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Research article

Artificial intelligence-based intelligent computing using circular q-rung orthopair fuzzy information aggregation

Jian Qi*

The Department of Digital Information, Hebei Institute of International Business and Economics, Qinhuangdao, Hebei Province, 066311, China

* **Correspondence:** Email: qijian@hbiibe.edu.cn.

Abstract: Artificial Intelligence (AI) based computing techniques play a transformative role in enhancing the capabilities of modern computing systems by enabling them to learn, adapt, optimize, and make decisions autonomously. These techniques are applied across various fields to improve performance, reduce human effort, and solve complex problems more efficiently. In this article, we explored a potent approach of the circular q-rung orthopair fuzzy for coping with uncertain and vague type information in life-life dilemmas because it covers extensive information in the form of degree of membership value, degree of non-membership value, and radius among both membership functions. Some flexible operations of Frank t-norms were formulated under the circular q-rung orthopair fuzzy (Crq-ROF) context. Based on these operations, we developed novel approaches of circular q-rung orthopair fuzzy Frank weighted average (Crq-ROFFWA) and circular q-rung orthopair fuzzy Frank weighted geometric (Crq-ROFFWG) operators with dominant properties. Additionally, we modified a novel theory of evaluation based on distance from the average solution (EDAS) method for the multi-attribute decision-making (MADM) problem. Later, we discussed an experimental case study related to artificial intelligence for measuring the performance of intelligent computing techniques. Using a numerical example, we explored the worth and compatibility of discussed methodologies and decision support systems. Finally, utilizing a comparison technique, we identified the supremacy and effectiveness of proposed theories.

Keywords: circular q-rung orthopair fuzzy sets; frank triangular norms; artificial intelligence; decision support system
Mathematics Subject Classification: 94D05, 68U35, 68T27

1. Introduction

Artificial intelligence (AI)-based intelligent computing techniques have become a driving force across industries, playing a transformative role in automation, decision-making, optimization, and innovation. These techniques enable machines to simulate human intelligence and autonomously improve system performance, revolutionizing how complex problems are solved. AI techniques also enhance personalization and user-centric experiences by tailoring services to individual user preferences and behaviors [1]. Moreover, predictive maintenance and risk management is another vital role where AI helps foresee issues before they occur, preventing costly equipment breakdowns and minimizing risks. In manufacturing, predictive maintenance systems using AI analyze sensor data to detect potential failures and perform timely interventions. AI-based intelligent computing systems are also adaptive, utilizing techniques like reinforcement learning to continuously learn and self-optimize, improving performance over time, as seen in self-driving cars that adjust to changing environments [2].

AI-based techniques also improve security and fraud detection through anomaly detection and pattern recognition, which are widely used in cybersecurity and finance to detect and respond to threats or fraudulent activities in real time [3]. For example, AI helps banks identify unusual transaction patterns, enhancing fraud detection capabilities. Additionally, decision support systems (DSS) rely on AI to provide intelligent recommendations and risk analyses, assisting decision-makers in complex scenarios like medical diagnosis or financial planning. Finally, the scalability and adaptability of AI-based intelligent computing techniques enable them to be applied to large-scale systems, such as smart cities and IoT networks [4,5]. These systems manage and process vast amounts of data in real time, optimizing infrastructure such as traffic flow and energy usage to improve public services. Figure 1 shows the characteristics of AI in computing strategies.

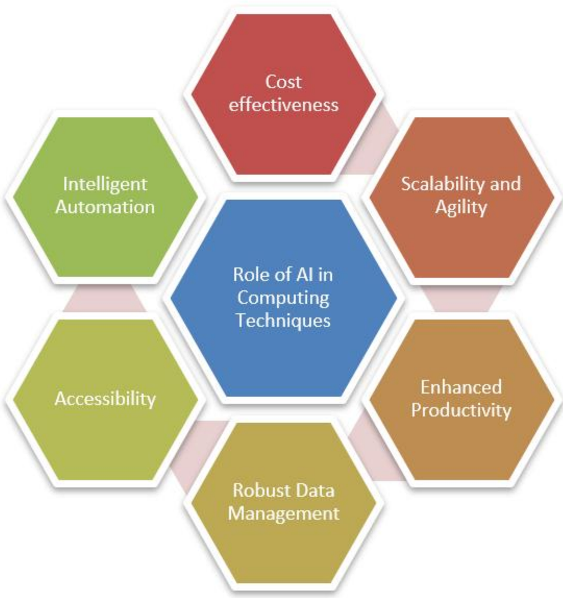

Figure 1. Characteristics of AI-based computing techniques.

Decision-making and AI are intricately connected, as AI technologies are designed to enhance and automate decision-making processes across domains. In traditional decision-making, human judgment relies on experience, intuition, and analysis of available data, often limited by cognitive biases and time constraints. AI, however, leverages algorithms, machine learning, and vast data processing capabilities to provide more accurate, data-driven decisions. AI systems can identify patterns, predict outcomes, and recommend optimal actions, thus reducing human error and improving efficiency. AI-based decision support systems offer predictive insights and automate repetitive tasks in healthcare, finance, and logistics.

The significance of the MADM problem in real-life situations lies in its ability to provide a structured approach for handling complex decisions involving multiple conflicting criteria. In real-world scenarios, such as business, healthcare, public policy, and resource management, decision-makers must evaluate multiple factors simultaneously. Moreover, the MADM enhances decision-making under uncertainty and supports collaboration in group decision-making contexts. This enables stakeholders to work together and reach a consensus by providing a common framework to evaluate alternatives. Additionally, MADM offers transparency and justification in decision processes, making explaining and defending decisions to stakeholders easier. This is particularly valuable in public policy and corporate governance sectors, where clear, data-driven decision-making is essential.

1.1. Literature review

In real-world decision-making, many problems involve ambiguity and vagueness. To handle such situations, Zadeh [6] developed a theory of fuzzy set (FS) by exploring classical set theory. The FS plays a critical role in decision-making problems by providing a way to handle uncertainty, vagueness, and imprecision in situations where crisp, binary decisions are not sufficient. In decision-making, FSs help model uncertainty and enable a more nuanced analysis of alternatives by incorporating linguistic terms like ‘‘high’’, ‘‘medium’’, and ‘‘low’’ into the decision process. This flexibility leads to more realistic and practical solutions, especially in complex situations of the MADM problems. Later, Atanassov [7] proposed an efficient technique of an intuitionistic fuzzy set (IFS) with two grades called the DoMV $\mu \in [0,1]$ and DoNMV $v \in [0,1]$ with the condition $0 \le \mu + v \le 1$. The IFS is a dominant and flexible model because of its wide range, whereas the FS is a fixed case of an IFS. Many research scholars have applied the theory of IFSs to complicated real-life applications and decision-making problems. Sometimes, when the grades of DoMV or DoNMV, such as $(0.65, 0.54)$ then $0.65 + 0.54 = 1.19 \notin [0,1]$. To handle such a situation, Yager [8] initiated the concepts of a pythagorean fuzzy set (PyFS) with the relaxed condition of the sum of the square of the DoMV and DoNMV bounded on a closed interval $[0,1]$. Yager [9] extended the concepts of a q-rung orthopair fuzzy set (q-ROFS). The mathematical shape of the q-ROFS is expressed as $0 \le \mu^{\tau} + v^{\tau} \le 1$. Several strategies and mathematical approaches to q-ROFS exist in the literature. The q-ROFS gained a lot of attention from research scholars and became a hot research framework to cope with uncertain judgments of experts or decision-makers.

However, Atanassov [10] introduced an extended version of IFS called circular IFS (Cr-IFS). The idea of Cr-IFS is more convenient and effective than the theory of FSs, IFSs, PyFSs, and q-ROFSs. A Cr-IFS has three grades such as DoMV, DoNMV, and a radius among them. Bozyigit et al. [11] initiated a new theory of circular (Cr-PyFS). After that, Yusoff et al. [12] proposed a dominant concept of circular q-ROFS (Crq-ROFS). The Crq-ROFS is a reliable and potent approach of fuzzy theory used to resolve a lot of complicated real-life applications and numerical examples. With the passage of time, the above-discussed theories have gained more attention from mathematicians and research scientists. For instance, Xu [13] discussed new distance measures for Cr-IFS and decision support systems. Chen [14] elaborated the theory of simple arithmetic mean to derive new mathematical approaches to Cr-IFSs and robust decision-making techniques. Alsattar et al. [15] assessed some sustainable smart living apartments considering the theory of Cr-PyFSs. For more understanding of the discussed terminologies of FSs, we provide the features and conditions of all discussed fuzzy terminologies in Table 1 and Figure 2.

Table 1. A detailed overview of discussed fuzzy methodologies.

Frameworks	Conditions	Descriptions
FS by Zadeh **[6]**	$0 \leq \mu(\mathrm{p}) \leq 1$	$\{(\mathrm{p}, \mu(\mathrm{p}))\|\mathrm{p} \in E\}$
IFS by Atanassov **[7]**	$0 \leq \mu(\mathrm{p}) + v(\mathrm{p}) \leq 1$	$\{(\mathrm{p}, (\mu(\mathrm{p}), v(\mathrm{p})))\|\mathrm{p} \in E\}$
PyFS by Yager **[8]**	$0 \leq \mu^2(\mathrm{p}) + v^2(\mathrm{p}) \leq 1$	$\{(\mathrm{p}, (\mu(\mathrm{p}), v(\mathrm{p})))\|\mathrm{p} \in E\}$
q-ROFS by Yager **[9]**	$0 \leq \mu^\tau(\mathrm{p}) + v^\tau(\mathrm{p}) \leq 1, \tau \geq 1$	$\{(\mathrm{p}, (\mu(\mathrm{p}), v(\mathrm{p})))\|\mathrm{p} \in E\}$
Cr-IFS by Atanassov **[10]**	$0 \leq \mu(\mathrm{p}) + v(\mathrm{p}) \leq 1$	$\{(\mathrm{p}, (\mu(\mathrm{p}), v(\mathrm{p}); r(\mathrm{p})))\|\mathrm{p} \in E\}$
Cr-PyFS by Bozyigit et al. **[11]**	$0 \leq \mu^2(\mathrm{p}) + v^2(\mathrm{p}) \leq 1$	$\{(\mathrm{p}, (\mu(\mathrm{p}), v(\mathrm{p}); r(\mathrm{p})))\|\mathrm{p} \in E\}$
Crq-ROFS by Yusoff et al. **[12]**	$0 \leq \mu^\tau(\mathrm{p}) + v^\tau(\mathrm{p}) \leq 1, \tau \geq 1$	$\{(\mathrm{p}, (\mu(\mathrm{p}), v(\mathrm{p}); r(\mathrm{p})))\|\mathrm{p} \in E\}$

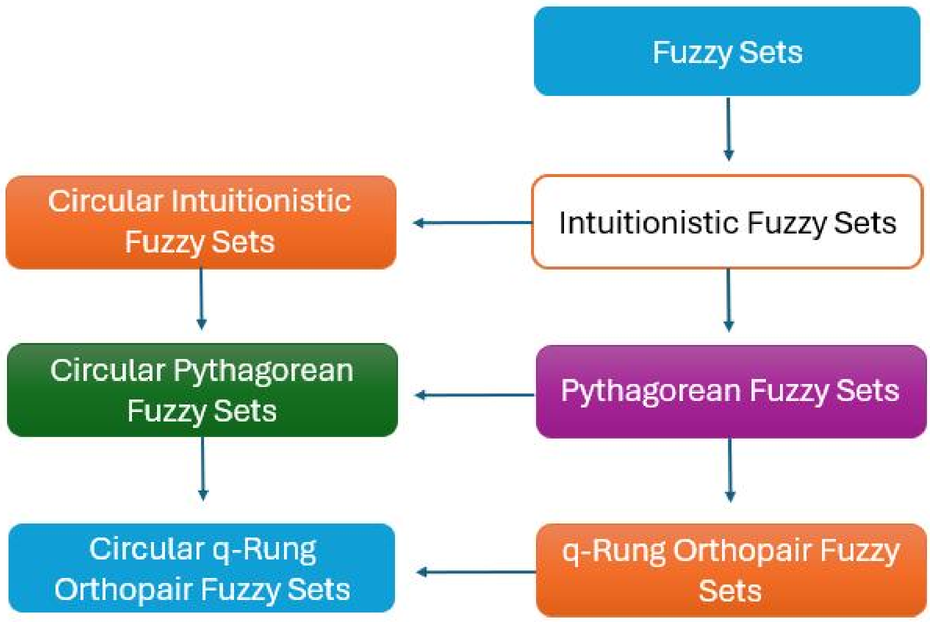

Figure 2. Extension of fuzzy frameworks.

