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Research article

Modeling uncertainties associated with multi-attribute decision-making based evaluation of cooling system using interval-valued complex intuitionistic fuzzy hypersoft settings

Muhammad Arshad[1], Muhammad Saeed[1], Atiqe Ur Rahman[1], Sanaa A. Bajri[2,*], Haifa Alqahtani[3] and Hamiden Abd El-Wahed Khalifa[4,5]

[1] Department of Mathematics, University of Management and Technology, Lahore 54000, Pakistan

[2] Department of Mathematical Sciences, College of Science, Princess Nourah bint Abdulrahman University, P.O. Box 84428, Riyadh 11671, Saudi Arabia

[3] Department of Statistics and Business Analytics, United Arab Emirates University, UAE

[4] Department of Mathematics, College of Science, Qassim University, Buraydah, 51452, Saudi Arabia

[5] Department of Operations and Management Research, Faculty of Graduate Studies for Statistical Research, Cairo University, Giza 12613, Egypt

* **Correspondence:** Email: sabajri@pnu.edu.sa.

Abstract: Academics encounter a challenge regulating data-driven unpredictability in numerous complicated decision scenarios. Regulating the cyclical nature of appraisal attributes, determining lower and higher limitations, granting multi-parametric values as a means of assessing argumentation, and modeling uncertainty are a few examples of these problems. It requires the incorporation of complex plane settings, interval-valued intuitionistic fuzzy settings, and hypersoft settings. Inspired by these kinds of scenarios, the goal of this research was to articulate a new theoretical framework, the interval-valued complex intuitionistic fuzzy hypersoft set (Γ-set), which can handle these kinds of problems as a whole under the umbrella of a single framework. First, the concepts of Γ-set, as well as its set operations and aggregations, such as decision matrix, cardinal matrix, aggregate matrix, and cardinality set, were examined. The second phase offers an appealing algorithm that consists of nine steps that go from taking into account necessary set construction to making the best choice. A prototype case study analyzing eighteen evaluation qualities and thirty-four sub-attributes for determining an optimal cooling system ($\mathbb{CSYS}$) for a factory validates the provided algorithm. Informative comparison analysis and preferred study features were provided as essential components of research to assist academics in making significant advances regarding their field and gradually, but thoroughly, advancing their specialization.

Keywords: complex fuzzy set; interval-valued fuzzy set; complex fuzzy soft set; hypersoft set; decision making; optimization
Mathematics Subject Classification: 03E72, 68T35, 90B50

1. Introduction

To guarantee ideal conditions for work and ensure efficient operation, a factory's cooling system ($\mathbb{CSYS}$) is crucial. It assists in controlling temperature in a variety of manufacturing processes, lowering the chance of machine failure or collapse and minimizing machines getting too hot. A $\mathbb{CSYS}$'s ability to regulate temperature also protects the integrity of goods being manufactured, which matters in industries where exact circumstances are critical, such as food processing and pharmaceutical. Furthermore, a properly operating $\mathbb{CSYS}$ improves worker well-being and security by reducing heat-related discomfort and fostering a more favorable workplace. All things considered, the significance of a $\mathbb{CSYS}$ in a factory cannot be emphasized because it significantly impacts employee well-being, profitability, and quality of goods [1–3]. Identifying a $\mathbb{CSYS}$ for business or commercial purposes is a complex procedure that requires careful consideration of many factors to make the best decision. The decision-making ($\mathbb{DMK}$) process heavily weighs a variety of factors, including the project's unique cooling specifications, energy consumption, ecological impact, startup and ongoing expenses, accessibility, and servicing considerations. The selection gets exacerbated by factors such as the $\mathbb{CSYS}$'s adaptability to handle potential expansion or variations of usage and its suitability with the current setup. To choose the best course of action, decisions must frequently be made because every consideration has a distinct significance and value. As a result, choosing a $\mathbb{CSYS}$ necessitates an in-depth evaluation that considers numerous functioning, technological, and financial aspects, making it a multi-criteria decision-making ($\mathbb{MCDM}$) process by nature [4–6]. Because of several variables, such as prospective cooling specifications, socioeconomic position, accessibility to resources, and concerns about technical improvements, choosing the right $\mathbb{CSYS}$ for corporate or commercial use is occasionally challenging [7,8]. So far, numerous scientific endeavors have been documented about the application of statistical frameworks and models to manage uncertainties concerning the information.

Interval-valued intuitionistic fuzzy set ($\mathbb{I}_v\mathbb{IFS}$) [9] is included in the list of such frameworks for dealing with informational ambiguities. The $\mathbb{I}_v\mathbb{IFS}$ provides a more adaptable and realistic context for dealing with imprecise and uncertain information, resulting in more accurate and significant decision outputs. It is projected to address the limitations of the fuzzy set ($\mathbb{FS}$) [10], interval-valued fuzzy set ($\mathbb{I}_v\mathbb{FS}$) [11–14], and intuitionistic fuzzy set ($\mathbb{IFS}$) [15] by providing the interval-valued membership and non-membership grades. In the scenario with a great deal of data, the decision-makers can convey their reluctance or lack of confidence in assigning accurate membership function ($\mathbb{MF}_n$) and non-membership function ($\mathbb{NF}_n$) by using $\mathbb{I}_v\mathbb{IFS}$s, which offer a more subtle and thorough representation of uncertainty. To deal with the periodicity of data, complex $\mathbb{I}_v\mathbb{IFS}$ [16] has been initiated as an extension of complex $\mathbb{FS}$ [17] and complex $\mathbb{IFS}$ [18]. Recently, Das et al. [19], Batool et al. [20], and Mani et al. [21] discussed the uncertainty quantification using different analytical and abstract approaches based on the ideas of $\mathbb{FS}$-like extensions. Zhang et al. [22] and Bai et al. [23] provided robust strategies to manage uncertainties associated with multi-granularity behavioral $\mathbb{DMK}$. Recently, a hypersoft set

($\mathbb{HSS}$) [24, 25] has been put forward as an extension of the soft set ($\mathbb{SS}$) [26] to handle uncertainty in a parametric way. Consequently, the hybrid set structures [27–33] have been put forward in hypersoft settings by extending the concepts [34–39].

1.1. Relevant literature

The problem has been creating a challenge for the scholars on how to control the issues like the redundancy of different factors in the information and consideration of multi-attribute based domain collectively. In this regard, Saeed et al. [40, 41] employed complex fuzzy $\mathbb{HSS}$ algorithmic approaches to the evaluation of solid wastage management strategies and procurement techniques. Similarly, using the same environment, Rahman et al. [42–44] discussed the susceptibility analysis for liver disorder, and other $\mathbb{MCDM}$ issues by proposing theoretical cum algorithmic strategies. Ying et al. [45] used the extensive idea of complex fuzzy $\mathbb{HSS}$ in $\mathbb{MCDM}$. Das and Samanta [46], Thirunavukarasu [47], Kumar and Bajaj [48], and Khan [49] employed $\mathbb{SS}$-environment to manage such informational repetition of evaluating features using complex plane settings. As the interval settings provide a broad domain for modeling a great deal of data with the identification of lower and upper limits, Selvachandran and Singh [50] employed an integrated context of $\mathbb{SS}$-environment and interval-valued settings to cope with such periodicity of features.

The theoretical frameworks like $\mathbb{FS}$, $\mathbb{IFS}$, $\mathbb{I}_v\mathbb{FS}$, $\mathbb{I}_v\mathbb{IFS}$, $\mathbb{SS}$, and $\mathbb{HSS}$ are considered as crucial tools in the selection of factory $\mathbb{CSYS}$s because it takes into account the inherent uncertainties and imprecisions that occur during $\mathbb{DMK}$. Energy efficiency, cost-effectiveness, environmental impact, and technical standards are just a few of the many considerations to take into account when choosing the best $\mathbb{CSYS}$. These structures take into account the inconsistencies and obscurity that result from inadequate knowledge or subjective viewpoints, enabling managers to describe these aspects and their connections in a more sophisticated manner. Managers can assess various cooling system solutions more thoroughly by utilizing fuzzy logic-based models, which take into account both statistical data and qualitative perspectives. This method makes it possible to have a more thorough $\mathbb{DMK}$ process that considers a range of variables and their relative weights, which finally results in the choice of the cooling system that best suits the unique requirements and limitations of the manufacturing.

As it has been reiterated in the previous section, there are many uncertainties and ambiguities in the selection of $\mathbb{CSYS}$, which makes the process very complex and difficult. In order to deal with such situations, many scholars have used fuzzy logic and related concepts, which have played a better role. Albahri et al. [51] evaluated $\mathbb{CSYS}$ using a fuzzy $\mathbb{MCDM}$ approach with multiple perspective integration. Kiran and Rajput [52] compared three fuzzy logic based strategies with the optimization of $\mathbb{CSYS}$. Aprea et al. [53] employed the idea of fuzzy control to optimize $\mathbb{CSYS}$. Martinez-Molina and Alamaniotis [54] employed a fuzzy inference system to evaluate the $\mathbb{CSYS}$ in the $\mathbb{MCDM}$ scenario.

1.2. Research gap and motivation

Upon reading the aforementioned paragraphs, it is evident that no reference has been made in the literature so far that looks into choosing a $\mathbb{CSYS}$ based on the criteria listed below:

(1) **Proper arrangement for modeling uncertainties:** To accurately represent and assess the unpredictability implicit in the $\mathbb{DMK}$ process, it requires incorporating multiple approaches. The managers will be able to build solid simulations that take into consideration the complex nature

of ambiguities in $\mathbb{CSYS}$ evaluation by integrating various techniques, which will eventually result in more knowledgeable and trustworthy judgments.

(2) **Entitlement of multi-attribute settings:** It describes the identification and evaluation of several factors or characteristics that are pertinent to the $\mathbb{DMK}$ process. This method recognizes that the selection of $\mathbb{CSYS}$ requires more than one criterion. Experts can take into consideration the various frequently contradictory goals that emerge during the assessment of $\mathbb{CSYS}$s by using a multi-attribute paradigm. This makes it possible to conduct a more thorough analysis that weighs the trade-offs between various characteristics and, in the end, selects a $\mathbb{CSYS}$ that best suits the unique requirements and objectives of the institution or organization.

(3) **Consideration of multi-argument domain:** It entails accounting for a wide range of variables and contributing factors that affect the system's efficacy and productivity. This method acknowledges that the assessment procedure involves an intricate relationship of components that go in addition to straightforward standards. About certain operative situations and targets, this permits a comprehensive awareness of the system's capacities and constraints, permitting intelligent choices that optimize effectiveness, adaptability, and longevity.

(4) **Proper arrangement for modeling data periodicity:** It entails putting strategies into practice to take into consideration the recurring pattern of data trends across time. To address such repetitive patterns, this incorporates the complex plane settings that involve amplitude and phase factors. The experts may forecast prospective system functionality with confidence, spot any anomalies or inefficiencies, and adjust scheduled repairs and operating tactics appropriately by knowing the recurring behavior of these influencing factors. Furthermore, flexible modeling that accounts for variations in frequency over time is made possible by the integration of historical data with real-time monitoring. This guarantees that $\mathbb{CSYS}$ evaluations continue to be correct and pertinent even in dynamic operational contexts.

(5) **Entitlement of interval-valued settings:** Instead of using a single, exact value to express parameters and factors, it enables the experts to use a range of feasible outcomes. Under these circumstances, decision-makers can carry out more thorough analyses that take into consideration the entire gamut of probable results while making well-informed choices that take operational modifications and unpredictability into consideration.

