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Research article

Innovative approaches to solar cell selection under complex intuitionistic fuzzy dynamic settings

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Abstract: The need to meet current energy demands while protecting the interests of future generations has driven people to adopt regulatory frameworks that promote the careful use of limited resources. Among these resources, the sun is an everlasting source of energy. Solar energy stands out as a prime example of a renewable and environmentally friendly energy source. An imperative requirement exists for precise and dependable decision-making methods for the selection of the most efficacious solar cell. We aimed to address this particular issue. The theory of complex intuitionistic fuzzy sets (CIFS) adeptly tackles ambiguity, encompassing complex problem formulations characterized by both intuitionistic uncertainty and periodicity. We introduced two aggregation operators: The complex intuitionistic fuzzy dynamic ordered weighted averaging (CIFDOWA) operator and the complex intuitionistic fuzzy dynamic ordered weighted geometric (CIFDOWG) operator. Noteworthy features of these operators were stated, and significant special cases were meticulously outlined. An updated score function was devised to address the deficiencies, identified in the current score function within the context of CIF knowledge. In addition, we devised a methodical strategy for managing multiple attribute decision-making (MADM) problems that involve CIF data by implementing the proposed operators. To demonstrate the efficacy of the formulated algorithm, we presented a numerical example involving the selection of solar cells together with a comparative analysis with several well-established methodologies.

Keywords: CIFS; CIFDOWA operator; CIFDOWG operator; decision making; optimization; algorithms
Mathematics Subject Classification: 90B50, 94D05

1. Introduction

1.1. Background

Solar energy is an economical and ecologically sustainable fuel option utilized in the production of electricity. By means of the photovoltaic effect, a solar cell converts the energy emitted by light into electrical energy. Fuzzy logic was employed by Kwi-Seong et al. [1] to assess hybrid systems that integrated fuel cells, batteries, DC/AC converters, and DC/AC inverters. To ascertain the most advantageous location for connecting solar photovoltaic installations to the power grid in the province of Granada, Spain, Arán-Carrión et al. [2] conducted a study. Several thermal solar concentrator devices were evaluated by Cavallaro [3] in the field of high temperature solar thermal energy using the PROMETHEE method. In [4], the fuzzy TOPSIS method was employed to assess the thermal energy storage capabilities of concentrated solar power systems. Cavallaro [5] conducted a comparative analysis of thin-film photovoltaic manufacturing processes using an ELECTRE III technique. The averaging aggregation operators of T-spherical fuzzy sets were introduced by Shouzhen Zeng et al. [6] for use in the solar cell selection process. Multiple attribute decision-making (MADM) is currently a prominent topic within the domain of decision-making processes aimed at determining the best alternative(s) from a set of feasible choices. In practical decision-making, decision-makers are required to convey their evaluations of attributes using various evaluation methods, including crisp numbers, interval numbers, and fuzzy numbers [7–10]. However, due to growing data uncertainty and the cognitive limitations experienced by decision-makers, it frequently proves challenging for them to represent their preferences using real numerical values. Fuzzy sets (FSs) were introduced by Zadeh [11] as a framework to evaluate indeterminate and ambiguous information. Fuzzy sets are supported by membership function with the range [0,1]. Recent studies have demonstrated that many domains compile characteristics and assess their distinctions, thereby augmenting their reliance on FSs [12–14]. Atanassov [15] introduced the concept of intuitionistic fuzzy sets (IFSs), which are composed of membership and non-membership functions. These functions ensure that the combined degrees of membership and non-membership fall within the range of values between 0 and 1. Researchers have proposed many techniques that use IFSs to address the difficulty of integrating and quantifying the differences among multiple attributes in distinct areas. Liu et al. [16] proposed a hybrid solution that combines flexible weighting for IFS. Thao [17] conducted an inquiry into the application of entropies and divergence metrics within the domain of IFS, incorporating the consideration of Archimedean norms. Building upon this investigation, Gohain et al. [18] extended the study to examine the similarity and distance metrics pertaining to IFS. Furthermore, Garg and Rani [19] formulated and examined similarity measures specifically designed for IFSs. Hayat et al. [20] delved into the exploration of novel aggregation operators strategically designed to effectively convey information within the framework of IFS. To attain a more comprehensive understanding of advancements in the realm of IFSs, readers are advised to refer to the sources listed as references [21–28].

The practical advantages of the knowledge mentioned in the preceding paragraph is constrained by its dependence on FS and IFS, which are utilized in the one-dimensional processing of data. This limitation may result in the loss of data for specialists. Instead of FS and IFS, there is an increasing demand for a method that can effectively manage two-dimensional data. In response to this requirement, Ramot et al. [29] introduced the complex fuzzy set (CFS) theoretical framework subsequent to an extensive investigation. Membership in the CFS is enhanced by the "phase term," a periodic component that is crucial for decision-making. Calculations of distance and averages between qualities continue to present challenges. As a result of these intricate contexts, the incorporation of CFS into various industries has been the subject of debate [30–33]. By including a complex-valued non-membership degree, [34] expands the notion of CFS and produces CIFS. To provide an example, the authors of [35] examined novel aggregation operators that operate within the CIFS domain. A comprehensive analysis of robust aggregation techniques within CIFS was performed in reference [36], whereas reference [37] explored generalized geometric aggregation strategies within the CIFS framework. In order to establish a systematic framework for handling MADM situations involving CIF data, Dilshad et al. [38] introduced and implemented two novel aggregation operators. Masmali et al. [39] proposed a method to determine the most effective water purification method under CIF framework. A comprehensive theoretical framework was established in reference [40] with regards to prioritized aggregation operators for CIF soft information.

In addition to these, numerous additional researchers introduced other types of aggregation operators and their utilization in the decision-making process. A few examples are linguistic q-rung orthopair fuzzy sets and their interactional partitioned Heronian mean aggregation operators [41], picture fuzzy interactional partitioned Heronian mean aggregation operators, an application to the MADM process [42], and multi-attribute group decision-making based on linguistic Pythagorean fuzzy interaction partitioned Bonferroni mean aggregation operators [43]. Yager [44] presented a generalized class of averaging operators with the ordered weighted averaging (OWA) operator. Located between the max and min operators, this operator is distinguished by a reordering operation that assigns the weight vector in accordance with the descending order of the input parameters. To obtain further information regarding the evolution of OWA operators across different domains, we refer to the references [45–54].

1.2. Research gap, motivations, and contributions

Xu and Yager [55] showed that the majority of current research focuses on decision-making scenarios in which all pertinent decision information is accessible simultaneously. On the other hand, Wei, G. W. [56] put forth a number of geometric aggregation functions and their implementation in dynamic MADM in IF settings. These dynamic aggregation operators were defined for IFS containing information with real-valued membership and non-membership degrees. But they only take into account the amplitude term and cannot handle the phase term. This research challenge highlights the importance of defining dynamic aggregation operators in a CIF environment. The idea of CIFS is a generalization of the theories of CFS and IFS that considers significantly more object-related information during the process and handles two-dimensional data in a single set. However, the existing studies related to CIF cannot be evaluated with time-dependent data. In the absence of a dynamic component, alterations in the significance of data points might not be reflected in the aggregated output.

This insensitivity could potentially lead to erroneous conclusions in situations where the relevance of data elements evolves over time.

One of the main advantages of the current study is that the prompt response of dynamic weighted aggregation operators to environmental or input data changes makes them ideal for time-sensitive decision-making. But when making different kinds of decisions, like multi-period investment decisions, dynamic medical diagnostics, dynamic evaluation of the efficiency of military systems, and personnel dynamic assessment, the important data for making decisions are often gathered at different times. Dynamic ordered weighted aggregation operators make decisions more accurate and flexible by combining data in a smart way that changes over time. They facilitate the ability to make decisions in real-time, mitigate risks, improve the allocation of resources, contribute to planning for the future, and have the potential to generate cost savings. Through an examination of the distinct challenges presented by dynamic decision knowledge, this research seeks to contribute significantly to the field of MADM. In this current study, we overcome the deficiency of existing literature by introducing two novel dynamic aggregation operators, namely, CIFDOWA and CIFDOWG, within the framework of CIF settings. The major contributions to this study are outlined as follows:

1) The limitations of the current score function has been identified and an improved score function is introduced. The development will help to improve the ranking system in CIF system
2) Two novel aggregation operators, CIFDOWA and CIFDOWA, have been developed to manage complex scenarios involving CIF data in decision-making.
3) The structural characteristics of CIFDOWA and CIFDOWG operators, specifically idempotency, monotonicity, and boundedness, have been proven. This highlights the logical existence of the proposed operators.
4) By utilizing newly defined operators, a methodical approach to tackling MADM issues within the context of CIF information is given.
5) By utilizing the proposed method to solve a real-world MADM problem such as the selection of solar cells, its practical application is highlighted.
6) An extensive comparative study is conducted to assess the viability of the suggested method in relation to various well-established strategies. The developed approach is consistent and reliable, as shown by the comparison findings.

The succeeding sections of this manuscript are organized as follows: Section 2 delivers a comprehensive exposition of fundamental definitions. In Section 3, a deficiency in the existing score function is addressed by introducing a specially built new score function that resolves this limitation within the CIF framework. Section 4 elaborates on dynamic aggregation operators expressly devised for CIFS and delves into an exploration of their fundamental properties. Section 5 elaborates on a method to tackle the complex issue of MADM with CIF information, utilizing CIF dynamic weighted aggregation operators. Section 6 depicts a numerical illustration of the proposed strategies with which the MADM problem of efficient solar cell selection is addressed. Moreover, this section conducts a comparative analysis with the objective of elucidating the efficacy and feasibility of these pioneering strategies compared to established methodologies. In Section 7, the paper concludes by summarizing the primary findings and discussing potential implications.

