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Research article

Modeling earthquake bond prices with correlated dual trigger indices and the approximate solution using the Monte Carlo algorithm

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Abstract: Countries prone to earthquakes face increasing seismic activity, often resulting in losses that exceed national budgets. To mitigate these losses, earthquake bonds present a promising alternative funding source; however, pricing them is complex, requiring simultaneous accounting for financial and seismic risks. Therefore, this study aimed to model earthquake bond pricing. The model incorporates earthquake intensity to account for rising seismic activity. It also includes depth and maximum magnitude as correlated dual trigger indices, making the bonds more attractive to investors, as claims are generated if both events occur. These three factors were modeled together as a compound stochastic process. The bond price was then formulated using a risk-neutral pricing measure with a stochastic interest rate under the Cox-Ingersoll-Ross model. Since the model lacks a closed-form solution, we employed an algorithm based on the Monte Carlo method for estimation. Through this algorithm, we showed that bond prices for terms of one to six years follow a normal distribution. The use of stochastic interest rates becomes significant as the bond term increases. We also found that earthquake intensity and bond terms negatively correlate with bond prices, while annual coupons positively correlate. Additionally, including dual triggers lowers claim probability and increases the bond demand, but is compensated by higher prices. This study can assist issuers in pricing earthquake bonds based on earthquake severity-maximum magnitude, depth, and intensity-and aid geological institutions in estimating earthquake risk in observed areas.

Keywords: earthquake bond; price; intensity; depth; maximum magnitude; risk-neutral pricing measure; Monte Carlo method
Mathematics Subject Classification: 91B70, 91G30, 91G70, 91G80

1. Introduction

Countries whose territories are located on the world's tectonic plates have a very high risk of earthquakes. For example, Japan, which sits atop the confluence of four tectonic plates (Pacific, Philippine, Eurasian, and North American), experiences more than 1500 earthquakes every year [1]. Indonesia, located on the confluence of three tectonic plates (Indo-Australian, Eurasian, and Pacific), often faces large earthquakes. An example is the Aceh earthquake in 2004, which caused a tsunami that killed around 230,000 people [2,3]. In South America, Chile also has a high risk of earthquakes because its territory is located on the meeting point of the Nazca and South American plates. The most well-known of this century is the great earthquake of 2010, which caused losses of around 30 billion USD [4,5].

A high earthquake risk is positively correlated with the losses experienced by the countries concerned. For example, the Tohoku earthquake in Japan in 2011 caused economic losses of around 235 billion USD, making it the costliest natural disaster in history [6,7]. In Nepal, a 7.8-magnitude (M_W) earthquake in 2015 caused losses of around 10 billion USD, equivalent to around 50% of the country's GDP [8,9]. In Indonesia, the earthquake and tsunami in Sulawesi in 2018 resulted in losses of more than 1.6 billion USD [10]. This data shows that countries with a high risk of earthquakes often experience substantial financial losses because of these disasters.

The significant losses from earthquakes experienced by these countries are often disproportionate to the budget they allocate to overcome them. For example, the earthquake and tsunami in Sulawesi in 2018 caused losses of more than 1.3 billion USD, while the Indonesian government only allocated around 0.4 billion USD for disaster management [11]. In Nepal, the 2015 earthquake caused losses of around 10 billion USD, while the country's annual budget for disaster management was only around 0.2 billion USD [12–14]. A similar situation occurred in Haiti after the 2010 earthquake, which caused losses of 7.8 billion USD, with a much smaller disaster management budget [15]. This data shows a large gap between losses from earthquakes and the funds available to address their impacts.

Earthquake financing from these countries requires new sources so that they do not only depend on the state budget or social assistance. One source of funds that can be used for this financing is earthquake bonds. This mechanism was developed from traditional earthquake insurance by the World Bank and several earthquake-prone countries to obtain greater risk coverage capacity and reduce financial pressure when claims occur [16]. In this way, the country and (re)insurers involve the community in the bond market to cover the risk of an earthquake jointly. In return for the community's willingness, significant returns from earthquake bonds that are greater than traditional bonds will be given to them [17]. Some countries have used earthquake bonds. For example, after the earthquake in 2017, Mexico issued these bonds, which raised more than 300 million USD for fast and efficient recovery [18]. Then, Chile used these bonds with a contingency fund of 500 million USD in 2018 to obtain additional funds after a major earthquake [19].

Although some countries have used earthquake bonds, the number is still tiny. One reason is that the fair pricing process is different from traditional bonds. In addition to considering financial factors (e.g., interest rates), earthquake risk factors are also calculated simultaneously. Several studies have examined the pricing of earthquake bonds. Zimbidis et al. [20] used extreme value theory to model the risk of maximum magnitude from earthquakes and apply it in pricing earthquake bonds. Shao et al. [21] proposed an earthquake bond pricing model considering interest, inflation, and coupon rates. They used the maximum magnitude and depth of the earthquakes to estimate the trigger event. Tang and Yuan [22] modeled earthquake bond prices based on the aggregate loss trigger index, formulated based on distorted and risk-neutral probability measures. Hofer et al. [23] provided a general process for creating catastrophe bond-based protection against natural hazard losses, such as earthquakes, for a spatially distributed portfolio. Mistry and Lombardi [24] designed an earthquake bond price based on aggregate losses, using high spatial resolution hazard and exposure models to calculate direct economic losses for each exposed asset. Anggraeni et al. [25] designed a bond pricing model for funding a single earthquake based on the earthquake disaster risk index. Mistry and Lombardi [26] introduced a stochastic method to address uncertainty in exposure model attributes and asset location. The method then was applied to model the earthquake bond. Wei et al. [27] proposed a catastrophe bond for earthquakes with aggregate loss and maximum magnitude indices. Aghdam et al. [28] presented a model for pricing catastrophe bonds applied to earthquakes, incorporating a jump sentence to indicate damage severity and probability. Ibrahim et al. [29] proposed pricing models for significant earthquakes, focusing on inconstant event intensity and maximum magnitude. Then, Anggraeni et al. [30] developed a disaster region decomposition using earthquake parameters and space-time-depth-magnitude distance to price earthquake bonds for a single period.

After conducting a literature review, we found gaps in previous studies regarding the pricing of earthquake bonds. In summary, there are no price models of earthquake bonds that use magnitude and depth trigger indices, which are assumed to be dependent. The assumption that the two indices can be dependent is crucial because this allows for a more accurate risk assessment, affects the frequency and number of payouts, and improves bond risk management [31]. Apart from that, no one has modeled the two trigger indices with intensity consideration. Involving this intensity is urgent because the frequency of earthquakes varies in each country each year. Furthermore, including this intensity increases the accuracy of the depiction of claims-triggering events and can reduce uncertainty in measuring earthquake risk when pricing bonds.

Based on gap analysis from previous studies related to pricing earthquake bonds, this study aims to model the price of earthquake bonds using magnitude and depth trigger indices. Both triggers are assumed to be dependent, and their values are determined based on the earthquake's intensity. This intensity is designed finitely through many significant earthquakes throughout the observation time. Mathematically, this intensity is designed in an earthquake frequency model based on a homogeneous Poisson process. Then, we design the coupon and redemption value payment schemes into a non-binary form to assess all possible severities of earthquakes that occur within the life of the bond. We utilize the concept of a risk-neutral pricing measure with stochastic interest rate assumptions to model bond prices. This stochasticity is accommodated using the Cox-Ingersoll-Ross formula, which guarantees the positivity of interest rates in accordance with actual conditions in almost every country worldwide. After the model was designed, we carried out a sensitivity analysis of the variables used on earthquake bond prices, especially the sensitivity of earthquake intensity.

Apart from that, we also compare the results of earthquake bond price estimation based on the proposed model and other models. This study can be employed by earthquake bond issuers worldwide to price earthquake bonds that reflect the severity of the earthquake with its maximum magnitude, depth, and intensity. Apart from that, this study can also be used by geological institutions and others to estimate the risk of maximum magnitude and depth of earthquakes in the observation area.

2. A brief description of earthquake bonds

Catastrophe (CAT) bonds are insurance-linked securities in the form of bonds issued to overcome budget limitations and coverage capacity against the risk of catastrophic loss from the sponsor (insured). One example of a CAT bond is an earthquake bond, which is specifically designed for the risk of loss due to earthquakes. Bonds are popular in this link because bonds can provide significant funds quickly [32]. This advantage reduces the sudden financial burden for sponsors when an earthquake that causes extreme losses occurs suddenly.

The simple structure of an earthquake bond consists of a sponsor, a special-purpose vehicle (SPV), and an investor [33,34]. The sponsor is typically a government entity (national or regional), an insurer, or a reinsurer. This sponsor is an insured party who transfers some or all of the risk of the earthquake to investors in the market of the bond. Then, the SPV is an independent entity established by the sponsor to transfer earthquake risk to investors. In summary, once the SPV is established, the sponsor remits the premium to the SPV, and the SPV issues earthquake bonds. The premiums and funds earned from the bonds are then reinvested into short-term safe securities. Investment income is then used for payments in earthquake bonds, one of which is for coupon payments (if any) to investors. Suppose the claim's trigger event for an earthquake bond occurs before the payment date of the periodical coupon or the redemption value. In that case, the investor receives the coupon and redemption value from the SPV, whose values correspond to the percentage determined at the time of bond issuance [35,36]. Then, suppose the event that triggers a claim from an earthquake bond does not occur after the payment date of the periodical coupon and the redemption value. In that case, the investor receives the entire coupon and redemption value from the SPV [37]. The structure of this type of earthquake bond is presented visually in Figure 1.

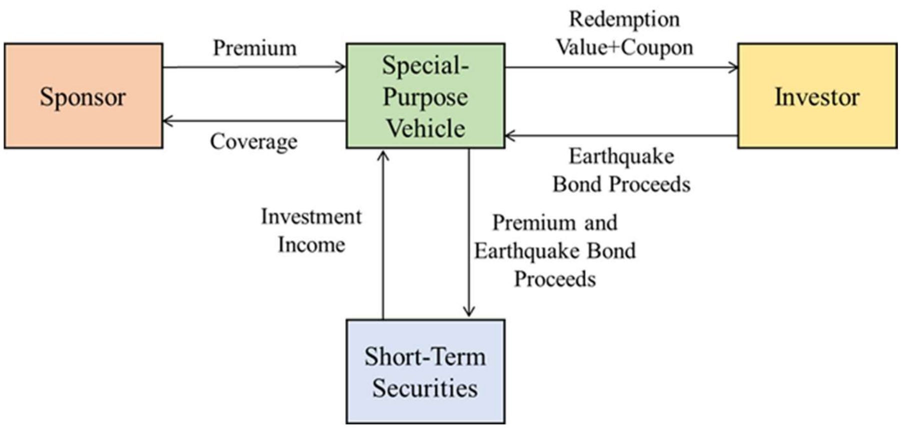

Figure 1. The simple structure of earthquake bonds.