It is evident from the description above how crucial the aforementioned elements are to arriving at the best choice. Based on the findings presented in the literature currently in publication, it can be said that researchers have either ignored or just partially considered these aspects. However, none has made use of them simultaneously. Even though including all of them could make the computations far more complicated, it is still possible to have a more trustworthy and superior $\mathbb{DMK}$ approach. Thus, with all of these considerations in mind, the purpose of this study is to talk about $\mathbb{CSYS}$ optimization using a contemporary mathematical framework called the interval-valued complex intuitionistic fuzzy hypersoft set (Γ-set) which incorporates interval-valued ($\mathbb{IV}$)-settings, $\mathbb{MF}_n$, and $\mathbb{NF}_n$ with complex numbers and offers a solid foundation to characterize and quantify uncertainty. Decision-makers are able to do this by expressing and capturing the inherent ambiguity and variety in choice scenarios. The $\mathbb{DMK}$ frequently entails taking into account a number of qualities or standards. The Γ-sets, which can accommodate $\mathbb{IV}$ $\mathbb{MF}_n$, $\mathbb{NF}_n$, make it easier to combine and aggregate many features. This offers a thorough assessment of options based on several features while taking into account their intricate interactions. The $\mathbb{DMK}$ frequently involves stakeholders with divergent viewpoints or desires. The

Γ-sets offer a versatile framework to address such disputes. Conflicts can be resolved or managed in this way, resulting in decisions that are more fair and driven by agreement. By taking into account both the real and imaginary parts of complex numbers, Γ-sets provide a thorough examination of choice scenarios. Incorporating phase information, oscillatory behavior, and intricate interactions between variables is therefore made possible, resulting in a more precise and sophisticated decision analysis. The Γ-sets offer a well-structured framework for weighing options, making them an effective decision-assistant tool.

To put it briefly, Γ-sets offer the experts a more complete and adaptable basis for handling complicated and ambiguous decision eventualities by incorporating various qualities, mitigating imprecision and reluctance, and providing a more precise and in-depth analysis of the $\mathbb{DMK}$ challenge. Overall, the Γ-sets improve $\mathbb{DMK}$ by providing an extensive structure for possibility assessment, allowing complex connections, and eliminating ambiguity. Specifically in complex $\mathbb{DMK}$ situations involving imperfection and inconsistencies, they enable the managers to reach better-informed and reliable opinions.

1.3. Prominent contributions

Here, the prominent contributions of the study have been highlighted:

(1) The suggested framework Γ-set is thought to be the most effective way to investigate the intricacies of $\mathbb{CSYS}$ assessment. Its ability to manage a wider range of membership grades is an advantage, particularly when working with attributes that have several sub-values. By far, the most effective approach to investigate the area of $\mathbb{CSYS}$ assessment is through the structure itself. Their ability to cater to a broad spectrum of membership subclasses is what makes them successful, especially when handling attributes with several sub-values and periodicity of features.

(2) Expert appraisals are a helpful instrument for showcasing practical applications and showing the significance of the recommended course of action. The assessments and insights of subject-matter professionals are used to show how the technique can be effectively applied in real-world scenarios. The paper provides expert opinions for approximating $\mathbb{CSYS}$s in terms of complex-valued intervals with lower and upper contexts.

(3) The notions of decision matrix, cardinality and its set, cardinal matrix, and aggregate matrix are among the aggregations of the Γ-set that are examined to construct an adaptive decision-assisted mechanism.

(4) Based on aggregations and its relevant aggregate matrices, a robust algorithm is put forward that includes nine steps starting from the consideration of essential set construction to optimal decision. The presented algorithm is validated by a prototype case study considering eighteen evaluating attributes and thirty-four sub-attributes to select an optimized $\mathbb{CSYS}$ for a factory.

(5) As the cornerstone of scientific inquiry, concise comparison analysis and preferential aspects of the study are offered to help academics make major improvements to their subject and rigorously and methodically progress expertise.

There are three sections in the remaining paper. In Section 2, the fundamental information is provided, along with explanations of key terminologies. Section 3 presents the concepts of the Γ-set and its set operations, the aggregations of the Γ-set such as decision matrix, cardinality and its set, cardinal matrix, and aggregate matrix, the decision-assisted mechanism including the algorithm proposal and case study validation, and the comparison analysis. The investigation is finally concluded in Section 4 with descriptions of the summary, constraints, and future scope.

2. Preliminaries

In this study, $\mathbb{P}(\mathbb{U})$ will represent the power set of the universal set $\mathbb{U}$, $\Im(\mathscr{I})$ will be taken as the set of all subintervals of unit closed interval $\mathscr{I}$, and ϖ is $[0, 2\pi]$.

The concept of $\mathbb{IFS}$ was proposed by Atanassov [15] to adequately compensate the $\mathbb{FS}$ with a non-membership grade entitlement. The total of the membership and non-membership grades in $\mathbb{IFS}$ falls between [0,1].

Definition 2.1. *[15] A set $\mathbb{B}$ is an $\mathbb{IFS}$ if $\mathbb{B}$ is characterized by a M_{fn}: $\mathbb{B}_m$ and N_{fn}: $\mathbb{B}_n$, and defined as*

$$\mathbb{B} = \{(\check{\curlyvee}, \langle \mathbb{B}_m(\check{\curlyvee}), \mathbb{B}_n(\check{\curlyvee})\rangle), \check{\curlyvee} \in \mathbb{U}\},$$

where $\mathbb{B}_m(\check{\curlyvee}), \mathbb{B}_n(\check{\curlyvee}) \in I$ and $0 \leq \mathbb{B}_m(\check{\curlyvee}) + \mathbb{B}_n(\check{\curlyvee}) \leq 1$ with hesitancy grade

$$\mathbb{B}_h(\check{\curlyvee}) = 1 - \mathbb{B}_m(\check{\curlyvee}) - \mathbb{B}_n(\check{\curlyvee}).$$

Atanassov [9] subsequently broadened his concept of $\mathbb{IFS}$ to $\mathbb{I}_v\mathbb{IFS}$ so that it may be suitable for situations where lower and upper bounds must be taken into account.

Definition 2.2. *[9] A set $\mathbb{D}$ is an $\mathbb{I}_v\mathbb{IFS}$ if $\mathbb{D}$ is characterized by a M_{fn}: $\mathbb{D}_m$ and N_{fn}: $\mathbb{D}_n$, and defined as*

$$\mathbb{D} = \{(\check{\curlyvee}, \langle \mathbb{D}_m(\check{\curlyvee}), \mathbb{D}_n(\check{\curlyvee})\rangle), \check{\curlyvee} \in \mathbb{U}\},$$

where $\mathbb{D}_m(\check{\curlyvee}), \mathbb{D}_n(\check{\curlyvee}) \in \Im(\mathscr{I})$ and $0 \leq \inf \mathbb{D}_m(\check{\curlyvee}) + \inf \mathbb{D}_n(\check{\curlyvee}) \leq \sup \mathbb{D}_m(\check{\curlyvee}) + \sup \mathbb{D}_n(\check{\curlyvee}) \leq 1$ with hesitancy grades $\mathbb{D}_h(\check{\curlyvee}) = [1 - \sup \mathbb{D}_m(\check{\curlyvee}) - \sup \mathbb{D}_n(\check{\curlyvee}), 1 - \inf \mathbb{D}_m(\check{\curlyvee}) - \inf \mathbb{D}_n(\check{\curlyvee})]$.

As an extrapolation of $\mathbb{IFS}$, Alkouri and Salleh [18] proposed the concept of complex-valued membership and non-membership grades to control the recurrence of assessing attributes in the data. Every grade is made up of phase and amplitude terms with the requirements that the total of the amplitude terms fall inside [0,1] and the total of the phase terms fall inside $[0, 2\pi]$.

Definition 2.3. *[18] A complex intuitionistic fuzzy set ($\mathbb{CIFS}$) $\mathbb{F}$ over $\mathbb{U}$ can be written as*

$$\mathbb{F}(\check{\curlyvee}) = \{(\check{\curlyvee}, \langle \mathbb{F}_m(\check{\curlyvee}), \mathbb{F}_n(\check{\curlyvee})\rangle) : \check{\curlyvee} \in \mathbb{U}\} = \left\{\left(\check{\curlyvee}, \langle A_m(\check{\curlyvee}) e^{iP_m(\check{\curlyvee})}, A_n(\check{\curlyvee}) e^{iP_n(\check{\curlyvee})}\rangle\right) : \check{\curlyvee} \in \mathbb{U}\right\}.$$

where $\mathbb{F}_m(\check{\curlyvee})$ and $\mathbb{F}_n(\check{\curlyvee})$ represent $\mathbb{MF}_n$ and $\mathbb{NF}_n$ of $\mathbb{F}(\check{\curlyvee})$, with $A_m(\check{\curlyvee}) \in \mathscr{I}$ as $\mathbb{A}$-term, and $P_m(\check{\curlyvee}) \in \varpi$ as $\mathbb{P}$-term of $\mathbb{MF}_n$, $A_n(\check{\curlyvee}) \in \mathscr{I}$ as amplitude term ($\mathbb{A}$-term), and $P_n(\check{\curlyvee}) \in \varpi$ as phase term ($\mathbb{P}$-term) of $\mathbb{NF}_n$, such that $0 \leq \mathbb{F}_m(\check{\curlyvee}) + \mathbb{F}_n(\check{\curlyvee}) \leq 1$, and the hesitancy grade is $\mathbb{F}_h(\check{\curlyvee}) = 1 - \mathbb{F}_m(\check{\curlyvee}) - \mathbb{F}_n(\check{\curlyvee})$.

The characterization of $\mathbb{IVCIFS}$ was studied by Garg and Rani [16], who combined the concept of $\mathbb{CIFS}$ with interval settings. As a result, every term in $\mathbb{CIFS}$, namely, the phase and amplitude terms, is expressed in intervals with descriptions of the upper and lower limits in $\mathbb{IVCIFS}$.

Definition 2.4. *[16] An $\mathbb{IVCIFS}$ $\mathbb{G}$ over $\mathbb{U}$ can be written as*

$$\mathbb{G}(\check{\curlyvee}) = \{(\check{\curlyvee}, \langle \mathbb{G}_m(\check{\curlyvee}), \mathbb{G}_n(\check{\curlyvee})\rangle) : \check{\curlyvee} \in \mathbb{U}\} = \left\{\left(\check{\curlyvee}, \langle A_m(\check{\curlyvee}) e^{iP_m(\check{\curlyvee})}, A_n(\check{\curlyvee}) e^{iP_n(\check{\curlyvee})}\rangle\right) : \check{\curlyvee} \in \mathbb{U}\right\},$$

where $\mathbb{G}_m(\check{\curlyvee})$ *represents* $\mathbb{MF}_n$ *of* $\mathbb{G}(\check{\curlyvee})$ *with* $A_m(\check{\curlyvee}) \in \Im(\mathscr{I})$ *as* $\mathbb{A}$*-term, and* $P_m(\check{\curlyvee}) \subseteq \varpi$ *as* $\mathbb{P}$*-term, and* $\mathbb{G}_n(\check{\curlyvee})$ *represents* $\mathbb{NF}_n$ *with* $A_n(\check{\curlyvee}) \in \Im(\mathscr{I})$ *as* $\mathbb{A}$*-term,* $P_n(\check{\curlyvee}) \subseteq \varpi$ *as* $\mathbb{P}$*-term, and* $0 \leq \inf \mathbb{G}_m(\check{\curlyvee}) + \inf \mathbb{G}_n(\check{\curlyvee}) \leq \sup \mathbb{G}_m(\check{\curlyvee}) + \sup \mathbb{G}_n(\check{\curlyvee}) \leq 1$ *with hesitancy grade*

$$\mathbb{G}_h(\check{\curlyvee}) = \left[1 - \sup \mathbb{G}_m(\check{\curlyvee}) - \sup \mathbb{G}_n(\check{\curlyvee}), 1 - \inf \mathbb{G}_m(\check{\curlyvee}) - \inf \mathbb{G}_n(\check{\curlyvee})\right].$$

