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Research article

Determination of medical emergency via new intuitionistic fuzzy correlation measures based on Spearman's correlation coefficient

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Abstract: Uncertainty in medical diagnosis is the main challenge in medical emergencies (MEs) experienced by triage nurses and physicians in the emergency department (ED). The intuitionistic fuzzy correlation coefficient (IFCC) approach is used to analyze and interpret the relationship between variables in an uncertain environment. Assorted methods that involve applying a correlation coefficient under intuitionistic fuzzy sets (IFSs) were constructed based on Pearson's correlation model with various drawbacks. In this work, we construct two new intuitionistic fuzzy correlation measures (IFCMs) based on Spearman's correlation model. It is demonstrated that the Spearman-based IFCMs are appropriate for measuring correlation coefficients without any drawbacks. In addition, we show that the Spearman-based IFCMs overcome all the shortcomings of the associated IFCC methods. Equally, the Spearman-based IFCMs satisfy the maxims of the correlation coefficient that have been delineated in the classical case of correlation coefficient. Due to the challenges that uncertainty in medical diagnosis pose to MEs and the proficiency of the IFCC approach, we discuss the application of the constructed IFCMs in a triage process for an effective medical diagnosis during an ME. The medical data for the triage process are obtained via a knowledge-based approach. Finally, comparative analyses are carried out to ascertain the validity and authenticity of the developed Spearman-based IFCMs relative to other IFCC approaches.

Keywords: triage; medical emergency; medical diagnosis; intuitionistic fuzzy sets; emergency department; correlation coefficient; decision-making
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1. Introduction

A medical emergency (ME) is defined as an illness or acute injury that presents a life-threatening or long-term health risk; it is also sometimes referred to as a "life or limb" situation. Many of these emergencies, such as gastrointestinal, cardiovascular (heart), and respiratory emergencies, cannot be handled by the patient alone; therefore, they may require help from a health expert [1]. When someone experiences an ME, getting them medical attention quickly can save their lives. Finding the location and quickest route to the closest emergency department (ED) is one of the main issues for MEs. Unstoppable bleeding, breathing issues (difficulty breathing, shortness of breath), head or spine injury, severe or persistent vomiting, abrupt injury from an accident, burns or smoke inhalation, near drowning, deep or large wounds/injuries, sudden intense pain anywhere in the body, sudden dizziness, weakness, swallowing of a poisonous substance, severe abdominal pain, unusual headache, seizure, bluish or grey skin coloration (cyanosis), shift in mental status, chest pain, choking, persistent coughing, vomiting of blood, fainting or losing consciousness, etc. are some of the warning signs of an ME [2].

The ED is peopled by physicians, nurses, and other medical professionals. A triage nurse or doctor is the first medical professional to treat a patient in an emergency; they assess the patient's condition and decide whether to call a consultant. The most seriously injured patients are treated right away. For this reason, some patients who arrive at the ED late might receive medical attention first. In the ED, the procedures to be followed are typically triage, registration, treatment, reevaluation, and discharge of the patient. When a patient is brought to the ED, emergency technicians use the patient's medical history and a quick physical examination to ascertain the reason for the visit and severity of the patient's illness. The stages in the ED are as follows: Level 1 (resuscitation immediate life-saving intervention); Level 2 (emergency medical attention); Level 3 (urgent medical attention); Level 4 (semi-urgent medical attention); and Level 5 (non-urgent medical attention). The triage registered nurse assigns a patient to a priority level based on their medical history and current condition. Millions of people are affected by MEs every year. Medical errors, insufficient access to timely care, and subpar healthcare systems can result in fatalities. To shorten the time that patients must wait for medical attention, an emergency registered nurse may occasionally begin diagnostic testing. Most of the time, errors in determining the severity of ME cases have led to the deaths of some patients who were waiting to consult a professional.

Accurate and prompt diagnosis is crucial for optimizing the possible advantages of therapy. Uncertainty must be controlled throughout the diagnostic process to enable a precise and prompt diagnosis and treatments. However, an inability to control diagnostic uncertainty may result in misdiagnosis or delayed conditions and/or postponed or unneeded tests and/or treatments [3, 4]. According to Sklar et al. [5], years of experience in practicing medicine reduces uncertainty in medical decision-making. In actuality, practicing medicine involves a certain amount of uncertainty. As stated by Platts-Mills et al. [6], uncertainty in an ME originates from a variety of sources. Some patients brought to the ED are unknowns; uncertainty about their history, especially the type and timing of symptoms prior to the ED visit, uncertainty about the current understanding of their signs and causes of their disease, uncertainty about the limitations of diagnostic tests, uncertainty about the advantages and disadvantages of their treatments, etc. are a few examples of the uncertainties that may exist. The introduction of fuzzy set theory [7], on the other hand, has significantly reduced uncertainty in decision-making.

The fuzzy set is described by the membership degree (MD) of elements defined in a closed unit interval $[0, 1]$. But, there are some decision-making problems that require a non-MD (NMD) compartment with the flexibility to accommodate hesitancy, which a fuzzy set is unable to handle. To address such challenges, Atanassov [8] introduced intuitionistic fuzzy sets (IFSs), which combine the MD and NMD in such a way that, 1-MD is not necessarily equal to the NMD and the intuitionistic fuzzy hesitation margin (IFHM) is one minus the sum of MD and NMD. Thses attributes position the theory of IFSs as a formidable soft computing tool for resolving problems of uncertainty and imprecision in everyday encounters. In [9–11], applications of IFSs in decision-making have been discussed from the perspective of aggregation operators, and Szmidt et al. [12] discussed the usefulness of IFSs in attributes selection. A ranking technique that applies information fusion under IFSs has been used to address threat assessment [13], and Zeng et al. [14] discussed pattern recognition by employing a distance method for IFSs. Some applications of the theory of IFSs have been discussed via composite relations [15], distance measures [16–19], and similarity measures [20–25] due to the practicality of IFSs. In addition, Alcantud [26] investigated multi-attribute group decision-making (MAGDM) under IFSs by using weighted geometric mean aggregation operators, and a figure skating application based on intuitionistic fuzzy divergences can be found in [27].

Many real-life applications of IFSs have been discussed from the perspective of IFCC. Correlation analysis is a statistical method that is used to ascertain the grade of connection between two sets of numerically continuous data. This type of analysis is used when an investigator needs to investigate the relationship between two variables. The correlation coefficient is a statistical tool that is applied to calculate the degree of association between two variables. To improve the applicability of IFSs in real-world scenarios, intuitionistic fuzzy information has been integrated into a correlation analysis construct [28]. Intuitionistic fuzzy correlation analysis has been conducted for probability spaces [29]. Huang and Guo [30] presented a strong method, but they did so by taking into account only two parameters of the IFSs, and Hung [31] examined IFCM from a statistical standpoint. Liu et al. [32] developed a statistical technique for computing IFCM via variance and covariance analyses and implemented for decision-making. The method in [31] was independently improved in [33, 34] by the inclusion of IFHMs to prevent omission error. Similarly, a statistical approach for computing IFCM has been developed and implemented for decision-making [35]. A novel IFCC method was presented by Xu [36] to discuss disease diagnosis. To improve accuracy, the methodology in [36] was altered by adding all the convention parameters of IFSs [37].

The study in [38] examined comparable methods for computing the IFCC. Zeng and Li [39] created a similar IFCC approach that incorporated IFHMs into the approach in [28] for the purposes of inclusion and relaibility. In [40], the approach developed by Huang and Guo [30] was enhanced by the inclusion of the complete parameters of IFSs and applied in pattern recognition; additionally, and Bajaj and Kumar [41] modified the approach in [40] and realized better performance. Some IFCMs have been constructed and applied in medical diagnosis [42], decision-making [43–46], and pattern recognition [47]. In addition, the study of correlation coefficients in the fuzzy domain has been extended to higher variants of fuzzy sets with relevant applications [48–52].

The existing IFCM approaches have some limitations with regards to the conditions of the correlation coefficient and reliable interpretation. Almost all of the existing approaches were developed based on Pearson's correlation coefficient model, and none of them were constructed by using Spearman's correlation coefficient model. The methods in [28, 38, 39] fails to provide reliable information if the MDs and NMDs are either one or zero. The methods in [28, 31, 33, 35] indicate that a perfect positive correlation exists, although the IFSs are not identical and thus violate a maxim of the correlation coefficient. In addition, the IFCMs in [30, 32, 36, 37, 40, 41] provide values that are not defined within either $[0, 1]$ or $[-1, 1]$, which violate another maxim of the correlation coefficient. When the IFSs are equal, the IFCMs of [36, 37] produce $0/0$, which is mathematically undefined. For equal IFSs, the correlation coefficient should be one. Furthermore, the methods of [36, 37] yield a perfect correlation coefficient in the absence of equality between the IFSs. To recap, all of the IFCMs fail the metric conditions of the correlation coefficient in some ways.

Because of the limitations of the existing IFCMs, we develop two reliable approaches for measuring IFCCs in this work. This study is aimed at creating two new Spearman correlation coefficient-based IFCM approaches that have solid mathematical correctness, a reasonable level of interpretation, and dependable precision. To prevent omission errors, the measures integrate all parameters of the IFSs. The objectives of the work are delineated as follows: (i) to restate and evaluate the extant methods of IFCM; (ii) to create new IFCMs based on Spearman's correlation coefficient, where all of the parameters of IFSs are incorporated to produce dependable output; (iii) to apply the created IFCMs in the determination of an ME by using a knowledge-based approach; (iv) to conduct comparative analysis between the available IFCMs and the new approaches to showcase the advantage of the former.

The paper is organized as follows: In Section 2, the basics of IFSs and some of the current IFCMs are covered. In Section 3, the new methods are introduced, along with their numerical proofs and the description of some of their properties. In Section 4, an ME designed based on a knowledge-based approach is determined to ascertain the most critically ill patient to enhance effective treatment. Finally, in Section 5, the findings are summarized and suggestions for additional research are provided.

2. Preliminaries

In this section, we reiterate the concept of IFSs and present IFCC together with some existing IFCMs.

2.1. Basics on IFSs

Throughout this work, we take X as a non-empty set, which is the underlying set of IFSs.

Definition 2.1. *[7] A set with the structure of the form, $\wp = \{\langle x_j, \delta_\wp(x_j)\rangle | x_j \in X\}$, where $\delta_\wp(x_j) \in [0, 1]$ is the MD of $x_j \in X$ to $\wp$, is called a fuzzy set.*

Definition 2.2. *[8] A set with the form $\ell = \{\langle x_j, \delta_\ell(x_j), \kappa_\ell(x_j)\rangle | x_j \in X\}$, where $\delta_\ell(x_j), \kappa_\ell(x_j) \in [0, 1]$ denote the MD and NMD of $x_j \in X$ to ℓ with the property, $0 \leq \delta_\ell(x_j) + \kappa_\ell(x_j) \leq 1$ is called an IFS. In addition, the IFHM of ℓ in X is described by $\varrho_\ell(x_j) = 1 - \delta_\ell(x_j) - \kappa_\ell(x_j)$. The IFHM indicates whether $x_j \in X$ or $x_j \notin X$.*

Definition 2.3. *[8] Suppose that ℓ and $\tilde{\ell}$ are IFSs in X. Then, the following are some basic operations on the IFSs:*

(i) $\ell^c = \{\langle x_j, \kappa_\ell(x_j), \delta_\ell(x_j)\rangle | x_j \in X\}$, $\tilde{\ell}^c = \{\langle x_j, \kappa_{\tilde{\ell}}(x_j), \delta_{\tilde{\ell}}(x_j)\rangle | x_j \in X\}$.
(ii) $\ell \cup \tilde{\ell} = \{\langle x_j, \max\{\delta_\ell(x_j), \delta_{\tilde{\ell}}(x_j)\}, \min\{\kappa_\ell(x_j), \kappa_{\tilde{\ell}}(x_j)\}\rangle | x_j \in X\}$.
(iii) $\ell \cap \tilde{\ell} = \{\langle x_j, \min\{\delta_\ell(x_j), \kappa_{\tilde{\ell}}(x_j)\}, \max\{\delta_\ell(x_j), \kappa_{\tilde{\ell}}(x_j)\}\rangle | x_j \in X\}$.
(iv) $\ell = \tilde{\ell}$ iff $\delta_\ell(x_j) = \delta_{\tilde{\ell}}(x_j)$ and $\kappa_\ell(x_j) = \kappa_{\tilde{\ell}}(x_j)$ $\forall$ $x_j \in X$.
(v) $\ell \subseteq \tilde{\ell}$ iff $\delta_\ell(x_j) \leq \delta_{\tilde{\ell}}(x_j)$ and $\kappa_\ell(x_j) \geq \kappa_{\tilde{\ell}}(x_j)$ $\forall$ $x_j \in X$.

Definition 2.4. *Suppose that ℓ and $\tilde{\ell}$ are IFSs in X. Then, the arithmetic average of the IFSs ℓ and $\tilde{\ell}$ denoted by $\hat{\ell}$ is defined as follows:*

$$\hat{\ell} = \{\langle x_j, \delta_{\hat{\ell}}(x_j), \kappa_{\hat{\ell}}(x_j)\rangle | x_j \in X\},$$

where

$$\delta_{\hat{\ell}}(x_j) = \text{Average}\Big(\delta_\ell(x_j), \delta_{\tilde{\ell}}(x_j)\Big) \text{ and } \kappa_{\hat{\ell}}(x_j) = \text{Average}\Big(\kappa_\ell(x_j), \kappa_{\tilde{\ell}}(x_j)\Big).$$

Definition 2.5. *[28] If ℓ and $\tilde{\ell}$ are IFSs in $X = \{x_1, x_2, \cdots, x_q\}$ and q is the cardinality of X, then the IFCC between ℓ and $\tilde{\ell}$ represented by $\rho(\ell, \tilde{\ell})$ is a function ρ: $IFS(X) \times IFS(X) \rightarrow [0, 1]$ or $[-1, 1]$ with the following properties:*

A1. $0 \leq \rho(\ell, \tilde{\ell}) \leq 1$ or $-1 \leq \rho(\ell, \tilde{\ell}) \leq 1$,
A2. $\rho(\ell, \tilde{\ell}) = 1$ iff $\ell = \tilde{\ell}$,
A3. $\rho(\ell, \tilde{\ell}) = \rho(\tilde{\ell}, \ell)$.