The triggering event for earthquake bonds can generally be measured by five indices: industry, indemnity, parametric, model, and multiple indices. The industry index measures trigger events based on actual losses due to earthquakes that occur throughout the insurance or reinsurance industry. The indemnity index measures the triggering event based on actual losses due to the earthquake that occurred to the sponsor. Then, the parametric index measures the triggering event based on specific characteristics of the earthquake, e.g., magnitude, depth, or radius. The model index measures trigger events using mathematical models that have been designed by companies or agencies specialized in earthquake risk analysis, e.g., universities, state research agencies, or private companies such as Risk Management Solutions (RMS) and CoreLogic. Finally, multiple indices measure triggering events using more than one earthquake severity measure from the same index or different indices. Multiple indices provide a smaller possibility of a claim occurring than a single index [38]. It is suitable for use in earthquake-prone countries to make earthquake bonds more attractive to investors. The number of indices used in multiple indices is usually only two or three. Too many indices used in multiple indices make measuring trigger events too complex. For that reason, dual-triggers is still tolerated.

This study uses dual-trigger indices. The characteristics measured by the severity of an earthquake are its maximum magnitude and depth assumed to be dependent. The uses of these indices have advantages, where both are objectively, quickly, and inexpensively measured. These advantages can speed up earthquake funding that is immediately needed when an earthquake occurs suddenly. The triggering events are designed based on the magnitude and depth of different types of earthquakes. We show the earthquake types based on magnitude and depth in Table 1.

Table 1. The types of earthquakes based on magnitude and depth.

Earthquake Type Based on Magnitude	Magnitude Interval (Richter Scale, M_L)	Earthquake Type Based on Depth	Depth Interval (Kilometers, km)
Nonsignificant[1] Earthquake	$[0,5)$	Shallow Earthquake	$[0,70)$
Moderate Earthquake	$[5,6)$	Intermediate Earthquake	$[70,300)$
Strong Earthquake	$[6,7)$	Deep Earthquake	$[300,\infty)$
Major Earthquake	$[7,8)$		
Great Earthquake	$[8,\infty)$		

*Note: We classify nonsignificant earthquakes as light, minor, micro, and super micro earthquakes.

3. The main results

3.1. *The model*

Before the model framework is explained, we determine that all variables in this study are modeled in $(\Theta,\mathcal{P},\mathcal{P}_t,\mathrm{P})$, where Θ represents the sample space, $\mathcal{P}$ represents the σ-algebra on Θ, $\mathcal{P}_t\subset\mathcal{P}$ with $t\in[0,T]$ represents increasing filtration, and P represents a probability measure on $\mathcal{P}$. In more detail, T represents a positive integer.

3.1.1. Trigger processes and their distributions

The trigger indices used for earthquake bonds in this study are parametric. Two variables measure this index: the maximum strength of a significant earthquake and its depth. The word “significant” here is crucial because it is an indication that we are only considering earthquakes greater than a particular magnitude rather than all the infinite number of earthquakes. In this study, earthquakes with a magnitude greater than or equal to 5 M_L (Richter scale) are defined as significant.

We employed a compound process to model the maximum magnitude of a significant earthquake until time $t \in [0, T]$ in this study. This compound process consists of the frequency process of significant earthquakes and the magnitude process of a single significant earthquake. Mathematically, the compound process of the maximum magnitude of significant earthquakes is denoted by $\{\mathcal{M}_t : t \in [0, T]\}$, which is formulated as follows:

$$\mathcal{M}_t = \max_{j \in J_t}\{M_j\},$$

where $J_t = \{1, 2, 3, \ldots, N_t\}$ is a set of indices that represent the sequence of significant earthquakes, $\{N_t : t \in [0, T]\}$ denotes the process of significant earthquake frequencies, and $\{M_j : j \in J_t\}$ represents the magnitude process of a single significant earthquake. Then, the depth of a significant earthquake is denoted as $\{D_j : j \in J_t\}$ corresponding to $\{M_j : j \in J_t\}$. The depth of a significant earthquake of magnitude $\mathcal{M}_t$ until time t is denoted by $\{\mathcal{D}_t : t \in [0, T]\}$, which is formulated as follows:

$$\mathcal{D}_t = D_S, \tag{1}$$

where

$$S = \arg\max_{j \in J_t}\{M_j\}.$$

We assume that the probability of more than one significant earthquake of magnitude $\mathcal{M}_t$ at the same depth is zero for $t \in [0, T]$. This is because, in reality, it rarely happens. In this study, we used the following assumptions:

a. $\{N_t : t \in [0, T]\}$ follows a homogeneous Poisson process with intensity $\lambda > 0$. In simple terms, this means that the frequency of significant earthquakes has a constant intensity of occurrence. Then, there is no earthquake at time $t = 0$. The frequencies of significant earthquakes between time intervals do not influence each other, and the frequencies of significant earthquakes depends on the time interval. Finally, there was no more than one significant earthquake in a very short time interval. The probability mass function (PMF) of N_t is formulated as follows [39]:

$$\mathrm{P}(N_t = n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, \tag{2}$$

with $n = 0, 1, 2, 3, \ldots$.

b. $\{M_j : j \in J_t\}$ are independent and identically distributed (i.i.d.). In simple terms, this means that the magnitudes between significant earthquakes do not influence each other and have the same characteristics. This implies that for every $j \in J_t$ and $\mathcal{m} \subset \mathbb{R}^+ \cup \{0\}$, M_j can be referred to as M to write simply,

$$\mathrm{P}(M_j \in \mathcal{m}) = \mathrm{P}(M \in \mathcal{m}),$$

and for every $j_1 \in J_t$, $j_2 \in J_t$, $\mathcal{m}_1 \subset \mathbb{R}^+ \cup \{0\}$, and $\mathcal{m}_2 \in \mathbb{R}^+ \cup \{0\}$,

$$\mathrm{P}(M_{j_1} \in \mathcal{m}_1, M_{j_2} \in \mathcal{m}_2) = \mathrm{P}(M \in \mathcal{m}_1)\mathrm{P}(M \in \mathcal{m}_2).$$

Note that when $m \to \infty$, $\mathrm{P}(M \le m) \to 1$, and when $m \to -\infty$, $\mathrm{P}(M \le m) \to 0$.

c. $\{D_j : j \in J_t\}$ are i.i.d. In simple terms, this means that the depths between significant earthquakes do not influence each other and have the same characteristics. In other words, for every $j \in J_t$ and $\mathcal{d} \subset \mathbb{R}^+ \cup \{0\}$, D_j can be referred to as D to write simply,

$$\mathrm{P}(D_j \in \mathcal{d}) = \mathrm{P}(D \in \mathcal{d}),$$

and for every $j_1 \in J_t$, $j_2 \in J_t$, $\mathcal{d}_1 \subset \mathbb{R}^+ \cup \{0\}$, and $\mathcal{d}_2 \in \mathbb{R}^+ \cup \{0\}$,

$$\mathrm{P}(D_{j_1} \in \mathcal{d}_1, D_{j_2} \in \mathcal{d}_2) = \mathrm{P}(D \in \mathcal{d}_1)\mathrm{P}(D \in \mathcal{d}_2).$$

Note that when $d \to \infty$, $\mathrm{P}(D \le d) \to 1$, when $d \to -\infty$, $\mathrm{P}(D \le d) \to 0$.

d. $\{N_t : t \in [0, T]\}$ and $\{M_j : j \in J_t\}$ are assumed to be independent. In simple terms, this means that the frequency and magnitude of significant earthquakes do not affect each other. In other words, for every $j \in J_t$, $t \in [0, T]$, $\mathcal{m} \subset \mathbb{R}^+ \cup \{0\}$, and $\mathcal{n} \subset \mathbb{N} \cup \{0\}$, the following equation holds:

$$\mathrm{P}(N_t \in \mathcal{n}, M_j \in \mathcal{m}) = \mathrm{P}(N_t \in \mathcal{n})\mathrm{P}(M_j \in \mathcal{m}).$$

e. $\{N_t : t \in [0, T]\}$ and $\{D_j : j \in J_t\}$ are assumed to be independent. In simple terms, this means that the frequency and depth of significant earthquakes do not affect each other. In other words, for every $j \in J_t$, $t \in [0, T]$, $\mathcal{d} \subset \mathbb{R}^+ \cup \{0\}$, and $\mathcal{n} \subset \mathbb{N} \cup \{0\}$, the following equation holds:

$$\mathrm{P}(N_t \in \mathcal{n}, D_j \in \mathcal{d}) = \mathrm{P}(N_t \in \mathcal{n})\mathrm{P}(D_j \in \mathcal{d}).$$

The CDF of $\{\mathcal{M}_t : t \in [0, T]\}$ for random N_t is formulated is as follows:

$$\mathrm{P}(\mathcal{M}_t \le m) = \sum_{n=0}^{\infty} \mathrm{P}(\mathcal{M}_t \le m, N_t = n) = e^{-\lambda t[1-\mathrm{P}(M \le m)]}. \tag{3}$$

Proof. See Appendix A. Then, the probability that $\mathcal{M}_t \in (m_1, m_2]$ with $m_1 < m_2$ is formulated as follows:

$$\mathrm{P}(m_1 < \mathcal{M}_t \leq m_2) = \sum_{n=0}^{\infty} \mathrm{P}(m_1 < \mathcal{M}_t \leq m_2, N_t = n) = \mathrm{P}(\mathcal{M}_t \leq m_2) - \mathrm{P}(\mathcal{M}_t \leq m_1). \tag{4}$$

Proof. See Appendix A. With similar formulation steps, the CDF of $\{\mathcal{D}_t : t \in [0, T]\}$ for random N_t is briefly formulated as follows:

$$\mathrm{P}(\mathcal{D}_t \leq d) = \sum_{n=0}^{\infty} \mathrm{P}(\mathcal{D}_t \leq d, N_t = n) = \mathrm{P}(D \leq d). \tag{5}$$

Proof. See Appendix A. Next, we model the joint CDF of $\{\mathcal{M}_t : t \in [0, T]\}$ and $\{\mathcal{D}_t : t \in [0, T]\}$ as follows:

$$\mathrm{P}(\mathcal{M}_t \leq m, \mathcal{D}_t \leq d) = \sum_{n=0}^{\infty} \mathrm{P}(\mathcal{M}_t \leq m, \mathcal{D}_t \leq d, N_t = n). \tag{6}$$

Then, the probability that $\mathcal{M}_t \in (m_1, m_2]$ with $m_1 < m_2$ and $\mathcal{D}_t \in (d_1, d_2]$ with $d_1 < d_2$ are formulated as follows:

$$\mathrm{P}(m_1 < \mathcal{M}_t \leq m_2, d_1 < \mathcal{D}_t \leq d_2) = \sum_{n=0}^{\infty} \mathrm{P}(m_1 \leq \mathcal{M}_t < m_2, d_1 \leq \mathcal{D}_t < d_2, N_t = n). \tag{7}$$