2. Preliminaries

This section provides a concise overview of the fundamental definitions that are necessary for understanding the content delineated within this article.

Definition 1. [15]. An IFS A of the universe W, is characterized as follows: $A = \{(x, \mu_A(x), \nu_A(x): x \in W\}$.
Here $\mu_A: W \longrightarrow [0,1]$ and $\nu_A: W \longrightarrow [0,1]$ are known as the membership and non-membership functions, respectively, such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The hesitancy margin of A is defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.
Definition 2. [29]. A CFS A of the universe W, is expressed as follows: $A = \{(x, \mu_A(x)) : x \in W\}$. Here $\mu_A: W \rightarrow \{c : c \epsilon C, |c| \leq 1\}$ signifies the complex-valued membership degree function, defined as $\mu_A(x) = \mathrm{r}_A(x) e^{i2\Pi\theta_A(x)}$. The parameters $\mathrm{r}_A(x)$ and $\theta_A(x)$ are amplitude and phase terms, respectively. Moreover, $\mathrm{r}_A(x), \theta_A(x) \in [0, 1]$.
Definition 3. [34]. A CIFS A of the universe W is delineated as: $A = \{(x, \mu_A(x), \nu_A(x)) : x \in W\}$. Here, μ_A and ν_A denote the complex-valued membership and non-membership functions, respectively. These functions allocate a complex number within the closed unit disk to each element, defined as: $\mu_A(x) = \mathrm{r}_A(x) e^{i2\Pi\theta_A(x)}$ and $\nu_A(x) = \mathrm{K}_A(x) e^{i2\pi\varphi_A(x)}$. Furthermore, $\mathrm{r}_A(x)$, $\mathrm{K}_A(x)$, $\theta_A(x)$ and $\varphi_A(x)$ satisfy $0 \leq \mathrm{r}_A(x),\ \mathrm{K}_A(x),\ \theta_A(x),\ \varphi_A(x),\ \mathrm{r}_A(x) + \mathrm{K}_A(x),\ \theta_A(x) + \varphi_A(x) \leq 1$.

To simplify the representation, the membership and non-membership degrees of $x \in W$ are denoted as $A = ((\mathrm{r}, \theta), (\mathrm{K}, \varphi))$ and are called complex intuitionistic fuzzy numbers (CIFNs). The CIFNs satisfy $0 \leq \mathrm{r}, \mathrm{K}, \mathrm{r} + \mathrm{K} \leq 1$, and $0 \leq \theta, \varphi, \theta + \varphi \leq 1$.

We highlight the idea of an intuitionistic fuzzy variable in the following definition.
Definition 4. [56]. In the context of time variable t, we establish the IF variable α_t as follows: $\alpha_t = (\mu_t, \nu_t)$, where μ_t and ν_t belong to [0,1] and $\mu_t + \nu_t \leq 1$. For the IF variable $\alpha_t = (\mu_t, \nu_t)$, if t takes values $t_1, t_2, \ldots, t_p$, then $\alpha_{t_1}, \alpha_{t_2}, \ldots, \alpha_{t_p}$ represent p fuzzy numbers that were obtained at p distinct time periods.

Aggregation operators are essential mathematical tools used to combine multiple inputs into a single output. Here, we describe the ideas of some important aggregation operators in CIF settings.
Definition 5. [36]. Let ψ be the collection of CIFNs $\alpha_\gamma = \left((\mathrm{r}_\gamma, \theta_\gamma), (\mathrm{K}_\gamma, \varphi_\gamma)\right)$, for $\gamma = 1,2, \ldots, n$. Additionally, let $w = [w_1, w_2, \ldots, w_n]^T$ represents the weight vector relative to these CIFNs, such that $w_\gamma \in [0,1]$ and $\sum_{\gamma=1}^n w_\gamma = 1$. For a collection of n CIFNs, $\alpha_1, \alpha_2, \ldots, \alpha_n$, the CIF weighted averaging operator (CIFWA) is a function $CIFWA: \psi^n \rightarrow \psi$, which is defined in the following way:

$$CIFWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \oplus_{\gamma=1}^n w_\gamma \alpha_\gamma = \begin{pmatrix} (1 - \prod_{\gamma=1}^n (1 - \mathrm{r}_\gamma)^{w_\gamma}, 1 - \prod_{\gamma=1}^n (1 - \theta_\gamma)^{w_\gamma}), \\ (\prod_{\gamma=1}^n (\mathrm{K}_\gamma)^{w_\gamma}, \prod_{\gamma=1}^n (\varphi_\gamma)^{w_\gamma}) \end{pmatrix}.$$

Definition 6. [36]. Let ψ be the collection of CIFNs $\alpha_\gamma = \left((\mathrm{r}_\gamma, \theta_\gamma), (\mathrm{K}_\gamma, \varphi_\gamma)\right)$, for $\gamma = 1,2, \ldots, n$. The weight vector $w = [w_1, w_2, \ldots \ldots \ldots, w_n]^T$ pertains to these CIFNs, adhering to $w_\gamma \in [0,1]$, and $\sum_{\gamma=1}^n w_\gamma = 1$. For a given set of n CIFNs, $\alpha_1, \alpha_2, \ldots, \alpha_n$, the mapping CIF weighted geometric operator (CIFWG), denoted as $CIFWG: \psi^n \rightarrow \psi$, could be described as follows:

$$CIFWG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \otimes_{\gamma=1}^n {\alpha_\gamma}^{w_\gamma} = \begin{pmatrix} (\prod_{\gamma=1}^n (\mathrm{r}_\gamma)^{w_\gamma}, \prod_{\gamma=1}^n (\theta_\gamma)^{w_\gamma}), \\ (1 - \prod_{\gamma=1}^n (1 - \mathrm{K}_\gamma)^{w_\gamma}, 1 - \prod_{\gamma=1}^n (1 - \varphi_\gamma)^{w_\gamma}) \end{pmatrix}.$$

Definition 7. [36]. Let ψ be the collection of CIFNs $\alpha_\gamma = \left((\mathrm{r}_\gamma, \theta_\gamma), (\mathrm{K}_\gamma, \varphi_\gamma)\right)$, for $\gamma = 1,2, \ldots, n$. Additionally, let $w = [w_1, w_2, \ldots \ldots \ldots, w_n]^T$ represents the weight vector of these CIFNs, such that $w_\gamma \in [0,1]$ and $\sum_{\gamma=1}^n w_\gamma = 1$. For a collection of n CIFNs, $\alpha_1, \alpha_2, \ldots, \alpha_n$, the mapping CIF ordered weighted averaging operator (CIFOWA), denoted as $CIFOWA: \psi^n \rightarrow \psi$, is described as follows:

$$CIFOWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \oplus_{\gamma=1}^{n} w_\gamma \alpha_{\sigma(\gamma)} = \begin{pmatrix} 1-\prod_{\gamma=1}^{n}(1-\text{г}_{\sigma(\gamma)})^{w_\gamma}, 1-\prod_{\gamma=1}^{n}(1-\theta_{\sigma(\gamma)})^{w_\gamma}, \\ (\prod_{\gamma=1}^{n}(\text{К}_{\sigma(\gamma)})^{w_\gamma}, \prod_{\gamma=1}^{n}(\varphi_{\sigma(\gamma)})^{w_\gamma}) \end{pmatrix}.$$

Here $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation such that $\alpha_{\sigma(\gamma-1)} \geq \alpha_{\sigma(\gamma)}$ for all γ.

Definition 8. [36]. Let ψ be the collection of CIFNs $\alpha_\gamma = ((\text{г}_\gamma, \theta_\gamma), (\text{К}_\gamma, \varphi_\gamma))$, for $\gamma = 1,2, \ldots, n$. The weight vector $w = [w_1, w_2, \ldots \ldots \ldots, w_n]^T$ corresponds to these CIFNs, adhering to $w_\gamma \in [0,1]$, and $\sum_{\gamma=1}^{n} w_\gamma = 1$. For a given set of n CIFNs, $\alpha_1, \alpha_2, \ldots, \alpha_n$, the mapping CIF ordered weighted geometric operator (CIFOWG), denoted as $CIFOWG: \psi^n \to \psi$, could be described as follows:

$$CIFOWG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \otimes_{\gamma=1}^{n} {\alpha_{\sigma(\gamma)}}^{w_\gamma} = \begin{pmatrix} (\prod_{\gamma=1}^{n}(\text{г}_{\sigma(\gamma)})^{w_\gamma}, \prod_{\gamma=1}^{n}(\theta_{\sigma(\gamma)})^{w_\gamma}), \\ (1-\prod_{\gamma=1}^{n}(1-\text{К}_{\sigma(\gamma)})^{w_\gamma}, 1-\prod_{\gamma=1}^{n}(1-\varphi_{\sigma(\gamma)})^{w_\gamma}) \end{pmatrix}.$$

Here $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation such that $\alpha_{\sigma(\gamma-1)} \geq \alpha_{\sigma(\gamma)}$ for all γ.

The next definition emphasizes the evaluation and ordering procedures that employ specific score and accuracy functions within the existing structure.