Definition 2.5. *[26] A* $\mathbb{SS}$ $(\mathbb{H}, \Lambda)$ *over* $\Cup$ *is a set of order pairs such that* $\mathbb{H} : \Lambda \to \mathbb{P}(\Cup)$ *is given by*

$$(\mathbb{H}, \Lambda) = \left\{(\lambda, \mathbb{H}(\check{\curlyvee})) : \lambda \in \Lambda, \check{\curlyvee} \in \Cup, \mathbb{H}(\check{\curlyvee}) \in \mathbb{P}(\Cup)\right\}.$$

Definition 2.6. *[48] A set* $(\mathbb{M}, \Lambda)$ *is called* $\mathbb{CIFSS}$ *over* $\Cup$ *if* $\mathbb{M}$ *is a parameterized gathering of* $\mathbb{CIF}$*-subsets of* $\Cup$ *and is given by* $\mathbb{M} : \Lambda \to \mathbb{P}(\Cup)$ *and is defined by*

$$(\mathbb{M}, \Lambda) = \left\{\left(\lambda, \left\{\frac{\mathbb{M}_m(\check{\curlyvee}), \mathbb{M}_n(\check{\curlyvee})}{\check{\curlyvee}}\right\}\right) : \check{\curlyvee} \in \Cup, \lambda \in \Lambda\right\},$$

where $\mathbb{M}_m(\check{\curlyvee}) = A_m(\check{\curlyvee})\, e^{iP_m(\check{\curlyvee})}$ *represents the* $\mathbb{MF}_n$ *of* $\mathbb{M}$ *with* $A_m(\check{\curlyvee}) \in \mathscr{I}$ *as* $\mathbb{A}$*-term,* $P_m(\check{\curlyvee}) \in \varpi$ *as* $\mathbb{P}$*-term, and* $\mathbb{M}_n(\check{\curlyvee}) = A_n(\check{\curlyvee})\, e^{iP_n(\check{\curlyvee})}$ *represents the* $\mathbb{NF}_n$ *of* $\mathbb{M}$ *with* $A_n(\check{\curlyvee}) \in \mathscr{I}$ *as* $\mathbb{A}$*-term,* $P_n(\check{\curlyvee}) \in \varpi$ *as* $\mathbb{P}$*-term, such that* $1 \leq \mathbb{M}_m(\check{\curlyvee}) + \mathbb{M}_n(\check{\curlyvee}) \leq 1$ *has a hesitancy grade* $\mathbb{M}_h(\check{\curlyvee}) = 1 - \mathbb{M}_m(\check{\curlyvee}) - \mathbb{M}_n(\check{\curlyvee})$.

Definition 2.7. *[24] The collection* $(\mathbb{O}, \Lambda)$ *is called* $\mathbb{HSS}$ *over* $\Cup$ *if* $\mathbb{O} : \Lambda \to \mathbb{P}(\Cup)$*, where* $\Lambda = \prod_{i=1}^{n} \Lambda_i$ *such that* Λ_i *are disjoint attribute-valued sets (*$\mathbb{DAVS}$*) of sub-parameters, each set corresponding to a unique parameter* $\lambda \in \Lambda$.

Definition 2.8. *[27] If* $\mathbb{C}_{\mathbb{IF}}(\Cup)$ *represents the collection of all* $\mathbb{IF}$*-subsets over* $\Cup$*, then the* $\mathbb{IFHSS}$ $(\mathbb{Q}, \Lambda)$ *is obtained when the mapping* $\mathbb{O} : \Lambda \to \mathbb{P}(\Cup)$ *in Definition 2.7 is replaced by* $\mathbb{Q} : \Lambda \to \mathbb{C}_{\mathbb{IF}}(\Cup)$ *and all other conditions of Definition 2.7 are remained valid.*

Definition 2.9. *[43] If* $\mathbb{C}_{\mathbb{CIF}}(\Cup)$ *represents the collection of all* $\mathbb{CIF}$*-subsets over* $\Cup$*, then* $\mathbb{CIFHSS}$ $(\mathbb{T}, \Lambda)$ *is obtained when the mapping* $\mathbb{O} : \Lambda \to \mathbb{P}(\Cup)$ *in Definition 2.7 is replaced by* $\mathbb{T} : \Lambda \to \mathbb{C}_{\mathbb{CIF}}(\Cup)$ *and all other conditions of Definition 2.7 are remained valid.*

3. The notions of Γ-set, its set operations, and aggregations

This portion provides the notional description of the Γ-set and its set operations as well as aggregations.

Definition 3.1. *A set* $(\mathbb{M}_{\mathbb{I}}, \Lambda)$ *is called* $\mathbb{IVCIFSS}$ *over* $\Cup$ *if* $\mathbb{M}_{\mathbb{I}}$ *is a parameterized gathering of* $\mathbb{IVCIF}$*-subsets of* $\Cup$*, is given by* $\mathbb{M}_{\mathbb{I}} : \Lambda \to \mathbb{P}(\Cup)$*, and is defined by*

$$(\mathbb{M}_{\mathbb{I}}, \Lambda) = \left\{\left(\lambda, \left\{\frac{\mathbb{M}_{\mathbb{I}m}(\check{\curlyvee}), \mathbb{M}_{\mathbb{I}n}(\check{\curlyvee})}{\check{\curlyvee}}\right\}\right) : \check{\curlyvee} \in \Cup, \lambda \in \Lambda\right\},$$

where $\mathbb{M}_{\mathbb{I}m}(\check{\curlyvee}) = A_m(\check{\curlyvee})\, e^{iP_m(\check{\curlyvee})}$ *represents the* $\mathbb{MF}_n$ *of* $\mathbb{M}_{\mathbb{I}}$ *with* $A_m(\check{\curlyvee}) \in \Im(\mathscr{I})$ *as* $\mathbb{A}$*-term,* $P_m(\check{\curlyvee}) \subseteq \varpi$ *as* $\mathbb{P}$*-term, and* $\mathbb{M}_{\mathbb{I}n}(\check{\curlyvee}) = A_n(\check{\curlyvee})\, e^{iP_n(\check{\curlyvee})}$ *represents the* $\mathbb{NF}_n$ *of* $\mathbb{M}_{\mathbb{I}}$ *with* $A_n(\check{\curlyvee}) \in \Im(\mathscr{I})$ *as* $\mathbb{A}$*-term,* $P_n(\check{\curlyvee}) \subseteq \varpi$ *as* $\mathbb{P}$*-term, such that* $1 \leq \inf \mathbb{M}_{\mathbb{I}m}(\check{\curlyvee}) + \inf \mathbb{M}_{\mathbb{I}n}(\check{\curlyvee}) \leq \sup \mathbb{M}_{\mathbb{I}m}(\check{\curlyvee}) + \sup \mathbb{M}_{\mathbb{I}n}(\check{\curlyvee}) \leq 1$ *and the hesitancy grade is* $\mathbb{M}_{\mathbb{I}h}(\check{\curlyvee}) = [1 - \sup \mathbb{M}_{\mathbb{I}m}(\check{\curlyvee}) - \sup \mathbb{M}_{\mathbb{I}n}(\check{\curlyvee}), 1 - \inf \mathbb{M}_{\mathbb{I}m}(\check{\curlyvee}) - \inf \mathbb{M}_{\mathbb{I}n}(\check{\curlyvee})]$.

Definition 3.2. *Let $\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3, ..., \mathcal{K}_n$ be $\mathbb{DAVS}$ having n distinct attributes $k_1, k_2, k_3, ..., k_n$, respectively, for $n \geq 1, \mathcal{K} = \mathcal{K}_1 \times \mathcal{K}_2 \times \mathcal{K}_3 \times ..... \times \mathcal{K}_n$, and let $\Lambda(\underline{\lambda})$ be an $\mathbb{IVCIFSS}$ over $\mathbb{U}$ for all $\underline{\lambda} = (b_1, b_2, b_3, ..., b_n) \in \mathcal{K}$. The Γ-set, denoted by $\Omega_{\mathcal{K}} = (\Lambda, \mathcal{K})$, over $\mathbb{U}$ is defined as*

$$\Omega_{\mathcal{K}} = \left\{(\underline{\lambda}, \Lambda(\underline{\lambda})) : \underline{\lambda} \in \mathcal{K}, \Lambda(\underline{\lambda}) \in C_{IV}(\mathbb{U})\right\},$$

where $\Lambda : \mathcal{K} \to C_{IV}(\mathbb{U}), \Lambda(\underline{\lambda}) = \emptyset if \underline{\lambda} \notin \mathcal{K}$ is an $\mathbb{IVCIF}$ approximate function ($\mathbb{A}_{fn}$) of $\Omega_{\mathcal{K}}$ and $\Lambda(\underline{\lambda}) = \langle[\overleftarrow{\Lambda}_1(\underline{\lambda}), \overrightarrow{\Lambda}_1(\underline{\lambda})], [\overleftarrow{\Lambda}_2(\underline{\lambda}), \overrightarrow{\Lambda}_2(\underline{\lambda})]\rangle$ with
(a) $\overleftarrow{\Lambda}_1(\underline{\lambda}) = \overleftarrow{\gamma} e^{i\overleftarrow{\theta}}$ and $\overrightarrow{\Lambda}_1(\underline{\lambda}) = \overrightarrow{\gamma} e^{i\overrightarrow{\theta}}$ are lower and upper bounds of the $\mathbb{MF}_n$ of $\Omega_{\mathcal{K}}$, respectively.
(b) $\overleftarrow{\Lambda}_2(\underline{\lambda}) = \overleftarrow{\gamma} e^{i\overleftarrow{\theta}}$ and $\overrightarrow{\Lambda}_2(\underline{\lambda}) = \overrightarrow{\gamma} e^{i\overrightarrow{\theta}}$ are lower and upper bounds of the $\mathbb{NF}_n$ of $\Omega_{\mathcal{K}}$, respectively, and its value $\Lambda(\underline{\lambda})$ is called the $\underline{\lambda}$-member of the Γ-set for all values of $\underline{\lambda} \in \mathcal{K}$.

Definition 3.3. *The* complement *of the Γ-set $(\Lambda, \mathcal{K})$, denoted by $(\Lambda, \mathcal{K})^c$ is stated as*

$$(\Lambda, \mathcal{K})^c = \{(\check{x}, (\Lambda(\check{x}))^c) : \check{x} \in \mathcal{K}, (\Lambda(\check{x}))^c \in C_{IV}(\mathbb{U})\},$$

where the $\mathbb{A}$-term and $\mathbb{P}$-terms of the $\mathbb{MF}_n$ $(\Lambda(\check{x}))^c$ are given by
$(\overleftarrow{\gamma}_{\mathcal{K}}(\check{x}))^c = 1 - \overleftarrow{\gamma}_{\mathcal{K}}(\check{x})$, $(\overrightarrow{\gamma}_{\mathcal{K}}(\check{x}))^c = 1 - \overrightarrow{\gamma}_{\mathcal{K}}(\check{x})$ and $(\overleftarrow{\theta}_{\mathcal{K}}(\check{x}))^c = 2\pi - \overleftarrow{\theta}_{\mathcal{K}}(\check{x})$, $(\overrightarrow{\theta}_{\mathcal{K}}(\check{x}))^c = 2\pi - \overrightarrow{\theta}_{\mathcal{K}}(\check{x})$ respectively.

3.1. Aggregation of Γ-set

This part establishes the aggregation procedures and their decision mechanism for the Γ-set. This leads to an aggregate fuzzy set that is fuzzy-like and is derived from a $\mathbb{CIFHSS}$ and its cardinal set. Definition 3.2 defines the concepts $\mathcal{D}, \mathfrak{E}, \xi_{\mathcal{D}}$, and $\uplus_{IVCIFHS}$.

