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Research article

Simultaneous confidence intervals for all pairwise differences of coefficients of variation of zero-inflated Birnbaum–Saunders distributions

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Abstract: The data used for the analysis were collected from multiple regions or years. Evaluating each region or year separately may be insufficient for drawing comprehensive inferences and may fail to reveal statistically significant differences. To ensure the reliability of the analysis and to enable overall conclusions, it is necessary to apply a statistical method known as simultaneous confidence intervals. This technique enables the simultaneous construction of confidence intervals for multiple parameters. Therefore, we proposed and evaluated methods for constructing simultaneous confidence intervals for all pairwise differences between the coefficients of variation in zero-inflated Birnbaum-Saunders distributions. The methods utilized for constructing simultaneous confidence intervals comprise the generalized confidence interval (GCI), the bootstrap confidence interval (BCI), the method of variance estimates recovery (MOVER), the MOVER based on GCI, the MOVER based on BCI, the Bayesian credible interval, and the highest posterior density interval (HPD). Monte Carlo simulations were employed to evaluate the performance of each method, which involved the assessment of coverage probabilities and average widths under a set of parameter configurations and sample sizes. The generalized confidence interval method was the most efficient overall, as indicated by the simulation results. Finally, all proposed methods were applied to real-world wind speed data to examine their practical applicability and to demonstrate the consistency of the results between the simulation study and real-world applications.

Keywords: average width; coefficient of variation; coverage probability; simultaneous confidence interval; zero-inflated Birnbaum-Saunders
Mathematics Subject Classification: 62F25, 62P12

1. Introduction

Interest in clean energy has been steadily increasing, as it provides an alternative to fossil fuels and helps mitigate environmental problems caused by greenhouse gas emissions and air pollution. Wind energy is one of the best renewable sources of energy because it comes from the movement of air. This natural flow of air results in the generation of wind, which in turn leads to environmental changes, particularly in the areas of wind energy and air quality. In terms of wind energy, wind turbines can turn powerful winds into electricity. Even a small increase in wind speed can lead to a big increase in the amount of energy created. Wind power plants are best built in places where the winds are strong and steady. These plants create clean energy without polluting the air. Wind speed is also crucial for managing the buildup and spread of pollutants such as fine particulate matter (PM2.5), nitrogen dioxide, and harmful smoke. On the other hand, when there is no wind or the wind speed is relatively low, pollutants tend to stay in the same place, which can have serious effects on public health. This phenomenon happens a lot in big cities with a lot of pollution sources, like Bangkok and Chiang Mai, where the air is often still and the winds are weak in the winter. Wind speed is very important for making energy and controlling pollution, so many academics have looked at the properties of wind speed data, especially the statistical distributions that can accurately characterize how wind speed changes over time. These kinds of studies are helpful for making wind energy planning more accurate. For instance, Mohammadi, Alavi, and McGowan [1] performed a study and discovered that the Birnbaum-Saunders (BS) distribution is one of the most effective statistical models for wind speed data. However, although the BS distribution can adequately describe the characteristics of wind speed data, it has a significant limitation in that it cannot analyze data with zero values. At certain times, wind speed may decrease to nearly zero. As a result, the BS model does not properly show what the data resembles. Consequently, when wind speed data encompasses positive and zero values, researchers have formulated a novel model termed the zero-inflated Birnbaum-Saunders (ZIBS) distribution. Many researchers have studied ZIBS distribution by constructing confidence intervals for various parameters. For example, Ratasukharom, Niwitpong, and Niwitpong [2] developed confidence intervals for the mean of the ZIBS distribution. In the same year, they also investigated methods for constructing confidence intervals for the variance [3]. Subsequently, Janthasuwan Niwitpong, and Niwitpong [4] proposed a method for constructing confidence intervals for the coefficient of variation. More recently, Thangjai et al. [5] examined the use of functions of percentiles to construct confidence intervals for ZIBS distribution.

In cases where wind speed data from multiple areas or regions need to be analyzed simultaneously, for example, comparing wind speeds in Thailand's northern, northeastern, and southern regions, analyzing each region separately may not be sufficient to draw comprehensive conclusions or to reveal statistically significant differences. Because of such circumstances, one must apply a statistical procedure called Simultaneous Confidence Intervals (SCIs). The use of SCIs enables the simultaneous comparison of wind speed parameters across regions. As a result, many researchers have investigated SCIs for all pairwise differences between parameters in various distributions. For instance, Li, Song, and Shi [6] suggested a parametric bootstrap method to create SCIs for the mean differences among several pairs of two-parameter exponential distributions. Subsequently, Thangjai Niwitpong and Niwitpong [7] introduced methods for estimating SCIs for the differences in means across several normal populations when the coefficients of variation are unknown. Their proposed techniques included the generalized confidence interval (GCI) and the method of variance estimates recovery (MOVER). Later, Malekzadeh and Kharrati-Kopaei [8] applied GCI, fiducial GCI (FGCI), and parametric bootstrap methods to construct SCIs for quantile differences among multiple two-parameter exponential distributions under a progressive Type II censoring scheme. Puggard Niwitpong and Niwitpong [9] presented estimation methods for SCIs that consider all potential pairwise differences between the coefficients of variation in BS distributions. These techniques involve the use of the highest posterior density interval, Bayesian credible intervals, the percentile bootstrap, the GCI, the MOVER based on the asymptotic confidence interval, and the MOVER based on the GCI. Furthermore, Ren, Liu, and Pu [10] developed three fiducial methods, one exact and two approximate, for constructing SCIs for differences in the means of multiple delta-gamma distributions.

One parameter of importance in statistical data analysis that has not been examined in relation to the generation of simultaneous confidence intervals for the ZIBS distribution is the coefficient of variation (CV). The CV is a statistical measure of how datasets are distributed in relation to each other. The CV is the standard deviation divided by the mean. A higher CV means that the data is more variable compared to other data, whereas a lower CV means that the data is less variable compared to other data. The CV is important because it has no units of measurement, which makes it easy to compare variability between datasets with various units of measurement. In many real-world situations, the coefficient of variation serves as a useful tool. For instance, environmental scientists utilize the coefficient of variation to study the variability in environmental data, such as rainfall patterns, temperature fluctuations, or pollutant levels [11–13]. Moreover, the difference in the coefficient of variation, which is a useful method for comparing the level of relative dispersion between data sets, enables us to assess and compare the consistency or volatility of data quantitatively with greater clarity. This is applicable in various scenarios, such as comparing the consistency of different processes, evaluating the impact of changes on data volatility, or identifying significant differences between groups. This information can be effectively used for practical decision-making.

Based on a comprehensive review of the literature, it has been found that no prior research has utilized the coefficients of variation of the ZIBS distributions for constructing simultaneous confidence intervals. Therefore, we propose seven statistical methods: The generalized confidence interval (GCI), the bootstrap confidence interval (BCI), the method of variance estimates recovery (MOVER), the MOVER based on GCI (MG), the MOVER based on BCI (MB), the Bayesian credible interval (BAY), and the highest posterior density (HPD). These methods will be employed to construct simultaneous confidence intervals for the differences in the coefficients of variation in the ZIBS distributions. Finally, the proposed approaches will be applied to wind speed data in Thailand.

2. Properties of the ZIBS distribution

Let $Y = (Y_1, Y_2, \ldots, Y_n)$ be a random sample drawn from the ZIBS distribution with the probability of zero δ, shape parameter α, and scale parameter β, denoted by $ZIBS(\alpha, \beta, \delta)$. The probability density function of the ZIBS distribution can be defined as

$$f(y; \alpha, \beta, \delta) = \delta \mathrm{I}_0[y] + (1-\delta)\frac{1}{2\alpha\beta\sqrt{2\pi}}\left[\left(\frac{\beta}{y}\right)^{\frac{1}{2}} + \left(\frac{\beta}{y}\right)^{\frac{3}{2}}\right]\exp\left[-\frac{1}{2\alpha^2}\left(\frac{y}{\beta} + \frac{\beta}{y} - 2\right)\right]\mathrm{I}_{(0,\infty)}[y], \quad (1)$$

where $\alpha, \beta > 0$, I is an indicator function, in which $\mathrm{I}_0[y]$ takes the value 1 when $y = 0$ and 0 when $y > 0$ and $\mathrm{I}_{(0,\infty)}[y]$ takes the value 0 when $y = 0$ and 1 when $y > 0$. The cumulative distribution function (CDF) of Y is

$$G(y; \alpha, \beta, \delta) = \begin{cases} \delta & ; y = 0 \\ \delta + (1-\delta)F(y; \alpha, \beta) & ; y > 0 \end{cases}, \quad (2)$$

where $F(y; \alpha, \beta)$ is the CDF of the Birnbaum-Saunders distribution. For $Y = 0$, the number of zero observations is distributed according to the binomial distribution denoted by $n_{(0)} \sim Bin(n, \delta)$. Given $n = n_{(1)} + n_{(0)}$, where $n_{(1)}$ and $n_{(0)}$ represent the numbers of positive and zero values, respectively, the maximum likelihood estimate of δ is $\hat{\delta} = n_{(0)}/n$. According to the Aitchison [14] concept, the population mean, variance, and coefficient of variation can be calculated as follows:

$$E(Y) = (1-\delta)\beta\left(1+\frac{\alpha^2}{2}\right),$$

$$V(Y) = (1-\delta)(\alpha\beta)^2\left(1+\frac{5\alpha^2}{4}\right) + \delta(1-\delta)\beta^2\left(1+\frac{\alpha^2}{2}\right)^2,$$

and

$$\upsilon = \frac{\sqrt{V(Y)}}{E(Y)} = \frac{1}{2+\alpha^2}\left[\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}\right]^{\frac{1}{2}}.$$

In this study, we are interested in all pairwise differences between the CVs of ZIBS populations, as follows: $\varpi_{ir} = \upsilon_i - \upsilon_r$, where $i, r = 1,2,3,\dots,k$ and $i \neq r$. Assume that $\hat{\alpha}_i$ and $\hat{\delta}_i$ are independent, then the maximum likelihood estimator of ϖ_{ir} can be determined as

$$\hat{\varpi}_{ir} = \hat{\upsilon}_i - \hat{\upsilon}_r = \frac{1}{2+\hat{\alpha}_i^2}\left[\frac{\hat{\alpha}_i^2(4+5\hat{\alpha}_i^2)+\hat{\delta}_i(2+\hat{\alpha}_i^2)^2}{1-\hat{\delta}_i}\right]^{\frac{1}{2}} - \frac{1}{2+\hat{\alpha}_r^2}\left[\frac{\hat{\alpha}_r^2(4+5\hat{\alpha}_r^2)+\hat{\delta}_r(2+\hat{\alpha}_r^2)^2}{1-\hat{\delta}_r}\right]^{\frac{1}{2}}, \quad (3)$$

where $\hat{\alpha}_i = \left\{2\left[\left(\sum_{j=1}^{n_{i(1)}}\frac{y_{ij}}{n_{i(1)}}\sum_{j=1}^{n_{i(1)}}\frac{y_{ij}^{-1}}{n_{i(1)}}\right)^{\frac{1}{2}} - 1\right]\right\}^{\frac{1}{2}}$ is the modified moment estimator of α_i proposed by Ng, Kundu, and Balakrishnan [15]. According to Janthasuwan, Niwitpong, and Niwitpong [4], the asymptotic variance of $\hat{\upsilon}_i$, derived using the Taylor series in the delta method, is given by

$$\hat{V}(\hat{\upsilon}_i) \approx \frac{1}{(2+\hat{\alpha}_i^2)^2(1-\hat{\delta}_i)\left[\hat{\alpha}_i^2(4+5\hat{\alpha}_i^2)+\hat{\vartheta}_i(2+\hat{\alpha}_i^2)^2\right]}\left\{\frac{32\hat{\alpha}_i^4(1+2\hat{\alpha}_i^2)^2}{n_{i(1)}(2+\hat{\alpha}_i^2)^2} + \frac{\hat{\delta}_i[2+\hat{\alpha}_i^2(4+3\hat{\alpha}_i^2)]^2}{n_i(1-\hat{\delta}_i)}\right\}. \quad (4)$$

Therefore, the estimated variance of $\hat{\varpi}_{ir}$ can be written as

$$\hat{V}(\hat{\varpi}_{ir}) = \hat{V}(\hat{\upsilon}_i - \hat{\upsilon}_r) = \hat{V}(\hat{\upsilon}_i) + \hat{V}(\hat{\upsilon}_r),$$

where $i, r = 1,2,3,\dots,k$, $i \neq r$, and $COV(\hat{\upsilon}_i, \hat{\upsilon}_r) = 0$.