Definition 9. [34]. For any CIFN α_0=$((\text{г}_0, \theta_0), (\text{К}_0, \varphi_0))$, wherein the score function is articulated as follows:

$$S(\alpha_0) = \text{г}_0 - \text{К}_0 + \theta_0 - \varphi_0,\ S(\alpha_0) \in [-2,2].$$

The accuracy function is specified as:

$$H(\alpha_0) = \text{г}_0 + \text{К}_0 + \theta_0 + \varphi_0, H(\alpha_0) \in [0,2].$$

Additionally, it is essential to emphasize that α_1 and α_2, representing distinct CIFNs, adhere to the stated comparison laws as follows:

i. If $S(\alpha_1) > S(\alpha_2)$, then $\alpha_1 > \alpha_2$;

ii. If $S(\alpha_1) < S(\alpha_2)$, then $\alpha_1 < \alpha_2$;

iii. If $S(\alpha_1) = S(\alpha_2)$, then if $H(\alpha_1) > H(\alpha_2) \Rightarrow \alpha_1 > \alpha_2$, $H(\alpha_1) < H(\alpha_2) \Rightarrow \alpha_1 < \alpha_2$ and $H(\alpha_1) = H(\alpha_2) \Rightarrow \alpha_1 = \alpha_2$.

3. An improvement of the existing score functions for CIFNs

This section identifies the shortcomings of the score function that was devised in reference [34] and proposes an improved version that is more suitable for CIFNS.

Example 1. Let $\alpha_1 = ((0.4,0.6), (0.3,0.2))$ and $\alpha_2 = ((0.7,0.3), (0.1,0.4))$ be two CIFNs. Applying definition 9 to CIFNs α_1 and α_2 yields $S(\alpha_1) = S(\alpha_2) = 0.5$ and $H(\alpha_1) = H(\alpha_2) = 1.5$. Using definition 9, it is easy to verify that CIFNs α_1 and α_2 are not comparable.

The example given above highlights the inherent deficiency of the existing score function within our designated domain of investigation. As a result, it motivates our endeavor to improve this score function, delineated in the following definition.

Definition 10. For any $\alpha_0 = ((\text{г}_0, \theta_0), (\text{К}_0, \varphi_0))$ signifies a CIFN. The updated score function $\acute{C}(\alpha_0)$ for CIFNs is defined in the following way:

$$Ć(\alpha_0) = \frac{1}{2}(\mathrm{r}_0 + \theta_0 - \mathrm{K}_0 + \varphi_0 - \mathrm{r}_0\mathrm{K}_0 + \theta_0\varphi_0).$$

Herein, $Ć(\alpha_0) \in [-2,2]$.

It is essential to mention that the given score function complies with the comparison rules, that is, $Ć(\alpha_1) > Ć(\alpha_2) \Longrightarrow \alpha_1 > \alpha_2$, $Ć(\alpha_1) < Ć(\alpha_2) \Longrightarrow \alpha_1 < \alpha_2$, and $Ć(\alpha_1) = Ć(\alpha_2) \Longrightarrow \alpha_1 = \alpha_2$.

The subsequent illustration is provided to clarify the accuracy and effectiveness that are intrinsic to the suggested score function designed for CIFNs.

Example 2. A pair of arbitrary-selected CIFNs are denoted as $\alpha_1 = \big((0.4,0.6),(0.3,0.2)\big)$ and $\alpha_2 = \big((0.7,0.3),(0.1,0.4)\big)$, respectively. Example 1 has already shown the limits of the existing score function when used with these CIFNs. Using the framework described in definition 10, the application of this framework to these CIFNs results in $Ć(\alpha_1) = 0.45$ and $Ć(\alpha_2) = 0.68$. Therefore, based on the principle explained in property 2 of definition 10, it is clear that $\alpha_1 < \alpha_2$. Based on this substantial evidence, it may be inferred that α_2 is indeed preferable to α_1.

4. Dynamic operations on CIFNs

The aggregation of information is a fundamental and vital element in the study of information fusion. Specifically, the operators CIFOWA and CIFOWG are designed to aggregate CIF information, particularly when dealing with time-independent parameters. However, when accounting for the time period, it is vital to acknowledge that the acquisition of CIF information occurs at varying intervals. Consequently, it becomes imperative to assure that both the aggregation operators and their respective weights are not held invariable. In the subsequent sections, we proceed by establishing the foundational framework of a CIF variable.

4.1. Dynamic operational laws of CIFNs

In this section, we present the concept of CIF variable and expound upon several fundamental dynamic operational laws governing these variable.

Definition 11. [38]. Let t denote a time variable. We define $\alpha_t = \big((\mathrm{r}_t, \theta_t), (\mathrm{K}_t, \varphi_t)\big)$ as a CIF variable, where r_t, K_t, θ_t, and φ_t lies within the closed interval [0,1], subject to the constraints $\mathrm{r}_t + \mathrm{K}_t \leq 1$ and $\theta_t + \varphi_t \leq 1$.

For the CIF variable $\alpha_t = \big((\mathrm{r}_t, \theta_t), (\mathrm{K}_t, \varphi_t)\big)$, when t assumes values $t_1, t_2, \ldots, t_p$, the expressions $\alpha_{t_1}, \alpha_{t_2}, \ldots, \alpha_{t_p}$ denote a set of p distinct CIF numbers observed at p different time periods.

Within the context of CIFNs, we elucidate the basic laws that govern their interactions by means of the definitions 12 and 13.

Definition 12. [38]. Consider two CIFNs, denoted as $\alpha_{t_1} = \Big(\big(\mathrm{r}_{t_1}, \theta_{t_1}\big), \big(\mathrm{K}_{t_1}, \varphi_{t_1}\big)\Big)$ and $\alpha_{t_2} = \Big(\big(\mathrm{r}_{t_2}, \theta_{t_2}\big), \big(\mathrm{K}_{t_2}, \varphi_{t_2}\big)\Big)$. The basic operational principles regulating their interplay can be expressed as follows:

i. $\alpha_{t_1} \leq \alpha_{t_2}$ *if* $\mathrm{r}_{t_1} \leq \mathrm{r}_{t_2}, \mathrm{K}_{t_1} \geq \mathrm{K}_{t_2}$ and $\theta_{t_1} \leq \theta_{t_2}, \varphi_{t_1} \geq \varphi_{t_2}$;

ii. $\alpha_{t_1} = \alpha_{t_2}$ if and only if $\alpha_{t_1} \subseteq \alpha_{t_2}$ and $\alpha_{t_2} \subseteq \alpha_{t_1}$;

iii. $\alpha_{t_1}^c = \Big(\big(\mathrm{K}_{t_1}, \varphi_{t_1}\big), \big(\mathrm{r}_{t_1}, \theta_{t_1}\big)\Big)$.

Definition 13. [38]. Consider two CIFNs, denoted as $\alpha_{t_1}=\left(\left(\Gamma_{t_1},\theta_{t_1}\right),\left(K_{t_1},\varphi_{t_1}\right)\right)$ and $\alpha_{t_2}=\left(\left(\Gamma_{t_2},\theta_{t_2}\right),\left(K_{t_2},\varphi_{t_2}\right)\right)$, alongside a positive real number denoted by λ, we precisely articulate the fundamental dynamic operations in the subsequent manner:

i. $\alpha_{t_1}\oplus\alpha_{t_2}=\left(\left(1-\prod_{k=1}^{2}\left(1-\Gamma_{t_k}\right),1-\prod_{k=1}^{2}\left(1-\theta_{t_k}\right)\right),\left(\prod_{k=1}^{2}K_{t_k},\prod_{k=1}^{2}\varphi_{t_k}\right)\right)$;

ii. $\alpha_{t_1}\otimes\alpha_{t_2}=\left(\left(\prod_{k=1}^{2}\Gamma_{t_k},\prod_{k=1}^{2}\theta_{t_k}\right),\left(1-\prod_{k=1}^{2}\left(1-K_{t_k}\right),1-\prod_{k=1}^{2}\left(1-\varphi_{t_k}\right)\right)\right)$;

iii. $\lambda\alpha_{t_1}=\left(1-\left(1-\Gamma_{t_1}\right)^{\lambda},1-\left(1-\theta_{t_1}\right)^{\lambda}\right),\left(\left(K_{t_1}\right)^{\lambda},\left(\varphi_{t_1}\right)^{\lambda}\right)$;

iv. ${\alpha_{t_1}}^{\lambda}=\left(\left(\Gamma_{t_1}\right)^{\lambda},\left(\theta_{t_1}\right)^{\lambda}\right),\left(1-\left(1-K_{t_1}\right)^{\lambda},1-\left(1-\varphi_{t_1}\right)^{\lambda}\right)$.

4.2. Structural properties of CIFDOWA operator

Within this subsection, we define the notion of the CIF dynamic ordered weighted averaging operator and show its basic structural features.

Definition 14. Consider ψ be the collection of CIFNs $\alpha_{t_k}=\left(\left(\Gamma_{t_k},\theta_{t_k}\right),\left(K_{t_k},\varphi_{t_k}\right)\right)$, at distinct time periods with k ranging from 1 to p. Furthermore, let $\lambda_t=\left[\lambda_{t_1},\lambda_{t_2},\dots\dots..,\lambda_{t_p}\right]^T$ represents the weight vector associated with time periods t_k, adhering to $\lambda_{t_k}\in[0,1]$, satisfying the constraint $\sum_{k=1}^{p}\lambda_{t_k}=1$. Within this context, we introduce a mapping $CIFDOWA:\ \psi^p\rightarrow\psi$ defined as follows:

$$CIFDOWA\left(\alpha_{t_1},\alpha_{t_2},\dots,\alpha_{t_p}\right)=\oplus_{k=1}^{p}\lambda_{t_k}\alpha_{\sigma(t_k)},$$

where $\left((\sigma(t_1),\sigma(t_2),\dots\dots,\sigma\left(t_p\right)\right)$ is a permutation of $(t_1,t_2,\dots\dots..,t_p)$ such that $\alpha_{\sigma(t_{k-1})}\geq\alpha_{\sigma(t_k)}$ for all k.