To enable better understanding of the IFCC, we present the following information: $\rho(\ell, \tilde{\ell})$ tending to 1 is an indication that ℓ and $\tilde{\ell}$ have strong correlation; $\rho(\ell, \tilde{\ell})$ tending to -1 or 0 is an indication that ℓ and $\tilde{\ell}$ have weak correlation; $\rho(\ell, \tilde{\ell}) = 1$ indicates a perfect positive correlation; and $\rho(\ell, \tilde{\ell}) = 0$ or -1 indicates no correlation or a perfect negative correlation.

2.2. Various existing IFCM methods

Given two IFSs ℓ and $\tilde{\ell}$ in $X = \{x_1, x_2, \cdots, x_q\}$, an existing IFCM can be described as follows [28]:

$$\rho_1(\ell, \tilde{\ell}) = \frac{\mathcal{K}(\ell, \tilde{\ell})}{\sqrt{\mathcal{I}(\ell)\mathcal{I}(\tilde{\ell})}}, \tag{2.1}$$

where

$$\left.\begin{aligned} \mathcal{K}(\ell, \tilde{\ell}) &= \sum_{j=1}^{q}\Big(\delta_\ell(x_j)\delta_{\tilde{\ell}}(x_j) + \kappa_\ell(x_j)\kappa_{\tilde{\ell}}(x_j)\Big) \\ \mathcal{I}(\ell) &= \sum_{j=1}^{q}\Big(\delta_\ell^2(x_j) + \kappa_\ell^2(x_j)\Big) \\ \mathcal{I}(\tilde{\ell}) &= \sum_{j=1}^{q}\Big(\delta_{\tilde{\ell}}^2(x_j) + \kappa_{\tilde{\ell}}^2(x_j)\Big) \end{aligned}\right\}. \tag{2.2}$$

Example 2.1. *Suppose that* $\ell = \{\langle x, \frac{1}{3}, \frac{1}{3}\rangle\}$ *and* $\tilde{\ell} = \{\langle x, \frac{1}{4}, \frac{1}{4}\rangle\}$ *are IFSs in* $X = \{x\}$. *Then,* $\rho_1(\ell, \tilde{\ell}) = 1$, *which is a violation of A2 of Definition 2.5 since* $\ell \neq \tilde{\ell}$.

According to Hung [31],

$$\rho_2(\ell, \tilde{\ell}) = \frac{\rho_m(\ell, \tilde{\ell}) + \rho_n(\ell, \tilde{\ell})}{2}, \tag{2.3}$$

where

$$\left.\begin{aligned} \rho_m(\ell, \tilde{\ell}) &= \frac{\Sigma_{j=1}^{q}(\delta_\ell(x_j) - \overline{\delta}_\ell)(\delta_{\tilde{\ell}}(x_j) - \overline{\delta}_{\tilde{\ell}})}{\sqrt{\Sigma_{j=1}^{q}(\delta_\ell(x_j) - \overline{\delta}_\ell)^2}\sqrt{\Sigma_{j=1}^{q}(\delta_{\tilde{\ell}}(x_j) - \overline{\delta}_{\tilde{\ell}})^2}} \\ \rho_n(\ell, \tilde{\ell}) &= \frac{\Sigma_{j=1}^{q}(\kappa_\ell(x_j) - \overline{\kappa}_\ell)(\kappa_{\tilde{\ell}}(x_j) - \overline{\kappa}_{\tilde{\ell}})}{\sqrt{\Sigma_{j=1}^{q}(\kappa_\ell(x_j) - \overline{\kappa}_\ell)^2}\sqrt{\Sigma_{j=1}^{q}(\kappa_{\tilde{\ell}}(x_j) - \overline{\kappa}_{\tilde{\ell}})^2}} \end{aligned}\right\}, \tag{2.4}$$

where

$$\left.\begin{aligned} \overline{\delta}_\ell &= \frac{\Sigma_{j=1}^{q}\delta_\ell(x_j)}{q}, \overline{\delta}_{\tilde{\ell}} = \frac{\Sigma_{j=1}^{q}\delta_{\tilde{\ell}}(x_j)}{q} \\ \overline{\kappa}_\ell &= \frac{\Sigma_{j=1}^{q}\kappa_\ell(x_j)}{q}, \overline{\kappa}_{\tilde{\ell}} = \frac{\Sigma_{j=1}^{q}\kappa_{\tilde{\ell}}(x_j)}{q} \end{aligned}\right\}. \tag{2.5}$$

for $j = 1, 2, \cdots, q$.

Example 2.2. *Let* $\ell = \{\langle x_1, \frac{1}{4}, \frac{1}{4}\rangle, \langle x_2, \frac{1}{8}, \frac{1}{8}\rangle\}$ *and* $\tilde{\ell} = \{\langle x_1, \frac{1}{2}, \frac{1}{2}\rangle, \langle x_2, \frac{1}{4}, \frac{1}{4}\rangle\}$ *be IFSs in* $X = \{x_1, x_2\}$; *then,* $\rho_2(\ell, \tilde{\ell}) = 1$, *which violates A2 of Definition 2.5 because* $\ell \neq \tilde{\ell}$.

According to Zeng and Li [39],

$$\rho_3(\ell, \tilde{\ell}) = \frac{\mathcal{K}(\ell, \tilde{\ell})}{\sqrt{\mathcal{I}(\ell)\mathcal{I}(\tilde{\ell})}}, \tag{2.6}$$

where

$$\left.\begin{aligned} \mathcal{K}(\ell, \tilde{\ell}) &= \frac{\sum_{j=1}^{q}\left(\delta_\ell(x_j)\delta_{\tilde{\ell}}(x_j) + \kappa_\ell(x_j)\kappa_{\tilde{\ell}}(x_j) + \varrho_\ell(x_j)\varrho_{\tilde{\ell}}(x_j)\right)}{q} \\ \mathcal{I}(\ell) &= \frac{\sum_{j=1}^{q}\left(\delta_\ell^2(x_j) + \kappa_\ell^2(x_j) + \varrho_\ell^2(x_j)\right)}{q} \\ \mathcal{I}(\tilde{\ell}) &= \frac{\sum_{j=1}^{q}\left(\delta_{\tilde{\ell}}^2(x_j) + \kappa_{\tilde{\ell}}^2(x_j) + \varrho_{\tilde{\ell}}^2(x_j)\right)}{q} \end{aligned}\right\}. \tag{2.7}$$

Example 2.3. *Let* $\ell = \{\langle x_1, 1, 0\rangle, \langle x_2, 0, 1\rangle\}$ *and* $\tilde{\ell} = \{\langle x_1, 0, 1\rangle, \langle x_2, 1, 0\rangle\}$ *be IFSs in a set* $X = \{x_1, x_2\}$; *then,* $\rho_3(\ell, \tilde{\ell}) = 0$, *which gives a misleading information since an imprecise correlation exists between the IFSs.*

According to Xu et al. [38],

$$\rho_4(\ell, \tilde{\ell}) = \frac{\mathcal{K}(\ell, \tilde{\ell})}{\max\{\sqrt{\mathcal{I}(\ell)}, \sqrt{\mathcal{I}(\tilde{\ell})}\}}, \tag{2.8}$$

where

$$\left.\begin{aligned}\mathcal{K}(\ell,\tilde{\ell}) &= \sum_{j=1}^{q}\Big(\delta_{\ell}(x_j)\delta_{\tilde{\ell}}(x_j)+\kappa_{\ell}(x_j)\kappa_{\tilde{\ell}}(x_j)+\varrho_{\ell}(x_j)\varrho_{\tilde{\ell}}(x_j)\Big)\\ \mathcal{I}(\ell) &= \sum_{j=1}^{q}\Big(\delta_{\ell}^2(x_j)+\kappa_{\ell}^2(x_j)+\varrho_{\ell}^2(x_j)\Big)\\ \mathcal{I}(\tilde{\ell}) &= \sum_{j=1}^{q}\Big(\delta_{\tilde{\ell}}^2(x_j)+\kappa_{\tilde{\ell}}^2(x_j)+\varrho_{\tilde{\ell}}^2(x_j)\Big)\end{aligned}\right\}. \tag{2.9}$$

Using Example 2.3, we get $\rho_4(\ell,\tilde{\ell}) = 0$, which is not true because an imprecise correlation exists between the IFSs.

According to Park et al. [33],

$$\rho_5(\ell,\tilde{\ell}) = \frac{\rho_m(\ell,\tilde{\ell})+\rho_n(\ell,\tilde{\ell})+\rho_h(\ell,\tilde{\ell})}{3}, \tag{2.10}$$

where

$$\left.\begin{aligned}\rho_m(\ell,\tilde{\ell}) &= \frac{\Sigma_{j=1}^{q}(\delta_{\ell}(x_j)-\overline{\delta}_{\ell})(\delta_{\tilde{\ell}}(x_j)-\overline{\delta}_{\tilde{\ell}})}{\sqrt{\Sigma_{j=1}^{q}(\delta_{\ell}(x_j)-\overline{\delta}_{\ell})^2}\sqrt{\Sigma_{j=1}^{q}(\delta_{\tilde{\ell}}(x_j)-\overline{\delta}_{\tilde{\ell}})^2}}\\ \rho_n(\ell,\tilde{\ell}) &= \frac{\Sigma_{j=1}^{q}(\kappa_{\ell}(x_j)-\overline{\kappa}_{\ell})(\kappa_{\tilde{\ell}}(x_j)-\overline{\kappa}_{\tilde{\ell}})}{\sqrt{\Sigma_{j=1}^{q}(\kappa_{\ell}(x_j)-\overline{\kappa}_{\ell})^2}\sqrt{\Sigma_{j=1}^{q}(\kappa_{\tilde{\ell}}(x_j)-\overline{\kappa}_{\tilde{\ell}})^2}}\\ \rho_h(\ell,\tilde{\ell}) &= \frac{\Sigma_{j=1}^{q}(\varrho_{\ell}(x_j)-\overline{\varrho}_{\ell})(\varrho_{\tilde{\ell}}(x_j)-\overline{\varrho}_{\tilde{\ell}})}{\sqrt{\Sigma_{j=1}^{q}(\varrho_{\ell}(x_j)-\overline{\varrho}_{\ell})^2}\sqrt{\Sigma_{j=1}^{q}(\varrho_{\tilde{\ell}}(x_j)-\overline{\varrho}_{\tilde{\ell}})^2}}\end{aligned}\right\}, \tag{2.11}$$

where

$$\left.\begin{aligned}\overline{\delta}_{\ell} &= \frac{\Sigma_{j=1}^{q}\delta_{\ell}(x_j)}{q}, \overline{\delta}_{\tilde{\ell}} = \frac{\Sigma_{j=1}^{q}\delta_{\tilde{\ell}}(x_j)}{q}\\ \overline{\kappa}_{\ell} &= \frac{\Sigma_{j=1}^{q}\kappa_{\ell}(x_j)}{q}, \overline{\kappa}_{\tilde{\ell}} = \frac{\Sigma_{j=1}^{q}\kappa_{\tilde{\ell}}(x_j)}{q}\\ \overline{\varrho}_{\ell} &= \frac{\Sigma_{j=1}^{q}\varrho_{\ell}(x_j)}{q}, \overline{\varrho}_{\tilde{\ell}} = \frac{\Sigma_{j=1}^{q}\varrho_{\tilde{\ell}}(x_j)}{q}\end{aligned}\right\} \tag{2.12}$$

for $j = 1, 2, \cdots, q$. Using Example 2.2, we get $\rho_5(\ell,\tilde{\ell}) = 1$ which violates A2 of Definition 2.5 because $\ell \neq \tilde{\ell}$.

According to Xu [36],

$$\rho_6(\ell,\tilde{\ell}) = \frac{1}{2q}\sum_{j=1}^{q}\Big(\frac{\Delta\delta_{\min}+\Delta\delta_{\max}}{\Delta\delta_j+\Delta\delta_{\max}}+\frac{\Delta\kappa_{\min}+\Delta\kappa_{\max}}{\Delta\kappa_j+\Delta\kappa_{\max}}\Big), \tag{2.13}$$

where

$$\left.\begin{aligned} \Delta\delta_j &= |\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j)| \\ \Delta\kappa_j &= |\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j)| \\ \Delta\delta_{\min} = \min_{1\le j\le q}\{\Delta\delta_j\}, \Delta\kappa_{\min} &= \min_{1\le j\le q}\{\Delta\kappa_j\} \\ \Delta\delta_{\max} = \max_{1\le j\le q}\{\Delta\delta_j\}, \Delta\kappa_{\max} &= \max_{1\le j\le q}\{\Delta\kappa_j\} \end{aligned}\right\}. \tag{2.14}$$

Using Example 2.2, we get $\rho_6(\ell, \tilde{\ell}) = 1.1667$ which violates A1 of Definition 2.5 because neither $1.1667 \notin [0, 1]$ nor $1.1667 \notin [-1, 1]$.