3.1.2. Dynamic interest rate

The interest rate in this study is assumed to follow the mean-reversion-square-root process model proposed by Cox-Ingersol-Ross [40]. In simple terms, this process guarantees the positivity of the interest rate if the sample of the interest rate is positive. Interest rate positivity generally applies to all countries in the world. There are several exceptions for Japan and Switzerland, where interest rates can be negative in these countries. Mathematically, the mean-reversion-square-root process for the interest rate is formulated as follows:

$$di_k = \mathcal{a}(\mathcal{b} - i_k)dk + \mathcal{g}\sqrt{i_k}dW_k, \tag{8}$$

where $\{i_k : k = 1, 2, 3, \dots, T\}$ is a sequence of random variables representing the interest rate at the end of k-th year, $\mathcal{a} > 0$ is the mean-reversion parameter, $\mathcal{b} > 0$ is the mean long-run parameter, $\mathcal{g} > 0$ is the volatility parameter, and $\{W_k : k = 1, 2, 3, \dots, T\}$ is the standard Weiner process initiated at zero under risk-neutral pricing measure Q. The positivity interest rate previously mentioned is guaranteed by the condition $2\mathcal{a}\mathcal{b} \geq \mathcal{g}^2$ [33]. Based on the interest rate model in Eq (8), the current value of one currency unit can be determined under the no-arbitrage assumption. Mathematically, the value at time $t = 0$ of one currency unit at time T is formulated as follows [24,41]:

$$B_T = \mathbb{E}_{\mathrm{Q}}\left(e^{-\sum_{k=1}^{T} i_k} \middle| \mathcal{P}_0\right), \tag{9}$$

where $\mathbb{E}_{\mathrm{Q}}$ is the expectation under the risk-neutral pricing measure.

3.1.3. Pricing model of earthquake bonds

Investors of earthquake bonds will receive an annual coupon each year and redemption value at maturity from the SPV. Both amounts depend on the maximum magnitude and depth of the earthquakes that occurred until the payment date. The greater the maximum magnitude of the earthquake, the smaller the coupon amount and redemption value paid, and vice versa. Then, the shallower the earthquake's epicenter, the smaller the coupon amount and redemption value paid, and vice versa. The annual coupon payment proportion is let as the set $\{c_1, c_2, c_3, \dots, c_{15} \in [0,1]: c_1 \leq c_2 \leq c_3 \leq \cdots \leq c_{15}\}$. Then, the proportion of the redemption value payment is let as the set $\{r_1, r_2, r_3, \dots, r_{15} \in [0,1]: r_1 \leq r_2 \leq r_3 \leq \cdots \leq r_{15}\}$. Annual coupon payments are assumed to be non-protected. This assumption means that investors have the worst-case possibility of an annual coupon payment of none. Then, the redemption value payment is assumed to be half-protected. This assumption means investors have a worst-case possibility of paying the redemption value of half. Descriptively, the following is the payment scheme for the annual coupon and the redemption value:

a. If the earthquake with the maximum magnitude before the payment date is of the nonsignificant and deep types, the investor gets r_1 part of the redemption value and c_1 part of the annual coupon. We assume $c_1 = 1$ and $r_1 = 1$. In other words, if the maximum magnitude of the earthquake until the payment date occurs is of the nonsignificant and deep earthquake types, the investor gets the total redemption value and the annual coupon.
b. If the earthquake with the maximum magnitude before the payment date is of the nonsignificant and intermediate types, the investor gets r_2 part of the redemption value and c_2 part of the annual coupon.
c. If the earthquake with the maximum magnitude before the payment date is of the nonsignificant and shallow types, the investor gets r_3 part of the redemption value and c_3 part of the annual coupon.
d. If the earthquake with the maximum magnitude before the payment date is of the moderate and deep types, the investor gets r_4 part of the redemption value and c_4 part of the annual coupon.
e. If the earthquake with the maximum magnitude before the payment date is the moderate and intermediate types, the investor gets r_5 part of the redemption value and c_5 part of the annual coupon.
f. If the earthquake with the maximum magnitude before the payment date is of the moderate and shallow types, the investor gets r_6 part of the redemption value and c_6 part of the annual coupon.
g. If the earthquake with the maximum magnitude before the payment date is of the strong and deep types, the investor gets r_7 part of the redemption value and c_7 part of the annual coupon.
h. If the earthquake with the maximum magnitude before the payment date is of the strong and intermediate types, the investor gets r_8 part of the redemption value and c_8 part of the annual coupon.
i. If the earthquake with the maximum magnitude before the payment date is of the strong and shallow types, the investor gets r_9 part of the redemption value and c_9 part of the annual coupon.
j. If the earthquake with the maximum magnitude before the payment date is of the major and

deep types, the investor gets r_{10} part of the redemption value and c_{10} part of the annual coupon.

k. If the earthquake with the maximum magnitude before the payment date is of the major and intermediate types, the investor gets r_{11} part of the redemption value and c_{11} part of the annual coupon.

l. If the earthquake with the maximum magnitude before the payment date is of the major and shallow types, the investor gets r_{12} part of the redemption value and c_{12} part of the annual coupon.

m. If the earthquake with the maximum magnitude before the payment date is of the great and deep types, the investor gets r_{13} part of the redemption value and c_{13} part of the annual coupon.

n. If the earthquake with the maximum magnitude before the payment date is of the great and intermediate types, the investor gets r_{14} part of the redemption value and c_{14} part of the annual coupon.

o. If the earthquake with the maximum magnitude before the payment date is of the great and shallow types, the investor gets r_{15} part of the redemption value and c_{15} part of the annual coupon. Because the annual coupon is non-protected, $c_{15} = 0$. Then, because the redemption value is half-protected, $r_{15} = 0.5$.

Mathematically, the annual coupon payment schemes at the end of the k-th year with $k = 1, 2, 3, \dots, T$ and the redemption value at the end of year T are, respectively, formulated as follows:

$$\mathcal{C}_k = \begin{cases} c_1 C & : 0 < \mathcal{M}_k \leq 5, \mathcal{D}_k > 300, \\ c_2 C & : 0 < \mathcal{M}_k \leq 5, 70 < \mathcal{D}_k \leq 300, \\ c_3 C & : 0 < \mathcal{M}_k \leq 5, \mathcal{D}_k \leq 70, \\ c_4 C & : 5 < \mathcal{M}_k \leq 6, \mathcal{D}_k > 300, \\ c_5 C & : 5 < \mathcal{M}_k \leq 6, 70 < \mathcal{D}_k \leq 300, \\ c_6 C & : 5 < \mathcal{M}_k \leq 6, \mathcal{D}_k \leq 70, \\ c_7 C & : 6 < \mathcal{M}_k \leq 7, \mathcal{D}_k > 300, \\ c_8 C & : 6 < \mathcal{M}_k \leq 7, 70 < \mathcal{D}_k \leq 300, \\ c_9 C & : 6 < \mathcal{M}_k \leq 7, \mathcal{D}_k \leq 70, \\ c_{10} C & : 7 < \mathcal{M}_k \leq 8, \mathcal{D}_k > 300, \\ c_{11} C & : 7 < \mathcal{M}_k \leq 8, 70 < \mathcal{D}_k \leq 300, \\ c_{12} C & : 7 < \mathcal{M}_k \leq 8, \mathcal{D}_k \leq 70, \\ c_{13} C & : \mathcal{M}_k > 8, \mathcal{D}_k > 300, \\ c_{14} C & : \mathcal{M}_k > 8, 70 < \mathcal{D}_k \leq 300, \\ c_{15} C & : \mathcal{M}_k > 8, \mathcal{D}_k \leq 70, \end{cases}$$

and

$$\mathcal{R}_T = \begin{cases} r_1 R & : 0 < \mathcal{M}_T \le 5, \mathcal{D}_T > 300, \\ r_2 R & : 0 < \mathcal{M}_T \le 5, 70 < \mathcal{D}_T \le 300, \\ r_3 R & : 0 < \mathcal{M}_T \le 5, \mathcal{D}_T \le 70, \\ r_4 R & : 5 < \mathcal{M}_T \le 6, \mathcal{D}_T > 300, \\ r_5 R & : 5 < \mathcal{M}_T \le 6, 70 < \mathcal{D}_T \le 300, \\ r_6 R & : 5 < \mathcal{M}_T \le 6, \mathcal{D}_T \le 70, \\ r_7 R & : 6 < \mathcal{M}_T \le 7, \mathcal{D}_T > 300, \\ r_8 R & : 6 < \mathcal{M}_T \le 7, 70 < \mathcal{D}_T \le 300, \\ r_9 R & : 6 < \mathcal{M}_T \le 7, \mathcal{D}_T \le 70, \\ r_{10} R & : 7 < \mathcal{M}_T \le 8, \mathcal{D}_T > 300, \\ r_{11} R & : 7 < \mathcal{M}_T \le 8, 70 < \mathcal{D}_T \le 300, \\ r_{12} R & : 7 < \mathcal{M}_T \le 8, \mathcal{D}_T \le 70, \\ r_{13} R & : \mathcal{M}_T > 8, \mathcal{D}_T > 300, \\ r_{14} R & : \mathcal{M}_T > 8, 70 < \mathcal{D}_T \le 300, \\ r_{15} R & : \mathcal{M}_T > 8, \mathcal{D}_T \le 70, \end{cases}$$

where $C \ge 0$ is the annual coupon, and $R > 0$ is the redemption value. If $C = 0$, then the earthquake bond is called a zero-coupon earthquake bond.

In this study, the price of earthquake bonds is formulated as the summation of the expected present value of the redemption value and the annual coupon. We model it using the risk-neutral pricing measure Q. Mathematically, the price model for T-year earthquake bonds purchased at time $t = 0$ is formulated as follows:

$$V_T = \sum_{k=1}^{T} \mathbb{E}_{\mathrm{Q}}\left(\mathcal{C}_k e^{-\sum_{s=1}^{k} i_s} \middle| \mathcal{P}_0\right) + \mathbb{E}_{\mathrm{Q}}\left(\mathcal{R}_T e^{-\sum_{s=1}^{T} i_s} \middle| \mathcal{P}_0\right)$$

$$= \sum_{k=1}^{T} \sum_{p=1}^{5} \sum_{q=1}^{3} c_{3(p-1)+q} \mathrm{P}\big(m_p < \mathcal{M}_k \le m_{p+1}, d_{q+1} < \mathcal{D}_k \le d_q\big) C B_k \tag{10}$$

$$+ \sum_{p=1}^{5} \sum_{q=1}^{3} r_{3(p-1)+q} \mathrm{P}\big(m_p < \mathcal{M}_T \le m_{p+1}, d_{q+1} < \mathcal{D}_T \le d_q\big) R B_T,$$

where $\mathbb{E}_{\mathrm{Q}}$ is the expectation under Q, B_k with $k = 1, 2, 3, \dots, T$ can be determined in Eq (9), $m_1 = 0$, $m_2 = 5$, $m_3 = 6$, $m_4 = 7$, $m_5 = 8$, $m_6 \to \infty$, $d_1 \to \infty$, $d_2 = 300$, $d_3 = 70$, and $d_4 = 0$.

