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Research article

Mathematical analysis of time-fractional nonlinear Kuramoto-Sivashinsky equation

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Abstract: This research explored fractional Kuramoto-Sivashinsky equation analytical solutions through application of the residual power series transform method (RPSTM). The Caputo derivative approach served as the basis to analyze fractional systems since it offered a strong foundation for modeling intricate nonlinear processes. Researchers extended the Kuramoto-Sivashinsky equation through fractional domain application to capture anomalous behavior and memory effects since it showed practical uses in turbulence and plasma dynamics and flame propagation. The RPSTM proposed solution united residual power series method capabilities with integral transforms features to develop an accurate and efficient method for fractional nonlinear partial differential equation solutions. The method enabled accurate approximate solution acquisition while the procedure underwent complete convergence examination. This paper demonstrated the effectiveness and reliability of the developed method through numerical simulation results. The RPSTM proved itself as an effective analytical method for fractional differential problems which reveals vital information about the fractional Kuramoto-Sivashinsky equation behavior. The research added value to existing fractional calculus studies focused on nonlinear science applications.

Keywords: residual power series transform method; factional Kuramoto-Sivashinsky equation; natural transform; Caputo operator
Mathematics Subject Classification: 34G20, 35A20, 35A22, 35R11

1. Introduction

Fractional nonlinear partial differential equations (FNLPDEs) represent an advanced mathematical model system that uses fractional derivatives to better model physical memory and inherited effects appearing across biological and engineering systems and physical systems. The focus on FNLPDEs increased substantially because these equations deliver enhanced modeling accuracy over traditional approaches [1, 2]. These models use fractional-order derivatives following the definitions from Caputo or Riemann-Liouville to include effects of long-range interactions together with nonlocal effects. The discipline of fluid mechanics along with quantum mechanics and biological systems together with financial mathematics uses FNLPDEs for their modeling needs [3, 4]. The complexity of these equations makes it extremely difficult to obtain exact solutions and their close counterparts [5, 6]. Various analytical and numerical techniques have been investigated different partial differential equations, such as the Adomian decomposition method [7], variational iteration method [8], iteration procedure [9], and Laplace decomposition method [10]. Computational advancements enable the FNLPDE study to yield modern predictions and improved predictive models for numerous scientific as well as engineering applications.

The Kuramoto-Sivashinsky (KS) equation serves as an effective mathematical model which researchers use to describe various nonlinear scientific and engineering processes. The practical application of the classical KS equation remains limited in revealing all behaviors of systems that exhibit anomalous diffusion or memory effects. Fractional calculus requires extension because it leads to the development of the fractional-order Kuramoto-Sivashinsky (FKS) equation. The fractional KS equation makes use of fractional derivatives because these derivatives enable sufficient system modeling of nonlocal phenomena with extended interaction ranges. The fractional KS equation becomes particularly applicable for fluid mechanics along with plasma physics and material science because of its essential features [11, 12]. Mathematical challenges appear in the fractional KS equation through its inclusion of fractional derivatives despite their physical importance. Advanced mathematical difficulties emerge from fractional derivatives due to their impact on analysis methods and numerical computations alongside the evaluation of fractional order effects on solution properties and practical applications for this fractional model. The efficient solution of the fractional KS equation continues to present challenges for researchers. The research examines deceptive parameters through new approaches followed by detailed studies as well as exploring real-world fractional KS equation implementations to address the existing knowledge voids. This work intends to improve knowledge and practical usefulness of this essential mathematical model across multiple scientific and engineering applications [13, 14].

The KS equation [15, 16] models plasma instabilities, chemical reaction-diffusion, flame front propagation viscous flow difficulties, and magnetized plasmas. This article focuses on studying the FKS equation [17].

$$D_t^{\varrho}\varphi(\zeta,t)+\varphi(\zeta,t)\frac{\partial}{\partial\zeta}\varphi(\zeta,t)+\alpha\frac{\partial^2}{\partial\zeta^2}\varphi(\zeta,t)+\beta\frac{\partial^3}{\partial\zeta^3}\varphi(\zeta,t)+\gamma\frac{\partial^4}{\partial\zeta^4}\varphi(\zeta,t)=0, \quad \text{where} \quad 0<\varrho\leq 1, \tag{1.1}$$

with the initial condition (IC):

$$\varphi(\zeta,0)=\varphi_0(\zeta), \tag{1.2}$$

where α, β, and γ are constants.

The KS equation together with fractional variants functions as a prominent modeling tool for physical, and engineering processes that involve turbulence, and thin film dynamics, and reaction-diffusion systems [18, 19]. Different analytical approaches along with numerical techniques have been used to investigate these equations which include the asymptotic expansion [20] and novel computational techniques [21]. New advanced numerical methods now employ both compact difference schemes on graded meshes [22] together with quintic B-spline-based methods [23]. The semi-analytical approaches together with integer-fractional time-derivative models have enhanced the understanding of the KS equation as per [24, 25]. Research studies reveal essential information about the complicated KS equation behavior, which aids in creating efficient computational techniques for solving it.

The residual power series transform method (RPSTM) is a new analytical method to approximate solutions of nonlinear differential equations that can be applied to various complex physical, engineering, and mathematical models. Extending the ideas of power series expansions and residual methods provides an efficient means for determining approximate solutions to various nonlinear problems. RPSTM applies a proper integral transform (Laplace or Fourier transform) to the nonlinear equation to obtain the problem more straightforwardly in the transformed domain. We then try to write the solution in the power series expansion form, which means the unknown function is approximated by a series of terms with powers of the independent variable, with unknown coefficients [26–28]. It, therefore, determines these coefficients by solving the transformed equation iteratively. The characteristic feature of RPSTM is that a residual is introduced, which measures the error between the approximation and the actual solution at each step. In each iteration, the residual enters to refine the series expansion and forces the method to converge to an accurate solution. The residual power series transform method is proper when dealing with complex conditions on the boundary or highly nonlinear systems. Like perturbation or linearization techniques, it does not need to linearize the nonlinear terms, allowing use for problems that are difficult to deal with through traditional perturbation or linearization techniques. RPSTM can be applied to a large class of equations, including those resulting in heat conduction, fluid dynamics, and nonlinear wave propagation; moreover, enormous theoretical and applied experience has been gained using RPSTM thanks to its success in efficiently and simply calculating accurate solutions [29–31].

2. Basic definitions

2.1. Definition

The fractional Rieman-Liouville integral of order $\varrho \in \mathbb{R}_{+}$of a function $h(\gamma) \in L([0,1],\mathbb{R})$ is expressed by [32–34]

$$I_0^{\varrho} h(\gamma) = \frac{1}{\Gamma(\varrho)} \int_0^t (t-s)^{\varrho-1} h(s)ds,$$

on the assumption that the integral on the right side of the equation is convergent.

2.2. Definition

For $\mu \in \mathbb{R}$, a function $f : \mathbb{R} \to \mathbb{R}^{+}$is said to be in the space C_{μ} if it can be written as $f(\zeta) = \zeta^{q} f_1(\zeta)$ with $q > \mu$, $f_1(\zeta) \in C[0,\infty)$, and it is in space $f(\zeta) \in C_{\mu}^{n}$ if $f^{(n)} \in C_{\mu}$ for $n \in \mathbb{N} \cup \{0\}$ [34].

2.3. Definition

The fractional Caputo derivative of a function $h \in C_{-1}^{n}$ with $n \in \mathbb{N} \cup \{0\}$ is given as [34]

$$D_t^{\varrho} h(t) = \begin{cases} I^{n-\varrho} f^{(n)}, & n-1 < \varrho \leq n, n \in \mathbb{N}, \\ \frac{d^n}{\mathrm{d}t^n} h(t), & \varrho = n, n \in \mathbb{N}. \end{cases}$$

2.4. Definition

The Mittag-Leffler function (MLF) of two-parameters is expressed by [34]:

$$E_{\varrho,\beta}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(k\varrho + \beta)}.$$

For $\varrho = \beta = 1, E_{1,1}(t) = e^t$ and $E_{1,1}(-t) = e^{-t}$.

2.5. Definition

The natural transform (NT) of a function $v(\zeta, t)$ for $t \geq 0$ is defined by [34]

$$\mathcal{N}[v(\zeta, t)] = R(\zeta, s, u) = \int_0^{\infty} e^{-st} v(\zeta, ut) \mathrm{d}t,$$

where s and u for the transform parameters are taken to be real and positive.