The aggregation operators and mathematical terminologies play an effective role in the decision-analysis process. We studied theories about many mathematical approaches and terminologies. For example, Ahmmad [16] utilized entropy measures to investigate the unknown degree of weights for classifying renewable energy sources. Bibi and Ali [17] exposed the theory of Neutrosophic fuzzy rough sets to resolve decision analysis applications. Hussain et al. [18] applied properties of Schweizer-Sklar t-norms to derive aggregation operators of Maclaurin symmetric mean models and decision-making processes. Hussain et al. [19] developed aggregation operators of a picture fuzzy framework to compute the degree of criterion and reliable optimal option. A family of Aczel Alsina aggregation models was developed by Hussain et al. [20]. An application related to digital security systems based on Sugeno-Weber aggregation operators was discussed by Hussain et al. [21]. Hussain et al. [22] exposed aggregation operators of Dombi Hamy mean models for exploring the theory of decision analysis problems. Hussain et al. [23] evaluated construction materials under some characteristics and derived Aczel Alsina aggregation operators. Hussain et al. [24] expanded the properties of Heronian mean models considering the theory of a t-spherical fuzzy framework. Senapati [25] introduced a robust ranking method based on a decision analysis process. Senapati and Yager [26] proposed a family of convincing mathematical approaches of weighted averaging operators taking into account Farmateen fuzzy situations. Senapati et al. [27] developed geometric aggregation models for intuitionistic fuzzy situations and decision-making applications. Garg [28] stated theories of exponential-logarithm and q-rung orthopair fuzzy domains. Garg [29] modified concepts of single-valued Neutrosophic sets with properties of exponential-logarithms. Garg [30] exposed a robust theory of Linguistic interval-valued pythagorean fuzzy fields and decision-making models. We also studied different mathematical approaches and decision-making terminologies in the literature [31–33].

Riaz and Farid [34] investigated reliable green supply chain enterprises using linear Diophantine fuzzy soft sets and the decision algorithm of the MADM problem. Riaz et al. [35] evaluated some appropriate third-party logistic providers under consideration of prioritized operators and q-rung orthopair fuzzy situations. Riaz et al. [36] exposed the theory of the VIKOR method for the q-ROFSs and real-life applications. Saeed and Shafique [37] investigated some sustainable agricultural techniques using relations between Farmatean and Neutrosophic soft theory. Saeed et al. [38] defined the properties of refined pythagorean fuzzy theory and formulated some axiomatic results. Zulqarnain et al. [39] enhanced the theory of the TOPSIS method under the concepts of an intuitionistic fuzzy environment. Ihsan et al. [40] integrated some robust mathematical approaches for evaluating human resource management enterprises and advanced optimization techniques of the TOPSIS method. Akram and Ali [41] demonstrated hybrid mathematical models of pythagorean fuzzy bipolar soft sets for handling ambiguous information of human opinions. Another extended version of the decision-making technique of the CRITIC-EDAS method was developed by Akram et al. [42]. Ashraf et al. [43] discussed a novel theory of the EDAS method and Aczel Alsina aggregation operators for resolving complicated real-life problems. Keshavarz Ghorabaee et al. [44] introduced a feasible decision-making approach of the EDAS method for choosing a suitable optimal option under consideration of different characteristics. Furthermore, positive distance average solution (PDAS) and negative distance average solution (NDAS) are discussed to define the relationship among estimated information. Based on the robustness of the EDAS method, several real-life applications resolved using the theory of the EDAS method, such as Dhumras and Bajaj [45] capturing various characteristics of robotic agri-farming systems using the Dombi aggregation operators and EDAS approach. Fan et al. [46] conducted a comprehensive Meta-analysis with bipolar fuzzy theory and power aggregation operators. Akram et al. [47] determined disease symptoms through the medical diagnosis process and advanced optimization techniques of the ELECTRE-I method. Al-Barakati et al. [48] discussed a novel theory WASPAS method for selecting sustainable renewable energy sources. Rao and Sujatha [49] discussed dominant techniques of reliable healthcare waste management considering theory of Farmatean fuzzy situations. Ali et al. [50] elaborated the TOPSIS technique for resolving real-life applications with Bonferroni Mean models.

1.2. Problem statement

In today's rapidly evolving technological landscape, AI-based computing tools are increasingly used to enhance decision-making across various domains, including healthcare, finance, education, and transportation. However, evaluating these tools effectively remains a critical challenge due to the complexity and subjectivity of performance metrics. Traditional evaluation methods often fail to address uncertainty, imprecision, and vagueness inherent in real-world decision-making environments. Therefore, we propose the application of fuzzy decision-making approaches to assess AI-based computing tools under multiple criteria. By incorporating fuzzy logic, the approach can model uncertainty more effectively and account for the varying degrees of importance and subjective preferences associated with different evaluation criteria. For this framework, we aim to provide a more comprehensive and nuanced assessment, facilitating better selection, optimization, and deployment of AI-driven solutions in complex, dynamic decision-making contexts.

1.3. Motivation behind the research work

The motivation behind the Crq-ROFS lies in addressing the complexities of decision-making under uncertainty. Traditional fuzzy sets often struggle to capture the full spectrum of vagueness and ambiguity that decision-makers face, especially when dealing with multiple criteria or conflicting information. The Crq-ROFS is designed to extend the capabilities of fuzzy systems by providing a more flexible and comprehensive framework for representing and processing uncertainty. This structure provides a powerful way to handle situations where uncertainty is high and cyclical or rotational, enabling more accurate decision support in dynamic environments where conditions evolve over time.

The aggregation operators based on Frank t-norms and t-conorms stem from the need to effectively combine individual preferences, criteria, or pieces of information in multi-criteria decision-making (MCDM) problems. Frank t-norms and t-conorms offer a way to model the interaction between decision criteria by combining them in a non-linear fashion that accounts for both optimism and pessimism in decision-making. These operators are particularly useful in scenarios where decision-makers exhibit varying degrees of risk tolerance and address conflicting criteria. The EDAS method in MADM problems is to offer a simple, yet effective approach for ranking and selecting alternatives in complex decision-making scenarios. The MADM problems often involve multiple alternatives evaluated against various criteria, with each criterion potentially having different importance and levels of uncertainty. The EDAS method is motivated by the need for an intuitive and computationally efficient technique that does not require complex mathematical operations or expert judgment, making it more accessible for real-world applications. By comparing alternatives based on their distance from an average solution, the EDAS method captures the relative performance of each alternative without the need for precise weight assignments, making it suitable for situations with limited or uncertain information.

1.4. Contributions of the article

mainour major objectives are as follows:

a) Expose a novel theory of Crq-ROFSs with feasible operations for the development of advanced methodologies.
b) Construct dominant operations of Frank triangular norms under the system of the Crq-ROF framework.
c) Derive Frank aggregation operators based on Crq-ROF information, namely Crq-ROFFWA and Crq-ROFFWG operators, with reasonable idempotency, monotonicity, and boundedness properties.
d) Based on initiated theories and mathematical strategies, expand the theory of the EDAS method with reliable methodologies for Crq-ROF information. We state real-life applications to prove the worth and compatibility of advanced decision-making techniques of the EDAS method and derived approaches. A numerical example discusses investigating suitable AI-based computing techniques under consideration for specific characteristics or attribute information.
e) Finally, a comparison technique shows the worth and reliability of pioneered approaches with existing terminologies. Some remarkable advantages and features of derived approaches are also mentioned.

1.5. Structure of the manuscript

The outline of this paper is as follows: In Section 2, we recall some fundamental concepts and basic preliminaries for the development of new theories. In Section 3, we state some feasible operations of Frank triangular norms in the light of the Crq-ROF context. In Section 4, we propose a family of new mathematical approaches based on formulated operations of Crq-ROF information. In Section 5, we demonstrate a novel technique of the EDAS method for the MADM problem and complex real-life problems. Section 6 contains the importance of AI in computing techniques. We also discuss an experimental case study with the help of numerical examples to evaluate an appropriate AI-based intelligent computing technique. In Section 7, we compare approaches to prove the validity and supremacy of diagnosed theories and decision-making problems. Finally, we state some concluding remarks in Section 8. Figure 3 illustrates the major contributions of this paper.

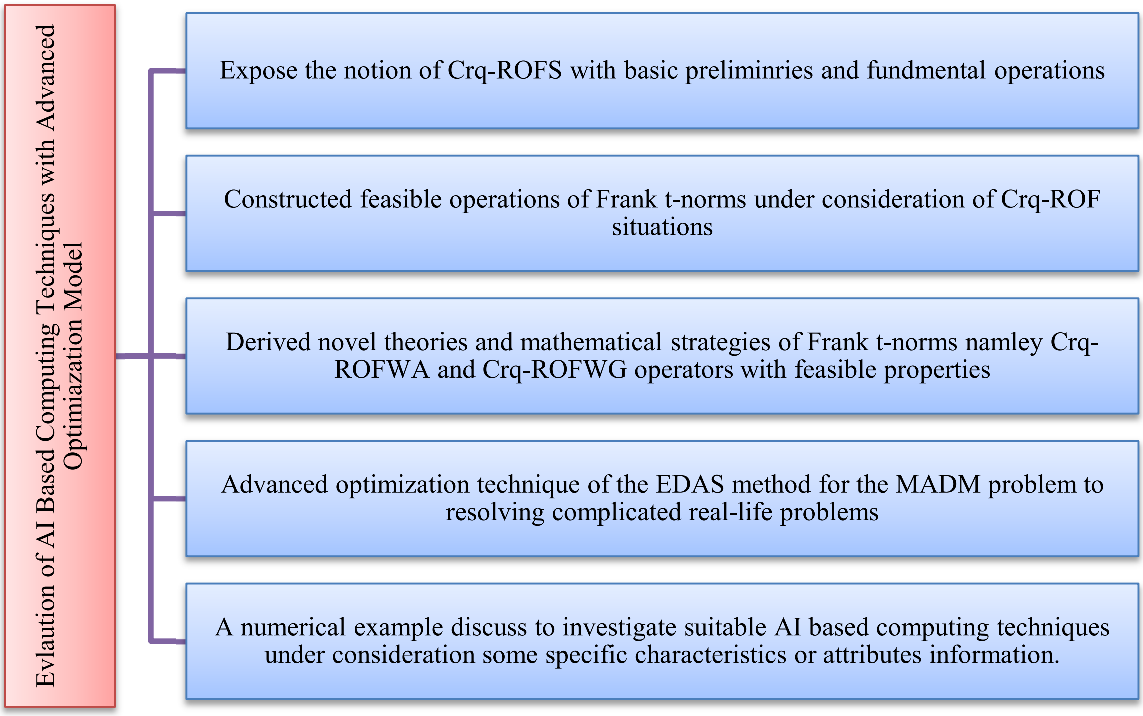

Figure 3. The outline of this paper.

