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Editorial

Special issue "Mathematical Foundations of Information Theory"

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This special issue of *AIMS Mathematics* is devoted to the theme *Mathematical Foundations of Information Theory*. The objective of the issue is to present recent research that advances the theoretical underpinnings of information theory through rigorous mathematical analysis, while also highlighting its interactions with neighboring areas such as combinatorics, graph theory, statistics, and optimization.

The eleven papers collected in this issue reflect the diversity of contemporary foundational research in information theory. Several contributions address long-standing open problems or develop new structural insights into classical objects, while others introduce refined analytical frameworks or provide exact characterizations of carefully formulated models. Taken together, the papers underscore the continuing role of mathematical depth and precision in shaping the development of information theory. The contributions in this special issue are partitioned into three categories.

1. Classical and quantum information theory, and statistical inference

The non-asymptotic region of conditional error probabilities for binary non-Bayesian hypothesis testing was characterized by Neyman and Pearson in 1933. In practice, the area of this fundamental region serves as a useful scalar proxy for the distinguishability between a pair of probability measures. Using information-theoretic methods, [1] shows that this area coincides with a new divergence, termed the NP-divergence, which is not an f-divergence. It is also shown that the asymptotic behavior of the area of that region is governed by the Bhattacharyya distance. Another contribution of [1] is the introduction of an information-theoretic sufficient-statistics criterion, termed I-sufficiency, which, for dominated data models, is shown to coincide with the sufficiency criteria introduced by Fisher, Kolmogorov and Blackwell. Paper [1] highlights the continuing relevance of spectral methods in modern information theory and statistical inference.

The fundamental question of how to causally describe the state of a channel succinctly to an encoder and how the latter should utilize the description to maximize throughput is a challenging problem even for single-user channels, let alone for multiple-access channels (MACs). The question of how the encoders of a MAC should instead utilize perfect causal state information is open. Paper [2] determines the capacity regions of a two-user memoryless state-dependent multiple-access channel (SD-MAC) with a helper that provides rate-limited (lossy) causal descriptions of the state sequence to one or both encoders. The authors focus on scenarios where the receiver is cognizant of the channel state and analyze optimal state quantization strategies. The exact single-letter characterization of the capacity regions is obtained by establishing matching achievability and converse results. The work in [2] makes a valuable theoretical contribution to the study of SD-MAC with causal state information, with observations on optimal quantization strategies and their impact on capacity regions.

The problem of approximating an output distribution over a noisy channel is well studied. Paper [3] studies a variant of this problem, where the approximation error need not tend to zero as the number of channel uses increases, and analyzes the tradeoff between the approximation error and the rate of the codebook. It considers the problem of generating channel output statistics that approximates a desired distribution via a finite-rate codebook, and characterizes the smallest achievable relative entropy, normalized by blocklength, between the induced and target output distributions. As applications, [3] presents a new proof of the rate-equivocation region for the wiretap channel, and a capacity formula for state masking with channel state information at the decoder. It offers a novel operational perspective on soft-covering and output approximation with insights into problems in information-theoretic security and state masking.

Correlation detection is a fundamental problem in statistical inference for database models. For two databases, it is formulated as a binary hypothesis testing problem in which, under the null hypothesis, the databases are independent, while under the alternative hypothesis they are correlated through an unknown permutation. The objective is to optimize the tradeoff between the false-alarm and missed-detection probabilities. Paper [4] proposes an efficient statistical test for correlation detection between two Gaussian databases. It introduces a new detection rule and derives upper bounds on the false-alarm and missed-detection probabilities in Gaussian settings. Paper [4] lies at the intersection of distributed statistical inference and information theory.

Finally, paper [5], in quantum information theory, introduces the notion of empirical coordination for quantum correlations. It studies the generation of separable quantum states using local quantum systems, shared randomness, and point-to-point classical communication, a task commonly known as coordination or state simulation. The authors consider an empirical coordination criterion that only requires the empirical average state to converge in probability to a target state, which is weaker than strong coordination based on direct estimation of the coordination error. The paper shows that shared randomness does not affect the achievable rates and characterizes the empirical coordination capacity for both the point-to-point and cascade network settings. The work [5] establishes the corresponding coordination capacities, compares the quantum and classical settings, and provides physical insights and illustrative examples. The implications for quantum cooperative games are also discussed.