The following result describes that when a finite number of CIFNs are combined using the CIFDOWA operator, the resulting value is also a CIFN.

Theorem 1. Let $\alpha_{t_k}=\left(\left(\Gamma_{t_k},\theta_{t_k}\right),\left(K_{t_k},\varphi_{t_k}\right)\right)$ be the set of CIFNs at p distinct time periods t_k, where $k=1,2,3,\dots,p$. Assume that $\lambda_t=\left[\lambda_{t_1},\lambda_{t_2},\dots\dots..,\lambda_{t_p}\right]^T$ represents the weight vector related to t_k such that $\lambda_{t_k}\in[0,1]$ and $\sum_{k=1}^{p}\lambda_{t_k}=1$. In the framework of the CIFDOWA operator, the aggregated value of these CIFNs is also represented as a CIFN, obtained through the following expression:

$$CIFDOWA\left(\alpha_{t_1},\alpha_{t_2},\dots,\alpha_{t_p}\right)=\begin{pmatrix}\left(1-\prod_{k=1}^{p}\left(1-\Gamma_{\sigma(t_k)}\right)^{\lambda_{t_k}},1-\prod_{k=1}^{p}\left(1-\theta_{\sigma(t_k)}\right)^{\lambda_{t_k}}\right),\\ \left(\prod_{k=1}^{p}\left(K_{\sigma(t_k)}\right)^{\lambda_{t_k}},\prod_{k=1}^{p}\left(\varphi_{\sigma(t_k)}\right)^{\lambda_{t_k}}\right)\end{pmatrix},$$

where $\left((\sigma(t_1),\sigma(t_2),\dots\dots,\sigma\left(t_p\right)\right)$ is a permutation of $(t_1,t_2,\dots\dots..,t_p)$ such that $\alpha_{\sigma(t_{k-1})}\geq\alpha_{\sigma(t_k)}$ for all k.

Proof: We demonstrate the theorem's validity utilizing the method of mathematical induction.

In the initial step, we take $p=2$, and consider a pair of CIFNs: $\alpha_{t_1}=\left(\left(\Gamma_{t_1},\theta_{t_1}\right),\left(K_{t_1},\varphi_{t_1}\right)\right)$ and $\alpha_{t_2}=\left(\left(\Gamma_{t_2},\theta_{t_2}\right),\left(K_{t_2},\varphi_{t_2}\right)\right)$. Through the prescribed operations specific to CIFNs, we derive the subsequent expressions:

$$\lambda_{t_1}\alpha_{\sigma(t_1)}=\left(\left(1-\left(1-\Gamma_{\sigma(t_1)}\right)^{\lambda_{t_1}},1-\left(1-\theta_{\sigma(t_1)}\right)^{\lambda_{t_1}}\right),\left(\left(K_{\sigma(t_1)}\right)^{\lambda_{t_1}},\left(\varphi_{\sigma(t_1)}\right)^{\lambda_{t_1}}\right)\right),$$

$$\lambda_{t_2}\alpha_{\sigma(t_2)} = \left(\left(1-(1-\Gamma_{\sigma(t_2)})^{\lambda_{t_2}}, 1-\left(1-\theta_{\sigma(t_2)}\right)^{\lambda_{t_2}}\right), \left(\left(\mathrm{K}_{\sigma(t_2)}\right)^{\lambda_{t_2}}, \left(\varphi_{\sigma(t_2)}\right)^{\lambda_{t_2}}\right)\right).$$

Next, employing the CIFDOWA operator on α_{t_1} and α_{t_2}, we combine these two CIFNs in the subsequent manner:

$$CIFDOWA\left(\alpha_{t_1}, \alpha_{t_2}\right) = \lambda_{t_1}\alpha_{\sigma(t_1)} \oplus \lambda_{t_2}\alpha_{\sigma(t_2)} = \left(\left(1-(1-\Gamma_{\sigma(t_1)})^{\lambda_{t_1}}, 1-\left(1-\theta_{\sigma(t_1)}\right)^{\lambda_{t_1}}\right), \left(\left(\mathrm{K}_{\sigma(t_1)}\right)^{\lambda_{t_1}}, \left(\varphi_{\sigma(t_1)}\right)^{\lambda_{t_1}}\right)\right) \oplus \left(\left(1-(1-\Gamma_{\sigma(t_2)})^{\lambda_{t_2}}, 1-\left(1-\theta_{\sigma(t_2)}\right)^{\lambda_{t_2}}\right), \left(\left(\mathrm{K}_{\sigma(t_2)}\right)^{\lambda_{t_2}}, \left(\varphi_{\sigma(t_2)}\right)^{\lambda_{t_2}}\right)\right).$$

It follows that:

$$CIFDOWA(\alpha_{t_1}, \alpha_{t_2}) = \left((1-\prod_{k=1}^{2}(1-\Gamma_{\sigma(t_k)})^{\lambda_{t_k}}, 1-\prod_{k=1}^{2}(1-\theta_{\sigma(t_k)})^{\lambda_{t_k}}), (\prod_{k=1}^{2}\left(\mathrm{K}_{\sigma(t_k)}\right)^{\lambda_{t_k}}, \prod_{k=1}^{2}\left(\varphi_{\sigma(t_k)}\right)^{\lambda_{t_k}})\right).$$

Therefore, the validity of the statement for $p = 2$ is verified.

Let the statement be valid for $p = m > 2$. This means that:

$$CIFDOWA\left(\alpha_{t_1}, \alpha_{t_2}, \ldots, \alpha_{t_m}\right) = \begin{pmatrix} \left(1-\prod_{k=1}^{m}(1-\Gamma_{\sigma(t_k)})^{\lambda_{t_k}}, 1-\prod_{k=1}^{m}(1-\theta_{\sigma(t_k)})^{\lambda_{t_k}}\right), \\ \left(\prod_{k=1}^{m}\left(\mathrm{K}_{\sigma(t_k)}\right)^{\lambda_{t_k}}, \prod_{k=1}^{m}\left(\varphi_{\sigma(t_k)}\right)^{\lambda_{t_k}}\right) \end{pmatrix}.$$

Consider

$$CIFDOWA\left(\alpha_{t_1}, \alpha_{t_2}, \ldots, \alpha_{t_m}, \alpha_{t_{m+1}}\right) = \lambda_{t_1}\alpha_{\sigma(t_1)} \oplus \lambda_{t_2}\alpha_{\sigma(t_2)} \oplus \ldots \ldots \oplus \lambda_{t_m}\alpha_{\sigma(t_m)} \oplus \lambda_{t_{m+1}}\alpha_{\sigma(t_{m+1})} = \left(\left(1-\prod_{k=1}^{m}(1-\Gamma_{\sigma(t_k)})^{\lambda_{t_k}}, 1-\prod_{k=1}^{m}\left(1-\theta_{\sigma(t_k)}\right)^{\lambda_{t_k}}\right), \left(\prod_{k=1}^{m}\left(\mathrm{K}_{\sigma(t_k)}\right)^{\lambda_{t_k}}, \prod_{k=1}^{m}\left(\varphi_{\sigma(t_k)}\right)^{\lambda_{t_k}}\right)\right) \oplus \left(\left(1-(1-\Gamma_{\sigma(t_{m+1})})^{\lambda_{t_{m+1}}}, 1-\left(1-\theta_{\sigma(t_{m+1})}\right)^{\lambda_{t_{m+1}}}\right), \left(\left(\mathrm{K}_{\sigma(t_{m+1})}\right)^{\lambda_{t_{m+1}}}, \left(\varphi_{\sigma(t_{m+1})}\right)^{\lambda_{t_{m+1}}}\right)\right).$$

This shows that:

$$CIFDOWA\left(\alpha_{t_1}, \alpha_{t_2}, \ldots, \alpha_{t_m}, \alpha_{t_{m+1}}\right) = \left((1-\prod_{k=1}^{m+1}(1-\Gamma_{\sigma(t_k)})^{\lambda_{t_k}}, 1-\prod_{k=1}^{m+1}(1-\theta_{\sigma(t_k)})^{\lambda_{t_k}}), \left(\prod_{k=1}^{m+1}\left(\mathrm{K}_{\sigma(t_k)}\right)^{\lambda_{t_k}}, \prod_{k=1}^{m+1}\left(\varphi_{\sigma(t_k)}\right)^{\lambda_{t_k}}\right)\right).$$

The statement stands valid for the case where $p = m + 1$. Hence, the assertion remains true for all positive integers p.

The subsequent example validates the assertion outlined in Theorem 1.