2. Preliminaries

In this section, we define some basic terminologies of C-IFS for further development of this research work.

Definition 1. [9] The q-ROFS B on universe of discourse E is expressed as follows:

$$B = \{ (p, (\mu_B(p), v_B(p)))|p \in E\}.$$

It is clear that $\mu_B(p) \in [0,1]$ and $v_B(p) \in [0,1]$ indicate the degree of membership value (DoMV) and degree of non-membership value (DoNMV) subject to the condition:

$$0 \leq \mu_B^{\tau}(p) + v_B^{\tau}(p) \leq 1, \tau \geq 1.$$

Furthermore, the hesitancy value of an element is denoted by $\pi_B(p) = \sqrt[\tau]{1 - \left(\mu_B^{\tau}(p) + v_B^{\tau}(p)\right)}$.

Definition 2. [10] For the universal set E, the Cr-IFS is invented by:

$$A = \{ (p, \mu_A(p), v_A(p), r_A)|p \in E\}.$$

Note that $\mu_A(p)$ and $v_A(p)$ denote DoMV and DoNMV, respectively, with subject to condition $0 \leq \mu_A(p) + v_A(p) \leq 1$, where the radius of the circle denoted by r is the point $(\mu_A(p), v_A(p))$ on the place. Additionally, $\pi_A(p) = 1 - \mu_A(p) - v_A(p)$ $(\forall p \in E)$ indicates the hesitancy value of p in A. Moreover, a circular-intuitionistic fuzzy value (Cr-IFV) is expressed by $\beta = (\mu_A(p), v_A(p): r_A)$.

Definition 3. [12] The Crq-ROFS A on a universe of discourse E is given by:

$$A = \{ (p, \mu_A(p), v_A(p), r_A)|p \in E\}.$$

Note that $\mu_A(p)$ and $v_A(p)$ denote the DoMV and DoNMV, respectively, with subject to condition

$0 \leq \mu_A^{\tau}(p) + v_A^{\tau}(p) \leq 1$, where the radius of the circle denoted by r_A is the point $(\mu_A(p), v_A(p))$ on the place. Additionally, $\pi_A(p) = \sqrt[\tau]{1 - \left(\mu_B^{\tau}(p) + v_B^{\tau}(p)\right)}$ indicates the hesitancy value of p in A. Additionally, a circular q-rung orthopair fuzzy value (Crq-ROFV) is expressed by $\beta = (\mu_A(p), v_A(p): r_A)$.

Definition 4. [10] Suppose that β is a Crq-ROFV. The mathematical shape of score function $\mathcal{S}(\beta)$ and accuracy function $\mathcal{S}(\mathcal{A})$ is given by:

$$\mathcal{S}(\beta) = \frac{1}{2}\left(\mu^{\tau}(p) - v^{\tau}(p) + \sqrt{2r}\right),$$

$$\mathcal{A}(\beta) = \mu^{\tau}(p) + v^{\tau}(p), \mathcal{A}(\beta) \in [0,1].$$

For simplification, we have some rules:

- If $\mathcal{S}(\beta_1) > \mathcal{S}(\beta_2)$, then $\beta_1 > \beta_2$;
- if $\mathcal{S}(\beta_1) = \mathcal{S}(\beta_2)$, then:
    1) if $\mathcal{A}(\beta_1) < \mathcal{A}(\beta_2)$, then $\mathcal{A}_1 < \mathcal{A}_2$,
    2) if $\mathcal{A}(\beta_2) > \mathcal{A}(\beta_2)$, then $\mathcal{A}_1 > \mathcal{A}_2$.

Definition 5. [14] Consider any two Crq-ROFVs, $\alpha_i = (\mu_i, v_i, r_i)$ $(i = 1, 2)$. Then:

1) $\alpha_1 \oplus_t \alpha_2 = (\mu_1 + \mu_2 - \mu_1\mu_2, v_1 v_2, r_1 + r_2 - r_1 r_2)$;
2) $\alpha_1 \oplus_{tc} \alpha_2 = (\mu_1 + \mu_2 - \mu_1\mu_2, v_1 v_2, r_1 r_2)$;
3) $\alpha_1 \otimes_t \alpha_2 = (\mu_1\mu_2, v_1 + v_2 - v_1 v_2, r_1 r_2)$;
4) $\alpha_1 \otimes_{tc} \alpha_2 = (\mu_1\mu_2, v_1 + v_2 - v_1 v_2, r_1 + r_2 - r_1 r_2)$;
5) $\lambda\alpha_{1_t} = (1 - (1 - \mu_1)^{\lambda}, v_1^{\lambda}, 1 - \left(1 - r_1\right)^{\lambda}), \lambda > 0$;
6) $\lambda\alpha_{1_{tc}} = (1 - (1 - \mu_1)^{\lambda}, v_1^{\lambda}, r_1^{\lambda}), \lambda > 0$;
7) $\alpha_{1_t}^{\lambda} = (\mu_1^{\lambda},\ 1 - (1 - v_1)^{\lambda},\ r_1^{\lambda}), \lambda > 0$;
8) $\alpha_{1_{tc}}^{\lambda} = (\mu_1^{\lambda},\ 1 - (1 - v_1)^{\lambda},\ 1 - \left(1 - r_1\right)^{\lambda}), \lambda > 0$.

Definition 6. [51] Consider two real numbers a and b. The Frank t-norm and t-conorm can be written as:

$$\mathrm{Fra}(a,b)_t = \log_{ב}\left(1 + \frac{(ב^a - 1)(ב^b - 1)}{ב - 1}\right),$$

$$\mathrm{Fra}(a,b)_{tc} = 1 - \log_{ב}\left(1 + \frac{(ב^{1-a} - 1)(ב^{1-b} - 1)}{ב - 1}\right),$$

where $(a,b) \in [0,1] \times [0,1]$ and $ב \neq 1$.

3. Frank operations for circular *q*-rung orthopair fuzzy information

This section illustrates some flexible operations of Frank triangular norms in the light Crq-ROFVs.

Definition 7. Let $\alpha = \left(\mu(p), v(p);\ r(p)\right)$, $\alpha_1 = \left(\mu_1(p), v_1(p); r_1(p)\right)$ and $\alpha_2 = \left(\mu_2(p), v_2(p);\ r_2(p)\right)$ be the three Crq-ROFVs, $ב > 1$ and $\psi > 0$ be any real number. Then, we have:

1) $$\alpha_1 \oplus_t \alpha_2 = \left(\begin{array}{c} \sqrt[\tau]{1 - log_{\beth}\left(1 + \frac{\left(\beth^{1-\mu_1^\tau}-1\right)\left(\beth^{1-\mu_2^\tau}-1\right)}{\beth-1}\right)}, \\ \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{v_1^\tau}-1\right)\left(\beth^{v_2^\tau}-1\right)}{\beth-1}\right)}, \\ \sqrt[\tau]{1 - log_{\beth}\left(1 + \frac{\left(\beth^{1-r_1^\tau}-1\right)\left(\beth^{1-r_2^\tau}-1\right)}{\beth-1}\right)}. \end{array} \right)$$

2) $$\alpha_1 \oplus_{tc} \alpha_2 = \left(\begin{array}{c} \sqrt[\tau]{1 - log_{\beth}\left(1 + \frac{\left(\beth^{1-\mu_1^\tau}-1\right)\left(\beth^{1-\mu_2^\tau}-1\right)}{\beth-1}\right)}, \\ \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{v_1^\tau}-1\right)\left(\beth^{v_2^\tau}-1\right)}{\beth-1}\right)}, \\ \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{r_1^\tau}-1\right)\left(\beth^{r_2^\tau}-1\right)}{\beth-1}\right)}. \end{array} \right)$$

3) $$\alpha_1 \otimes_t \alpha_2 = \left(\begin{array}{c} \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{\mu_1^\tau}-1\right)\left(\beth^{\mu_2^\tau}-1\right)}{\beth-1}\right)}, \\ \sqrt[\tau]{1 - log_{\beth}\left(1 + \frac{\left(\beth^{1-v_1^\tau}-1\right)\left(\beth^{1-v_2^\tau}-1\right)}{\beth-1}\right)}, \\ \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{r_1^\tau}-1\right)\left(\beth^{r_2^\tau}-1\right)}{\beth-1}\right)}. \end{array} \right)$$

4) $$\alpha_1 \otimes_{tc} \alpha_2 = \left(\begin{array}{c} \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{\mu_1^\tau}-1\right)\left(\beth^{\mu_2^\tau}-1\right)}{\beth-1}\right)}, \\ \sqrt[\tau]{1 - log_{\beth}\left(1 + \frac{\left(\beth^{1-v_1^\tau}-1\right)\left(\beth^{1-v_2^\tau}-1\right)}{\beth-1}\right)}, \\ \sqrt[\tau]{1 - log_{\beth}\left(1 + \frac{\left(\beth^{1-r_1^\tau}-1\right)\left(\beth^{1-r_2^\tau}-1\right)}{\beth-1}\right)}. \end{array} \right)$$

5) $$\psi\, \alpha_t = \left(\begin{array}{c} \sqrt[\tau]{1 - log_{\beth}\left(1 + \frac{\left(\beth^{1-\mu^\tau}-1\right)^{\psi}}{(\beth-1)^{\psi-1}}\right)}, \\ \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{v^\tau}-1\right)^{\psi}}{(\beth-1)^{\psi-1}}\right)}, \\ \sqrt[\tau]{1 - log_{\beth}\left(1 + \frac{\left(\beth^{1-r^\tau}-1\right)^{\psi}}{(\beth-1)^{\psi-1}}\right)}. \end{array} \right)$$

6) $$\psi\,\alpha_{tc} = \left(\begin{array}{c} \sqrt[\tau]{1 - log_{\beth}\left(1 + \frac{\left(\beth^{1-\mu^{\tau}} - 1\right)^{\psi}}{(\beth-1)^{\psi-1}}\right)}, \\ \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{v^{\tau}} - 1\right)^{\psi}}{(\beth-1)^{\psi-1}}\right)}, \\ \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{r^{\tau}} - 1\right)^{\psi}}{(\beth-1)^{\psi-1}}\right)}. \end{array}\right)$$

7) $$\alpha_t^{\psi} = \left(\begin{array}{c} \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{\mu^{\tau}} - 1\right)^{\psi}}{(\beth-1)^{\psi-1}}\right)}, \\ \sqrt[\tau]{1 - log_{\beth}\left(1 + \frac{\left(\beth^{1-v^{\tau}} - 1\right)^{\psi}}{(\beth-1)^{\psi-1}}\right)}, \\ \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{r^{\tau}} - 1\right)^{\psi}}{(\beth-1)^{\psi-1}}\right)}. \end{array}\right)$$

8) $$\alpha_{tc}^{\psi} = \left(\begin{array}{c} \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{\mu^{\tau}} - 1\right)^{\psi}}{(\beth-1)^{\psi-1}}\right)}, \\ \sqrt[\tau]{1 - log_{\beth}\left(1 + \frac{\left(\beth^{1-v^{\tau}} - 1\right)^{\psi}}{(\beth-1)^{\psi-1}}\right)}, \\ \sqrt[\tau]{1 - log_{\beth}\left(1 + \frac{\left(\beth^{1-r^{\tau}} - 1\right)^{\psi}}{(\beth-1)^{\psi-1}}\right)}. \end{array}\right)$$

4. Circular *q*-rung orthopair fuzzy frank aggregation operators

Besides the theory of Cr-IF information, we delve into advanced arithmetic aggregation operators by utilizing operational laws of Frank triangular norms, including Crq-ROFFWA and Crq-ROFWG operators.