3. Interval estimation

The concept of the method to construct simultaneous confidence intervals for ϖ_{ir} is explained in detail as follows:

3.1. The generalized confidence interval (GCI)

The GCI method, which is predicated on the idea of a generalized pivotal quantity (GPQ), was suggested by Weerahandi [16] for creating confidence intervals. The generalized pivotal quantities for the parameters β_i, α_i, and δ_i are obtained to construct the simultaneous confidence interval for ϖ_{ir} using GCI. According to Sun [17], the GPQ of β_i can be defined as

$$Q_{\beta_i}(y_{ij};\Lambda_i)=\begin{cases} max(\beta_{i1},\beta_{i2})\,;\Lambda_i\le 0;\\ min(\beta_{i1},\beta_{i2})\,;\Lambda_i>0,\end{cases} \tag{5}$$

where Λ_i follows the t-distribution with $n_{i(1)}-1$ degrees of freedom, and β_{i1} and β_{i2} are the two solutions of the following quadratic equation:

$$\Delta_{i1}{\beta_i}^2-2\Delta_{i2}\beta_i+\left(n_{i(1)}-1\right){C_i}^2-\left(1/n_{i(1)}\right)D_i\Lambda_i^2=0,$$

where $\Delta_{i1}=\left[\left(n_{i(1)}-1\right)A_i^2-\frac{1}{n_{i(1)}}B_i\Lambda_i^2\right]$, $\Delta_{i2}=\left(n_{i(1)}-1\right)A_iC_i-(1-A_iC_i)\Lambda_i^2$,

$A_i=\frac{1}{n_{i(1)}}\sum_{j=1}^{n_{i(1)}}\frac{1}{\sqrt{Y_{ij}}}$, $B_i=\sum_{j=1}^{n_{i(1)}}\left(\frac{1}{\sqrt{Y_{ij}}}-A_i\right)^2$, $C_i=\frac{1}{n_{i(1)}}\sum_{j=1}^{n_{i(1)}}\sqrt{Y_{ij}}$, and $D_i=\sum_{j=1}^{n_{i(1)}}\left(\sqrt{Y_{ij}}-C_i\right)^2$. Then, according to Wang [18], the GPQ of α_i is derived as

$$Q_{\alpha_i}(y_{ij};H_i,\Lambda_i)=\left\{\frac{\sum_{j=1}^{n_{i(1)}}Y_{ij}+\left\{\sum_{j=1}^{n_{i(1)}}[Y_{ij}]^{-1}\right\}G_{\beta_i}^2(y_{ij};\Lambda_i)-2n_{i(1)}Q_{\beta_i}(y_{ij};\Lambda_i)}{Q_{\beta_i}(y_{ij};\Lambda_i)H_i}\right\}^{\frac{1}{2}}, \tag{6}$$

where H_i follows the Chi-squared distribution with $n_{i(1)}$ degrees of freedom. Note that Λ_i and H_i denote the generalized pivotal quantities for the scale and shape parameters, respectively, of the Birnbaum–Saunders component in the *i*-th ZIBS population. Both quantities are constructed as functions of the observed data and auxiliary random variables, and their distributions are free of unknown parameters. They are used as intermediate components in deriving the GPQ of the coefficient of variation.

For the GPQ of δ_i, we use variance stabilized transformation (VST). According to Wu and Hsieh [19], the GPQ of δ_i is defined as

$$Q_{\delta_i}=\sin^2\left[\arcsin\sqrt{\hat{\delta}_i}-\frac{\mathrm{K}_i}{2\sqrt{n_i}}\right], \tag{7}$$

where $K_i=2\sqrt{n_i}\left(\arcsin\sqrt{\hat{\delta}_i}-\arcsin\sqrt{\delta_i}\right)\sim N(0,1)$. The GPQs for β_i, α_i, and δ_i, defined in Eqs (5)–(7), can be used to calculate the GPQ for v_i as

$$\begin{aligned}Q_{v_i}&=\left\{2+\left[Q_{\alpha_i}(y_{ij};H_i,\Lambda_i)\right]^2\right\}^{-1}\left(1-Q_{\delta_i}\right)^{-\frac{1}{2}}\\&\times\left\{\left[Q_{\alpha_i}(y_{ij};H_i,\Lambda_i)\right]^2\left[4+5\left[Q_{\alpha_i}(y_{ij};H_i,\Lambda_i)\right]^2\right]+Q_{\delta_i}\left\{2+\left[Q_{\alpha_i}(y_{ij};H_i,\Lambda_i)\right]^2\right\}^2\right\}^{\frac{1}{2}}.\end{aligned} \tag{8}$$

Now, the GPQ for ϖ_{ir} as $Q_{\varpi_{ir}}=Q_{v_i}-Q_{v_r}$, where $i,r=1,2,3,\dots,k$ and $i\neq r$. The $(1-\rho)100\%$ SCI for ϖ_{ir} using the GCI is given by

$$CI_{GCI:\varpi_{ir}}=\left[L_{GCI:\varpi_{ir}},U_{GCI:\varpi_{ir}}\right]=\left[Q_{\varpi_{ir}}(\rho/2),Q_{\varpi_{ir}}(1-\rho/2)\right], \tag{9}$$

where $Q_{\varpi_{ir}}(\rho/2)$ and $Q_{\varpi_{ir}}(1-\rho/2)$ denote the $100(\rho/2)th$ and $100(1-\rho/2)th$ percentiles of $Q_{\varpi_{ir}}$, respectively. The procedure for constructing SCIs for ϖ_{ir} based on the GCI method is presented in Algorithm 1 in the Appendix.

3.2. The bootstrap confidence interval (BCI)

The bootstrap technique, introduced by Efron [20], is a resampling method that involves repeatedly drawing samples with replacement from the original dataset to approximate the sampling distribution of a statistic. Lemonte, Simas, and Cribari-Neto [21] found that the constant-bias-correcting parametric bootstrap is the most effective approach for bias reduction. Therefore, this method was employed to construct the confidence interval for ϖ_{ir}. Given that B bootstrap samples are obtained, the corresponding $\hat{\alpha}_i$ series for these samples can be derived and represented as $\hat{\alpha}_{i1}^{\#}, \hat{\alpha}_{i2}^{\#}, \ldots, \hat{\alpha}_{iB}^{\#}$. In this context, $\hat{\alpha}_{il}^{\#}$ refers to the sequence of bootstrap maximum likelihood estimates (MLEs) of α_{il}, where $i = 1,2,\ldots,k$ and $l = 1,2,\ldots,B$. The estimation of the MLE for α_{il} is carried out using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) quasi-Newton method, a widely used algorithm for solving nonlinear optimization problems. Accordingly, the bias of the estimator α_i can be expressed as $B(\hat{\alpha}_i, \alpha_i) = E(\hat{\alpha}_i) - \alpha_i$. The bootstrap expectation $E(\hat{\alpha}_i)$ is estimated using the average $\hat{\alpha}_i^{/} = (1/B)\sum_{l=1}^{B} \hat{\alpha}_{il}^{\#}$. Thus, the bootstrap-based bias estimate for B replications of $\hat{\alpha}_i$ is derived as $\hat{B}(\hat{\alpha}_i, \alpha_i) = \hat{\alpha}_i^{/} - \alpha_i$. According to Mackinnon and Smith [22], the bias-corrected estimate of $\hat{\alpha}_i^{\#}$ is obtained by incorporating the bootstrap bias estimate, resulting in

$$\hat{\alpha}_i^{*} = \hat{\alpha}_i^{\#} - 2\hat{B}(\hat{\alpha}_i, \alpha_i). \tag{10}$$

Let $\hat{\delta}_i^{\#}$ be the bootstrap sample-based observed values of $\hat{\delta}_i$. According to Brown, Cai, and DasGupta [23], $\hat{\delta}_i^{*} \sim Beta\left(n_i\hat{\delta}_i^{\#} + 0.5, n_i\left(1 - \hat{\delta}_i^{\#}\right) + 0.5\right)$ is the bootstrap estimator of δ_i. Hence, the bootstrap estimators of ϖ_{ir} can be defined as

$$\hat{\varpi}_{ir}^{*} = \frac{1}{2+(\hat{\alpha}_i^{*})^2}\left\{\frac{(\hat{\alpha}_i^{*})^2\left[4+5(\hat{\alpha}_i^{*})^2\right]+\hat{\delta}_i^{*}\left[2+(\hat{\alpha}_i^{*})^2\right]^2}{1-\hat{\delta}_i^{*}}\right\}^{\frac{1}{2}} - \frac{1}{2+(\hat{\alpha}_r^{*})^2}\left\{\frac{(\hat{\alpha}_r^{*})^2\left[4+5(\hat{\alpha}_r^{*})^2\right]+\hat{\delta}_r^{*}\left[2+(\hat{\alpha}_r^{*})^2\right]^2}{1-\hat{\delta}_r^{*}}\right\}^{\frac{1}{2}}. \tag{11}$$

Consequently, the $(1-\rho)100\%$ SCI for ϖ_{ir} using the BCI method is provided by

$$CI_{BCI:\varpi_{ir}} = \left[L_{BCI:\varpi_{ir}}, U_{BCI:\varpi_{ir}}\right] = \left[\hat{\varpi}_{ir}^{*}(\rho/2), \hat{\varpi}_{ir}^{*}(1-\rho/2)\right], \tag{12}$$

where $\hat{\varpi}_{ir}^{*}(\rho/2)$ and $\hat{\varpi}_{ir}^{*}(1-\rho/2)$ denote the $100(\rho/2)th$ and $100(1-\rho/2)th$ percentiles of $\hat{\varpi}_{ir}^{*}$, respectively. The procedure for constructing SCIs for ϖ_{ir} based on the BCI method is presented in Algorithm 2 in the Appendix.

3.3. The method of variance estimates recovery (MOVER)

The MOVER method was used to derive a closed-form approximation for the confidence intervals of parameter differences $\theta_i - \theta_r$ (for $i, r = 1,2,3, \ldots, k$ and $i \neq r$). The confidence intervals of $\theta_i - \theta_r$ rely on the confidence intervals of the individual parameters. Let $[l_i, u_i]$ represent the $(1-\rho)100\%$ confidence interval for θ_i. As proposed by Zou and Donner [24], the confidence interval for the difference between parameters can be expressed as follows: The lower limit for $\theta_i - \theta_r$ is

$$L_{ir} = \left(\hat{\theta}_i - \hat{\theta}_r\right) - \sqrt{\left(\hat{\theta}_i - l_i\right)^2 + \left(u_r - \hat{\theta}_r\right)^2}$$

and the upper limit for $\theta_i - \theta_r$ is

$$U_{ir} = \left(\hat{\theta}_i - \hat{\theta}_r\right) + \sqrt{\left(u_i - \hat{\theta}_i\right)^2 + \left(\hat{\theta}_r - l_r\right)^2},$$

where $i, r = 1,2,3, \dots, k$ and $i \neq r$.

To begin, we consider the asymptotic confidence interval for the CV under the ZIBS distribution, which can be computed as

$$[l_i, u_i] = \left[\hat{\upsilon}_i - z_{1-\rho/2}\sqrt{\hat{V}(\hat{\upsilon}_i)}, \hat{\upsilon}_i + z_{1-\rho/2}\sqrt{\hat{V}(\hat{\upsilon}_i)}\right], \tag{13}$$

where $\hat{\upsilon}_i = (2 + \hat{\alpha}_i^2)^{-1}\left\{\left(1 - \hat{\delta}_i\right)^{-1}\left[\hat{\alpha}_i^2(4 + 5\hat{\alpha}_i^2) + \hat{\delta}_i(2 + \hat{\alpha}_i^2)^2\right]\right\}^{\frac{1}{2}}$, and $\hat{V}(\hat{\upsilon}_i)$ is calculated based on Eq (4). Therefore, the $(1-\rho)100\%$ SCI for ϖ_{ir} using the MOVER method can be expressed as

$$CI_{MOVER:\varpi_{ir}} = \left[L_{MOVER:\varpi_{ir}}, U_{MOVER:\varpi_{ir}}\right], \tag{14}$$

where $L_{MOVER:\varpi_{ir}} = (\hat{\upsilon}_i - \hat{\upsilon}_r) - \sqrt{(\hat{\upsilon}_i - l_i)^2 + (u_r - \hat{\upsilon}_r)^2}$,

$U_{MOVER:\varpi_{ir}} = (\hat{\upsilon}_i - \hat{\upsilon}_r) + \sqrt{(u_i - \hat{\upsilon}_i)^2 + (\hat{\upsilon}_r - l_r)^2}$, $i, r = 1,2,3, \dots, k$, and $i \neq r$.