According to Xu and Cai [37],

$$\rho_7(\ell, \tilde{\ell}) = \frac{1}{3q}\sum_{j=1}^{q}\Big(\frac{\Delta\delta_{\min} + \Delta\delta_{\max}}{\Delta\delta_j + \Delta\delta_{\max}} + \frac{\Delta\kappa_{\min} + \Delta\kappa_{\max}}{\Delta\kappa_j + \Delta\kappa_{\max}} + \frac{\Delta\varrho_{\min} + \Delta\varrho_{\max}}{\Delta\varrho_j + \Delta\varrho_{\max}}\Big), \tag{2.15}$$

where

$$\left.\begin{aligned} \Delta\delta_j &= |\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j)| \\ \Delta\kappa_j &= |\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j)| \\ \Delta\varrho_j &= |\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j)| \\ \Delta\delta_{\min} = \min_{1\le j\le q}\{\Delta\delta_j\}, \Delta\kappa_{\min} &= \min_{1\le j\le q}\{\Delta\kappa_j\} \\ \Delta\varrho_{\min} = \min_{1\le j\le q}\{\Delta\varrho_j\}, \Delta\delta_{\max} &= \max_{1\le j\le q}\{\Delta\delta_j\} \\ \Delta\kappa_{\max} = \max_{1\le j\le q}\{\Delta\kappa_j\}, \Delta\varrho_{\max} &= \max_{1\le j\le q}\{\Delta\varrho_j\} \end{aligned}\right\}. \tag{2.16}$$

Example 2.4. *Let* $\ell = \{\langle x_1, \frac{2}{5}, \frac{3}{10}\rangle, \langle x_2, \frac{3}{10}, \frac{1}{5}\rangle\}$ *and* $\tilde{\ell} = \{\langle x_1, \frac{3}{10}, \frac{1}{5}\rangle, \langle x_2, \frac{1}{5}, \frac{1}{10}\rangle\}$ *be IFSs in* $X = \{x_1, x_2\}$*; then,* $\rho_7(\ell, \tilde{\ell}) = 1$ *although* $\ell \neq \tilde{\ell}$*, which violates A2 of Definition 2.5. If* $\ell = \tilde{\ell}$*, we have* $\rho_7(\ell, \tilde{\ell}) = \frac{0}{0}$*, which violates A1 of Definition 2.5.*

According to Liu et al. [32],

$$\rho_8(\ell, \tilde{\ell}) = \frac{\phi(\ell, \tilde{\ell})}{\sqrt{\psi(\ell)\psi(\tilde{\ell})}}, \tag{2.17}$$

where

$$\left.\begin{aligned} \psi(\ell) = \frac{\Sigma_{j=1}^{q}\mathrm{D}^2(\ell)}{q-1}, \psi(\tilde{\ell}) &= \frac{\Sigma_{j=1}^{q}\mathrm{D}^2(\tilde{\ell})}{q-1} \\ \phi(\ell, \tilde{\ell}) &= \frac{\Sigma_{j=1}^{q}\mathrm{D}(\ell)\mathrm{D}(\tilde{\ell})}{q-1} \end{aligned}\right\} \tag{2.18}$$

for

$$\left.\begin{aligned} \mathrm{D}(\ell) &= \Big(\delta_\ell(x_j) - \overline{\delta}_\ell\Big) - \Big(\kappa_\ell(x_j) - \overline{\kappa}_\ell\Big) \\ \mathrm{D}(\tilde{\ell}) &= \Big(\delta_{\tilde{\ell}}(x_j) - \overline{\delta}_{\tilde{\ell}}\Big) - \Big(\kappa_{\tilde{\ell}}(x_j) - \overline{\kappa}_{\tilde{\ell}}\Big) \end{aligned}\right\}, \tag{2.19}$$

where $\overline{\delta}_\ell$, $\overline{\kappa}_\ell$, $\overline{\delta}_{\tilde{\ell}}$, and $\overline{\kappa}_{\tilde{\ell}}$ are as in (2.5).

Example 2.5. Let $\ell = \{\langle x_1, \frac{1}{3}, \frac{1}{3}\rangle, \langle x_2, \frac{1}{2}, \frac{1}{2}\rangle\}$ and $\tilde{\ell} = \{\langle x_1, \frac{1}{4}, \frac{1}{4}\rangle, \langle x_2, \frac{1}{2}, \frac{1}{3}\rangle\}$ be IFSs in $X = \{x_1, x_2\}$, then we get $\rho_8(\ell, \tilde{\ell}) = \frac{0}{\sqrt{0\times 0}} = \infty$, which violates A1 of Definition 2.5.

According to Thao et al. [35],

$$\rho_9(\ell, \tilde{\ell}) = \frac{\phi(\ell, \tilde{\ell})}{\sqrt{\psi(\ell)\psi(\tilde{\ell})}}, \tag{2.20}$$

where

$$\left.\begin{aligned} \psi(\ell) &= \frac{1}{q-1}\Sigma_{j=1}^{q}\Big((\delta_\ell(x_j) - \overline{\delta}_\ell)^2 + (\kappa_\ell(x_j) - \overline{\kappa}_\ell)^2\Big) \\ \psi(\tilde{\ell}) &= \frac{1}{q-1}\Sigma_{j=1}^{q}\Big((\delta_{\tilde{\ell}}(x_j) - \overline{\delta}_{\tilde{\ell}})^2 + (\kappa_{\tilde{\ell}}(x_j) - \overline{\kappa}_{\tilde{\ell}})^2\Big) \\ \phi(\ell, \tilde{\ell}) &= \frac{1}{q-1}\Sigma_{j=1}^{q}\Big((\delta_\ell(x_j) - \overline{\delta}_\ell)(\delta_{\tilde{\ell}}(x_j) - \overline{\delta}_{\tilde{\ell}}) \\ &\qquad + (\kappa_\ell(x_j) - \overline{\kappa}_\ell)(\kappa_{\tilde{\ell}}(x_j) - \overline{\kappa}_{\tilde{\ell}})\Big) \end{aligned}\right\}, \tag{2.21}$$

where $\overline{\delta}_\ell$, $\overline{\kappa}_\ell$, $\overline{\delta}_{\tilde{\ell}}$, and $\overline{\kappa}_{\tilde{\ell}}$ are as in (2.5).

Example 2.6. *Let $\ell = \{\langle x_1, \frac{1}{2}, \frac{1}{2}\rangle, \langle x_2, \frac{1}{2}, \frac{1}{2}\rangle\}$ and $\tilde{\ell} = \{\langle x_1, \frac{1}{3}, \frac{1}{3}\rangle, \langle x_2, \frac{1}{3}, \frac{1}{3}\rangle\}$ be IFSs in $X = \{x_1, x_2\}$; then, $\rho_9(\ell, \tilde{\ell}) = \frac{0}{\sqrt{0\times 0}} = \infty$, which violates A1 of Definition 2.5.*

According to Huang and Guo [30],

$$\rho_{10}(\ell, \tilde{\ell}) = \frac{1}{2q}\sum_{j=1}^{q}\Big(\alpha_j(1 - \Delta\delta_j) + \beta_j(1 - \Delta\kappa_j)\Big), \tag{2.22}$$

where

$$\left.\begin{aligned} \alpha_j &= \frac{c - \Delta\delta_j - \Delta\delta_{\max}}{c - \Delta\delta_{\min} - \Delta\delta_{\max}} \\ \beta_j &= \frac{c - \Delta\kappa_j - \Delta\kappa_{\max}}{c - \Delta\kappa_{\min} - \Delta\kappa_{\max}} \end{aligned}\right\} \tag{2.23}$$

for $c > 2$, and

$$\left.\begin{aligned} \Delta\delta_j &= |\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j)| \\ \Delta\kappa_j &= |\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j)| \\ \Delta\delta_{\min} &= \min_{1\le j\le q}\{\Delta\delta_j\}, \Delta\kappa_{\min} = \min_{1\le j\le q}\{\Delta\kappa_j\} \\ \Delta\delta_{\max} &= \max_{1\le j\le q}\{\Delta\delta_j\}, \Delta\kappa_{\max} = \max_{1\le j\le q}\{\Delta\kappa_j\} \end{aligned}\right\}. \tag{2.24}$$

Example 2.7. *Suppose that we have IFSs*

$$\ell = \Big\{\langle x_1, \frac{2}{5}, \frac{31}{60}\rangle, \langle x_2, \frac{11}{15}, \frac{11}{60}\rangle, \langle x_3, \frac{5}{6}, \frac{7}{60}\rangle, \langle x_4, \frac{3}{5}, \frac{19}{60}\rangle, \langle x_5, \frac{13}{30}, \frac{1}{2}\rangle, \langle x_6, \frac{7}{10}, \frac{1}{5}\rangle, \langle x_7, \frac{1}{15}, \frac{1}{4}\rangle\Big\},$$

$$\tilde{\ell} = \Big\{\langle x_1, \frac{13}{30}, \frac{7}{15}\rangle, \langle x_2, \frac{23}{30}, \frac{3}{20}\rangle, \langle x_3, \frac{5}{6}, \frac{7}{60}\rangle, \langle x_4, \frac{2}{3}, \frac{13}{60}\rangle, \langle x_5, \frac{23}{30}, \frac{1}{6}\rangle, \langle x_6, \frac{7}{10}, \frac{1}{5}\rangle, \langle x_7, \frac{1}{15}, \frac{1}{4}\rangle\Big\}$$

defined in $X = \{x_1, x_2, \cdots, x_7\}$. Then, $\rho_{10}(\ell, \tilde{\ell}) = 1.0399$, which is not within $[0, 1]$. Thus, it violates A1 of Definition 2.5.

According to Ejegwa et al. [40],

$$\rho_{11}(\ell, \tilde{\ell}) = \frac{1}{3q} \sum_{j=1}^{q} \Big(\alpha_j(1 - \Delta\delta_j) + \beta_j(1 - \Delta\kappa_j) + \gamma_j(1 - \Delta\varrho_j)\Big), \tag{2.25}$$

where

$$\left.\begin{aligned} \alpha_j &= \frac{c - \Delta\delta_j - \Delta\delta_{\max}}{c - \Delta\delta_{\min} - \Delta\delta_{\max}} \\ \beta_j &= \frac{c - \Delta\kappa_j - \Delta\kappa_{\max}}{c - \Delta\kappa_{\min} - \Delta\kappa_{\max}} \\ \gamma_j &= \frac{c - \Delta\varrho_j - \Delta\varrho_{\max}}{c - \Delta\varrho_{\min} - \Delta\varrho_{\max}} \end{aligned}\right\} \tag{2.26}$$

for $c > 2$, and

$$\left.\begin{aligned} \Delta\delta_j &= |\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j)| \\ \Delta\kappa_j &= |\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j)| \\ \Delta\varrho_j &= |\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j)| \\ \Delta\delta_{\min} = \min_{1\le j\le q}\{\Delta\delta_j\}, \Delta\kappa_{\min} &= \min_{1\le j\le q}\{\Delta\kappa_j\} \\ \Delta\varrho_{\min} = \min_{1\le j\le q}\{\Delta\varrho_j\}, \Delta\delta_{\max} &= \max_{1\le j\le q}\{\Delta\delta_j\} \\ \Delta\kappa_{\max} = \max_{1\le j\le q}\{\Delta\kappa_j\}, \Delta\varrho_{\max} &= \max_{1\le j\le q}\{\Delta\varrho_j\} \end{aligned}\right\}. \tag{2.27}$$

Using Example 2.7, we get $\rho_{11}(\ell, \tilde{\ell}) = 1.0272$, which is not within $[0, 1]$. Thus, it violates A1 of Definition 2.5.

According to Bajaj and Kumar [41],

$$\rho_{12}(\ell, \tilde{\ell}) = \frac{1}{3q} \sum_{j=1}^{q} \Big(\alpha_j(1 - \Delta\delta_j) + \beta_j(1 - \Delta\kappa_j) + \gamma_j(1 - \Delta\varrho_j)\Big), \tag{2.28}$$

where α_j, β_j and γ_j are as in (2.26), and

$$\left.\begin{aligned} \Delta\delta_j &= \frac{|\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j)| + |\delta_\ell^2(x_j) - \delta_{\tilde{\ell}}^2(x_j)|}{2} \\ \Delta\kappa_j &= \frac{|\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j)| + |\kappa_\ell^2(x_j) - \kappa_{\tilde{\ell}}^2(x_j)|}{2} \\ \Delta\varrho_j &= \frac{|\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j)| + |\varrho_\ell^2(x_j) - \varrho_{\tilde{\ell}}^2(x_j)|}{2} \\ \Delta\delta_{\min} = \min_{1\le j\le q}\{\Delta\delta_j\}, \Delta\kappa_{\min} &= \min_{1\le j\le q}\{\Delta\kappa_j\} \\ \Delta\varrho_{\min} = \min_{1\le j\le q}\{\Delta\varrho_j\}, \Delta\delta_{\max} &= \max_{1\le j\le q}\{\Delta\delta_j\} \\ \Delta\kappa_{\max} = \max_{1\le j\le q}\{\Delta\kappa_j\}, \Delta\varrho_{\max} &= \max_{1\le j\le q}\{\Delta\varrho_j\} \end{aligned}\right\}. \tag{2.29}$$

Using Example 2.7, we get $\rho_{12}(\ell, \tilde{\ell}) = 1.0266$, which violates A1 of Definition 2.5.

3. New IFCMs based on Spearman's correlation model

In consideration of the shortcomings of the existing IFCMs, we develop new IFCMs based on the classical Spearman correlation coefficient as follows:

Definition 3.1. *Suppose that ℓ and $\tilde{\ell}$ are IFSs in $X = \{x_1, x_2, \cdots, x_q\}$. Then, the new IFCMs are as follows:*

$$\rho_a(\ell, \tilde{\ell}) = \frac{1}{3}\Big(\rho_m(\ell, \tilde{\ell}) + \rho_n(\ell, \tilde{\ell}) + \rho_h(\ell, \tilde{\ell})\Big), \tag{3.1}$$

where

$$\left.\begin{aligned} \rho_m(\ell, \tilde{\ell}) &= 1 - \frac{6\sum_{j=1}^{q}\left(\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j)\right)^2}{q(q^2+1)} \\ \rho_n(\ell, \tilde{\ell}) &= 1 - \frac{6\sum_{j=1}^{q}\left(\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j)\right)^2}{q(q^2+1)} \\ \rho_h(\ell, \tilde{\ell}) &= 1 - \frac{6\sum_{j=1}^{q}\left(\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j)\right)^2}{q(q^2+1)} \end{aligned}\right\}. \tag{3.2}$$

In the case of the classical Spearman correlation model, we have $q(q^2 - 1)$ instead of $q(q^2 + 1)$. If $q(q^2 - 1)$ is used and $q = 1$, we get

$$\rho(\ell, \tilde{\ell}) = \infty,$$

since

$$\left.\begin{aligned} \rho_m(\ell, \tilde{\ell}) &= 1 - \frac{6\sum_{j=1}^{q}\left(\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j)\right)^2}{q(q^2-1)} = \infty \\ \rho_n(\ell, \tilde{\ell}) &= 1 - \frac{6\sum_{j=1}^{q}\left(\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j)\right)^2}{q(q^2-1)} = \infty \\ \rho_h(\ell, \tilde{\ell}) &= 1 - \frac{6\sum_{j=1}^{q}\left(\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j)\right)^2}{q(q^2-1)} = \infty \end{aligned}\right\}.$$

In the case of $q(q^2 - 1)$, the new IFCM is undefined when $q = 1$. Hence, we used $q(q^2 + 1)$ instead. Similarly, we have

$$\rho_b(\ell, \tilde{\ell}) = \frac{1}{3}\Big(\rho_{m*}(\ell, \tilde{\ell}) + \rho_{n*}(\ell, \tilde{\ell}) + \rho_{h*}(\ell, \tilde{\ell})\Big), \tag{3.3}$$

where

$$\left.\begin{aligned} \rho_{m*}(\ell, \tilde{\ell}) &= 1 - \frac{6\sum_{j=1}^{q}\left(\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j)\right)^2}{(q+1)^3 - (q+1)} \\ \rho_{n*}(\ell, \tilde{\ell}) &= 1 - \frac{6\sum_{j=1}^{q}\left(\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j)\right)^2}{(q+1)^3 - (q+1)} \\ \rho_{h*}(\ell, \tilde{\ell}) &= 1 - \frac{6\sum_{j=1}^{q}\left(\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j)\right)^2}{(q+1)^3 - (q+1)} \end{aligned}\right\}. \tag{3.4}$$

Now, we discuss the properties of the new IFCMs through the following theorems.