Proof. See Appendix B. Although Eq (10) has been reduced to its simplest form, its solution cannot be determined analytically. However, we can approximate it numerically. It is discussed in Subsection 3.4.

3.1.4. Monte Carlo algorithm to approximate the model solution

The earthquake bond price in Eq (10) can be estimated numerically using the Monte Carlo method. Suppose that:

a. $\mathcal{S}$ represents the number of simulations, and $\mathcal{s} \in \{1, 2, \dots, \mathcal{S}\}$ is a set of indices representing the simulations' sequence.

b. $\hat{V}_T^{(s)}$ represents the price of earthquake bonds in the s-th simulation.

c. $\mathcal{M}_{\mathcal{W},k}^{(s)}$ is a set of random numbers from the random variable $\mathcal{M}_k$ of size $\mathcal{W}$ in the s-th simulation.

d. $\mathcal{D}_{\mathcal{W},k}^{(s)}$ is a set of random numbers from the random variable $\mathcal{D}_k$ of size $\mathcal{W}$ in the s-th simulation.

e. $\mathfrak{p}_{pq,k}^{(s)}$ is the value of $\mathrm{P}\left(m_p < \mathcal{M}_k^{(s)} \leq m_{p+1}, d_{q+1} \leq \mathcal{D}_k^{(s)} < d_q\right)$ in the s-th simulation with $p = 1, 2, \dots, 5$ and $q = 1, 2, 3$.

f. $B_k^{(s)}$ is the discount factor from $t = 0$ to the end of the k-th year in the s-th simulation.

In simple terms, the solution to Eq (10) can be approximated using the following equation:

$$\hat{V}_T = \lim_{\mathcal{S}\to\infty} \frac{1}{\mathcal{S}} \sum_{s=1}^{\mathcal{S}} \hat{V}_T^{(s)}, \tag{11}$$

where

$$\hat{V}_T^{(s)} = \sum_{k=1}^{T} \sum_{p=1}^{5} \sum_{q=1}^{3} c_{3(p-1)+q} \mathfrak{p}_{pq,k}^{(s)} C B_k^{(s)} + \sum_{p=1}^{5} \sum_{q=1}^{3} r_{3(p-1)+q} \mathfrak{p}_{pq,T}^{(s)} R B_T^{(s)}. \tag{12}$$

Practically, the steps for using Eqs (11) and (12) are as follows:

a. Determine the values of $\mathcal{S}$, $\mathcal{W}$, C, R, T, and $(c_f,\ r_f\colon\ f = 1,\ 2, \dots, 15)$.

b. Generate $N_k^{(s)}$ for each k and s, where $N_k^{(s)}$ represents the frequency of significant earthquakes until the end of the k-year in the s-th simulation.

c. Generate $M_j^{(s)}$ and $D_j^{(s)}$ for each $j = 1, 2, \dots, N_k^{(s)}$, where $M_j^{(s)}$ and $D_j^{(s)}$ represent the strength and depth of the j-th earthquake in the s-th simulation, respectively.

d. Determine the values of $\mathcal{M}_k^{(s)}$ and $\mathcal{D}_k^{(s)}$ for each k and s.

e. Determine the value $\mathfrak{p}_{pq,k}^{(s)}$ for each k, p, q, and s.

f. Generate $i_k^{(s)}$ for each k and s through the random error using Eq (8), where $i_k^{(s)}$ is the interest rate at the end of the k-th year in the s-th simulation.

g. Determine $B_k^{(s)}$ for each k and s.

h. Determine the value of $\hat{V}_T^{(s)}$.

i. Determine the value of $\hat{V}_T$.

3.2. *Model application to actual data*

3.2.1. Data description

This section contains simulation of earthquake bond price model used in Indonesia's earthquake and financial data. The data is on the significant earthquake's magnitude and depth in Indonesia from January 1, 2009 to December 31, 2023. Both sets of data were sourced from the official website of the Meteorological, Climatological, and Geophysical Agency of the Republic of Indonesia (https://repogempa.bmkg.go.id/eventcatalog) on June 10, 2024. The size of both data is the same, namely 5153. Then, other data is on annual interest rates in Indonesia from 1992 to 2023. This data was accessed from the official Bank Indonesia website (https://www.bi.go.id/en/iru/economic-market-data/default.aspx) on June 10, 2024. The size of this data is 32.

3.2.2. Estimating Cox-Ingersoll-Ross model parameters

This section begins by estimating the Cox-Ingersoll-Ross force of interest model parameters in Eq (8). To estimate these parameters, we use a discrete version of the model with one time step as follows [21]:

$$i_{k+1} - i_k = a(b - i_k) + g\sqrt{i_k}\varepsilon_k, \tag{13}$$

where ε_k for all $k = 1, 2, 3, \ldots T - 1$ are independent and have a normal distribution $\mathrm{N}(0, 1)$. To estimate the parameters of Eq (13), we perform the ordinary least squares (OLS) method proposed by Kladivko [42]. The first step is to reformulate Eq (13) as follows:

$$\frac{i_{k+1} - i_k}{\sqrt{i_k}} = \frac{ab}{\sqrt{i_k}} - a\sqrt{i_k} + g\varepsilon_k. \tag{14}$$

The solution to Eq (14) is obtained by solving the following minimization problem:

$$\min. \sum_{k=1}^{T-1} (g\varepsilon_k)^2 = \min. \sum_{k=1}^{T-1} \left(\frac{i_{k+1} - i_k}{\sqrt{i_k}} - \frac{ab}{\sqrt{i_k}} + a\sqrt{i_k} \right)^2 = \min.(\mathrm{y} - \mathrm{Xb})^2,$$

where

$$y = \begin{bmatrix} \frac{i_2 - i_1}{\sqrt{i_1}} \\ \frac{i_3 - i_2}{\sqrt{i_2}} \\ \frac{i_4 - i_3}{\sqrt{i_3}} \\ \vdots \\ \frac{i_T - i_{T-1}}{\sqrt{i_{T-1}}} \end{bmatrix}, X = \begin{bmatrix} \frac{1}{\sqrt{i_1}} & -\sqrt{i_1} \\ \frac{1}{\sqrt{i_2}} & -\sqrt{i_2} \\ \frac{1}{\sqrt{i_3}} & -\sqrt{i_3} \\ \vdots & \vdots \\ \frac{1}{\sqrt{i_{T-1}}} & -\sqrt{i_{T-1}} \end{bmatrix}, \text{and } b = \begin{bmatrix} a\mathscr{b} \\ a \end{bmatrix}.$$

Briefly, the estimated parameter b is as follows:

$$\hat{b} = (X'X)^{-1}X'y. \tag{15}$$

In other word,

$$\hat{a} = b_{2,1} \text{ and } \hat{\mathscr{b}} = \frac{b_{1,1}}{b_{2,1}},$$

where $b_{\mathscr{p},\mathscr{q}}$ is the element of b in the $\mathscr{p}$-th row and $\mathscr{q}$-th column with $\mathscr{p} = 1, 2$ and $\mathscr{q} = 1$. Then, $\hat{\mathscr{g}}_1$ is obtained from the following equation:

$$\hat{\mathscr{g}} = \sqrt{\frac{1}{T-2}\sum_{\mathscr{k}=1}^{T-1}(\mathscr{g}\varepsilon_{\mathscr{k}} - \bar{y})^2} = \sqrt{\frac{1}{T-2}\sum_{t=1}^{T-1}\left(\frac{i_{\mathscr{k}+1} - i_{\mathscr{k}}}{\sqrt{i_{\mathscr{k}}}} - \frac{\hat{a}\hat{\mathscr{b}}}{\sqrt{i_{\mathscr{k}}}} + \hat{a}\sqrt{i_{\mathscr{k}}} - \bar{y}\right)^2}, \tag{16}$$

where $\bar{y}$ is the average of $\mathscr{g}\varepsilon_{\mathscr{k}}$ with $\mathscr{k} = 1, 2, 3, \ldots, T-1$. Based on the data we use, by applying Eqs (15) and (16), we obtain $\hat{a} = 0.20845$, $\hat{\mathscr{b}} = 0.08285$, and $\hat{\mathscr{g}} = 0.10944$.

3.2.3. Estimating significant earthquake intensity

To estimate λ, the OLS method can be used. In summary, the estimate of λ, denoted $\hat{\lambda}$, is formulated as follows [39]:

$$\hat{\lambda} = \frac{\mathcal{N}}{\mathcal{T}}, \tag{17}$$

where $\mathcal{T}$ is the length of the observation time, and $\mathcal{N}$ is the number of significant earthquakes during the observation time. The length of the observation time can have specific units, e.g., days, months, or years. We chose the unit of year to match the term of earthquake bonds, which also have units of years. Based on the data obtained, the values were $\mathcal{N} = 5153$ and $\mathcal{T} = 15$ years. Therefore, we obtain $\hat{\lambda} = \frac{5153}{15} \approx 343.3333$ significant earthquakes per year.

3.2.4. Fitting earthquake magnitude and depth distributions

This section involves identifying the theoretical distribution to represent the data distribution of significant earthquake magnitudes and depths in Indonesia. This identification was performed using the Anderson-Darling test with a 0.01 level of significance, and the analysis was carried out using EasyFit Professional software version 5.5. Briefly, using this software, the most significant distribution representing significant earthquake magnitude is the Weibull distribution (α, β, γ) with $\hat{\alpha} = 0.99308$, $\hat{\beta} = 0.41869$, and $\hat{\gamma} = 5$. Meanwhile, for significant earthquake depths, the most significant is the generalized Pareto distribution (GPD) (ξ, σ) with $\hat{\xi} = 0.4672$ and $\hat{\sigma} = 46.902$. In other words, we have a null hypothesis for the goodness-of-fit test between magnitude and depth data distributions as follows:

$$\mathrm{P}(M \leq m) = 1 - e^{-\left(\frac{m-\gamma}{\beta}\right)^{\alpha}} \approx 1 - e^{-\left(\frac{m-5}{0.41869}\right)^{0.99308}} \tag{18}$$

and

$$\mathrm{P}(D \leq d) = 1 - \left(1 + \xi \frac{d}{\sigma}\right)^{-\frac{1}{\xi}} \approx 1 - \left(1 + 0.4672 \frac{d}{46.902}\right)^{-\frac{1}{0.4672}}, \tag{19}$$

where $m \geq 5$ and $d \geq 0$. The Weibull and generalized Pareto distributions have test values of 3.6689 and 147.73, respectively. With a critical value of 3.9074, the null hypothesis for the fitting test between the Weibull and significant earthquake magnitude data distributions is not rejected, while the fitting test between the Weibull and significant earthquake depth data distributions is rejected. These results show that the distribution of significant earthquake magnitude data in Indonesia can be represented by the Weibull distribution (0.99308, 0.41869, 5). In contrast, the distribution of significant earthquake depth data in Indonesia cannot be represented by the GPD (2.1406, 100.4). Although the distribution of earthquake depth data does not match the GPD, we tolerate this mismatch in the context of numerical simulations as conducted by Shao et al. [21]. The primary focus in this section is on model validation using a simulation approach. Therefore, this discrepancy does not reduce the reliability of the simulation results because the method used is designed to produce valid results.