3.4. The MOVER based on GCI (MG)

Based on Eq (8), the lower and upper limits of υ_i can be derived using the GCI method, expressed as

$$\left[l_{GCI:\upsilon_i}, u_{GCI:\upsilon_i}\right] = \left[Q_{\upsilon_i}(\rho/2), Q_{\upsilon_i}(1-\rho/2)\right], \tag{15}$$

where $Q_{\upsilon_i}(\rho/2)$ and $Q_{\upsilon_i}(1-\rho/2)$ denote the $100(\rho/2)th$ and $100(1-\rho/2)th$ percentiles of Q_{υ_i}, respectively. Hence, the $(1-\rho)100\%$ SCI for ϖ_{ir} can be obtained using the MG method, as shown in Eq (16), which is

$$CI_{MG:\varpi_{ir}} = \left[L_{MG:\varpi_{ir}}, U_{MG:\varpi_{ir}}\right], \tag{16}$$

where $L_{MG:\varpi_{ir}} = (\hat{\upsilon}_i - \hat{\upsilon}_r) - \sqrt{\left(\hat{\upsilon}_i - l_{GCI:\upsilon_i}\right)^2 + \left(u_{GCI:\upsilon_r} - \hat{\upsilon}_r\right)^2}$,

$U_{MG:\varpi_{ir}} = (\hat{\upsilon}_i - \hat{\upsilon}_r) + \sqrt{\left(u_{GCI:\upsilon_i} - \hat{\upsilon}_i\right)^2 + \left(\hat{\upsilon}_r - l_{GCI:\upsilon_r}\right)^2}$, $i, r = 1,2,3, \dots, k$ and $i \neq r$.

3.5. The MOVER based on BCI (MB)

The bootstrap estimators for υ_i can be expressed as

$$\hat{\upsilon}_i^* = \frac{1}{2 + (\hat{\alpha}_i^*)^2}\left\{\frac{(\hat{\alpha}_i^*)^2\left[4 + 5(\hat{\alpha}_i^*)^2\right] + \hat{\delta}_i^*\left[2 + (\hat{\alpha}_i^*)^2\right]^2}{1 - \hat{\delta}_i^*}\right\}^{\frac{1}{2}}.$$

The lower and upper limits of υ_i can be obtained using the BCI method, given by

$$\left[l_{BCI:\upsilon_i}, u_{BCI:\upsilon_i}\right] = \left[\hat{\upsilon}_i^*(\rho/2), \hat{\upsilon}_i^*(1-\rho/2)\right], \tag{17}$$

where $\hat{\upsilon}_i^*(\rho/2)$ and $\hat{\upsilon}_i^*(1-\rho/2)$ denote the $100(\rho/2)th$ and $100(1-\rho/2)th$ percentiles of $\hat{\upsilon}_i^*$, respectively. Consequently, the $(1-\rho)100\%$ SCI for ϖ_{ir} using the BCI method is provided by

$$CI_{MB:\varpi_{ir}} = \left[L_{MB:\varpi_{ir}}, U_{MB:\varpi_{ir}}\right], \tag{18}$$

where $L_{MB:\varpi_{ir}} = (\hat{v}_i - \hat{v}_r) - \sqrt{\left(\hat{v}_i - l_{BCI:v_i}\right)^2 + \left(u_{BCI:v_r} - \hat{v}_r\right)^2}$,

$$U_{MB:\varpi_{ir}} = (\hat{v}_i - \hat{v}_r) + \sqrt{\left(u_{BCI:v_i} - \hat{v}_i\right)^2 + \left(\hat{v}_r - l_{BCI:v_r}\right)^2},\ i,r = 1,2,3,\dots,k \text{ and } i \neq r.$$

The procedure for constructing SCIs for ϖ_{ir} based on the MOVER, MG, and MB methods is presented in Algorithm 3 in the Appendix.

3.6. The Bayesian credible interval (BAY)

Bayes' theorem, which is sometimes referred to as Bayes' rule, is a method for updating probability estimates when new information becomes available. It is vital to realize that Bayes' theorem applies to sequences of events, where fresh information from a later event is used to modify the probability of an earlier occurrence. Markov chain Monte Carlo (MCMC) sampling techniques can be used when the posterior distribution is difficult to compute and complicated. These samples are then used to compute Bayesian estimates and to construct consistent credible intervals for the parameter of interest. In the case of the ZIBS distributions, represented as $Y_{ij} \sim ZIBS(\alpha_i, \beta_i, \delta_i)$, where $i = 1,2,\dots,k$ and $j = 1,2,\dots,n_i$. The joint likelihood function of k independent ZIBS distributions is

$$L(y_{ij}; \alpha_i, \beta_i, \delta_i) \propto \prod_{i=1}^{k} \left\{ \delta_i^{n_{i(0)}} (1-\delta_i)^{n_{i(1)}} \frac{1}{(\alpha_i\beta_i)^{n_{i(1)}}} \prod_{j=1}^{n_{i(1)}} \left[\left(\frac{\beta_i}{y_{ij}}\right)^{\frac{1}{2}} + \left(\frac{\beta_i}{y_{ij}}\right)^{\frac{3}{2}} \right] \times exp\left[-\sum_{j=1}^{n_{i(1)}} \frac{1}{2\alpha_i^2}\left(\frac{y_{ij}}{\beta_i} + \frac{\beta_i}{y_{ij}} - 2\right) \right] \right\}.$$

Wang, Sun, and Park [25] recommended the use of proper priors with known hyperparameters when constructing confidence intervals for the parameters of the Birnbaum-Saunders distribution using Bayesian inference. Parameter β_i is assumed to follow an inverse gamma distribution with parameters p_i and q_i, denoted $IG(p_i, q_i)$, and similarly, α_i^2 is modeled as $IG(t_i, s_i)$. Furthermore, under the assumption that δ_i follows a binomial distribution, the Jeffrey's prior for the binomial parameter is given by $P(\delta_i) \propto \delta_i^{-1/2}(1-\delta_i)^{-1/2}$, which corresponds to a Beta distribution, denoted as $Beta(1/2, 1/2)$. As a result, the joint posterior density function for α_i^2, β_i, and δ_i is expressed as follows:

$$H(\alpha_i^2, \beta_i, \delta_i | y_{ij}) \propto \prod_{i=1}^{k} \left\{ \delta_i^{n_{i(0)}-1/2} (1-\delta_i)^{n_{i(1)}-1/2} \frac{1}{(\alpha_i^2)^{n_{i(1)}/2} \beta_i^{n_{i(1)}}} \prod_{j=1}^{n_{i(1)}} \left[\left(\frac{\beta_i}{y_{ij}}\right)^{\frac{1}{2}} + \left(\frac{\beta_i}{y_{ij}}\right)^{\frac{3}{2}} \right] \times exp\left[-\sum_{j=1}^{n_{i(1)}} \frac{1}{2\alpha_i^2}\left(\frac{y_{ij}}{\beta_i} + \frac{\beta_i}{y_{ij}} - 2\right) \right] (\alpha_i^2)^{-t_i-1} exp\left(-\frac{s_i}{\beta_i}\right) \beta_i^{-p_i-1} exp\left(-\frac{q_i}{\beta_i}\right) \right\}.$$

The marginal posterior distribution of $\beta_i | y_{ij}$ can be obtained by integrating the joint posterior density function with respect to α_i, resulting in

$$P(\beta_i | y_{ij}) \propto \beta_i^{-(n_{i(1)}+p_i+1)} exp\left(-\frac{q_i}{\beta_i}\right) \prod_{j=1}^{n_{i(1)}} \left[\left(\frac{\beta_i}{y_{ij}}\right)^{\frac{1}{2}} + \left(\frac{\beta_i}{y_{ij}}\right)^{\frac{3}{2}} \right] \times \left[\sum_{j=1}^{n_{i(1)}} \frac{1}{2}\left(\frac{y_{ij}}{\beta_i} + \frac{\beta_i}{y_{ij}} - 2\right) + s_i \right]^{-\frac{n_{i(1)}+1}{2-t_i}}, \tag{19}$$

while the conditional posterior distribution of $\alpha_i^2|\beta_i, y_{ij}$ is

$$\alpha_i^2|(\beta_i, y_{ij}) \sim IG\left(\frac{n_{i(1)}}{2} + t_i, \sum_{j=1}^{n_{i(1)}} \frac{1}{2}\left(\frac{y_{ij}}{\beta_i} + \frac{\beta_i}{y_{ij}} - 2\right) + s_i\right). \tag{20}$$

Additionally, the marginal posterior distribution of $\delta_i|y_{ij}$ is given by

$$\delta_i|y_{ij} \sim Beta\left(n_{i(0)} + \frac{1}{2}, n_{i(1)} + \frac{1}{2}\right). \tag{21}$$

The posterior samples of α_i^2 and δ_i can be readily obtained using the *LearnBayes* package in R software. Parameter α_i is then computed as the square root of α_i^2. Moreover, Eq (19) cannot be expressed in closed form and are analytically intractable; it is not possible to generate posterior samples of β_i using conventional methods. Therefore, the generalized ratio-of-uniforms method, as proposed by Wakefield, Gelfand, and Smith [26], is utilized for the posterior sampling of β_i. The following details describe the procedure for the sampling algorithm based on the generalized ratio-of-uniforms method.

Let (a_i, b_i) be a pair of random variables uniformly distributed over the set

$$W(c_i) = \left\{(a_i, b_i): 0 < a_i \leq \left[P\left(\frac{b_i}{a_i^{c_i}}|y_{ij}\right)\right]^{\frac{1}{c_i+1}}\right\}, \tag{22}$$

where $c_i \geq 0$ is a constant and $P(\cdot|y_{ij})$ is defined in Eq (19). Define $\beta_i = b_i/a_i^{c_i}$. It follows that β_i has a probability density function of the form

$$f_{\beta_i}(\beta_i|y_{ij}) = \frac{P(\beta_i|y_{ij})}{\int P(\beta_i|y_{ij})d\beta_i}.$$

Random samples uniformly distributed over $W(c_i)$ are generated via the accept-reject method, employing a suitably chosen one-dimensional enclosing rectangle $[0, p(c_i)] \times [q^-(c_i), q^+(c_i)]$, with all bounds $p(c_i) = \sup_{\beta_i>0}\left\{[P(\beta_i|y_{ij})]^{\frac{1}{c_i+1}}\right\}$, $q^-(c_i) = \inf_{\beta_i>0}\left\{[P(\beta_i|y_{ij})]^{\frac{c_i}{c_i+1}}\right\}$, and $q^+(c_i) = \sup_{\beta_i>0}\left\{[P(\beta_i|y_{ij})]^{\frac{c_i}{c_i+1}}\right\}$, assumed finite. An appropriate value for c_i occurs when $q^-(c_i) = 0$, while $p(c_i)$ and $q^+(c_i)$ are finite, as proposed by Wang, Sun, and Park [25]. In the generalized ratio-of-uniforms method, the computation proceeds as follows: First, compute $p(c_i)$ and $q^+(c_i)$. Next, generate a_i from $Uni(0, p(c_i))$, b_i from $Uni(0, q^+(c_i))$, and compute $\vartheta_i = b_i/a_i^{c_i}$. If the condition $a_i \leq [P(\beta_i|y_{ij})]^{\frac{1}{c_i+1}}$ is satisfied, it is accepted; otherwise, the process is repeated.