Theorem 3.1. *The new IFCM $\rho_a(\ell,\tilde{\ell})$ between IFSs ℓ and $\tilde{\ell}$ in X is comparable to*

$$1-\frac{6\sum_{j=1}^{q}}{3q(q^2+1)}\begin{Bmatrix}\left(\delta_\ell(x_j)-\delta_{\tilde{\ell}}(x_j)\right)^2+\left(\kappa_\ell(x_j)-\kappa_{\tilde{\ell}}(x_j)\right)^2\\ +\left(\varrho_\ell(x_j)-\varrho_{\tilde{\ell}}(x_j)\right)^2\end{Bmatrix}.$$

Proof. Recall that

$$\rho_a(\ell,\tilde{\ell})=\frac{1}{3}\begin{Bmatrix}1-\frac{6\sum_{j=1}^{q}\left(\delta_\ell(x_j)-\delta_{\tilde{\ell}}(x_j)\right)^2}{q(q^2+1)}+\\ 1-\frac{6\sum_{j=1}^{q}\left(\kappa_\ell(x_j)-\kappa_{\tilde{\ell}}(x_j)\right)^2}{q(q^2+1)}+\\ 1-\frac{6\sum_{j=1}^{q}\left(\varrho_\ell(x_j)-\varrho_{\tilde{\ell}}(x_j)\right)^2}{q(q^2+1)}\end{Bmatrix}.$$

Then, we have

$$\begin{aligned}\rho_a(\ell,\tilde{\ell}) &= \frac{1}{3}\begin{Bmatrix}\frac{q(q^2+1)-6\sum_{j=1}^{q}\left(\delta_\ell(x_j)-\delta_{\tilde{\ell}}(x_j)\right)^2}{q(q^2+1)}\\ +\frac{q(q^2+1)-6\sum_{j=1}^{q}\left(\kappa_\ell(x_j)-\kappa_{\tilde{\ell}}(x_j)\right)^2}{q(q^2+1)}\\ +\frac{q(q^2+1)-6\sum_{j=1}^{q}\left(\varrho_\ell(x_j)-\varrho_{\tilde{\ell}}(x_j)\right)^2}{q(q^2+1)}\end{Bmatrix}\\ &= \frac{1}{3q(q^2+1)}\begin{Bmatrix}3q(q^2+1)-6\sum_{j=1}^{q}\left(\delta_\ell(x_j)-\delta_{\tilde{\ell}}(x_j)\right)^2\\ -6\sum_{j=1}^{q}\left(\kappa_\ell(x_j)-\kappa_{\tilde{\ell}}(x_j)\right)^2\\ -6\sum_{j=1}^{q}\left(\varrho_\ell(x_j)-\varrho_{\tilde{\ell}}(x_j)\right)^2\end{Bmatrix}\\ &= \frac{1}{3q(q^2+1)}\begin{Bmatrix}3q(q^2+1)-6\sum_{j=1}^{q}\left(\left(\delta_\ell(x_j)-\delta_{\tilde{\ell}}(x_j)\right)^2+\right.\\ \left.\left(\kappa_\ell(x_j)-\kappa_{\tilde{\ell}}(x_j)\right)^2+\left(\varrho_\ell(x_j)-\varrho_{\tilde{\ell}}(x_j)\right)^2\right)\end{Bmatrix}\\ &= 1-\frac{6\sum_{j=1}^{q}}{3q(q^2+1)}\begin{Bmatrix}\left(\delta_\ell(x_j)-\delta_{\tilde{\ell}}(x_j)\right)^2+\left(\kappa_\ell(x_j)-\kappa_{\tilde{\ell}}(x_j)\right)^2\\ +\left(\varrho_\ell(x_j)-\varrho_{\tilde{\ell}}(x_j)\right)^2\end{Bmatrix}\end{aligned}$$

as expected. □

Theorem 3.2. *The IFCM $\rho_b(\ell,\tilde{\ell})$ between IFSs ℓ and $\tilde{\ell}$ in X is comparable to*

$$1-\frac{6\sum_{j=1}^{q}}{3((q+1)^3-(q+1))}\begin{Bmatrix}\left(\delta_\ell(x_j)-\delta_{\tilde{\ell}}(x_j)\right)^2+\left(\kappa_\ell(x_j)-\kappa_{\tilde{\ell}}(x_j)\right)^2\\ +\left(\varrho_\ell(x_j)-\varrho_{\tilde{\ell}}(x_j)\right)^2\end{Bmatrix}.$$

Proof. The proof is as in Theorem 3.1. □

Theorem 3.3. *The new IFCM $\rho_a(\ell,\tilde{\ell})$ between IFSs ℓ and $\tilde{\ell}$ in X satisfies the correlation coefficient conditions.*

Proof. We shall prove the following correlation coefficient conditions:

(i) $|\rho_a(\ell,\tilde{\ell})|\leq 1$,
(ii) $\rho_a(\ell,\tilde{\ell})=1$ iff $\ell=\tilde{\ell}$,

(iii) $\rho_a(\ell,\tilde{\ell}) = \rho_a(\tilde{\ell},\ell)$.

Now, $|\rho_a(\ell,\tilde{\ell})| \leq 1$ implies that $0 \leq \rho_a(\ell,\tilde{\ell}) \leq 1$. First, since $(\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j))^2 \geq 0$, $(\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j))^2 \geq 0$ and $(\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j))^2 \geq 0$, then $\rho_a(\ell,\tilde{\ell}) \geq 0$ follows immediately.

Second, assume that

$$6\sum_{j=1}^{q}(\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j))^2 = \Theta_1,\ 6\sum_{j=1}^{q}(\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j))^2 = \Theta_2$$

$$6\sum_{j=1}^{q}(\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j))^2 = \Theta_3.$$

Then,

$$\begin{aligned}\rho_a(\ell,\tilde{\ell}) &= 1 - \frac{6\sum_{j=1}^{q}}{3q(q^2+1)}\left\{\begin{array}{c}(\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j))^2 + (\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j))^2\\ +(\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j))^2\end{array}\right\}\\ &\leq 1 - \frac{1}{3q(q^2+1)}\left\{\begin{array}{c}6\sum_{j=1}^{q}(\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j))^2 + 6\sum_{j=1}^{q}(\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j))^2\\ +6\sum_{j=1}^{q}(\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j))^2\end{array}\right\}\\ &= 1 - \frac{(\Theta_1 + \Theta_1 + \Theta_1)}{3q(q^2+1)}.\end{aligned}$$

Thus,

$$\rho_a(\ell,\tilde{\ell}) - 1 = -\frac{(\Theta_1 + \Theta_1 + \Theta_1)}{3q(q^2+1)} \leq 0,$$

and so $\rho_a(\ell,\tilde{\ell}) \leq 1$. Because $\rho_a(\ell,\tilde{\ell}) \geq 0$ and $\rho_a(\ell,\tilde{\ell}) \leq 1$, we have $|\rho_a(\ell,\tilde{\ell})| \leq 1$, which proves (i).

Suppose that $\rho_a(\ell,\tilde{\ell}) = 1$; then, we get

$$\frac{6\sum_{j=1}^{q}}{3q(q^2+1)}\left\{\begin{array}{c}(\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j))^2 + (\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j))^2\\ +(\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j))^2\end{array}\right\} = 0,$$

i.e., $(\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j))^2 = 0$, $(\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j))^2 = 0$ and $(\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j))^2 = 0$. Thus, $\delta_\ell(x_j) = \delta_{\tilde{\ell}}(x_j)$, $\kappa_\ell(x_j) = \kappa_{\tilde{\ell}}(x_j)$ and $\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j)$. Hence, $\ell = \tilde{\ell}$.

Conversely, if $\ell = \tilde{\ell}$, then,

$$\frac{6\sum_{j=1}^{q}}{3q(q^2+1)}\left\{\begin{array}{c}(\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j))^2 + (\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j))^2\\ +(\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j))^2\end{array}\right\} = 0,$$

and hence, $\rho_a(\ell,\tilde{\ell}) = 1$, which proves (ii).

Finally, we prove (iii). Now, since

$$\rho_a(\ell,\tilde{\ell}) = \frac{1}{3}\left\{\begin{array}{l}1 - \dfrac{6\sum_{j=1}^{q}(\delta_\ell(x_j) - \delta_{\tilde{\ell}}(x_j))^2}{q(q^2+1)} +\\ 1 - \dfrac{6\sum_{j=1}^{q}(\kappa_\ell(x_j) - \kappa_{\tilde{\ell}}(x_j))^2}{q(q^2+1)} +\\ 1 - \dfrac{6\sum_{j=1}^{q}(\varrho_\ell(x_j) - \varrho_{\tilde{\ell}}(x_j))^2}{q(q^2+1)}\end{array}\right\}$$

$$
\begin{aligned}
&= \frac{1}{3}\left\{\begin{array}{l}
\dfrac{q(q^2+1)-6\sum_{j=1}^{q}\left(\delta_{\ell}(x_j)-\delta_{\tilde{\ell}}(x_j)\right)^2}{q(q^2+1)} \\
+\dfrac{q(q^2+1)-6\sum_{j=1}^{q}\left(\kappa_{\ell}(x_j)-\kappa_{\tilde{\ell}}(x_j)\right)^2}{q(q^2+1)} \\
+\dfrac{q(q^2+1)-6\sum_{j=1}^{q}\left(\varrho_{\ell}(x_j)-\varrho_{\tilde{\ell}}(x_j)\right)^2}{q(q^2+1)}
\end{array}\right\} \\
&= \frac{1}{3}\left\{\begin{array}{l}
\dfrac{q(q^2+1)-6\sum_{j=1}^{q}\left(\delta_{\tilde{\ell}}(x_j)-\delta_{\ell}(x_j)\right)^2}{q(q^2+1)} \\
+\dfrac{q(q^2+1)-6\sum_{j=1}^{q}\left(\kappa_{\tilde{\ell}}(x_j)-\kappa_{\ell}(x_j)\right)^2}{q(q^2+1)} \\
+\dfrac{q(q^2+1)-6\sum_{j=1}^{q}\left(\varrho_{\tilde{\ell}}(x_j)-\varrho_{\ell}(x_j)\right)^2}{q(q^2+1)}
\end{array}\right\} \\
&= \rho_a(\tilde{\ell},\ell),
\end{aligned}
$$

then, (iii) is proved. □

Theorem 3.4. *The function $\rho_b(\ell,\tilde{\ell})$ between IFSs ℓ and $\tilde{\ell}$ in X satisfies the correlation coefficient conditions.*

Proof. The proof is analogous to Theorem 3.3. □

3.1. Numerical analysis of the new IFCMs

Here, we justify the superiority of the developed IFCMs over the existing methods [28, 30–33, 35–41]. Table 1 lists the results of the numerical analysis.

Table 1. Results of the Numerical Analysis.

Examples	Existing IFCMs	New IFCMs
Example 2.1	$\rho_1(\ell,\tilde{\ell}) = 1$	$\rho_a(\ell,\tilde{\ell}) = 0.9584$, $\rho_b(\ell,\tilde{\ell}) = 0.9861$
Example 2.2	$\rho_1(\ell,\tilde{\ell}) = 1, \rho_2(\ell,\tilde{\ell}) = 1$, $\rho_5(\ell,\tilde{\ell}) = 1, \rho_6(\ell,\tilde{\ell}) = 1.1667$, $\rho_7(\ell,\tilde{\ell}) = 1.1667, \rho_8(\ell,\tilde{\ell}) = 0$, $\rho_9(\ell,\tilde{\ell}) = 1$	$\rho_a(\ell,\tilde{\ell}) = 0.9063$, $\rho_b(\ell,\tilde{\ell}) = 0.9609$
Example 2.3	$\rho_1(\ell,\tilde{\ell}) = 0, \rho_2(\ell,\tilde{\ell}) = -1$, $\rho_3(\ell,\tilde{\ell}) = 0, \rho_4(\ell,\tilde{\ell}) = 0$, $\rho_5(\ell,\tilde{\ell}) = 0, \rho_6(\ell,\tilde{\ell}) = 1$, $\rho_7(\ell,\tilde{\ell}) = 0, \rho_8(\ell,\tilde{\ell}) = -1$, $\rho_9(\ell,\tilde{\ell}) = -1, \rho_{10}(\ell,\tilde{\ell}) = 0$	$\rho_a(\ell,\tilde{\ell}) = 0.2, \rho_b(\ell,\tilde{\ell}) = 0.6667$
Example 2.4	$\rho_6(\ell,\tilde{\ell}) = \rho_7(\ell,\tilde{\ell}) = 1$. If $\ell = \tilde{\ell}$, $\rho_6(\ell,\tilde{\ell}) = \rho_7(\ell,\tilde{\ell}) = \frac{0}{0} = \infty$	$\rho_a(\ell,\tilde{\ell}) = 0.976, \rho_b(\ell,\tilde{\ell}) = 0.99$
Example 2.5	$\rho_8(\ell,\tilde{\ell}) = \frac{0}{\sqrt{0\times 0}} = \infty$	$\rho_a(\ell,\tilde{\ell}) = 0.9806$, $\rho_b(\ell,\tilde{\ell}) = 0.9919$
Example 2.6	$\rho_9(\ell,\tilde{\ell}) = \frac{0}{\sqrt{0\times 0}} = \infty$	$\rho_a(\ell,\tilde{\ell}) = 0.9333$, $\rho_b(\ell,\tilde{\ell}) = 0.9722$
Example 2.7	$\rho_{10}(\ell,\tilde{\ell}) = 1.0399$, $\rho_{11}(\ell,\tilde{\ell}) = 1.0272$, $\rho_{12}(\ell,\tilde{\ell}) = 1.0266$	$\rho_a(\ell,\tilde{\ell}) = 0.9986$, $\rho_b(\ell,\tilde{\ell}) = 0.9990$

Next, we discuss the effectiveness of the new IFCMs as compare to the existing IFCMs using the information in Table 1 as follows:

(i) In Example 2.1, the IFCM ρ_1 [28] fails because it gives a perfect positive correlation whereas ℓ and $\tilde{\ell}$ are not identical, which violates A2 of Definition 2.5. On the other hand, the new measures indicate that a strong correlation exists between the IFSs ℓ and $\tilde{\ell}$.