2.6. Definition

The MLF of NT $E_{\varrho,\beta}$ is given as [34]

$$\mathcal{N}[v(\zeta, t)] = \int_0^{\infty} e^{-st} v(\zeta, ut) \mathrm{d}t = \sum_{k=0}^{\infty} \frac{u^{k+1}\Gamma(k+1)}{s^{k+1}\Gamma(k\varrho + \beta)}.$$

2.7. Definition

The Miller and Ross sense of the NT of $D^{\varrho} f(t)$ is expressed by the following [34]:

$$\mathcal{N}\left(D^{\varrho} f(t)\right) = \frac{s^{\varrho}}{u^{\varrho}} R(s, u) - \sum_{k=0}^{n-1} \frac{s^{n-k-1}}{u^{n-k}} f^{(k)}(0), n-1 < \varrho \leq n.$$

2.8. Lemma

The NT of $\frac{\partial^{\varrho} f(\zeta,t)}{\partial t^{\varrho}}$ with respect to t can be defined as [34]:

$$\mathcal{N}\left[\frac{\partial^{\varrho} f(\zeta, t)}{\partial t^{\varrho}}\right] = \frac{s^{\varrho}}{u^{\varrho}} R(\zeta, s, u) - \sum_{k=0}^{n-1} \frac{s^{n-k-1}}{u^{n-k}} \left[\lim_{t \to 0} \frac{\partial^{\varrho} f(\zeta, t)}{\partial t^{\varrho}}\right].$$

2.9. Lemma

The NT of the ϱ order partial derivative of $f(\zeta, t)$ with respect to ζ is denoted by [34]

$$\mathcal{N}\left[\frac{\partial^{\varrho} f(\zeta, t)}{\partial \zeta^{\varrho}}\right] = \frac{d^{\varrho}}{d\zeta^{\varrho}} R(\zeta, s, u).$$

2.10. Lemma

The dual relationship between Laplace and NTs is expressed by [34]

$$\mathcal{N}[f(\zeta,t)] = R(\zeta,s,u) = \frac{1}{u}\int_0^\infty e^{\frac{-u}{\zeta}} f(\zeta,t)\mathrm{d}t = \frac{1}{u}\mathcal{L}\{f(\zeta,t)\},$$

where $\mathcal{L}$ is the laplace transform. As a conclusion from the above lemma, it can be noted that the Natural transform can be seen as an extension of both the Sumudu and Laplace transforms. In particular, when $u = 1$ then the NT reduces to the Laplace transform and in same way, for $s = 1$, the generalization leads us to the Sumudu transform.

3. Methodology

3.1. The general implementation of RPSTM

The RPSTM collection of principles, upon which our overall model solution was built, is described in this section.

Step 1. Consider the following PDE of general form:

$$D_t^{\varrho}\varphi(\zeta,t) + \vartheta(\zeta)N(\varphi) - \zeta(\zeta,\varphi) = 0. \tag{3.1}$$

Step 2. Both sides of Eq (3.1) are subjected to the NT in order to obtain:

$$\mathcal{N}[D_t^{q\varrho}\varphi(\zeta,t) + \vartheta(\zeta)N(\varphi) - \zeta(\zeta,\varphi)] = 0. \tag{3.2}$$

Apply the transformation to Eq (3.2) as

$$\Psi(\zeta,s) = \sum_{j=0}^{q-1}\frac{D_t^j\varphi(\zeta,0)}{s^{q\varrho+2}} + \frac{\vartheta(\zeta)Y(s)}{s^{q\varrho}} - \frac{F(\zeta,s)}{s^{q\varrho}}, \tag{3.3}$$

where, $N[\zeta(\zeta,\varphi)] = F(\zeta,s), N[N(\varphi)] = Y(s)$.

Step 3. The expression representing the solution to Eq (3.3) is as follows:

$$\Psi(\zeta,s) = \sum_{r=0}^{\infty}\frac{\hbar_r(\zeta)}{s^{r\varrho+2}},\ s > 0.$$

Step 4. Follow this procedure:

$$\hbar_0(\zeta) = \lim_{s\to\infty} s^2\Psi(\zeta,s) = \varphi(\zeta,0).$$

The following outcome can be achieved as

$$\hbar_1(\zeta) = D_t^{\varrho}\varphi(\zeta,0),$$

$$\hbar_2(\zeta) = D_t^{2\varrho}\varphi(\zeta,0),$$

$$\vdots$$

$$\hbar_w(\zeta) = D_t^{w\varrho}\varphi(\zeta, 0).$$

Step 5. After the Kth truncation, the $\Psi(\zeta, s)$ series can be obtained using the subsequent formula:

$$\Psi_K(\zeta, s) = \sum_{r=0}^{K} \frac{\hbar_r(\zeta)}{s^{r\varrho+2}}, \ s > 0.$$

$$\Psi_K(\zeta, s) = \frac{u^\varrho \hbar_0(\zeta)}{s} + \frac{u^\varrho \hbar_1(\zeta)}{s^{\varrho+2}} + \cdots + \frac{u^\varrho \hbar_w(\zeta)}{s^{w\varrho+2}} + \sum_{r=w+1}^{K} \frac{\hbar_r(\zeta)}{s^{r\varrho+2}}.$$

Step 6. For the subsequent results, it is necessary to consider the natural residual function (NRF) as depicted in 3.3 and the *Kth*-truncated NRF independently.

$$\mathcal{N}Res(\zeta, s) = \Psi(\zeta, s) - \sum_{j=0}^{q-1} \frac{u^\varrho D_t^j \varphi(\zeta, 0)}{s^{j\varrho+2}} + \frac{u^\varrho \vartheta(\zeta) Y(s)}{s^{j\varrho}} - \frac{u^\varrho F(\zeta, s)}{s^{j\varrho}},$$

and

$$NRes_K(\zeta, s) = \Psi_K(\zeta, s) - \sum_{j=0}^{q-1} \frac{u^\varrho D_t^j \varphi(\zeta, 0)}{s^{j\varrho+2}} + \frac{u^\varrho \vartheta(\zeta) Y(s)}{s^{j\varrho}} - \frac{u^\varrho F(\zeta, s)}{s^{j\varrho}}. \quad (3.4)$$

Step 7. In Eq (3.4), $\Psi_K(\zeta, s)$ should be substituted for its expansion form.

$$\begin{aligned} ARes_K(\zeta, s) = &\Big(\frac{u\hbar_0(\zeta)}{s} + \frac{u^\varrho \hbar_1(\zeta)}{s^{\varrho+2}} + \cdots + \frac{u^\varrho \hbar_w(\zeta)}{s^{w\varrho+2}} + \sum_{r=w+1}^{K} \frac{u^\varrho \hbar_r(\zeta)}{s^{r\varrho+2}}\Big) \\ &- \sum_{j=0}^{q-1} \frac{u^\varrho D_t^j \varphi(\zeta, 0)}{s^{j\varrho+2}} + \frac{u^\varrho \vartheta(\zeta) Y(s)}{s^{j\varrho}} - \frac{F(\zeta, s)}{s^{j\varrho}}. \end{aligned} \quad (3.5)$$

Step 8. To obtain the following result, multiply each side of Eq (3.5) by s^{Kp+2}.

$$\begin{aligned} s^{K\varrho+2} NRes_K(\zeta, s) = &s^{K\varrho+2}\Big(\frac{u\hbar_0(\zeta)}{s} + \frac{\hbar_1(\zeta)}{s^{\varrho+2}} + \cdots + \frac{\hbar_w(\zeta)}{s^{w\varrho+2}} + \sum_{r=w+1}^{K} \frac{\hbar_r(\zeta)}{s^{r\varrho+2}} \\ &- \sum_{j=0}^{q-1} \frac{D_t^j \varphi(\zeta, 0)}{s^{j\varrho+2}} + \frac{\vartheta(\zeta) Y(s)}{s^{j\varrho}} - \frac{F(\zeta, s)}{s^{j\varrho}}\Big). \end{aligned} \quad (3.6)$$

Step 9. Calculate Eq (3.6) in order to derive the following, assuming that $\lim_{s\to\infty}$.