Definition 3.4. *Let $\xi_{\mathcal{D}} \in \uplus_{IVCIFHS}$. Assume that $\mathbb{U} = \{\check{\mathrm{Y}}_1, \check{\mathrm{Y}}_2, ..., \check{\mathrm{Y}}_m\}$ and $\mathfrak{E} = \{\mathcal{L}_1, \mathcal{L}_2, ..., \mathcal{L}_n\}$ with $\mathcal{L}_1 = \{e_{11}, e_{12}, ..., e_{1n}\}, \mathcal{L}_2 = \{e_{21}, e_{22}, ..., e_{2n}\}, ..., \mathcal{L}_n = \{e_{n1}, e_{n2}, ..., e_{nn}\}$, and $\mathcal{D} = \mathcal{L}_1 \times \mathcal{L}_2 \times ..... \times \mathcal{L}_n = \{\check{x}_1, \check{x}_2, ..., \check{x}_n, ..., \check{x}_{n^n} = \check{x}_r\}$, each $\check{x}_i$ is an n-tuple element of $\mathcal{D}$ and $|\mathcal{D}| = r = n^n$, then the $\xi_{\mathcal{D}}$ can be presented in the following tabular notation (see Table 1).*

Table 1. Tabular representation of $\xi_{\mathcal{D}}$.

$\xi_{\mathcal{D}}$	$\check{x}_1$	$\check{x}_2$	...	$\check{x}_r$
$\check{\mathrm{Y}}_1$	$\begin{pmatrix} \tau^1_{\chi_{\mathcal{D}}(\check{x}_1)}(\check{\mathrm{Y}}_1), \\ \tau^2_{\chi_{\mathcal{D}}(\check{x}_1)}(\check{\mathrm{Y}}_1), \end{pmatrix}$	$\begin{pmatrix} \tau^1_{\chi_{\mathcal{D}}(\check{x}_2)}(\check{\mathrm{Y}}_1), \\ \tau^2_{\chi_{\mathcal{D}}(\check{x}_2)}(\check{\mathrm{Y}}_1), \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \tau^1_{\chi_{\mathcal{D}}(\check{x}_r)}(\check{\mathrm{Y}}_1), \\ \tau^2_{\chi_{\mathcal{D}}(\check{x}_r)}(\check{\mathrm{Y}}_1), \end{pmatrix}$
$\check{\mathrm{Y}}_2$	$\begin{pmatrix} \tau^1_{\chi_{\mathcal{D}}(\check{x}_1)}(\check{\mathrm{Y}}_2), \\ \tau^2_{\chi_{\mathcal{D}}(\check{x}_1)}(\check{\mathrm{Y}}_2), \end{pmatrix}$	$\begin{pmatrix} \tau^1_{\chi_{\mathcal{D}}(\check{x}_2)}(\check{\mathrm{Y}}_2), \\ \tau^2_{\chi_{\mathcal{D}}(\check{x}_2)}(\check{\mathrm{Y}}_2), \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \tau^1_{\chi_{\mathcal{D}}(\check{x}_r)}(\check{\mathrm{Y}}_2), \\ \tau^2_{\chi_{\mathcal{D}}(\check{x}_r)}(\check{\mathrm{Y}}_2), \end{pmatrix}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\check{\mathrm{Y}}_m$	$\begin{pmatrix} \tau^1_{\chi_{\mathcal{D}}(\check{x}_1)}(\check{\mathrm{Y}}_m), \\ \tau^2_{\chi_{\mathcal{D}}(\check{x}_1)}(\check{\mathrm{Y}}_m), \end{pmatrix}$	$\begin{pmatrix} \tau^1_{\chi_{\mathcal{D}}(\check{x}_2)}(\check{\mathrm{Y}}_m), \\ \tau^2_{\chi_{\mathcal{D}}(\check{x}_2)}(\check{\mathrm{Y}}_m), \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \tau^1_{\chi_{\mathcal{D}}(\check{x}_r)}(\check{\mathrm{Y}}_m), \\ \tau^2_{\chi_{\mathcal{D}}(\check{x}_r)}(\check{\mathrm{Y}}_m), \end{pmatrix}$

Where $\tau^1_{\chi_{\mathcal{D}}(x)}$ and $\tau^2_{\chi_{\mathcal{D}}(x)}$ are $\mathbb{MF}_n$ and $\mathbb{NF}_n$ of $\chi_{\mathcal{D}}$, respectively, with interval-valued intuitionistic fuzzy values. If $\alpha_{ij} = (\tau^1_{\chi_{\mathcal{D}}(\check{x}_j)}(\check{\mathrm{Y}}_i), \tau^2_{\chi_{\mathcal{D}}(\check{x}_j)}(\check{\mathrm{Y}}_i))$, for $i = \mathbb{N}_1^m$ and $j = \mathbb{N}_1^r$, then the Γ-set $\xi_{\mathcal{D}}$ is uniquely *characterized by a matrix,*

$$[\alpha_{ij}] = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1r} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mr} \end{bmatrix},$$

which is called an $m \times r$ Γ-set matrix.

Definition 3.5. *If $\xi_{\mathcal{D}} \in \uplus_{IVCIFHS}$, then the cardinal set of $\xi_{\mathcal{D}}$ is defined as*

$$\|\xi_{\mathcal{D}}\| = \left\{(\tau^1_{\|\xi_{\mathcal{D}}\|}(\check{x}), \tau^2_{\|\xi_{\mathcal{D}}\|}(\check{x}))/\check{x} : \check{x} \in \mathcal{D}\right\},$$

where $\tau^1_{\|\xi_{\mathcal{D}}\|}, \tau^2_{\|\xi_{\mathcal{D}}\|} : \mathcal{D} \to [0, 1]$ are $\mathbb{MF}_n$ and $\mathbb{NF}_n$ of $\|\xi_{\mathcal{D}}\|$ with

$$\tau^1_{\|\xi_{\mathcal{D}}\|}(\check{x}), \tau^2_{\|\xi_{\mathcal{D}}\|}(\check{x}) = \frac{|\chi_{\mathcal{D}}(\check{x})|}{|\Uplus|},$$

respectively. These have interval-valued intuitionistic fuzzy values. Note that $\|C_{ivcifhs}(\Uplus)\|$ is the collection of all cardinal sets of Γ-sets and $\|C_{ivcifhs}(\Uplus)\| \subseteq \mathbb{IVIF}(\mathcal{D})$.

Definition 3.6. *Let $\xi_{\mathcal{D}} \in C_{ivcifhs}(\Uplus)$ and $\|\xi_{\mathcal{D}}\| \in \|C_{ivcifhs}(\Uplus)\|$. Consider $\mathfrak{E}$ as in Definition 3.2, then tabular representation of $\|\xi_{\mathcal{D}}\|$ is presented in Table 2.*

Table 2. Tabular representation of $\|\xi_{\mathcal{D}}\|$.

$\mathcal{D}$	$\check{x}_1$	$\check{x}_2$	$\cdots$	$\check{x}_r$
$\tau_{\|\xi_{\mathcal{D}}\|}$	$\begin{pmatrix} \tau^1_{\|\xi_{\mathcal{D}}\|}(\check{x}_1), \\ \tau^2_{\|\xi_{\mathcal{D}}\|}(\check{x}_1) \end{pmatrix}$	$\begin{pmatrix} \tau^1_{\|\xi_{\mathcal{D}}\|}(\check{x}_2), \\ \tau^2_{\|\xi_{\mathcal{D}}\|}(\check{x}_2) \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \tau^1_{\|\xi_{\mathcal{D}}\|}(\check{x}_r), \\ \tau^2_{\|\xi_{\mathcal{D}}\|}(\check{x}_r) \end{pmatrix}$

If $\alpha_{1j} = (\tau^1_{\|\xi_{\mathcal{D}}\|}(\check{x}_j), \tau^2_{\|\xi_{\mathcal{D}}\|}(\check{x}_j))$, for $j = \mathbb{N}^r_1$, then the cardinal set $\|\xi_{\mathcal{D}}\|$ is represented by a matrix,

$$[\alpha_{ij}]_{1\times r} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1r} \end{bmatrix},$$

and is called cardinal matrix *of $\|\xi_{\mathcal{D}}\|$.*

Definition 3.7. *Let $\xi_{\mathcal{D}} \in C_{ivcifhs}(\Uplus)$ and $\|\xi_{\mathcal{D}}\| \in \|C_{ivcifhs}(\Uplus)\|$ then the Γ-aggregation operator is defined as*

$$\overbrace{\xi_{\mathcal{D}}} = A_{ivcfhs}\left(\|\xi_{\mathcal{D}}\|, \xi_{\mathcal{D}}\right),$$

where

$$A_{ivcifhs} : \|C_{ivcifhs}(\Uplus)\| \times C_{ivcfhs}(\Uplus) \to \mathbb{F}(\Uplus).$$

$\overbrace{\xi_{\mathcal{D}}}$ *is called the aggregate fuzzy set of Γ-set $\xi_{\mathcal{D}}$.*
Its $\mathbb{MF}_n$ is given as

$$\tau_{\overbrace{\xi_{\mathcal{D}}}} : \Uplus \to [0, 1]$$

with

$$\tau_{\overbrace{\xi_{\mathcal{D}}}}(\nu) = \frac{1}{|\mathcal{D}|} \sum_{\check{x} \in \mathcal{D}} \tau_{Card(\xi_{\mathcal{D}})}(\check{x}) \tau_{Card(\chi_{\mathcal{D}})}(\nu).$$

Definition 3.8. *Let* $\xi_{\mathcal{D}} \in C_{ivcifhs}(\Bbbk)$ *and* $\overbrace{\xi_{\mathcal{D}}}$ *be its aggregate fuzzy set. Assume that* $\Bbbk = \{\check{\gamma}_1, \check{\gamma}_2, ..., \check{\gamma}_m\}$*, then* $\overbrace{\xi_{\mathcal{D}}}$ *can be presented as*

$$\begin{bmatrix} \xi_{\mathcal{D}} & \vdots & \tau_{\underbrace{}_{\xi_{\mathcal{D}}}} \\ \cdots\cdots\cdots & \vdots & \cdots\cdots\cdots\cdots \\ \check{\gamma}_1 & \vdots & \tau_{\underbrace{}_{\xi_{\mathcal{D}}}}(\check{\gamma}_1) \\ \check{\gamma}_2 & \vdots & \tau_{\underbrace{}_{\xi_{\mathcal{D}}}}(\check{\gamma}_2) \\ \vdots & \vdots & \vdots \\ \check{\gamma}_m & \vdots & \tau_{\underbrace{}_{\xi_{\mathcal{D}}}}(\check{\gamma}_m) \end{bmatrix}.$$

If $\alpha_{i1} = \tau_{\underbrace{}_{\xi_{\mathcal{D}}}}(\check{\gamma}_i)$ *for* $i = \mathbb{N}_1^m$ *then* $\overbrace{\xi_{\mathcal{D}}}$ *is represented by the matrix,*

$$[\alpha_{i1}]_{m\times 1} = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{bmatrix},$$

which is called an aggregate matrix of $\overbrace{\xi_{\mathcal{D}}}$ *over* $\Bbbk$.

3.2. Applications of Γ*-set*

In this portion of the study, an algorithm is described for assisting the decision-support framework, taking into account the terms given in Subsection 3.1. The algorithm has been verified using a case study from an actual scenario.

The flow chart of Algorithm 3.1 is displayed in Figure 1. The summarized version of Algorithm 3.1 is displayed in Figure 2.

Algorithm 3.1. ***$\mathcal{DS}$ Algorithm Based on Aggregations of* Γ*-set.***

▹ ***Start***

▹ ***Input stage.***

———*(1). Assume* $\Bbbk$ *as initial space of objects.*

———*(2). Assume* $\mathfrak{E}$ *as attributive set (*$\mathbb{SP}$*).*

———*(3). Categorize* $\mathbb{SP}$ *into* $\mathbb{DAVS}$ $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, ..., \mathcal{L}_n$.

▹ ***Construction stage.***

———*(4).* $\mathcal{D} = \mathcal{L}_1 \times \mathcal{L}_2 \times \mathcal{L}_3 \times ... \times \mathcal{L}_n$.