Example 3. Suppose $\alpha_{t_1} = ((0.5,0.8), (0.3,0.2))$, $\alpha_{t_2} = ((0.5,0.7), (0.4,0.2))$, $\alpha_{t_3} = ((0.80,0.30), (0.20,0.60))$ and $\alpha_{t_4} = ((0.60,0.90), (0.20,0.10))$ CIFNs, and $\lambda_t = [0.350,0.150,0.300,0.200]^T$ represents the weight vector of the periods t_1, t_2, t_3, and t_4. To aggregate these values using the CIFDOWA operator, we initiate the process by permuting these numbers according to definition 10, obtaining the subsequent data.

$$\acute{C}\left(\alpha_{t_1}\right) = 0.605,\ \acute{C}\left(\alpha_{t_2}\right) = 0.47,\ \acute{C}\left(\alpha_{t_3}\right) = 0.75 \text{ and } \acute{C}\left(\alpha_{t_4}\right) = 0.685.$$

Utilizing definition 10, the permuted values of the CIFNs are computed in the subsequent manner: $\alpha_{\sigma(t_1)} = ((0.8,0.3),(0.2,0.6))$, $\alpha_{\sigma(t_2)} = ((0.6,0.9),(0.2,0.1))$, $\alpha_{\sigma(t_3)} = ((0.5,0.8),(0.3,0.2)$ and $\alpha_{\sigma(t_4)} = ((0.5,0.7),(0.4,0.2))$. Then,

$$\prod_{k=1}^{4}(1-\mathrm{r}_{\sigma(t_k)})^{\lambda(t_k)} = 0.351, \quad \prod_{k=1}^{4}(1-\theta_{\sigma(t_k)})^{\lambda(t_k)} = 0.303,$$

$$\prod_{k=1}^{4}\left(\mathrm{K}_{\sigma(t_k)}\right)^{\lambda(t_k)} = 0.259, \quad \prod_{k=1}^{4}\left(\varphi_{\sigma(t_k)}\right)^{\lambda(t_k)} = 0.265.$$

It follows that

$$CIFDOWA\left(\alpha_{t_1},\alpha_{t_2},\alpha_{t_3},\alpha_{t_4}\right) = \oplus_{k=1}^{4} \lambda_{t_k}\alpha_{\sigma(t_k)}$$
$$= \left((0.649,0.697),(0.259,0.265)\right).$$

Therefore, we can infer that the result of the preceding discussion is a CIFN.

The subsequent result verifies the idempotency property of CIFNs in the framework of CIFDOWA operator.

Theorem 2. Consider a collection of CIFNs $\alpha_{t_k} = \left(\left(\mathrm{r}_{t_k},\theta_{t_k}\right),\left(\mathrm{K}_{t_k},\varphi_{t_k}\right)\right)$, for $k = 1,2,3,\dots,p$, adheres to the constraint $\alpha_{\sigma(t_k)} = \alpha_{t_0}$ for all k, where $\alpha_{t_0} = \left(\left(\mathrm{r}_{t_0},\theta_{t_0}\right),\left(\mathrm{K}_{t_0},\varphi_{t_0}\right)\right)$ is itself a CIFN and $CIFDOWA\left(\alpha_{t_1},\alpha_{t_2},\dots,\alpha_{t_p}\right) = \alpha_{t_0}$.

Proof: Given that $\alpha_{\sigma(t_k)} = \alpha_{t_o}$ for all k, we may deduce from Definition 12 that $\mathrm{r}_{\sigma(t_k)} = \mathrm{r}_{t_0}$, $\theta_{\sigma(t_k)} = \theta_{t_0}$, $\mathrm{K}_{\sigma(t_k)} = \mathrm{K}_{t_0}$, and $\varphi_{\sigma(t_k)} = \varphi_{t_0}$ for all k. By replacing these relationships in Theorem 1, we derive the following:

$$CIFDOWA\left(\alpha_{t_1},\alpha_{t_2},\dots,\alpha_{t_p}\right) = \begin{pmatrix} \left(1-\prod_{k=1}^{p}(1-\mathrm{r}_{t_0})^{\lambda_{t_k}},1-\prod_{k=1}^{p}(1-\theta_{t_0})^{\lambda_{t_k}}\right), \\ \left(\prod_{k=1}^{p}(\mathrm{K}_{t_0})^{\lambda_{t_k}},\prod_{k=1}^{p}(\varphi_{t_0})^{\lambda_{t_k}}\right) \end{pmatrix} =$$
$$\begin{pmatrix} \left(1-(1-\mathrm{r}_{t_0})^{\sum_{k=1}^{p}\lambda_{t_k}},1-(1-\theta_{t_0})^{\sum_{k=1}^{p}\lambda_{t_k}}\right), \\ \left((\mathrm{K}_{t_0})^{\sum_{k=1}^{p}\lambda_{t_k}},(\varphi_{t_0})^{\sum_{k=1}^{p}\lambda_{t_k}}\right) \end{pmatrix} = \left(\left(\mathrm{r}_{t_0},\theta_{t_0}\right),\left(\mathrm{K}_{t_0},\varphi_{t_0}\right)\right).$$

Hence, we conclude that $CIFDOWA\left(\alpha_{t_1},\alpha_{t_2},\dots,\alpha_{t_p}\right) = \alpha_{t_0}$.

In the context of the CIFDOWA operator, the following result demonstrates that every set of CIFNs satisfies the monotonicity property.

Theorem 3. Consider p number of CIFNs $\alpha_{t_k} = \left(\left(\mathrm{r}_{t_k},\theta_{t_k}\right),\left(\mathrm{K}_{t_k},\varphi_{t_k}\right)\right)$ and $\alpha'_{t_k} = \left(\left(\mathrm{r}'_{t_k},\theta'_{t_k}\right),\left(\mathrm{K}'_{t_k},\varphi'_{t_k}\right)\right)$ for all $k = 1,2,3,\dots.,p$. Furthermore, let $\lambda_t = \left[\lambda_{t_1},\lambda_{t_2},\dots\dots.,\lambda_{t_p}\right]^T$ represents the weight vector related to t_k such that $\lambda_{t_k} \in [0,1]$, and $\sum_{k=1}^{p}\lambda_{t_k} = 1$. Moreover, $\left((\sigma(t_1),\sigma(t_2),\dots,\sigma(t_p)\right)$ is a permutation of $(t_1,t_2,\dots\dots.,t_p)$ such that $\alpha_{\sigma(t_{k-1})} \geq \alpha_{\sigma(t_k)}$ and $\alpha'_{\sigma(t_{k-1})} \geq \alpha'_{\sigma(t_k)}$ for all k. Provided that the subsequent conditions are satisfied for each k: $\mathrm{r}_{\sigma(t_k)} \leq \mathrm{r}'_{\sigma(t_k)}$, $\mathrm{K}_{\sigma(t_k)} \geq \mathrm{K}'_{\sigma(t_k)}$, $\theta_{\sigma(t_k)} \leq \theta'_{\sigma(t_k)}$, and $\varphi_{\sigma(t_k)} \geq \varphi'_{\sigma(t_k)}$, then we can rigorously establish the following: $CIFDOWA\left(\alpha_{t_1},\alpha_{t_2},\dots,\alpha_{t_p}\right) \leq CIFDOWA\left(\alpha'_{t_1},\alpha'_{t_2},\dots.,\alpha'_{t_p}\right)$.

Proof: In light of the provided representations of α_{t_k} and α'_{t_k}, the respective outcomes in the context of CIFDOWA are delineated as follows:

$$CIFDOWA\left(\alpha_{t_1},\alpha_{t_2},\dots,\alpha_{t_p}\right) = \left((\mathrm{r}_t,\theta_t),(\mathrm{K}_t,\varphi_t)\right),$$

$$CIFDOWA\left(\alpha'_{t_1},\alpha'_{t_2},\ldots\ldots,\alpha'_{t_p}\right)=\left((\Gamma'_t,\theta'_t),(K'_t,\varphi'_t)\right).$$

By employing the observation that $\Gamma_{\sigma(t_k)} \leq \Gamma'_{\sigma(t_k)}$, we can infer that $1-\Gamma_{\sigma(t_k)} \geq 1-\Gamma'_{\sigma(t_k)}$. Hence, this implies that $\prod_{k=1}^{p}(1-\Gamma_{\sigma(t_k)})^{\lambda_{t_k}} \geq \prod_{k=1}^{p}(1-\Gamma'_{\sigma(t_k)})^{\lambda_{t_k}} \Rightarrow 1-\prod_{k=1}^{p}(1-\Gamma_{\sigma(t_k)})^{\lambda_{t_k}} \leq 1-\prod_{k=1}^{p}(1-\Gamma'_{\sigma(t_k)})^{\lambda_{t_k}}$. Consequently, we can affirm that $\Gamma_t \leq \Gamma'_t$.

Similarly, by considering $K_{\sigma(t_k)} \geq K'_{\sigma(t_k)}$, we deduce that $\prod_{k=1}^{p}\left(K_{\sigma(t_k)}\right)^{\lambda_{t_k}} \geq \prod_{k=1}^{p}\left(K'_{\sigma(t_k)}\right)^{\lambda_{t_k}}$, which further indicates that $K_t \geq K'_t$.

Thus, by utilizing Definition 12, we get the desired outcome.

The following result proves that any finite set of CIFNs adheres to the boundedness property in the framework of the CIFDOWA operator.

Theorem 4. Consider a p number of CIFNs, denoted as $\alpha_{t_k}=\left((\Gamma_{t_k},\theta_{t_k}),(K_{t_k},\varphi_{t_k})\right)$ for all $k=1,2,\ldots,p$. Let $\lambda_t=\left[\lambda_{t_1},\lambda_{t_2},\ldots\ldots\ldots,\lambda_{t_p}\right]^T$ represents the weight vector related to t_k, where $\lambda_{t_k}\in[0,1]$, and the constraint $\sum_{k=1}^{p}\lambda_{t_k}=1$ holds.