$$\begin{aligned} \lim_{s\to\infty} s^{K\varrho+2} NRes_K(\zeta, s) = &\lim_{s\to\infty} s^{K\varrho+2}\Big(\frac{u\hbar_0(\zeta)}{s} + \frac{u^\varrho \hbar_1(\zeta)}{s^{\varrho+2}} + \cdots + \frac{u^\varrho \hbar_w(\zeta)}{s^{w\varrho+2}} + \sum_{r=w+1}^{K} \frac{u^\varrho \hbar_r(\zeta)}{s^{r\varrho+2}} \\ &- \sum_{j=0}^{q-1} \frac{u^\varrho D_t^j \varphi(\zeta, 0)}{s^{j\varrho+2}} + \frac{u^\varrho \vartheta(\zeta) Y(s)}{s^{j\varrho}} - \frac{u^\varrho F(\zeta, s)}{s^{j\varrho}}\Big). \end{aligned}$$

Step 10. In order find $\hbar_K(\zeta)$, we solve the given equation.

$$\lim_{s\to\infty} (s^{K\varrho+2} NRes_K(\zeta, s)) = 0,$$

where $K = w+1, w+2, \cdots$.
Step 11. In order to derive the K-approximate solution to Eq (3.3), replace $\hbar_K(\zeta)$ with a truncated $\Psi(\zeta, s)$ series.
Step 12. To obtain the final solution, apply the natural inverse transform (NIT), and solve $\Psi_K(\zeta, s)$ in order to acquire the necessary function $\varphi_K(\zeta, t)$.

3.1.1. Problem 1

Consider the fractional KS equation is given as

$$D_t^{\varrho}\varphi(\zeta,t) + \varphi(\zeta,t)\frac{\partial}{\partial\zeta}\varphi(\zeta,t) - \frac{\partial^2}{\partial\zeta^2}\varphi(\zeta,t) + \frac{\partial^4}{\partial\zeta^4}\varphi(\zeta,t) = 0, \quad \text{where} \quad 0 < \varrho \le 1, \tag{3.7}$$

with the initial condition

$$\varphi(\zeta,0) = \frac{15\tanh^3(l(\zeta-\lambda)) - 45\tanh(l(\zeta-\lambda))}{19\sqrt{19}} + \xi, \tag{3.8}$$

and exact solution

$$\varphi(\zeta,t) = \frac{15\tanh^3(l(-\lambda-\xi t+\zeta)) - 45\tanh(l(-\lambda-\xi t+\zeta))}{19\sqrt{19}} + \xi.$$

After applying NT to Eq (3.7) and using Eq (3.8), we obtain

$$\varphi(\zeta,s) - \frac{u\frac{15\tanh^3(l(\zeta-\lambda))-45\tanh(l(\zeta-\lambda))}{19\sqrt{19}} + \xi}{s} + \frac{u^{\varrho}}{s^{\varrho}}\mathcal{N}_t\Big[\mathcal{N}_t^{-1}\varphi(\zeta,s)\times\frac{\partial}{\partial\zeta}\mathcal{N}_t^{-1}\varphi(\zeta,s)\Big] - \frac{u^{\varrho}}{s^{\varrho}}\Big[\frac{\partial^2}{\partial\zeta^2}\varphi(\zeta,s)\Big] + \frac{u^{\varrho}}{s^{\varrho}}\Big[\frac{\partial^4}{\partial\zeta^4}\varphi(\zeta,s)\Big] = 0. \tag{3.9}$$

Consequently, the k^{th}-truncated term series are as follows:

$$\varphi(\zeta,s) = \frac{u\frac{15\tanh^3(l(\zeta-\lambda))-45\tanh(l(\zeta-\lambda))}{19\sqrt{19}} + \xi}{s} + \sum_{r=1}^{k}\frac{u^{\varrho}f_r(\zeta,s)}{s^{r\varrho+1}}, \quad r = 1,2,3,4\cdots \tag{3.10}$$

The NRF is as follows:

$$\mathcal{N}_t Res(\zeta,s) = \varphi(\zeta,s) - \frac{u\frac{15\tanh^3(l(\zeta-\lambda))-45\tanh(l(\zeta-\lambda))}{19\sqrt{19}} + \xi}{s} + \frac{u^{\varrho}}{s^{\varrho}}\mathcal{N}_t\Big[\mathcal{N}_t^{-1}\varphi(\zeta,s)\times\frac{\partial}{\partial\zeta}\mathcal{N}_t^{-1}\varphi(\zeta,s)\Big] - \frac{u^{\varrho}}{s^{\varrho}}\Big[\frac{\partial^2}{\partial\zeta^2}\varphi(\zeta,s)\Big] + \frac{u^{\varrho}}{s^{\varrho}}\Big[\frac{\partial^4}{\partial\zeta^4}\varphi(\zeta,s)\Big] = 0, \tag{3.11}$$

and the k^{th}-NRFs are:

$$\mathcal{N}_t Res_k(\zeta,s) = \varphi_k(\zeta,s) - \frac{u\frac{15\tanh^3(l(\zeta-\lambda))-45\tanh(l(\zeta-\lambda))}{19\sqrt{19}} + \xi}{s} + \frac{u^{\varrho}}{s^{\varrho}}\mathcal{N}_t\Big[\mathcal{N}_t^{-1}\varphi_k(\zeta,s)\times\frac{\partial}{\partial\zeta}\mathcal{N}_t^{-1}\varphi_k(\zeta,s)\Big] - \frac{u^{\varrho}}{s^{\varrho}}\Big[\frac{\partial^2}{\partial\zeta^2}\varphi_k(\zeta,s)\Big] + \frac{u^{\varrho}}{s^{\varrho}}\Big[\frac{\partial^4}{\partial\zeta^4}\varphi_k(\zeta,s)\Big] = 0. \tag{3.12}$$

Determining $f_r(\zeta, s)$ for $r = 1, 2, 3, ...$ needs some computation. Follow these steps, take the r^{th}-NRF Eq (3.12), and substitute it for the r^{th}-truncated series Eq (3.10); I solve $\lim_{s\to\infty}(s^{r\varrho+1})$ by multiplying the final equation by $s^{r\varrho+1}$. $\mathcal{N}_t Res_{\varphi,r}(\zeta, s)) = 0$, and $r = 1, 2, 3, \cdots$. Few terms are obtained through the proposed method is as follows:

$$f_1(\zeta, s) = \frac{45l\operatorname{sech}^4(l(\zeta-\lambda))}{6859}\Big(\tanh(l(\zeta-\lambda))\Big(15\Big(152\sqrt{19}l^3-1\Big)\operatorname{sech}^2(l(\zeta-\lambda))+76\sqrt{19}l\Big(1-16l^2\Big) - 30\Big)+19\sqrt{19}\xi\Big), \tag{3.13}$$

$$\begin{aligned} f_2(\zeta, s) = {} & \frac{90l^2\operatorname{sech}^4(l(\zeta-\lambda))}{2476099}\Big(-225\Big(5776\Big(1596\sqrt{19}l^3-13\Big)l^3+5\sqrt{19}\Big)\tanh(l(\zeta-\lambda))\operatorname{sech}^6(l(\zeta-\lambda)) \\ & -2888\Big(38\sqrt{19}l\Big(16l^2-1\Big)+15\Big)\xi+2\Big(5776l\Big(16l^2-1\Big)\Big(19\sqrt{19}l\Big(16l^2-1\Big)+15\Big) \\ & +\sqrt{19}\Big(6859\xi^2+900\Big)\Big)\tanh(l(\zeta-\lambda))+7220\operatorname{sech}^2(l(\zeta-\lambda))\Big(\Big(19\sqrt{19}l\Big(52l^2-1\Big)+3\Big)\xi \\ & -3l\Big(76l^2-1\Big)\Big(19\sqrt{19}l\Big(28l^2-1\Big)+6\Big)\tanh(l(\zeta-\lambda))\Big)-15\operatorname{sech}^4(l(\zeta-\lambda))\Big(2527\Big(152\sqrt{19}l^3-1\Big)\xi \\ & -4\Big(361l\Big(112\Big(19\sqrt{19}l\Big(58l^2-1\Big)-2\Big)l^2+11\Big)-45\sqrt{19}\Big)\tanh(l(\zeta-\lambda))\Big)\Big), \end{aligned} \tag{3.14}$$

and so on.