3.2.5. Estimating earthquake bond prices

Equation (11) is used to approximate the earthquake bond prices. We assume that the number of simulations is $\mathcal{S} = 100{,}000$ and the number of generated $\mathcal{M}_k$ and $\mathcal{D}_k$ values is $\mathcal{W} = 100$. Then, the annual coupon is $C = 0.1$ USD, the redemption value is $R = 1$ USD, and the term is $T = 3$ years. The estimation of earthquake bond prices in this study assumes that $c_1 = 1$, $c_2 = \frac{13}{14}$, $c_3 = \frac{12}{14}$, $c_4 = \frac{11}{14}$, $c_5 = \frac{10}{14}$, $c_6 = \frac{9}{14}$, $c_7 = \frac{8}{14}$, $c_8 = \frac{7}{14}$, $c_9 = \frac{6}{14}$, $c_{10} = \frac{5}{14}$, $c_{11} = \frac{4}{14}$, $c_{12} = \frac{3}{14}$, $c_{13} = \frac{2}{14}$, $c_{14} = \frac{1}{14}$, $c_{15} = 0$, $d_1 = 1$, $d_2 = \frac{27}{28}$, $d_3 = \frac{26}{28}$, $d_4 = \frac{25}{28}$, $d_5 = \frac{24}{28}$, $d_6 = \frac{23}{28}$, $d_7 = \frac{22}{28}$, $d_8 = \frac{21}{28}$, $d_9 = \frac{20}{28}$, $d_{10} = \frac{19}{28}$, $d_{11} = \frac{18}{28}$, $d_{12} = \frac{17}{28}$, $d_{13} = \frac{16}{28}$, $d_{14} = \frac{15}{28}$, and $d_{15} = \frac{14}{28}$. Based on determining parameters, the price of earthquake bonds at time $t = 0$ resulting from the $\mathcal{S}$ simulation is visualized in a histogram in Figure 2. 15.692% of results exceeded the mean plus standard deviation, while 15.801% of results were below the mean minus standard deviation. The remaining 68.507% fell between these two limits. These percentages are an indication that the simulation results are normally distributed. Then, the skewness of the results is 0.2159, and the kurtosis is 3.1029. The skewness value is close to zero and the kurtosis value is close to three, which also indicates that the simulation results are normally distributed. The indication of normality can also be seen in the $\mathcal{S}$ simulation of bond pricing results with a term of one to six years, whose histograms are also illustrated in Figure 2. Finally, based on the average of the simulation results obtained, the estimated price of earthquake bonds at $t = 0$ is $\hat{V}_3 = 0.5033$ USD. Further details are presented in Table 2.

Table 2 demonstrates that the expected annual coupon each year is positive. This positivity indicates that, on average, investors will always receive an annual coupon every year. Then, annual coupon expectations always decrease every year. This decrease happens because the longer the term of an earthquake bond, the higher the possibility of a significant earthquake event. Then, the higher the possibility, the smaller the investor receiving the annual coupon. As a result, the expected value of the annual coupon continuously decreases every year.

Table 2. Details of earthquake bond price estimations with a term of three years in Indonesia.

Payment	Value (USD)	Present Value (USD)
Expected coupon in year 1	0.0197	0.0185
Expected coupon in year 2	0.0146	0.0129
Expected coupon in year 3	0.0116	0.0096
Expected redemption value in year 3	0.5579	0.4623
$\hat{V}_3$		0.5033

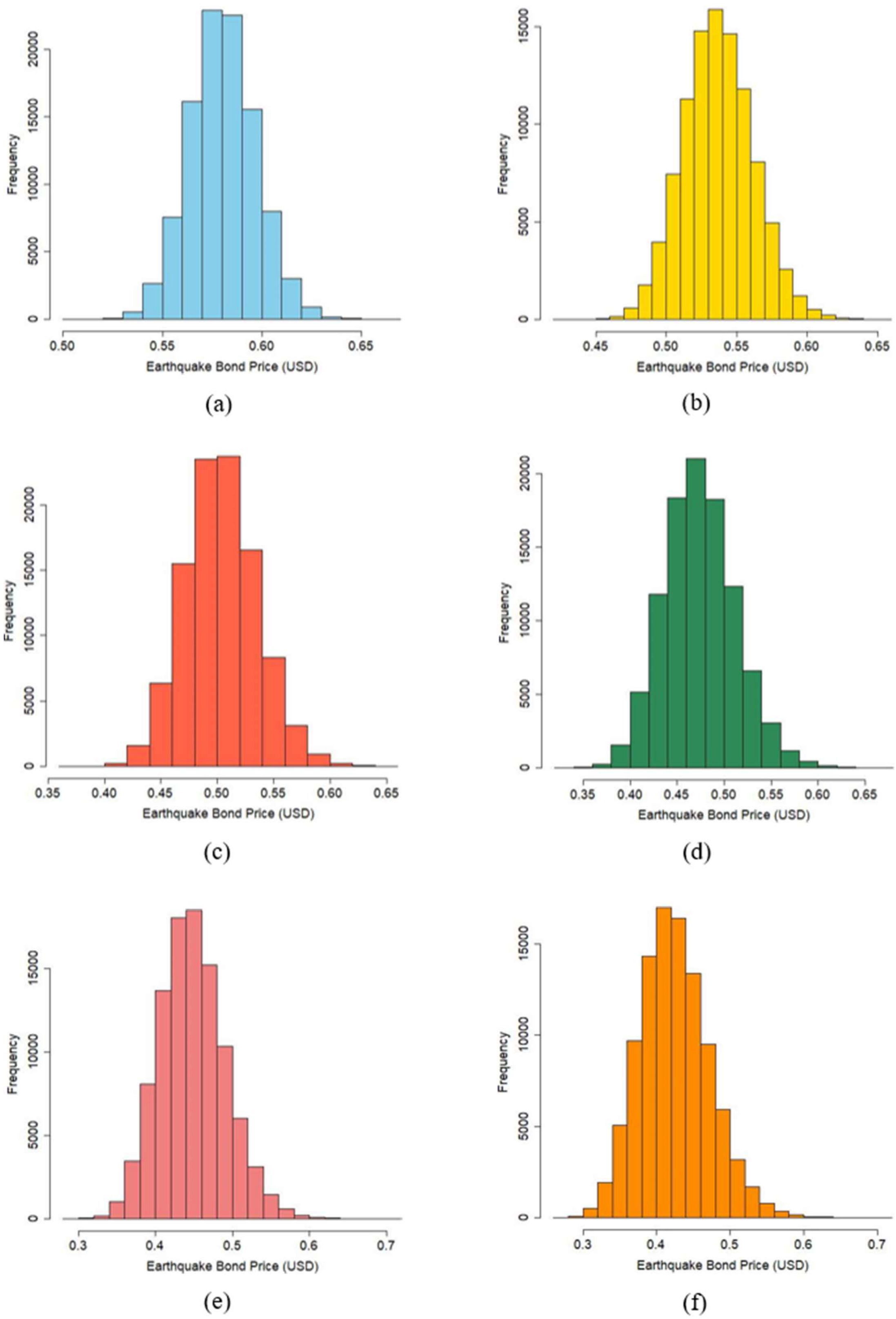

Figure 2. Histogram displaying the results of 100,000 simulations for calculating earthquake bond prices with a term of one (a), two (b), three (c), four (d), five (e), and six (f) years.

4. Discussion

4.1. *Sensitivity analysis*

We first analyze how the difference between the earthquake bond price is estimated using constant and stochastic interest rates. The stochastic interest rate model and its parameters are given in Subsection 4.2. Then, the constant interest rate used is 0.0583. This value is adjusted to the actual interest rate in Indonesia at the beginning of 2024. Using Eq (11) and the same values of other variables in Subsections 4.2 to 4.4, the analysis results of prices in earthquake bonds with one- to seven-year terms under both interest rate assumptions are visualized in Figure 3. Figure 3 illustrates that if the term of the earthquake bond is long, the difference in bond prices with the two assumptions of constant and stochastic interest rates tends to be more significant. Suppose the earthquake bond price with the constant interest rate is the approximate value for the earthquake bond price with the stochastic interest rate. In that case, the absolute percentage errors (APE) of the approximations for one to five years are 0.4722%, 0.9050%, 1.2710%, 1.6378%, 1.9182%, 2.2085%, and 2.4319%, respectively. These APE values are crucial for large amounts of money in bonds.

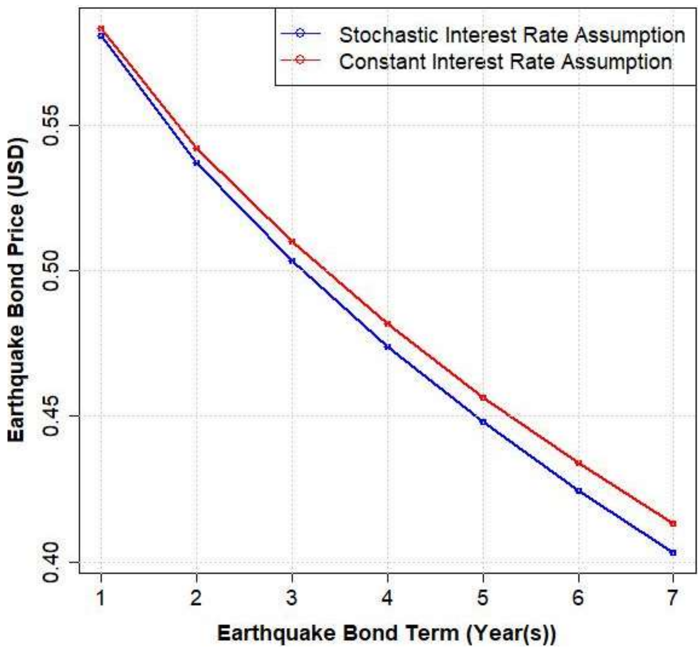

Figure 3. The prices in earthquake bonds with one to seven-year terms under constant and stochastic interest rate assumptions.