Note that the posterior samples of α_i, β_i, and δ_i are denoted by α_i', β_i', and δ_i', respectively. Accordingly, the Bayesian estimate of ϖ_{ir} is given by

$$\varpi_{ir}' = \frac{1}{2+(\alpha_i')^2}\left\{\frac{(\alpha_i')^2\left[4+5(\alpha_i')^2\right]+\delta_i'\left[2+(\alpha_i')^2\right]^2}{1-\delta_i'}\right\}^{\frac{1}{2}} - \frac{1}{2+(\alpha_r')^2}\left\{\frac{(\alpha_r')^2\left[4+5(\alpha_r')^2\right]+\delta_r'\left[2+(\alpha_r')^2\right]^2}{1-\delta_r'}\right\}^{\frac{1}{2}}, \tag{23}$$

where $i, r = 1,2,3, \ldots, k$ and $i \neq r$. Now, the $(1-\rho)100\%$ SCI for ϖ_{ir} using the BAY method is provided by

$$CI_{BAY:\varpi_{ir}} = [L_{BAY:\varpi_{ir}}, U_{BAY:\varpi_{ir}}] = [\varpi_{ir}'(\rho/2), \varpi_{ir}'(1-\rho/2)], \tag{24}$$

where $\varpi'_{ir}(\rho/2)$ and $\varpi'_{ir}(1-\rho/2)$ denote the $100(\rho/2)th$ and $100(1-\rho/2)th$ percentiles of ϖ'_{ir}, respectively.

3.7. The highest posterior density (HPD)

The HPD interval in Bayesian statistics is a credible interval that shows the narrowest range that includes a certain probability mass from the posterior distribution. For constructing the HPD interval under the ZIBS distribution, the posterior distribution of ϖ_{ir} in Eq (23) is considered. Accordingly, the $(1-\rho)100\%$ SCI for ϖ_{ir} using the HPD method is given by

$$CI_{HPD:\varpi_{ir}} = \left[L_{HPD:\varpi_{ir}}, U_{HPD:\varpi_{ir}}\right], \tag{25}$$

where $L_{HPD:\varpi_{ir}}$ and $U_{HPD:\varpi_{ir}}$ are computed using the *hdi* function in the *HDInterval* package of the R statistical software. The procedure for constructing SCIs for ϖ_{ir} based on the BAY and HPD methods is presented in Algorithm 4 in the Appendix.

4. Simulation studies

To assess the performance of each method, we conducted an extensive Monte Carlo simulation study using R software. Random samples were drawn from the ZIBS model under different sample sizes and parameters, thereby enabling an evaluation of each method's performance across scenarios. The sample sizes and parameter settings were specified as shown in Table 1. Parameter β_i was set to 1. A total of 3,000 replications were conducted, with 3,000 iterations for the GCI method, 500 iterations for the BCI method, and 1,000 iterations for the BAY and HPD methods. For the simulation procedures of the BAY and HPD methods, we set $c_i = 2$. The hyperparameters were chosen as $p_i = q_i = t_i = s_i = 10^{-4}$, which are values close to zero, as recommended by Congdon [27]. These weakly informative priors were adopted to reflect limited prior information and to enable the data to primarily drive the inference. A qualitative sensitivity assessment indicated that moderate variations in these hyperparameters do not materially affect the coverage probabilities and average widths of the BAY and HPD methods. Two major criteria were used for performance evaluation: The coverage probabilities (CPs) equal to or greater than the nominal confidence level of 0.95, together with the narrowest average width (AW). In simulations, let $C_s = 1$ if the parameter values fall within the confidence interval range, else $C_s = 0$. The coverage probability and average width can be calculated by

$$CP = \frac{1}{M}\sum_{s=1}^{M} C_s \tag{I}$$

and

$$AW = \frac{1}{M}\sum_{s=1}^{M}(U_s - L_s), \tag{II}$$

where L_s and U_s are the lower and upper bounds of the confidence interval for loop s, respectively, and M represents the total number of simulations runs. The computational procedure for estimating the coverage probability and the average width of all methods is outlined in Algorithm 5 in the Appendix.

For the case of k=3 (Table 2 and Figure 1), the simulation results revealed that, with respect to CPs, nearly all methods achieved values close to or greater than the nominal level of 0.95. In particular, the GCI method consistently maintained CPs at or above the nominal threshold while producing the narrowest AWs in nearly all scenarios. For instance, when the sample sizes were equal (S=1), GCI attained a CP of 0.952 with an AW of 0.391, which was markedly narrower than those obtained from the competing methods. In some cases, such as S=12, the MG method yielded narrower AWs while preserving CPs above the nominal level. Nevertheless, GCI remained the most efficient method overall. In contrast, the MB method consistently produced the widest intervals among the approaches considered.

Table 1. Parameter settings for k = 3, 5, 10.

Scenarios (S)	$(n_1, n_2, \dots, n_k)$	$(\alpha_1, \alpha_2, \dots, \alpha_k)$	$(\delta_1, \delta_2, \dots, \delta_k)$
***k*=3**			
1-8	(30^3)	(0.5^3), (1.0^3)	(0.1^3), (0.3^3), (0.1,0.3,0.5), (0.5^3)
9-16	(30,50,100)	(0.5^3), (1.0^3)	(0.1^3), (0.3^3), (0.1,0.3,0.5), (0.5^3)
17-24	(50^3)	(0.5^3), (1.0^3)	(0.1^3), (0.3^3), (0.1,0.3,0.5), (0.5^3)
25-32	(100^3)	(0.5^3), (1.0^3)	(0.1^3), (0.3^3), (0.1,0.3,0.5), (0.5^3)
***k*=5**			
33-42	$(30^3, 50^2)$	(0.5^5), (1.0^5)	(0.1^5), (0.3^5), $(0.1^2, 0.3^2, 0.5)$, $(0.1, 0.3^2, 0.5^2)$, (0.5^5)
43-52	$(30^2, 50, 100^2)$	(0.5^5), (1.0^5)	(0.1^5), (0.3^5), $(0.1^2, 0.3^2, 0.5)$, $(0.1, 0.3^2, 0.5^2)$, (0.5^5)
53-62	(50^5)	(0.5^5), (1.0^5)	(0.1^5), (0.3^5), $(0.1^2, 0.3^2, 0.5)$, $(0.1, 0.3^2, 0.5^2)$, (0.5^5)
63-72	$(50^2, 100^3)$	(0.5^5), (1.0^5)	(0.1^5), (0.3^5), $(0.1^2, 0.3^2, 0.5)$, $(0.1, 0.3^2, 0.5^2)$, (0.5^5)
73-82	(100^5)	(0.5^5), (1.0^5)	(0.1^5), (0.3^5), $(0.1^2, 0.3^2, 0.5)$, $(0.1, 0.3^2, 0.5^2)$, (0.5^5)
***k*=10**			
83-92	$(30^5, 50^5)$	(0.5^{10}), (1.0^{10})	(0.1^{10}), $(0.1^5, 0.3^5)$, (0.3^{10}), $(0.3^5, 0.5^5)$, (0.5^{10})
93-102	$(30^5, 50^3, 100^2)$	(0.5^{10}), (1.0^{10})	(0.1^{10}), $(0.1^5, 0.3^5)$, (0.3^{10}), $(0.3^5, 0.5^5)$, (0.5^{10})
103-112	$(50^5, 100^5)$	(0.5^{10}), (1.0^{10})	(0.1^{10}), $(0.1^5, 0.3^5)$, (0.3^{10}), $(0.3^5, 0.5^5)$, (0.5^{10})

Notes: $n^k = n_1, n_2, \dots, n_k$

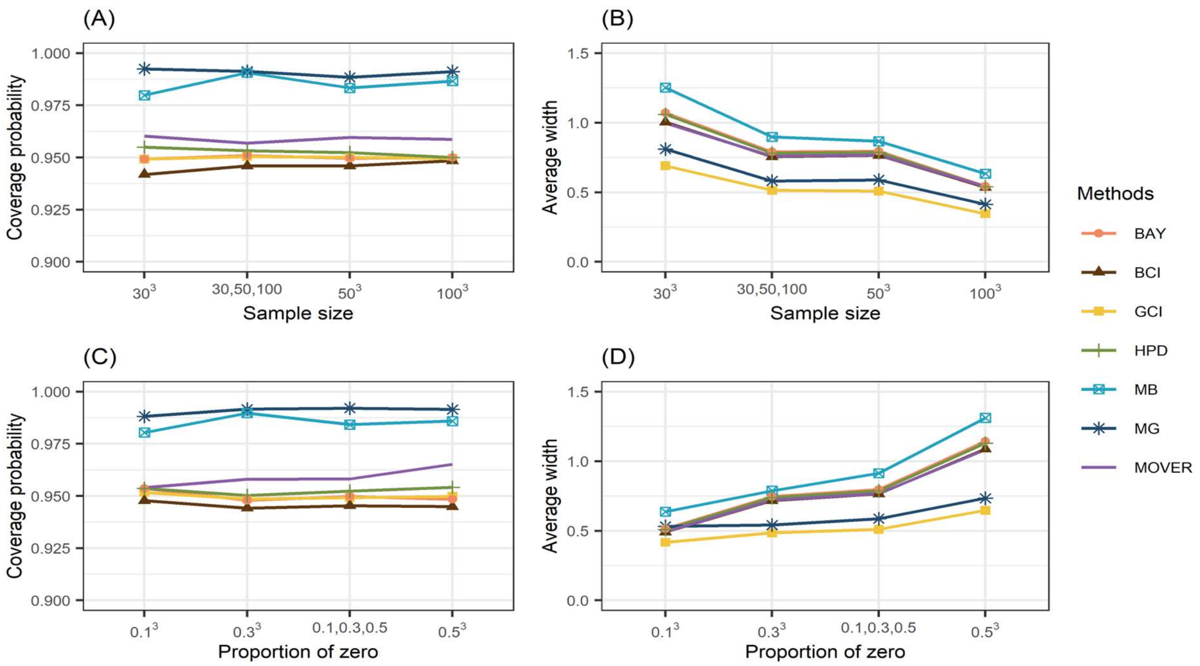

Figure 1. Comparison of the performance of the proposed methods for k = 3 in terms of CP with respect to (A) sample size and (C) proportion of zeros, and in terms of AW with respect to (B) sample size and (D) proportion of zeros.

Table 2. The CPs and AWs for the 95% SCIs of ϖ_{ir} : k = 3.