Put $f_r(\zeta, s)$, for $r = 1, 2, 3, \cdots$, in Eq (3.10):

$$\begin{aligned} \varphi(\zeta, s) = {} & \frac{\frac{15\tanh^3(l(\zeta-\lambda))-45\tanh(l(\zeta-\lambda))}{19\sqrt{19}}+\xi}{s^2}+\frac{45l\operatorname{sech}^4(l(\zeta-\lambda))}{6859s^{p+1}}\Big(\tanh(l(\zeta-\lambda))\Big(15\Big(152\sqrt{19}l^3 \\ & -1\Big)\operatorname{sech}^2(l(\zeta-\lambda))+76\sqrt{19}l\Big(1-16l^2\Big)-30\Big)+19\sqrt{19}\xi\Big)+\frac{90l^2\operatorname{sech}^4(l(\zeta-\lambda))}{2476099s^{2p+1}} \\ & \times\Big(-225\Big(5776\Big(1596\sqrt{19}l^3-13\Big)l^3+5\sqrt{19}\Big)\tanh(l(\zeta-\lambda))\operatorname{sech}^6(l(\zeta-\lambda)) \\ & -2888\Big(38\sqrt{19}l\Big(16l^2-1\Big)+15\Big)\xi+2\Big(5776l\Big(16l^2-1\Big)\Big(19\sqrt{19}l\Big(16l^2-1\Big)+15\Big) \\ & +\sqrt{19}\Big(6859\xi^2+900\Big)\Big)\tanh(l(\zeta-\lambda))+7220\operatorname{sech}^2(l(\zeta-\lambda))\Big(\Big(19\sqrt{19}l\Big(52l^2-1\Big)+3\Big)\xi \\ & -3l\Big(76l^2-1\Big)\Big(19\sqrt{19}l\Big(28l^2-1\Big)+6\Big)\tanh(l(\zeta-\lambda))\Big)-15\operatorname{sech}^4(l(\zeta-\lambda))\Big(2527\Big(152\sqrt{19}l^3-1\Big)\xi \\ & -4\Big(361l\Big(112\Big(19\sqrt{19}l\Big(58l^2-1\Big)-2\Big)l^2+11\Big)-45\sqrt{19}\Big)\tanh(l(\zeta-\lambda))\Big)\Big)+\cdots \end{aligned} \tag{3.15}$$

Apply NIT to obtain:

$$\varphi(\zeta,t)=\frac{15\tanh^3(l(\zeta-\lambda))-45\tanh(l(\zeta-\lambda))}{19\sqrt{19}}+\xi+\frac{45lt^{\varrho}\mathrm{sech}^4(l(\zeta-\lambda))}{6859\Gamma(p+1)}\Big(\tanh(l(\zeta-\lambda))\Big(15\Big(152\sqrt{19}l^3$$
$$-1\Big)\mathrm{sech}^2(l(\zeta-\lambda))+76\sqrt{19}l\Big(1-16l^2\Big)-30\Big)+19\sqrt{19}\xi\Big)+\frac{90l^2\mathrm{sech}^4(lt^{2p}(\zeta-\lambda))}{2476099\Gamma(2p+1)}$$
$$\times\Big(-225\Big(5776\Big(1596\sqrt{19}l^3-13\Big)l^3+5\sqrt{19}\Big)\tanh(l(\zeta-\lambda))\mathrm{sech}^6(l(\zeta-\lambda))$$
$$-2888\Big(38\sqrt{19}l\Big(16l^2-1\Big)+15\Big)\xi+2\Big(5776l\Big(16l^2-1\Big)\Big(19\sqrt{19}l\Big(16l^2-1\Big)+15\Big)$$
$$+\sqrt{19}\Big(6859\xi^2+900\Big)\Big)\tanh(l(\zeta-\lambda))+7220\mathrm{sech}^2(l(\zeta-\lambda))\Big(\Big(19\sqrt{19}l\Big(52l^2-1\Big)+3\Big)\xi$$
$$-3l\Big(76l^2-1\Big)\Big(19\sqrt{19}l\Big(28l^2-1\Big)+6\Big)\tanh(l(\zeta-\lambda))\Big)-15\mathrm{sech}^4(l(\zeta-\lambda))\Big(2527\Big(152\sqrt{19}l^3-1\Big)\xi$$
$$-4\Big(361l\Big(112\Big(19\sqrt{19}l\Big(58l^2-1\Big)-2\Big)l^2+11\Big)-45\sqrt{19}\Big)\tanh(l(\zeta-\lambda))\Big)\Big)+\cdots \tag{3.16}$$

3.1.2. Problem 2

Consider the fractional KS equation is given as

$$D_t^{\varrho}\varphi(\zeta,t)+\varphi(\zeta,t)\frac{\partial}{\partial\zeta}\varphi(\zeta,t)+\frac{\partial^2}{\partial\zeta^2}\varphi(\zeta,t)+\frac{\partial^4}{\partial\zeta^4}\varphi(\zeta,t)=0, \quad \text{where} \quad 0<\varrho\le 1, \tag{3.17}$$

with the initial condition is

$$\varphi(\zeta,0)=\frac{15}{9}\sqrt{\frac{11}{9}}\Big(11\tanh^3(l(\zeta-\lambda))-9\tanh(l(\zeta-\lambda))\Big)+\xi, \tag{3.18}$$

and exact solution

$$\varphi(\zeta,t)=\frac{15}{9}\sqrt{\frac{11}{9}}\Big(11\tanh^3(l(-\lambda-\xi t+\zeta))-9\tanh(l(-\lambda-\xi t+\zeta))\Big)+\xi.$$

NT is applied to Eq (3.17), using Eq (3.18) to obtain

$$\varphi(\zeta,s)-\frac{u\frac{15}{9}\sqrt{\frac{11}{9}}\Big(11\tanh^3(l(\zeta-\lambda))-9\tanh(l(\zeta-\lambda))\Big)+\xi}{s}+\frac{u^{\varrho}}{s^{\varrho}}\mathcal{N}_t\Big[\mathcal{N}_t^{-1}\varphi(\zeta,s)\times\frac{\partial}{\partial\zeta}\mathcal{N}_t^{-1}\varphi(\zeta,s)\Big]$$
$$+\frac{u^{\varrho}}{s^{\varrho}}\Big[\frac{\partial^2}{\partial\zeta^2}\varphi(\zeta,s)\Big]+\frac{u^{\varrho}}{s^{\varrho}}\Big[\frac{\partial^4}{\partial\zeta^4}\varphi(\zeta,s)\Big]=0. \tag{3.19}$$

Consequently, the series which is k^{th}-truncated is:

$$\varphi(\zeta,s)=\frac{\frac{15}{9}\sqrt{\frac{11}{9}}\Big(11\tanh^3(l(\zeta-\lambda))-9\tanh(l(\zeta-\lambda))\Big)+\xi}{s^2}+\sum_{r=1}^{k}\frac{f_r(\zeta,s)}{s^{rp+1}},\quad r=1,2,3,4\cdots \tag{3.20}$$

The NRF is as follows:

$$\mathcal{N}_t Res(\zeta,s)=\varphi(\zeta,s)-\frac{\frac{15}{9}\sqrt{\frac{11}{9}}\Big(11\tanh^3(l(\zeta-\lambda))-9\tanh(l(\zeta-\lambda))\Big)+\xi}{s^2}+\frac{u^{\varrho}}{s^{\varrho}}\mathcal{N}_t\Big[\mathcal{N}_t^{-1}\varphi(\zeta,s)\times\frac{\partial}{\partial\zeta}\mathcal{N}_t^{-1}\varphi(\zeta,s)\Big]+\frac{u^{\varrho}}{s^{\varrho}}\Big[\frac{\partial^2}{\partial\zeta^2}\varphi(\zeta,s)\Big]+\frac{u^{\varrho}}{s^{\varrho}}\Big[\frac{\partial^4}{\partial\zeta^4}\varphi(\zeta,s)\Big]=0, \tag{3.21}$$

and the k^{th}-NRFs are:

$$\mathcal{N}_t Res_k(\zeta,s)=\varphi_k(\zeta,s)-\frac{\frac{15}{9}\sqrt{\frac{11}{9}}\Big(11\tanh^3(l(\zeta-\lambda))-9\tanh(l(\zeta-\lambda))\Big)+\xi}{s^2}+\frac{u^{\varrho}}{s^{\varrho}}\mathcal{N}_t\Big[\mathcal{N}_t^{-1}\varphi_k(\zeta,s)\times\frac{\partial}{\partial\zeta}\mathcal{N}_t^{-1}\varphi_k(\zeta,s)\Big]+\frac{u^{\varrho}}{s^{\varrho}}\Big[\frac{\partial^2}{\partial\zeta^2}\varphi_k(\zeta,s)\Big]+\frac{u^{\varrho}}{s^{\varrho}}\Big[\frac{\partial^4}{\partial\zeta^4}\varphi_k(\zeta,s)\Big]=0. \tag{3.22}$$