Next is a sensitivity analysis of the annual significant earthquake intensity on earthquake bond prices. We make the intensity a set $\{25, 50, 75, \dots, 375, 400\}$. Using Eq (11) and the same values of other variables as in Subsections 4.2 to 4.4, the analysis results of the influence of significant earthquake intensity on earthquake bond prices with one- to five-year terms are given visually in Figure 4. Figure 4 demonstrates that the greater the intensity of a significant earthquake, the lower the price of earthquake bonds, and vice versa. This case makes sense, where the possibility of losing the coupon and redemption value of earthquake bonds will be high if the intensity of the significant earthquake is also high. As a result, investor demand for bonds is low, so bond prices are also low.

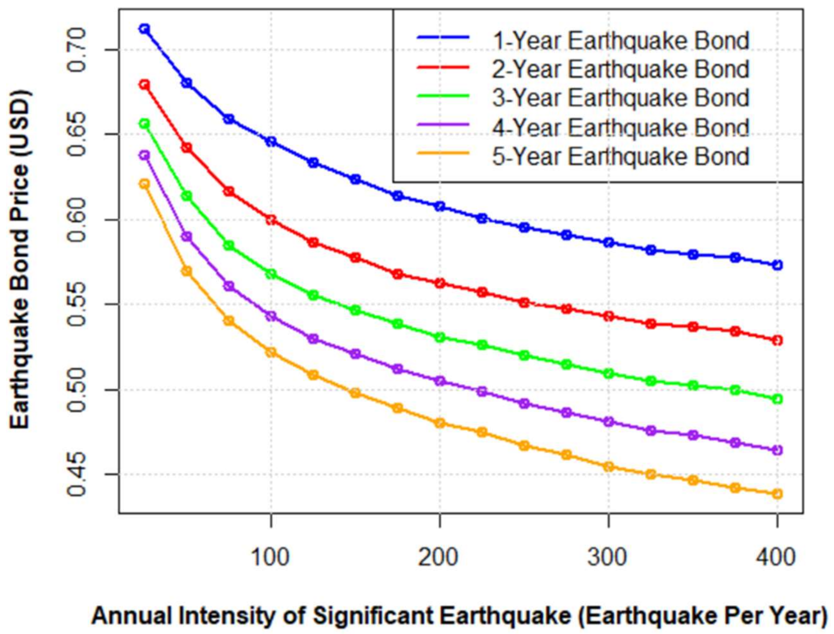

Figure 4. Earthquake bond prices with one- to five-year terms under the intensity of $\{25, 50, 75, \dots, 375, 400\}$ earthquakes per year.

Next is an analysis of the influence of the annual coupon amount of earthquake bonds on its prices. We make the number of annual coupons into the set $\{0, 0.025, 0.05, \dots, 0.225, 0.25\}$ USD. We estimate the earthquake bond price for each annual coupon value on a one- to five-year earthquake bond using Eq (11). Except for the coupon value, the other variable values are the same as in Subsections 4.2 to 4.4. The analysis results of the influence of the annual coupon on earthquake bond prices are illustrated visually in Figure 5. Figure 5 demonstrates that the higher the annual coupon amount of the earthquake bond, the higher its price, and vice versa. This case makes sense, where if the annual coupon amount is high, there is a high demand from investors for the bond. As a result, bond prices are also high.

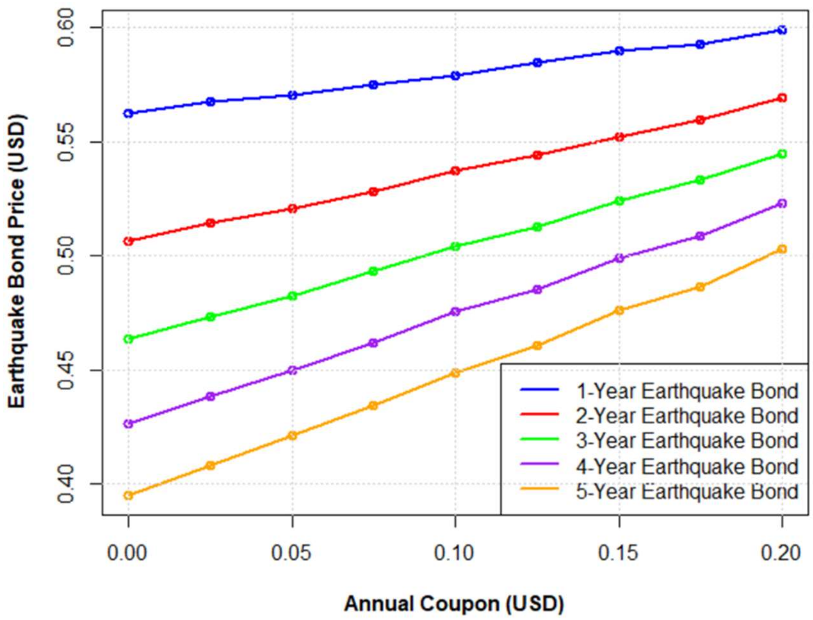

Figure 5. One- to five-year earthquake bond prices with an annual coupon of $\{0, 0.025, 0.05, \ldots, 0.225, 0.25\}$ USD.

The following is an analysis of how the term of the earthquake bond affects its price. The analysis results are displayed visually in Figures 4 and 5. Figures 4 and 5 demonstrate that the price of an earthquake bond decreases with its term and vice versa. This case makes sense, where the possibility of losing the coupon and redemption value of the earthquake bond will be high if the bond term is long. As a result, investor demand for these bonds is low, so bond prices are also low.

The last is an analysis of the influence of the correlation rates between the magnitude and depth of earthquakes on earthquake bond prices. We set this correlation to $\{-1, -0.8, -0.6, \ldots, 0.8, 1\}$. We estimate the earthquake bond price for each correlation rate in terms of one to five years using Eq (11). We also assume that the other variable values are the same as in Subsections 4.2 to 4.4. The results of the analysis of these correlation rate effects on earthquake bond prices are illustrated in Figure 6. Figure 6 demonstrates the correlation rates significantly affect the earthquake bond prices. The more negative the correlation rates between magnitude and depth, the smaller the earthquake bond prices, and vice versa. This trend makes sense because when the correlation rates between magnitude and depth are highly negative, the potential for severe damage due to earthquakes is more significant. As a result, the perceived risk of harm increases significantly. Investors, therefore, will view the bond as riskier, reducing demand for the bond. As a result, bond prices decrease as investors seek higher returns to compensate for the greater risk or avoid purchasing such bonds altogether. These findings support the urgency of using the assumed link between magnitude and depth in modeling earthquake bond prices used in this study.

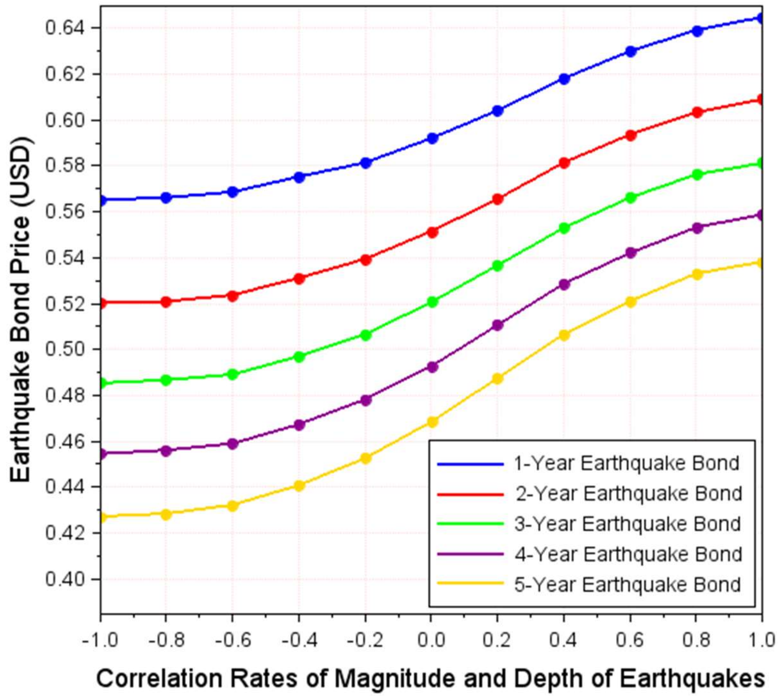

Figure 6. One- to five-year earthquake bond prices with correlation rates between magnitude and depth $\{-1, -0.8, -0.6, \ldots, 0.8, 1\}$.

4.2. *Model comparison*

The earthquake bond pricing model with the maximum magnitude and depth trigger indices in this study (which we will call model A) is compared with the model with the maximum magnitude trigger index only (which we will call model B). The payment schemes of the annual coupon and the redemption value of the earthquake bonds in model B are, respectively, formulated as follows:

$$\mathcal{C}_k = \begin{cases} c_1 C & : 0 < \mathcal{M}_k \leq 5, \\ c_2 C & : 5 < \mathcal{M}_k \leq 6, \\ c_3 C & : 6 < \mathcal{M}_k \leq 7, \\ c_4 C & : 7 < \mathcal{M}_k \leq 8, \\ c_5 C & : \mathcal{M}_k > 8, \end{cases}$$

and

$$\mathcal{R}_T = \begin{cases} r_1 R & : 0 < \mathcal{M}_T \leq 5, \\ r_2 R & : 5 < \mathcal{M}_T \leq 6, \\ r_3 R & : 6 < \mathcal{M}_T \leq 7, \\ r_4 R & : 7 < \mathcal{M}_T \leq 8, \\ r_5 R & : \mathcal{M}_T > 8, \end{cases}$$

where $k = 1, 2, 3, \ldots, T$. Then, the earthquake bond price at time $t = 0$ from model B is formulated as follows:

$$V_T = \sum_{k=1}^{T} \mathbb{E}_{\mathrm{Q}}\left(\mathcal{C}_k e^{-\sum_{s=1}^{k} i_s} \middle| \mathcal{P}_0\right) + \mathbb{E}_{\mathrm{Q}}\left(\mathcal{R}_T e^{-\sum_{s=1}^{T} i_s} \middle| \mathcal{P}_0\right),$$

$$= \sum_{k=1}^{T} \sum_{p=1}^{5} c_p C\mathrm{P}\left(m_p < \mathcal{M}_k \leq m_{p+1}\right) B_k + \sum_{p=1}^{5} r_p R\mathrm{P}\left(m_p < \mathcal{M}_T \leq m_{p+1}\right) B_T. \quad (20)$$

The proof of Eq (20) can be carried out similarly to the proof of Eq (10). Next, using both models, we calculate the earthquake bond price for one to five years using the similar way as in Eqs (11) and (12). We use $c_1 = 1$, $c_2 = 0.75$, $c_3 = 0.5$, $c_4 = 0.25$, $c_5 = 0$, $r_1 = 1$, $r_2 = 0.875$, $r_3 = 0.75$, $r_4 = 0.625$, and $r_5 = 0.5$. With the same variable values as in Subsections 4.2 to 4.4, the results of estimating earthquake bond prices with models A and B are displayed visually in Figure 7. Figure 7 demonstrates that the estimated earthquake bond price obtained from model A is always higher than that obtained from model B. In other words, using two trigger indices tends to make earthquake bond prices higher than using one index. This case makes sense, where earthquake bonds with two trigger indices have a lower possibility of claims occurring than those with one trigger index. As a result, earthquake bonds with two trigger indices tend to have higher demand than bonds with one trigger index. As a result, the price of earthquake bonds with two trigger indices tends to be higher than those with one trigger index.