———*(5). Using Definition 3.2, construct* Γ*-set* $\chi_{\mathcal{D}}$ *over* $\Bbbk$*, in accordance with.*

▹ ***Computation stage.***

———*(6). By Definition 3.5, calculate* $\| \xi_{\mathcal{D}} \|$ *for* $\mathbb{A}$*-term and* $\mathbb{P}$*-term individually.*

———*(7). By Definition 3.7, calculate* $\overbrace{\xi_{\mathcal{D}}}$ *for* $\mathbb{A}$*-term and* $\mathbb{P}$*-term individually.*

———*(8). By Definition 3.7, calculate* $\tau_{\underbrace{}_{\xi_{\mathcal{D}}}}(\nu)$.

▹ ***Output stage.***

———(9). *Based on Definition 3.8, determine max modulus of* $\tau_{\underbrace{}_{\xi_{\mathcal{D}}}}(v)$ *to have optimal selection.*

▹***End***

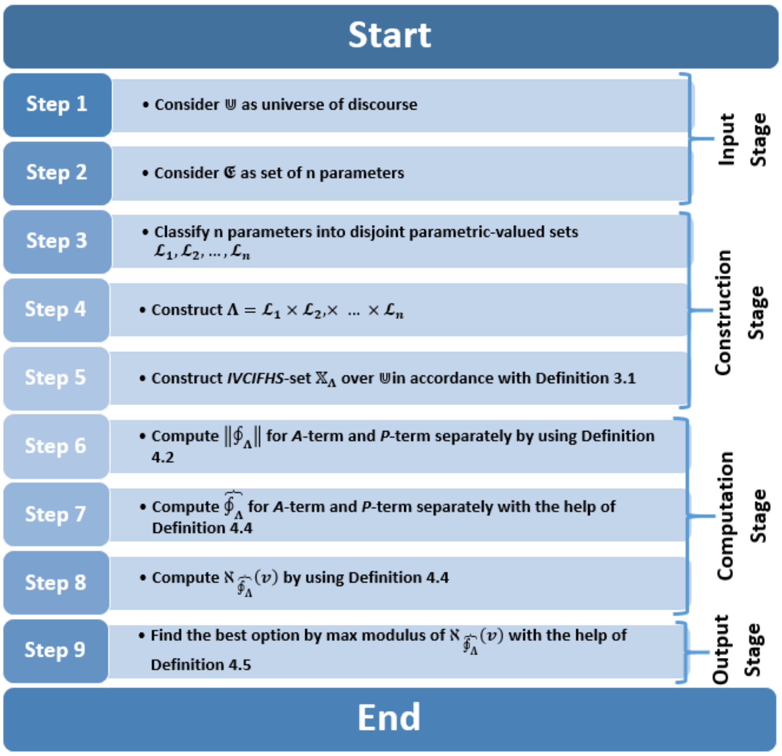

Figure 1. $\mathcal{DS}$ Algorithm based on aggregations of Γ-set.

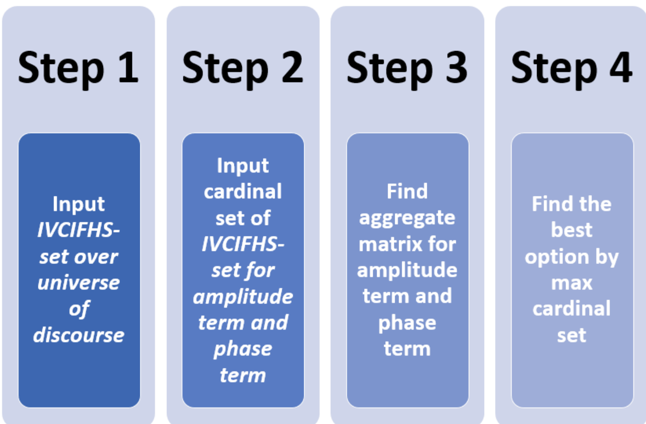

Figure 2. $\mathcal{DS}$ Summarized algorithm based on aggregations of Γ-set.

The algorithm is now explained with the help of the following example:

Example 3.1. The $\mathbb{CSYS}$ of a plant is critical for ensuring a productive and safe workplace. Cooling towers are used to dissipate heat, chillers are used to cool industrial processes, HVAC systems are used to keep employees comfortable, and adequate ventilation is used to remove pollutants and excess heat. Adequate insulation is also important for reducing cooling demands. Efficient $\mathbb{CSYS}$s are critical for maximum output, equipment durability, and worker comfort. Maintenance on a regular basis and enhanced monitoring assist in improving performance, energy efficiency, and sustainability. Some similar systems in the light of fuzzy set-like structures have been discussed in [51–54]. Consider the daily life scenario of $\mathbb{CSYS}$ of a factory. The factory owner (say Mr. B) wants to install the $\mathbb{CSYS}$ in his factory. Mr. B hires a team of experts for the said purpose. The team enlisted attributes and sub-attributes that must be taken into consideration for $\mathbb{CSYS}$ of the factory. The expert team identified effective attributes and sub-attributes for $\mathbb{CSYS}$ that have been summarized hereafter: The performance and efficiency of an industrial $\mathbb{CSYS}$ are influenced by a variety of qualities and sub-attributes that make up its complex infrastructure. The key characteristics and related characteristics of a factory $\mathbb{CSYS}$ are listed below:

(1) **Cooling Capacity (CC):** The cooling capacity (CC) of a factory's $\mathbb{CSYS}$ is crucial since it impacts how well the system can lower and maintain the facility's temperature. It calculates the rate of heat removal by the $\mathbb{CSYS}$, commonly expressed in British thermal units (BTUs) or tonnes of refrigeration. To avoid equipment overheating, preserve product quality, and create a comfortable working atmosphere, it is essential to ensure an adequate CC. While a correctly sized system contributes to energy efficiency and effective manufacturing operations, insufficient CC can result in inefficiencies, equipment breakdowns, and decreased productivity.

(2) **Energy Efficiency (EE):** In order to save operating costs, protect the environment, and maintain competitiveness over the long run, a factory's $\mathbb{CSYS}$ must be as energy-efficient (EE) as possible. A $\mathbb{CSYS}$ that uses little energy to effectively maintain the required temperature is called EE. As a result, there are fewer carbon emissions, lower utility costs, and a smaller environmental impact. Along with financial savings, it improves the factory's overall sustainability initiatives and complies with legal standards and corporate social responsibility (CSR) objectives. A stable $\mathbb{CSYS}$ is also ensured by increased energy efficiency, minimizing downtime and production interruptions and eventually boosting the factory's productivity, profitability, and marketability.

(3) **Temperature Control (TC):** An important part of a factory's $\mathbb{CSYS}$ is temperature regulation because it has a direct impact on worker comfort, machine performance, and product quality. In order to avoid equipment overheating, product rotting, or production halts, it makes sure that the environment stays within designated temperature limits. For businesses with strict temperature requirements, such as pharmaceutical or food processing, maintaining exact temperature control (TC) is crucial. As excessive cooling can result in energy waste, it also helps to increase energy efficiency. A factory's $\mathbb{CSYS}$'s ability to regulate temperature effectively ensures ideal working conditions, increases equipment longevity, reduces production errors, and ultimately supports the factory's productivity and product quality standards.

(4) **Redundancy (R):** A $\mathbb{CSYS}$'s redundancy (R) is an essential defense against unanticipated failures or downtime. It entails keeping extra or backup parts, like chillers or cooling towers, ready to go in case the main system experiences issues. Redundancy guarantees continuous cooling, avoiding expensive production halts brought on by equipment breakdowns or maintenance. Particularly in crucial areas like data centers or pharmaceutical manufacturing, where continual cooling is crucial, it improves system dependability and resilience. Additionally, redundancy gives you the freedom to make repairs and maintenance without disrupting business as usual. Overall, it helps to keep the cooling process steady and effective, protecting the factory's output and reducing potential losses.

(5) **Environmental Impact (EI):** In terms of sustainability and corporate responsibility, a factory's $\mathbb{CSYS}$ has a considerable environmental influence. Systems for cooling frequently use a lot of energy and water. Therefore, reducing this effect is essential. A factory's carbon footprint and water usage can be reduced through the use of effective cooling technology and ethical water management, easing the burden on nearby ecosystems. Additionally, dangerous emissions can be avoided by using eco-friendly refrigerants and following the right disposal procedures. By addressing the environmental impact (EI), regulations may be adhered to, the public's perspective is improved, and there may be financial benefits due to lessening resource use and incentives. In the end, a responsible approach to $\mathbb{CSYS}$ design and operation helps the environment as well as the reputation of the plant.

(6) **Remote Monitoring and Control (RMC):** The key role of remote monitoring and control (RMC) in a factory's $\mathbb{CSYS}$ is that they increase operational efficiency and decrease downtime by enabling remote, real-time monitoring and control of cooling processes. Potential problems can be identified early with remote monitoring, preventing equipment breakdowns and guaranteeing optimum performance. To maintain temperature and energy efficiency, operators can make quick adjustments, increasing total output. Additionally, remote control makes predictive maintenance possible, lowering repair costs and lengthening the lifespan of the system. By allowing off-peak cooling operation, it also aids in energy conservation. Overall, RMC improves $\mathbb{CSYS}$ performance, cuts costs associated with running the plant, and makes it more competitive.

(7) **Air Quality (AQ):** The effectiveness of a factory's $\mathbb{CSYS}$ depends on the quality of the air because it affects workers' productivity and health as well as the smooth operations of machinery. By removing pollutants, humidity, and excess heat, an efficient $\mathbb{CSYS}$ should provide sufficient airflow to provide a secure and comfortable working environment. Air pollution can affect one's health and productivity. In addition to preventing the accumulation of dust and other particles that might harm delicate equipment, proper air quality control also minimizes maintenance costs and downtime. Monitoring and maintaining the $\mathbb{CSYS}$'s air quality (AQ) results in a healthier workforce, longer equipment life, and greater factory operational effectiveness.

(8) **Maintenance Requirements (MR):** The $\mathbb{CSYS}$ in a factory needs to be maintained regularly since it affects the system's dependability, lifespan, and operational effectiveness. Coils should be cleaned, filters should be changed, and parts should be inspected on a regular basis to keep the system running smoothly. Lack of maintenance can result in less effective cooling, more energy use, and expensive breakdowns. Preventive maintenance also aids in the early detection of possible problems, minimizing production losses and downtime. A $\mathbb{CSYS}$ that is properly maintained lasts longer and requires fewer costly repairs or replacements. In general, taking maintenance needs into account is crucial for a $\mathbb{CSYS}$ that is affordable, trouble-free, and successfully supports manufacturing operations.

(9) **Safety Features (SF):** To safeguard employees, machinery, and the building itself, a factory's $\mathbb{CSYS}$ must have numerous safety elements. These characteristics include pressure relief valves, fire suppression systems, and emergency shutdown procedures. These safety precautions stop accidents, equipment damage, and potential fires in the event of a malfunction or dangerous condition. Safety systems that are properly developed also adhere to industry norms and laws, ensuring a secure workplace. Safety features (SF) must undergo routine maintenance and inspections to ensure their effectiveness. In general, safety measures are an essential component

of a $\mathbb{CSYS}$, offering peace of mind, lowering risks, and ensuring the safety of everyone involved in manufacturing activities.

(10) **Scalability (S):** Scalability, which determines a $\mathbb{CSYS}$'s capacity to adapt to changing needs and growth, is a key consideration. As the facility expands or production needs change, a scalable system can easily handle growing cooling demands. Without the need for expensive and disruptive overhauls, it enables the effective addition or adjustment of CC, equipment, or infrastructure. Scalability guarantees that the $\mathbb{CSYS}$ will continue to be affordable, EE, and able to accommodate changing demands. This adaptability improves the factory's current operations as well as its ability to compete, develop, and successfully respond to market dynamics in the future.