(ii) In Example 2.2, the IFCMs ρ_1, ρ_2, ρ_5, and ρ_9 [28, 31, 33, 35] indicate that a perfect positive correlation exists between ℓ and $\tilde{\ell}$, whereas ℓ and $\tilde{\ell}$ are not identical. This is a violation of A2 of Definition 2.5. In addition, the IFCMs ρ_6 [36] and ρ_7 [37] provide a value that is not defined within either $[0, 1]$ or $[-1, 1]$, which violates A1 of Definition 2.5. Surprisingly, the IFCM ρ_8 [32] shows that no correlation exists between the IFSs, which is a misleading information because there exists an imprecise correlation between the IFSs. The new measures, ρ_a and ρ_b give more precise results.

(iii) In Example 2.3, the IFCMs $\rho_1, \rho_3, \rho_4, \rho_5, \rho_7$, and ρ_{10} [28,30,33,37–39] indicate that no correlation exists between the IFSs, which is misleading because a somewhat uncertain correlation exists. In addition, the IFCMs ρ_2, ρ_8, and ρ_9 [31, 32, 35] indicate that a perfect negative correlation exists between the IFSs, which is not true because a somewhat uncertain correlation exists. Unexpectedly, the IFCM ρ_6 [36] indicates that a perfect positive correlation exists between the IFSs, which is again misleading because the IFSs are not identical. On the contrary, the new measures suggest that a positive correlation exists between the IFSs.

(iv) In Example 2.4, the IFCMs ρ_6 [36] and ρ_7 [37] show that a perfect positive relationship exists between the IFSs although $\ell \neq \tilde{\ell}$. In addition, If $\ell = \tilde{\ell}$, we can see that $\rho_6(\ell, \tilde{\ell}) = \rho_7(\ell, \tilde{\ell}) = \frac{0}{0} = \infty$. These IFCMs violate A1 and A2 of Definition 2.5. On the other hand, the new measures show that a strong positive correlation exists between the IFSs; also, and whenever $\ell = \tilde{\ell}$, they show a perfect positive correlation that is in agreement with the IFCC metric conditions.

(v) In Example 2.5, the IFCM ρ_8 [32] gives a meaningless correlation value, which is not within either $[0, 1]$ or $[-1, 1]$, i.e., it violates A1 of Definition 2.5. However, the new measures show that a strong positive correlation exists between the IFSs. Similarly, for Example 2.6, the IFCM ρ_9 [35] yields an inappropriate result similar to ρ_8 [32]. Also, the new measures indicate that a strong positive correlation exists between the IFSs.

(vi) Finally, in Example 2.7, the IFCM ρ_{10}, ρ_{11}, and ρ_{12} [30, 40, 41] give correlation values that are greater than 1, which violate A1 of Definition 2.5. On the contrary, the new measures suggest that a positive correlation exists between the IFSs, which satisfies A1 of Definition 2.5.

To recap, all of the IFCMs fail to satisfy the IFCM metric conditions, except for the new IFCMs. In addition, the new measures yield a more precise result than all of the existing techniques.

4. Triage process for prompt treatment under uncertainty

Triage is the medical process used to determine the patients in the ED who are most in need of urgent treatment. This process is carried out by a triage nurse in the ED who assesses the patients' medical conditions and decides whether to call a consultant for the most critical case. For this reason, some patients who arrive at the ED late might receive medical attention first. Often, in a medical facility, some patients urgently require medical attention to address their debilitating medical conditions. However, the inability to control diagnostic uncertainty may result in misdiagnosis or delayed conditions and/or postponed or unnecessary tests and/or treatments. Owing to the inherent uncertainty in medical diagnoses, it is necessary to deploy an intuitionistic fuzzy approach to eliminate diagnostic imprecision and uncertainty. To achieve a reliable diagnosis, we propose employing a knowledge-based diagnostic process that captures the linguistic variables of patients' symptoms through the use of intuitionistic fuzzy numbers (IFNs), which are presented in Table 2.

The symptoms for this type of ME case are represented by a set:

$$\breve{S} = \{\breve{S}_1, \breve{S}_2, \breve{S}_3, \breve{S}_4, \breve{S}_5, \breve{S}_6, \breve{S}_7\},$$

where $\breve{S}_1$ is body temperature, $\breve{S}_2$ is pulse, $\breve{S}_3$ is prostration, $\breve{S}_4$ is dehydration, $\breve{S}_5$ is blood pressure, $\breve{S}_6$ is dyspnea, and $\breve{S}_7$ is paleness.

Table 2. Linguistic variables for symptoms evaluation.

Linguistic variables	IFNs
Extremely high (EH)	$(1, 0)$
Very very high (VVH)	$(0.9, 0.05)$
Very high (VH)	$(0.8, 0.15)$
High (H)	$(0.7, 0.2)$
Medium high (MH)	$(0.6, 0.25)$
Medium (M)	$(0.5, 0.4)$
Medium low (ML)	$(0.4, 0.5)$
Low (L)	$(0.3, 0.65)$
Very low (VL)	$(0.2, 0.75)$
Very very low (VVL)	$(0.05, 0.9)$
Extremely low (EL)	$(0, 1)$

Suppose that six patients represented by a set $\breve{P} = \{\breve{P}_1, \breve{P}_2, \breve{P}_3, \breve{P}_4, \breve{P}_5, \breve{P}_6\}$, are brought to ED for treatment. However, due to the limited number of consultants, all patients cannot receive medical attention at the same time, so, it is expedient to attend to the patient with the most serious medical case to avoid fatality. Three triage nurses attend to the patients to determine which of the patients' cases is most in need of emergency medical attention. The opinions of the triage nurses are presented in terms of linguistic variables in Tables 3–5, respectively.

Table 3. Linguistic variables for triage nurse I.

Patients	$\breve{S}_1$	$\breve{S}_2$	$\breve{S}_3$	$\breve{S}_4$	$\breve{S}_5$	$\breve{S}_6$	$\breve{S}_7$
$\breve{P}_1$	M	H	VH	VL	MH	VH	H
$\breve{P}_2$	VVL	M	VH	MH	MH	M	VL
$\breve{P}_3$	ML	H	VH	VVH	ML	H	ML
$\breve{P}_4$	VL	H	M	ML	MH	VVH	H
$\breve{P}_5$	L	MH	VH	M	VL	H	VH
$\breve{P}_6$	M	L	MH	VH	VVH	H	ML

Table 4. Linguistic variables for triage nurse II.

Patients	$\breve{S}_1$	$\breve{S}_2$	$\breve{S}_3$	$\breve{S}_4$	$\breve{S}_5$	$\breve{S}_6$	$\breve{S}_7$
$\breve{P}_1$	M	H	M	VVH	VVH	VL	VVL
$\breve{P}_2$	ML	VH	H	H	L	MH	MH
$\breve{P}_3$	L	VH	VVH	MH	ML	VH	H
$\breve{P}_4$	VL	M	L	M	VL	ML	H
$\breve{P}_5$	VVL	VH	H	L	MH	M	VH
$\breve{P}_6$	M	M	ML	MH	VVH	MH	H

Table 5. Linguistic variables for triage nurse III.

Patients	$\check{S}_1$	$\check{S}_2$	$\check{S}_3$	$\check{S}_4$	$\check{S}_5$	$\check{S}_6$	$\check{S}_7$
$\check{P}_1$	ML	L	VH	VVH	H	VL	H
$\check{P}_2$	L	MH	L	H	VVL	M	VH
$\check{P}_3$	M	H	VVH	L	M	MH	VVH
$\check{P}_4$	VL	VH	H	MH	H	VH	M
$\check{P}_5$	M	VVH	VVL	H	VH	H	VL
$\check{P}_6$	VVL	H	H	L	M	VH	MH

Using the information in Table 2, we obtained the results presented in Tables 6–8 from Tables 3–5, respectively.

Table 6. IFNs for triage nurse I.

Patients	$\check{S}_1$	$\check{S}_2$	$\check{S}_3$	$\check{S}_4$	$\check{S}_5$	$\check{S}_6$	$\check{S}_7$
$\check{P}_1$	$(0.5, 0.4)$	$(0.7, 0.2)$	$(0.8, 0.15)$	$(0.2, 0.75)$	$(0.6, 0.25)$	$(0.8, 0.15)$	$(0.7, 0.2)$
$\check{P}_2$	$(0.05, 0.9)$	$(0.5, 0.4)$	$(0.8, 0.15)$	$(0.6, 0.25)$	$(0.6, 0.25)$	$(0.5, 0.4)$	$(0.2, 0.75)$
$\check{P}_3$	$(0.4, 0.5)$	$(0.7, 0.2)$	$(0.8, 0.15)$	$(0.9, 0.05)$	$(0.4, 0.6)$	$(0.7, 0.2)$	$(0.4, 0.5)$
$\check{P}_4$	$(0.2, 0.75)$	$(0.7, 0.2)$	$(0.5, 0.4)$	$(0.4, 0.5)$	$(0.6, 0.25)$	$(0.9, 0.05)$	$(0.7, 0.2)$
$\check{P}_5$	$(0.3, 0.65)$	$(0.6, 0.25)$	$(0.8, 0.15)$	$(0.5, 0.4)$	$(0.2, 0.75)$	$(0.7, 0.2)$	$(0.8, 0.15)$
$\check{P}_6$	$(0.5, 0.4)$	$(0.3, 0.65)$	$(0.6, 0.25)$	$(0.8, 0.15)$	$(0.9, 0.05)$	$(0.7, 0.2)$	$(0.4, 0.5)$

Table 7. IFNs for triage nurse II.

Patients	$\check{S}_1$	$\check{S}_2$	$\check{S}_3$	$\check{S}_4$	$\check{S}_5$	$\check{S}_6$	$\check{S}_7$
$\check{P}_1$	$(0.5, 0.4)$	$(0.7, 0.2)$	$(0.5, 0.4)$	$(0.9, 0.05)$	$(0.9, 0.05)$	$(0.2, 0.75)$	$(0.05, 0.9)$
$\check{P}_2$	$(0.4, 0.5)$	$(0.8, 0.15)$	$(0.7, 0.2)$	$(0.7, 0.2)$	$(0.3, 0.65)$	$(0.6, 0.25)$	$(0.6, 0.25)$
$\check{P}_3$	$(0.3, 0.65)$	$(0.8, 0.15)$	$(0.8, 0.15)$	$(0.6, 0.25)$	$(0.4, 0.5)$	$(0.8, 0.15)$	$(0.7, 0.2)$
$\check{P}_4$	$(0.2, 0.75)$	$(0.5, 0.4)$	$(0.3, 0.65)$	$(0.5, 0.4)$	$(0.2, 0.75)$	$(0.4, 0.5)$	$(0.7, 0.2)$
$\check{P}_5$	$(0.05, 0.9)$	$(0.8, 0.15)$	$(0.7, 0.2)$	$(0.3, 0.65)$	$(0.6, 0.25)$	$(0.5, 0.4)$	$(0.8, 0.15)$
$\check{P}_6$	$(0.5, 0.4)$	$(0.5, 0.4)$	$(0.4, 0.5)$	$(0.6, 0.25)$	$(0.9, 0.05)$	$(0.6, 0.25)$	$(0.7, 0.2)$

Table 8. IFNs for triage nurse III.

Patients	$\check{S}_1$	$\check{S}_2$	$\check{S}_3$	$\check{S}_4$	$\check{S}_5$	$\check{S}_6$	$\check{S}_7$
$\check{P}_1$	$(0.4, 0.5)$	$(0.3, 0.65)$	$(0.8, 0.15)$	$(0.9, 0.05)$	$(0.7, 0.2)$	$(0.2, 0.75)$	$(0.7, 0.2)$
$\check{P}_2$	$(0.3, 0.65)$	$(0.6, 0.25)$	$(0.3, 0.25)$	$(0.7, 0.2)$	$(0.05, 0.9)$	$(0.5, 0.4)$	$(0.8, 0.15)$
$\check{P}_3$	$(0.5, 0.4)$	$(0.7, 0.2)$	$(0.9, 0.05)$	$(0.3, 0.65)$	$(0.5, 0.4)$	$(0.6, 0.25)$	$(0.9, 0.05)$
$\check{P}_4$	$(0.2, 0.75)$	$(0.8, 0.15)$	$(0.7, 0.2)$	$(0.6, 0.25)$	$(0.7, 0.2)$	$(0.8, 0.15)$	$(0.5, 0.4)$
$\check{P}_5$	$(0.5, 0.4)$	$(0.9, 0.05)$	$(0.05, 0.9)$	$(0.7, 0.2)$	$(0.8, 0.15)$	$(0.7, 0.2)$	$(0.2, 0.75)$
$\check{P}_6$	$(0.05, 0.9)$	$(0.7, 0.2)$	$(0.7, 0.2)$	$(0.3, 0.65)$	$(0.5, 0.4)$	$(0.8, 0.15)$	$(0.6, 0.25)$

By using Definition 2.4, we combined the opinions of the three triage nurses as presented in Table 9.

Table 9. Medical information.

