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Research article

Generalized confidence interval for the common coefficient of variation of several zero-inflated Birnbaum-Saunders distributions with an application to wind speed data

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Abstract: Wind speed is a critical factor that affects various aspects of life in Thailand, particularly agriculture, which is a fundamental component of the Thai economy. Therefore, studying and understanding wind speed is essential for planning and developing the country's economy sustainably. Wind speed data often include both positive and zero values, which are consistent with the zero-inflated Birnbaum-Saunders distribution. Additionally, the inherent variability of wind speed poses challenges for accurate prediction. When analyzing data from multiple weather stations, the common coefficient of variation helps compare the wind variability at each station, even if the average wind speeds differ. Therefore, the estimation of the common coefficient of variation allows for reliable statistical inference and decision-making. In this article, we proposed five methods to construct confidence intervals for the common coefficient of variation of several zero-inflated Birnbaum-Saunders distributions. These methods include the generalized confidence interval, the method of variance estimation recovery, the large sample approximation, the bootstrap confidence interval, and the fiducial generalized confidence interval. We evaluated the performance of these methods using a comprehensive simulation study and compared them in terms of coverage probabilities and average widths. The results revealed that overall, the generalized confidence interval and the bootstrap confidence interval are the most effective and perform better than other methods in various situations. Finally, we applied these proposed methods to wind speed data from Thailand.

Keywords: average width; confidence interval; coefficient of variation; coverage probability; zero-inflated Birnbaum-Saunders; wind speed

Mathematics Subject Classification: 62F25, 62P12

1. Introduction

In meteorology, wind speed is a fundamental atmospheric quantity caused by the movement of air from high to low pressure, usually due to temperature variations. Alongside wind speed, wind direction plays a pivotal role in the analysis and prediction of weather patterns and the global climate. Wind speed and direction significantly affect factors such as evaporation rates, sea surface turbulence, and the formation of oceanic waves and storms. Moreover, these factors have substantial impacts on water quality, water levels, and various fields, including weather forecasting, climatology, renewable energy, environmental monitoring, aviation, and agriculture. Therefore, a comprehensive understanding of wind speed is important for managing potential impacts. Additionally, using wind speed data for analysis can help researchers and experts understand and address issues related to wind speed across various areas. For wind speed data, a normal distribution may not be appropriate, even though the normal distribution is one of the most widely used statistical distributions. If the data exhibits skewness, it is advisable to consider alternative distributions. Many new distributions have been developed using certain transformations from the normal distribution. One such distribution is the Birnbaum-Saunders distribution. Importantly, Mohammadi, Alavi, and McGowan [1] investigated the application of the two-parameter Birnbaum-Saunders distribution for analyzing wind speed and wind energy density at ten different stations in Canada. Their results demonstrated that the Birnbaum-Saunders distribution was especially effective at all the chosen locations. The Birnbaum-Saunders distribution was introduced by Birnbaum and Saunders [2] for the purpose of modeling the fatigue life of metals subjected to periodic stress. As a result, this distribution is sometimes referred to as the fatigue life distribution. The Birnbaum-Saunders distribution has been applied in various contexts, such as engineering, testing, medical sciences, and environmental studies. It is well known that the Birnbaum-Saunders distribution is a positive skewed one. However, some data to be analyzed may have both positive and zero values. Therefore, if zero observations follow a binomial distribution combined with the Birnbaum-Saunders distribution, the resulting distribution is the zero-inflated Birnbaum-Saunders (ZIBS) distribution, which is a new and interesting distribution. This ZIBS distribution was inspired by Aitchison [3], and several researchers have studied the combination of zero observations with other distributions to form new distributions, such as the zero-inflated lognormal distribution [4], the zero-inflated gamma distribution [5], and the zero-inflated two-parameter exponential distribution [6].

The coefficient of variation (CV) of wind speed is important for several reasons. Since the CV measures the dispersion of data relative to the mean, it is expressed as the ratio of the standard deviation to the mean. The CV assesses the variability of a dataset, regardless of the unit of measurement. Additionally, using the CV to evaluate wind speed is beneficial in various contexts. For instance, calculating the CV helps in understanding how much wind speed fluctuates compared to its average. If the CV is high, it indicates that the wind speed is highly variable, making it more difficult to predict wind conditions. In the context of wind energy, the CV can help assess the reliability of energy sources. If wind speed variability is high, it may result in inconsistent energy production, which could affect the stability of energy output from wind farms. Additionally, the coefficient of variation has been used in many fields, including life insurance, science, economics, and medicine. Importantly, many researchers have constructed confidence intervals (CIs) for the coefficient of variation, which have been applied to various distributions. For example, Vangel [7] constructed the CIs for a normal distribution coefficient of variation. Buntao and Niwitpong [8] introduced the CIs for the coefficient of variation of zero-inflated lognormal and lognormal distributions. D'Cunha and Rao [9] proposed the Bayes estimator and created CIs for the coefficient of variation of the lognormal distribution. Sangnawakij and Niwitpong [10] developed CIs for coefficients of variation in two-parameter exponential distributions. Janthasuwan, Niwitpong, and Niwitpong [11] established CIs for the coefficients of variation in the zero-inflated Birnbaum-Saunders distribution.

In the analysis and comparison of wind variability across multiple weather stations or wind directions, without needing to account for the differences in average wind speed at each station or direction, it is necessary to use the common CV. The common CV provides a single indicator representing the overall variability of wind speed, which is crucial when planning wind energy projects, designing wind turbines, or calculating the power production of wind farms that require knowledge of wind stability across different areas. Additionally, the common CV is useful in meteorological and climatological research, as it allows for the analysis of wind variability across multiple regions simultaneously. It can also assist in examining the relationship between wind variability and long-term climate changes or recurring events, such as storms or shifts in wind patterns. Therefore, the common coefficient of variation is a crucial aspect when making inferences for more than one population. This holds particularly true when collecting independent samples from various situations. Consequently, numerous researchers have investigated methods for computing the common coefficient of variation in several populations from variety distributions. For instance, Tian [12] made inferences about the coefficient of variation of a common population within a normal distribution. Then, Forkman [13] studied methods for constructing CIs and statistical tests based on McKay's approximation for the common coefficient of variation in several populations with normal distributions. Sangnawakij and Niwitpong [14] proposed the method of variance of estimate recovery to construct CIs for the common coefficient of variation for several gamma distributions. Next, Singh et al. [15] used several inverse Gaussian populations to estimate the common coefficient of variation, test the homogeneity of the coefficient of variation, and test for a specified value of the common coefficient of variation. After that, Yosboonruang, Niwitpong, and Niwitpong [16] presented methods to construct CIs for the common coefficient of variation of zero-inflated lognormal distributions, employing the method of variance estimate recovery, equal-tailed Bayesian intervals, and the fiducial generalized confidence interval. Finally, Puggard, Niwitpong, and Niwitpong [17] introduced Bayesian credible intervals, highest posterior density intervals, the method of variance estimate recovery, generalized confidence intervals, and large-sample methods to construct confidence intervals for the common coefficient of variation in several Birnbaum-Saunders distributions. Previous research has shown that no studies have investigated the estimation of the common coefficient of variation in the context of several ZIBS distributions. Therefore, the primary objective of this article is to determine the CIs for the common coefficient of variation of several ZIBS distributions. The article presents five distinct methods: the generalized confidence interval, the method of variance estimates recovery, the large sample approximation, the bootstrap confidence interval, and the fiducial generalized confidence interval.

2. Materials and methods

Let $Y_{ij}, i = 1,2, \dots, k$ and $j = 1,2, \dots, m_i$ be a random sample drawn from the ZIBS distributions. The density function of Y_{ij} is given by

$$f(y_{ij}; \vartheta_i, \alpha_i, \beta_i) = \vartheta_i \mathrm{I}_0[y_{ij}] + (1-\vartheta_i)\frac{1}{2\alpha_i\beta_i\sqrt{2\pi}}\left[\left(\frac{\beta_i}{y_{ij}}\right)^{1/2} + \left(\frac{\beta_i}{y_{ij}}\right)^{3/2}\right]$$

$$\times \exp\left[-\frac{1}{2}\left(\frac{y_{ij}}{\beta_i} + \frac{\beta_i}{y_{ij}} - 2\right)\right]\mathrm{I}_{(0,\infty)}[y_{ij}],$$

where ϑ_i, α_i, and β_i are the proportion of zero, shape, and scale parameters, respectively. I is an indicator function, with $\mathrm{I}_0[y_{ij}] = \begin{cases} 1; & y_{ij} = 0, \\ 0; & \text{otherwise,} \end{cases}$ and $\mathrm{I}_{(0,\infty)}[y_{ij}] = \begin{cases} 0; & y_{ij} = 0, \\ 1; & y_{ij} > 0 \end{cases}$. This distribution is a combination of Birnbaum-Saunders and binomial distributions. Suppose that $m_i = m_{i(1)} + m_{i(0)}$ is the sample size, where $m_{i(1)}$ and $m_{i(0)}$ are the numbers of positive and zero values, respectively. For the expected value and variance of Y_{ij}, we have applied the concepts from Aitchison [3], which can be expressed as follows:

$$E(Y_{ij}) = (1-\vartheta_i)\beta_i\left(1 + \frac{\alpha_i^2}{2}\right)$$

and

$$V(Y_{ij}) = (1-\vartheta_i)(\alpha_i\beta_i)^2\left(1 + \frac{5\alpha_i^2}{4}\right) + \vartheta_i(1-\vartheta_i)\beta_i^2\left(1 + \frac{\alpha_i^2}{2}\right)^2,$$

respectively. Hence, the coefficient of variation of Y_{ij} is defined as

$$\theta = \frac{1}{2+\alpha_i^2}\sqrt{\frac{\alpha_i^2(4+5\alpha_i^2) + \vartheta_i(2+\alpha_i^2)^2}{1-\vartheta_i}}.$$

The asymptotic distribution of $\hat{\vartheta}_i$ is calculated by using the delta method, which is given by $\sqrt{m_i}(\hat{\vartheta}_i - \vartheta_i) \sim N(0, \vartheta_i(1-\vartheta_i))$, where $\hat{\vartheta}_i = m_{i(0)}/m_i$. According to Ng, Kundu, and Balakrishnan [18], the asymptotic joint distribution of $\hat{\alpha}_i$ and $\hat{\beta}_i$ is obtained as

$$\begin{pmatrix} \hat{\alpha}_i \\ \hat{\beta}_i \end{pmatrix} \sim N\left(\begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix}, \begin{pmatrix} \frac{\alpha_i^2}{2m_{i(1)}} & 0 \\ 0 & \frac{(\alpha_i\beta_i)^2}{m_{i(1)}}\left(\frac{1+\frac{3}{4}\alpha_i^2}{\left(1+\frac{1}{2}\alpha_i^2\right)^2}\right) \end{pmatrix}\right),$$

where $\hat{\alpha}_i = \left\{2\left[\left(\bar{y}_i \sum_{j=1}^{m_{i(1)}} \frac{y_{ij}^{-1}}{m_{i(1)}}\right)^{\frac{1}{2}} - 1\right]\right\}^{\frac{1}{2}}$, $\hat{\beta}_i = \left\{\bar{y}_i \left(\sum_{j=1}^{m_{i(1)}} \frac{y_{ij}^{-1}}{m_{i(1)}}\right)^{-1}\right\}^{\frac{1}{2}}$, and $\bar{y}_i = \sum_{j=1}^{m_{i(1)}} \frac{y_{ij}}{m_{i(1)}}$. The estimator of θ_i is given by

$$\hat{\theta}_i = \frac{1}{2+\hat{\alpha}_i^2}\sqrt{\frac{\hat{\alpha}_i^2(4+5\hat{\alpha}_i^2)+\hat{\vartheta}_i(2+\hat{\alpha}_i^2)^2}{1-\hat{\vartheta}_i}}. \tag{1}$$