Furthermore, let

$$\alpha^-=\left(\left(\min_{t_k}\{\Gamma_{\sigma(t_k)}\},\min_{t_k}\{\theta_{\sigma(t_k)}\}\right),\left(\max_{t_k}\{K_{\sigma(t_k)}\},\max_{t_k}\{\varphi_{\sigma(t_k)}\}\right)\right),$$

and

$$\alpha^+=\left(\left(\max_{t_k}\{\Gamma_{\sigma(t_k)}\},\max_{t_k}\{\theta_{\sigma(t_k)}\}\right),\left(\min_{t_k}\{K_{\sigma(t_k)}\},\min_{t_k}\{\varphi_{\sigma(t_k)}\}\right)\right)$$

be the lower and upper bounds of these CIFNs. Moreover, $\left((\sigma(t_1),\sigma(t_2),\ldots\ldots,\sigma(t_p)\right)$ is a permutation of $(t_1,t_2,\ldots\ldots\ldots,t_p)$ such that $\alpha^-_{\sigma(t_{k-1})} \geq \alpha^-_{\sigma(t_k)}$ and $\alpha^+_{\sigma(t_{k-1})} \geq \alpha^+_{\sigma(t_k)}$ for all k. Then,

$$\alpha^- \leq CIFDOWA\left(\alpha_{t_1},\alpha_{t_2},\ldots,\alpha_{t_p}\right) \leq \alpha^+.$$

Proof: Consider the outcome of the CIFDOWA operator to the set of CIFNs as follows:

$$CIFDOWA\left(\alpha_{t_1},\alpha_{t_2},\ldots,\alpha_{t_p}\right)=\left((\Gamma_t,\theta_t),(K_t,\varphi_t)\right).$$

For each CIFN α_{t_k},

$$\min_{t_k}\{\Gamma_{\sigma(t_k)}\} \leq \Gamma_{\sigma(t_k)} \leq \max_{t_k}\{\Gamma_{\sigma(t_k)}\} \Rightarrow 1-\max_{t_k}\{\Gamma_{\sigma(t_k)}\} \leq 1-\Gamma_{\sigma(t_k)} \leq 1-\min_{t_k}\{\Gamma_{\sigma(t_k)}\} \Rightarrow$$
$$\prod_{k=1}^{p}(1-\max_{t_k}\{\Gamma_{\sigma(t_k)}\})^{\lambda_{t_k}} \leq \prod_{k=1}^{p}\left(1-\Gamma_{\sigma(t_k)}\right)^{\lambda_{t_k}} \leq \prod_{k=1}^{p}(1-\min_{t_k}\{\Gamma_{\sigma(t_k)}\})^{\lambda_{t_k}} \Rightarrow$$
$$(1-\max_{t_k}\{\Gamma_{\sigma(t_k)}\})^{\sum_{k=1}^{p}\lambda_{t_k}} \leq \prod_{k=1}^{p}\left(1-\Gamma_{\sigma(t_k)}\right)^{\lambda_{t_k}} \leq (1-\min_{t_k}\{\Gamma_{\sigma(t_k)}\})^{\sum_{k=1}^{p}\lambda_{t_k}} \Rightarrow 1-$$
$$\max_{t_k}\{\Gamma_{\sigma(t_k)}\} \leq \prod_{k=1}^{p}\left(1-\Gamma_{\sigma(t_k)}\right)^{\lambda_{t_k}} \leq 1-\min_{t_k}\{\Gamma_{\sigma(t_k)}\} \Rightarrow \min_{t_k}\{\Gamma_{\sigma(t_k)}\} \leq 1-\prod_{k=1}^{p}\left(1-\Gamma_{\sigma(t_k)}\right)^{\lambda_{t_k}} \leq$$
$$\max_{t_k}\{\Gamma_{\sigma(t_k)}\}.$$

Hence,

$$\min_{t_k}\{\Gamma_{\sigma(t_k)}\} \leq \Gamma_t \leq \max_{t_k}\{\Gamma_{\sigma(t_k)}\}.$$

Moreover,

$$\min_{t_k}\{\mathrm{K}_{\sigma(t_k)}\} \leq \mathrm{K}_{\sigma(t_k)} \leq \max_{t_k}\{\mathrm{K}_{\sigma(t_k)}\} \Rightarrow \prod_{k=1}^{p}(\min_{t_k}\{\mathrm{K}_{\sigma(t_k)}\})^{\lambda_{t_k}} \leq \prod_{k=1}^{p}\{\mathrm{K}_{\sigma(t_k)}\}^{\lambda_{t_k}} \leq \prod_{k=1}^{p}\left(\max_{t_k}\{\mathrm{K}_{\sigma(t_k)}\}\right)^{\lambda_{t_k}} \Rightarrow \left(\min_{t_k}\{\mathrm{K}_{\sigma(t_k)}\}\right)^{\sum_{k=1}^{p}\lambda_{t_k}} \leq \prod_{k=1}^{p}\left(\mathrm{K}_{\sigma(t_k)}\right)^{\lambda_{t_k}} \leq \left(\max_{t_k}\{\mathrm{K}_{\sigma(t_k)}\}\right)^{\sum_{k=1}^{p}\lambda_{t_k}} \Rightarrow \min_{t_k}\{\mathrm{K}_{\sigma(t_k)}\} \leq \mathrm{K}_t \leq \max_{t_k}\{\mathrm{K}_{\sigma(t_k)}\}.$$

Similarly we can obtain that $\min_{t_k}\{\theta_{\sigma(t_k)}\} \leq \theta_t \leq \max_{t_k}\{\theta_{\sigma(t_k)}\}$ and $\min_{t_k}\{\varphi_{\sigma(t_k)}\} \leq \varphi_t \leq \max_{t_k}\{\varphi_{\sigma(t_k)}\}$. Thus, through the utilization of Definition 12, it follows that $\alpha_t^- \leq CIFDWA\left(\alpha_{t_1}, \alpha_{t_2}, \ldots, \alpha_{t_p}\right) \leq \alpha_t^+$.

4.3. Structural properties of CIFDOWG operator

In this subsection, we present the concept of the CIF dynamic ordered weighted geometric operator and meticulously establish its inherent structural properties of foundational significance.

Definition 15. Consider ψ be the collection of CIFNs $\alpha_{t_k} = \left((\Gamma_{t_k}, \theta_{t_k}), (\mathrm{K}_{t_k}, \varphi_{t_k})\right)$, observed at distinct periods t_k, where k ranges from 1 to p. Moreover, let $\lambda_t = \left[\lambda_{t_1}, \lambda_{t_2}, \ldots \ldots .., \lambda_{t_p}\right]^T$ signifies the weight vector associated with these time periods t_k, adhering to the constraint that λ_{t_k} lies within the interval [0,1], while satisfying the condition $\sum_{k=1}^{p}\lambda_{t_k} = 1$. The CIFDOWG operator $CIFDOWG: \psi^p \rightarrow \psi$, is a mapping defined by $CIFDOWG\left(\alpha_{t_1}, \alpha_{t_2}, \ldots, \alpha_{t_p}\right) = \otimes_{k=1}^{p} {\alpha_{\sigma(t_k)}}^{\lambda_{t_k}}$, where $\left((\sigma(t_1), \sigma(t_2), \ldots \ldots, \sigma(t_p)\right)$ is a permutation of $(t_1, t_2, \ldots, t_p)$ such that $\alpha_{\sigma(t_{k-1})} \geq \alpha_{\sigma(t_k)}$ for all k.

The following result describes that when a finite number of CIFNs are combined using the CIFDOWG operator, the resulting value is also a CIFN.

Theorem 5. Let ψ be the collection of CIFNs $\alpha_{t_k} = \left((\Gamma_{t_k}, \theta_{t_k}), (\mathrm{K}_{t_k}, \varphi_{t_k})\right)$, at p different periods t_k $(k = 1,2, \ldots, p)$. The weight vector related to t_k is denoted as $\lambda_t = \left[\lambda_{t_1}, \lambda_{t_2}, \ldots \ldots .., \lambda_{t_p}\right]^T$, satisfying $\sum_{k=1}^{p}\lambda_{t_k} = 1$. In the framework of the CIFDOWG operator, the aggregated value of these CIFNs is also represented as a CIFN, determined through the following expression:

$$CIFDOWG\left(\alpha_{t_1}, \alpha_{t_2}, \ldots, \alpha_{t_p}\right) = \begin{pmatrix} \left(\prod_{k=1}^{p}(\Gamma_{\sigma(t_k)})^{\lambda_{t_k}}, \prod_{k=1}^{p}(\theta_{\sigma(t_k)})^{\lambda_{t_k}}\right), \\ \left(1-\prod_{k=1}^{p}\left(1-\mathrm{K}_{\sigma(t_k)}\right)^{\lambda_{t_k}}, 1-\prod_{k=1}^{p}(1-\varphi_{\sigma(t_k)})^{\lambda_{t_k}}\right) \end{pmatrix},$$

where, $\left((\sigma(t_1), \sigma(t_2), \ldots \ldots, \sigma(t_p)\right)$ is a permutation of $(t_1, t_2, \ldots \ldots .., t_p)$ such that $\alpha_{\sigma(t_{k-1})} \geq \alpha_{\sigma(t_k)}$ for all k.

Proof: We substantiate the theorem's validity by employing the mathematical induction technique.