S	Coverage probabilities							Average widths						
	GCI	BCI	MOVER	MG	MB	BAY	HPD	GCI	BCI	MOVER	MG	GB	BAY	HPD
1	**0.952**	**0.951**	**0.962**	**0.993**	**0.992**	**0.958**	**0.963**	***0.391***	0.521	0.523	0.574	0.690	0.553	0.547
2	**0.952**	0.945	**0.970**	**0.994**	**0.989**	0.948	**0.956**	***0.434***	0.794	0.803	0.495	0.916	0.828	0.820
3	**0.952**	0.946	**0.967**	**0.992**	0.944	**0.951**	**0.957**	***0.474***	0.880	0.881	0.625	1.308	0.920	0.906
4	0.947	0.945	**0.978**	**0.993**	**0.988**	0.949	**0.961**	0.588	1.224	1.220	***0.668***	1.565	1.291	1.275
5	0.948	0.938	0.943	**0.993**	**0.991**	0.949	0.947	0.704	0.740	0.741	***0.907***	1.132	0.795	0.788
6	0.947	0.939	**0.953**	**0.991**	**0.989**	0.948	**0.952**	0.852	1.062	1.053	***0.930***	1.148	1.139	1.128
7	0.947	0.934	0.946	**0.993**	**0.988**	0.944	0.949	0.918	1.181	1.161	***0.967***	1.284	1.272	1.252
8	0.948	0.937	**0.963**	**0.992**	**0.957**	0.946	**0.954**	1.165	1.634	1.590	***1.321***	1.974	1.778	1.756
9	**0.952**	0.948	**0.955**	**0.991**	**0.992**	**0.955**	**0.957**	***0.299***	0.409	0.414	0.337	0.454	0.427	0.421
10	0.949	0.948	**0.964**	**0.991**	**0.991**	**0.951**	**0.953**	0.324	0.615	0.630	***0.357***	0.678	0.635	0.627
11	**0.952**	0.948	**0.964**	**0.992**	**0.992**	**0.951**	**0.954**	***0.323***	0.586	0.599	0.364	0.719	0.602	0.596
12	**0.952**	0.947	**0.970**	**0.992**	**0.992**	0.949	**0.955**	0.422	0.936	0.950	***0.399***	1.167	0.972	0.958
13	**0.951**	0.946	0.946	**0.991**	**0.991**	**0.955**	**0.953**	***0.549***	0.586	0.589	0.675	0.781	0.619	0.612
14	0.946	0.942	0.947	**0.991**	**0.990**	0.949	**0.951**	0.661	0.837	0.834	***0.710***	0.876	0.882	0.872
15	**0.950**	0.946	**0.953**	**0.992**	**0.990**	**0.950**	**0.950**	***0.647***	0.810	0.809	0.739	0.962	0.847	0.840
16	**0.953**	0.943	**0.956**	**0.991**	**0.988**	0.949	**0.954**	***0.888***	1.265	1.248	1.073	1.547	1.350	1.330
17	**0.953**	**0.952**	**0.963**	**0.993**	**0.993**	**0.956**	**0.957**	***0.290***	0.401	0.407	0.393	0.476	0.414	0.411
18	0.949	0.948	**0.965**	**0.992**	**0.991**	0.949	0.949	0.312	0.603	0.618	***0.347***	0.641	0.616	0.610
19	**0.953**	0.946	**0.964**	**0.992**	**0.989**	0.948	**0.953**	***0.336***	0.662	0.675	0.343	0.695	0.677	0.669
20	**0.951**	0.943	**0.969**	**0.992**	**0.990**	0.945	**0.950**	***0.402***	0.916	0.932	0.511	1.119	0.939	0.930
21	**0.951**	0.943	0.947	**0.963**	**0.931**	**0.950**	0.949	***0.536***	0.578	0.579	0.638	0.659	0.602	0.597
22	0.946	0.943	**0.953**	**0.992**	**0.991**	0.946	0.947	0.643	0.821	0.818	***0.736***	0.902	0.856	0.848
23	**0.951**	0.948	**0.955**	**0.993**	**0.991**	**0.953**	**0.955**	***0.687***	0.909	0.901	0.791	1.055	0.948	0.937
24	0.947	0.945	**0.960**	**0.991**	**0.990**	**0.951**	**0.957**	0.859	1.235	1.219	***0.954***	1.382	1.300	1.287
25	**0.950**	**0.952**	**0.961**	**0.990**	**0.962**	**0.954**	**0.953**	***0.198***	0.282	0.289	0.264	0.365	0.286	0.284
26	**0.950**	0.945	**0.962**	**0.992**	**0.986**	0.947	0.947	***0.208***	0.420	0.435	0.243	0.459	0.423	0.420
27	0.945	0.947	**0.962**	**0.991**	**0.990**	0.949	0.949	0.222	0.460	0.475	***0.302***	0.601	0.464	0.459
28	**0.953**	**0.951**	**0.970**	**0.991**	**0.992**	0.949	**0.950**	0.259	***0.630***	0.651	0.299	0.718	0.636	0.630
29	**0.957**	**0.952**	**0.954**	**0.992**	**0.991**	**0.954**	**0.952**	0.374	***0.410***	0.411	0.470	0.540	0.418	0.415
30	**0.950**	0.945	**0.950**	**0.992**	**0.990**	0.946	0.947	0.445	***0.579***	0.578	0.512	0.681	0.591	0.586
31	0.945	0.948	**0.954**	**0.992**	**0.990**	**0.952**	**0.951**	0.474	0.637	0.635	***0.569***	0.685	0.650	0.643
32	0.947	0.948	**0.957**	**0.991**	**0.992**	0.949	**0.952**	0.587	0.864	0.859	***0.646***	1.017	0.884	0.877

Note: Bold values indicate coverage probabilities ≥0.95, and bold italic values indicate the optimal average widths for each scenario.

For the case of k=5 (Table 3 and Figure 2), the overall pattern remains consistent with the results for k=3. The GCI maintained CPs that were nearly 0.95 while resulting in AWs that were narrower than those of the other approaches. Although the MG method generally yielded relatively high CPs and occasionally provided narrower intervals than GCI, several scenarios showed CPs falling below 0.95, preventing it from consistently meeting the evaluation criteria. Other methods, the MOVER, MB, BAY, and HPD, maintained CPs above 0.95 more reliably; however, they produced noticeably wider AWs.

Table 3. The CPs and AWs for the 95% SCIs of ϖ_{ir} : $k = 5$.

S	Coverage probabilities							Average widths						
	GCI	BCI	MOVER	MG	MB	BAY	HPD	GCI	BCI	MOVER	MG	GB	BAY	HPD
33	**0.950**	0.949	**0.957**	**0.984**	**0.974**	**0.955**	**0.958**	***0.352***	0.475	0.527	0.371	0.545	0.499	0.494
34	**0.954**	0.946	**0.965**	**0.981**	**0.996**	0.948	**0.953**	0.389	0.719	0.758	***0.386***	0.781	0.747	0.739
35	0.948	0.948	**0.966**	**0.979**	**0.971**	**0.952**	**0.956**	***0.387***	0.681	0.716	0.520	0.777	0.707	0.699
36	**0.951**	0.947	**0.968**	**0.975**	**0.968**	**0.950**	**0.955**	0.414	0.796	0.817	***0.400***	0.961	0.823	0.815
37	0.945	0.944	**0.975**	**0.984**	**0.996**	0.946	**0.957**	0.517	1.108	1.102	***0.552***	1.289	1.159	1.144
38	**0.951**	0.941	0.945	**0.987**	**0.996**	**0.950**	**0.950**	***0.640***	0.677	0.707	0.721	0.756	0.721	0.715
39	0.949	0.940	**0.952**	**0.977**	**0.982**	0.948	**0.951**	0.771	0.967	0.964	***0.883***	1.208	1.030	1.020
40	**0.950**	0.942	**0.953**	**0.985**	**0.996**	**0.951**	**0.953**	***0.755***	0.931	0.925	0.827	1.011	0.987	0.976
41	0.949	0.944	**0.957**	**0.985**	**0.997**	**0.951**	**0.953**	0.825	1.075	1.047	***0.888***	1.175	1.141	1.129
42	0.948	0.942	**0.962**	**0.985**	**0.997**	**0.950**	**0.959**	1.044	1.473	1.398	***1.257***	1.613	1.587	1.568
43	**0.950**	0.949	**0.952**	**0.982**	**0.992**	**0.953**	**0.954**	***0.300***	0.410	0.475	0.319	0.484	0.428	0.423
44	0.948	0.945	**0.962**	**0.985**	**0.997**	0.948	**0.952**	0.327	0.618	0.676	***0.386***	0.681	0.638	0.631
45	**0.950**	0.946	**0.960**	**0.985**	**0.987**	0.949	**0.951**	***0.317***	0.543	0.597	0.480	0.902	0.560	0.555
46	0.949	0.947	**0.964**	**0.983**	**0.997**	0.949	**0.952**	0.334	0.638	0.685	***0.394***	0.809	0.656	0.650
47	**0.951**	0.944	**0.967**	**0.985**	**0.989**	0.946	**0.953**	***0.425***	0.940	0.969	0.438	1.048	0.977	0.962
48	0.947	0.939	0.939	**0.983**	**0.991**	0.949	0.947	0.549	0.586	0.632	***0.635***	0.687	0.619	0.612
49	0.949	0.944	**0.950**	**0.985**	**0.998**	**0.950**	**0.952**	0.661	0.836	0.856	***0.761***	0.941	0.881	0.871
50	**0.951**	0.947	**0.952**	**0.983**	**0.993**	**0.952**	**0.953**	***0.620***	0.751	0.775	0.692	0.962	0.787	0.779
51	**0.951**	0.947	**0.954**	**0.972**	**0.985**	**0.951**	**0.952**	0.678	0.871	0.879	***0.655***	0.961	0.913	0.904
52	0.947	0.942	**0.953**	**0.979**	**0.987**	0.948	**0.953**	0.889	1.265	1.236	***1.181***	1.619	1.350	1.331
53	0.949	0.948	**0.958**	**0.977**	**0.995**	**0.952**	**0.954**	0.290	0.402	0.462	***0.378***	0.513	0.415	0.411
54	**0.951**	0.947	**0.967**	**0.984**	**0.995**	**0.950**	**0.953**	***0.313***	0.604	0.655	0.353	0.738	0.617	0.612
55	0.947	0.949	**0.965**	**0.983**	**0.956**	**0.951**	**0.954**	0.323	0.601	0.647	***0.361***	0.824	0.616	0.609
56	**0.950**	0.945	**0.964**	**0.986**	**0.988**	0.946	**0.950**	***0.344***	0.704	0.736	0.394	0.819	0.721	0.712
57	**0.953**	0.949	**0.974**	**0.986**	**0.968**	**0.950**	**0.957**	0.402	0.912	0.933	***0.399***	1.092	0.936	0.927
58	**0.951**	0.945	**0.950**	**0.980**	**0.997**	**0.953**	**0.952**	***0.535***	0.577	0.615	0.604	0.708	0.602	0.597
59	0.948	0.946	**0.955**	**0.983**	**0.996**	**0.950**	**0.952**	0.641	0.818	0.829	***0.685***	0.951	0.853	0.846
60	0.948	0.941	0.948	**0.965**	**0.936**	0.946	0.948	0.648	0.826	0.833	***0.705***	1.120	0.861	0.852
61	**0.950**	0.945	**0.953**	**0.986**	**0.958**	**0.950**	**0.953**	***0.714***	0.960	0.942	0.718	1.074	1.002	0.991
62	**0.952**	0.945	**0.962**	**0.984**	**0.997**	**0.951**	**0.957**	***0.859***	1.236	1.194	0.979	1.438	1.301	1.288
63	0.949	0.949	**0.957**	**0.985**	**0.997**	**0.952**	**0.953**	0.237	0.332	0.406	***0.264***	0.373	0.340	0.337
64	**0.950**	0.944	**0.959**	**0.984**	**0.992**	0.946	0.947	***0.253***	0.498	0.567	0.262	0.571	0.506	0.501
65	**0.950**	**0.950**	**0.963**	**0.986**	**0.998**	**0.952**	**0.952**	***0.252***	0.460	0.526	0.267	0.527	0.467	0.463
66	0.947	0.945	**0.962**	**0.985**	**0.992**	0.946	0.947	0.266	0.543	0.600	***0.291***	0.691	0.551	0.545
67	0.948	0.945	**0.965**	**0.983**	**0.997**	0.945	0.949	0.319	0.750	0.802	***0.371***	0.828	0.765	0.756
68	**0.950**	0.943	0.945	**0.964**	**0.951**	0.948	0.947	***0.442***	0.481	0.536	0.578	0.636	0.497	0.492
69	**0.950**	0.947	**0.952**	**0.971**	**0.957**	**0.950**	**0.950**	***0.529***	0.681	0.717	0.751	0.972	0.703	0.696
70	0.949	0.945	**0.951**	**0.985**	**0.986**	0.949	**0.950**	0.512	0.641	0.678	***0.561***	0.725	0.660	0.654
71	0.948	0.946	**0.954**	**0.985**	**0.986**	0.949	**0.950**	0.562	0.749	0.768	***0.580***	0.861	0.771	0.764
72	**0.950**	0.941	**0.952**	**0.982**	**0.997**	0.946	0.948	***0.702***	1.020	1.021	0.833	1.187	1.060	1.049
73	**0.953**	0.949	**0.959**	**0.963**	**0.996**	**0.952**	**0.951**	***0.198***	0.282	0.356	0.219	0.319	0.286	0.283

S	Coverage probabilities							Average widths						
	GCI	BCI	MOVER	MG	MB	BAY	HPD	GCI	BCI	MOVER	MG	GB	BAY	HPD
74	0.948	0.947	**0.963**	**0.984**	**0.989**	0.949	0.949	0.208	0.420	0.488	***0.251***	0.521	0.424	0.420
75	0.948	0.947	**0.962**	**0.985**	**0.998**	0.949	0.949	0.215	0.418	0.484	***0.256***	0.485	0.421	0.417
76	**0.952**	**0.950**	**0.964**	**0.986**	**0.998**	**0.950**	**0.950**	***0.227***	0.489	0.546	0.226	0.529	0.492	0.488
77	0.947	**0.950**	**0.968**	**0.982**	**0.996**	**0.951**	**0.952**	0.259	0.628	0.682	***0.332***	0.768	0.634	0.629
78	**0.950**	0.945	0.948	**0.985**	**0.997**	0.949	0.947	***0.374***	0.410	0.465	0.413	0.471	0.418	0.415
79	**0.951**	0.947	**0.953**	**0.965**	**0.983**	**0.951**	0.949	***0.445***	0.579	0.616	0.475	0.679	0.590	0.585
80	**0.952**	0.948	**0.952**	**0.971**	**0.981**	**0.950**	**0.950**	***0.448***	0.582	0.618	0.503	0.770	0.593	0.588
81	0.947	0.945	**0.951**	**0.985**	**0.997**	0.948	0.948	0.491	0.674	0.694	***0.484***	0.770	0.687	0.681
82	0.949	0.944	**0.954**	**0.985**	**0.996**	0.946	0.948	0.587	0.865	0.870	***0.687***	1.013	0.885	0.877

Note: Bold values indicate coverage probabilities $\geq$0.95, and bold italic values indicate the optimal average widths for each scenario.