According to Janthasuwan, Niwitpong, and Niwitpong [11], the asymptotic variance of $\hat{\theta}_i$, derived using the Taylor series in the delta method, is given by

$$V(\hat{\theta}_i) \approx \frac{1}{\Psi_i}\left\{\frac{32\alpha_i^4(1+2\alpha_i^2)^2}{m_{i(1)}(2+\alpha_i^2)^2} + \frac{\vartheta_i[2+\alpha_i^2(4+3\alpha_i^2)]^2}{m_i(1-\vartheta_i)}\right\}, \tag{2}$$

where $\Psi_i = (2+\alpha_i^2)^2(1-\vartheta_i)[\alpha_i^2(4+5\alpha_i^2)+\vartheta_i(2+\alpha_i^2)^2]$. According to Graybill and Deal [19], the common CV of several ZIBS distributions can be written as

$$\hat{\theta} = \frac{\sum_{i=1}^k \hat{\theta}_i/\hat{V}(\hat{\theta}_i)}{\sum_{i=1}^k 1/\hat{V}(\hat{\theta}_i)}, \tag{3}$$

where $\hat{V}(\hat{\theta}_i)$ denotes the estimator of $V(\hat{\theta}_i)$, which is defined in Eq (2) with α_i and ϑ_i replaced by $\hat{\alpha}_i$ and $\hat{\vartheta}_i$, respectively. This can be expressed as follows:

$$\hat{V}(\hat{\theta}_i) \approx \frac{1}{\hat{\Psi}_i}\left\{\frac{32\hat{\alpha}_i^4(1+2\hat{\alpha}_i^2)^2}{m_{i(1)}(2+\hat{\alpha}_i^2)^2} + \frac{\hat{\vartheta}_i[2+\hat{\alpha}_i^2(4+3\hat{\alpha}_i^2)]^2}{m_i(1-\hat{\vartheta}_i)}\right\},$$

where $\hat{\Psi}_i = (2+\hat{\alpha}_i^2)^2(1-\hat{\vartheta}_i)[\hat{\alpha}_i^2(4+5\hat{\alpha}_i^2)+\hat{\vartheta}_i(2+\hat{\alpha}_i^2)^2]$.

The following subsection provides detailed explanations of the methods employed for constructing confidence intervals.

2.1. Generalized confidence interval

Weerahandi [20] recommended the generalized confidence interval (GCI) method for constructing confidence intervals, which is based on the concept of a generalized pivotal quantity (GPQ). To construct the confidence interval for θ using the GCI, we get the generalized pivotal quantities for the parameters β_i, α_i, and ϑ_i. Sun [21] introduced the GPQ for the scale parameter β_i, which can be derived as

$$G_{\beta_i}(y_{ij};\Lambda_i) = \begin{cases} max(\beta_{i1},\beta_{i2})\,; \Lambda_i \le 0; \\ min(\beta_{i1},\beta_{i2})\,; \Lambda_i > 0, \end{cases} \tag{4}$$

where Λ_i follows the t-distribution with $m_{i(1)} - 1$ degrees of freedom. β_{i1} and β_{i2} are the two solutions of the following quadratic equation:

$$\Omega_1\beta_i^{\,2} - 2\Omega_2\beta_i + \left(m_{i(1)} - 1\right)C_i^{\,2} - \frac{1}{m_{i(1)}}D_i\Lambda_i^2 = 0,$$

where $\Omega_1 = \left(m_{i(1)} - 1\right)A_i^2 - \frac{1}{m_{i(1)}}B_i\Lambda_i^2$, $\Omega_2 = \left(m_{i(1)} - 1\right)A_iC_i - (1 - A_iC_i)\Lambda_i^2$, $A_i = \frac{1}{m_{i(1)}}\sum_{j=1}^{m_{i(1)}}\frac{1}{\sqrt{Y_{ij}}}$, $B_i = \sum_{j=1}^{m_{i(1)}}\left(\frac{1}{\sqrt{Y_{ij}}} - A_i\right)^2$, $C_i = \frac{1}{m_{i(1)}}\sum_{j=1}^{m_{i(1)}}\sqrt{Y_{ij}}$, and $D_i = \sum_{j=1}^{m_{i(1)}}\left(\sqrt{Y_{ij}} - C_i\right)^2$. Next, considering the GPQ for the shape parameter α_i as proposed by Wang [22], the GPQ for α_i is derived as

$$G_{\alpha_i}\left(y_{ij}; K_i, \Lambda_i\right) = \sqrt{\frac{E_{i1} + E_{i2}G_{\beta_i}^2\left(y_{ij};\Lambda_i\right) - 2m_{i(1)}G_{\beta_i}\left(y_{ij};\Lambda_i\right)}{G_{\beta_i}\left(y_{ij};\Lambda_i\right)K_i}}, \tag{5}$$

where $E_{i1} = \sum_{j=1}^{m_{i(1)}}Y_{ij}$, $E_{i2} = \sum_{j=1}^{m_{i(1)}}\frac{1}{Y_{ij}}$, and K_i follows the chi-squared distribution with $m_{i(1)}$ degrees of freedom. Subsequently, the GPQ for the proportion of zero ϑ_i was recommended by Wu and Hsieh [23], who proposed using the GPQ based on the variance stabilized transformation to construct confidence intervals. Therefore, the GPQ for ϑ_i is defined as

$$G_{\vartheta_i} = \sin^2\left[\arcsin\sqrt{\hat{\vartheta}_i} - \frac{W_i}{2\sqrt{m_i}}\right], \tag{6}$$

where $W_i = 2\sqrt{m_i}\left(\arcsin\sqrt{\hat{\vartheta}_i} - \arcsin\sqrt{\vartheta_i}\right) \sim N(0,1)$. Now, we can calculate the GPQs for θ_i and the variance of $\hat{\theta}_i$ using Eqs (5) and (6), resulting in

$$G_{\theta_i} = \frac{1}{2 + G_{\alpha_i}^2}\sqrt{\frac{G_{\alpha_i}^2\left(4 + 5G_{\alpha_i}^2\right) + G_{\vartheta_i}\left(2 + G_{\alpha_i}^2\right)^2}{1 - G_{\vartheta_i}}} \tag{7}$$

and

$$G_{V(\hat{\theta}_i)} = \frac{1}{G_{\Psi_i}}\left\{\frac{32G_{\alpha_i}^4\left(1 + 2G_{\alpha_i}^2\right)^2}{m_{i(1)}\left(2 + G_{\alpha_i}^2\right)^2} + \frac{G_{\vartheta_i}\left[2 + G_{\alpha_i}^2\left(4 + 3G_{\alpha_i}^2\right)\right]^2}{m_i\left(1 - G_{\vartheta_i}\right)}\right\}, \tag{8}$$

where $G_{\Psi_i} = \left(2 + G_{\alpha_i}^2\right)^2\left(1 - G_{\vartheta_i}\right)\left[G_{\alpha_i}^2\left(4 + 5G_{\alpha_i}^2\right) + G_{\vartheta_i}\left(2 + G_{\alpha_i}^2\right)^2\right]$. Therefore, the GPQ for θ_i is the weighted average of the GPQ G_{θ_i} based on k individual samples, given by

$$G_\theta = \frac{\sum_{i=1}^{k} G_{\theta_i}/G_{V(\hat{\theta}_i)}}{\sum_{i=1}^{k} 1/G_{V(\hat{\theta}_i)}}. \tag{9}$$

Then, the $(1-\rho)100\%$ CI for the common CV of several ZIBS distributions employing the GCI method is given by

$$[L_{GCI}, U_{GCI}] = [G_\theta(\rho/2), G_\theta(1 - \rho/2)], \tag{10}$$

where $G_\theta(\rho/2)$ and $G_\theta(1-\rho/2)$ denote the $100(\rho/2)$th and $100(1-\rho/2)$th percentiles of G_θ, respectively.

Algorithm 1 is used to construct the GCI for the common coefficient of variation of several ZIBS distributions.

Algorithm 1.

For $g = 1$ to n, where n is the number of generalized computations:

1) Compute $A_i, B_i, C_i, D_i, E_{i1}$, and E_{i2}.
2) At the p step:

   a) Generate $\Lambda_i \sim t(m_{i(1)}-1)$, and then compute $G_{\beta_i}(y_{ij};\Lambda_i)$ from Eq (4);

   b) If $G_{\beta_i}(y_{ij};\Lambda_i) < 0$, regenerate $\Lambda_i \sim t(m_{i(1)}-1)$;

   c) Generate $K_i \sim \chi^2_{m_{i(1)}}$, and then compute $G_{\alpha_i}(y_{ij};K_i,\Lambda_i)$ from Eq (5);

   d) Compute G_{ϑ_i}, G_{θ_i}, and $G_{V(\hat{\theta}_i)}$ from Eqs (6)–(8), respectively;

   e) Compute G_θ from Eq (9).

End g loop.

3) Repeat step 2, a total of G times;
4) Compute L_{GCI} and U_{GCI} from Eq (10).

2.2. Method of variance estimates recovery

The method of variance estimates recovery (MOVER) estimates a closed-form confidence interval. Let $\hat{\omega}_i$ be an unbiased estimator of ω_i. Furthermore, let $[l_i, u_i]$ represent the $(1-\rho)100\%$ confidence interval for $\omega_i, i = 1,2,\ldots,k$. Assume that $\sum_{i=1}^{k} c_i\omega_i$ is a linear combination of the parameters ω_i, where c_i are constants. According to Zou, Huang, and Zhang [24], the lower and upper limits of the confidence interval for $\sum_{i=1}^{k} c_i\omega_i$ are defined by

$$L = \sum_{i=1}^{k} c_i\hat{\omega}_i - \sqrt{\sum_{i=1}^{k}[c_i\hat{\omega}_i - \min(c_il_i, c_iu_i)]^2}$$

and

$$U = \sum_{i=1}^{k} c_i\hat{\omega}_i + \sqrt{\sum_{i=1}^{k}[c_i\hat{\omega}_i - \max(c_il_i, c_iu_i)]^2}.$$

Considering Eq (7), the $(1-\rho)100\%$ CI for θ_i based on the GPQs has become

$$[l_i, u_i] = \left[G_{\theta_i}(\rho/2), G_{\theta_i}(1-\rho/2)\right], \tag{11}$$

where $G_{\theta_i}(\rho/2)$ and $G_{\theta_i}(1-\rho/2)$ represent the $100(\rho/2)$th and $100(1-\rho/2)$th percentiles of G_{θ_i}, respectively. Hence, the $(1-\rho)100\%$ CI for the common CV of several ZIBS distributions employing the MOVER method is given by

$$L_{MOVER} = \sum_{i=1}^{k} c_i^{\#}\hat{\theta}_i - \sqrt{\sum_{i=1}^{k}\left[c_i^{\#}\hat{\theta}_i - min(c_i^{\#}l_i, c_i^{\#}u_i)\right]^2} \tag{12}$$

and

$$U_{MOVER} = \sum_{i=1}^{k} c_i^{\#}\hat{\theta}_i + \sqrt{\sum_{i=1}^{k}\left[c_i^{\#}\hat{\theta}_i - max(c_i^{\#}l_i, c_i^{\#}u_i)\right]^2}, \tag{13}$$

where $c_i^{\#} = \frac{\eta_i}{\sum_{j=1}^{k}\eta_j}$ and $\eta_i = \frac{1}{\hat{V}(\hat{\theta}_i)}$.

