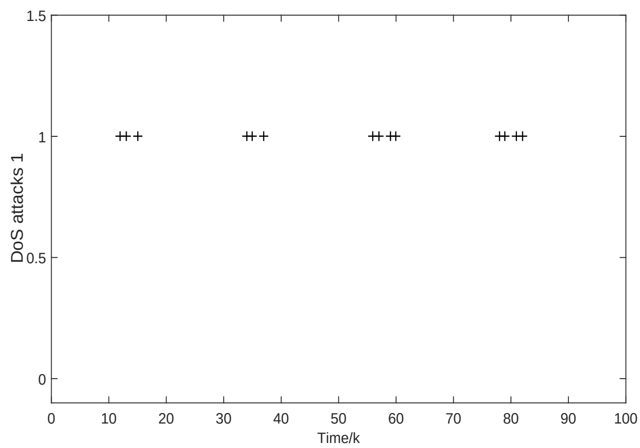

Figure 4. The IDB DoS signals for the slow transmission channel.

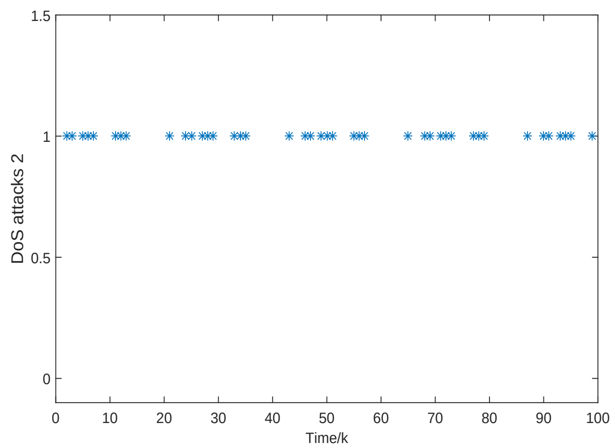

Figure 5. The IDB DoS signals for the fast transmission channel.

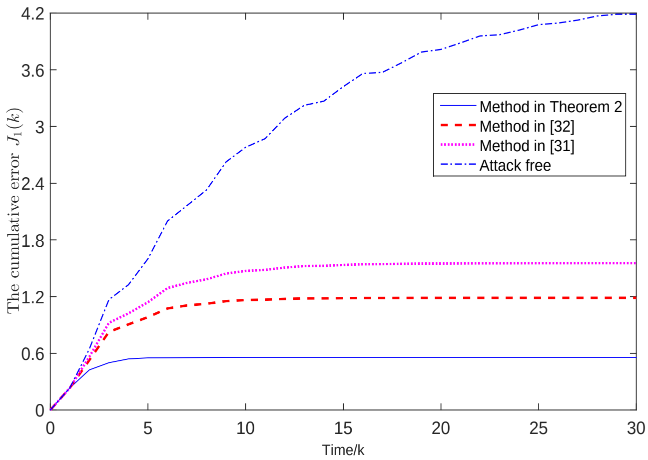

Figure 6. The cumulative error $J_1(k)$.

Next, by solving (4.13)–(4.18), the γ_{opt} is computed as 3.3166, then, we conclude that

$$\sqrt{\frac{\sum_{k=0}^{30} \|\tilde{z}(k)\|^2}{\sum_{k=0}^{30} \|w(k)\|^2}} = 1.0476 \leq 3.3166,$$

which satisfies the H_∞ performance (2.4). The simulation validates the consistency with our theoretical outcomes.

Comparison.

From another perspective, we define the cumulative error indexes $J_1(k) = \sum_{k=0}^{T} |e_1(k)|$ and $J_2(k) = \sum_{k=0}^{T} |e_2(k)|$. According to [32], under periodic DoS attacks, the estimator gain is computed as follows:

$$\hat{K}_{11} = \begin{bmatrix} 0.7011 \\ -1.1932 \end{bmatrix}, \ \hat{K}_{21} = \begin{bmatrix} -0.0062 \\ 0.6543 \end{bmatrix}, \ \hat{K}_{12} = \begin{bmatrix} 0.0042 \\ -0.4961 \end{bmatrix}, \ \hat{K}_{22} = \begin{bmatrix} -0.0062 \\ 0.6543 \end{bmatrix}.$$