Patients	$\breve{S}_1$	$\breve{S}_2$	$\breve{S}_3$	$\breve{S}_4$	$\breve{S}_5$	$\breve{S}_6$	$\breve{S}_7$
$\breve{P}_1$	$(\frac{7}{15}, \frac{13}{30})$	$(\frac{17}{30}, \frac{7}{20})$	$(\frac{7}{10}, \frac{7}{30})$	$(\frac{2}{3}, \frac{17}{60})$	$(\frac{11}{15}, \frac{1}{6})$	$(\frac{2}{5}, \frac{11}{20})$	$(\frac{29}{60}, \frac{13}{30})$
$\breve{P}_2$	$(\frac{1}{4}, \frac{41}{60})$	$(\frac{19}{30}, \frac{4}{15})$	$(\frac{3}{5}, \frac{1}{3})$	$(\frac{2}{3}, \frac{13}{60})$	$(\frac{19}{60}, \frac{3}{5})$	$(\frac{8}{15}, \frac{7}{20})$	$(\frac{8}{15}, \frac{23}{60})$
$\breve{P}_3$	$(\frac{2}{5}, \frac{31}{60})$	$(\frac{11}{15}, \frac{11}{60})$	$(\frac{5}{6}, \frac{7}{60})$	$(\frac{3}{5}, \frac{19}{60})$	$(\frac{13}{30}, \frac{1}{2})$	$(\frac{7}{10}, \frac{1}{5})$	$(\frac{1}{15}, \frac{1}{4})$
$\breve{P}_4$	$(\frac{1}{5}, \frac{3}{4})$	$(\frac{2}{3}, \frac{1}{4})$	$(\frac{1}{2}, \frac{5}{12})$	$(\frac{1}{2}, \frac{23}{60})$	$(\frac{1}{2}, \frac{2}{5})$	$(\frac{7}{10}, \frac{7}{30})$	$(\frac{19}{30}, \frac{4}{15})$
$\breve{P}_5$	$(\frac{17}{60}, \frac{39}{60})$	$(\frac{23}{30}, \frac{3}{20})$	$(\frac{31}{60}, \frac{5}{12})$	$(\frac{1}{2}, \frac{5}{12})$	$(\frac{8}{15}, \frac{23}{60})$	$(\frac{19}{30}, \frac{20}{75})$	$(\frac{3}{5}, \frac{21}{60})$
$\breve{P}_6$	$(\frac{7}{20}, \frac{17}{30})$	$(\frac{1}{2}, \frac{5}{12})$	$(\frac{17}{30}, \frac{19}{60})$	$(\frac{17}{30}, \frac{7}{20})$	$(\frac{23}{30}, \frac{1}{6})$	$(\frac{7}{10}, \frac{1}{5})$	$(\frac{17}{30}, \frac{19}{60})$

Summaries of the triage and data collection processes are as follows: (i) three triage nurses provide the linguistic variables as seen in Tables 3–5; (ii) the linguistic variables are converted to IFNs (as shown in Tables 6–8) based on the information in Table 2; and (iii) the IFNs from the linguistic variables are compressed into one by taking the arithmetic average of the IFNs (as shown in Table 9).

The linguistic variables of a healthy person as determined based on expert knowledge are MH, MH, EL, EL, M, EL, and EL for body temperature, pulse, prostration, dehydration, blood pressure, dyspnea, and paleness, respectively. Assuming that a healthy person is represented by an IFS, denoted as $\breve{H}$, the medical information for $\breve{H}$ is taken as follows:

$$\breve{H} = \{\langle \breve{S}_1, 0.6, 0.25\rangle, \langle \breve{S}_2, 0.6, 0.25\rangle, \langle \breve{S}_3, 0, 1\rangle, \langle \breve{S}_4, 0, 1\rangle, \langle \breve{S}_5, 0.5, 0.4\rangle, \langle \breve{S}_6, 0, 1\rangle, \langle \breve{S}_7, 0, 1\rangle\}.$$

4.1. Approach I

An innovative method has been developed for the triage process to determine the most urgent case among the six sick patients described in Table 9.

Target

Choose the most critically ill patient to optimize emergency treatment.

Algorithm for approach I

The triage process algorithm that chooses the most critically sick patient is as follows:

Step 1: Compute $\rho(\breve{P}_j, \breve{H})$ for $j = 1, 2, \cdots, 7$ by using the measures given by (2.1, 2.3, 2.6, 2.8, 2.10, 2.13, 2.15, 2.17, 2.20, 2.22, 2.25, 2.28, 3.1, 3.3), where $\breve{H}$ is the medical information of a healthy person.

Step 2: Find

$$\rho^*(\breve{P}_j, \breve{H}) = \max_{1\le j\le 7}\{\rho(\breve{P}_j, \breve{H})\}. \tag{4.1}$$

Step 3: Compute the degree of confidence (DoC), defined by

$$\diamond = \sum_{j}^{7} |\rho^*(\breve{P}_j, \breve{H}) - \rho(\breve{P}_j, \breve{H})|, \tag{4.2}$$

where a small value of $\diamond$ shows precision and reliability.
Step 4: The value of $\min\{\rho(\breve{P}_j, \breve{H})\}$ determines the most critically ill patient for emergency treatment.

Now, we implement Step 1 to compute the correlation coefficients between each patient and the healthy person to determine which of the patients has the weakest correlation with the healthy person by using the new approaches. We obtained the following results from the computations:

$$\rho_a(\breve{P}_1, \breve{H}) = 0.9820, \rho_a(\breve{P}_2, \breve{H}) = 0.9790,$$
$$\rho_a(\breve{P}_3, \breve{H}) = 0.9736, \rho_a(\breve{P}_4, \breve{H}) = 0.9789,$$
$$\rho_a(\breve{P}_5, \breve{H}) = 0.9814, \rho_a(\breve{P}_6, \breve{H}) = 0.9781,$$
$$\rho_b(\breve{P}_1, \breve{H}) = 0.9875, \rho_b(\breve{P}_2, \breve{H}) = 0.9854,$$
$$\rho_b(\breve{P}_3, \breve{H}) = 0.9817, \rho_b(\breve{P}_4, \breve{H}) = 0.9853,$$
$$\rho_b(\breve{P}_5, \breve{H}) = 0.9871, \rho_b(\breve{P}_6, \breve{H}) = 0.9848.$$

By applying Steps 2 and 3, the DoCs of the correlation coefficients were calculated to be 0.019 and 0.0132, respectively. The ordering of the correlation coefficients is as follows:

$$\rho_a(\breve{P}_1, \breve{H}) > \rho_a(\breve{P}_5, \breve{H}) > \rho_a(\breve{P}_2, \breve{H}) > \rho_a(\breve{P}_4, \breve{H}) > \rho_a(\breve{P}_6, \breve{H}) > \rho_a(\breve{P}_3, \breve{H}),$$
$$\rho_b(\breve{P}_1, \breve{H}) > \rho_b(\breve{P}_5, \breve{H}) > \rho_b(\breve{P}_2, \breve{H}) > \rho_b(\breve{P}_4, \breve{H}) > \rho_b(\breve{P}_6, \breve{H}) > \rho_b(\breve{P}_3, \breve{H}).$$

The order shows that, $\breve{P}_3$ has the weakest correlation with $\breve{H}$, which means that the patient with the most critical health challenge is $\breve{P}_3$. In a situation in which there is only one medical consultant, $\breve{P}_3$ should be given treatment priority.

4.1.1. Comparative analysis under approach I

To demonstrate the superiority of the new IFCMs over the existing IFCMs, we present a comparative study. The results of the comparison are listed in Table 10.

From Table 10, we see that the IFCMs in [36–38] are not appropriate because their results do not fall within $[0, 1]$ or $[-1, 1]$. Notably, our IFCMs yielded the most robust results. The ordering and DoC results for the measures listed in Table 10 are presented in Table 11.

Table 10. Results of comparative for approach I.

Measures	$(\breve{P}_1, \breve{H})$	$(\breve{P}_2, \breve{H})$	$(\breve{P}_3, \breve{H})$	$(\breve{P}_4, \breve{H})$	$(\breve{P}_5, \breve{H})$	$(\breve{P}_6, \breve{H})$
ρ_1 [28]	0.6535	0.5915	0.4926	0.5905	0.6422	0.5738
ρ_2 [31]	0.1028	−0.5747	−0.5650	−0.4324	−0.1207	−0.3933
ρ_3 [39]	0.6548	0.5910	0.4943	0.5898	0.6422	0.5730
ρ_4 [38]	1.2093	1.0773	0.9489	1.0893	1.1851	1.0506
ρ_5 [33]	0.3196	−0.4856	−0.3649	−0.4335	−0.0634	−0.4822
ρ_6 [36]	1.3522	1.2994	1.3572	1.3542	1.3114	1.2775
ρ_7 [37]	1.3288	1.2832	1.3056	1.3250	1.2853	1.2489
ρ_8 [32]	0.1034	−0.5749	−0.5655	−0.4334	−0.1206	−0.3955
ρ_9 [35]	0.1089	−0.5725	−0.5614	−0.4330	−0.1195	−0.3972
ρ_{10} [30]	0.7612	0.6890	0.7437	0.7353	0.7137	0.6656
ρ_{11} [40]	0.8268	0.7719	0.8096	0.8028	0.7907	0.7531
ρ_{12} [41]	0.8432	0.7898	0.8271	0.8194	0.8077	0.7763
ρ_a	0.9820	0.9790	0.9736	0.9789	0.9814	0.9781
ρ_b	0.9875	0.9854	0.9817	0.9853	0.9871	0.9848

Table 11. Order and DoC results for the measures.

Measures	Orderings	Triage	DoCs
ρ_1 [28]	$\rho_1(\breve{P}_1, \breve{H}) \succ \rho_1(\breve{P}_5, \breve{H}) \succ \rho_1(\breve{P}_2, \breve{H}) \succ \rho_1(\breve{P}_4, \breve{H}) \succ \rho_1(\breve{P}_6, \breve{H}) \succ \rho_1(\breve{P}_3, \breve{H})$	$\breve{P}_3$	0.3769
ρ_2 [31]	$\rho_2(\breve{P}_1, \breve{H}) \succ \rho_2(\breve{P}_5, \breve{H}) \succ \rho_2(\breve{P}_6, \breve{H}) \succ \rho_2(\breve{P}_4, \breve{H}) \succ \rho_2(\breve{P}_3, \breve{H}) \succ \rho_2(\breve{P}_2, \breve{H})$	$\breve{P}_2$	2.5501
ρ_3 [39]	$\rho_3(\breve{P}_1, \breve{H}) \succ \rho_3(\breve{P}_5, \breve{H}) \succ \rho_3(\breve{P}_2, \breve{H}) \succ \rho_3(\breve{P}_4, \breve{H}) \succ \rho_3(\breve{P}_6, \breve{H}) \succ \rho_3(\breve{P}_3, \breve{H})$	$\breve{P}_3$	0.3837
ρ_4 [38]	fails A1	N/A	N/A
ρ_5 [33]	$\rho_5(\breve{P}_1, \breve{H}) \succ \rho_5(\breve{P}_5, \breve{H}) \succ \rho_5(\breve{P}_3, \breve{H}) \succ \rho_5(\breve{P}_4, \breve{H}) \succ \rho_5(\breve{P}_6, \breve{H}) \succ \rho_5(\breve{P}_2, \breve{H})$	$\breve{P}_2$	3.4276
ρ_6 [36]	fails A1	N/A	N/A
ρ_7 [37]	fails A1	N/A	N/A
ρ_8 [32]	$\rho_8(\breve{P}_1, \breve{H}) \succ \rho_8(\breve{P}_5, \breve{H}) \succ \rho_8(\breve{P}_6, \breve{H}) \succ \rho_8(\breve{P}_4, \breve{H}) \succ \rho_8(\breve{P}_3, \breve{H}) \succ \rho_8(\breve{P}_2, \breve{H})$	$\breve{P}_2$	2.6069
ρ_9 [35]	$\rho_9(\breve{P}_1, \breve{H}) \succ \rho_9(\breve{P}_5, \breve{H}) \succ \rho_9(\breve{P}_6, \breve{H}) \succ \rho_9(\breve{P}_4, \breve{H}) \succ \rho_9(\breve{P}_3, \breve{H}) \succ \rho_9(\breve{P}_2, \breve{H})$	$\breve{P}_2$	2.6281
ρ_{10} [30]	$\rho_{10}(\breve{P}_1, \breve{H}) \succ \rho_{10}(\breve{P}_3, \breve{H}) \succ \rho_{10}(\breve{P}_4, \breve{H}) \succ \rho_{10}(\breve{P}_5, \breve{H}) \succ \rho_{10}(\breve{P}_2, \breve{H}) \succ \rho_{10}(\breve{P}_6, \breve{H})$	$\breve{P}_6$	0.2587
ρ_{11} [40]	$\rho_{11}(\breve{P}_1, \breve{H}) \succ \rho_{11}(\breve{P}_3, \breve{H}) \succ \rho_{11}(\breve{P}_4, \breve{H}) \succ \rho_{11}(\breve{P}_5, \breve{H}) \succ \rho_{11}(\breve{P}_2, \breve{H}) \succ \rho_{11}(\breve{P}_6, \breve{H})$	$\breve{P}_6$	0.2059
ρ_{12} [41]	$\rho_{12}(\breve{P}_1, \breve{H}) \succ \rho_{12}(\breve{P}_3, \breve{H}) \succ \rho_{12}(\breve{P}_4, \breve{H}) \succ \rho_{12}(\breve{P}_5, \breve{H}) \succ \rho_{12}(\breve{P}_2, \breve{H}) \succ \rho_{12}(\breve{P}_6, \breve{H})$	$\breve{P}_6$	0.1957
ρ_a	$\rho_a(\breve{P}_1, \breve{H}) \succ \rho_a(\breve{P}_5, \breve{H}) \succ \rho_a(\breve{P}_2, \breve{H}) \succ \rho_a(\breve{P}_4, \breve{H}) \succ \rho_a(\breve{P}_6, \breve{H}) \succ \rho_a(\breve{P}_3, \breve{H})$	$\breve{P}_3$	0.019
ρ_b	$\rho_b(\breve{P}_1, \breve{H}) \succ \rho_b(\breve{P}_5, \breve{H}) \succ \rho_b(\breve{P}_2, \breve{H}) \succ \rho_b(\breve{P}_4, \breve{H}) \succ \rho_b(\breve{P}_6, \breve{H}) \succ \rho_b(\breve{P}_3, \breve{H})$	$\breve{P}_3$	0.0132

From the results in Table 11, it is evident that patients $\breve{P}_3$ and $\breve{P}_6$ have the weakest correlation with the healthy person. Thus, patients who needed the most urgent medical attention are $\breve{P}_3$ and $\breve{P}_6$. Although the measures in [30–33, 35, 40, 41] yield conflicting interpretations, their interpretations cannot be trusted because they were found to be unrealistic, as shown in Table 1. We have used the information in Table 11 to plot the DoC graph as shown in Figure 1.