In the initial case where $p = 2$, we are presented with two CIFNs, denoted as $\alpha_{t_1} = \left((\Gamma_{t_1}, \theta_{t_1}), (\mathrm{K}_{t_1}, \varphi_{t_1})\right)$ and $\alpha_{t_2} = \left((\Gamma_{t_2}, \theta_{t_2}), (\mathrm{K}_{t_2}, \varphi_{t_2})\right)$. By applying the prescribed operations associated with CIFNs, we derive the subsequent mathematical expressions:

$${\alpha_{\sigma(t_1)}}^{\lambda_{t_1}} = \left(\left(\left(\Gamma_{\sigma(t_1)}\right)^{\lambda_{t_1}}, \left(\theta_{\sigma(t_1)}\right)^{\lambda_{t_1}}\right), \left(1-(1-\mathrm{K}_{\sigma(t_1)})^{\lambda_{t_1}}, 1-\left(1-\varphi_{\sigma(t_1)}\right)^{\lambda_{t_1}}\right)\right),$$

$${\alpha_{\sigma(t_2)}}^{\lambda_{t_2}} = \left(\left(\left(\Gamma_{\sigma(t_2)}\right)^{\lambda_{t_2}}, \left(\theta_{\sigma(t_2)}\right)^{\lambda_{t_2}}\right), \left(1-(1-\mathrm{K}_{\sigma(t_2)})^{\lambda_{t_2}}, 1-\left(1-\varphi_{\sigma(t_2)}\right)^{\lambda_{t_2}}\right)\right).$$

Subsequently, employing the CIFDOWG operator on α_{t_1} and α_{t_2}, we amalgamate these CIFNs in the subsequent manner:

$$CIFDOWG(\alpha_{t_1},\alpha_{t_2}) = \alpha_{\sigma(t_1)}{}^{\lambda_{t_1}} \otimes \alpha_{\sigma(t_2)}{}^{\lambda_{t_2}}$$

$$= \left(\left(\left(\Gamma_{\sigma(t_1)}\right)^{\lambda_{t_1}}, \left(\theta_{\sigma(t_1)}\right)^{\lambda_{t_1}}\right), \left(1-(1-\mathrm{K}_{\sigma(t_1)})^{\lambda_{t_1}}, 1-\left(1-\varphi_{\sigma(t_1)}\right)^{\lambda_{t_1}}\right)\right)$$
$$\otimes \left(\left(\left(\Gamma_{\sigma(t_2)}\right)^{\lambda_{t_2}}, \left(\theta_{\sigma(t_2)}\right)^{\lambda_{t_2}}\right), \left(1-(1-\mathrm{K}_{\sigma(t_2)})^{\lambda_{t_2}}, 1-\left(1-\varphi_{\sigma(t_2)}\right)^{\lambda_{t_2}}\right)\right).$$

It follows that:

$$CIFDOWG(\alpha_{t_1},\alpha_{t_2}) = \left(\left(\prod_{k=1}^{2}\left(\Gamma_{\sigma(t_k)}\right)^{\lambda_{t_k}}, \prod_{k=1}^{2}\left(\theta_{\sigma(t_k)}\right)^{\lambda_{t_k}}\right), \left(1-\prod_{k=1}^{2}(1-\mathrm{K}_{\sigma(t_k)})^{\lambda_{t_k}}, 1-\prod_{k=1}^{2}(1-\varphi_{\sigma(t_k)})^{\lambda_{t_k}}\right)\right).$$

Therefore, we have duly demonstrated the validity of the theorem for the foundational case where $p = 2$.

Next, we advance to the induction step, wherein we posit the theorem's veracity for $p = m > 2$. Consequently, it follows that:

$$CIFDOWG\left(\alpha_{t_1},\alpha_{t_2},\ldots,\alpha_{t_p}\right) = \begin{pmatrix} \left(\prod_{k=1}^{m}(\Gamma_{t_k})^{\lambda_{t_k}}, \prod_{k=1}^{m}(\theta_{t_k})^{\lambda_{t_k}}\right), \\ \left(1-\prod_{k=1}^{m}\left(1-\mathrm{K}_{t_k}\right)^{\lambda_{t_k}}, 1-\prod_{k=1}^{m}(1-\varphi_{t_k})^{\lambda_{t_k}}\right) \end{pmatrix}.$$

Consider

$$CIFDOWG\left(\alpha_{t_1},\alpha_{t_2},\ldots,\alpha_{t_m},\alpha_{t_{m+1}}\right) = \alpha_{t_1}{}^{\lambda_{t_1}} \otimes \alpha_{t_2}{}^{\lambda_{t_2}} \otimes \ldots \otimes \alpha_{t_m}{}^{\lambda_{t_m}} \otimes \alpha_{t_{m+1}}{}^{\lambda_{t_{m+1}}} =$$
$$\left(\left(\prod_{k=1}^{m}\left(\Gamma_{\sigma(t_k)}\right)^{\lambda_{t_k}}, \prod_{k=1}^{m}\left(\theta_{\sigma(t_k)}\right)^{\lambda_{t_k}}\right), \left(1-\prod_{k=1}^{m}(1-\mathrm{K}_{\sigma(t_k)})^{\lambda_{t_k}}, 1-\prod_{k=1}^{m}\left(1-\varphi_{\sigma(t_k)}\right)^{\lambda_{t_k}}\right)\right) \otimes$$
$$\left(\left(\left(\Gamma_{\sigma(t_{m+1})}\right)^{\lambda_{t_{m+1}}}, \left(\theta_{\sigma(t_{m+1})}\right)^{\lambda_{t_{m+1}}}\right), \left(1-(1-\mathrm{K}_{\sigma(t_{m+1})})^{\lambda_{t_{m+1}}}, 1-\left(1-\varphi_{\sigma(t_{m+1})}\right)^{\lambda_{t_{m+1}}}\right)\right).$$

It follows that:

$$CIFDOWG\left(\alpha_{t_1},\alpha_{t_2},\ldots,\alpha_{t_m},\alpha_{t_{m+1}}\right)$$
$$= \left(\left(\prod_{k=1}^{m+1}\left(\Gamma_{\sigma(t_k)}\right)^{\lambda_{t_k}}, \prod_{k=1}^{m+1}\left(\theta_{\sigma(t_k)}\right)^{\lambda_{t_k}}\right), \left(1-\prod_{k=1}^{m+1}(1-\mathrm{K}_{\sigma(t_k)})^{\lambda_{t_k}}, 1 - \prod_{k=1}^{m+1}(1-\varphi_{\sigma(t_k)})^{\lambda_{t_k}}\right)\right).$$

Hence, the statement is true for every positive integer p.

The subsequent example validates the assertion outlined in Theorem 5.

Example 4. Suppose $\alpha_{t_1} = ((0.7,0.6),(0.3,0.2))$, $\alpha_{t_2} = ((0.8,0.3),(0.1,0.5))$, $\alpha_{t_3} = ((0.2,0.7),(0.8,0.2))$ and $\alpha_{t_4} = ((0.6,0.6),(0.3,0.4))$ are any four CIFNs, and $\lambda_t = [0.350,0.150,0.300,0.200]^T$ represents the weight vector of the periods t_1, t_2, t_3, and t_4. To aggregate these values using the CIFDOWG operator, we initiate the process by permuting these numbers according to Definition 10, obtaining the subsequent data.

$$C\left(\alpha_{t_1}\right) = 0.555,\ C\left(\alpha_{t_2}\right) = 0.785,\ C\left(\alpha_{t_3}\right) = 0.140 \text{ and } C\left(\alpha_{t_4}\right) = 0.680.$$

Utilizing Definition 10, the permuted values of the CIFNs are computed in the subsequent manner: $\alpha_{\sigma(t_1)} = ((0.8,0.3),(0.1,0.5))$, $\alpha_{\sigma(t_2)} = ((0.6,0.6),(0.3,0.4))$, $\alpha_{\sigma(t_3)} = ((0.7,0.6),(0.3,0.2)$ and $\alpha_{\sigma(t_4)} = ((0.2,0.7),(0.8,0.2))$. Then, we have

$$\prod_{k=1}^{4}(\Gamma_{t_k})^{\lambda_{t_k}} = 0.558,\ \prod_{k=1}^{4}(\theta_{t_k})^{\lambda_{t_k}} = 0.485,$$

$$\prod_{k=1}^{4}\left(1-\mathrm{K}_{t_k}\right)^{\lambda_{t_k}} = 0.595,\ \prod_{k=1}^{4}(1-\varphi_{t_k})^{\lambda_{t_k}}) = 0.650.$$

This implies that

$$CIFDOWG\left(\alpha_{t_1},\alpha_{t_2},\alpha_{t_3},\alpha_{t_4}\right) = \otimes_{k=1}^{p} \alpha_{\sigma(t_k)}{}^{\lambda_{t_k}}$$

$$= ((0.558,0.485),(0.405,0.350)).$$

Therefore, we can infer that the result of the preceding discussion is a CIFN.

The subsequent result verifies the idempotency property of CIFNs in the framework of CIFDOWG operator.

Theorem 6. Consider p number of CIFNs denoted by $\alpha_{t_k} = \left((\Gamma_{t_k},\theta_{t_k}),(\mathrm{K}_{t_k},\varphi_{t_k})\right)$ where $k = 1,2,\dots,p$, and it satisfies the condition $\alpha_{\sigma(t_k)} = \alpha_{t_0}$ for all k, where $\alpha_{t_0} = \left((\Gamma_{t_0},\theta_{t_0}),(\mathrm{K}_{t_0},\varphi_{t_0})\right)$ being a CIFN itself. Additionally, we consider a weight vector $\lambda_t = \left[\lambda_{t_1},\lambda_{t_2},\dots\dots..,\lambda_{t_p}\right]^T$ related to t_k , where $\lambda_{t_k} \in [0,1]$, and $\sum_{k=1}^{p}\lambda_{t_k} = 1$. Then, it follows:

$$CIFDOWG\left(\alpha_{t_1},\alpha_{t_2},\dots,\alpha_{t_p}\right) = \alpha_{t_0}.$$

Proof: The demonstration of this theorem proceeds analogously to the reasoning employed in establishing Theorem 2.