Definition 8. Consider a set of Crq-ROFVs $\alpha_i = \left(\mu_i(p), v_i(p);\ r_i(p)\right)$ $(i = 1,2,\dots,n)$. The Crq-ROFFWA operator is characterized by a specific function of $\alpha^n \to \alpha$ as follows:

$$\mathrm{Crq-ROFFWA}(\alpha_1, \alpha_2, \dots, \alpha_n)_t = \bigoplus_{i=1}^{n}{}_t\, (w_i\alpha_i),$$

$$\mathrm{Crq-ROFFWA}(\alpha_1, \alpha_2, \dots, \alpha_n)_{tc} = \bigoplus_{i=1}^{n}{}_{tc}\, (w_i\alpha_i),$$

where $w = (w_1, w_2, \dots, w_n)^t$ is the weight vector of $\alpha_i (i = 1,2,\dots,n)$, $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$.

Theorem 1. *Consider a set of Crq-ROFVs* $\alpha_i = \left(\mu_i(p), v_i(p);\ r_i(p)\right)$ $(i = 1,2,\dots,n)$. *The values investigated by the Crq-ROFFWA operators are also Crq-ROFVs, such that:*

$$Crq-ROFFWA(\alpha_1,\alpha_2,\dots,\alpha_n)_t=\begin{pmatrix}\sqrt[\tau]{1-log_{ב}\left(1+\prod_{i=1}^{n}\left(ב^{1-\mu_i^\tau}-1\right)^{w_i}\right)},\\ \sqrt[\tau]{log_{ב}\left(1+\prod_{i=1}^{n}(ב^{v_i^\tau}-1)^{w_i}\right)},\\ \sqrt[\tau]{1-log_{ב}\left(1+\prod_{i=1}^{n}\left(ב^{1-r_i^\tau}-1\right)^{w_i}\right)}.\end{pmatrix}$$

$$Crq-ROFFWA(\alpha_1,\alpha_2,\dots,\alpha_n)_{tc}=\begin{pmatrix}\sqrt[\tau]{1-log_{ב}\left(1+\prod_{i=1}^{n}\left(ב^{1-\mu_i^\tau}-1\right)^{w_i}\right)},\\ \sqrt[\tau]{log_{ב}\left(1+\prod_{i=1}^{n}(ב^{v_i^\tau}-1)^{w_i}\right)},\\ \sqrt[\tau]{log_{ב}\left(1+\prod_{i=1}^{n}(ב^{r_i^\tau}-1)^{w_i}\right)}.\end{pmatrix}$$

Proof. Based on mathematical induction, we can prove $n=2$, and we have:

$$Crq-ROFFWA(\alpha_1,\alpha_2)_t=\bigoplus_{i=1}^{2}{}_t\,(w_i\alpha_i)(w_i\alpha_i)=w_1\alpha_1\oplus_t w_2\alpha_2$$

$$=\begin{pmatrix}\sqrt[\tau]{1-log_{ב}\left(1+\frac{\left(ב^{1-\mu_1^\tau}-1\right)^{w_1}}{(ב-1)^{w_1-1}}\right)},\\ \sqrt[\tau]{log_{ב}\left(1+\frac{\left(ב^{v_1^\tau}-1\right)^{w_1}}{(ב-1)^{w_1-1}}\right)},\\ \sqrt[\tau]{log_{ב}\left(1+\frac{\left(ב^{r_1^\tau}-1\right)^{w_1}}{(ב-1)^{w_1-1}}\right)}\end{pmatrix}$$

$$\oplus_t \begin{pmatrix} \sqrt[\tau]{1 - log_{\beth}\left(1 + \frac{\left(\beth^{1-\mu_2^{\tau}} - 1\right)^{w_2}}{(\beth - 1)^{w_2 - 1}}\right)}, \\ \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{v_2^{\tau}} - 1\right)^{w_2}}{(\beth - 1)^{w_2 - 1}}\right)}, \\ \sqrt[\tau]{log_{\beth}\left(1 + \frac{\left(\beth^{r_2^{\tau}} - 1\right)^{w_2}}{(\beth - 1)^{w_2 - 1}}\right)} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[\tau]{1 - log_{\beth}\left(1 + \prod_{i=1}^{2}\left(\beth^{1-\mu_i^{\tau}} - 1\right)^{w_i}\right)}, \\ \sqrt[\tau]{log_{\beth}\left(1 + \prod_{i=1}^{2}(\beth^{v_i^{\tau}} - 1)^{w_i}\right)}, \\ \sqrt[\tau]{1 - log_{\beth}\left(1 + \prod_{i=1}^{2}\left(\beth^{1-r_i^{\tau}} - 1\right)^{w_i}\right)} \end{pmatrix}, \left[\because \sum_{i=1}^{2} w_i = 1\right].$$

Hence, the result is valid for $n = 2$.

Now, suppose that the given result is true for $n = s$, so we have:

$$Crq - ROFFWA(\alpha_1, \alpha_2, \dots \alpha_s)_t = \bigoplus_{i=1}^{2}{}_t\, (w_i \alpha_i) = \begin{pmatrix} \sqrt[\tau]{1 - log_{\beth}\left(1 + \prod_{i=1}^{s}\left(\beth^{1-\mu_i^{\tau}} - 1\right)^{w_i}\right)}, \\ \sqrt[\tau]{log_{\beth}\left(1 + \prod_{i=1}^{s}(\beth^{v_i^{\tau}} - 1)^{w_i}\right)}, \\ \sqrt[\tau]{1 - log_{\beth}\left(1 + \prod_{i=1}^{s}\left(\beth^{1-r_i^{\tau}} - 1\right)^{w_i}\right)}. \end{pmatrix}$$

Now, for $n = s + 1$:

$$Crq-ROFFWA(\alpha_1,\alpha_2,\ldots\alpha_s,\alpha_{s+1})_t=\bigoplus_{i=1}^{S+1}{}_t\,(w_i\alpha_i)=\bigoplus_{i=1}^{S}{}_t\,w_i\alpha_i\oplus_t w_{s+1}\alpha_{s+1}$$

$$=\begin{pmatrix}\sqrt[\tau]{1-log_{ב}\left(1+\frac{\prod_{i=1}^{s}\left(ב^{1-\mu_i^\tau}-1\right)^{w_i}}{(ב-1)^{\sum_{i=1}^{s}w_i-1}}\right)},\\ \sqrt[\tau]{log_{ב}\left(1+\frac{\prod_{i=1}^{s}(ב^{v_i^\tau}-1)^{w_i}}{(ב-1)^{\sum_{i=1}^{s}w_i-1}}\right)},\\ \sqrt[\tau]{1-log_{ב}\left(1+\frac{\prod_{i=1}^{s}\left(ב^{1-r_i^\tau}-1\right)^{w_i}}{(ב-1)^{\sum_{i=1}^{s}w_i-1}}\right)}\end{pmatrix}$$

$$\oplus_t\begin{pmatrix}\sqrt[\tau]{1-log_{ב}\left(1+\frac{\left(ב^{1-\mu_{s+1}^\tau}-1\right)^{w_{s+1}}}{(ב-1)^{w_{s+1}-1}}\right)},\\ \sqrt[\tau]{log_{ב}\left(1+\frac{(ב^{v_{s+1}^\tau}-1)^{w_{s+1}}}{(ב-1)^{w_{s+1}-1}}\right)},\\ \sqrt[\tau]{1-log_{ב}\left(1+\frac{\left(ב^{1-r_{s+1}^\tau}-1\right)^{w_{s+1}}}{(ב-1)^{w_{s+1}-1}}\right)}\end{pmatrix}$$

$$=\begin{pmatrix}\sqrt[\tau]{1-log_{ב}\left(1+\prod_{i=1}^{s+1}\left(ב^{1-\mu_i^\tau}-1\right)^{w_i}\right)},\\ \sqrt[\tau]{log_{ב}\left(1+\prod_{i=1}^{s+1}(ב^{v_i^\tau}-1)^{w_i}\right)},\\ \sqrt[\tau]{1-log_{ב}\left(1+\prod_{i=1}^{s+1}\left(ב^{1-r_i^\tau}-1\right)^{w_i}\right)}\end{pmatrix},\left[\because\sum_{i=1}^{s+1}w_i=1\right].$$

The result is true for $n=s+1$ if it is true for $n=s,$ and true for $n=2$. Hence, by the method of induction, the given result is true for any natural number n.
Similarly, we can prove t-conorm:

$$Crq-ROFFWA(\alpha_1,\alpha_2,\ldots,\alpha_n)_{tc}=\bigoplus_{i=1}^{n}{}_{tc}\,(w_i\alpha_i)=\left(\begin{array}{c}\sqrt[\tau]{1-log_{\beth}\left(1+\prod_{i=1}^{n}\left(\beth^{1-\mu_i^{\tau}}-1\right)^{w_i}\right)},\\ \sqrt[\tau]{log_{\beth}\left(1+\prod_{i=1}^{n}(\beth^{v_i^{\tau}}-1)^{w_i}\right)},\\ \sqrt[\tau]{log_{\beth}\left(1+\prod_{i=1}^{n}(\beth^{r_i^{\tau}}-1)^{w_i}\right)}.\end{array}\right)$$

Property 1. *Let* $\alpha_i=\left(\mu_i(\mathrm{p}),v_i(\mathrm{p});\ r_i(\mathrm{p})\right)\ (i=1,2,\ldots,n)$ *be a family of identical Crq-ROFVs. Then, we can easily delve into the following expression:*

$$Cr-IFFWA_{min}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\alpha,$$
$$Cr-IFFWA_{map}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\alpha.$$

Proof. Since $\alpha_i=\left(\mu_i(\mathrm{p}),v_i(\mathrm{p});\ r_i(\mathrm{p})\right)\ (i=1,2,\ldots,n)$ is a family of identical Crq-ROFVs, then we can prove the above expression as follows:

$$Crq-ROFFWA(\alpha_1,\alpha_2,\ldots,\alpha_n)_t=\left(\begin{array}{c}\sqrt[\tau]{1-log_{\beth}\left(1+\prod_{i=1}^{n}\left(\beth^{1-\mu_i^{\tau}}-1\right)^{w_i}\right)},\\ \sqrt[\tau]{log_{\beth}\left(1+\prod_{i=1}^{n}(\beth^{v_i^{\tau}}-1)^{w_i}\right)},\\ \sqrt[\tau]{1-log_{\beth}\left(1+\prod_{i=1}^{n}\left(\beth^{1-r_i^{\tau}}-1\right)^{w_i}\right)}\end{array}\right)$$

$$= \left(\begin{array}{c} \sqrt[\tau]{1 - log_{ב}\left(1 + \prod_{i=1}^{n} (ב^{1-\mu^{\tau}} - 1)^{w_i}\right)}, \\ \sqrt[\tau]{log_{ב}\left(1 + \prod_{i=1}^{n} (ב^{v^{\tau}} - 1)^{w_i}\right)}, \\ \sqrt[\tau]{1 - log_{ב}\left(1 + \prod_{i=1}^{n} (ב^{1-r^{\tau}} - 1)^{w_i}\right)} \end{array} \right)$$

$$= \left(\begin{array}{c} \sqrt[\tau]{1 - log_{ב}\left(1 + \prod_{i=1}^{n} (ב^{1-\mu^{\tau}} - 1)^{\sum_{i=1}^{n} w_i}\right)}, \\ \sqrt[\tau]{log_{ב}\left(1 + \prod_{i=1}^{n} (ב^{v^{\tau}} - 1)^{\sum_{i=1}^{n} w_i}\right)}, \\ \sqrt[\tau]{1 - log_{ב}\left(1 + \prod_{i=1}^{n} (ב^{1-r^{\tau}} - 1)^{\sum_{i=1}^{n} w_i}\right)} \end{array} \right) = \left(\begin{array}{c} \sqrt[\tau]{1 - log_{ב}\left(1 + (ב^{1-\mu^{\tau}} - 1)\right)}, \\ \sqrt[\tau]{log_{ב}\left(1 + (ב^{v^{\tau}} - 1)\right)}, \\ \sqrt[\tau]{1 - log_{ב}\left(1 + (ב^{1-r^{\tau}} - 1)\right)} \end{array} \right)$$