(11) **Noise Control (NC):** For a number of reasons, noise management in a factory's $\mathbb{CSYS}$ is important. First, excessive noise can have a negative effect on factory workers' health and well-being, which could result in stress and lower production. Second, it might hinder communication and result in a hostile workplace. Furthermore, noise pollution might harm areas outside the factory's boundaries, which could result in legal or regulatory problems. A $\mathbb{CSYS}$ ensures a quieter and more peaceful workplace, compliance with noise regulations, and a good reputation in the community, all of which contribute to overall operational efficiency and social responsibility. Noise control (NC) measures include acoustic enclosures, sound insulation, and vibration dampening.

(12) **Water Management (WM):** For a number of reasons, water management (WM) is essential in a factory's $\mathbb{CSYS}$. First off, it makes sure that water resources are used sustainably and efficiently, lowering consumption and minimizing environmental damage. The lifespan of cooling equipment is increased and maintenance costs are decreased by efficient water treatment and filtration, which also prevents scale formation, corrosion, and microbial growth. Additionally, by ensuring optimum heat transfer efficiency, efficient water management improves $\mathbb{CSYS}$ performance. Finally, according to local water restriction, disposing of wastewater responsibly is necessary to avoid obligations for the environment and the law. In conclusion, WM in a $\mathbb{CSYS}$ promotes the preservation of resources, the durability of equipment, operational effectiveness, and environmental responsibility.

(13) **Integration with building management system (IBMS):** For smooth and effective operations, the $\mathbb{CSYS}$ in a plant must be integrated with the building management system (BMS). It makes it possible to monitor and manage $\mathbb{CSYS}$s, temperature settings, and energy consumption in real time. According to production and occupancy schedules, the BMS may automatically regulate cooling, improving EE and lowering operating costs. Additionally, integration makes data-driven $\mathbb{DMK}$, preventive maintenance, and early fault detection possible. Additionally, by coordinating multiple systems, including lighting and security, with the $\mathbb{CSYS}$, it improves total building automation and safety. Fundamentally, integration with the BMS improves energy management, streamlines operations, and ensures a comfortable and effective factory environment.

(14) **Compliance with Regulations (CR):** The $\mathbb{CSYS}$ of a plant must follow laws in order to maintain compliance with environmental, safety, and efficiency standards. In order to avoid expensive fines and legal problems, regulatory compliance ensures that $\mathbb{CSYS}$s adhere to legal standards, such as emissions limitations and environmental impact assessments. By following standards for pressure vessels, handling of refrigerants, and fire safety codes, it also supports worker safety.

Additionally, compliance guarantees that the $\mathbb{CSYS}$ runs responsibly and effectively, reducing energy use and environmental impact. For overall legal and ethical operations, environmental stewardship, and the safety of industrial workers and surrounding communities, regulatory standards in a $\mathbb{CSYS}$ must be met.

(15) **Emergency Preparedness (EP):** The $\mathbb{CSYS}$ in a factory must be prepared for emergencies in order to protect workers and operations. It entails preparing for unforeseen events like equipment breakdowns, power outages, or natural disasters. A $\mathbb{CSYS}$ can avoid expensive production interruptions, equipment damage, and worker safety concerns by having backup cooling options, emergency shutdown protocols, and contingency plans in place. Regular training and drills are also a part of effective disaster preparedness to make sure that staff members are trained to act promptly and safely in emergency situations. In the end, it is a crucial part of business continuity because it reduces disruptions and safeguards the factory's resources and employees.

(16) **Budget and Cost Analysis (BCA):** For the $\mathbb{CSYS}$ of a factory to be effective and cost-effective, budget and cost analysis (BCA) are significant. The original investment, ongoing costs, and $\mathbb{CSYS}$ maintenance costs are all taken into account in a thorough budget plan. Making knowledgeable selections about the kind of cooling technology, equipment sizing, and EE solutions are facilitated by this. Cost analysis evaluates the overall cost of ownership, which takes into account energy use, upkeep, and potential long-term savings. The factory can choose a $\mathbb{CSYS}$ based on this information to meet its budgetary needs while maximizing long-term cost savings and return on investment. It ensures sustainability and the best possible use of resources.

(17) **Training and Documentation (TD):** A factory's $\mathbb{CSYS}$ depends on training and documentation (TD) to ensure safe and effective operations. In-depth training programs instruct staff members on how to handle, maintain, and troubleshoot cooling equipment. Employees are more equipped to see problems and take swift action, lowering the possibility of expensive failures and safety risks. Procedures, manuals, and records that have been thoroughly documented are invaluable resources for system operation and maintenance. They support uniformity, adherence to safety rules, and efficient knowledge sharing among workers. Ultimately, spending money on training and documentation improves the $\mathbb{CSYS}$'s dependability, lifespan, and performance, which benefits the plant as a whole.

(18) **Future-Proofing (FP):** Designing and implementing $\mathbb{CSYS}$ solutions that foresee and react to changing needs and problems is necessary to future-proof (FP) a factory's $\mathbb{CSYS}$. It guarantees that the $\mathbb{CSYS}$ will continue to be effective and pertinent as laws, rules, and business standards evolve. Long-term savings come from this proactive strategy since it reduces the need for pricey system replacements or upgrades. Incorporating EE technologies, choosing scalable equipment, and anticipating environmental rules are all examples of FP. Additionally, it promotes sustainability initiatives, improves operational resilience, and places the business in a competitive position in a market that is always changing, assuring the long-term success of cooling operations.

The following tables (Tables 3–5) summarize the attributes and sub-attributes.

Each of these attributes, as well as their sub-attributes, should be carefully considered when designing and installing a factory $\mathbb{CSYS}$ to ensure the system fits the facility's unique requirements and operates successfully and consistently.

The expert committee prioritized the attributes and sub-attributes based on factory-specific needs, budget, available space, and committee preferences as selection parameters to find the $\mathbb{CSYS}$ available in market that best suits the factory.

Table 3. Attributes and sub-attributes for $\mathbb{CSYS}$ of the factory.

Sr. No.	Attribute	Abbrev.	Sub-attribute	Abbrev.
(1)	Cooling Capacity	C Cap	Cooling Load Assessment	CL As
			Capacity Sizing	C Size
(2)	Energy Efficiency	E Eff	High-Efficiency Components	HE Comp
			Variable Speed Control	VS Cont
			Heat Recovery	H Rec
			Insulation	Ins
(3)	Temperature Control	T Cont	Precision Temperature Control	PT Cont
			Zoning	Zon
(4)	Redundancy	Red	Backup Systems	B Sys
			Redundant Cooling Paths	RC Pth
(5)	Environmental Impact	E Imp	Eco-Friendly Refrigerants	EF Ref
			Water Conservation	W Con
			Waste Heat Utilization	WH Utl
(6)	Remote Monitoring & Control	RM Cont	Monitoring Sensors	M Sen
			Remote Control	R Cont
			Alert Systems	A Sys
(7)	Air Quality	A Qty	Air Filtration	A Fil
			Ventilation	Vent
(8)	Maintenance Requirements	M Req	Scheduled Maintenance	S Man
			Easy Access	E Acc
(9)	Safety Features	S Fea	Temperature Alarms	T Al
			Pressure Relief Valves	PR Val
			Emergency Shutdown	E Sd
(10)	Scalability	Sca	Flexible Design	F Desn
(11)	Noise Control	N Cont	Acoustic Insulation	A Ins
(12)	Water Management	W Man	Water Recycling	W Rec
			Water Treatment	W Tmt
(13)	Integration with building management system	IBM Sys	Communication Protocols	C Pro
(14)	Compliance with Regulations	C Reg	Regulatory Compliance	R Com
(15)	Emergency Preparedness	E Prep	Emergency Response Plans	ER Pln
(16)	Budget and Cost Analysis	BC Ana	Life Cycle Cost Analysis	LCC Ana
(17)	Training and Documentation	T Doc	Personnel Training	P Tra
			Documentation	Doc
(18)	Future-Proofing	F Prof	Technology Updates	T Up

Table 4. Attributes and sub-attributes for refrigerator selection.

Attrib.	**Sub attrib**	**Description**
C Cap	CL As	Evaluate the cooling requirements of the factory
	C Size	Properly size the $\mathbb{CSYS}$ to meet the calculated cooling load
E Eff	HE Comp	Use energy-efficient chillers, pumps, fans, and heat exchangers
	VS Cont	Implement variable frequency drives to adjust the speed of fans
	H Rec	Consider systems that can recover waste heat for other processes
	Ins	Proper insulation of $\mathbb{CSYS}$'s components to reduce energy losses
T Cont	PT Cont	Precise temperature control mechanisms to maintain setpoints
	Zon	Divide the factory into cooling zones on the basis of cooling needs
Red	B Sys	Install backup chillers to prevent downtime in equipment failure
	RC Pth	Design redundant cooling paths to maintain continuous operation
E Imp	EF Ref	Environmentally friendly refrigerants with low global warming potential
	W Con	Employ water-saving technologies to minimize water usage
	WH Utl	Explore options for using waste heat to reduce environmental impact
RM Cont	M Sen	Sensors to monitor temperature, humidity, & system performance
	R Cont	Remote control capabilities for adjustments & troubleshooting
	A Sys	Set up automated alert systems to notify personnel of issues
A Qty	A Fil	Use air filtration systems to maintain air quality
	Vent	Ensure proper ventilation to maintain fresh air circulation

Table 5. Attributes and sub-attributes for refrigerator selection.

Attrib.	**Sub attrib**	**Description**
M Req	S Man	Maintenance schedule for regular equipment checks/cleaning
	E Acc	Easy access to components for maintenance purposes
S Fea	T Al	Install alarms for temperature deviations outside safe ranges
	PR Val	Include safety valves to prevent overpressure
	E Sd	Implement emergency shutdown systems for critical situations
Sca	F Desn	Plan for system expansion/modification for future factory growth
N Cont	A Ins	Use soundproofing materials/designs to reduce noise pollution
W Man	W Rec	Implement water recycling systems to reduce water consumption
	W Tmt	Include water treatment systems to maintain water quality
IBM Sys	C Pro	Compatibility/integration with factory's Building Management System
C Reg	R Com	The $\mathbb{CSYS}$ complies with safety/energy efficiency regulations
E Prep	ER Pln	Develop/maintain emergency response plans for $\mathbb{CSYS}$ failures
BC Ana	LCC Ana	Evaluate long-term costs like installation, operation & maintenance
T Doc	P Tra	Provide training for operating & maintaining $\mathbb{CSYS}$
	Doc	Maintain comprehensive documentation like maintenance records etc.
F Prof	T Up	Stay informed about advancements in cooling technology