Under the proposed method in [33], the estimator gain is computed as follows:

$$\hat{K}_{11} = \begin{bmatrix} 0.1505 \\ -0.0417 \end{bmatrix}, \ \hat{K}_{21} = \begin{bmatrix} -0.1034 \\ 0.2513 \end{bmatrix}, \ \hat{K}_{12} = \begin{bmatrix} 0.1021 \\ -0.3715 \end{bmatrix}, \ \hat{K}_{22} = \begin{bmatrix} -0.0062 \\ 0.6543 \end{bmatrix}.$$

When no DoS attacks are present, the designed estimator gain is computed as follows:

$$\check{K}_{11} = \begin{bmatrix} 0.0042 \\ -0.4961 \end{bmatrix}, \ \check{K}_{21} = \begin{bmatrix} -0.0019 \\ -1.5305 \end{bmatrix}, \ \check{K}_{12} = \begin{bmatrix} 0.1017 \\ 0.2044 \end{bmatrix}, \ \check{K}_{22} = \begin{bmatrix} -0.1012 \\ -0.7706 \end{bmatrix}.$$

In order to further validate the feasibility of the designed resilient H_∞ estimator, we provide some specific comparisons as shown in Figures 6 and 7. By applying the estimated gains obtained from the DoS attack models in references [32, 33] and without DoS attack to the IT2 fuzzy SPSs with the asynchronous IDB DoS attack model, the proposed estimator shows the minimum cumulative error. That is, the proposed H_∞ estimator offers more effective mitigation of the proposed IDB DoS attacks.

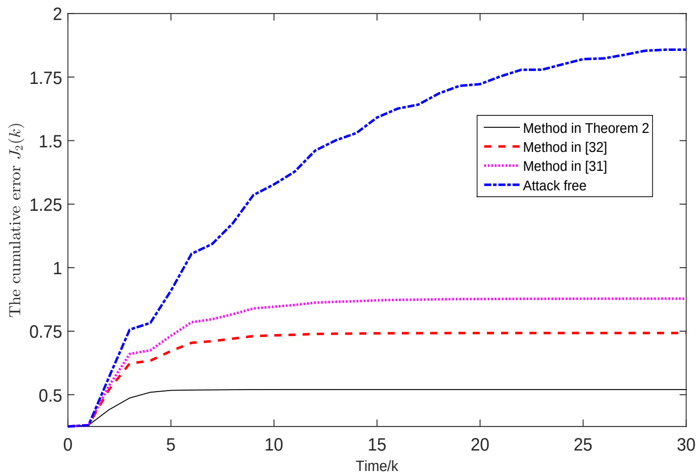

Figure 7. The cumulative error $J_2(k)$.

6. Conclusions

In this paper, the H_∞ security state estimation problem was discussed for IT2 fuzzy SPSs. An asynchronous IDB DoS attack model was proposed to characterize the cyber-attack behavior of different channels. Then, an IT2 fuzzy resilient estimator was designed to tackle this cyber-attack behavior. By constructing an SPP-dependent Lyapunov function, a matrix-inequality-based sufficient condition was derived to ensure the asymptotic stability of the augmented system. Through simulation verification, we found that the proposed resilient estimator can effectively resist asynchronous IDB DoS attacks. Further research can be concentrated on the stabilization of networked SPSs with energy-constrained DoS attacks.

Author contributions

Yue Hu: Investigation, Writing-original draft; Chaoqun Guo: Conceptualization, Methodology; Oh-Min Kwon: Writing-review editing, Validation. Seung Hoon Lee: Validation. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

All authors declare that they have not used the Artificial Intelligence (AI) in the creation of this article.

Acknowledgements

This work was supported by the Shandong Provincial Natural Science Foundation under grant no. ZR2026QC0750 and in part by the Shandong Provincial Natural Science Foundation under grant no. ZR2025QC1625. This work was also supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (RS-2020-NR049604), by the Regional Innovation System & Education (RISE) program through the Chungbuk

Regional Innovation System & Education Center, funded by the Ministry of Education (MOE) and the Chungcheongbuk-do, Republic of Korea (2026-RISE-11-014-03), and partially supported by Innovative Human Resource Development for Local Intellectualization program through the Institute of Information & Communications Technology Planning & Evaluation (IITP) grant funded by the Korea government (MSIT) (IITP-2026-RS-2020-II201462, 20%).