Determining $f_r(\zeta,s)$ for $r=1,2,3,...$ needs some computation. Follow these steps, take the r^{th}-NRF Eq (3.22), and put it for the r^{th}-truncated series Eq (3.20); I solve $\lim_{s\to\infty}(s^{r\varrho+1})$ by multiplying the final equation by $s^{r\varrho+1}$. $\mathcal{N}_t Res_{\varphi,r}(\zeta,s))=0$, and $r=1,2,3,\cdots$. Few terms are obtained through the proposed method is as follows:

$$f_1(\zeta,s)=\frac{5}{27}l\,\text{sech}^2(l(\zeta-\lambda))\Big(16\Big(9\sqrt{11}l\Big(4l^2+1\Big)-55\Big)\tanh(l(\zeta-\lambda))+\text{sech}^2(l(\zeta-\lambda))\Big(\tanh(l(\zeta-\lambda))\Big(55\Big(216\sqrt{11}l^3-121\Big)\text{sech}^2(l(\zeta-\lambda))-36\sqrt{11}l\Big(224l^2+11\Big)+6050\Big)+99\sqrt{11}\xi\Big)-72\sqrt{11}\xi\Big), \tag{3.23}$$

$$f_2(\zeta,s)=\frac{10}{243}l^2\text{sech}^2(l(\zeta-\lambda))\Big(-8\Big(36l\Big(4l^2+1\Big)\Big(9\sqrt{11}l\Big(4l^2+1\Big)-110\Big)+\sqrt{11}\Big(81\xi^2+1100\Big)\Big)\tanh(l(\zeta-\lambda))+2\text{sech}^2(l(\zeta-\lambda))\Big(\Big(144l\Big(2l\Big(4l\Big(9\sqrt{11}l\Big(478l^2+59\Big)-4565\Big)+63\sqrt{11}\Big)-2695\Big)+11\sqrt{11}\Big(81\xi^2+10700\Big)\Big)\tanh(l(\zeta-\lambda))-36\Big(36\sqrt{11}l\Big(118l^2+7\Big)-3355\Big)\xi\Big)+180\text{sech}^4(l(\zeta-\lambda))\Big(\Big(9\sqrt{11}l\Big(620l^2+11\Big)-3509\Big)\xi-\Big(3l\Big(l\Big(8l\Big(9\sqrt{11}l\Big(3262l^2+155\Big)-24299\Big)+99\sqrt{11}\Big)-5566\Big)+8470\sqrt{11}\Big)\tanh(l(\zeta-\lambda))\Big)+288\Big(9\sqrt{11}l\Big(4l^2+1\Big)-55\Big)\xi-275\Big(979776\sqrt{11}l^6-679536l^3+6655\sqrt{11}\Big)\tanh(l(\zeta-\lambda))\text{sech}^8(l(\zeta-\lambda))-5\text{sech}^6(l(\zeta-\lambda))\Big(693\Big(216\sqrt{11}l^3-121\Big)\xi-4\Big(9l\Big(16l^2\Big(189\sqrt{11}l\Big(662l^2+11\Big)-92686\Big)-14641\Big)+153065\sqrt{11}\Big)\tanh(l(\zeta-\lambda))\Big)\Big), \tag{3.24}$$

and so on.

Put $f_r(\zeta, s)$, for $r = 1, 2, 3, \cdots$, in Eq (3.20):

$$\begin{aligned}
\varphi(\zeta, s) = & \frac{\frac{15}{9}\sqrt{\frac{11}{9}}\Big(11\tanh^3(l(\zeta-\lambda)) - 9\tanh(l(\zeta-\lambda))\Big) + \xi}{s^2} \\
& + \frac{5}{27s^{p+1}} l\mathrm{sech}^2(l(\zeta-\lambda))\Big(16\Big(9\sqrt{11}l\Big(4l^2+1\Big)-55\Big)\tanh(l(\zeta-\lambda)) + \mathrm{sech}^2(l(\zeta-\lambda))\Big(\tanh(l(\zeta \\
& - \lambda))\Big(55\Big(216\sqrt{11}l^3-121\Big)\mathrm{sech}^2(l(\zeta-\lambda)) - 36\sqrt{11}l\Big(224l^2+11\Big)+6050\Big)+99\sqrt{11}\xi\Big)-72\sqrt{11}\xi\Big) \\
& + \frac{10}{243s^{2p+1}} l^2\mathrm{sech}^2(l(\zeta-\lambda))\Big(-8\Big(36l\Big(4l^2+1\Big)\Big(9\sqrt{11}l\Big(4l^2+1\Big)-110\Big)+\sqrt{11}\Big(81\xi^2 \\
& + 1100\Big)\Big)\tanh(l(\zeta-\lambda)) + 2\mathrm{sech}^2(l(\zeta-\lambda))\Big(\Big(144l\Big(2l\Big(4l\Big(9\sqrt{11}l\Big(478l^2+59\Big)-4565\Big)+63\sqrt{11}\Big) \\
& - 2695\Big)+11\sqrt{11}\Big(81\xi^2+10700\Big)\Big)\tanh(l(\zeta-\lambda)) - 36\Big(36\sqrt{11}l\Big(118l^2+7\Big)-3355\Big)\xi\Big) \\
& + 180\mathrm{sech}^4(l(\zeta-\lambda))\Big(\Big(9\sqrt{11}l\Big(620l^2+11\Big)-3509\Big)\xi - \Big(3l\Big(l\Big(8l\Big(9\sqrt{11}l\Big(3262l^2+155\Big)-24299\Big) \\
& + 99\sqrt{11}\Big)-5566\Big)+8470\sqrt{11}\Big)\tanh(l(\zeta-\lambda))\Big) + 288\Big(9\sqrt{11}l\Big(4l^2+1\Big)-55\Big)\xi - 275\Big(979776\sqrt{11}l^6 \\
& - 679536l^3+6655\sqrt{11}\Big)\tanh(l(\zeta-\lambda))\mathrm{sech}^8(l(\zeta-\lambda)) - 5\mathrm{sech}^6(l(\zeta-\lambda))\Big(693\Big(216\sqrt{11}l^3-121\Big)\xi \\
& - 4\Big(9l\Big(16l^2\Big(189\sqrt{11}l\Big(662l^2+11\Big)-92686\Big)-14641\Big)+153065\sqrt{11}\Big)\tanh(l(\zeta-\lambda))\Big)\Big) + \cdots
\end{aligned} \tag{3.25}$$