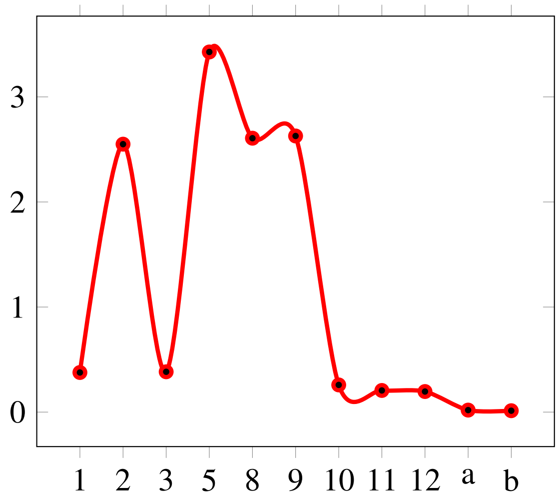

Figure 1. DoC results.

From Figure 1, we see that the new IFCMs have the lowest DoC. This shows the reliability of the newly developed IFCMs.

4.2. Approach II

Here, we apply the multiple criteria decision-making (MCDM) approach to the triage process because the MCDM is more reputable than the approach in Subsection 4.1.

Algorithm for Approach II

Step 1: Frame the intuitionistic fuzzy decision matrix (IFDM) $\tilde{\breve{P}}_j = \{\breve{S}_i(\breve{P}_j)\}_{(n\times q)}$, where $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, q$ and $\breve{P}_j$ and $\breve{S}_j$ represent the patients and symptoms, respectively.
Step 2: Determine the cost criterion (CC) (i.e., the lowest $\breve{S}_i$) and the benefit criteria (BC) (i.e., the non-lowest $\breve{S}_i$).
Step 3: Normalize the IFDM to get the normalized IFDM denoted by $\tilde{\breve{P}} = \langle\delta_{\tilde{\breve{P}}_j}(\breve{S}_i), \kappa_{\tilde{\breve{P}}_j}(\breve{S}_i)\rangle_{n\times q}$, where $\langle\delta_{\tilde{\breve{P}}_j}(\breve{S}_i), \kappa_{\tilde{\breve{P}}_j}(\breve{S}_i)\rangle$ are IFNs, and $\tilde{\breve{P}}_j$ is defined as follows:

$$\tilde{\breve{P}}_j = \begin{cases} \langle\delta_{\tilde{\breve{P}}_j}(\breve{S}_i), \kappa_{\tilde{\breve{P}}_j}(\breve{S}_i)\rangle & \text{for BC of } \breve{P}_j; \\ \langle\langle\kappa_{\tilde{\breve{P}}_j}(\breve{S}_i), \delta_{\tilde{\breve{P}}_j}(\breve{S}_i)\rangle & \text{for CC of } \breve{P}_j. \end{cases} \tag{4.3}$$

Step 4: Compute the positive ideal solution (PIS) and negative ideal solution (NIS) as follows:

$$\begin{aligned} \tilde{\breve{P}}^+ &= \{\tilde{\breve{P}}_1^+, \tilde{\breve{P}}_2^+, \cdots, \tilde{\breve{P}}_q^+\}, \\ \tilde{\breve{P}}^- &= \{\tilde{\breve{P}}_1^-, \tilde{\breve{P}}_2^-, \cdots, \tilde{\breve{P}}_q^-\}, \end{aligned} \tag{4.4}$$

where

$$\tilde{\breve{P}}^+ = \begin{cases} \langle\max\{\delta_{\tilde{\breve{P}}_j}(\breve{S}_i)\}, \min\{\kappa_{\tilde{\breve{P}}_j}(\breve{S}_i)\}\rangle, & \text{if } \breve{S}_i \text{ is a BC;} \\ \langle\min\{\delta_{\tilde{\breve{P}}_j}(\breve{S}_i)\}, \max\{\kappa_{\tilde{\breve{P}}_j}(\breve{S}_i)\}\rangle, & \text{if } \breve{S}_i \text{ is a CC,} \end{cases} \tag{4.5}$$

$$\tilde{\breve{P}}^- = \begin{cases} \langle\min\{\delta_{\tilde{\breve{P}}_j}(\breve{S}_i)\}, \max\{\kappa_{\tilde{\breve{P}}_j}(\breve{S}_i)\}\rangle, & \text{if } \breve{S}_i \text{ is a BC;} \\ \langle\max\{\delta_{\tilde{\breve{P}}_j}(\breve{S}_i)\}, \min\{\kappa_{\tilde{\breve{P}}_j}(\breve{S}_i)\}\rangle, & \text{if } \breve{S}_i \text{ is a CC.} \end{cases} \tag{4.6}$$

Step 5: Obtain the correlation coefficients $\rho(\breve{P}_j, \tilde{\breve{P}}^{-})$ and $\rho(\breve{P}_j, \tilde{\breve{P}}^{+})$ based on the IFCMs.
Step 6: Determine the closeness coefficients, $\Delta_j(\breve{P}_j)$ by using (4.7):

$$\Delta_j(\breve{P}_j) = \frac{\rho(\breve{P}_j, \tilde{\breve{P}}^{+})}{\rho(\breve{P}_j, \tilde{\breve{P}}^{+}) + \rho(\breve{P}_j, \tilde{\breve{P}}^{-})}, \tag{4.7}$$

for $j = 1, \cdots, n$. For the case of correlation values defined in $[-1, 1]$, we first compute

$$\Delta_j^{+}(\breve{P}_j) = \frac{\rho(\breve{P}_j, \tilde{\breve{P}}^{+}) - \rho_{\min}(\breve{P}_j, \tilde{\breve{P}}^{+})}{\rho_{\max}(\breve{P}_j, \tilde{\breve{P}}^{+}) - \rho_{\min}(\breve{P}_j, \tilde{\breve{P}}^{+})}, \tag{4.8}$$

$$\Delta_j^{-}(\breve{P}_j) = \frac{\rho(\breve{P}_j, \tilde{\breve{P}}^{-}) - \rho_{\min}(\breve{P}_j, \tilde{\breve{P}}^{-})}{\rho_{\max}(\breve{P}_j, \tilde{\breve{P}}^{-}) - \rho_{\min}(\breve{P}_j, \tilde{\breve{P}}^{-})}, \tag{4.9}$$

before calculating the closeness coefficients, $\Delta_j(\breve{P}_j)$ as follows:

$$\Delta_j(\breve{P}_j) = \frac{\Delta_j^{+}(\breve{P}_j)}{\Delta_j^{+}(\breve{P}_j) + \Delta_j^{-}(\breve{P}_j)}. \tag{4.10}$$

Step 7: Choose the largest closeness coefficient for the triage process.

Following the algorithm (i.e., Steps 1 and 2), the IFDM is obtained as presented in Table 9, and the CC is $\breve{S}_1$. By Step 3, the normalized IFDM values are obtained as presented in Table 12.

Table 12. Normalized IFDM.

Patients	$\breve{S}_1$	$\breve{S}_2$	$\breve{S}_3$	$\breve{S}_4$	$\breve{S}_5$	$\breve{S}_6$	$\breve{S}_7$
$\breve{P}_1$	$(\frac{13}{30}, \frac{7}{15})$	$(\frac{17}{30}, \frac{7}{20})$	$(\frac{7}{10}, \frac{7}{30})$	$(\frac{2}{3}, \frac{17}{60})$	$(\frac{11}{15}, \frac{1}{6})$	$(\frac{2}{5}, \frac{11}{20})$	$(\frac{29}{60}, \frac{13}{30})$
$\breve{P}_2$	$(\frac{41}{60}, \frac{1}{4})$	$(\frac{19}{30}, \frac{4}{15})$	$(\frac{3}{5}, \frac{1}{3})$	$(\frac{2}{3}, \frac{13}{60})$	$(\frac{19}{60}, \frac{3}{5})$	$(\frac{8}{15}, \frac{7}{20})$	$(\frac{8}{15}, \frac{23}{60})$
$\breve{P}_3$	$(\frac{31}{60}, \frac{2}{5})$	$(\frac{11}{15}, \frac{11}{60})$	$(\frac{5}{6}, \frac{7}{60})$	$(\frac{3}{5}, \frac{19}{60})$	$(\frac{13}{30}, \frac{1}{2})$	$(\frac{7}{10}, \frac{1}{5})$	$(\frac{1}{15}, \frac{1}{4})$
$\breve{P}_4$	$(\frac{3}{4}, \frac{1}{5})$	$(\frac{2}{3}, \frac{1}{4})$	$(\frac{1}{2}, \frac{5}{12})$	$(\frac{1}{2}, \frac{23}{60})$	$(\frac{1}{2}, \frac{2}{5})$	$(\frac{7}{10}, \frac{7}{30})$	$(\frac{19}{30}, \frac{4}{15})$
$\breve{P}_5$	$(\frac{39}{60}, \frac{17}{60})$	$(\frac{23}{30}, \frac{3}{20})$	$(\frac{31}{60}, \frac{5}{12})$	$(\frac{1}{2}, \frac{5}{12})$	$(\frac{8}{15}, \frac{23}{60})$	$(\frac{19}{30}, \frac{20}{75})$	$(\frac{3}{5}, \frac{21}{60})$
$\breve{P}_6$	$(\frac{17}{30}, \frac{7}{20})$	$(\frac{1}{2}, \frac{5}{12})$	$(\frac{17}{30}, \frac{19}{60})$	$(\frac{17}{30}, \frac{7}{20})$	$(\frac{23}{30}, \frac{1}{6})$	$(\frac{7}{10}, \frac{1}{5})$	$(\frac{17}{30}, \frac{19}{60})$

By following Step 4, we get Table 13.

Table 13. $\tilde{\breve{P}}^{+}$ and $\tilde{\breve{P}}^{-}$ results.

Patients	$\breve{S}_1$	$\breve{S}_2$	$\breve{S}_3$	$\breve{S}_4$	$\breve{S}_5$	$\breve{S}_6$	$\breve{S}_7$
$\tilde{\breve{P}}^{+}$	$(\frac{13}{30}, \frac{7}{15})$	$(\frac{23}{30}, \frac{3}{20})$	$(\frac{5}{6}, \frac{7}{60})$	$(\frac{2}{3}, \frac{13}{60})$	$(\frac{23}{30}, \frac{1}{6})$	$(\frac{7}{10}, \frac{1}{5})$	$(\frac{1}{15}, \frac{1}{4})$
$\tilde{\breve{P}}^{-}$	$(\frac{3}{4}, \frac{1}{5})$	$(\frac{1}{2}, \frac{5}{12})$	$(\frac{1}{2}, \frac{5}{12})$	$(\frac{1}{2}, \frac{5}{12})$	$(\frac{19}{60}, \frac{3}{5})$	$(\frac{2}{5}, \frac{11}{20})$	$(\frac{29}{60}, \frac{13}{30})$

Now, we use Step 5 and get the following results:

$$\rho_a(\tilde{\breve{P}}^{+}, \breve{P}_1) = 0.9977, \rho_a(\tilde{\breve{P}}^{+}, \breve{P}_2) = 0.9960, \rho_a(\tilde{\breve{P}}^{+}, \breve{P}_3) = 0.9986,$$

$$\rho_a(\check{\tilde{P}}^+, \check{P}_4) = 0.9969, \rho_a(\check{\tilde{P}}^+, \check{P}_5) = 0.9975, \rho_a(\check{\tilde{P}}^+, \check{P}_6) = 0.9982,$$
$$\rho_a(\check{\tilde{P}}^-, \check{P}_1) = 0.9964, \rho_a(\check{\tilde{P}}^-, \check{P}_2) = 0.9961, \rho_a(\check{\tilde{P}}^-, \check{P}_3) = 0.9951,$$
$$\rho_a(\check{\tilde{P}}^-, \check{P}_4) = 0.9944, \rho_a(\check{\tilde{P}}^-, \check{P}_5) = 0.9953, \rho_a(\check{\tilde{P}}^-, \check{P}_6) = 0.9946.$$

$$\rho_b(\check{\tilde{P}}^+, \check{P}_1) = 0.9984, \rho_b(\check{\tilde{P}}^+, \check{P}_2) = 0.9973, \rho_b(\check{\tilde{P}}^+, \check{P}_3) = 0.9990,$$
$$\rho_b(\check{\tilde{P}}^+, \check{P}_4) = 0.9978, \rho_b(\check{\tilde{P}}^+, \check{P}_5) = 0.9982, \rho_b(\check{\tilde{P}}^+, \check{P}_6) = 0.9987,$$
$$\rho_b(\check{\tilde{P}}^-, \check{P}_1) = 0.9975, \rho_b(\check{\tilde{P}}^-, \check{P}_2) = 0.9973, \rho_b(\check{\tilde{P}}^-, \check{P}_3) = 0.9966,$$
$$\rho_b(\check{\tilde{P}}^-, \check{P}_4) = 0.9961, \rho_b(\check{\tilde{P}}^-, \check{P}_5) = 0.9968, \rho_b(\check{\tilde{P}}^-, \check{P}_6) = 0.9962.$$

Next, we compute Δ_j for ρ_a and ρ_b; the results are listed in Table 14.

Table 14. Closeness coefficients, $\Delta_j(\check{P}_j)$ for $j = 1, 2, \cdots, 6$.

Iterations	$\Delta_j(\check{P}_j)$ for ρ_a	Ranking for ρ_a	$\Delta_j(\check{P}_j)$ for ρ_b	Ranking for ρ_b
1	0.50032	5	0.50022	5
2	0.49998	6	0.49999	6
3	0.50088	2	0.50061	2
4	0.50062	3	0.50043	3
5	0.50054	4	0.50037	4
6	0.50090	1	0.50062	1

From the results in Table 14, we can state that patients $\check{P}_6$ and $\check{P}_3$ should be given urgent medical attention to avoid death if there is a limited number of consultants.

4.2.1. Comparative analysis under approach II

Next, we show the superiority of the new IFCMs via comparative analysis. By following Step 5 of the MCDM algorithm, we get the information in Table 15, which is illustrated in Figure 2.

Table 15. Results for $\rho(\tilde{\breve{P}}^{+}, \breve{P}_j)$, $j = 1, 2, \cdots, 6$.