Example 3.2. Suppose an individual (Mr. B) wants to purchase and install $\mathbb{CSYS}$, as in Example 3.1, for his factory. He hires a team of experts for the said purpose. The committee considered

certain attributes and sub-attributes. For the sake of convenience and simplicity of calculations, the committee ignored some attributes/ sub-attributes and preferred some sub-attributes over others. There are four $\mathbb{CSYS}$s available in market that fulfill the criteria of expert committee and form the universe of discourse $\mathbb{U} = \{\check{\gamma}_1, \check{\gamma}_2, \check{\gamma}_3, \check{\gamma}_4\}$. The expert committee considered the set of parameters $\mathfrak{E} = \{e_1, e_2, ..., e_{18}\}$. For i = 1, 2,..., 18, the attributes e_i stand for "cooling capacity", "energy efficiency", "temperature control", "redundancy", "environmental impact", "remote monitoring and control", "air quality", "maintenance requirements", "safety features", "scalability", "noise control", "water management", "integration with building management system", "compliance with regulations", "emergency preparedness", "budget and cost analysis", "training and documentation", and "future proofing", respectively. Corresponding to each attribute, the sets of attribute-values are: $\mathcal{L}_1 = \{e_{11}, e_{12}\}$; $\mathcal{L}_2 = \{e_{21}, e_{22}, e_{23}, e_{24}\}$; $\mathcal{L}_3 = \{e_{31}, e_{32}\}$; $\mathcal{L}_4 = \{e_{41}, e_{42}\}$; $\mathcal{L}_5 = \{e_{51}, e_{52}, e_{53}\}$; $\mathcal{L}_6 = \{e_{61}, e_{62}, e_{63}\}$; $\mathcal{L}_7 = \{e_{71}, e_{72}\}$; $\mathcal{L}_8 = \{e_{81}, e_{82}\}$; $\mathcal{L}_9 = \{e_{91}, e_{92}, e_{93}\}$; $\mathcal{L}_{10} = \{e_{101}\}$; $\mathcal{L}_{11} = \{e_{111}\}$; $\mathcal{L}_{12} = \{e_{121}, e_{122}\}$; $\mathcal{L}_{13} = \{e_{131}\}$; $\mathcal{L}_{14} = \{e_{141}\}$; $\mathcal{L}_{15} = \{e_{151}\}$; $\mathcal{L}_{16} = \{e_{161}\}$; $\mathcal{L}_{17} = \{e_{171}, e_{172}\}$, and $\mathcal{L}_{18} = \{e_{181}\}$. The expert committee preferred some attributes/ sub-attributes over others. e_{11} was preferred over e_{12}; e_{21} and e_{23} were preferred over e_{22} and e_{24}; e_{31} was preferred over e_{32}; e_{42} was preferred over e_{41}; e_{51} and e_{53} were preferred over e_{52}; e_{62} was preferred over e_{61} and e_{63}; e_{72} was preferred over e_{71}; e_{81} was preferred over e_{82}; e_{92} was preferred over e_{92} and e_{93}; e_{121} was preferred over e_{122}; and e_{171} was preferred over e_{172}, respectively. The set is $\mathcal{D} = \mathcal{L}_1 \times \mathcal{L}_2 \times ... \times \mathcal{L}_{18} = \{\delta_1, \delta_2, \delta_3, \delta_4\}$, where each δ_i is an 18-tuple element. We construct Γ-sets $\psi_{\mathcal{D}}(\delta_1), \psi_{\mathcal{D}}(\delta_2), \psi_{\mathcal{D}}(\delta_3), \psi_{\mathcal{D}}(\delta_4)$ as follows:

$$\psi_{\mathcal{D}}(\delta_1) = \left\{ \begin{array}{c} \frac{([0.4,0.5],[0.1,0.2])e^{i([0.1,0.2],[0.1,0.4])\pi}}{\check{\gamma}_1}, \frac{([0.2,0.3],[0.3,0.4])e^{i([0.1,0.3],[0.2,0.3])\pi}}{\check{\gamma}_2}, \\ \frac{([0.1,0.2],[0.1,0.4])e^{i([0.3,0.4],[0.1,0.2])\pi}}{\check{\gamma}_3}, \frac{([0.1,0.2],[0.4,0.5])e^{i([0.1,0.4],[0.1,0.2])\pi}}{\check{\gamma}_4} \end{array} \right\},$$

$$\psi_{\mathcal{D}}(\delta_2) = \left\{ \begin{array}{c} \frac{([0.1,0.2],[0.3,0.5])e^{i([0.1,0.2],[0.2,0.4])\pi}}{\check{\gamma}_1}, \frac{([0.1,0.5],[0.1,0.2])e^{i([0.1,0.2],[0.4,0.5])\pi}}{\check{\gamma}_2}, \\ \frac{([0.2,0.3],[0.1,0.5])e^{i([0.3,0.4],[0.2,0.3])\pi}}{\check{\gamma}_3}, \frac{([0.1,0.2],[0.2,0.4])e^{i([0.2,0.3],[0.1,0.3])\pi}}{\check{\gamma}_4} \end{array} \right\},$$

$$\psi_{\mathcal{D}}(\delta_3) = \left\{ \begin{array}{c} \frac{([0.3,0.4],[0.2,0.3])e^{i([0.1,0.4],[0.2,0.3])\pi}}{\check{\gamma}_1}, \frac{([0.1,0.2],[0.2,0.3])e^{i([0.2,0.3],[0.3,0.4])\pi}}{\check{\gamma}_2}, \\ \frac{([0.2,0.4],[0.1,0.3])e^{i([0.2,0.3],[0.1,0.2])\pi}}{\check{\gamma}_3}, \frac{([0.1,0.2],[0.2,0.3])e^{i([0.1,0.2],[0.2,0.5])\pi}}{\check{\gamma}_4} \end{array} \right\},$$

$$\psi_{\mathcal{D}}(\delta_4) = \left\{ \begin{array}{c} \frac{([0.1,0.3],[0.3,0.4])e^{i([0.1,0.2],[0.4,0.5])\pi}}{\check{\gamma}_1}, \frac{([0.4,0.5],[0.1,0.3])e^{i([0.1,0.4],[0.2,0.3])\pi}}{\check{\gamma}_2}, \\ \frac{([0.1,0.4],[0.1,0.2])e^{i([0.2,0.3],[0.0,0.3])\pi}}{\check{\gamma}_3}, \frac{([0.1,0.2],[0.4,0.5])e^{i([0.2,0.4],[0.2,0.3])\pi}}{\check{\gamma}_4} \end{array} \right\},$$

Step 1. Γ-set $\chi_{\mathcal{D}}$ can also be written as, $\chi_{\mathcal{D}} =$

$$\left\{ \begin{array}{c} \left(\delta_1, \left\{ \begin{array}{c} \frac{([0.4,0.5],[0.1,0.2])e^{i([0.1,0.2],[0.1,0.4])\pi}}{\check{\gamma}_1}, \frac{([0.2,0.3],[0.3,0.4])e^{i([0.1,0.3],[0.2,0.3])\pi}}{\check{\gamma}_2}, \\ \frac{([0.1,0.2],[0.1,0.4])e^{i([0.3,0.4],[0.1,0.2])\pi}}{\check{\gamma}_3}, \frac{([0.1,0.2],[0.4,0.5])e^{i([0.1,0.4],[0.1,0.2])\pi}}{\check{\gamma}_4} \end{array} \right\} \right), \\ \left(\delta_2, \left\{ \begin{array}{c} \frac{([0.1,0.2],[0.3,0.5])e^{i([0.1,0.2],[0.2,0.4])\pi}}{\check{\gamma}_1}, \frac{([0.1,0.5],[0.1,0.2])e^{i([0.1,0.2],[0.4,0.5])\pi}}{\check{\gamma}_2}, \\ \frac{([0.2,0.3],[0.1,0.5])e^{i([0.3,0.4],[0.2,0.3])\pi}}{\check{\gamma}_3}, \frac{([0.1,0.2],[0.2,0.4])e^{i([0.2,0.3],[0.1,0.3])\pi}}{\check{\gamma}_4} \end{array} \right\} \right), \\ \left(\delta_3, \left\{ \begin{array}{c} \frac{([0.3,0.4],[0.2,0.3])e^{i([0.1,0.4],[0.2,0.3])\pi}}{\check{\gamma}_1}, \frac{([0.1,0.2],[0.2,0.3])e^{i([0.2,0.3],[0.3,0.4])\pi}}{\check{\gamma}_2}, \\ \frac{([0.2,0.4],[0.1,0.3])e^{i([0.2,0.3],[0.1,0.2])\pi}}{\check{\gamma}_3}, \frac{([0.1,0.2],[0.2,0.3])e^{i([0.1,0.2],[0.2,0.5])\pi}}{\check{\gamma}_4} \end{array} \right\} \right), \\ \left(\delta_4, \left\{ \begin{array}{c} \frac{([0.1,0.3],[0.3,0.4])e^{i([0.1,0.2],[0.4,0.5])\pi}}{\check{\gamma}_1}, \frac{([0.4,0.5],[0.1,0.3])e^{i([0.1,0.4],[0.2,0.3])\pi}}{\check{\gamma}_2}, \\ \frac{([0.1,0.4],[0.1,0.2])e^{i([0.2,0.3],[0.0,0.3])\pi}}{\check{\gamma}_3}, \frac{([0.1,0.2],[0.4,0.5])e^{i([0.2,0.4],[0.2,0.3])\pi}}{\check{\gamma}_4} \end{array} \right\} \right) \end{array} \right\}.$$

Step 2. The cardinal set is computed as,
$\| \chi_{\mathfrak{D}} \| (\mathbb{A} - term) =$

$$\left\{ \begin{array}{l} ([0.200, 0.300], [0.225, 0.375])/\delta_1, ([0.125, 0.300], [0.175, 0.400])/\delta_2, \\ ([0.175, 0.300], [0.175, 0.300])/\delta_3, ([0.175, 0.350], [0.225, 0.350])/\delta_4, \end{array} \right\}$$

$\| \chi_{\mathfrak{D}} \| (\mathbb{P} - term) =$

$$\left\{ \begin{array}{l} ([0.150, 0.325], [0.125, 0.275])/\delta_1, ([0.175, 0.275], [0.225, 0.375])/\delta_2, \\ ([0.150, 0.300], [0.200, 0.350])/\delta_3, ([0.150, 0.325], [0.200, 0.350])/\delta_4, \end{array} \right\}$$

Step 3. The set $\overbrace{\chi_{\mathfrak{D}}}$ can be determined as,
$\overbrace{\chi_{\mathfrak{D}}}(\mathbb{A} - term) =$

$$\frac{1}{4}\begin{bmatrix} [0.4,0.5],[0.1,0.2] & [0.1,0.2],[0.3,0.5] & [0.3,0.4],[0.2,0.3] & [0.1,0.3],[0.3,0.4] \\ [0.2,0.3],[0.3,0.4] & [0.1,0.5],[0.1,0.2] & [0.1,0.2],[0.2,0.3] & [0.4,0.5],[0.1,0.3] \\ [0.1,0.2],[0.1,0.4] & [0.2,0.3],[0.1,0.5] & [0.2,0.4],[0.1,0.3] & [0.1,0.4],[0.1,0.2] \\ [0.1,0.2],[0.4,0.5] & [0.1,0.2],[0.2,0.4] & [0.1,0.2],[0.2,0.3] & [0.1,0.2],[0.4,0.5] \end{bmatrix}$$

$$\times \begin{bmatrix} [0.200,0.300],[0.225,0.375] \\ [0.125,0.300],[0.175,0.400] \\ [0.175,0.300],[0.175,0.300] \\ [0.175,0.350],[0.225,0.350] \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 0.4 & 0.1 & 0.2 & 0.0 \\ 0.0 & 0.4 & 0.0 & 0.4 \\ 0.1 & 0.2 & 0.3 & 0.3 \\ 0.2 & 0.0 & 0.0 & 0.2 \end{bmatrix}\begin{bmatrix} 0.075 \\ 0.125 \\ 0.125 \\ 0.125 \end{bmatrix} = \begin{bmatrix} 0.016875 \\ 0.025000 \\ 0.026875 \\ 0.010000 \end{bmatrix},$$

$\overbrace{\chi_{\mathfrak{D}}}(\mathbb{P} - term) =$

$$\frac{1}{4}\begin{bmatrix} [0.1,0.2],[0.1,0.4] & [0.1,0.2],[0.2,0.4] & [0.1,0.4],[0.2,0.3] & [0.1,0.2],[0.4,0.5] \\ [0.1,0.3],[0.2,0.3] & [0.1,0.2],[0.4,0.5] & [0.2,0.3],[0.3,0.4] & [0.1,0.4],[0.2,0.3] \\ [0.3,0.4],[0.1,0.2] & [0.3,0.4],[0.2,0.3] & [0.2,0.3],[0.1,0.2] & [0.2,0.3],[0.0,0.3] \\ [0.1,0.4],[0.1,0.2] & [0.2,0.3],[0.1,0.3] & [0.1,0.2],[0.2,0.5] & [0.2,0.4],[0.2,0.3] \end{bmatrix}$$

$$\times \begin{bmatrix} [0.150,0.325],[0.125,0.275] \\ [0.175,0.275],[0.225,0.375] \\ [0.150,0.300],[0.200,0.350] \\ [0.150,0.325],[0.200,0.350] \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 0.1 & 0.0 & 0.2 & 0.2 \\ 0.1 & 0.2 & 0.0 & 0.2 \\ 0.3 & 0.2 & 0.2 & 0.3 \\ 0.3 & 0.2 & 0.0 & 0.2 \end{bmatrix}\begin{bmatrix} 0.200 \\ 0.050 \\ 0.100 \\ 0.125 \end{bmatrix} = \begin{bmatrix} 0.016250 \\ 0.013750 \\ 0.031875 \\ 0.023750 \end{bmatrix},$$

$$\overbrace{\chi_{\mathfrak{D}}} = \left\{ \begin{array}{l} 0.016875e^{i0.016250\pi}/\check{\curlyvee}_1, 0.025000e^{i0.013750\pi}/\check{\curlyvee}_2, \\ 0.026875e^{i0.031875\pi}/\check{\curlyvee}_3, 0.010000e^{i0.023750\pi}/\check{\curlyvee}_4, \end{array} \right\}$$

Consider the modulus value of
$\max(\tau_{\underbrace{\quad}_{\chi_{\mathfrak{D}}}}) = \max\{0.0171514589/\check{\gamma}_1, 0.02534612415/\check{\gamma}_2, 0.02774543956/\check{\gamma}_3, 0.01024034277/\check{\gamma}_4\}$ $= 0.02774543956/\check{\gamma}_3$. This means that the $\mathbb{CSYS}$ $\check{\gamma}_3$ may be recommended for the factory. The pictorial version of ranking can be seen in Figure 3.