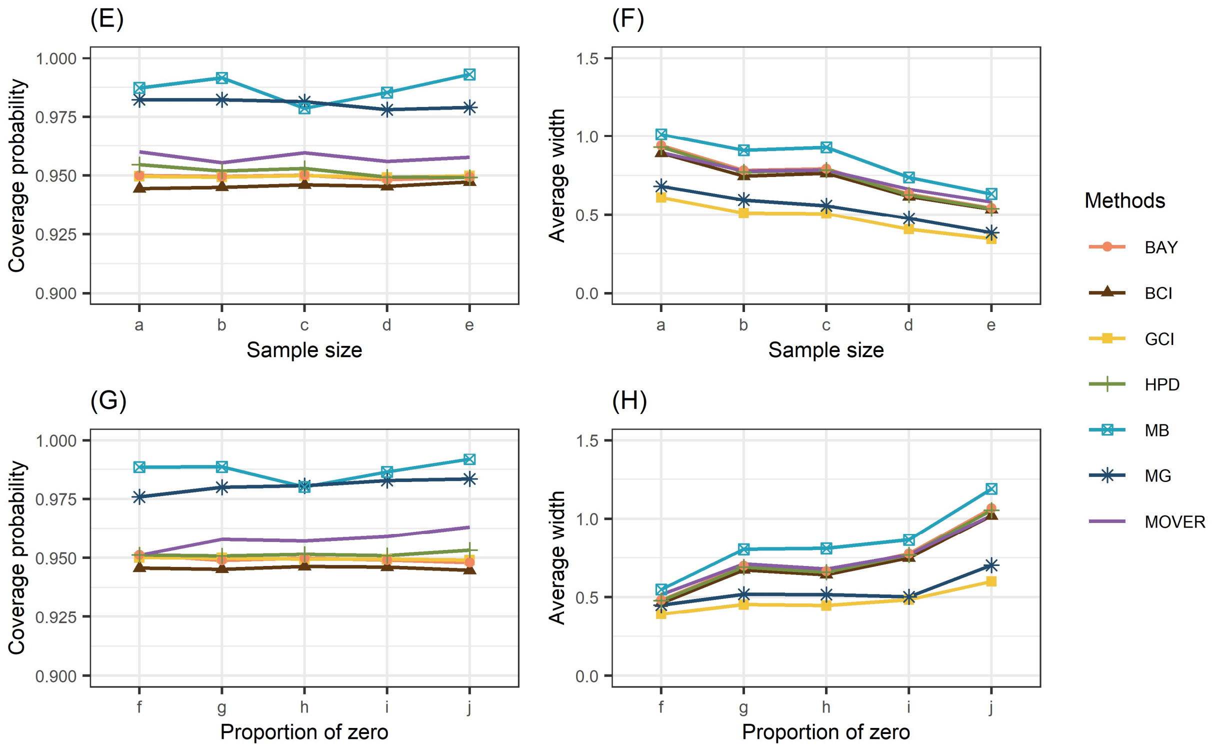

Figure 2. Comparison of the performance of the proposed methods for k = 5 in terms of CP with respect to (E) sample size and (G) proportion of zeros, and in terms of AW with respect to (F) sample size and (H) proportion of zeros (a=(30^3,50^2), b=(30^2,50,100^2), c=(50^5), d=(50^2,100^3), e=(100^5), f=(0.1^5), g=(0.3^5), h=(0.1^2,0.3^2,0.5), i=(0.1,0.3^2,0.5^2), and j=(0.5^5)).

For the case of k=10 (Table 4 and Figure 3), the results became more complex. The GCI method began to show limitations, as its CP dropped below 0.95 in several scenarios (e.g., S=96 and S=101), thereby failing to meet the first evaluation criteria. Nevertheless, in scenarios where GCI maintained CPs above 0.95, it continued to produce the narrowest AWs compared with the other methods. In contrast, the MG method, which consistently achieved CPs greater than 0.95, emerged as the more suitable choice in many cases where GCI failed, as it provided narrower AWs than BAY or MOVER. For instance, at S=96, the MG method achieved a CP of 0.995 with an AW of 0.477, which was substantially smaller than those obtained from the competing methods.

The simulation results in Figures 1–3 reveal a distinct pattern concerning the effects of the sample size and the proportion of zeros on the methods' performance. As the sample size increased, the efficiency of all methods tended to improve, as evidenced by a noticeable reduction in AWs. In contrast, as the proportion of zeros in the data increased, the performance of all methods deteriorated markedly. This indicated that a higher proportion of zeros inflates the variability of the estimators.

Table 4. The CPs and AWs for the 95% SCIs of ϖ_{ir} : k = 10.

S	Coverage probabilities							Average widths						
	GCI	BCI	MOVER	MG	MB	BAY	HPD	GCI	BCI	MOVER	MG	GB	BAY	HPD
83	**0.950**	0.949	**0.977**	**0.996**	**0.999**	**0.955**	**0.957**	***0.342***	0.463	0.525	0.435	0.598	0.486	0.481
84	**0.950**	0.945	**0.980**	**0.996**	**0.994**	**0.950**	**0.953**	***0.353***	0.563	0.659	0.376	0.700	0.585	0.580
85	0.948	0.947	**0.983**	**0.991**	**0.997**	**0.950**	**0.955**	0.374	0.700	0.801	***0.400***	0.796	0.725	0.717
86	0.949	0.944	**0.985**	**0.994**	**0.995**	0.948	**0.954**	0.417	0.855	1.001	***0.462***	0.951	0.884	0.874
87	**0.950**	0.944	**0.988**	**0.986**	**0.994**	0.946	**0.956**	***0.497***	1.073	1.211	0.517	1.373	1.119	1.104
88	**0.951**	0.940	**0.967**	**0.994**	**0.998**	0.949	0.949	***0.622***	0.660	0.744	0.732	0.780	0.701	0.695
89	0.949	0.942	**0.968**	**0.993**	**0.994**	**0.950**	**0.951**	0.674	0.781	0.869	***0.793***	0.926	0.825	0.818
90	0.947	0.941	**0.971**	**0.991**	**0.985**	0.948	**0.951**	0.751	0.944	1.054	***0.865***	1.187	1.002	0.992
91	**0.950**	0.943	**0.974**	**0.989**	**0.985**	0.949	**0.954**	***0.856***	1.151	1.274	1.047	1.424	1.221	1.209
92	0.949	0.942	**0.978**	**0.992**	**0.998**	0.948	**0.956**	1.015	1.436	1.579	***1.149***	1.594	1.542	1.524
93	**0.951**	0.947	**0.982**	**0.984**	**0.984**	**0.952**	**0.954**	***0.316***	0.430	0.526	0.366	0.522	0.451	0.445
94	**0.952**	0.947	**0.986**	**0.997**	**0.991**	**0.950**	**0.953**	***0.324***	0.517	0.659	0.400	0.616	0.537	0.530
95	0.949	0.946	**0.988**	**0.996**	**0.987**	0.948	**0.952**	0.345	0.651	0.802	***0.360***	0.726	0.674	0.665
96	0.945	**0.950**	**0.991**	**0.995**	**0.998**	**0.951**	**0.957**	0.379	0.780	0.997	***0.477***	0.959	0.806	0.796
97	0.949	0.948	**0.993**	**0.992**	**0.991**	**0.950**	**0.958**	0.450	0.986	1.202	***0.531***	1.111	1.029	1.011
98	**0.950**	0.940	**0.974**	**0.990**	**0.990**	0.948	0.948	***0.577***	0.615	0.746	0.755	0.773	0.652	0.645
99	**0.950**	0.944	**0.978**	**0.991**	**0.997**	**0.951**	**0.951**	***0.621***	0.718	0.869	0.711	0.824	0.758	0.749
100	**0.952**	0.942	**0.980**	**0.988**	**0.969**	**0.950**	**0.951**	***0.694***	0.876	1.055	0.865	1.201	0.928	0.916
101	0.946	0.941	**0.981**	**0.996**	**0.999**	0.947	**0.952**	0.782	1.051	1.268	***0.883***	1.184	1.112	1.098
102	**0.950**	0.940	**0.984**	**0.995**	**0.998**	0.947	**0.953**	***0.934***	1.324	1.571	1.096	1.536	1.419	1.396
103	**0.950**	0.948	**0.979**	**0.970**	**0.953**	**0.951**	**0.952**	***0.246***	0.344	0.408	0.311	0.439	0.353	0.350
104	**0.950**	0.949	**0.985**	**0.994**	**0.999**	**0.951**	**0.953**	***0.251***	0.411	0.510	0.290	0.497	0.419	0.415
105	0.948	0.945	**0.983**	**0.996**	**0.986**	0.947	0.949	0.263	0.515	0.618	***0.308***	0.609	0.525	0.520
106	0.949	0.944	**0.986**	**0.991**	**0.995**	0.946	0.948	0.286	0.617	0.769	***0.367***	0.757	0.626	0.621
107	**0.951**	0.947	**0.987**	**0.997**	**0.999**	0.949	**0.953**	***0.334***	0.777	0.927	0.369	0.897	0.794	0.786
108	0.949	0.943	**0.973**	**0.944**	**0.953**	0.949	0.947	0.458	0.497	0.580	***0.576***	0.633	0.514	0.510
109	0.947	0.944	**0.973**	**0.981**	**0.977**	0.949	0.948	0.491	0.579	0.676	***0.612***	0.710	0.597	0.592

S	Coverage probabilities							Average widths						
	GCI	BCI	MOVER	MG	MB	BAY	HPD	GCI	BCI	MOVER	MG	GB	BAY	HPD
110	0.948	0.947	**0.977**	**0.989**	**0.991**	**0.950**	**0.951**	0.548	0.705	0.819	***0.628***	0.848	0.730	0.723
111	0.949	0.942	**0.976**	**0.995**	**0.995**	0.945	0.946	0.615	0.842	0.983	***0.757***	1.039	0.869	0.861
112	**0.951**	0.944	**0.978**	**0.996**	**0.990**	0.947	**0.951**	***0.731***	1.062	1.222	0.819	1.239	1.106	1.095

Note: Bold values indicate coverage probabilities $\geq$0.95, and bold italic values indicate the optimal average widths for each scenario.