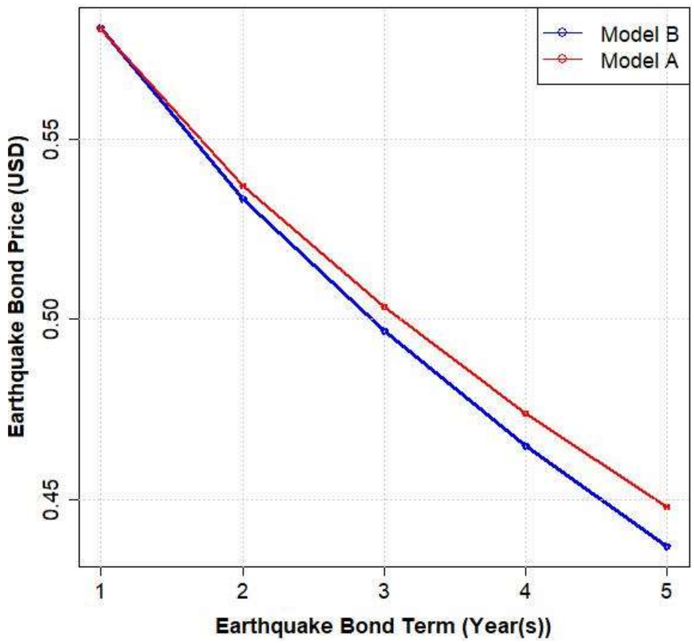

Figure 7. Earthquake bond price estimates for one to five years using models A and B.

5. Conclusions

This study aims to model the price of earthquake bonds using maximum magnitude and depth trigger indices. Both triggers are assumed to be dependent to enable more accurate risk assessment and higher revenue payments. Apart from that, the values of both triggers are determined based on the earthquake's intensity. Intensity addresses rising seismic activity, while depth and magnitude are designed as dual-trigger events that can enhance the bond's appeal. All three factors are modeled simultaneously using a compound process of earthquake frequency, magnitude, and depth. Then, we design a coupon payment scheme and redemption value into a non-binary form to assess all possible severities of earthquakes that occur within the life of the bond. To model bond prices, we use the concept of a risk-neutral pricing measure with stochastic assumptions of interest rates. This stochasticity is accommodated using the Cox-Ingersoll-Ross model, which guarantees the positivity of interest rates in accordance with actual conditions in almost every country worldwide.

After the model was designed, we carried out a sensitivity analysis of the variables used in earthquake bond prices. There is a notable difference between earthquake bond prices with stochastic versus constant interest rates. The longer the bond's term, the more pronounced the price difference becomes. We then found that the greater the term and the earthquake intensity, the lower the price of earthquake bonds, and vice versa. It makes sense, as the possibility of losing the coupon and redemption value of the earthquake bond is higher when both the term and intensity are also high. As a result, investor demand for bonds decreases, and bond prices also decline. We also found that the higher the annual coupon amount of the earthquake bond, the higher the price of the bond, and vice versa. It makes sense, as a higher annual coupon amount leads to greater demand from investors for the bond, resulting in higher bond prices. Additionally, we found that the more negative the correlation between magnitude and depth, the lower the earthquake bond prices, and vice versa. It occurs because a highly negative correlation indicates a higher potential for damage, increasing perceived risk and reducing demand for the bonds. These findings emphasize the importance of considering the relationship between magnitude and depth in modeling earthquake bond prices. Finally, we compared the results of estimating earthquake bond prices based on the proposed model with another model using one trigger. The results show that using two trigger indices tends to make earthquake bond prices higher than using just one trigger index. It is reasonable, as earthquake bonds with two trigger indices have a lower likelihood of claims occurring than those with one trigger index. As a result, earthquake bonds with two trigger indices tend to have higher demand and, consequently, higher prices than those with one trigger index.

This study can be employed by earthquake bond issuers worldwide to price earthquake bonds that reflect the severity of the earthquake with its maximum magnitude, depth, and intensity. Apart from that, this study can also be used by geological institutions and others to estimate the risk of maximum magnitude and depth of earthquakes in the observation area. This study has areas for improvement, such as measuring the severity of the earthquake geologically and financially. Considering aggregate loss can be an opportunity for future study. Additionally, the inflation rate to obtain actual returns is not included, which also presents a chance for future study.

Author contributions

Riza Andrian Ibrahim: Conceptualization, methodology, software, formal analysis, investigation, data curation, writing-original draft preparation, visualization, supervision; Sukono: Conceptualization, validation, formal analysis, investigation, resources, writing-review and editing, supervision, funding acquisition; Herlina Napitupulu: Conceptualization, methodology, validation, data curation, writing-review and editing, supervision; Rose Irnawaty Ibrahim: Conceptualization, validation, writing-review and editing, supervision, project administration. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Appendix A

Proof of Eq (3)

The cumulative distribution function (CDF) of $\{\mathcal{M}_t : t \in [0, T]\}$ is formulated as follows:

$$\mathrm{P}(\mathcal{M}_t \le m) = \sum_{n=0}^{\infty} \mathrm{P}(\mathcal{M}_t \le m, N_t = n) = \sum_{n=0}^{\infty} \mathrm{P}(N_t = n)\mathrm{P}(\mathcal{M}_t \le m | N_t = n). \tag{A1}$$

Recall that $\{M_j : j \in J_t\}$ are i.i.d., and $\{N_t : t \in [0, T]\}$ and $\{M_j : j \in J_t\}$ are independent. Then, for a fixed number $n = 0, 1, 2, 3, \dots,$ the CDF of $\{\mathcal{M}_t : t \in [0, T]\}$ is formulated as follows [43]:

$$\begin{gathered}\mathrm{P}(\mathcal{M}_t \le m | N_t = n) = \mathrm{P}\left(\max_{j \in J_t}\{M_j\} \le m \middle| N_t = n\right) = \mathrm{P}\left(\max_{j \in \mathcal{J}}\{M_j\} \le m\right) \\ = \mathrm{P}(M_1 \le m, M_2 \le m, \dots, M_n \le m) = \mathrm{P}(M_1 \le m)\mathrm{P}(M_2 \le m) \dots \mathrm{P}(M_n \le m) \\ = \mathrm{P}^n(M \le m),\end{gathered} \tag{A2}$$

where $\mathcal{J} = \{1, 2, \dots, n\}$. Substitute Eq (A2) into (A1) so that:

$$\mathrm{P}(\mathcal{M}_t \le m) = \sum_{n=0}^{\infty} \mathrm{P}(N_t = n)\mathrm{P}^n(M \le m). \tag{A3}$$

Substitute Eq (2) into (A3) to obtain the following equation:

$$\mathrm{P}(\mathcal{M}_t \le m) = \sum_{n=0}^{\infty} \frac{(\lambda t)^n}{n!} e^{-\lambda t} \mathrm{P}^n(M \le m) = \sum_{n=0}^{\infty} \frac{[\lambda t \mathrm{P}(M \le m)]^n}{n!} e^{-\lambda t}$$

$$= \sum_{n=0}^{\infty} \frac{[\lambda t \mathrm{P}(M \le m)]^n}{n!} e^{-\lambda t} \frac{e^{\lambda t[1-\mathrm{P}(M \le m)]}}{e^{\lambda t[1-\mathrm{P}(M \le m)]}}$$

$$= \frac{1}{e^{\lambda t[1-\mathrm{P}(M \le m)]}} \sum_{n=0}^{\infty} \frac{[\lambda t \mathrm{P}(M \le m)]^n}{n!} e^{-\lambda t \mathrm{P}(M \le m)} = e^{-\lambda t[1-\mathrm{P}(M \le m)]}.$$

Proof of Eq (4)

The probability that $\mathcal{M}_t \in (m_1, m_2]$ is formulated as follows:

$$\mathrm{P}(m_1 < \mathcal{M}_t \le m_2) = \sum_{n=0}^{\infty} \mathrm{P}(m_1 < \mathcal{M}_t \le m_2, N_t = n)$$

$$= \sum_{n=0}^{\infty} \mathrm{P}(N_t = n)\mathrm{P}(m_1 < \mathcal{M}_t \le m_2 | N_t = n)$$

$$= \sum_{n=0}^{\infty} \mathrm{P}(N_t = n)[\mathrm{P}(\mathcal{M}_t \le m_2 | N_t = n) - \mathrm{P}(\mathcal{M}_t \le m_1 | N_t = n)]$$

$$= \sum_{n=0}^{\infty} [\mathrm{P}(N_t = n)\mathrm{P}(\mathcal{M}_t \le m_2 | N_t = n) - \mathrm{P}(N_t = n)\mathrm{P}(\mathcal{M}_t \le m_1 | N_t = n)]$$

$$= \sum_{n=0}^{\infty} \mathrm{P}(N_t = n)\mathrm{P}(\mathcal{M}_t \le m_2 | N_t = n) - \sum_{n=0}^{\infty} \mathrm{P}(N_t = n)\mathrm{P}(\mathcal{M}_t < m_1 | N_t = n)$$

$$= \mathrm{P}(\mathcal{M}_t \le m_2) - \mathrm{P}(\mathcal{M}_t \le m_1).$$

Proof of Eq (5)

The CDF of $\{\mathcal{D}_t : t \in [0, T]\}$ is formulated as follows:

$$\mathrm{P}(\mathcal{D}_t \le d) = \mathrm{P}(D_S \le d) = \sum_{n=0}^{\infty} \sum_{s=1}^{n} \mathrm{P}(D_S \le d, S = s, N_t = n).$$

The equation can be expanded as follows:

$$\mathrm{P}(\mathcal{D}_t \le d) = \sum_{n=0}^{\infty} \sum_{s=1}^{n} \mathrm{P}(N_t = n)\mathrm{P}(S = s | N_t = n)\mathrm{P}(D_S \le d | S = s, N_t = n). \tag{A4}$$

Since $\{M_j : j \in J_t\}$ and $\{N_t : t \in [0, T]\}$ are independent, and $S = s$ given $N_t = n$ indicates that M_s is the maximum value $\{M_j : \in \mathcal{J}\}$ with $\mathcal{J} = \{1, 2, 3, \dots, n\}$, we have the following:

$$\mathrm{P}(S = s | N_t = n) = \mathrm{P}\left(M_s = \max_{\mathcal{j} \in \mathcal{J}} \{M_{\mathcal{j}}\}\right).$$