Algorithm 2 is used to construct the MOVER for the common coefficient of variation of several ZIBS distributions.

Algorithm 2.

1) Compute $\hat{\alpha}_i$ and $\hat{\vartheta}_i$;
2) Compute $\hat{\theta}_i$ and $\hat{V}(\hat{\theta}_i)$;
3) Compute l_i and u_i from Eq (11);
4) Compute L_{MOVER} from Eq (12);
5) Compute U_{MOVER} from Eq (13).

2.3. Large sample approximation

Recall that the estimator of θ_i from Eq (1) is

$$\hat{\theta}_i = \frac{1}{2+\hat{\alpha}_i^2}\sqrt{\frac{\hat{\alpha}_i^2(4+5\hat{\alpha}_i^2)+\hat{\vartheta}_i(2+\hat{\alpha}_i^2)^2}{1-\hat{\vartheta}_i}},$$

and the estimated variance of $\hat{\theta}_i$ is

$$\hat{V}(\hat{\theta}_i) \approx \frac{1}{\hat{\Psi}_i}\left\{\frac{32\hat{\alpha}_i^4(1+2\hat{\alpha}_i^2)^2}{m_{i(1)}(2+\hat{\alpha}_i^2)^2} + \frac{\hat{\vartheta}_i[2+\hat{\alpha}_i^2(4+3\hat{\alpha}_i^2)]^2}{m_i(1-\hat{\vartheta}_i)}\right\},$$

where $\hat{\Psi}_i = (2+\hat{\alpha}_i^2)^2(1-\hat{\vartheta}_i)\left[\hat{\alpha}_i^2(4+5\hat{\alpha}_i^2)+\hat{\vartheta}_i(2+\hat{\alpha}_i^2)^2\right]$. The large sample (LS) estimate of the CV for the ZIBS distribution is a pooled estimate, as described in Eq (3). Accordingly, the $(1-\rho)100\%$ CI for the common CV of several ZIBS distributions employing the LS method is as follows:

$$[L_{LS}, U_{LS}] = \left[\hat{\theta} - z_{1-\frac{\rho}{2}}\sqrt{\frac{1}{\sum_{i=1}^{k}\eta_i}}, \hat{\theta} + z_{1-\frac{\rho}{2}}\sqrt{\frac{1}{\sum_{i=1}^{k}\eta_i}}\right], \tag{14}$$

where $\eta_i = \frac{1}{\hat{V}(\hat{\theta}_i)}$.

Algorithm 3 is used to construct the LS for the common coefficient of variation of several ZIBS distributions.

Algorithm 3.

1) Compute $\hat{\alpha}_i$ and $\hat{\vartheta}_i$;
2) Compute $\hat{\theta}_i$ and $\hat{V}(\hat{\theta}_i)$;
3) Compute L_{LS} and U_{LS} from Eq (14).

2.4. Bootstrap confidence interval

Efron [25] introduced the bootstrap method, which involves repeated resampling of existing data. According to Lemonte, Simas, and Cribari-Neto [26], the constant-bias-correcting parametric bootstrap is the most efficient method for reducing bias. As a result, we used it to estimate the confidence interval for θ. Assuming that there are D bootstrap samples available, the $\hat{\alpha}_i$ series for those samples can be computed, which is shown as $\hat{\alpha}_{i1}^{\#}, \hat{\alpha}_{i2}^{\#}, \ldots, \hat{\alpha}_{iD}^{\#}$. Here, $\hat{\alpha}_{ir}^{\#}$ is a sequence of the bootstrap maximum likelihood estimation (MLE) of α_{ir} for $i = 1,2,\ldots,k$ and $r = 1,2,\ldots,D$. The MLE of α_{ir} can be calculated using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) quasi-Newton nonlinear optimization algorithm. The bias of the estimator α_i is defined as

$$D(\hat{\alpha}_i, \alpha_i) = E(\hat{\alpha}_i) - \alpha_i,$$

and then the bootstrap expectation $E(\hat{\alpha}_i)$ could be approximated using the mean $\hat{\alpha}_i^{/} = \frac{1}{D}\sum_{r=1}^{D} \hat{\alpha}_{ir}^{\#}$. As a result, the bootstrap bias estimate for D replications of $\hat{\alpha}_i$ is derived as $\hat{D}(\hat{\alpha}_i, \alpha_i) = \hat{\alpha}_i^{/} - \hat{\alpha}_i$. According to Mackinnon and Smith [27], the corrected estimate for $\hat{\alpha}_i^{\#}$ is obtained by applying the bootstrap bias estimate, which is

$$\hat{\alpha}_i^{*} = \hat{\alpha}^{\#}{}_i - 2\hat{D}(\hat{\alpha}_i, \alpha_i). \tag{15}$$

Let $\hat{\vartheta}_i^{\#}$ be observed values of $\hat{\vartheta}_i$ based on bootstrap samples. In accordance with Brown, Cai, and DasGupta [28], the bootstrap estimator of ϑ_i is given by

$$\hat{\vartheta}_i^{*} \sim beta\left(m_i\hat{\vartheta}_i^{\#} + \frac{1}{2}, m_i\left(1 - \hat{\vartheta}_i^{\#}\right) + \frac{1}{2}\right). \tag{16}$$

By using Eqs (15) and (16), the bootstrap estimators of θ_i and the variance of $\hat{\theta}_i$ can be written as

$$\hat{\theta}_i^{*} = \frac{1}{2+\left(\hat{\alpha}_i^{*}\right)^2}\sqrt{\frac{\left(\hat{\alpha}_i^{*}\right)^2\left(4+5\left(\hat{\alpha}_i^{*}\right)^2\right)+\hat{\vartheta}_i^{*}\left(2+\left(\hat{\alpha}_i^{*}\right)^2\right)^2}{1-\hat{\vartheta}_i^{*}}} \tag{17}$$

and

$$\hat{V}^{*}\left(\hat{\theta}_i\right) \approx \frac{1}{\hat{\Psi}_i^{*}}\left\{\frac{32\left(\hat{\alpha}_i^{*}\right)^4\left(1+2\left(\hat{\alpha}_i^{*}\right)^2\right)^2}{m_{i(1)}\left(2+\left(\hat{\alpha}_i^{*}\right)^2\right)^2} + \frac{\hat{\vartheta}_i^{*}\left[2+\left(\hat{\alpha}_i^{*}\right)^2\left(4+3\left(\hat{\alpha}_i^{*}\right)^2\right)\right]^2}{m_i\left(1-\hat{\vartheta}_i^{*}\right)}\right\}, \tag{18}$$

where $\hat{\Psi}_i^{*} = (2 + (\hat{\alpha}_i^{*})^2)^2\left(1 - \hat{\vartheta}_i^{*}\right)\left[(\hat{\alpha}_i^{*})^2(4 + 5(\hat{\alpha}_i^{*})^2) + \hat{\vartheta}_i^{*}(2 + (\hat{\alpha}_i^{*})^2)^2\right]$. Now, the common θ

based on k individual sample is obtained by

$$\hat{\theta}^* = \frac{\sum_{i=1}^{k} \hat{\theta}_i^* / \hat{V}^*(\hat{\theta}_i)}{\sum_{i=1}^{k} 1/\hat{V}^*(\hat{\theta}_i)}. \tag{19}$$

Consequently, the $(1-\rho)100\%$ CI for the common CV of several ZIBS distributions employing the bootstrap confidence interval (BCI) method is provided by

$$[L_{BCI}, U_{BCI}] = \left[\hat{\theta}^*(\rho/2), \hat{\theta}^*(1-\rho/2)\right], \tag{20}$$

where $\hat{\theta}^*(\rho/2)$ and $\hat{\theta}^*(1-\rho/2)$ denote the $100(\rho/2)\text{th}$ and $100(1-\rho/2)\text{th}$ percentiles of $\hat{\theta}^*$, respectively.

Algorithm 4 is used to construct the BCI for the common coefficient of variation of several ZIBS distributions.

Algorithm 4.

For $b = 1$ to n:

1) At the q step:

   a) Generate y_{ij}^*, with replacement from y_{ij} where $i = 1,2,\ldots,k$ and $j = 1,2,\ldots,m_i$;

   b) Compute $\hat{\alpha}_i^{/}$ and $\widehat{D}(\hat{\alpha}_i, \alpha_i)$;

   c) Compute $\hat{\alpha}_i^*$ from Eq (15);

   d) Generate $\hat{\vartheta}_i^*$ from Eq (16);

   e) Compute $\hat{\theta}_i^*$ from Eq (17);

   f) Compute $\hat{V}^*(\hat{\theta}_i)$ from Eq (18);

   g) Compute $\hat{\theta}^*$ from Eq (19).

End b loop.

2) Repeat step 1, a total of B times;
3) Compute L_{BCI} and U_{BCI} from Eq (20).

2.5. Fiducial generalized confidence interval

Hannig [29] and Hannig [30] introduced the concept of the generalized fiducial distribution by assuming a functional relationship $R_j = Q_j(\delta, \boldsymbol{U})$ for $j = 1,2,\ldots,m$, where $\boldsymbol{Q} = (Q_1,\ldots,Q_m)$ are the structural equations. Then, assume that $\boldsymbol{U} = (U_1,\ldots,U_m)$ are independent and identically distributed samples from a uniform distribution $U(0,1)$ and that the parameter $\delta \in \Xi \subseteq R^p$ is p-dimensional. Consequently, the generalized fiducial distribution is absolutely continuous with a density

$$\psi(\delta) = \frac{J(r,\delta)L(r,\delta)}{\int_{\Xi} J(r,\delta')L(r,\delta')d\delta'}, \tag{21}$$

where $L(\boldsymbol{r}, \boldsymbol{\delta})$ represents the joint likelihood function of the observed data and

$$J(r,\delta)=\sum_{\substack{j=(j_1,\dots,j_p)\\ 1\le j_i<\dots<j_p\le m}}\left|\det\left(\left(\frac{d}{dr}\boldsymbol{Q}^{-1}(r,\delta)\right)^{-1}\frac{d}{d\delta}\boldsymbol{Q}^{-1}(r,\delta)\right)_j\right|,$$

where $\frac{d}{dr}\boldsymbol{Q}^{-1}(r,\delta)$ and $\frac{d}{d\delta}\boldsymbol{Q}^{-1}(r,\delta)$ are $m\times p$ and $m\times m$ Jacobian matrices, respectively. In addition, Hannig [29] deduced that if the sample r was independently and identically distributed from an absolutely continuous distribution with cumulative distribution function $F_\delta(r)$, then $\boldsymbol{Q}^{-1}=\left(F_\delta(R_1),\dots,F_\delta(R_m)\right)$. Let $Z_{ij}, i=1,2,\dots,k, j=1,2,\dots,m_{i(1)}$, be a random sample drawn from the Birnbaum-Saunders distribution. The likelihood function can be written as

$$L\left(z_{ij}|\alpha_i,\beta_i\right)\propto\frac{1}{\alpha_i^{m_{i(1)}}\beta_i^{m_{i(1)}}}\prod_{j=1}^{m_{i(1)}}\left[\left(\frac{\beta_i}{z_{ij}}\right)^{\frac{1}{2}}+\left(\frac{\beta_i}{z_{ij}}\right)^{\frac{3}{2}}\right]exp\left[-\frac{1}{2\alpha_i^2}\sum_{j=1}^{m_{i(1)}}\left(\frac{z_{ij}}{\beta_i}+\frac{\beta_i}{z_{ij}}-2\right)\right].$$