Apply NIT to obtain:

$$\begin{aligned}
\varphi(\zeta, t) = & \frac{15}{9}\sqrt{\frac{11}{9}}\Big(11\tanh^3(l(\zeta-\lambda)) - 9\tanh(l(\zeta-\lambda))\Big) + \xi \\
& + \frac{5t^{\varrho}}{27\Gamma(p+1)} l\mathrm{sech}^2(l(\zeta-\lambda))\Big(16\Big(9\sqrt{11}l\Big(4l^2+1\Big)-55\Big)\tanh(l(\zeta-\lambda)) + \mathrm{sech}^2(l(\zeta-\lambda))\Big(\tanh(l(\zeta \\
& - \lambda))\Big(55\Big(216\sqrt{11}l^3-121\Big)\mathrm{sech}^2(l(\zeta-\lambda)) - 36\sqrt{11}l\Big(224l^2+11\Big)+6050\Big)+99\sqrt{11}\xi\Big)-72\sqrt{11}\xi\Big) \\
& + \frac{10t^{2p+1}}{243s\Gamma(2p+1)} l^2\mathrm{sech}^2(l(\zeta-\lambda))\Big(-8\Big(36l\Big(4l^2+1\Big)\Big(9\sqrt{11}l\Big(4l^2+1\Big)-110\Big)+\sqrt{11}\Big(81\xi^2 \\
& + 1100\Big)\Big)\tanh(l(\zeta-\lambda)) + 2\mathrm{sech}^2(l(\zeta-\lambda))\Big(\Big(144l\Big(2l\Big(4l\Big(9\sqrt{11}l\Big(478l^2+59\Big)-4565\Big)+63\sqrt{11}\Big) \\
& - 2695\Big)+11\sqrt{11}\Big(81\xi^2+10700\Big)\Big)\tanh(l(\zeta-\lambda)) - 36\Big(36\sqrt{11}l\Big(118l^2+7\Big)-3355\Big)\xi\Big) \\
& + 180\mathrm{sech}^4(l(\zeta-\lambda))\Big(\Big(9\sqrt{11}l\Big(620l^2+11\Big)-3509\Big)\xi - \Big(3l\Big(l\Big(8l\Big(9\sqrt{11}l\Big(3262l^2+155\Big)-24299\Big) \\
& + 99\sqrt{11}\Big)-5566\Big)+8470\sqrt{11}\Big)\tanh(l(\zeta-\lambda))\Big) + 288\Big(9\sqrt{11}l\Big(4l^2+1\Big)-55\Big)\xi - 275\Big(979776\sqrt{11}l^6 \\
& - 679536l^3+6655\sqrt{11}\Big)\tanh(l(\zeta-\lambda))\mathrm{sech}^8(l(\zeta-\lambda)) - 5\mathrm{sech}^6(l(\zeta-\lambda))\Big(693\Big(216\sqrt{11}l^3-121\Big)\xi \\
& - 4\Big(9l\Big(16l^2\Big(189\sqrt{11}l\Big(662l^2+11\Big)-92686\Big)-14641\Big)+153065\sqrt{11}\Big)\tanh(l(\zeta-\lambda))\Big)\Big) + \cdots
\end{aligned} \tag{3.26}$$

4. Discussion of tables and figures

A comparison of fractional-order solutions named $\Phi(\zeta, t)$ emerges from using our proposed method and the Natural transform decomposition method (NTDM) [35] on Problem 1 appears in

Table 1. Our method shows exceptional accuracy and efficiency for different fractional orders throughout the solution process. The visual representation of Figure 1 displays subfigure (a) showing the RPSTM solution when $\varrho = 1$ and subfigure (b) showing the exact solution. Our methodology demonstrates reliability through the matching results between the obtained solutions. The fractional-order comparisons of parameter $\varphi(\zeta, t)$ appear in Figure 2 while using various values of ϱ using RPSTM. The different results show that fractional parameters affect the solution dynamics in specific ways. The Figure 3 presents both the 3D and 2D graphical representations of the fractional-order comparison which includes different values of ϱ. The representation effectively illustrates the temporal modifications of $\varphi(\zeta, t)$ which provide data about fractional-order effects. The research in Table 2 expands the analysis of fractional orders across Problem 2 through a comparison between our proposed method and NTDM results from [35]. The findings reaffirm the superior accuracy and applicability of our approach. The RPSTM solution of Problem 2 exists in Figure 4, showing the calculated solution for the subfigure (a) shows the computed solution when ϱ equals one whereas subfigure (b) presents the exact numerical solution. The matching visual presentations between laboratory solutions supports the performance of RPSTM as an effective method. The fractional-order graph displaying the function $\varphi(\zeta, t)$ appears in Figure 5 while changing its ϱ value in Problem 2. The shown figure illustrates how different fractional orders transform the solutions' characteristics. The figure named Figure 6 contains 3D as well as 2D visual representations of fractional-order comparisons running from different ϱ values using RPSTM in Problem 2. The visual presentations illustrate how fractional parameters heavily affect the solution dynamics while showing a complete overview of the solution composition. Our proposed solution method shows high precision alongside operational speed and operational versatility within fractional-order problem domains according to the presented tables and figures.

Table 1. Fractional order comparison of $\varphi(\zeta, t)$ of our proposed method with the NTDM [35] for Problem 1.

t	ζ	$RPSTM$ $\varrho = 0.48$	$RPSTM$ $\varrho = 0.68$	$RPSTM$ $\varrho = 1.00$	$Exact$	$RPSTM$ $Error_{\varrho=1.00}$	$NTDM_{CF}$ [35] $Error_{\varrho=1.00}$	$NTDM_{ABC}$ [35] $Error_{\varrho=1.00}$
0.1	0.2	0.00994769	0.00994691	0.00994621	0.00994621	2.901441×10^{-9}	5.2357898×10^{-9}	5.2357898×10^{-9}
	0.4	0.00983899	0.00983823	0.00983753	0.00983754	8.762935×10^{-9}	5.1930879×10^{-9}	5.1930879×10^{-9}
	0.6	0.00973030	0.00972954	0.00972885	0.00972887	1.462442×10^{-8}	4.1981581×10^{-9}	4.1981581×10^{-9}
	0.8	0.00962161	0.00962086	0.00962018	0.00962020	2.048590×10^{-8}	4.2586011×10^{-9}	4.2586011×10^{-9}
	1.0	0.00951292	0.00951218	0.00951150	0.00951153	2.634736×10^{-8}	3.7555413×10^{-9}	3.7555413×10^{-9}
0.3	0.2	0.00994909	0.00994830	0.00994729	0.00994729	8.529431×10^{-9}	4.9107369×10^{-8}	4.9107369×10^{-8}
	0.4	0.00984038	0.00983960	0.00983860	0.00983862	2.611579×10^{-8}	4.5079263×10^{-8}	4.5079263×10^{-8}
	0.6	0.00973167	0.00973090	0.00972991	0.00972995	4.370214×10^{-8}	4.0994474×10^{-8}	4.0994474×10^{-8}
	0.8	0.00962297	0.00962220	0.00962122	0.00962128	6.128846×10^{-8}	3.7675803×10^{-8}	3.7675803×10^{-8}
	1.0	0.00951426	0.00951350	0.00951253	0.00951261	7.887474×10^{-8}	3.4266623×10^{-8}	3.4266623×10^{-8}
0.5	0.2	0.00995004	0.00994939	0.00994837	0.00994838	1.392422×10^{-8}	1.3857894×10^{-7}	1.3857894×10^{-7}
	0.4	0.00984132	0.00984068	0.00983967	0.00983971	4.323797×10^{-8}	1.2686543×10^{-7}	1.2686543×10^{-7}
	0.6	0.00973261	0.00973197	0.00973097	0.00973104	7.255169×10^{-8}	1.1569079×10^{-7}	1.1569079×10^{-7}
	0.8	0.00962389	0.00962326	0.00962227	0.00962237	1.018653×10^{-7}	1.0559300×10^{-7}	1.0559300×10^{-7}
	1.0	0.00951517	0.00951455	0.00951357	0.00951370	1.311789×10^{-7}	9.6577706×10^{-8}	9.6577706×10^{-8}

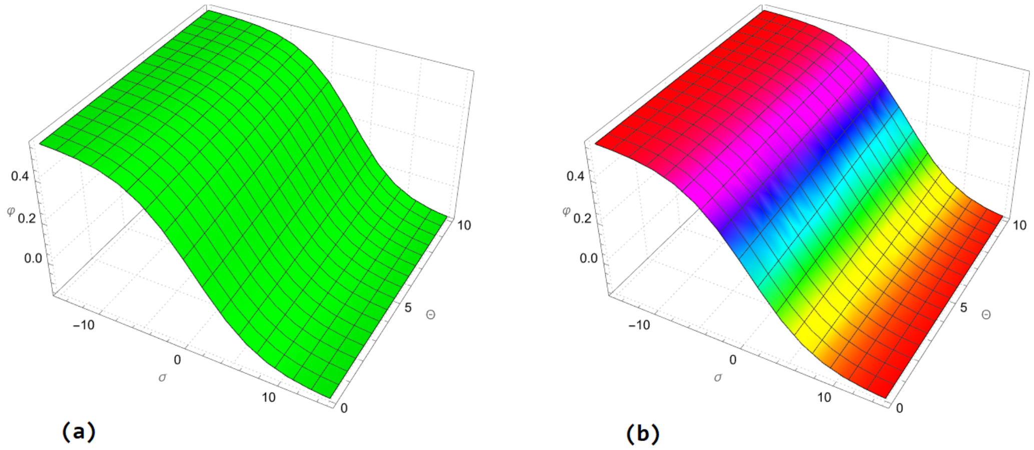


Figure 1. (a) Shows RPSTM solution for $\varrho = 1$ and (b) shows Exact solution.