$= (\mu, v, r) = \alpha.$

Property 2. *Let* $\alpha_i = \left(\mu_i(\mathrm{p}), v_i(\mathrm{p});\ r_i(\mathrm{p})\right)\ (i = 1,2, \dots, n)$ *be the system of Crq-ROFVs. If* $(\alpha_i^-)_{min} = (min\{\mu_i\}, max\{v_i\}, max\{r_i\}), (\alpha_i^-)_{max} = (min\{\mu_i\}, max\{v_i\}, min\{r_i\})$ *and* $(\alpha_i^+)_{min} = (max\{\mu_i\}, min\{v_i\}, max\{r_i\}), (\alpha_i^+)_{max} = (max\{\mu_i\}, min\{v_i\}, min\{r_i\})$, *then the following axiom is expressed as follows:*

$$(\alpha_i^-)_{min} \le Crq - ROFFWA_{min}(\alpha_1, \alpha_2, \dots, \alpha_n) \le (\alpha_i^+)_{min},$$

$$(\alpha_i^-)_{max} \le Crq - ROFFWA_{max}(\alpha_1, \alpha_2, \dots, \alpha_n) \le (\alpha_i^+)_{max}.$$

Property 3. *Suppose two sets of Crq-ROFVs* α_i *and* α_i' $(i = 1,2, \dots, n)$, *if* $\alpha_i \le \alpha_i'$. *Then, we have:*

$$Crq - ROFFWA_{min}(\alpha_1, \alpha_2, \dots, \alpha_n) \le Crq - ROFFWA_{min}(\alpha_1', \alpha_2', \dots, \alpha_n'),$$

$$Crq - ROFFWA_{max}(\alpha_1, \alpha_2, \dots, \alpha_n) \le Crq - ROFFWA_{max}(\alpha_1', \alpha_2', \dots, \alpha_n').$$

Definition 9. Consider a set of Crq-ROFVs $\alpha_i = \left(\mu_i(p), v_i(p);\ r_i(p)\right)\ (i = 1,2, \dots, n)$. The Crq-ROFFWG operators are characterized by a specific function of $\alpha^n \to \alpha$ as follows:

$$\mathrm{Crq - ROFFWG}(\alpha_1, \alpha_2, \dots, \alpha_n)_t = \bigotimes_{t\, i=1}^{n} \alpha_i^{w_i},$$

$$\mathrm{Crq - ROFFWG}(\alpha_1, \alpha_2, \dots, \alpha_n)_{tc} = \bigotimes_{tc\, i=1}^{n} \alpha_i^{w_i},$$

where $w = (w_1, w_2, \ldots, w_n)^t$ is the weight vector of $\alpha_i (i = 1,2, \ldots, n)$, $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$.

Theorem 2. *Consider a set of Crq-ROFVs* $\alpha_i = \left(\mu_i(p), v_i(p);\ r_i(p)\right) (i = 1,2, \ldots, n)$. *The values investigated by the Crq-ROFFWG operators are also Crq-ROFVs such that:*

$$Crq - ROFFWG(\alpha_1, \alpha_2, \ldots, \alpha_n)_t = \begin{pmatrix} \sqrt[\tau]{log_{ב}\left(1 + \prod_{i=1}^{n} (ב^{\mu_i^\tau} - 1)^{w_i}\right)}, \\ \sqrt[\tau]{1 - log_{ב}\left(1 + \prod_{i=1}^{n} \left(ב^{1-v_i^\tau} - 1\right)^{w_i}\right)}, \\ \sqrt[\tau]{1 - log_{ב}\left(1 + \prod_{i=1}^{n} \left(ב^{1-r_i^\tau} - 1\right)^{w_i}\right)}. \end{pmatrix}$$

$$Crq - ROFFWG(\alpha_1, \alpha_2, \ldots, \alpha_n)_{tc} = \begin{pmatrix} \sqrt[\tau]{log_{ב}\left(1 + \prod_{i=1}^{n} (ב^{\mu_i^\tau} - 1)^{w_i}\right)}, \\ \sqrt[\tau]{1 - log_{ב}\left(1 + \prod_{i=1}^{n} \left(ב^{1-v_i^\tau} - 1\right)^{w_i}\right)}, \\ \sqrt[\tau]{log_{ב}\left(1 + \prod_{i=1}^{n} (ב^{r_i^\tau} - 1)^{w_i}\right)}. \end{pmatrix}$$

Property 4. *Let* $\alpha_i = \left(\mu_i(p), v_i(p);\ r_i(p)\right) (i = 1,2, \ldots, n)$ *be a family of identical Crq-ROFVs. Then, we can easily delve into the following expression:*

$$Cr - IFFWG_{min}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha,$$

$$Cr - IFFWG_{max}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha.$$

Property 5. *Let* $\alpha_i = \left(\mu_i(p), v_i(p);\ r_i(p)\right) (i = 1,2, \ldots, n)$ *be the system of Crq-ROFVs. If* $(\alpha_i^-)_{min} = (min\{\mu_i\}, max\{v_i\}, max\{r_i\}), (\alpha_i^-)_{max} = (min\{\mu_i\}, max\{v_i\}, min\{r_i\})$ *and* $(\alpha_i^+)_{min} = (max\{\mu_i\}, min\{v_i\}, max\{r_i\}), (\alpha_i^+)_{max} = (max\{\mu_i\}, min\{v_i\}, min\{r_i\})$. *Then, the following axiom is expressed as follows:*

$$(\alpha_i^-)_{min} \leq Crq - ROFFWG_{min}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq (\alpha_i^+)_{min},$$

$$(\alpha_i^-)_{max} \leq Crq - ROFFWG_{max}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq (\alpha_i^+)_{max}.$$

Property 6. *Suppose two sets of Crq-ROFVs* α_i *and* α_i' $(i = 1,2, \ldots, n)$, *if* $\alpha_i \leq \alpha_i'$. *Then, we have:*

$$Crq - ROFFWG_{min}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq Crq - ROFFWG_{min}(\alpha_1', \alpha_2', \ldots, \alpha_n'),$$
$$Crq - ROFFWG_{max}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq Crq - ROFFOWG_{max}(\alpha_1', \alpha_2', \ldots, \alpha_n').$$

5. Advanced decision-making problem based on circular *q*-rung orthopair fuzzy environments

Due to its balanced and intuitive approach, the EDAS method holds significant value in decision-making, particularly for the MADM problems. This approach enables decision-makers to consider how each option performs relative to a central benchmark, offering a more nuanced and realistic evaluation.

5.1. Algorithm of the EDAS method for the MADM problem

Consider a finite class of alternatives $\{Đ_1, Đ_2, \ldots, Đ_m\}$ and a collection of attributes $\{\breve{G}_1, \breve{G}_2, \ldots, \breve{G}_n\}$ with degree of weight to each attribute $(\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_n)$ such that $\hat{w}_\ell > 0$ and $\sum_{\ell=1}^{n} \hat{w}_\ell = 1$. The decision maker presents their opinion for the evaluation of each alternative under different types of attribute information. In this decision algorithm, we consider the information in the form of Crq-ROFVs $\alpha_{j\ell} = \left(\mu_{j\ell}(p), v_{j\ell}(p);\ r_{j\ell}(p)\right)$ and list in a decision matrix $\mathfrak{W} = \left[\alpha_{j\ell}\right]_{m\times n}, j = 1,2,\ldots,m\ \&\ \ell = 1,2,\ldots,n$. The advanced decision algorithm for the EDAS method is constructed as follows:

Step 1. The professional expert collects information for each alternative based on various types of attributes information, and the decision matrix is given by:

$$\mathfrak{W} = \left[\alpha_{j\ell}\right]_{m\times n}, j = 1,2,\ldots,m\ \&\ \ell = 1,2,\ldots,n,$$

where $\alpha_{j\ell}$ indicate the Crq-ROFVs with alternatives and attributes $Đ_j$ and $\breve{G}_\ell, j = 1,2,\ldots,m\ \&\ \ell = 1,2,\ldots,n$, respectively.

Step 2. Normalize the decision matrix as follows:

$$\mathfrak{W} = \left[\alpha_{j\ell}\right]_{m\times n} = \begin{cases} \left(\mu_i(p), v_i(p);\ r_i(p)\right) & if\ beneficial\ attributes, \\ \left(v_i(p), \mu_i(p);\ r_i(p)\right) & if\ non-beneficial\ attributes. \end{cases}$$

Step 3. Compute averaging solutions using the following expression of the derived approach:

$$Crq - ROFFWA(\alpha_1, \alpha_2, \ldots, \alpha_n)_t = \begin{pmatrix} \sqrt[\tau]{1 - log_{\beth}\left(1 + \prod_{i=1}^{n}\left(\beth^{1-\mu_i^\tau} - 1\right)^{w_i}\right)}, \\ \sqrt[\tau]{log_{\beth}\left(1 + \prod_{i=1}^{n}\left(\beth^{v_i^\tau} - 1\right)^{w_i}\right)}, \\ \sqrt[\tau]{1 - log_{\beth}\left(1 + \prod_{i=1}^{n}\left(\beth^{1-r_i^\tau} - 1\right)^{w_i}\right)}. \end{pmatrix}$$

Step 4. Based on the above theory of averaging solution and score values of each attribute, we compute the results of positive distance average solution (PDAS) and negative distance average solution (NDAS) as follows:

$$PDAS_{\jmath\ell} = \frac{\max\left(0, \left(\mathcal{S}(\mathbb{R}_{\jmath\ell}) - \mathcal{S}(\mathbb{Q}_{\jmath})\right)\right)}{\mathcal{S}(\mathbb{Q}_{\jmath})},$$

and

$$NDAS_{\jmath\ell} = \frac{\max\left(0, \left(\mathcal{S}(\mathbb{Q}_{\jmath}) - \mathcal{S}(\mathbb{R}_{\jmath\ell})\right)\right)}{\mathcal{S}(\mathbb{Q}_{\jmath})}.$$

Step 5. Here, we calculate the positive weighted distance (PWD) and negative weighted distance (NWD) based on the following expressions:

$$PWD_{\ell} = \sum_{\ell=1}^{n} \mathfrak{w}_{\ell} PDAS_{\jmath\ell} \text{ and } NWD_{\ell} = \sum_{\ell=1}^{n} \mathfrak{w}_{\ell} NDAS_{\jmath\ell}.$$

Step 6. Obtain normalized results of PWD_{ℓ} and NWD_{ℓ} based on the following expressions:

$$NPWD_{\ell} = \frac{PWD_{\ell}}{\max(PWD_{\ell})} \text{ and } NNWD_{\ell} = 1 - \frac{NWD_{\ell}}{\max(NWD_{\ell})}.$$

Step 7. Apply the following theory to investigate the results of the appraisal solution as follows:

$$AS_i = \frac{1}{2}(NPWD_{\ell} + NNWD_{\ell}).$$

Step 8. Rank the obtained values of the appraisal solution; the highest value is known as the superior one.

5.2. Decision algorithm for the MADM problem

To evaluate an appropriate optimal option from the group of alternatives, an algorithm of the MADM problem is constructed under the system of the Crq-ROF framework. Some essential steps of the MADM problem are initiated as follows:

Step 1. First, the decision maker arranges their judgments using alternatives based on various attributes and information under the Crq-ROF context.