Therefore, from Eq (20), the generalized fiducial distribution of (α_i,β_i) is

$$p\left(\alpha_i,\beta_i|z_{ij}\right)\propto J\left(z_{ij},(\alpha_i,\beta_i)\right)L\left(z_{ij}|\alpha_i,\beta_i\right),$$

where

$$J\left(z_{ij},(\alpha_i,\beta_i)\right)=\sum_{1\le j<l\le m_{i(1)}}\frac{4\left|z_{ij}-z_{il}\right|}{\alpha_i\left(1+\beta_i/z_{ij}\right)\left(1+\beta_i/z_{il}\right)}$$

as obtained by Li and Xu [31]. Let $\alpha_i^\#$ and $\beta_i^\#$ be the generalized fiducial samples for α_i and β_i, respectively. According to Li and Xu [31], the adaptive rejection Metropolis sampling (ARMS) method was used to obtain the fiducial estimates of α_i and β_i from the generalized fiducial distribution. Thus, the calculation of $\alpha_i^\#$ and $\beta_i^\#$ can be implemented using the function *arms* in the package *dlm* of R software. Additionally, Hannig [29] recommended methods for estimating the fiducial generalized pivotal quantities for binomial proportion ϑ_i, with simulation results indicating that the best option is the mixture distribution of two beta distributions with weight ½, which is

$$\vartheta_i^\#\sim\frac{1}{2}beta\left(m_{i(0)},m_{i(1)}+1\right)+\frac{1}{2}beta\left(m_{i(0)}+1,m_{i(1)}\right). \tag{22}$$

Currently, the approximate fiducial generalized pivotal quantities for θ_i and the variance of $\hat{\theta}_i$ can be computed by

$$\theta_i^\#=\frac{1}{2+\left(\alpha_i^\#\right)^2}\sqrt{\frac{\left(\alpha_i^\#\right)^2\left(4+5\left(\alpha_i^\#\right)^2\right)+\vartheta_i^\#\left(2+\left(\alpha_i^\#\right)^2\right)^2}{1-\vartheta_i^\#}}, \tag{23}$$

and

$$V^{\#}(\hat{\theta}_i) \approx \frac{1}{\Psi_i^{\#}}\left\{\frac{32(\alpha_i^{\#})^4\left(1+2(\alpha_i^{\#})^2\right)^2}{m_{i(1)}\left(2+(\alpha_i^{\#})^2\right)^2} + \frac{\vartheta_i^{\#}\left[2+(\alpha_i^{\#})^2\left(4+3(\alpha_i^{\#})^2\right)\right]^2}{m_i(1-\vartheta_i^{\#})}\right\}, \quad (24)$$

where $\Psi_i^{\#} = \left(2+(\alpha_i^{\#})^2\right)^2(1-\vartheta_i^{\#})\left[(\alpha_i^{\#})^2\left(4+5(\alpha_i^{\#})^2\right)+\vartheta_i^{\#}\left(2+(\alpha_i^{\#})^2\right)^2\right]$. As a result, the common θ based on k individual samples is calculated as

$$\theta^{\#} = \frac{\sum_{i=1}^{k}\theta_i^{\#}/V^{\#}(\hat{\theta}_i)}{\sum_{i=1}^{k}1/V^{\#}(\hat{\theta}_i)}. \quad (25)$$

The $(1-\rho)100\%$ confidence interval for the common CV of several ZIBS distributions employing the fiducial generalized confidence interval (FGCI) method is obtained by

$$[L_{FGCI}, U_{FGCI}] = [\theta^{\#}(\rho/2), \theta^{\#}(1-\rho/2)], \quad (26)$$

where $\theta^{\#}(\rho/2)$ and $\theta^{\#}(1-\rho/2)$ denote the $100(\rho/2)$th and $100(1-\rho/2)$th percentiles of $\theta^{\#}$, respectively.

The algorithm 5 is used to construct the FGCI for the common coefficient of variation of several ZIBS distributions.

Algorithm 5.

For $g = 1$ to n:

1) Generate G samples of α_i and β_i by using the *arms* function in the *dlm* package of R software;
2) Burn-in Fsamples (the number of remaining samples is $G-F$);
3) Thin the samples by applying sampling lag $L > 1$, and the final number of samples is $G' = (G-F)/L$. Because the generated samples are not independent, we must reduce the autocorrelation by thinning them;
4) Generate $\vartheta_i^{\#}$ from Eq (22);
5) Compute $\theta_i^{\#}$ and $V^{\#}(\hat{\theta}_i)$ from Eqs (23) and (24), respectively;
6) Compute $\theta^{\#}$ from Eq (25);

End g loop.

7) Repeat steps 1–6, a total of G times;
8) Compute L_{FGCI} and U_{FGCI} from Eq (26).

3. Simulation results and discussion

To evaluate the performance of the proposed methods, Monte Carlo simulations in R software were conducted under various scenarios using different sample sizes, proportions of zeros, and shape parameters, as shown in Table 1. The scale parameter was consistently fixed at 1.0 in all scenarios. In generating a simulation, we set the total number of replications to 1000 replicates, 3000 replications for the GCI and FGCI, and 500 replications for the BCI. The performance comparison was based on a coverage probability (CP) greater than or equal to the nominal confidence level of 0.95, as well as the narrowest average width (AW). Algorithm 6 shows the computational steps to estimate the coverage probability and average width performances of all the methods.

Table 1. Parameter settings for k = 3, 5, 10.

Scenarios	$(m_1,\ldots,m_k)$	$(\alpha_1,\alpha_2,\ldots,\alpha_k)$	$(\vartheta_1,\vartheta_2,\ldots,\vartheta_k)$
k = 3			
1–12	(30_3)	(2.0_3), (2.5_3), (3.0_3)	(0.1_3), (0.1,0.3,0.5), (0.3_3), (0.5_3)
13–24	(30, 50,100)	(2.0_3), (2.5_3), (3.0_3)	(0.1_3), (0.1,0.3,0.5), (0.3_3), (0.5_3)
25–36	(50_3)	(2.0_3), (2.5_3), (3.0_3)	(0.1_3), (0.1,0.3,0.5), (0.3_3), (0.5_3)
37–48	(100_3)	(2.0_3), (2.5_3), (3.0_3)	(0.1_3), (0.1,0.3,0.5), (0.3_3), (0.5_3)
k = 5			
49–60	(30_3, 50_2)	(2.0_5), (2.5_5), (3.0_5)	(0.1_5), (0.1_2,0.3_2,0.5), (0.3_5), (0.5_5)
61–72	(30_2, 50_2, 100)	(2.0_5), (2.5_5), (3.0_5)	(0.1_5), (0.1_2,0.3_2,0.5), (0.3_5), (0.5_5)
73–84	(30, 50_2, 100_2)	(2.0_5), (2.5_5), (3.0_5)	(0.1_5), (0.1_2,0.3_2,0.5), (0.3_5), (0.5_5)
85–96	(50_3, 100_2)	(2.0_5), (2.5_5), (3.0_5)	(0.1_5), (0.1_2,0.3_2,0.5), (0.3_5), (0.5_5)
k = 10			
97–108	(30_5, 50_5)	(2.0_{10}), (2.5_{10}), (3.0_{10})	(0.1_{10}), (0.1_5,0.3_3,0.5_2), (0.3_{10}), (0.5_{10})
109–120	(30_5, 50_3, 100_2)	(2.0_{10}), (2.5_{10}), (3.0_{10})	(0.1_{10}), (0.1_5,0.3_3,0.5_2), (0.3_{10}), (0.5_{10})
121–132	(50_6, 100_4)	(2.0_{10}), (2.5_{10}), (3.0_{10})	(0.1_{10}), (0.1_5,0.3_3,0.5_2), (0.3_{10}), (0.5_{10})
133–144	(100_{10})	(2.0_{10}), (2.5_{10}), (3.0_{10})	(0.1_{10}), (0.1_5,0.3_3,0.5_2), (0.3_{10}), (0.5_{10})

*Note: (30_3) stands for (30,30,30).

The simulation results for k = 3 are shown in Table 2 and Figure 1. The coverage probabilities of the confidence intervals for the GCI method are greater than the nominal confidence level of 0.95 in almost all scenarios, while the coverage probabilities for the MOVER method are close to the specified coverage probability value when proportions of zeros equal 0.1_3. For the BCI method, they are close to the target, especially when the sample size is large. For the LS and FGCI methods, they provide coverage probability values lower than 0.95 in all scenarios. In terms of average width, the LS and MOVER methods have narrower confidence intervals than other methods in most scenarios. However, the coverage probabilities of both confidence intervals are less than 0.95 in almost all scenarios, so they do not meet the requirements. Among the remaining methods, the GCI method has the shortest average width in all scenarios studied, while the BCI method has the widest.

The simulation results for k = 5 are shown in Table 3 and Figure 2. The coverage probabilities of the LS and BCI methods are close to the nominal confidence level of 0.95 in almost all scenarios. In contrast, the MOVER and FGCI methods have values below the specified target. For the GCI method, the coverage probability meets the target when the proportions of zeros are unequal. In terms of the average width, the confidence interval of the MOVER method is the narrowest. However, this method has a coverage probability lower than 0.95 in all scenarios, thus failing to meet the criteria. The LS and BCI methods have the widest confidence intervals compared to the other methods.

Table 2. Performance measures of the 95% confidence intervals for the common CV; k = 3.