2. Coding theory and majorization theory

The coding theory paper [6] investigates the structure of Reed–Muller codes, which are of longstanding interest from both practical and theoretical perspectives. The article addresses the problem of determining as many Hamming weights as possible in binary Reed–Muller codes of arbitrary lengths and orders, a task that is crucial for characterizing their weight spectra (i.e., the number of codewords of each possible Hamming weight). Determining the weight spectra of Reed–Muller codes is a notoriously difficult and important problem that has remained open since the 1960s. This work derives new results on the identification of codewords and the determination of their Hamming weights for general parameters of binary Reed–Muller codes, thereby advancing the understanding of a classical and technically challenging family of codes. These contributions are particularly timely in light of the renewed interest in Reed–Muller codes following the introduction of polar codes and polarization in coding theory, as well as recent results demonstrating that Reed–Muller codes can achieve the capacity of binary-input, output-symmetric memoryless channels.

The work in paper [7] revisits the concept of majorization, which plays a central role in entropy inequalities and information-theoretic comparisons. In particular, the majorization framework enables the derivation of sharp inequalities that are based on comparisons of probability distributions via their ordered structures. By establishing new equivalent conditions for majorization, the paper leverages these novel characterizations to derive an improved entropy inequality. This contribution enhances the understanding of a fundamental mathematical tool that is widely used across information theory, probability theory, and many other subfields of applied and pure mathematics.

3. Graph-theoretic foundations and Shannon capacity

Four contributions in the special issue focus on graph-theoretic aspects of information theory, particularly those arising in zero-error information theory.

The Shannon capacity of a graph is a central notion in zero-error information theory, capturing the largest effective input alphabet size achievable with zero error for a discrete memoryless channel modeled by a graph. Paper [8] investigates the Shannon capacity for several families of graphs. It derives exact values and new bounds, constructs a countably infinite family of connected graphs whose Shannon capacity is not attained by the independence number of any finite strong power, and establishes sufficient conditions under which the Shannon capacity of graph polynomials—formed using disjoint unions and strong products—can be expressed in terms of the capacities of their components. An additional inequality relating the capacities of strong products and disjoint unions yields alternative proofs of known bounds and new tightness conditions. The paper is also intended to serve as an accessible introduction to the theory of the Shannon capacity of graphs.

The Lovász ϑ-function is a fundamental graph invariant with deep connections to combinatorial optimization, semidefinite programming, and spectral graph theory, and applications far beyond zero-error information theory. In paper [9], the Lovász ϑ-function is studied as a graph invariant beyond its role as an upper bound on the Shannon capacity of graphs. The paper analyzes its relationships with other graph invariants, examines its behavior for various graph families (including cospectral and non-isomorphic graphs), and explores its connections with spectral and algebraic graph theory. It also establishes Shannon capacity results for several graph families and presents a novel result demonstrating the utility of the Lovász ϑ-function in distinguishing between cospectral and non-isomorphic graphs that share identical associated matrices (i.e., adjacency, Laplacian, normalized Laplacian, and signless Laplacian matrices), as well as identical independence, clique, and chromatic numbers. Owing to its breadth and depth, the paper serves as a comprehensive reference of mutual interest to researchers in zero-error information theory and algebraic graph theory.

The letter [10], following an outline presented in [9], resolves a question that had remained open since the introduction, in the early 1980s, of Schrijver's strengthening of the Lovász ϑ-function as an upper bound on the independence number of graphs. By providing an explicit example, it demonstrates that this variant does not universally upper bound the Shannon capacity of a graph, thereby clarifying a subtle yet significant distinction between these two closely related graph invariants, both of whose computation relies on semidefinite programming.

Finally, paper [11] studies families of graphs defined by intersection properties and derives upper bounds on the size of such families. Extending earlier work of Chung, Graham, Frankl, and Shearer, the analysis employs combinatorial and probabilistic forms of Shearer's lemma to obtain bounds in terms of structural parameters of a given graph and relaxations based on the Lovász ϑ-function. This paper also establishes related results on counting graph homomorphisms, illustrating connections between combinatorial methods and information-theoretic techniques.

4. Summary

The contributions in this special issue collectively highlight the depth and breadth of current research in the mathematical foundations of information theory. This special issue received a total of 30 submissions, all of which were processed under the journal's standard peer-review procedures to ensure scientific rigor, novelty, matching with the scope of the special issue, and thematic coherence. Finally, 11 papers were published. It is our hope that this special issue will serve as a useful reference for researchers and students interested in the mathematical foundations of information theory, and that it will stimulate further research work.

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Use of Generative-AI tools declaration

The author declares that no Artificial Intelligence (AI) tools were utilized in creating this editorial article.

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Conflict of interest

Igal Sason served as the Guest Editor of the special issue on "Mathematical Foundations of Information Theory" for *AIMS Mathematics* and was not involved in the editorial review or the decision to publish the articles [8–11] that he authored or co-authored in this special issue. He declares no conflicts of interest.

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