Step 2. Before integrating experts' opinions, types of attributes should be the same as beneficial or non-beneficial under the following expression:

$$\mathfrak{W} = \left[\alpha_{\jmath\ell}\right]_{m\times n} = \begin{cases} \left(\mu_i(\mathrm{p}), v_i(\mathrm{p});\ r_i(\mathrm{p})\right) & if\ beneficial\ attributes, \\ \left(v_i(\mathrm{p}), \mu_i(\mathrm{p});\ r_i(\mathrm{p})\right) & if\ non-beneficial\ attributes. \end{cases}$$

Step 3. Apply strategies derived from the Frank aggregation operators considering the theory of Crq-ROF information.

Step 4. Obtain score values of all alternatives based on Definition 4.

Step 5. Rank alternatives based on the estimated score values of each alternate and the highest value of the alternate known as the best optimal option. In order to provide clarity in the proposed methods, we have provided a flow chart in Figure 4.

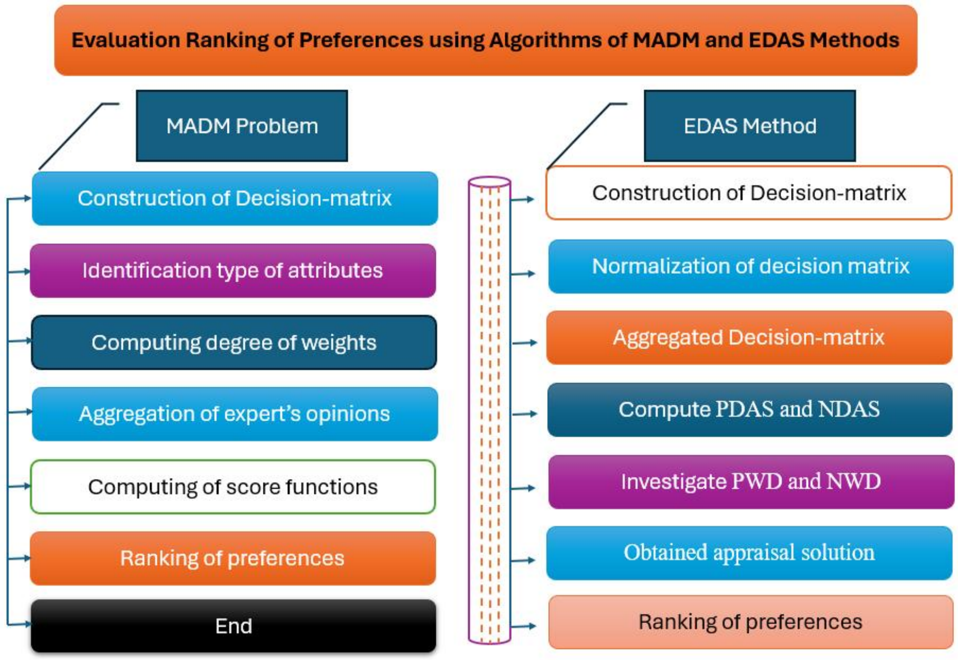

Figure 4. Flow chart of decision algorithms.

6. AI-based computing intelligence techniques

AI plays a transformative role across fields of life by enhancing efficiency, accuracy, and decision-making capabilities. In healthcare, AI algorithms analyze patient data to assist in diagnostics, predict disease outbreaks, and develop personalized treatment plans. In finance, AI-driven analytics streamline risk assessment, fraud detection, and quicker enabling [52]. Furthermore, AI enhances customer experience in retail and marketing by providing personalized recommendations based on consumer behavior and preferences. In transportation, AI optimizes logistics, enhances route planning, and supports the development of autonomous vehicles, leading to safer and more efficient travel.

In data computing, AI significantly improves how organizations manage and analyze large volumes of information. Traditional data analysis methods can be time-consuming and often struggle to derive meaningful insights from complex datasets [53]. However, AI techniques like machine learning and deep learning enable automated data processing and predictive analytics, enabling businesses to uncover patterns and trends that inform strategic decisions. By leveraging AI for data computing, organizations can achieve real-time insights, optimize operations, and drive innovation, enhancing their competitiveness in the market. This powerful synergy between AI and data computing is reshaping industries and paving the way for more innovative, data-driven solutions in everyday life. Furthermore, Figure 5 demonstrates the features and reliability of AI in computing strategies.

Figure 5. Reliability of AI in computing techniques.

6.1. Numerical example

Intelligent computing techniques offer several compelling advantages that enhance efficiency, decision-making, and innovation across domains. These techniques automate repetitive and time-consuming tasks, enabling human resources to focus on more complex and creative activities. To integrate reliable computing techniques, we discuss some dominant AI-based intelligent computing techniques as follows:

Hybrid Intelligent Systems $\mathbb{Q}_1$:

Hybrid Intelligent Systems are advanced computational models that integrate multiple artificial intelligence (AI) techniques to leverage the strengths of different methods in solving complex problems. Hybrid Intelligent Systems are applied in robotics, decision-making, control systems, and optimization problems. In practical applications, a hybrid system might combine machine learning algorithms with expert systems to provide recommendations or integrate genetic algorithms with neural networks to optimize learning in dynamic environments.

Deep Reinforcement Learning (DRL) $\mathbb{Q}_2$:

Deep Reinforcement Learning (DRL) combines deep learning and reinforcement learning, where neural networks approximate the complex decision-making process in environments where an agent learns to maximize rewards through trial and error. In reinforcement learning, an agent interacts with an environment, making decisions that influence future states and receives feedback through rewards or penalties.

Machine Learning (ML) $\mathbb{Q}_3$:

ML is a branch of artificial intelligence that focuses on enabling machines to learn from data and improve their performance on tasks without being explicitly programmed. It involves the development of algorithms that can recognize patterns, make predictions, and adapt based on experience.

Artificial Neural Networks (ANNs) $\mathbb{Q}_4$:

ANNs are computational models inspired by the structure and function of the human brain. They consist of interconnected layers of nodes (or neurons) that work together to process information and make decisions. Each neuron receives inputs, applies weights, and passes the result through an activation function to determine its output, which is then passed on to the next layer of neurons. The most common architecture involves an input layer, one or more hidden layers, and an output layer.

Quantum Computing for AI $\mathbb{Q}_5$:

Quantum Computing for AI refers to applying quantum computing principles to enhance artificial intelligence algorithms, particularly in solving complex problems that are challenging for classical computers. Quantum computers operate based on the principles of quantum mechanics, using quantum bits (qubits) that can represent and process multiple states simultaneously due to superposition and entanglement. Additionally, quantum machine learning is an emerging field that seeks to develop quantum algorithms capable of outperforming traditional methods for tasks like data classification, regression, and anomaly detection.

To assess the above-discussed computing techniques, we discuss some characteristics as follows:

Data-Driven and Data Management $\mathbb{R}_1$:

AI techniques rely heavily on data for training and decision-making, using large datasets to identify patterns, trends, and insights. Advanced computing techniques enable storing, retrieving, and analyzing vast amounts of data to facilitate a better decision-making process.

Natural Interaction $\mathbb{R}_2$:

AI systems often incorporate natural language processing (NLP) and computer vision, enabling more intuitive user interactions through voice, text, and visual inputs.

Enhanced Decision-Making $\mathbb{R}_3$:

AI algorithms can quickly analyze vast amounts of data, providing insights and recommendations that aid in informed decision-making.

Enhanced Security and Advanced Problem-Solving $\mathbb{R}_4$:

AI can quickly detect anomalies and potential threats, improving cybersecurity measures and protecting sensitive data. AI can tackle complex problems that are difficult for humans to solve, such as optimizing logistics, supply chains, and manage resources.

Simulation and Modeling $\mathbb{R}_5$:

Computing enables the simulation of complex systems, enabling researchers and engineers to test scenarios without the need for physical prototypes.

The evaluation processes for the suitable optimal option are completed under the following decision algorithms of the EDAS method and MADM problem.

6.2. Evaluation procedure based on the EDAS method

To assess an appropriate optimal option under-discussed attributes or characteristics in the case study, we utilize the steps of an algorithm from the EDAS method as follows.

Step 1. The expert organizes their opinion in the form of Crq-ROFVs, where this information has different attributes corresponding to each alternative. We arrange Cqr-ROFVs in the decision matrix of Table 2. Here, i^{th} and j^{th} indicate alternatives and criteria.

Table 2. Expert opinion in the form of Crq-ROFVs.

	$\mathbb{R}_1$	$\mathbb{R}_2$	$\mathbb{R}_3$	$\mathbb{R}_4$	$\mathbb{R}_5$
$\mathbb{Q}_1$	$(0.37, 0.65, 0.42)$	$(0.37, 0.36, 0.73)$	$(0.81, 0.28, 0.14)$	$(0.28, 0.38, 0.29)$	$(0.23, 0.47, 0.17)$
$\mathbb{Q}_2$	$(0.38, 0.53, 0.27)$	$(0.34, 0.54, 0.65)$	$(0.73, 0.36, 0.29)$	$(0.36, 0.87, 0.39)$	$(0.34, 0.64, 0.32)$
$\mathbb{Q}_3$	$(0.76, 0.34, 0.36)$	$(0.27, 0.43, 0.84)$	$(0.81, 0.45, 0.54)$	$(0.65, 0.54, 0.32)$	$(0.42, 0.56, 0.39)$
$\mathbb{Q}_4$	$(0.65, 0.65, 0.76)$	$(0.54, 0.87, 0.54)$	$(0.72, 0.27, 0.25)$	$(0.43, 0.49, 0.29)$	$(0.54, 0.32, 0.54)$
$\mathbb{Q}_5$	$(0.43, 0.32, 0.55)$	$(0.43, 0.64, 0.42)$	$(0.54, 0.54, 0.35)$	$(0.37, 0.54, 0.53)$	$(0.18, 0.58, 0.38)$

Step 2. In the discussed case study, the decision maker organized their opinion in the form of beneficial attributes. Thus, there is no need to normalize given information for the same type of attributes.

Step 3. Compute averaging solutions associated with each alternative under different attribute information using the expression of Crq-ROFFWA$_t$ operator. Table 3 presents the evaluated information. We also copmute score values of each averaging solution and attribute information, which are also listed in Table 3.

Table 3. Results by the Crq-ROFFWA$_t$ operator and score values of each attribute.

	Results of averaging solution	$\mathcal{S}(\mathbb{Q}_i)$	$\mathcal{S}(\mathbb{R}_1)$	$\mathcal{S}(\mathbb{R}_2)$	$\mathcal{S}(\mathbb{R}_3)$	$\mathcal{S}(\mathbb{R}_4)$	$\mathcal{S}(\mathbb{R}_5)$
$\mathbb{Q}_1$	$(0.5385, 0.4123, 0.4784)$	0.5321	0.3463	0.6062	0.5193	0.3643	0.2457
$\mathbb{Q}_2$	$(0.4976, 0.5695, 0.4419)$	0.4393	0.3204	0.5110	0.5520	0.1357	0.2886
$\mathbb{Q}_3$	$(0.6616, 0.4575, 0.5907)$	0.6403	0.6241	0.6182	0.7398	0.4586	0.3908
$\mathbb{Q}_4$	$(0.5979, 0.4804, 0.5548)$	0.5781	0.6164	0.2691	0.5303	0.3617	0.5820
$\mathbb{Q}_5$	$(0.4227, 0.5120, 0.4617)$	0.4511	0.5478	0.3669	0.41830	0.4614	0.3412

Step 4. Based on the above-computed results of averaging solution and score values of each attribute information, the obtained results of PDAS and NDAS based on the following expression are:

$$PDAS_{j\ell} = \frac{\max\left(0, \left(\mathcal{S}(\mathbb{R}_{j\ell}) - \mathcal{S}(\mathbb{Q}_j)\right)\right)}{\mathcal{S}(\mathbb{Q}_j)},$$

and

$$NDAS_{j\ell} = \frac{\max\left(0, \left(\mathcal{S}(\mathbb{Q}_j) - \mathcal{S}(\mathbb{R}_{j\ell})\right)\right)}{\mathcal{S}(\mathbb{Q}_j)}.$$

Step 5. Now, we calculate the PWD and NWD based on the following expressions. The results are shown in Table 4.

$$PWD_\ell = \sum_{\ell=1}^{n} \mathfrak{w}_\ell PDAS_{j\ell} \text{ and } NWD_\ell = \sum_{\ell=1}^{n} \mathfrak{w}_\ell NDAS_{j\ell}.$$

Table 4. Representation of PDAS and NDAS matrices.