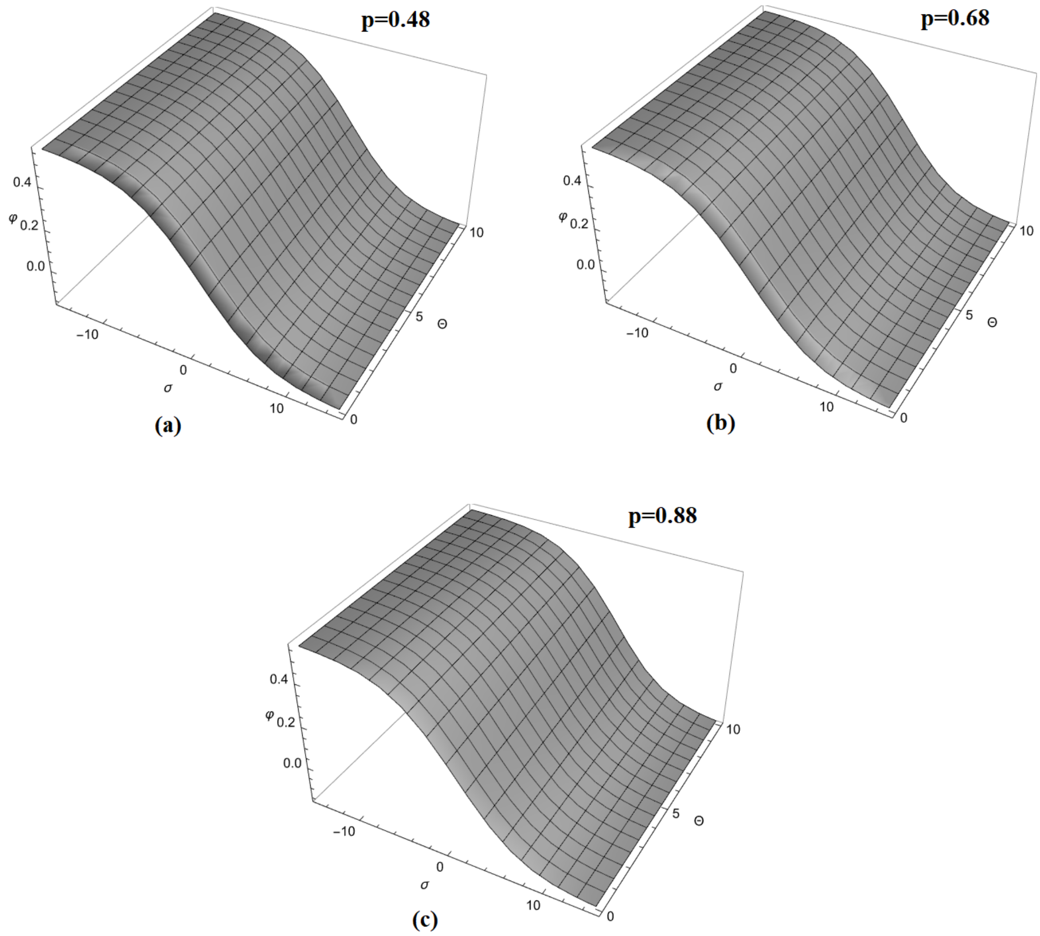
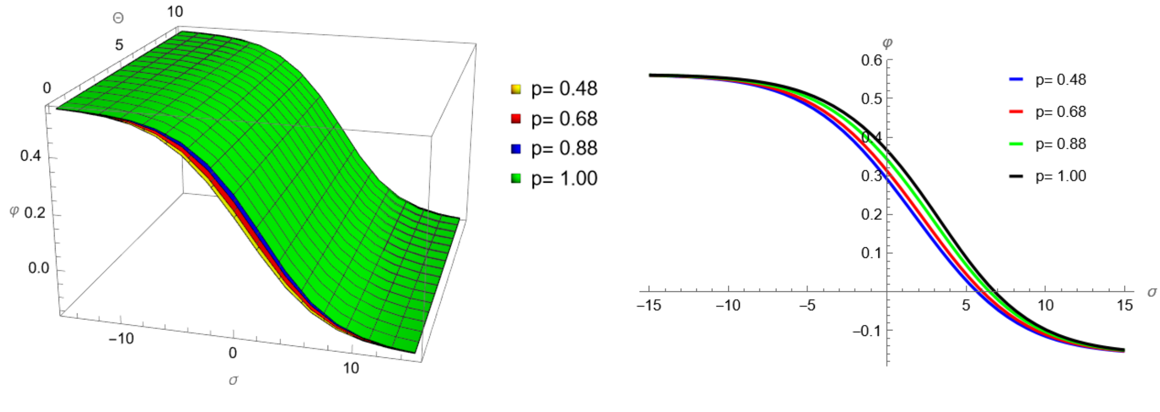


Figure 2. Fractional order comparison for various values of ϱ for $\varphi(\zeta, t)$ using RPSTM.



(a) (b)

Figure 3. 3D and 2D fractional order comparison for various values of ϱ for $\varphi(\zeta, t)$ using RPSTM.

Table 2. Fractional order comparison of $\varphi(\zeta, t)$ of our proposed method with the NTDM [35] for Problem 2.

t	ζ	$RPSTM$ $\varrho = 0.48$	$RPSTM$ $\varrho = 0.68$	$RPSTM$ $\varrho = 1.00$	$Exact$	$RPSTM$ $Error_{\varrho=1.00}$	$NTDM_{CF}$ [35] $Error_{\varrho=1.00}$	$NTDM_{ABC}$ [35] $Error_{\varrho=1.00}$
	0.2	0.00984027	0.00983794	0.00983580	0.00983583	2.723109×10^{-8}	6.87047039×10^{-8}	6.87047039×10^{-8}
	0.4	0.00950840	0.00950615	0.00950408	0.00950416	8.224331×10^{-8}	6.45480984×10^{-8}	6.45480984×10^{-8}
0.1	0.6	0.00917653	0.00917436	0.00917236	0.00917250	1.372555×10^{-7}	5.20957156×10^{-8}	5.20957156×10^{-8}
	0.8	0.00884467	0.00884257	0.00884065	0.00884084	1.922677×10^{-7}	4.59658139×10^{-8}	4.59658139×10^{-8}
	1.0	0.00851280	0.00851078	0.00850893	0.00850918	2.472799×10^{-7}	3.88305434×10^{-8}	3.88305434×10^{-8}
	0.2	0.00984452	0.00984212	0.00983906	0.00983914	8.007055×10^{-8}	1.19114111×10^{-7}	1.19114111×10^{-7}
	0.4	0.00951250	0.00951019	0.00950724	0.00950748	2.451619×10^{-7}	1.06644297×10^{-7}	1.06644297×10^{-7}
0.3	0.6	0.00918049	0.00917826	0.00917541	0.00917582	4.102533×10^{-7}	8.62871468×10^{-8}	8.62871468×10^{-8}
	0.8	0.00884848	0.00884633	0.00884358	0.00884416	5.753446×10^{-7}	7.38974440×10^{-8}	7.38974440×10^{-8}
	1.	0.008516470	0.00851440	0.00851175	0.00851249	7.404360×10^{-7}	6.34916304×10^{-8}	6.34916304×10^{-8}
	0.2	0.00984740	0.00984543	0.00984233	0.00984246	1.307463×10^{-7}	9.44764808×10^{-8}	9.44764808×10^{-8}
	0.4	0.00951529	0.00951339	0.00951039	0.00951080	4.059899×10^{-7}	8.12595049×10^{-8}	8.12595049×10^{-8}
0.5	0.6	0.00918318	0.00918135	0.00917845	0.00917914	6.812334×10^{-7}	6.65214220×10^{-8}	6.65214220×10^{-8}
	0.8	0.00885107	0.00884931	0.00884652	0.00884747	9.564769×10^{-7}	5.91709303×10^{-8}	5.91709303×10^{-8}
	1.0	0.00851896	0.00851726	0.00851458	0.00851581	1.231720×10^{-6}	4.98472826×10^{-8}	4.98472826×10^{-8}