Measures	$(\tilde{\breve{P}}^{+}, \breve{P}_1)$	$(\tilde{\breve{P}}^{+}, \breve{P}_2)$	$(\tilde{\breve{P}}^{+}, \breve{P}_3)$	$(\tilde{\breve{P}}^{+}, \breve{P}_4)$	$(\tilde{\breve{P}}^{+}, \breve{P}_5)$	$(\tilde{\breve{P}}^{+}, \breve{P}_6)$
ρ_1 [28]	0.9471	0.9066	0.9681	0.9277	0.9416	0.9594
ρ_2 [31]	0.5013	0.6032	0.6573	0.7250	0.7239	0.6689
ρ_3 [39]	0.9468	0.9077	0.9684	0.9279	0.9419	0.9590
ρ_4 [38]	1.7486	1.6545	1.8593	1.7137	1.7383	1.7584
ρ_5 [33]	0.2181	0.6153	0.7110	0.4789	0.5854	0.3528
ρ_6 [36]	1.5012	1.4672	1.7428	1.4008	1.4187	1.4974
ρ_7 [37]	1.5280	1.4697	1.7485	1.4146	1.4060	1.5348
ρ_8 [32]	0.5087	0.6002	0.6570	0.7262	0.7238	0.6731
ρ_9 [35]	0.5023	0.6002	0.6572	0.7215	0.7212	0.6668
ρ_{10} [30]	0.9458	0.9237	1.0399	0.8982	0.9139	0.9460
ρ_{11} [40]	0.9608	0.9465	1.0272	0.9272	0.9379	0.9621
ρ_{12} [41]	0.9629	0.9488	1.0266	0.9446	0.9441	0.9661
ρ_a	0.9977	0.9960	0.9986	0.9969	0.9975	0.9982
ρ_b	0.9984	0.9973	0.9990	0.9978	0.9982	0.9987

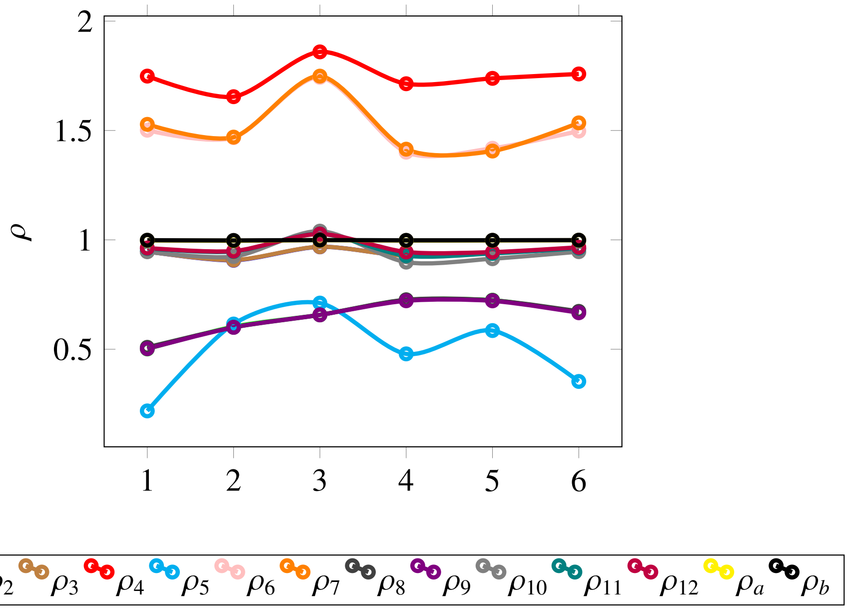

Figure 2. Illustration of the results presented in Table 15.

Similarly, we can obtain the results presented in Table 16, which are illustrated in Figure 3.

Table 16. Results for $\rho(\tilde{\breve{P}}^{-}, \breve{P}_j)$, $j = 1, 2, \cdots, 6$.

Measures	$(\tilde{\breve{P}}^{-}, \breve{P}_1)$	$(\tilde{\breve{P}}^{-}, \breve{P}_2)$	$(\tilde{\breve{P}}^{-}, \breve{P}_3)$	$(\tilde{\breve{P}}^{-}, \breve{P}_4)$	$(\tilde{\breve{P}}^{-}, \breve{P}_5)$	$(\tilde{\breve{P}}^{-}, \breve{P}_6)$
ρ_1 [28]	0.9055	0.8964	0.8769	0.8518	0.8768	0.8557
ρ_2 [31]	−0.2972	−0.2501	−0.2177	−0.7211	−0.6205	−0.9690
ρ_3 [39]	0.9060	0.8970	0.8777	0.8536	0.8777	0.8570
ρ_4 [38]	1.6352	1.6190	1.5840	1.5406	1.5841	1.5467
ρ_5 [33]	−0.1851	−0.1799	−0.3329	−0.2028	−0.5144	−0.6315
ρ_6 [36]	1.5473	1.5726	1.2443	1.5423	1.5017	1.4681
ρ_7 [37]	1.5649	1.5786	1.3483	1.5521	1.5342	1.4598
ρ_8 [32]	−0.2993	−0.2504	−0.2170	−0.7265	−0.6210	−0.9732
ρ_9 [35]	−0.2976	−0.2496	−0.2181	−0.7202	−0.6199	−0.9687
ρ_{10} [30]	0.9591	0.9881	0.8160	0.9652	0.9301	0.9043
ρ_{11} [40]	0.9710	0.9901	0.8745	0.9752	0.9517	0.9312
ρ_{12} [41]	0.9702	0.9920	0.8828	0.9775	0.9544	0.9360
ρ_a	0.9964	0.9961	0.9951	0.9944	0.9953	0.9946
ρ_b	0.9975	0.9973	0.9966	0.9961	0.9968	0.9962

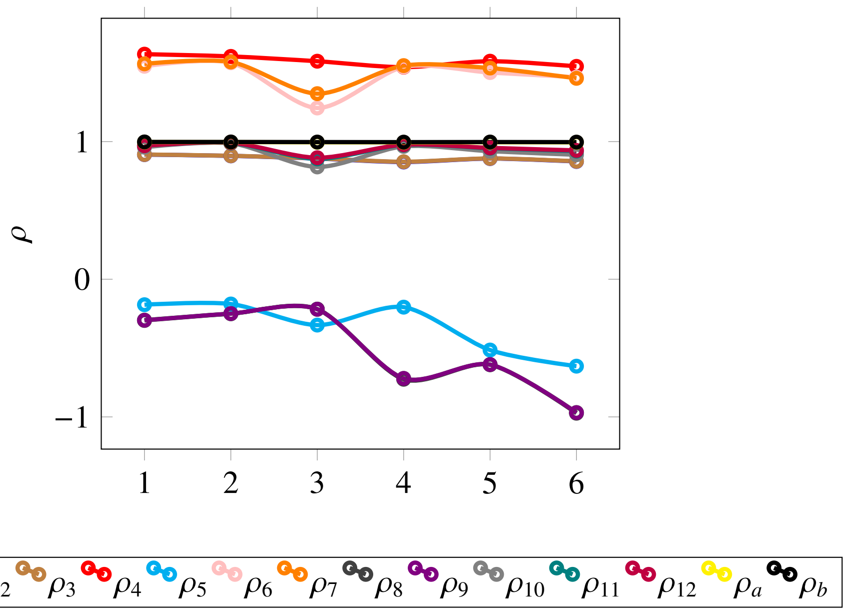

Figure 3. Illustration of the results presented in Table 16.

Comparison of the results in Tables 15 and 16 reveals that the new IFCMs yield the most precise results, and that ρ_4, ρ_6, ρ_7, ρ_{10}, ρ_{11}, and ρ_{12} give results that are not defined within the scope of correlation coefficient values.

Next, we computed the closeness coefficients for the correlation coefficients. Because ρ_4 [38], ρ_6 [36], ρ_7 [37], ρ_{10} [30], ρ_{11} [40], and ρ_{12} [41] in Tables 15 and 16 yielded correlation values that were not within [0, 1] and [−1, 1], we excluded them from the computations of the closeness coefficients.

Using the information in Tables 15 and 16, we obtained the closeness coefficients as shown in Table

17, as well as and their ordering, as shown in Table 18.

Table 17. Closeness coefficients.

Measures	$\Delta_1(\check{P}_1)$	$\Delta_2(\check{P}_2)$	$\Delta_3(\check{P}_3)$	$\Delta_4(\check{P}_4)$	$\Delta_5(\check{P}_5)$	$\Delta_6(\check{P}_6)$
ρ_1 [28]	0.51125	0.50282	0.52474	0.52133	0.51781	0.52858
ρ_2 [31]	0.0000	0.32246	0.41092	0.75189	0.68210	1.0000
ρ_3 [39]	0.51100	0.50294	0.52458	0.52085	0.51764	0.52808
ρ_5 [33]	0.0000	0.44625	0.60194	0.35789	0.74190	1.0000
ρ_8 [32]	0.0000	0.30555	0.40538	0.75400	0.67979	1.0000
ρ_9 [35]	0.0000	0.31781	0.41394	0.75125	0.68238	1.0000
ρ_a	0.50032	0.49998	0.50088	0.50062	0.50054	0.50090
ρ_b	0.50022	0.49999	0.50061	0.50043	0.50037	0.50062

From Table 18, we infer that patient $\check{P}_6$ had the most critical medical case that required urgent medical attention. Because the new measures have been shown to be the most reliable IFCMs, it is necessary to state that the patients should be queued as in the following order: $\check{P}_6$, $\check{P}_3$, $\check{P}_4$, $\check{P}_5$, $\check{P}_1$, and $\check{P}_2$, respectively, for emergency treatment.

Table 18. Closeness coefficient ordering results.

Measures	Orderings
ρ_1 [28]	$\Delta_6(\check{P}_6) \succ \Delta_3(\check{P}_3) \succ \Delta_4(\check{P}_4) \succ \Delta_5(\check{P}_5) \succ \Delta_1(\check{P}_1) \succ \Delta_2(\check{P}_2)$
ρ_2 [31]	$\Delta_6(\check{P}_6) \succ \Delta_4(\check{P}_4) \succ \Delta_5(\check{P}_5) \succ \Delta_3(\check{P}_3) \succ \Delta_2(\check{P}_2) \succ \Delta_1(\check{P}_1)$
ρ_3 [39]	$\Delta_6(\check{P}_6) \succ \Delta_3(\check{P}_3) \succ \Delta_4(\check{P}_4) \succ \Delta_5(\check{P}_5) \succ \Delta_2(\check{P}_2) \succ \Delta_1(\check{P}_1)$
ρ_5 [33]	$\Delta_6(\check{P}_6) \succ \Delta_5(\check{P}_5) \succ \Delta_3(\check{P}_3) \succ \Delta_2(\check{P}_2) \succ \Delta_4(\check{P}_4) \succ \Delta_1(\check{P}_1)$
ρ_8 [32]	$\Delta_6(\check{P}_6) \succ \Delta_4(\check{P}_4) \succ \Delta_5(\check{P}_5) \succ \Delta_3(\check{P}_3) \succ \Delta_2(\check{P}_2) \succ \Delta_1(\check{P}_1)$
ρ_9 [35]	$\Delta_6(\check{P}_6) \succ \Delta_4(\check{P}_4) \succ \Delta_5(\check{P}_5) \succ \Delta_3(\check{P}_3) \succ \Delta_2(\check{P}_2) \succ \Delta_1(\check{P}_1)$
ρ_a	$\Delta_6(\check{P}_6) \succ \Delta_3(\check{P}_3) \succ \Delta_4(\check{P}_4) \succ \Delta_5(\check{P}_5) \succ \Delta_1(\check{P}_1) \succ \Delta_2(\check{P}_2)$
ρ_b	$\Delta_6(\check{P}_6) \succ \Delta_3(\check{P}_3) \succ \Delta_4(\check{P}_4) \succ \Delta_5(\check{P}_5) \succ \Delta_1(\check{P}_1) \succ \Delta_2(\check{P}_2)$

5. Conclusions

Uncertainty in medical diagnoses is a fundamental problem that is faced by triage nurses and physicians. In this study, a novel approach was developed for the triage process, and it involves the use of new IFCMs via Spearman's correlation coefficient approach to eliminate all possible uncertainties that may prevent the user from obtaining reliable triage results. To justify the establishment of new IFCMs, various existing IFCMs were investigated and their shortcomings were identified. Furthermore, all of the extant IFCMs were established based on the classical Pearson correlation coefficient approach, and none were constructed by using the Spearman's correlation coefficient. Because of this oversight, new IFCMs were constructed based on the classical Spearman's correlation coefficient, and we have shown how the new measures overcame all of the limitations of the extant measures. To verify the validity of the new measures, some theoretical results were proved, which were found to satisfy the conditions of the correlation coefficient. Because of the ease of use of the developed

IFCMs, we applied them to solve the problem of triage processes in a typical ED to eliminate medical diagnostic uncertainty. The data for the analysis were obtained via a knowledge-based system, where the symptoms of the considered ailments were apportioned linguistic variables with corresponding IFNs. To unequivocally show the merits of the new correlation coefficient models, we compared the new IFCC methods with 12 extant IFCC methods [28, 30–33, 35–41]. It was observed that the new IFCC methods are the most reliable, consistent and precise, and that sufficiently satisfy the conditions of the correlation coefficient. This new triage process based on IFCMs can conveniently manage all of the uncertainties associated with an ME. However, the developed IFCMs could only function in an environment in which the sum of the MD and NMD is at most one. The novel IFCMs are restricted, because they cannot be directly applied to other settings with higher fuzziness like the Pythagorean fuzzy setting [53], Fermatean fuzzy setting [54], q-rung orthopair fuzzy setting [55], complemental fuzzy setting [56], etc. without alterations since the new measures were not developed to consider the properties of the aforementioned settings. The new IFCMs and the novel application should be investigated in other fuzzy environments for further research.

Author contributions

P. A. Ejegwa: conceptualization, methodology, software, and writing – original draft; N. Kausar: writing – review & editing, supervision, funding acquisition, and validation; J. A. Agba: data curation, visualization, and writing – review & editing; F. Ugwuh: resources, writing – review & editing, and validation; E. Özbilge: funding acquisition, supervision, and visualization; E. Ozbilge: supervision, visualization, writing – review & editing, and funding acquisition.

Use of AI tools declaration

The authors declare they do not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that they have no competing interests.

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