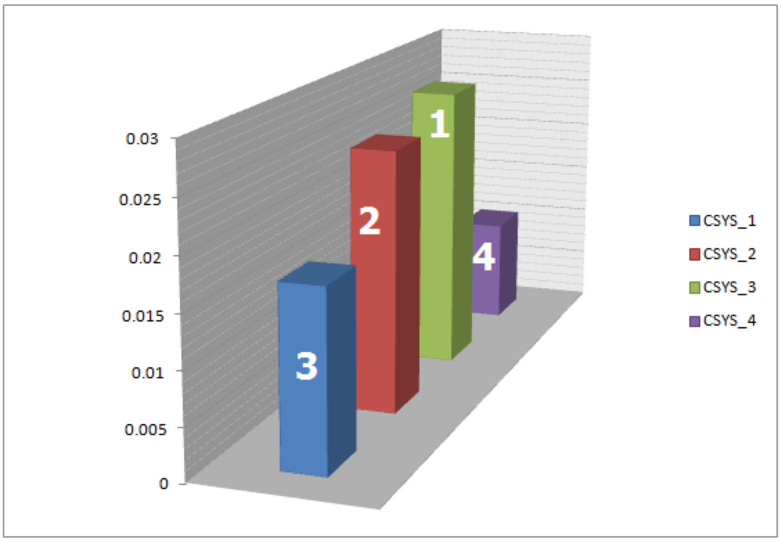

Figure 3. Ranking of alternatives.

3.3. Discussion and comparative analysis

Many different $\mathbb{DMK}$ methods for algorithmic computation using hybridized structures of complex sets with $\mathbb{FS}$ and $\mathbb{IFS}$ in the $\mathbb{SS}$ environment have been previously described in the academic literature by [28, 36–39]. $\mathbb{DMK}$ suffers greatly as a result of various properties that play important functions being absent. It is much more appropriate to further classify these parameters into their $\mathbb{DAVS}$. For instance, it is inadequate to examine simple attributes, neglecting sub-attributes, in the selection of $\mathbb{CSYS}$ of a manufacturing facility. In contrast to single-argument approximate mapping ($\mathbb{SAA}$-mapping), the aforementioned current $\mathbb{DMK}$ frameworks are unsuitable for interval-valued data or multi-argument approximate mapping ($\mathbb{MAA}$-mapping). However, the shortcomings of these models have been addressed in the suggested model. By taking $\mathbb{MAA}$-mapping into account, the $\mathbb{DMK}$ mechanism will become more reliable as well as credible. Table 6 provides a comparative analysis of our suggested structure and the relevant current structures mentioned above.

This part demonstrates how our suggested framework Γ-set is more versatile and generalized than the current pertinent mathematical models, which are specific cases [28, 36–39], by leaving out one or more characteristics such as $\mathbb{MF}_n$, $\mathbb{NF}_n$, $\mathbb{SAA}$ mapping, $\mathbb{MAA}$-mapping, periodic nature of data ($\mathbb{PN}$-data), and $\mathbb{IV}$-data.

Table 6. Comparison.

References	Type of A_{fn}	Limitations
Thirunavukarasu et al. [36]	$\mathbb{SAA}$-mapping	Insufficient for $\mathbb{IV}$ data, $\mathbb{NF}_n$ and partitioning $\mathbb{SP}$ to $\mathbb{DAVS}$.
Kumar et al. [37]	$\mathbb{SAA}$-mapping	Insufficient for $\mathbb{IV}$ data and partitioning $\mathbb{SP}$ to $\mathbb{DAVS}$
Khan et al. [39]	$\mathbb{SAA}$-mapping	Insufficient for $\mathbb{IV}$ data and partitioning $\mathbb{SP}$ to $\mathbb{DAVS}$
Selvachandran et al. [38]	$\mathbb{SAA}$-mapping	Insufficient for $\mathbb{NF}_n$ and partitioning $\mathbb{SP}$ to $\mathbb{DAVS}$
Rahman et al. [28]	$\mathbb{MAA}$-mapping	Insufficient for $\mathbb{IV}$ data, $\mathbb{NF}_n$.
Rahman et al. [28]	$\mathbb{MAA}$-mapping	Insufficient for $\mathbb{IV}$ data.
Rahman et al. [29]	$\mathbb{MAA}$-mapping	Insufficient for $\mathbb{NF}_n$.
Proposed Structure	$\mathbb{MAA}$-mapping	Addresses the limitations and shortcomings of above structures.

3.4. Advantageous features of the proposed framework

The proposed model is beneficial for $\mathbb{DMK}$ and uncertainty modeling due to a number of reasons. Here are some important benefits:

(1) A thorough framework for modeling and representing uncertainty is offered by Γ-sets. In the setting of complex numbers, they better express the inherent ambiguity and variety in choice scenarios by employing $\mathbb{IV}$ $\mathbb{MF}_n$ and $\mathbb{NF}_n$.

(2) The Γ-sets may manage complicated interactions among variables thanks to the inclusion of complex numbers. Decision-makers may now record and examine the phase information, oscillatory behavior, and interactions that are essential in complicated $\mathbb{DMK}$ scenarios because of this flexibility.

(3) The $\mathbb{CIFS}$s with $\mathbb{IV}$ make it easier to integrate and assess a number of traits or criteria. They provide a more thorough evaluation of complicated choice situations by enabling decision-makers to thoroughly analyze options based on interval-valued $\mathbb{MF}_n$ and $\mathbb{NF}_n$.

(4) As an effective decision-support tool, Γ-sets are used. They give decision-makers a well-organized framework for evaluating options, taking uncertainty and intricate linkages into account, and producing thorough rankings or assessments based on $\mathbb{IV}$ data.

(5) The Γ-set improves the accuracy and dependability of $\mathbb{DMK}$ processes by handling complicated choice situations with uncertainty and imprecision efficiently.

(6) The suggested method considered the importance of the Γ-set concept in addressing contemporary $\mathbb{DMK}$ problems. This relationship has enormous possibilities in the true portrayal inside the domain of cognitive invasions since the theory offered allows the investigators to confront an actual situation where the frequency of data in the form of intervals exists.

(7) The structure that is suggested, increases flexibility and improves the reliability of the $\mathbb{DMK}$ process by emphasizing in-depth analysis of attributes (partitioning $\mathbb{SP}$ to $\mathbb{DAVS}$) instead of concentrating on attributes alone.

(8) It covers the characteristics and properties of the existing relevant structures, i.e., $\mathbb{IVCFHSS}$,

$\mathbb{CFHSS}$, $\mathbb{CIFHSS}$, $\mathbb{IVCFSS}$, $\mathbb{IVCIFSS}$, $\mathbb{CFSS}$, $\mathbb{CIFSS}$, etc., so it is not unreasonable to call it the generalized form of all these structures.

The advantages of the proposed study can easily be judged from Tables 6 and 7. The comparison is evaluated on the basis of two different aspects:

(1) Main features discussed in the study (see Table 6).
(2) Features like $\mathbb{MF}_n$, $\mathbb{NF}_n$, $\mathbb{SAA}$-mapping, $\mathbb{MAA}$-mapping, $\mathbb{PN}$-data, and $\mathbb{IV}$-data (see Table 7).

Table 7. Comparison with existing models under appropriate features.

Authors	$\mathbb{MF}_n$	$\mathbb{NF}_n$	$\mathbb{SAA}$-mapping	$\mathbb{MAA}$-mapping	$\mathbb{PN}$-data	$\mathbb{IV}$-data
Thirunavukarasu et al. [36]	✓	×	✓	×	✓	×
Kumar et al. [37]	✓	✓	✓	×	✓	×
Khan et al. [39]	✓	✓	✓	×	✓	×
Selvachandran et al. [38]	✓	×	✓	×	✓	✓
Rahman et al. [28]	✓	×	✓	✓	✓	×
Rahman et al. [28]	✓	✓	✓	✓	✓	×
Rahman et al. [29]	✓	×	✓	✓	✓	✓
Proposed Structure	✓	✓	✓	✓	✓	✓

3.5. *Managerial implications*

The results of this study have important managerial ramifications for those making decisions across a range of businesses.

(1) Managers now have an effective tool to tackle complicated $\mathbb{DMK}$ problems comprehensively thanks to the invention of the Γ-set framework. Managers may make more informed and efficient decisions by better analyzing and interpreting complicated datasets by grasping and utilizing the Γ-set ideas.
(2) Managers are guided from set formation to effective decisions by the investigation's algorithm, which provides an organized method of $\mathbb{DMK}$.
(3) The technique's assessment through a hypothetical case study highlights how applicable and useful it is in everyday circumstances.

4. Conclusions

This study presents a novel theoretical context, Γ-set, that incorporates complex plane settings, interval-valued intuitionistic fuzzy settings, and hypersoft settings collectively to manage data driven ambiguities. An investigation is carried out on the notions of the Γ-set and its set operations and aggregations, including decision matrix, cardinal matrix, aggregate matrix, and cardinality set. Subsequently, a compelling algorithm is presented that contains nine phases for having final selection. Additionally, the algorithm has been validated by presenting a hypothetical case study that examines eighteen evaluation qualities and thirty-four sub-attributes for identifying an ideal $\mathbb{CSYS}$ for manufacturing. As crucial components of research, informative comparative analysis and chosen study features are offered to help academics make important advancements in their sector and progressively advance their specialization. While it represents a viable method for addressing intricate issues within a cohesive structure, it is not devoid of limitations. To begin, for researchers who are not acquainted with the theoretical framework, the intricacy of the Γ-set notions, set operations, and aggregations may provide difficulties in terms of practical application and comprehension. Furthermore, the framework's applicability to different disciplines or situations may be restricted by the hypothetical case study used to validate it. Even if the research offers insightful solutions and useful tools for handling complicated issues, more testing in a range of situations and ongoing framework improvement is required to guarantee the framework's wide applicability and efficacy in several domains. The future prospects of this study will focus on various important areas to improve the resilience and practicality of the Γ-set. First, to support a larger range of challenging domains and enhance the accessibility for practitioners, further investigation and development of the Γ-set notions and operations is required. Further study should concentrate on developing more sophisticated algorithms and techniques that can make efficient use of the Γ-set framework to solve challenging $\mathbb{DMK}$ issues in a variety of settings. Additionally, there is the possibility of carrying out more case studies in various contexts and industries to confirm the applicability and efficacy of the suggested strategy.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflicts of interest

Authors declare no conflicts of interest.

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