Conflict of interest

The authors declare that they have no conflicts of interest.

References

1. Q. Lin, L. Ding, Z. M. Kong, Z.-W. Yu, X. Li, H. J. Wang, Multi-time scale model predictive control-based demand side management for a microgrid, *IEEE T. Smart Grid*, **16** (2025), 1181–1193. https://doi.org/10.1109/TSG.2024.3493958
2. L. H. Zhong, F. W. Li, J. W. Zhang, B. Z. Yuan, J. Chen, W. J. Yang, et al., Multi-time scale coordinated dispatch of integrated electricity-hydrogen-heat microgrids with waste heat recovery, *Int. J. Elec. Power*, **175** (2026), 111634. https://doi.org/10.1016/j.ijepes.2026.111634
3. Y. S. Liu, W. Wang, J. B. Zhu, X. Y. Ma, Y. B. Che, Multiple time scale control scheme for the power distribution network with DC energy balancing interconnection, *IEEE T. Ind. Appl.*, **62** (2026), 2002–2012. https://doi.org/10.1109/TIA.2025.3601067
4. H. Shen, C. J. Peng, H. C. Yan, S. Y. Xu. Data-driven near optimization for fast sampling singularly perturbed systems, *IEEE T. Automat. Contr.*, **69** (2024), 4689–4694. https://doi.org/10.1109/TAC.2024.3352703
5. Q. Li, Z. H. Jin, L. Y. Qiao, A. C. Du, G. Liu, Distributed optimization of nonlinear singularly perturbed multi-agent systems via a small-gain approach and sliding mode control, *AIMS Mathematics*, **9** (2024), 20865–20886. https://doi.org/10.3934/math.20241015
6. M. Tang, M. Krstić, J. I. Poveda, Fixed-time input-to-state stability for singularly perturbed systems via composite lyapunov functions, *IEEE T. Automat. Contr.*, **71** (2026), 2644–2651. https://doi.org/10.1109/TAC.2025.3631559
7. W. H. Qi, N. Zhang, G. D. Zong, C. K. Ahn, SMC for discrete semi-Markov switching slow sampling singularly perturbed models with applications, *IEEE T. Circuits-I*, **71** (2024), 1910–1919. https://doi.org/10.1109/TCSI.2024.3355445
8. J. Wang, J. C. Wu, H. Shen, J. D. Cao, L. Rutkowski, Fuzzy H_∞ control of discrete-time nonlinear Markov jump systems via a novel hybrid reinforcement Q-learning method, *IEEE T. Cybernetics*, **53** (2023), 7380–7391. https://doi.org/ 10.1109/TCYB.2022.3220537
9. Y. Y. Wang, P. Shi, H. C. Yan, Reliable control of fuzzy singularly perturbed systems and its application to electronic circuits, *IEEE T. Circuits-I*, **65** (2018), 3519–3528. https://doi.org/ 10.1109/TCSI.2018.2834481
10. T. C. Zhang, W. X. Fang, L. Y. Wang, Y. L. Lu, Finite-time stability of fractional-order fuzzy inertial neural networks via a new finite-time inequality, *AIMS Mathematics*, **11** (2026), 5936–5953. https://doi.org/10.3934/math.2026245

11. Y. H. Zhao, L. K. Wang, X. P. Xie, H.-K. Lam, Finite-time asynchronous switching control for fuzzy Markov jump systems by applying polynomial membership functions, *IEEE T. Circuits-I*, **71** (2024), 5607–5617. https://doi.org/10.1109/TCSI.2024.3448629

12. T. T. Zhang, H. S. Zhang, X. P. Xie, Region stability/stabilization and H_∞ control for discrete-time impulsive Takagi-Sugeno fuzzy systems, *IEEE T. Fuzzy Syst.*, **32** (2024), 3410–3419. https://doi.org/10.1109/TFUZZ.2024.3372936

13. W. H. Qi, R. K. Li, J. H. Park, Z.-G. Wu, H. C. Yan, Observer-based stabilization of networked IT2 fuzzy semi-Markov jump models with redundant channels and applications to tunnel circuit model, *IEEE T. Circuits-I*, **71** (2024), 6316–6326. https://doi.org/10.1109/TCSI.2024.3400536