	Outcomes of the PDAS matrix					Outcomes of the NDAS matrix				
	$\mathbb{R}_1$	$\mathbb{R}_2$	$\mathbb{R}_3$	$\mathbb{R}_4$	$\mathbb{R}_5$	$\mathbb{R}_1$	$\mathbb{R}_2$	$\mathbb{R}_3$	$\mathbb{R}_4$	$\mathbb{R}_5$
$\mathbb{Q}_1$	0.0000	0.1391	0.0000	0.0000	0.0000	0.3493	0.0000	0.0241	0.3154	0.5382
$\mathbb{Q}_2$	0.0000	0.0000	0.0373	0.0000	0.0000	0.3979	0.0397	0.0000	0.7451	0.4577
$\mathbb{Q}_3$	0.1728	0.1616	0.3902	0.0000	0.0000	0.0000	0.0000	0.0000	0.1382	0.2656
$\mathbb{Q}_4$	0.1584	0.0000	0.0000	0.0000	0.0936	0.0000	0.4943	0.0034	0.3203	0.0000
$\mathbb{Q}_5$	0.0294	0.0000	0.0000	0.0000	0.0000	0.0000	0.3104	0.2139	0.1330	0.3587

Step 6. Normalized results of PWD_ℓ and NWD_ℓ based on the following expressions (the aggregated output is listed in Table 5):

$$NPWD_\ell = \frac{PWD_\ell}{\max(PWD_\ell)} \text{ and } NNWD_\ell = 1 - \frac{NWD_\ell}{\max(NWD_\ell)}.$$

Table 5. Normalized results and appraisal values.

	SP_i	SN_i	NSP_i	NSN_i	AS_i	Ranking
$\mathbb{Q}_1$	0.0278	0.2454	0.1919	0.2520	0.2220	3
$\mathbb{Q}_2$	0.0075	0.3281	0.0514	0.0000	0.0257	5
$\mathbb{Q}_3$	0.1449	0.0808	1.0000	0.7538	0.8769	1
$\mathbb{Q}_4$	0.0504	0.1636	0.3478	0.5013	0.4246	2
$\mathbb{Q}_5$	0.0059	0.2032	0.0405	0.3806	0.2106	4

Step 7. Based on the above results, we compute the results of appraisal solution AS_i. Theaggregated outcomes are listed in Table 5.

Using the above procedure, we also investigate the results of all alternatives based on other derived approaches like $\mathrm{Crq}-\mathrm{ROFFWA}_{\mathrm{tc}}$, $\mathrm{Crq}-\mathrm{ROFFWG}_{\mathrm{t}}$, and $\mathrm{Crq}-\mathrm{ROFFWG}_{\mathrm{tc}}$ operators. Table 6 carries the ranking of all individuals based on computed results by the derived terminologies. Furthermore, the graphic representation also demonstrates the results of all individuals in Figure 6. This figure offers more understanding of the aggregated outcomes shown in Table 6.

Table 6. Ranking of preferences.

Aggregation Operators	Ranking of preferences by EDAS method
$Crq-ROFFWA_t$	$\mathbb{Q}_3 \succ \mathbb{Q}_4 \succ \mathbb{Q}_1 \succ \mathbb{Q}_5 \succ \mathbb{Q}_2$
$Crq-ROFFWA_{tc}$	$\mathbb{Q}_3 \succ \mathbb{Q}_4 \succ \mathbb{Q}_5 \succ \mathbb{Q}_1 \succ \mathbb{Q}_2$
$Crq-ROFFWG_t$	$\mathbb{Q}_3 \succ \mathbb{Q}_4 \succ \mathbb{Q}_5 \succ \mathbb{Q}_1 \succ \mathbb{Q}_2$
$Crq-ROFFWG_{tc}$	$\mathbb{Q}_3 \succ \mathbb{Q}_4 \succ \mathbb{Q}_5 \succ \mathbb{Q}_1 \succ \mathbb{Q}_2$

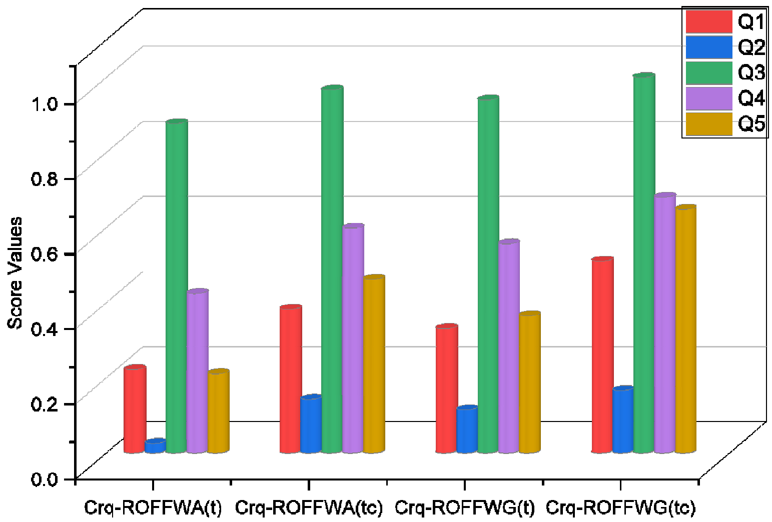

Figure 6. The results of score values corresponding to each alternative.

6.3. Evaluation procedure based on the MADM method

To prove the validity and effectiveness of the derived mathematical terminologies, we also

investigate an ideal solution using the discussed algorithm of the MADM problem.
Step 1. Aggregate judgments of the expert's opinion listed in Table 2 using proposed methodologies and the steps of the MADM problem. Table 7 contains aggregated outputs by the derived approaches.

Table 7. Representation of accumulated outcomes.

	$Crq-ROFFWA_t$	$Crq-ROFFWA_{tc}$	$Crq-ROFFWG_t$	$Crq-ROFFWG_{tc}$
$\mathbb{Q}_1$	(0.5385,0.4123,0.4784)	(0.5385,0.4123,0.2944)	(0.3758,04703,0.4784)	(0.3758,0.4703,0.2944)
$\mathbb{Q}_2$	(0.4976,0.5695,0.4419)	(0.4976,0.5695,0.3650)	(0.4119,0.6598,0.4419)	(0.4119,0.6598,0.3650)
$\mathbb{Q}_3$	(0.6616,0.4575,0.5907)	(0.6616,0.4575,0.4634)	(0.5456,0.4786,0.5907)	(0.5456,0.4786,0.4634)
$\mathbb{Q}_4$	(0.5979,0.4804,0.5548)	(0.5979,0.4804,0.4415)	(0.5688,0.6372,0.5548)	(0.5688,0.6372,0.4415)
$\mathbb{Q}_5$	(0.4227,0.5120,0.4617)	(0.4227,0.5120,0.4395)	(0.3680,0.5467,0.4617)	(0.3680,0.5467,0.4395)

Step 2. Obtain score values of all individuals using the theory of score function discussed in Definition 4. The estimated results are shown in Table 8.

Table 8. Score functions by the derived methodologies.

	$Crq-ROFFWA_t$	$Crq-ROFFWA_{tc}$	$Crq-ROFFWG_t$	$Crq-ROFFWG_{tc}$
$\mathbb{Q}_1$	0.5321	0.4267	0.4636	0.3582
$\mathbb{Q}_2$	0.4393	0.3965	0.3614	0.3185
$\mathbb{Q}_3$	0.6403	0.5782	0.5698	0.5077
$\mathbb{Q}_4$	0.5781	0.5213	0.4894	0.4325
$\mathbb{Q}_5$	0.4511	0.4394	0.4237	0.4120

Step 3. In this step, we maintain the ranking of alternatives in descending order, as shown in Table 9.

From Table 9, the most preferable decision is $\mathbb{Q}_3$, which represents the reliable, optimal option. Furthermore, we also list all computed results in a graphical representation in Figure 7.

Table 9. Ordering of preferences.

Aggregation operators	Ranking of preferences
$Crq-ROFFWA_t$	$\mathbb{Q}_3 \succ \mathbb{Q}_4 \succ \mathbb{Q}_1 \succ \mathbb{Q}_5 \succ \mathbb{Q}_2$
$Crq-ROFFWA_{tc}$	$\mathbb{Q}_3 \succ \mathbb{Q}_4 \succ \mathbb{Q}_5 \succ \mathbb{Q}_1 \succ \mathbb{Q}_2$
$Crq-ROFFWG_t$	$\mathbb{Q}_3 \succ \mathbb{Q}_4 \succ \mathbb{Q}_1 \succ \mathbb{Q}_5 \succ \mathbb{Q}_2$
$Crq-ROFFWG_{tc}$	$\mathbb{Q}_3 \succ \mathbb{Q}_4 \succ \mathbb{Q}_5 \succ \mathbb{Q}_1 \succ \mathbb{Q}_2$

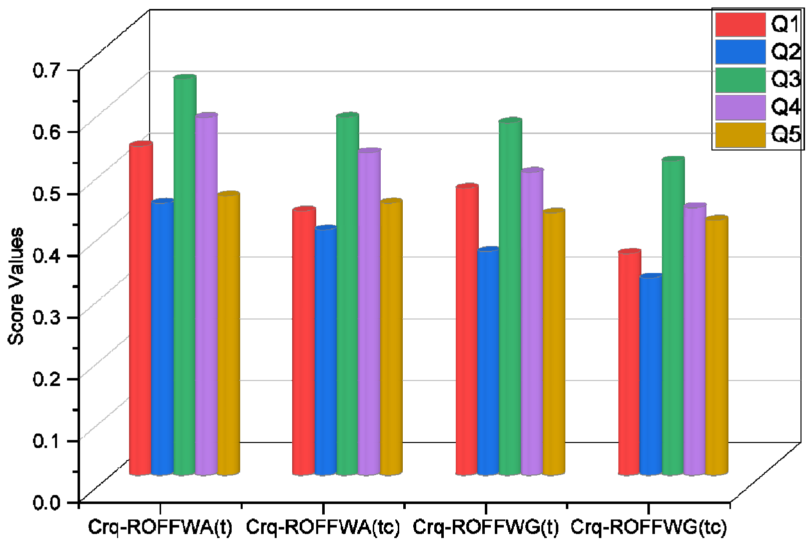

Figure 7. The results of score values corresponding to each alternative.

From the above analysis, we conclude that the machine learning $\mathbb{Q}_3$ is an appropriate computing technique based on the discussed methodologies. Machine learning $\mathbb{Q}_3$ is a feasible computing technique investigating both decision-making techniques of the EDAS method and MADM problem using derived mathematical approaches. Thus, proposed theories and terminologies are superior to existing approaches.