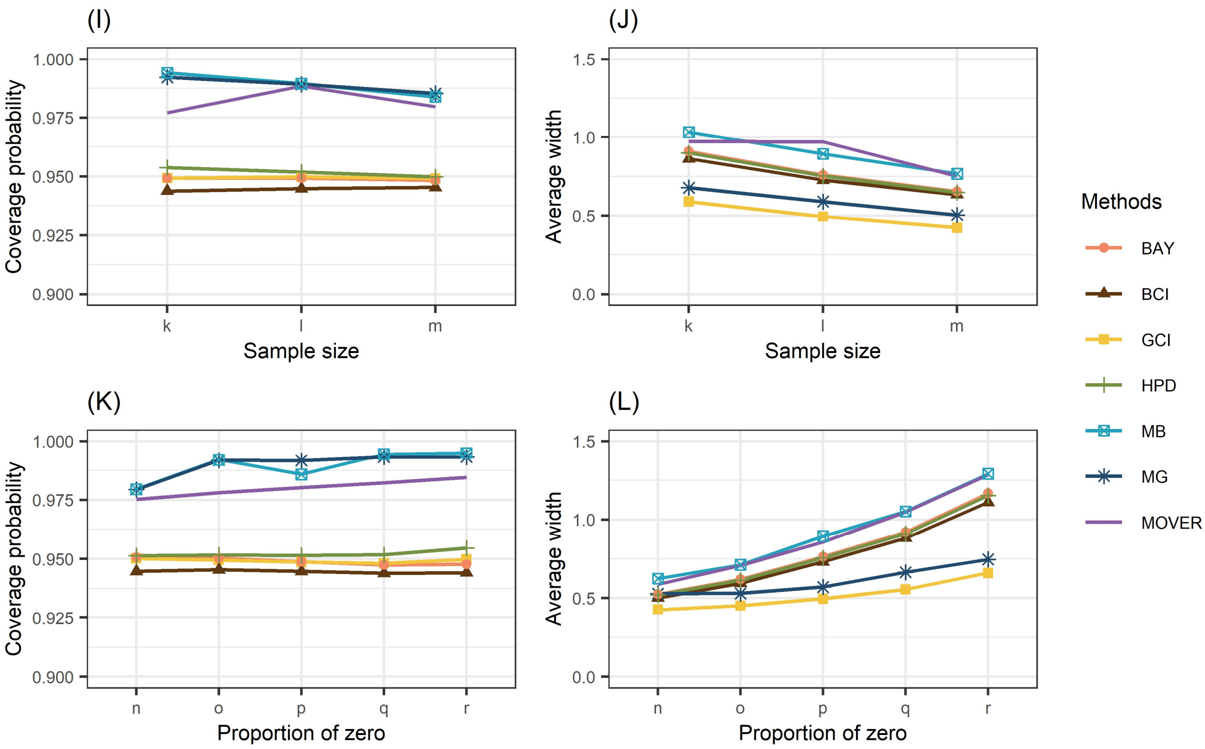

Figure 3. Comparison of the performance of the proposed methods for k = 10 in terms of CP with respect to (I) sample size and (K) proportion of zeros, and in terms of AW with respect to (J) sample size and (L) proportion of zeros (k=(30^5,50^5), l=(30^5,50^5,100^2), m=(50,$^5100^5$), n=(0.1^{10}), o=(0.1^5,0.3^5), p=(0.3^{10}), q=(0.3^5,0.5^5), and r=(0.5^{10})).

5. An empirical application

The empirical application in this study uses wind speed data, a key variable that influences multiple domains, including the environment, energy, and public health. Wind speed also plays a significant role in transportation and aviation, particularly with respect to flight safety and punctuality. For this reason, wind speed data from the Phuket Airport Weather Observing Station were selected as a case study for constructing SCIs for the coefficients of variation under zero-inflated Birnbaum–Saunders distributions. The dataset consists of wind speed observations from all directions, collected between January 1 and 7 of the years 2021 to 2025. Since the data were collected from different years, the corresponding estimators were considered approximately independent with respect to wind direction, reflecting year-to-year variability in prevailing wind patterns. Although wind speed components from different directions may exhibit dependence at a given time due to the influence of the same atmospheric system, such directional dependence was mitigated in this study by aggregating the data into daily representative values. Moreover, as all observations originated from a single monitoring station, spatial directional dependence was not present. These data were obtained from the Thai Meteorological Department's Automatic Weather System (http://www.aws-observation.tmd.go.th /main/main) and are presented in Table 5. The dataset included positive and zero wind speed values. To illustrate the distribution of the data, histograms of wind speed for all five years are plotted in Figure 4. In addition, Table 6 presents the summary statistics of wind speed for each year. For the positive observations, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) were employed to assess the goodness of fitness of the distributions. The criterion was calculated as

$$AIC = 2\ln(L) + 2p \text{ and } BIC = 2\ln(L) + 2pln(r),$$

where p is the number of parameters estimated, r is the number of observations, and L is the likelihood function. The Birnbaum–Saunders distribution exhibited the lowest AIC and BIC values among the other candidate distributions, as illustrated in Table 7. This suggested that it is the most suitable model for the positive wind speed data. Furthermore, to validate that the positive wind speed data follow the Birnbaum–Saunders distribution, we plotted the cumulative distribution function (CDF) derived from the observed positive wind speed data alongside the fitted CDF of the Birnbaum–Saunders distribution, as shown in Figure 5. The close agreement between the two curves indicated a satisfactory fit. Hence, the wind speed data consisted of positive and zero values and could be appropriately modeled using the zero-inflated Birnbaum–Saunders distribution. This distribution was employed to construct simultaneous confidence intervals for all pairwise differences in the coefficients of variation in the wind speed data. Table 8 presents the 95% simultaneous confidence intervals for all pairwise differences in the coefficients of variation in wind speed data across the five years, obtained using the GCI, BCI, MOVER, MG, MB, BAY, and HPD methods. The results indicated that the GCI method provided an interval of [0.2054, 0.55654], with a width of 0.3510, which was the narrowest among all methods. This suggested that GCI is the most suitable method for analyzing the wind speed data. Moreover, these findings are consistent with the simulation results reported in Table 3, Scenario 80, which most closely resembles the empirical dataset.

Table 5. Wind speed data (knots) from the Phuket Airport Weather Observing Station for each year from 2021 to 2025.

Wind Speed (knots)										
In 2021										
0.0	0.0	0.0	0.0	1.3	0.1	0.7	0.0	0.0	0.0	0.0
0.1	0.1	0.1	0.1	1.7	0.4	1.0	0.0	0.0	0.1	0.0
9.2	6.2	7.2	2.4	9.6	5.5	10.1	0.0	0.0	0.1	0.0
76.8	48.3	62.6	22.5	38.3	33.1	32.5	0.0	0.0	0.0	0.0
12.2	9.7	12.0	12.4	3.8	12.4	11.6	5.8	0.6	3.3	0.0
0.1	0.1	0.5	4.5	0.1	4.0	2.9	0.1	0.0	0.5	0.0
0.0	0.0	0.0	0.1	0.0	0.3	0.1	0.3	0.6	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3	0.0
0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0									

Wind Speed (knots)										
In 2022										
0.1	0.1	1.3	1.7	0.4	0.0	0.0	0.0	0.0	0.0	0.0
1.8	0.8	1.6	0.8	0.8	0.0	0.1	0.0	0.0	0.0	0.0
22.3	23.1	18.8	11.5	5.8	16.7	9.0	2.9	2.4	0.0	0.1
67.1	65.1	44.0	34.5	28.0	66.7	30.1	1.8	3.0	0.0	0.2
8.7	9.0	11.2	17.7	18.3	16.7	22.2	6.1	1.6	11.7	0.1
0.1	1.7	3.1	7.9	7.3	0.0	5.3	0.0	0.0	1.3	0.0
0.0	0.2	1.4	1.8	2.7	0.0	0.2	0.0	0.0	0.0	0.0
0.0	0.0	0.3	0.5	1.1	0.0	0.0	0.0	0.0	0.0	0.1
0.0	0.0	0.8	1.1	0.8	0.0	0.0	0.0	0.0	0.2	0.5
0.0	0.0	0.2	2.0	0.4	0.0	0.0	0.2	1.4	0.0	0.1
0.0	0.0									
In 2023										
0.6	0.0	1.3	0.8	0.1	1.3	0.9	6.5	1.5	0.0	0.0
0.8	0.6	2.3	3.3	0.8	4.4	2.9	0.0	0.0	0.0	0.0
8.8	9.6	17.2	14.9	14.0	12.8	13.4	3.1	1.3	9.2	7.8
56.3	74.3	47.4	50.4	33.5	24.9	42.6	6.7	2.1	0.6	0.8
14.8	15.3	13.8	9.6	15.9	15.6	25.8	4.1	0.3	8.6	0.0
1.1	0.0	1.8	0.6	7.1	8.1	6.0	7.1	0.8	2.3	0.2
0.0	0.0	0.1	0.1	1.4	2.1	1.3	0.1	0.2	0.0	1.6
0.0	0.0	0.0	0.1	1.0	0.6	0.3	1.0	7.6	0.7	0.0
0.0	0.0	0.1	0.1	1.0	0.5	0.0	0.0	0.0	0.6	0.0
0.0	0.0	0.1	0.0	1.0	0.2	0.0	0.3	4.5	0.1	0.1
0.0	0.0									
In 2024										
1.5	1.7	1.0	3.1	0.8	0.5	0.1	4.7	6.0	0.0	0.1
1.0	1.5	1.8	1.4	0.9	0.9	0.6	8.5	4.2	0.0	0.1
4.5	8.6	6.8	11.6	14.0	7.2	10.7	2.0	1.3	0.9	0.3
21.0	23.6	22.6	27.5	27.9	25.7	21.3	2.3	7.8	0.4	0.3
22.3	16.3	34.2	21.9	21.9	12.6	31.5	3.8	2.1	2.8	0.1
14.5	6.2	14.4	11.9	6.2	6.9	15.9	7.0	1.9	0.6	0.0
2.3	0.9	1.8	0.6	0.5	2.2	2.5	0.0	0.1	0.0	0.0
0.3	0.1	0.3	0.0	0.0	1.0	0.4	0.1	0.8	0.1	0.0
0.2	0.0	0.0	0.0	0.1	0.2	0.1	0.9	7.0	0.0	0.0
0.2	0.1	0.1	0.1	0.1	0.0	0.0	0.3	3.3	0.0	0.1
0.2	0.0									

Wind Speed (knots)										
In 2025										
0.1	0.5	0.6	0.3	0.6	0.5	2.2	0.0	0.0	0.5	0.1
0.5	1.3	1.3	0.6	0.9	2.4	0.7	7.8	2.6	0.0	0.1
7.5	12.7	7.6	11.8	9.6	12.0	7.2	8.3	2.2	0.0	0.0
32.2	38.8	43.3	55.6	67.5	54.2	51.5	6.7	1.2	0.0	0.1
24.4	17.0	20.3	9.7	12.9	28.0	19.8	2.0	0.4	1.7	8.6
8.1	5.8	3.0	1.3	1.5	1.9	6.1	0.0	0.0	0.9	1.5
1.4	0.6	0.2	0.1	0.1	0.2	0.3	2.9	3.0	0.1	0.0
0.3	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	3.5	0.3	0.4
0.0	0.0	0.0	0.1	0.1	0.0	0.1	0.0	5.8	0.2	0.0
0.0	0.0									

Table 6. Summary statistics for the wind speed data.

Statistics	Wind speed (knots)				
	2021	2022	2023	2024	2025
n_i	112	112	112	112	112
$n_{i(1)}$	52	69	84	94	78
$n_{i(0)}$	60	43	28	18	34
$\hat{\delta}_i$	0.535	0.383	0.250	0.161	0.304
$\hat{\alpha}_i$	2.960	2.579	2.414	2.376	2.571
$\hat{\beta}_i$	1.675	2.106	2.024	1.611	1.931
$\hat{\upsilon}_i$	2.991	2.449	2.130	1.975	2.276

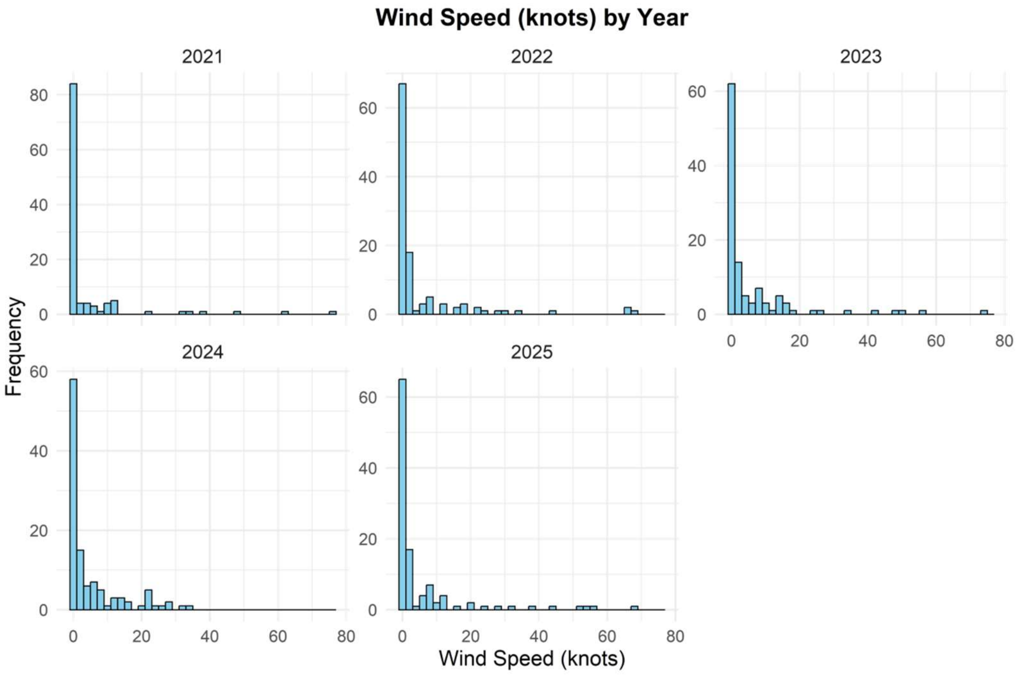

Figure 4. Histograms of wind speed data from the Phuket Airport Weather Observing Station for each year from 2021 to 2025.

