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Correction

Correction: Relative information spectra with applications to statistical inference

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A correction on

Relative information spectra with applications to statistical inference
by Sergio Verdú. AIMS Mathematics, 2024, 9(12), 35038–35090.
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The author would like to make the following correction to the published paper [1].

In Theorem 21-(d) of [1], min in (8.29) should be max. In other words, for $\nu \in (0, \underline{\pi}_{1|0})$, the fundamental tradeoff function is given by

$$\alpha_\nu(P_1, P_0) = \max_{\gamma \in \mathbb{R}} \left\{ \mathbb{F}_{P_1 \| P_0}(\gamma) + \exp(\gamma) \left(1 - \nu - \overline{\mathbb{F}}_{P_1 \| P_0}(\gamma) \right) \right\}, \tag{8.29}$$

where the maximum is achieved by $\gamma^\star$ defined in (8.30) and (8.32).

Proof. With the aid of (4.24) we can express the function within {} in (8.29) as

$$\begin{aligned} f_\nu(\gamma) &= \mathbb{F}_{P_1 \| P_0}(\gamma) + \exp(\gamma) \left(1 - \nu - \overline{\mathbb{F}}_{P_1 \| P_0}(\gamma) \right) \\ &= (1 - \nu) \exp(\gamma) - \frac{1}{\log \mathrm{e}} \int_{-\infty}^{\gamma} \exp(t)\, \overline{\mathbb{F}}_{X \| Y}(t)\, \mathrm{d}t. \end{aligned}$$

Its right- and left-derivatives at $\gamma \in \mathbb{R}$ are

$$\begin{aligned} \dot{f}_\nu^+(\gamma) &= \lim_{\epsilon \downarrow 0} \frac{f_\nu(\gamma + \epsilon) - f_\nu(\gamma)}{\epsilon} = \left(1 - \nu - \overline{\mathbb{F}}_{X \| Y}(\gamma) \right) \frac{\exp(\gamma)}{\log \mathrm{e}}, \\ \dot{f}_\nu^-(\gamma) &= \lim_{\epsilon \downarrow 0} \frac{f_\nu(\gamma) - f_\nu(\gamma - \epsilon)}{\epsilon} = \left(1 - \nu - \lim_{x \uparrow \gamma} \overline{\mathbb{F}}_{X \| Y}(x) \right) \frac{\exp(\gamma)}{\log \mathrm{e}}, \end{aligned}$$

respectively. Consequently,

1. $\dot{f}_\nu^+(\gamma) > 0$ and $\dot{f}_\nu^-(\gamma) > 0$ at those $\gamma \in \mathbb{R}$ such that $\overline{\mathbb{F}}_{X\|Y}(\gamma) < 1 - \nu$;
2. $\dot{f}_\nu^+(\gamma) < 0$ and $\dot{f}_\nu^-(\gamma) < 0$ at those $\gamma \in \mathbb{R}$ such that $\lim_{x\uparrow\gamma} \overline{\mathbb{F}}_{X\|Y}(x) > 1 - \nu$;
3. If $\overline{\mathbb{F}}^{-1}_{P_1\|P_0}(1-\nu) \neq \emptyset$, then $\dot{f}_\nu^+(\gamma^\star) = \dot{f}_\nu^-(\gamma^\star) = 0$ at any solution of (8.30);
4. If $\overline{\mathbb{F}}^{-1}_{P_1\|P_0}(1-\nu) = \emptyset$, then at the unique $\gamma^\star$ that satisfies (8.32), $\dot{f}_\nu^-(\gamma^\star) > 0$ and $\dot{f}_\nu^+(\gamma^\star) < 0$.

Therefore, we have shown that the non-concave function to be maximized satisfies $f_\nu(\gamma) < f_\nu(\gamma^\star)$ for any γ that does not satisfy either (8.30) or (8.32). The fact that $f_\nu(\gamma^\star) = \alpha_\nu(P_1, P_0)$ follows from (8.30)–(8.31) if $\overline{\mathbb{F}}^{-1}_{P_1\|P_0}(1-\nu) \neq \emptyset$. Otherwise, it follows from (8.33)–(8.34) taking into account that, in view of (4.25),

$$\exp(\gamma^\star) = \frac{\mathbb{F}_{P_1\|P_0}(\gamma^\star) - \lim_{x\uparrow\gamma^\star}\mathbb{F}_{P_1\|P_0}(x)}{\overline{\mathbb{F}}_{P_1\|P_0}(\gamma^\star) - \lim_{x\uparrow\gamma^\star}\overline{\mathbb{F}}_{P_1\|P_0}(x)}.$$

□

The change does not affect anything else in [1]. In particular in Appendix A, the reference to Theorem 21-(d) only uses the expressions of the fundamental tradeoff function in (8.31) and (8.33), which in turn are derived from Item 96.

The original manuscript will be updated [1]. We apologize for any inconvenience caused to our readers by this change.

Conflict of interest

The author declares there is no conflict of interest.

References

1. S. Verdú, Relative information spectra with applications to statistical inference, *AIMS Math.*, **9** (2024), 35038–35090. https://doi.org/10.3934/math.20241668