Scenarios	Coverage probability					Average width				
	GCI	MOVER	LS	BCI	FGCI	GCI	MOVER	LS	BCI	FGCI
1	0.960	0.954	0.898	0.943	0.941	0.3124	0.2772	0.2826	0.3127	0.3190
2	0.956	0.931	0.866	0.934	0.937	0.4539	0.3607	0.3706	0.4235	0.4313
3	0.964	0.941	0.857	0.935	0.920	0.4097	0.3738	0.3921	0.4678	0.4459
4	0.975	0.920	0.786	0.923	0.909	0.6208	0.5442	0.5843	0.7380	0.6854
5	0.952	0.952	0.864	0.948	0.931	0.2648	0.2344	0.2354	0.2779	0.2756
6	0.968	0.947	0.818	0.933	0.928	0.4445	0.3103	0.3108	0.4040	0.4133
7	0.968	0.935	0.797	0.953	0.923	0.3517	0.3257	0.3325	0.4519	0.4129
8	0.987	0.908	0.747	0.943	0.910	0.5304	0.4918	0.5044	0.7448	0.6674
9	0.965	0.953	0.865	0.950	0.929	0.2193	0.1965	0.1938	0.2527	0.2407
10	0.967	0.907	0.758	0.943	0.926	0.4259	0.2698	0.2620	0.3955	0.3937
11	0.976	0.922	0.715	0.937	0.901	0.2917	0.2885	0.2770	0.4365	0.3813
12	0.982	0.911	0.682	0.945	0.910	0.4374	0.4473	0.4247	0.7475	0.6405
13	0.960	0.953	0.900	0.942	0.937	0.2055	0.1927	0.1977	0.2176	0.2136
14	0.956	0.926	0.886	0.938	0.917	0.2770	0.2877	0.2998	0.3452	0.3159
15	0.965	0.936	0.863	0.941	0.920	0.2647	0.2590	0.2761	0.3312	0.3012
16	0.965	0.925	0.843	0.941	0.902	0.3964	0.3780	0.4159	0.5280	0.4647
17	0.957	0.944	0.898	0.945	0.934	0.1730	0.1618	0.1647	0.1950	0.1849
18	0.953	0.906	0.811	0.939	0.902	0.2519	0.2507	0.2525	0.3395	0.3026
19	0.978	0.933	0.834	0.946	0.911	0.2259	0.2267	0.2326	0.3188	0.2774
20	0.966	0.916	0.767	0.943	0.892	0.3358	0.3371	0.3515	0.5252	0.4429
21	0.954	0.944	0.859	0.948	0.908	0.1427	0.1356	0.1344	0.1761	0.1609
22	0.956	0.905	0.717	0.956	0.922	0.2403	0.2170	0.2092	0.3444	0.3006
23	0.970	0.918	0.759	0.952	0.893	0.1872	0.1996	0.1926	0.3086	0.2570
24	0.977	0.912	0.696	0.948	0.886	0.2877	0.3083	0.2973	0.5290	0.4305
25	0.962	0.944	0.894	0.946	0.930	0.2247	0.2130	0.2172	0.2394	0.2345
26	0.969	0.960	0.914	0.950	0.941	0.2951	0.2764	0.2841	0.3215	0.3095
27	0.965	0.934	0.844	0.940	0.910	0.2898	0.2888	0.3035	0.3631	0.3295
28	0.963	0.922	0.832	0.930	0.898	0.4228	0.4224	0.4543	0.5779	0.5085
29	0.967	0.954	0.900	0.942	0.923	0.1878	0.1796	0.1804	0.2135	0.2037
30	0.968	0.942	0.838	0.938	0.916	0.2754	0.2367	0.2390	0.3086	0.2920
31	0.964	0.932	0.813	0.940	0.902	0.2438	0.2494	0.2546	0.3483	0.3037
32	0.978	0.901	0.775	0.943	0.892	0.3631	0.3816	0.3885	0.5766	0.4877
33	0.951	0.939	0.842	0.930	0.908	0.1543	0.1498	0.1478	0.1930	0.1768
34	0.969	0.907	0.790	0.953	0.922	0.2610	0.2050	0.1972	0.2997	0.2773
35	0.977	0.919	0.723	0.941	0.882	0.2022	0.2227	0.2128	0.3380	0.2830
36	0.978	0.916	0.772	0.916	0.854	0.3030	0.3462	0.3280	0.5797	0.4705
37	0.953	0.944	0.917	0.945	0.926	0.1508	0.1500	0.1537	0.1685	0.1604
38	0.951	0.955	0.926	0.946	0.916	0.1871	0.1973	0.2013	0.2248	0.2076
39	0.936	0.937	0.893	0.950	0.906	0.1917	0.2062	0.2144	0.2582	0.2271

Continued on next page

Scenarios	Coverage probability					Average width				
	GCI	MOVER	LS	BCI	FGCI	GCI	MOVER	LS	BCI	FGCI
40	0.921	0.915	0.873	0.943	0.891	0.2691	0.3055	0.3206	0.4083	0.3453
41	0.952	0.939	0.887	0.943	0.928	*0.1249*	0.1262	0.1270	0.1502	0.1390
42	0.947	0.943	0.876	0.955	0.926	0.1677	0.1701	0.1679	*0.2150*	0.1936
43	0.948	0.913	0.838	0.957	0.909	0.1601	0.1799	0.1800	*0.2476*	0.2091
44	0.951	0.899	0.804	0.956	0.894	*0.2276*	0.2749	0.2708	0.4040	0.3288
45	0.955	0.943	0.879	0.954	0.916	*0.1016*	0.1053	0.1034	0.1354	0.1205
46	0.954	0.900	0.810	0.959	0.902	*0.1540*	0.1452	0.1381	0.2101	0.1849
47	0.938	0.909	0.763	0.951	0.880	0.1309	0.1614	0.1486	*0.2379*	0.1937
48	0.915	0.890	0.727	0.958	0.883	0.1877	0.2503	0.2267	*0.4030*	0.3163

*Note: Italics indicate the most suitable average width.

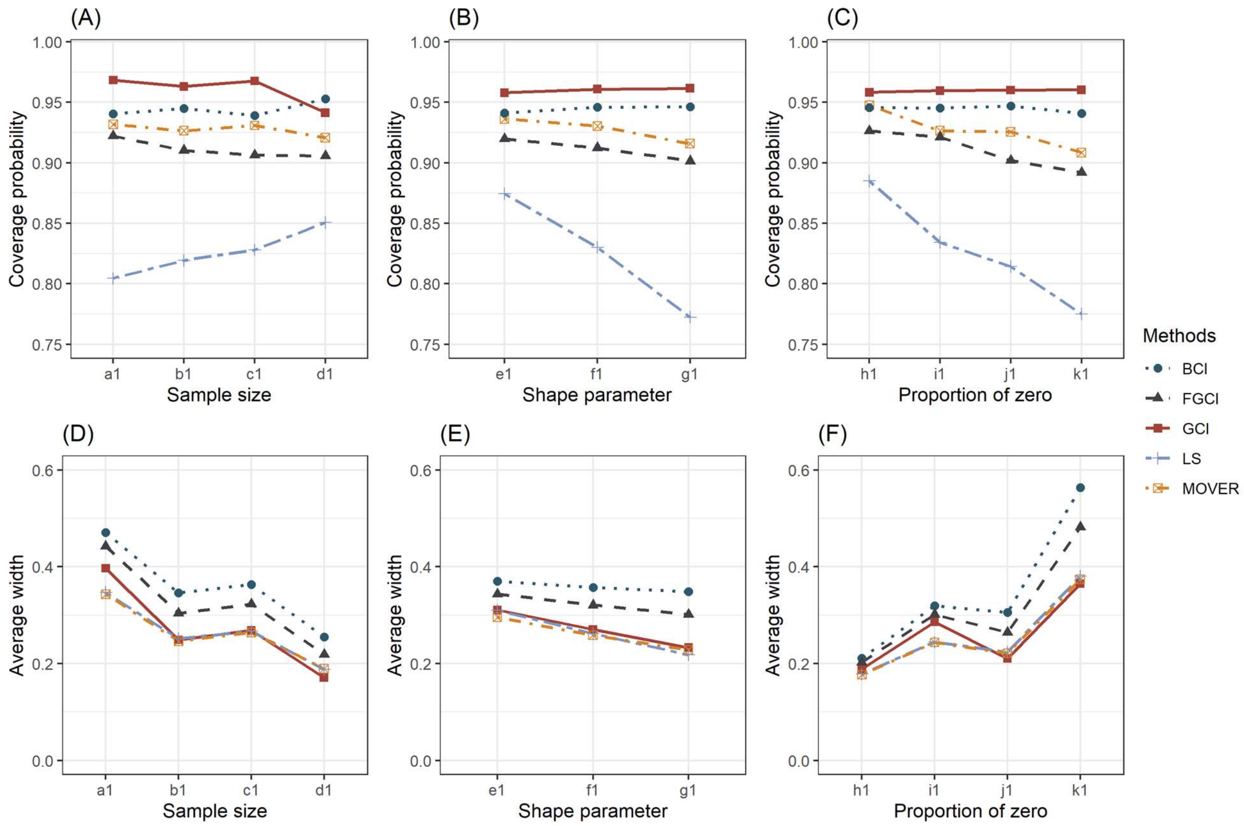

Figure 1. Comparison of the performance of the proposed method for k = 3 in terms of coverage probability with respect to (A) sample size, (B) shape parameter, (C) proportion of zero, and in terms of average width with respect to (D) sample sizes, (E) shape parameter, and (F) proportion of zero (a1 = (30_3), b1 = (30, 50, 100), c1 = (50_3), d1 = (100_3), e1 = (2.0_3), f1 = (2.5_3), g1 = (3.0_3), h1 = (0.1_3), i1 = (0.1, 0.3, 0.5), j1 = (0.3_3), k1 = (0.5_3)).

Table 3. Performance measures of the 95% confidence intervals for the common CV; k = 5.