14. R. Sakthivel, R. Kavikumar, A. Mohammadzadeh, O.-M. Kwon, B. Kaviarasan, Fault estimation for mode-dependent IT2 fuzzy systems with quantized output signals, *IEEE T. Fuzzy Syst.*, **29** (2021), 298–309. https://doi.org/10.1109/TFUZZ.2020.3018509

15. R. Kavikumar, O.-M. Kwon, B. Kaviarasan, R. Sakthivel, Antidisturbance control design for interval type-2 fuzzy stochastic systems with input quantization, *IEEE T. Fuzzy Syst.*, **31** (2023), 1806–1818. https://doi.org/10.1109/TFUZZ.2022.3215277

16. Y. Hu, O.-M. Kwon, S.-H. Lee, H_∞ Control for interval type-2 fuzzy singularly perturbed systems with multi-node round-robin protocol and packet dropouts, *Fuzzy Set. Syst.*, **500** (2025), 109202. https://doi.org/10.1016/j.fss.2024.109202

17. Y. Hu, C. X. Cai, H. Lin, O.-M. Kwon, Fuzzy-model-based H_∞ control for networked singularly perturbed systems: The asynchronous weighted try-once-discard protocol case, *IEEE T. Fuzzy Syst.*, **32** (2024), 2713–2724. https://doi.org/10.1109/TFUZZ.2024.3359287

18. J. Wang, X. M. Liu, J. W. Xia, H. Shen, J. H. Park, Quantized interval type-2 fuzzy control for persistent dwell-time switched nonlinear systems with singular perturbations, *IEEE T. Cybernetics*, **52** (2022), 6638–6648, https://doi.org/10.1109/TCYB.2021.3049459

19. H. Shen, G. Q. Wang, J. W. Xia, J. H. Park, X.-P. Xie, Interval type-2 fuzzy H_∞ filtering for nonlinear singularly perturbed jumping systems: A semi-Markov kernel method, *Fuzzy Set. Syst.*, **505** (2025), 109264. https://doi.org/10.1016/j.fss.2025.109264

20. C. L. Ye, G. Z. Zhang, F. Li, J. Wang, H. Shen, H_∞ control for interval type-2 fuzzy singularly perturbed nonlinear systems with Markov jumping parameters, *Int. J. Fuzzy Syst.*, **25** (2023), 2712–2721. https://doi.org/10.1007/s40815-023-01520-2

21. J. G. Zhu, G. M. Zhuang, G. L. Chen, X. P. Xie, Static output feedback quantized dissipative security control of singular hybrid systems subject to multiple cyber attacks: A dynamic-memory event-triggered strategy, *Commun. Nonlinear Sci.*, **128** (2024), 107652. https://doi.org/10.1016/j.cnsns.2023.107652

22. G. M. Zhuang, Q. Y. Zhu, X.-P. Xie, J. W. Xia, Event-based H_∞ fuzzy control for implicit hybrid AVS systems embedding acceleration characteristic under random deception and DoS attacks, *IEEE T. Autom. Sci. Eng.*, **22** (2025), 6154–6167. https://doi.org/10.1109/TASE.2024.3439705

23. E. G. Tian, H. Chen, C. S. Wang, L. C. Wang, Security-ensured state of charge estimation of lithium-ion batteries subject to malicious attacks, *IEEE T. Smart Grid*, **14** (2023), 2250–2261. https://doi.org/10.1109/TSG.2022.3202811

24. L. C. Wang, E. G. Tian, C. S. Wang, S. Liu, Secure estimation against malicious attacks for lithium-ion batteries under cloud environments, *IEEE T. Circuits-I*, **69** (2022), 4237–4247. https://doi.org/10.1109/TCSI.2022.3187725

25. F. Y. Yang, X. P. Xie, E. G. Tian, J. Y. Sun, Co-design of improved fuzzy switching-type state-FDI estimation and resilient control for DC microgrids under malicious attacks, *IEEE T. Autom. Sci. Eng.*, **22** (2025), 2671–2681. https://doi.org/10.1109/TASE.2024.3382755

26. H. Louati, R. Arooj, A. U. K. Niazi, M. E. Dalam, M. M. A. Almazah, A. Smerat, Finite-time fuzzy control strategy for nonlinear MASs with actuator faults and deception attacks, *AIMS Mathematics*, **10** (2025), 20113–20139. https://doi.org/10.3934/math.2025899