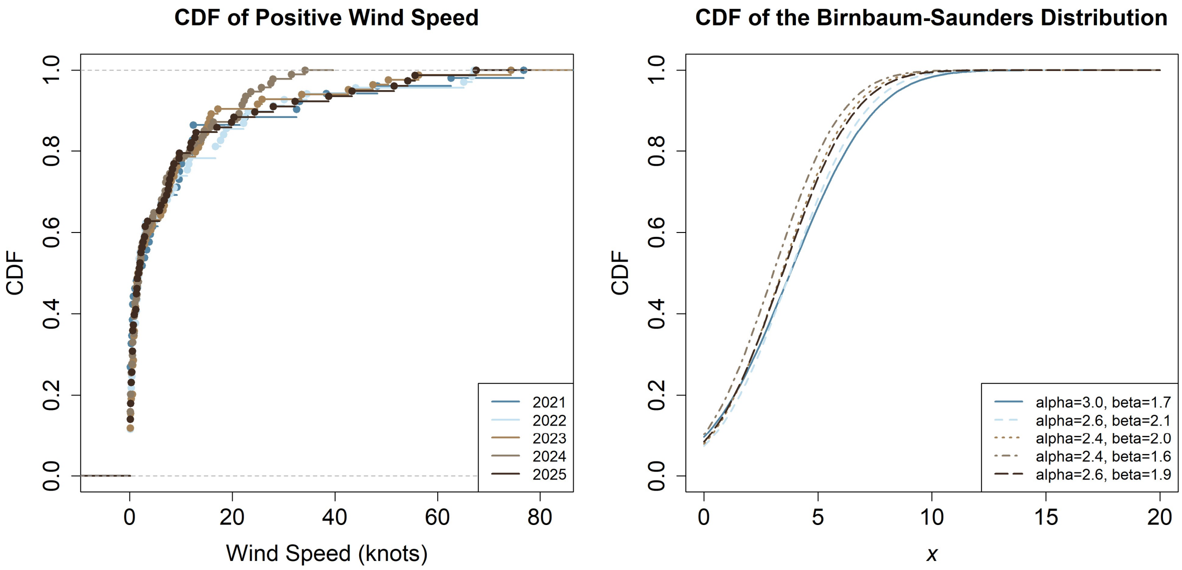

Figure 5. The CDF of the positive wind speed data compared with the estimated CDF from the Birnbaum–Saunders distribution.

Table 7. The AIC and BIC values of each distribution for wind speed data.

Distributions	Criterion	Wind speed (knots)				
		2021	2022	2023	2024	2025
Normal	AIC	440.260	576.834	681.145	671.851	641.617
	BIC	444.162	581.302	686.006	676.937	646.330
Lognormal	AIC	278.837	391.842	463.951	488.307	423.804
	BIC	282.739	396.310	468.813	493.394	428.517
Exponential	AIC	334.622	444.896	517.713	531.721	488.353
	BIC	336.574	447.130	520.144	534.264	490.709
Gamma	AIC	286.136	402.284	476.613	492.343	437.676
	BIC	290.039	406.753	481.475	497.429	442.390
Logistic	AIC	420.256	553.834	646.027	661.769	614.515
	BIC	424.158	558.302	650.888	666.856	619.229
Cauchy	AIC	371.745	485.059	571.346	618.019	532.519
	BIC	375.648	489.527	576.208	623.106	537.233
Weibull	AIC	282.298	397.006	470.048	489.824	430.833
	BIC	286.201	401.474	474.909	494.911	435.547
Birnbaum-Saunders	**AIC**	**264.940**	**383.066**	**457.083**	**469.653**	**413.405**
	BIC	**270.794**	**389.768**	**464.375**	**477.283**	**420.475**

Notes: The bold number indicates the lowest AIC and BIC of distribution.

Table 8. The SCIs of all pairwise differences between the CVs for the wind speed data.

Methods	Confidence interval for ϖ_{ir}		Length of intervals
	Lower	Upper	
GCI	0.2054	0.5565	0.3510
BCI	0.1609	0.6006	0.4397
MOVER	0.0528	0.7478	0.6951
MG	0.2030	0.5663	0.3633
MB	0.0354	0.7277	0.6924
BAY	0.0643	0.7527	0.6883
HPD	0.0533	0.7343	0.6810

6. Conclusions

Our objective of this investigation was to evaluate the construction of simultaneous confidence intervals for all pairwise differences in the coefficients of variation under zero-inflated Birnbaum–Saunders distributions using two critical criteria: Coverage probability and average width. The simulation results demonstrated that the GCI method performed most effectively when the number of samples was small to moderate (k = 3, 5), as it consistently maintained coverage probability close to the nominal level and produced the narrowest intervals. However, the GCI method demonstrated under-coverage in certain scenarios when the number of samples became large (k = 10). Although it yielded the shortest confidence intervals, its coverage probability fell below the nominal level in certain scenarios. This performance deterioration can be attributed to the accumulation of sampling errors associated with the estimation of nuisance parameters across groups. As k increased, the compounded variability inherent in GPQ construction led to less precise interval estimation. As a result, the MG was a more appropriate alternative in certain scenarios, as it consistently maintained a coverage probability above 0.95 and provided relatively narrow intervals. In contrast, the MOVER and MB methods, while conservative in maintaining coverage probability above the nominal level, produced excessively wide intervals. The BCI often failed to achieve adequate coverage, with coverage probability values often below 0.95, thus raising concerns regarding its reliability. The BAY and the HPD usually generated coverage probability values that were nearly 0.95; nevertheless, their AWs were wider than those of the GCI and the MG. Overall, the findings suggested that GCI is the most appropriate method when the number of samples is small and sample sizes are sufficient, whereas MG is preferable in scenarios with a large number of samples or a high proportion of zeros. Moreover, the application of all proposed methods to empirical wind speed data from Thailand demonstrated their practical applicability. The results of this real-data analysis corroborated the simulation results, further confirming the reliability of the suggested methods.

Author contributions

Usanee Janthasuwan: Data analysis, software, and drafted manuscript; Suparat Niwitpong: Supervision, conceptualization, review, and editing; Sa-Aat Niwitpong: Analytical methodologies, validated the final version, and obtained funding. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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Appendix

The procedure for constructing simultaneous confidence intervals for ϖ_{ir} based on the GCI method is as follows:

Algorithm 1.

1) Compute $A_i, B_i, C_i,$ and D_i
2) At the g step
   i. Generate $\Lambda_i \sim t_{(n_{i(1)}-1)}$ and then compute $Q_{\beta_i}(y_{ij}; \Lambda_i)$ from Eq (5).
   ii. If $Q_{\beta_i}(y_{ij}; \Lambda_i) < 0$ regenerate $\Lambda_i \sim t_{(n_{i(1)}-1)}$.
   iii. Compute $Q_{\alpha_i}(y_{ij}; H_i, \Lambda_i)$ and Q_{δ_i} from Eqs (6) and (7), respectively.
   iv. Compute Q_{υ_i} from Eq (8).
3) Repeat step 2), a total *G*=3000 times.
4) Compute $CI_{GCI:\varpi_{ir}}$ from Eq (9).

The procedure for constructing simultaneous confidence intervals for ϖ_{ir} based on the BCI method is as follows:

Algorithm 2.

1) At the b step
   i. Generate y_{ij}^*, with replacement from y_{ij} where $i = 1,2,\ldots,k$ and $j = 1,2,\ldots,n_i$.
   ii. Compute $\hat{\alpha}_i^{/}$ and $\hat{B}(\hat{\alpha}_i, \alpha_i)$.
   iii. Compute $\hat{\alpha}_i^*$ from Eq (10).
   iv. Generate $\hat{\delta}_i^*$ and compute $\hat{\varpi}_{ir}^*$ from Eq (11).
2) Repeat step 1), a total *B*=500 times.
3) Compute $CI_{BCI:\varpi_{ir}}$ from Eq (12).

The procedure for constructing simultaneous confidence intervals for ϖ_{ir} based on the MOVER, MG, and MB methods is as follows:

Algorithm 3.

1) Compute $\hat{\alpha}_i$ and $\hat{\delta}_i$.
2) Compute $\hat{\upsilon}_i$ and $\hat{V}(\hat{\upsilon}_i)$.

For MOVER:
   i. Compute $[l_i, u_i]$ from Eq (13).
   ii. Compute $CI_{MOVER:\varpi_{ir}}$ from Eq (14).

For MG:
   i. Compute $[l_{GCI:\upsilon_i}, u_{GCI:\upsilon_i}]$ from Eq (15).
   ii. Compute $CI_{MG:\varpi_{ir}}$ from Eq (16).

For MB:
   i. Compute $\hat{\upsilon}_i^*$.

ii. Compute $[l_{BCI:v_i}, u_{BCI:v_i}]$ from Eq (17).
iii. Compute $CI_{MB:\varpi_{ir}}$ from Eq (18).

The procedure for constructing simultaneous confidence intervals for ϖ_{ir} based on the BAY and HPD methods is as follows:

Algorithm 4.

1) Set $p_i, q_i, t_i,$ and s_i.
2) Compute $p(c_i)$ and $q^+(c_i)$
3) At the l step,
   i. Θ Generate $a_i \sim Uni(0, p(c_i))$ and $b_i \sim Uni(0, q^+(c_i))$, and compute $\vartheta_i = b_i / a_i^{c_i}$;
   if $a_i \leq [P(\beta_i | y_{ij})]^{\frac{1}{c_i+1}}$ **then** $\beta_i^{(k)} = \vartheta_i$;
   else
   Go back to Θ ; **end**
   ii. Generate $\alpha_i^2 \sim IG\left(\frac{n_{i(1)}}{2} + t_i, \sum_{j=1}^{n_{i(1)}} \frac{1}{2}\left(\frac{y_{ij}}{\beta_i^{(k)}} + \frac{\beta_i^{(k)}}{y_{ij}} - 2\right) + s_i\right)$ and set $\alpha_i^{(k)} = \sqrt{\alpha_i^2}$.
   iii. Generate $\delta_i | y_{ij}$ from Eq (21).
   iv. Compute ϖ'_{ir} from Eq (23).
4) Repeat step 3), a total L=1000 times.
5) Compute $CI_{BAY:\varpi_{ir}}$ and $CI_{HPD:\varpi_{ir}}$ from Eqs (24) and (25).

The computational procedure for estimating the coverage probability and the average width of all methods is outlined as follows:

Algorithm 5.

Define $n_i, \alpha_i, \beta_i,$ and δ_i.
For $m = 1$ to M;
1) Generate sample from the ZIBS distributions with parameters $\alpha_i, \beta_i,$ and δ_i.
2) Compute $\hat{\alpha}_i$ and $\hat{\delta}_i$.
3) Compute the $(1-\rho)100\%$ SCI for ϖ_{ir} based on the GCI, BCI, MOVER, MG, MB, BAY, and HPD methods, as implemented in Algorithms 1–4.
4) If $[L_s \leq \varpi_{ir} \leq U_s]$, set $C_s = 1$; else set $C_s = 0$;
   End m loop
5) Compute the coverage probability and average width from Eqs (I) and (II).