Recall that $\{M_j : j \in J_t\}$ is i.i.d. so that the probability that M_s is the maximum value $\{M_{\mathcal{j}} : \mathcal{j} \in \mathcal{J}\}$ is the same for each $s \in \mathcal{J}$, namely $\frac{1}{n}$. It can be mathematically expressed as follows:

$$\mathrm{P}(S = s | N_t = n) = \mathrm{P}\left(M_s = \max_{\mathcal{j} \in \mathcal{J}} \{M_{\mathcal{j}}\}\right) = \frac{1}{n}. \tag{A5}$$

Then, since $\{D_j : j \in J_t\}$ and $\{N_t : t \in [0, T]\}$ are independent, and $D_S \leq d$ given $S = s$ and $N_t = n$ indicates that the depth of an earthquake with maximum strength $M_s = \max_{\mathcal{j} \in \mathcal{J}} \{M_{\mathcal{j}}\}$ is not more than d, we have the following:

$$\mathrm{P}(D_S \leq d | S = s, N_t = n) = \mathrm{P}(D_s \leq d).$$

Remember that $\{D_j : j \in J_t\}$ are i.i.d. so that:

$$\mathrm{P}(D_S \leq d | S = s, N_t = n) = \mathrm{P}(D_s \leq d) = \mathrm{P}(D \leq d). \tag{A6}$$

Substitute Eqs (A5) and (A6) into (A4) to obtain the following equation:

$$\mathrm{P}(\mathcal{D}_t \leq d) = \sum_{n=0}^{\infty} \sum_{s=1}^{n} \mathrm{P}(N_t = n) \frac{1}{n} \mathrm{P}(D \leq d) = \mathrm{P}(D \leq d) \sum_{n=0}^{\infty} \mathrm{P}(N_t = n) = \mathrm{P}(D \leq d).$$

Appendix B

The value at time $t = 0$ of the annual coupon in year k and the redemption value in year T of the earthquake bond modeled using the risk-neutral pricing measure Q are, respectively, formulated as follows:

$$\mathfrak{C}_k = \mathbb{E}_{\mathrm{Q}}\left(\mathcal{C}_k e^{-\sum_{s=1}^{k} i_s} \middle| \mathcal{P}_0\right) \tag{A7}$$

and

$$\mathfrak{R}_T = \mathbb{E}_{\mathrm{Q}}\left(\mathcal{R}_T e^{-\sum_{s=1}^{T} i_s} \middle| \mathcal{P}_0\right), \tag{A8}$$

where $\mathbb{E}_{\mathrm{Q}}$ is the expectation under Q. To solve Eqs (A7) and (A8), we refer to Cox and Pedersen [44], where the joint process of the maximum magnitude and depth of earthquakes maintains its original distributional characteristics even after transitioning from the probability measure P to the risk-neutral measure Q. Under the risk-neutral pricing measure Q, events dependent only on financial variables are independent of those related to earthquake risk variables. Consequently, Eq (A7) can be rewritten as follows:

$$\mathfrak{C}_k \quad = \mathbb{E}_{\mathrm{Q}}\left(\mathcal{C}_k e^{-\sum_{s=1}^{k} i_s} \middle| \mathcal{P}_0\right)$$

$$= \mathbb{E}_{\mathrm{Q}}(\mathcal{C}_k|\mathcal{P}_0)\mathbb{E}_{\mathrm{Q}}\left(e^{-\sum_{s=1}^{k} i_s}\middle|\mathcal{P}_0\right)$$

$$= \mathbb{E}_{\mathrm{P}}(\mathcal{C}_k|\mathcal{P}_0)\mathbb{E}_{\mathrm{Q}}\left(e^{-\sum_{s=1}^{k} i_s}\middle|\mathcal{P}_0\right)$$

$$= \mathbb{E}_{\mathrm{P}}(c_1 C 1_{\{0<\mathcal{M}_k\le 5,\mathcal{D}_k>300\}} + c_2 C 1_{\{0<\mathcal{M}_k\le 5,70\quad \mathcal{D}_k\le 300\}} + c_3 C 1_{\{0<\mathcal{M}_k\le 5,\mathcal{D}_k\le 70\}} + \cdots$$
$$+ c_{13} C 1_{\{\mathcal{M}_k>8,\mathcal{D}_k>30\quad\}} + c_{14} C 1_{\{\mathcal{M}_k>8,70<\mathcal{D}_k\le 300\}} + c_{15} C 1_{\{\mathcal{M}_k>8,\mathcal{D}_k\le 70\}}|\mathcal{P}_0) B_k$$

$$\begin{aligned} = [&c_1 C\mathrm{P}(0<\mathcal{M}_k\le 5,\mathcal{D}_k>300) + c_2 C\mathrm{P}(0<\mathcal{M}_k\le 5,70<\mathcal{D}_k\le 300) \\ &+ c_3 C\mathrm{P}(0<\mathcal{M}_k\le 5,\mathcal{D}_k\le 70) + \cdots + c_{13} C\mathrm{P}(\mathcal{M}_k>8,\mathcal{D}_k>300) \\ &+ c_{14} C\mathrm{P}(\mathcal{M}_k>8,70<\mathcal{D}_k\le 300) + c_{15} C\mathrm{P}(\mathcal{M}_k>8,\mathcal{D}_k\le 70)] B_k \end{aligned}$$

$$\begin{aligned} = [&c_1 \mathrm{P}(0<\mathcal{M}_k\le 5,\mathcal{D}_k>300) + c_2 \mathrm{P}(0<\mathcal{M}_k\le 5,70<\mathcal{D}_k\le 300) \\ &+ c_3 \mathrm{P}(0<\mathcal{M}_k\le 5,\mathcal{D}_k\le 70) + \cdots + c_{13} \mathrm{P}(\mathcal{M}_k>8,\mathcal{D}_k>300) \\ &+ c_{14} \mathrm{P}(\mathcal{M}_k>8,70<\mathcal{D}_k\le 300) + c_{15} \mathrm{P}(\mathcal{M}_k>8,\mathcal{D}_k\le 70)] C B_k \end{aligned}$$

$$= \sum_{p=1}^{5}\sum_{q=1}^{3} c_{3(p-1)+q}\mathrm{P}\left(m_p<\mathcal{M}_k\le m_{p+1}, d_{q+1}<\mathcal{D}_k\le d_q\right) C B_k,$$

where $m_1 = 0$, $m_2 = 5$, $m_3 = 6$, $m_4 = 7$, $m_5 = 8$, $m_6 \to \infty$, $d_1 \to \infty$, $d_2 = 300$, $d_3 = 70$, and $d_4 = 0$. Then, Eq (A8) can be rewritten as follows:

$$\Re_T = \mathbb{E}_{\mathrm{Q}}\left(\mathcal{R}_T e^{-\sum_{s=1}^{T} i_s}\middle|\mathcal{P}_0\right)$$

$$= \mathbb{E}_{\mathrm{Q}}(\mathcal{R}_T|\mathcal{P}_0)\mathbb{E}_{\mathrm{Q}}\left(e^{-\sum_{s=1}^{T} i_s}\middle|\mathcal{P}_0\right)$$

$$= \mathbb{E}_{\mathrm{P}}(\mathcal{R}_T|\mathcal{P}_0)\mathbb{E}_{\mathrm{Q}}\left(e^{-\sum_{s=1}^{T} i_s}\middle|\mathcal{P}_0\right)$$

$$\begin{aligned} = \mathbb{E}_{\mathrm{P}}(&r_1 R 1_{\{0<\mathcal{M}_T\le 5,\mathcal{D}_T>300\}} + r_2 R 1_{\{0<\mathcal{M}_T\le 5,70<\mathcal{D}_T\le 300\}} + r_3 R 1_{\{0<\mathcal{M}_T\le 5,\mathcal{D}_T\le 70\}} + \cdots \\ &+ r_{13} R 1_{\{\mathcal{M}_T>8,\mathcal{D}_T>300\}} + r_{14} R 1_{\{\mathcal{M}_T>8,70<\mathcal{D}_T\le 300\}} + r_{15} R 1_{\{\mathcal{M}_T>8,\mathcal{D}_T\le 70\}}|\mathcal{P}_0) B_T \end{aligned}$$

$$\begin{aligned} = [&r_1 R\mathrm{P}(0<\mathcal{M}_T\le 5,\mathcal{D}_T>300) + r_2 R\mathrm{P}(0<\mathcal{M}_T\le 5,70<\mathcal{D}_T\le 300) \\ &+ r_3 R\mathrm{P}(0<\mathcal{M}_T\le 5,\mathcal{D}_T\le 70) + \cdots + r_{13} R\mathrm{P}(\mathcal{M}_T>8,\mathcal{D}_T>300) \\ &+ r_{14} R\mathrm{P}(\mathcal{M}_T>8,70<\mathcal{D}_T\le 300) + r_{15} R\mathrm{P}(\mathcal{M}_T>8,\mathcal{D}_T\le 70)]\ B_T \end{aligned}$$

$$= [r_1\mathrm{P}(0 < \mathcal{M}_T \leq 5, \mathcal{D}_T > 300) + r_2\mathrm{P}(0 < \mathcal{M}_T \leq 5, 70 < \mathcal{D}_T \leq 300) + r_3\mathrm{P}(0 < \mathcal{M}_T \leq 5, \mathcal{D}_T \leq 70) + \cdots + r_{13}\mathrm{P}(\mathcal{M}_T > 8, \mathcal{D}_T > 300) + r_{14}\mathrm{P}(\mathcal{M}_T > 8, 70 < \mathcal{D}_T \leq 300) + r_{15}\mathrm{P}(\mathcal{M}_T > 8, \mathcal{D}_T \leq 70)]RB_T$$

$$= \sum_{p=1}^{5}\sum_{q=1}^{3} r_{3(p-1)+q}\mathrm{P}\big(m_p < \mathcal{M}_T \leq m_{p+1}, d_{q+1} < \mathcal{D}_T \leq d_q\big)RB_T.$$

The price of earthquake bonds is modeled as the sum of $\mathfrak{C}_k$ with $k = 1, 2, 3, \dots, T$ and $\mathfrak{R}_T$, which is mathematically written as follows:

$$V_T = \sum_{k=1}^{T} \mathfrak{C}_k + \mathfrak{R}_T$$

$$= \sum_{k=1}^{T}\sum_{p=1}^{5}\sum_{q=1}^{3} c_{3(p-1)+q}\mathrm{P}\big(m_p < \mathcal{M}_k \leq m_{p+1}, d_{q+1} < \mathcal{D}_k \leq d_q\big)CB_k$$

$$+ \sum_{p=1}^{5}\sum_{q=1}^{3} r_{3(p-1)+q}\mathrm{P}\big(m_p < \mathcal{M}_T \leq m_{p+1}, d_{q+1} < \mathcal{D}_T \leq d_q\big)RB_T.$$