6.4. Impact of different variables on the results of decision-making problems

In this subsection, we illustrate the validation and compatibility of mathematical approaches and decision-making problems. To serve this purpose, we apply different parametric variables during the aggregation process using algorithms of the MADM problem. By setting various variables from 2 to 100 in discussed aggregation operators and decision-making problems, we obtain results of score functions corresponding to each alternative. To better explain this, we demonstrate the results of score functions as geometric representations in Figures 8–11. From geometric representations, we see consistency in aggregated results and ranking of preferences. This consistency shows the reliability and efficiency of the derived mathematical methodologies and optimization techniques.

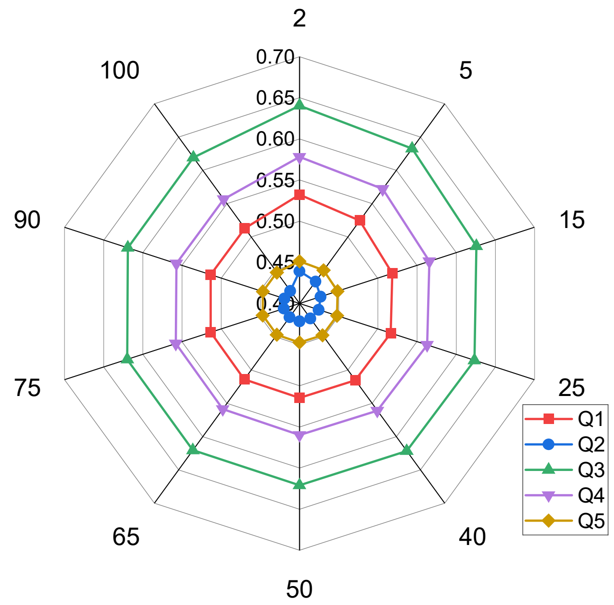

Figure 8. Results obtained by the $Crq-ROFFWA_t$ operator.

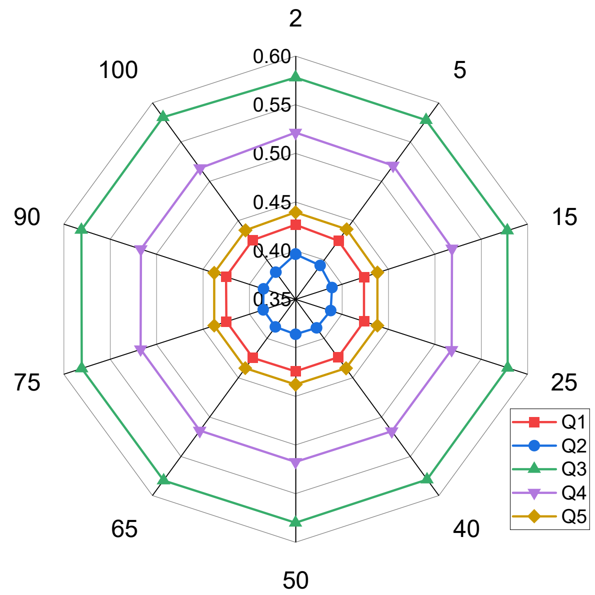

Figure 9. Results obtained by the $Crq-ROFFWA_{tc}$ operator.

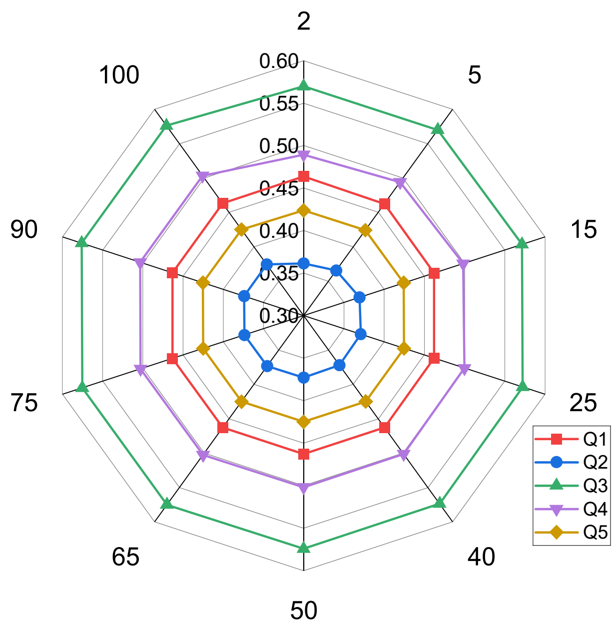

Figure 10. Results obtained by the $Crq-ROFFWG_t$ operator.

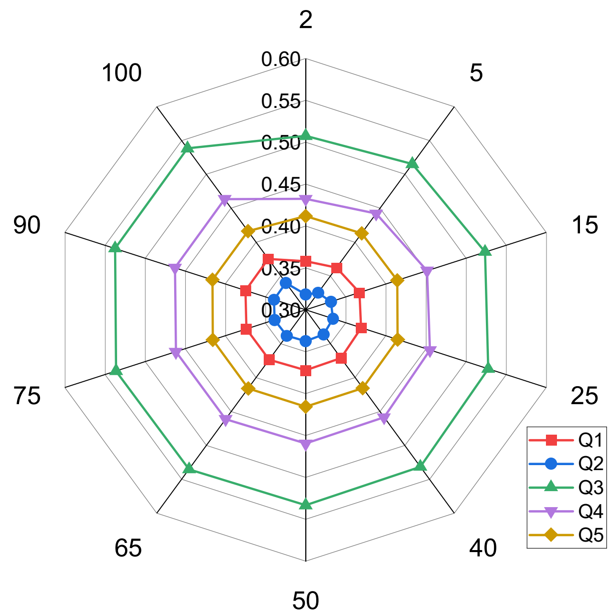

Figure 11. Results obtained by the $Crq-ROFFWG_{tc}$ operator.

7. Comparative study

To prove the validation and supremacy of discussed theories, we compare the ranking of results obtained by terminologies and pioneered aggregation operators. In this section, we also explore some feasible advantages of advanced and robust decision-making problems like the EDAS method for the MADM problem. To serve this purpose, we apply existing approaches [54–57] for the judgment of expert opinions listed in Table 2. Ali and Yang [54] proposed mathematical methodologies of Dombi t-norms considering the theory of the Crq-ROF situation, including Crq-ROF Dombi weighted averaging (Crq-ROFDWA) and Crq-ROF Dombi weighted geometric (Crq-ROFDWG) operators. Hussain et al. [55] proposed Aczel Alsina aggregation operators for a complex spherical fuzzy theory and real-life applications. Ma et al. [56] utilized the properties of Bonferroni mean models to derive a new family of picture-fuzzy approaches with the decision support system. Garg et al. [57] developed Aczel Alsina aggregation operators to investigate the unknown degree of weights by considering bipolar fuzzy information. We applied existing mathematical approaches on given information of Table 2 strategies, and Table 10 presents the ranking of alternatives based on the estimated output from previous mathematical approaches.

Table 10. Results of previous methodologies.

Aggregation Operators	Ranking of preferences
$\boldsymbol{Crq-ROFFWA_t}$	$\mathbb{Q}_3 \succ \mathbb{Q}_4 \succ \mathbb{Q}_1 \succ \mathbb{Q}_5 \succ \mathbb{Q}_2$
$\boldsymbol{Crq-ROFFWA_{tc}}$	$\mathbb{Q}_3 \succ \mathbb{Q}_4 \succ \mathbb{Q}_5 \succ \mathbb{Q}_1 \succ \mathbb{Q}_2$
$\boldsymbol{Crq-ROFFWG_t}$	$\mathbb{Q}_3 \succ \mathbb{Q}_4 \succ \mathbb{Q}_1 \succ \mathbb{Q}_5 \succ \mathbb{Q}_2$
$\boldsymbol{Crq-ROFFWG_{tc}}$	$\mathbb{Q}_3 \succ \mathbb{Q}_4 \succ \mathbb{Q}_5 \succ \mathbb{Q}_1 \succ \mathbb{Q}_2$
$\boldsymbol{Crq-ROFDWA_t}$ **[54]**	$\mathbb{Q}_3 \succ \mathbb{Q}_2 \succ \mathbb{Q}_4 \succ \mathbb{Q}_1 \succ \mathbb{Q}_5$
$\boldsymbol{Crq-ROFDWA_{tc}}$ **[54]**	$\mathbb{Q}_3 \succ \mathbb{Q}_2 \succ \mathbb{Q}_4 \succ \mathbb{Q}_1 \succ \mathbb{Q}_5$
$\boldsymbol{Crq-ROFDWG_t}$ **[54]**	$\mathbb{Q}_3 \succ \mathbb{Q}_2 \succ \mathbb{Q}_4 \succ \mathbb{Q}_1 \succ \mathbb{Q}_5$
$\boldsymbol{Crq-ROFDWG_{tc}}$ **[54]**	$\mathbb{Q}_3 \succ \mathbb{Q}_2 \succ \mathbb{Q}_4 \succ \mathbb{Q}_1 \succ \mathbb{Q}_5$
Hussain et al. **[55]**	Limited structure
Ma et al. **[56]**	Limited structure
Garg et al. **[57]**	Limited structure

From Table 10, we observe some theories that failed due to the limited structures of fuzzy models. However, the Crq-ROFS is an effective and robust model for accurately handling vague information because this model has extensive information about any object with the DoMV, DoNMV, and the radius of both terms. The EDAS method is also a reliable optimization decision-making technique based on Crq-ROFS. Keeping in mind the significance of the proposed models, we concluded that the diagnosed methodologies are compatible and superior to existing mathematical models.

Advantages of diagnosed mathematical methodologies:

a) The terminologies of Crq-ROFS are a more efficient approach to coping with uncertain and vague expert judgments. The IFSs and q-ROFSs are restricted environments and may lose information during aggregation.
b) Mathematical approaches of Frank aggregation operators are used to integrate expert's opinions more accurately than others.
c) The theory of the EDAS method is applied to investigate the ranking of alternatives based on the MADM problem and different criteria. This approach investigates ranking alternatives using

positive and negative solutions using different criteria or attribute information.

d) In this article, we discuss different AI computing techniques under consideration of various criteria and beneficial key features.

8. Conclusions

We propose some robust mathematical strategies of the Crq-ROF framework to handle incomplete and redundant human information using advanced optimization theories. The Crq-ROFS is more convenient and effective than IFSs, q-ROFSs, and Cr-IFSs. Further, we discuss the drawbacks of existing operations and terminologies of Frank triangular norms. Based on modified operations of Frank triangular norms, we develop appropriate mathematical models of the Crq-ROF context, such as the Crq-ROFFWA and Crq-ROFFWG operators. Some novel properties and special cases also prove the worth and applicability of discussed approaches. In this article, we discuss two decision-making approaches, the EDAS method and the MADM problem, for resolving real-life applications and enhancing the worth of diagnosed theories. Some numerical examples show the reliability and effectiveness of the presented optimization models. Finally, we evaluate an appropriate optimal option from different artificial intelligence techniques to measure the performance of intelligent computing. Furthermore, a robust comparison method is conducted to compare the results of existing terminologies with pioneered approaches.

From the discussed evaluation process and decision-making problems, we investigate machine learning and artificial intelligence neural networks as the best and 2nd best AI-based computing tools, respectively. The experimental case study is examined under some criteria and diagnosed mathematical terminologies.

In the future, we can modify the theory of circular q-rung orthopair fuzzy Hamacher operators, Bonferroni mean models, and fuzzy graph theory. Furthermore, we can apply derived mathematical approaches to resolve real-life applications related to artificial intelligence, healthcare systems, and supplier selection to enhance the worth and applicability of the discussed theories.

Use of Generative-AI tools declaration

The author declares he has not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The author declares no conflict of interest.

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