Scenarios	Coverage probability					Average width				
	GCI	MOVER	LS	BCI	FGCI	GCI	MOVER	LS	BCI	FGCI
49	0.944	0.933	0.978	0.944	0.938	0.2182	0.1734	0.2820	0.2145	0.2196
50	0.954	0.935	0.942	0.954	0.951	0.2685	0.2293	0.3080	0.2834	0.2831
51	0.937	0.902	0.965	0.944	0.933	0.2862	0.2319	0.3922	0.3217	0.3080
52	0.940	0.893	0.910	0.904	0.893	0.4433	0.3356	0.5854	0.5083	0.4783
53	0.924	0.930	0.977	0.941	0.934	0.1860	0.1490	0.2356	0.1903	0.1904
54	0.951	0.920	0.910	0.946	0.930	0.2355	0.1987	0.2566	0.2657	0.2582
55	0.915	0.905	0.927	0.941	0.912	0.2485	0.2079	0.3326	0.3074	0.2841
56	0.910	0.882	0.895	0.934	0.919	0.3868	0.3079	0.5006	0.5068	0.4596
57	0.900	0.928	0.961	0.941	0.917	0.1570	0.1266	0.1946	0.1711	0.1655
58	0.959	0.914	0.813	0.935	0.920	0.2159	0.1730	0.2119	0.2573	0.2457
59	0.881	0.879	0.896	0.955	0.920	0.2094	0.1866	0.2774	0.2993	0.2633
60	0.866	0.832	0.803	0.922	0.888	0.3235	0.2896	0.4272	0.5123	0.4468
61	0.915	0.918	0.986	0.940	0.937	0.1794	0.1482	0.2542	0.1822	0.1824
62	0.959	0.935	0.933	0.931	0.913	0.2317	0.2076	0.2873	0.2584	0.2453
63	0.898	0.900	0.975	0.937	0.911	0.2318	0.1991	0.3535	0.2751	0.2559
64	0.897	0.879	0.944	0.912	0.891	0.3514	0.2837	0.5282	0.4348	0.3954
65	0.907	0.917	0.988	0.937	0.928	0.1526	0.1264	0.2120	0.1617	0.1583
66	0.950	0.923	0.909	0.946	0.921	0.2032	0.1796	0.2397	0.2467	0.2274
67	0.878	0.895	0.948	0.946	0.904	0.2016	0.1778	0.3009	0.2640	0.2367
68	0.875	0.873	0.922	0.939	0.905	0.3101	0.2663	0.4551	0.4360	0.3821
69	0.877	0.911	0.979	0.951	0.920	0.1266	0.1053	0.1743	0.1456	0.1376
70	0.966	0.888	0.827	0.945	0.908	0.1865	0.1581	0.1996	0.2466	0.2211
71	0.816	0.859	0.912	0.958	0.906	0.1664	0.1565	0.2496	0.2550	0.2195
72	0.833	0.816	0.863	0.928	0.887	0.2587	0.2442	0.3822	0.4364	0.3693
73	0.938	0.946	0.996	0.950	0.944	0.1531	0.1339	0.2344	0.1615	0.1588
74	0.959	0.937	0.954	0.940	0.932	0.1894	0.1762	0.2585	0.2225	0.2082
75	0.919	0.918	0.982	0.942	0.926	0.1967	0.1761	0.3266	0.2440	0.2228
76	0.884	0.880	0.955	0.916	0.884	0.2931	0.256	0.4900	0.3874	0.3432
77	0.928	0.920	0.996	0.944	0.923	0.1296	0.1134	0.1945	0.1429	0.1377
78	0.965	0.898	0.935	0.952	0.925	0.1656	0.1574	0.216	0.2128	0.1940
79	0.870	0.881	0.966	0.948	0.917	0.1684	0.1581	0.2772	0.2346	0.2055
80	0.875	0.835	0.932	0.932	0.894	0.2572	0.2371	0.4154	0.3859	0.3298
81	0.923	0.935	0.980	0.942	0.905	0.1074	0.0949	0.1594	0.1289	0.1197
82	0.952	0.928	0.882	0.952	0.922	0.1488	0.1371	0.1773	0.2100	0.1868
83	0.843	0.907	0.933	0.953	0.892	0.1417	0.1401	0.2284	0.2271	0.1910
84	0.844	0.866	0.896	0.945	0.882	0.2195	0.2161	0.3534	0.3878	0.3185
85	0.931	0.925	0.984	0.948	0.929	0.1346	0.1221	0.1881	0.1460	0.1415
86	0.950	0.939	0.959	0.953	0.937	0.1661	0.1606	0.2165	0.1994	0.1852
87	0.907	0.916	0.961	0.939	0.899	0.1701	0.1615	0.2626	0.2221	0.1999

Continued on next page

Scenarios	Coverage probability					Average width				
	GCI	MOVER	LS	BCI	FGCI	GCI	MOVER	LS	BCI	FGCI
88	0.883	0.895	0.954	0.922	0.891	0.2420	0.2330	*0.3924*	0.3520	0.3043
89	0.928	0.926	0.978	0.948	0.924	0.1123	0.1025	*0.1558*	0.1297	0.1228
90	0.952	0.935	0.925	0.955	0.919	*0.1431*	0.1379	0.1806	0.1898	0.1712
91	0.861	0.903	0.945	0.946	0.905	0.1432	0.1424	0.2215	0.2126	0.1837
92	0.862	0.864	0.903	0.938	0.894	0.2063	0.2140	0.3319	0.3503	0.2915
93	0.909	0.907	0.953	0.939	0.903	0.0919	0.0858	*0.1269*	0.1167	0.1064
94	0.958	0.900	0.847	0.947	0.918	*0.1290*	0.1225	0.1481	0.1857	0.1631
95	0.860	0.889	0.897	0.952	0.914	0.1172	0.1258	0.1831	*0.2054*	0.1705
96	0.819	0.832	0.847	0.937	0.881	0.1713	0.2029	0.2787	0.3511	0.2824

*Note: Italics indicate the most suitable average width.

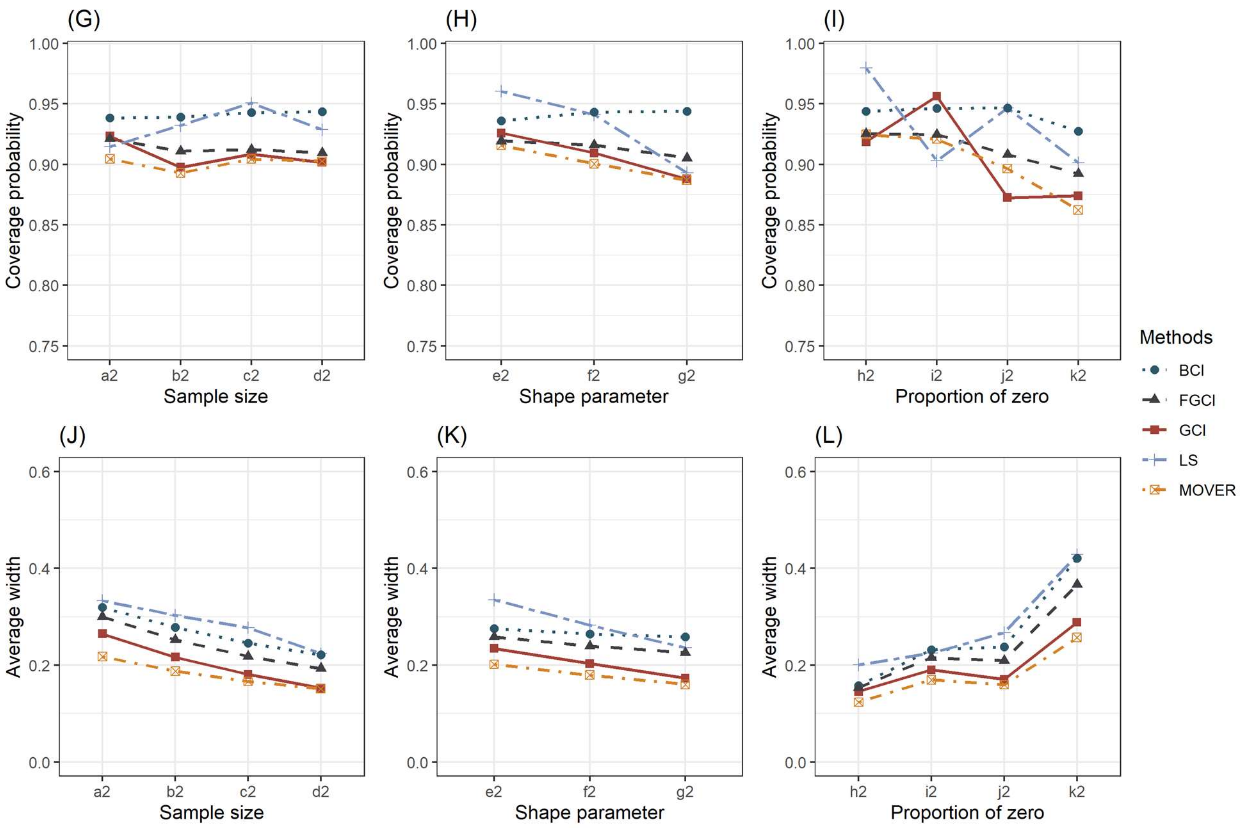

Figure 2. Comparison of the performance of the proposed method for $k = 5$ in terms of coverage probability with respect to (G) sample size, (H) shape parameter, (I) proportion of zero, and in terms of average width with respect to (J) sample sizes, (K) shape parameter, and (L) proportion of zero (a2 = $(30_3, 50_2)$, b2 = $(30_2, 50_2, 100)$, c2 = $(30, 50_2, 100_2)$, d2 = $(50_3, 100_2)$, e2 = (2.0_5), f2 = (2.5_5), g2 = (3.0_5), h2 = (0.1_5), i2 = $(0.1_2, 0.3_2, 0.5)$, j2 = (0.3_5), k2 = (0.5_5)).

The simulation results for $k = 10$ are shown in Table 4 and Figure 3. In almost all scenarios, the LS and BCI methods have coverage probabilities greater than or close to 0.95, except when the proportions of zeros equal 0.5_{10}. Both methods have wider average widths compared to the other methods. The GCI method has coverage probabilities close to 0.95 when the sample size is large and the shape parameters are equal to 2.0. For the MOVER method, even though it has the narrowest average width, it has coverage probabilities lower than 0.95 in all scenarios.

Figures 1–3 exhibit similar patterns, showing consistent trends. As the sample size increases, all the proposed methods tend to decrease. Similarly, as the shape parameter increases, all the proposed methods also tend to decrease. Conversely, when the proportion of zeros increases, all the proposed methods tend to increase. These observations are all based on the average width.

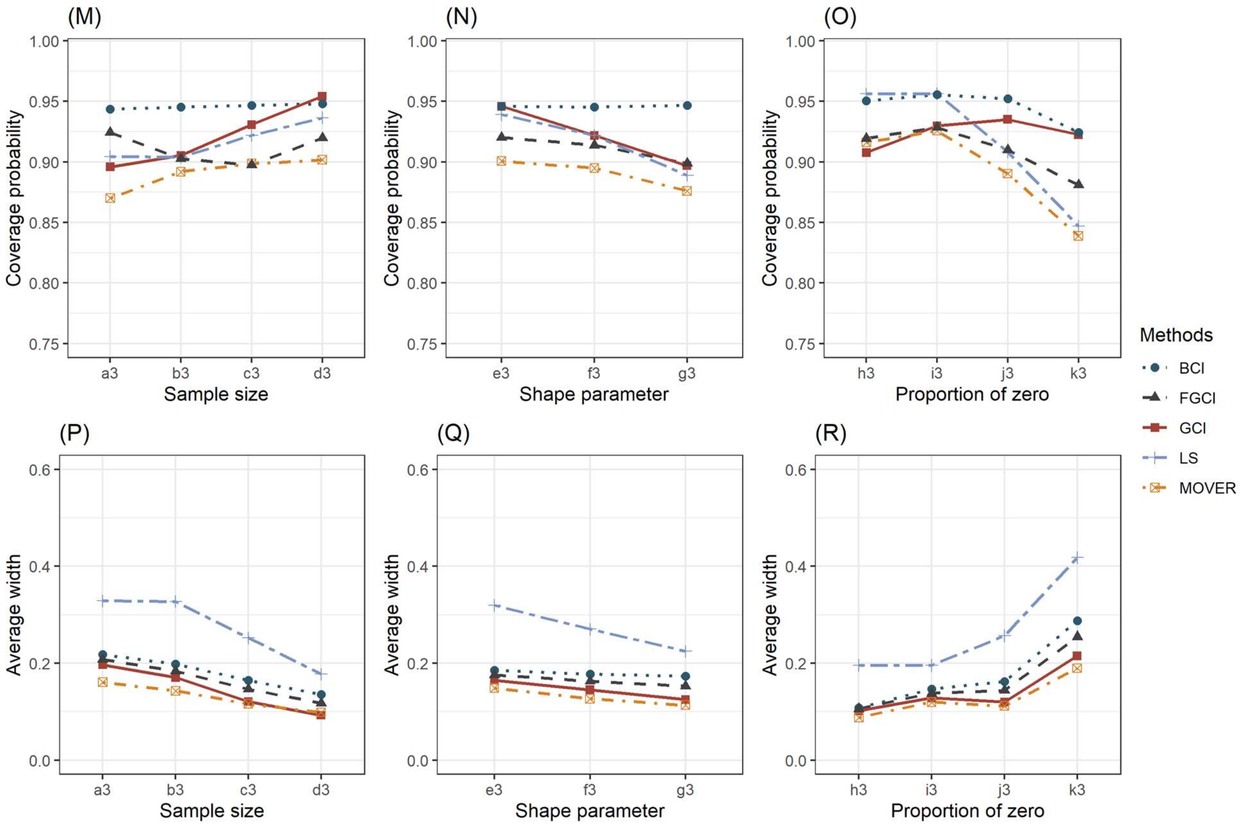

Figure 3. Comparison of the performance of the proposed method for $k = 10$ in terms of coverage probability with respect to (M) sample size, (N) shape parameter, (O) proportion of zero, and in terms of average width with respect to (P) sample sizes, (Q) shape parameter, and (R) proportion of zero (a3 = $(30_5, 50_5)$, b3 = $(30_5, 50_3, 100_2)$, c3 = $(50_6, 100_4)$, d3 = (100_{10}), e3 = (2.0_{10}), f3 = (2.5_{10}), g3 = (3.0_{10}), h3 = (0.1_{10}), i3 = $(0.1_5, 0.3_3, 0.5_2)$, j3 = (0.3_{10}), k3 = (0.5_{10})).