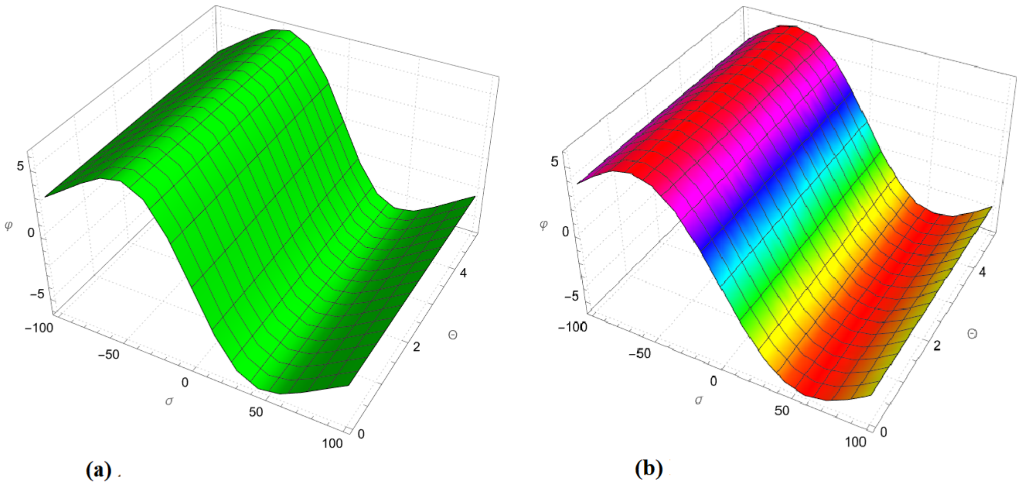


Figure 4. (a) Shows RPSTM solution for $\varrho = 1$ and (b) shows Exact solution.

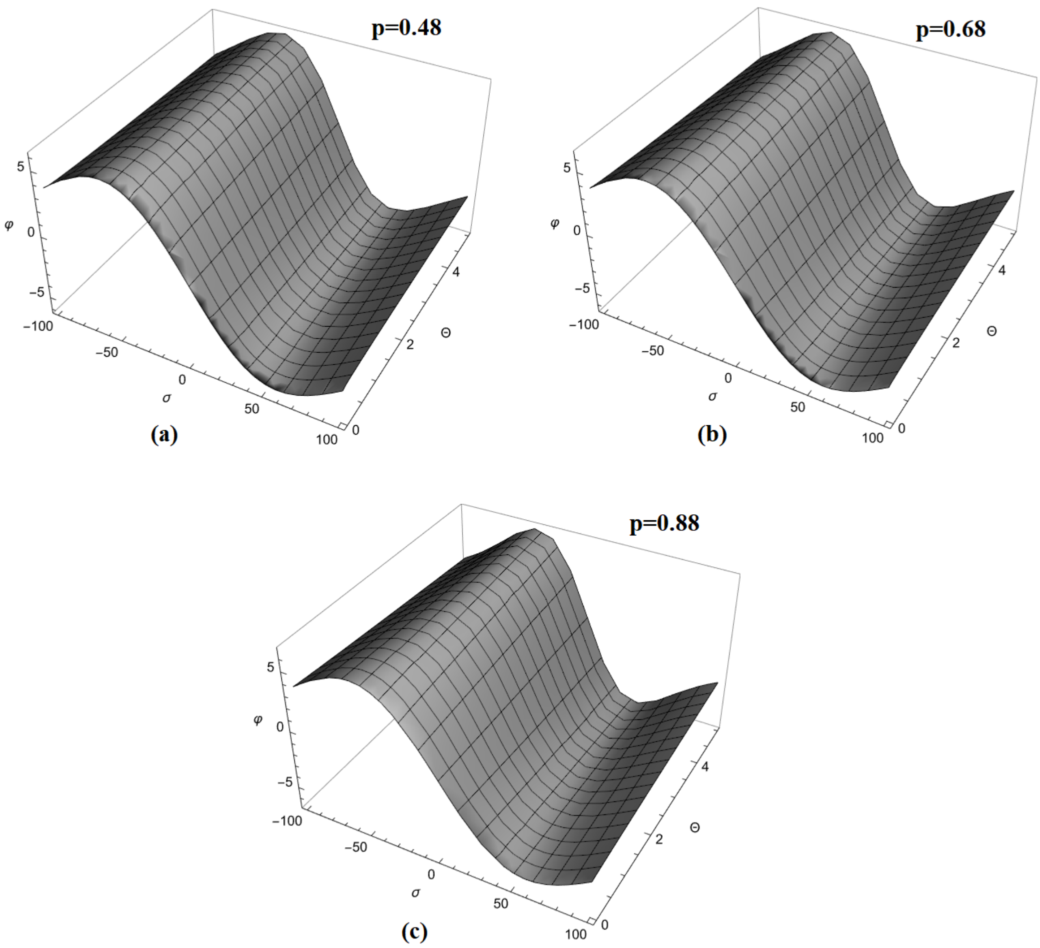


Figure 5. Fractional order comparison for various values of ϱ for $\varphi(\zeta, t)$ using RPSTM.

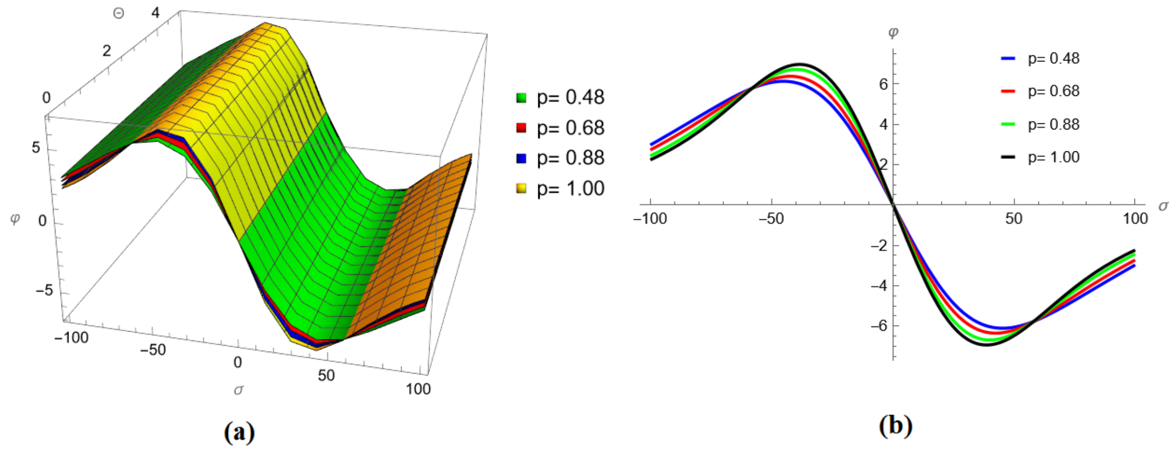


Figure 6. 3D and 2D fractional order comparison for various values of ϱ for $\varphi(\zeta, t)$ using RPSTM.

5. Conclusions

In this study, we successfully applied the residual power series transform method RPSTM to obtain analytical solutions for the fractional KS equation, incorporating the Caputo fractional operator. The proposed method demonstrated its effectiveness in handling the complexities of FNLPDEs, providing a systematic and efficient approach for deriving approximate solutions. The convergence analysis confirmed the reliability of the RPSTM, while numerical simulations illustrated its accuracy and applicability in capturing the dynamics of the fractional KS equation. The results highlight the potential of the RPSTM as a powerful analytical tool for solving a wide range of fractional differential equations, particularly those arising in nonlinear science and engineering. By extending the classical KS equation to the fractional domain, this work contributes to a deeper understanding of systems exhibiting memory effects and anomalous behavior. Future research could explore the application of this method to other fractional models and investigate its computational efficiency for more complex systems. Overall, this study underscores the significance of fractional calculus and advanced analytical techniques in addressing real-world problems.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that they have no conflicts of interest.

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