27. S. Han, H. Zhu, L. F. Hua, K. B. Shi, Z. N. Peng, H. Cheng, Adaptive hierarchical event-triggered H_∞ output tracking of IT2 fuzzy heterogeneous multiagent systems under multiple-channel DoS attacks, *IEEE T. Cybernetics*, **56** (2026), 3187–3199. https://doi.org/10.1109/TCYB.2026.3650819

28. X. K. Tang, J. Cheng, B. Zhang, L. Rutkowski, W. T. Huang, Security control for fuzzy singularly perturbation systems under DoS attacks, *IEEE T. Autom. Sci. Eng.*, **22** (2025), 19685–19693. https://doi.org/10.1109/TASE.2025.3596508

29. J. Cheng, Y. Y. Wang, J. H. Park, J. D. Cao, K. B. Shi, Static output feedback quantized control for fuzzy Markovian switching singularly perturbed systems with deception attacks, *IEEE T. Fuzzy Syst.*, **30** (2022), 1036–1047. https://doi.org/10.1109/TFUZZ.2021.3052104

30. L. Ma, Y. Zhang, C. Y. Yang, L. N. Zhou, Security control for two-time-scale cyber physical systems with multiple transmission channels under DoS attacks: The input-to-state stability, *J. Franklin I.*, **358** (2021), 6309–6325. https://doi.org/10.1016/j.jfranklin.2021.05.017

31. D. T. Wang, Y. Q. Wang, G. M. Zhuang, J. Chen, Dynamic event-triggered asynchronous and resilient dissipative filtering for discrete-time Markov jump singularly perturbed systems with hybrid cyber attacks, *Nonlinear Dyn.*, **112** (2024), 15461–15482. https://doi.org/10.1007/s11071-024-09820-4

32. L. Ma, H. Y. Liu, L. N. Zhou, C. Y. Yang, W. Dai, G. Q. Wang, Security control for multi-time-scale CPSs under DoS attacks: An improved dynamic event-triggered mechanism, *IEEE T. Netw. Sci. Eng.*, **9** (2022), 1813–1826. https://doi.org/10.1109/TNSE.2022.3153489

33. Y. Zhang, L. Ma, G. Q. Wang, C. Y. Yang, L. N. Zhou, W. Dai, Observer-based control for the two-time-scale cyber-physical systems: The dual-scale DoS attacks case, *IEEE T. Netw. Sci. Eng.*, **8** (2021), 3369–3379. https://doi.org/ 10.1109/TNSE.2021.3112809

34. X. Wang, E. G. Tian, W. X. Zheng, X. P. Xie, Important-data-based DoS attack mechanism and resilient H_∞ filter design for networked T-S fuzzy systems, *IEEE T. Cybernetics*, **54** (2024), 3352–3362. https://doi.org/10.1109/TCYB.2023.3285526

35. E. G. Tian, M. G. Fan, L. F. Ma, D. Yue, Stochastic important-data-based attack power allocation against remote state estimation in sensor networks, *IEEE T. Automat. Contr.*, **70** (2025), 2012–2019. https://doi.org/10.1109/TAC.2024.3477009

36. J. Cheng, S. Liu, H.-K. Lam, H. C. Yan, D. Zhang, Sliding mode control for fuzzy singularly perturbed systems with improved protocol, *IEEE T. Circuits-II*, **70** (2023), 3039–3043. https://doi.org/10.1109/TCSII.2023.3255400

37. J. X. Liu, C. W. Wu, Z. H. Wang, L. G. Wu, Reliable filter design for sensor networks using type-2 fuzzy framework, *IEEE T. Ind. Inform.*, **13** (2017), 1742–1752. https://doi.org/10.1109/TII.2017.2654323
38. Z. N. Zhang, S.-F. Su, Y. G. Niu, Dynamic event-triggered control for interval type-2 fuzzy systems under fading channel, *IEEE T. Cybernetics*, **51** (2021), 5342–5351. https://doi.org/10.1109/TCYB.2020.2996296
39. H. Kando, T. Iwazumt, Multirate digital control design of anoptimal regulator via singular perturbation theory, *Int. J. Control*, **44** (1986), 1555–1578. https://doi.org/10.1080/00207178608933686