Table 4. Performance measures of the 95% confidence intervals for the common CV; k = 10.

Scenarios	Coverage probability					Average width				
	GCI	MOVER	LS	BCI	FGCI	GCI	MOVER	LS	BCI	FGCI
97	0.927	0.894	0.961	0.953	0.953	0.1535	0.1264	0.2812	*0.1481*	0.1525
98	0.951	0.940	0.935	0.959	0.952	*0.1855*	0.1628	0.2812	0.1867	0.1872
99	0.954	0.850	0.943	0.958	0.954	*0.2056*	0.1670	0.3932	0.2208	0.2145
100	0.929	0.789	0.854	0.902	0.891	0.3392	0.2946	0.5878	0.3486	0.3355
101	0.903	0.905	0.950	0.953	0.929	0.1336	0.1075	0.2353	*0.1307*	0.1336
102	0.929	0.931	0.948	0.953	0.944	0.1707	0.1443	0.2344	*0.1745*	0.1707
103	0.909	0.862	0.904	0.957	0.936	0.1814	0.1472	0.3341	0.2105	0.1967
104	0.888	0.803	0.826	0.907	0.896	0.3046	0.2221	0.5069	0.3499	0.3245
105	0.824	0.914	0.951	0.937	0.920	0.1122	0.0900	*0.1926*	0.1165	0.1148
106	0.866	0.908	0.952	0.957	0.940	0.1581	0.1275	0.1935	*0.1688*	0.1587
107	0.834	0.853	0.864	0.957	0.906	0.1547	0.1313	0.2802	*0.2057*	0.1827
108	0.836	0.791	0.764	0.931	0.870	0.2584	0.2081	0.4254	0.3564	0.3196
109	0.920	0.908	0.961	0.959	0.933	0.1333	0.1111	0.2818	*0.1316*	0.1339
110	0.944	0.931	0.965	0.963	0.915	0.1626	0.1540	0.2816	*0.1782*	0.1728
111	0.934	0.895	0.927	0.956	0.925	0.1774	0.1455	0.3911	*0.1979*	0.1876
112	0.934	0.874	0.856	0.920	0.889	0.2932	0.2572	0.5836	0.3132	0.2929
113	0.887	0.926	0.953	0.953	0.920	0.1160	0.0948	0.2347	*0.1167*	0.1172
114	0.913	0.915	0.963	0.952	0.931	0.1417	0.1362	0.2356	*0.1686*	0.1563
115	0.904	0.895	0.904	0.958	0.908	0.1584	0.1290	0.3312	*0.1889*	0.1732
116	0.892	0.865	0.840	0.917	0.871	0.2691	0.1937	0.5054	0.3140	0.2841
117	0.873	0.920	0.936	0.944	0.892	0.0983	0.0786	0.1927	0.1039	0.1007
118	0.926	0.905	0.959	0.955	0.912	0.1309	0.1214	0.1922	*0.1667*	0.1490
119	0.861	0.879	0.836	0.958	0.905	0.1361	0.1148	0.2756	*0.1833*	0.1598
120	0.875	0.788	0.747	0.910	0.830	0.2287	0.1802	0.4273	0.3159	0.2729
121	0.943	0.921	0.962	0.958	0.929	0.1039	0.0941	0.2170	*0.1103*	0.1090
122	0.949	0.931	0.957	0.954	0.923	0.1250	0.1221	0.2174	*0.1426*	0.1357
123	0.950	0.903	0.951	0.947	0.903	*0.1332*	0.1243	0.3034	0.1674	0.1532
124	0.958	0.900	0.917	0.932	0.883	*0.1959*	0.1793	0.4527	0.2642	0.2349
125	0.880	0.919	0.961	0.941	0.889	0.0877	0.0797	*0.1804*	0.0979	0.0944
126	0.924	0.930	0.954	0.956	0.927	0.1100	0.1071	0.1808	*0.1341*	0.1237
127	0.955	0.907	0.920	0.948	0.887	*0.1134*	0.1092	0.2552	0.1599	0.1408
128	0.950	0.859	0.865	0.943	0.882	*0.1697*	0.1626	0.3878	0.2634	0.2246
129	0.894	0.923	0.952	0.943	0.866	0.0723	0.0671	*0.1480*	0.0880	0.0819
130	0.920	0.904	0.959	0.956	0.910	0.1008	0.0943	0.1468	*0.1301*	0.1158
131	0.921	0.863	0.873	0.950	0.906	0.0941	0.0986	0.2125	*0.1551*	0.1309
132	0.924	0.823	0.789	0.934	0.862	0.1446	0.1531	0.3257	0.2649	0.2175
133	0.959	0.933	0.970	0.954	0.936	*0.0836*	0.0798	0.1534	0.0920	0.0888
134	0.953	0.949	0.961	0.951	0.923	*0.0957*	0.1005	0.1538	0.1100	0.1037
135	0.964	0.907	0.959	0.940	0.919	*0.1057*	0.1063	0.2146	0.1404	0.1257

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Scenarios	Coverage probability					Average width				
	GCI	MOVER	LS	BCI	FGCI	GCI	MOVER	LS	BCI	FGCI
136	0.964	0.883	0.943	0.930	0.895	*0.1490*	0.1541	0.3189	0.2215	0.1904
137	0.946	0.909	0.969	0.952	0.931	0.0696	0.0678	0.1266	*0.0817*	0.0769
138	0.942	0.946	0.964	0.955	0.946	0.0838	0.0886	0.1274	*0.1022*	0.0934
139	0.968	0.888	0.927	0.954	0.929	*0.0886*	0.0946	0.1798	0.1343	0.1154
140	0.957	0.859	0.904	0.927	0.895	*0.1261*	0.1418	0.2706	0.2205	0.1824
141	0.936	0.921	0.947	0.959	0.934	0.0568	0.0590	0.1038	*0.0733*	0.0666
142	0.940	0.918	0.958	0.957	0.916	0.0751	0.0782	0.1036	*0.0968*	0.0858
143	0.958	0.875	0.877	0.958	0.905	*0.0727*	0.0841	0.1486	0.1302	0.1073
144	0.962	0.831	0.858	0.940	0.907	*0.1047*	0.1298	0.2275	0.2218	0.1763

*Note: Italics indicate the most suitable average width.

Algorithm 6.

For a given $(m_1, m_2, \ldots, m_k)$, $(\alpha_1, \alpha_2, \ldots, \alpha_k)$, $(\vartheta_1, \vartheta_2, \ldots, \vartheta_k)$, and $\beta_1 = \beta_2 = \ldots = \beta_k = 1$, for $r = 1$ to M

1) Generate sample from the ZIBS distribution;
2) Compute the unbiased estimates $\hat{\alpha}_i$ and $\hat{\vartheta}_i$;
3) Compute the 95% confidence intervals for θ based on the GCI, MOVER, LS, BCI, and FGCI via Algorithms 1–5, respectively;
4) If $[L_r \leq \theta \leq U_r]$, set $D_r = 1$; else set $D_r = 0$;

End r loop.

5) The coverage probability and average width for each method are obtained by $CP = \frac{1}{M}\sum_{r=1}^{M} D_r$ and $AW = \frac{U_r - L_r}{M}$, where U_r and L_r are the upper and lower confidence limits, respectively.

4. An empirical application

In this study, we leverage wind speed data from all directions to construct CIs for the common coefficient of variation of several ZIBS distributions. The data were collected from January 1 to 7, 2024, from three weather stations: Chanthaburi Weather Observing Station in Chanthaburi Province, Chumphon Weather Observing Station in Chumphon Province, and Songkhla Weather Observing Station in Songkhla Province. The selection of these three stations is due to their proximity to the Gulf of Thailand, which makes them directly influenced by sea breezes and tropical storms. This results in high wind speed fluctuations and also impacts the livelihoods, economy, and environment of the surrounding communities. All data were collected by the Thai Meteorological Department and are presented in Table 5 (Thai Meteorological Department Automatic Weather System, https://www.tmd.go.th/ service/tmdData). To visualize the data distribution, we plotted histograms of wind speed data from all three stations, as shown in Figure 4. Table 6 provides statistical summaries for wind speed data at each station, revealing that the coefficients of variation of the wind speed data for the Chanthaburi Weather Observing Station, Chumphon Weather Observing Station, and Songkhla Weather Observing Station are 2.6799, 2.5111, and 2.7118, respectively. When considering the entire wind speed dataset, we observe a mixture of zero values (no wind) and positive values. For the positive values, we evaluate the suitability of the data distribution using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) calculated as $AIC = 2ln(L) + 2p$ and $BIC = 2ln(L) + 2pln(o)$, respectively, where p is the number of parameters estimated, o is the number of observations, and L is the likelihood function. From Table 7, it is evident that the Birnbaum-Saunders distribution exhibits the lowest AIC and BIC values compared to other distributions, indicating its best fit for positive wind speed data. Additionally, to confirm that the positive wind speed data follows the Birnbaum-Saunders distribution, we plotted the cumulative distribution function (CDF) derived from the positive wind speed data and the estimated CDF from the Birnbaum-Saunders distribution. As shown in Figure 5, both graphs are similar, indicating a good fit. Therefore, the wind speed data comprises both positive and zero values and follows the ZIBS distribution. This distribution was thus used to compute the CIs for the common coefficient of variation of the wind speed data. Table 8 presents the 95% confidence intervals for the common coefficient of variation of wind speed data from the three weather observing stations using the GCI, MOVER, LS, BCI, and FGCI methods. We compared wind speed data with parameters generated from simulation using a sample size of $m_i = 100_3$, parameter $\alpha_i = 2.5_3$, and parameter $\vartheta_i = 0.5_3$, as shown in Table 2. The simulation results indicate that the GCI and BCI methods meet the criterion of coverage probability greater than or equal to the nominal confidence level of 0.95. When considering the average width, the GCI method provides the narrowest confidence interval. The results in Table 8 show that the confidence interval for the common coefficient of variation for the wind speed data using the GCI method is [2.5001, 2.7224], with a confidence interval width of 0.2224, which is the narrowest among all methods. This leads to the conclusion that the appropriate method for the wind speed data is consistent with the simulation results.

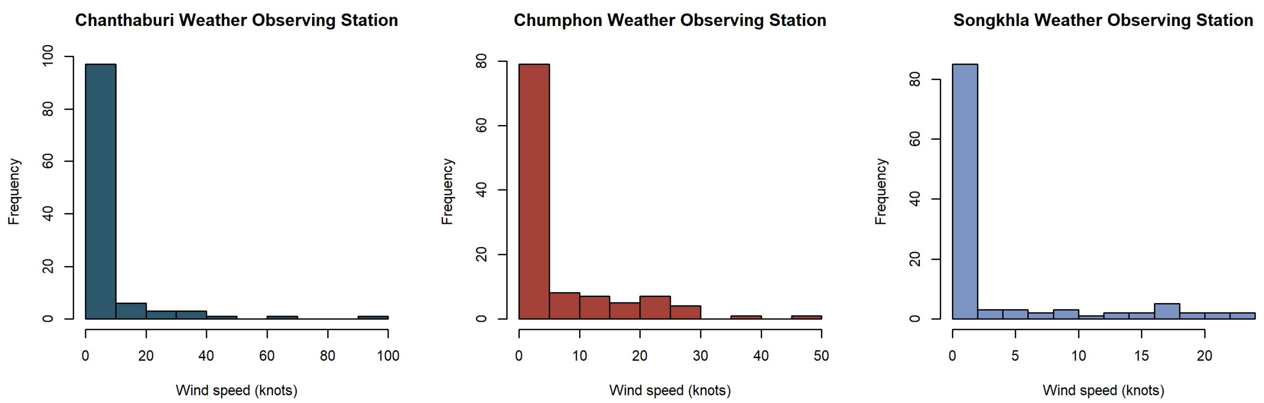

Figure 4. The histograms of wind speed data for each weather observing station.

Table 5. Data on the wind speed (knots) from the Chanthaburi Weather Observing Station, Chumphon Weather Observing Station, and Songkhla Weather Observing Station, Thailand.

N	NNE	NE	ENE	E	ESE	SE	SSE	S	SSW	SW	WSW	W	WNW	NW	NNW
Chanthaburi Weather Observing Station															
4.7	19.4	25.7	10.2	1.7	0.9	0.3	2.2	0.7	0.3	1.5	1.5	0.3	0.0	0.0	1.8
9.2	18.2	20.6	8.2	0.7	0.1	0.7	1.9	0.4	0.0	0.6	1.7	0.3	0.3	2.2	7.4
9.8	31.8	36.5	5.6	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	1.2	5.4
8.3	30.4	43.9	5.6	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	2.6
2.3	25.0	60.7	11.3	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
19.2	9.9	1.2	0.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.9	13.3
Chumphon Weather Observing Station															
0.0	0.0	0.0	0.0	0.8	21.4	14.6	2.8	7.6	11.0	13.2	6.3	1.5	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.5	17.6	13.9	3.5	0.9	1.5	26.5	18.1	0.3	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.2	15.6	23.5	5.9	0.9	4.6	21.0	14.0	0.4	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	11.5	25.2	7.9	0.6	1.9	16.1	29.2	0.3	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	17.8	36.2	1.9	0.3	2.0	22.2	13.1	0.1	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	8.7	26.6	6.2	1.0	2.9	22.6	24.2	0.2	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.3	5.4	2.3	1.8	7.1	46.9	23.0	0.1	0.0	0.0	0.0
Songkhla Weather Observing Station															
0.9	4.8	12.2	17.0	11.3	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.4
0.3	5.8	19.2	16.9	7.2	1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2	3.7	13.8	21.0	14.9	2.8	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.4	0.1
0.1	1.4	6.9	17.6	16.3	5.3	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1	1.5	8.3	21.9	17.0	1.5	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
0.4	1.8	9.4	22.1	14.6	2.1	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.3	0.1
0.3	1.3	8.8	22.2	18.8	1.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.2

Table 6. Summary statistics for the wind speed data.

Data	m_i	$m_{i(0)}$	$m_{i(1)}$	$\hat{\vartheta}_i$	$\hat{\alpha}_i$	$\hat{\beta}_i$	$\hat{\theta}_i$
Chanthaburi	112	55	57	0.4911	2.4263	2.5385	2.6799
Chumphon	112	53	59	0.4732	2.1425	3.1567	2.5111
Songkhla	112	56	56	0.5000	2.4389	1.6118	2.7118

Table 7. The AIC and BIC values of each distribution for the wind speed data.

Distribution	Chanthaburi		Chumphon		Songkhla	
	AIC	BIC	AIC	BIC	AIC	BIC
Normal	490.51	494.60	450.80	454.96	388.26	392.31
Lognormal	341.50	345.59	400.64	404.79	301.91	305.96
Logistic	466.00	470.09	450.81	454.96	391.04	395.09
Cauchy	418.12	422.20	467.59	471.75	390.09	394.14
Exponential	378.61	380.66	396.35	398.43	322.00	324.02
Gamma	350.13	354.22	390.40	394.55	299.53	303.58
Birnbaum-Saunders	336.10	342.23	386.43	392.66	285.49	291.56
Weibull	345.83	349.92	391.75	395.91	300.21	304.26

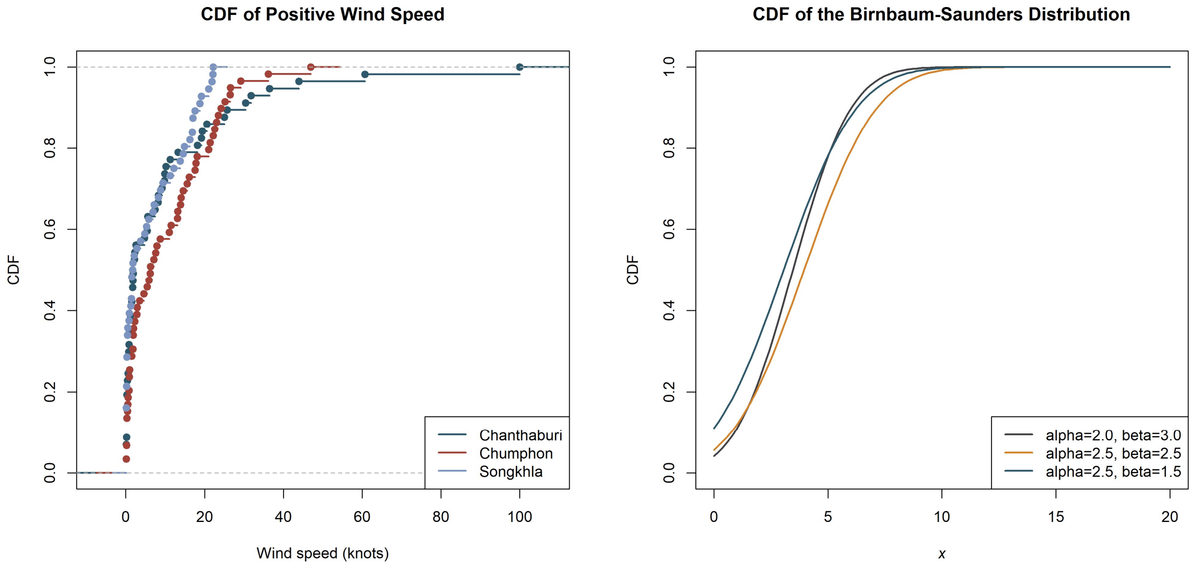

Figure 5. The CDF of the positive wind speed data and the estimated CDF from the Birnbaum-Saunders distribution.

Table 8. The 95% CIs for the common coefficients of variation for the wind speed data.

Methods	Interval [*L*, *U*]	Width
GCI	[2.5001, 2.7224]	0.2224
MOVER	[2.5050, 2.7283]	0.2233
LS	[2.5077, 2.7690]	0.2613
BCI	[2.4880, 2.8142]	0.3262
FGCI	[2.4979, 2.7816]	0.2837

5. General discussion

Based on the study results, it is evident that the GCI demonstrates good performance in almost all scenarios, as the coverage probability is greater than or close to the nominal confidence level of 0.95, which is consistent with the previous research by Ye, Ma, and Wang [32], Thangjai, Niwitpong, and Niwitpong [33], Janthasuwan, Niwitpong, and Niwitpong [11]. When k is large, both LS and BCI perform well. In most scenarios, MOVER and FGCI have coverage probabilities below acceptable levels, indicating that these methods may not be suitable for many situations, which aligns with the previous research by Puggard, Niwitpong, and Niwitpong [17]. Considering the average width, all proposed methods tend to decrease as the sample size and shape parameters increase, which improves their efficiency. Conversely, when the proportion of zeros increases, all proposed methods tend to decrease, leading to reduced efficiency. In our case, the simulation results showed that the MOVER method provided the narrowest confidence intervals for most scenarios and performed well with small sample sizes combined with a low proportion of zeros. However, the MOVER method yielded coverage probabilities lower than the specified confidence level in almost all scenarios. Similarly, the FGCI method achieves a coverage probability close to the specified confidence level in scenarios with a low proportion of zeros. This could be attributed to certain weaknesses that affect the fiducial generalized pivotal quantities for the proportion of zeros. Additionally, the issues with both the MOVER and FGCI methods likely arise from the upper and lower bounds for zero values used in constructing the confidence intervals and the combined effect with other parameters; this results in insufficient coverage probability. Finally, wind energy is a vital, renewable source of power, primarily generated by capturing wind speed. However, fluctuations in wind speed can introduce uncertainty. According to Lee, Fields, and Lundquist [34], understanding these variations is crucial for assessing wind resource potential.

6. Conclusions

This article presents an estimation of the common coefficient of variation of several ZIBS distributions. The methods proposed include GCI, MOVER, LS, BCI, and FGCI. The performance of each method was evaluated through Monte Carlo simulations, comparing their coverage probabilities and average widths. The simulation results for $k = 3$ recommend the GCI method due to its acceptable coverage probability and narrow confidence intervals in almost all scenarios, while the BCI method is another option for situations with large sample sizes. For $k = 5$, we recommend the GCI method when ϑ_i is unequal, the LS method when ϑ_i is small and the sample size is large, and the BCI method when ϑ_i is large. For $k = 10$, the BCI and GCI methods are recommended: the BCI method (for small to medium sample sizes) and the GCI method for large sample sizes. Additionally, in all sample cases ($k = 3$, 5, and 10), the MOVER method has the narrowest confidence intervals but a coverage probability below the acceptable level in most situations, and the FGCI method has a coverage probability below the acceptable level in almost all situations. Therefore, these two methods are not recommended. Finally, all the proposed methods were applied to wind speed data in Thailand and yielded results consistent with the simulation findings. In future research, we will explore new methods for constructing confidence intervals, potentially using Bayesian and highest posterior density (HPD) approaches to enhance their effectiveness. Additionally, we will use other real-world data to conduct a more comprehensive study.

Author contributions

Usanee Janthasuwan conducted the data analysis, drafted the initial manuscript, and contributed to the writing. Suparat Niwitpong developed the research framework, designed the experiment, and reviewed the manuscript. Sa-Aat Niwitpong provided analytical methodologies, validated the final version, and obtained funding.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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