In the context of the CIFDOWG operator, the subsequent result demonstrates that every collection of CIFNs satisfies the monotonicity property.

Theorem 7. Consider p number of CIFNs denoted as $\alpha_{t_k} = \left((\Gamma_{t_k},\theta_{t_k}),(\mathrm{K}_{t_k},\varphi_{t_k})\right)$ and $\alpha'_{t_k} = \left((\Gamma'_{t_k},\theta'_{t_k}),(\mathrm{K}'_{t_k},\varphi'_{t_k})\right)$ for all $k = 1,2,3,\dots.,p$. Let $\lambda_t = \left[\lambda_{t_1},\lambda_{t_2},\dots\dots..,\lambda_{t_p}\right]^T$ represents the weight vector of time periods t_k, where $\lambda_{t_k} \in [0,1]$, subject to the constraint $\sum_{k=1}^{p}\lambda_{t_k} = 1$. Moreover, $\left((\sigma(t_1),\sigma(t_2),\dots\dots,\sigma(t_p)\right)$ is a permutation of $(t_1,t_2,\dots\dots..,t_p)$ such that $\alpha_{\sigma(t_{k-1})} \geq \alpha_{\sigma(t_k)}$ $\alpha'_{\sigma(t_{k-1})} \geq \alpha'_{\sigma(t_k)}$ and for all k. If for all k, $\Gamma_{\sigma(t_k)} \leq \Gamma'_{\sigma(t_k)}$, $\mathrm{K}_{\sigma(t_k)} \geq \mathrm{K}'_{\sigma(t_k)}$, $\theta_{\sigma(t_k)} \leq \theta'_{\sigma(t_k)}$, and $\varphi_{\sigma(t_k)} \geq \varphi'_{\sigma(t_k)}$, then we can establish that:

$$CIFDOWG\left(\alpha_{t_1},\alpha_{t_2},\dots,\alpha_{t_p}\right) \leq CIFDOWG\left(\alpha'_{t_1,}\alpha'_{t_2},\dots..,\alpha'_{t_p}\right).$$

Proof: The demonstration of this theorem proceeds analogously to the reasoning employed in establishing Theorem 3.

The following result proves that any finite set of CIFNs adheres to the boundedness property for CIFDOWG operator.

Theorem 8. Consider p number of CIFNs, denoted as $\alpha_{t_k} = \left((\Gamma_{t_k},\theta_{t_k}),(\mathrm{K}_{t_k},\varphi_{t_k})\right)$ for all $k = 1,2,\dots,p$. Let $\lambda_t = \left[\lambda_{t_1},\lambda_{t_2},\dots\dots..,\lambda_{t_p}\right]^T$ denotes the weight vector related to t_k, where $\lambda_{t_k} \in [0,1]$, and the constraint $\sum_{k=1}^{p}\lambda_{t_k} = 1$ holds.

Furthermore, let

$$\alpha^- = \left(\left(\min_{t_k}\{\Gamma_{\sigma(t_k)}\}, \min_{t_k}\{\theta_{\sigma(t_k)}\}\right), \left(\max_{t_k}\{\mathrm{K}_{\sigma(t_k)}\}, \max_{t_k}\{\varphi_{\sigma(t_k)}\}\right)\right),$$

and

$$\alpha^+ = \left(\left(\max_{t_k}\{\Gamma_{\sigma(t_k)}\}, \max_{t_k}\{\theta_{\sigma(t_k)}\}\right), \left(\min_{t_k}\{\mathrm{K}_{\sigma(t_k)}\}, \min_{t_k}\{\varphi_{\sigma(t_k)}\}\right)\right)$$

be the lower and upper bounds of these CIFNs. Moreover, $\left((\sigma(t_1), \sigma(t_2), \dots\dots, \sigma(t_p)\right)$ is a permutation of $(t_1, t_2, \dots\dots\dots, t_p)$ such that $\alpha^-_{\sigma(t_{k-1})} \geq \alpha^-_{\sigma(t_k)}$ and $\alpha^+_{\sigma(t_{k-1})} \geq \alpha^+_{\sigma(t_k)}$ for all k. Then,

$$\alpha^- \leq CIFDOWG\left(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}\right) \leq \alpha^+.$$

Proof: The demonstration of this theorem proceeds analogously to the reasoning employed in establishing Theorem 4.

5. Proposed method employing dynamic ordered weighted aggregation operators for handling MADM challenges.

This section introduces a methodology to address MADM problems employing dynamic aggregation operators for CIF information.

Let us designate a discrete collection of alternatives as $\kappa = \{\kappa_1, \kappa_2, \dots, \kappa_m\}$. Furthermore, we contemplate a set of attributes denoted by $\pi = \{\pi_1, \pi_2, \dots, \pi_n\}$, accompanied by their respective weight vector denoted as $w = [w_1, w_2, \dots., w_n]^T$, where $w_\gamma \geq 0$ for $\gamma = 1,2, \dots\dots, n$, and $\sum_{\gamma=1}^n w_\gamma = 1$. Moreover, for p distinct time periods t_k, consider $\lambda = [\lambda_{t_1}, \lambda_{t_2}, \dots\dots\dots, \lambda_{t_p}]^T$ is a weight vector associated to these time periods, characterized by $\lambda_{t_k} \in [0,1]$, and subject to the constraint $\sum_{k=1}^p \lambda_{t_k} = 1$.

Let $R_{t_k} = \left(\kappa_{ij(t_k)}\right)_{m\times n} = \left(\left(\Gamma_{ij(t_k)}, \theta_{ij(t_k)}\right), \left(\mathrm{K}_{ij(t_k)}, \varphi_{ij(t_k)}\right)\right)$ denote the CIF decision matrices across p distinct time periods t_k for $k = 1, 2, \dots, p$. In these matrices, $\Gamma_{ij(t_k)}$ and $\theta_{ij(t_k)}$ signify the extent to which the alternative κ_i meets the attribute π_j during time periods t_k, whereas $\mathrm{K}_{ij(t_k)}$ and $\varphi_{ij(t_k)}$ indicate the extent to which alternative κ_i does not fulfill the attribute π_j during time periods t_k. It is noteworthy that these values are confined within the interval [0,1] and conform to the constraints $0 \leq \Gamma_{ij(t_k)} + \mathrm{K}_{ij(t_k)}, \theta_{ij(t_k)} + \varphi_{ij(t_k)} \leq 1$, where $i = 1,2, \dots, m$ and $j = 1, 2, \dots, n$.

To address the MADM problem, the subsequent algorithms are devised.

5.1. Algorithm for CIFDOWA

Step 1. In order to obtain the CIF permuted decision matrices $R_{\sigma(t_k)} = \left(\kappa_{\sigma(ij(t_k))}\right)_{m\times n} = \left(\left(\Gamma_{\sigma(ij(t_k))}, \theta_{\sigma(ij(t_k))}\right), \left(\mathrm{K}_{\sigma(ij(t_k))}, \varphi_{\sigma(ij(t_k))}\right)\right)$, we adopt the following two stages:

1) Obtain the score values of all π_j, corresponding to each alternative κ_i of each matrix R_{t_k} at time periods t_k by means of Definition 10.

2) Obtain the CIF permuted decision matrices by arranging the computed values from above stage of all criteria π_j, corresponding to each alternative κ_i of each matrix R_{t_k} at time periods t_k, in descending order.

Step 2. Apply the CIFDOWA operator to obtain the collected CIF permuted decision matrix Ɗ as follows:

$$CIFDOWA\left(\kappa_{\sigma(ij(t_1))},\kappa_{\sigma(ij(t_2))},\ldots,\kappa_{\sigma(ij(t_p))}\right)=\begin{pmatrix}\left(1-\prod_{k=1}^{p}(1-\Gamma_{\sigma(ij(t_k))})^{\lambda t_k},(1-\prod_{k=1}^{p}(1-\theta_{\sigma(ij(t_k))})^{\lambda t_k}\right),\\ (\prod_{k=1}^{p}\left(\mathrm{K}_{\sigma(ij(t_k))}\right)^{\lambda t_k},\prod_{k=1}^{p}\left(\varphi_{\sigma(ij(t_k))}\right)^{\lambda t_k})\end{pmatrix}.$$

Step 3. Apply CIFWA operator to each row $Ɗ_i$ of the collective matrix Ɗ, to determine the preference value (κ_i) of each alternative κ_i, where $i=1,2,\ldots,m$ as follows:

$$CIFWA(\kappa_{i1},\kappa_{i2},\ldots,\kappa_{in})=\begin{pmatrix}(1-\prod_{\gamma=1}^{n}(1-\Gamma_{i\gamma})^{w_\gamma},1-\prod_{\gamma=1}^{n}(1-\theta_{i\gamma})^{w_\gamma}),\\ (\prod_{\gamma=1}^{n}(\mathrm{K}_{i\gamma})^{w_\gamma},\prod_{\gamma=1}^{n}(\varphi_{i\gamma})^{w_\gamma})\end{pmatrix}=\left((\Gamma_i,\theta_i),(\mathrm{K}_i,\varphi_i)\right)=(\kappa_i).$$

Step 4. Calculate the score values $\acute{C}(\kappa_i)$ for every alternative κ_i using Definition 10.
Step 5. Determine the optimal option from the alternatives κ_i where $i=1,2,\ldots,m$ by ranking them according to their $\acute{C}(\kappa_i)$.

5.2. Algorithm for CIFDOWG

Step 1. In order to obtain the CIF permuted decision matrices $R_{\sigma(t_k)}=\left(\kappa_{\sigma(ij(t_k))}\right)_{m\times n}=\left(\left(\Gamma_{\sigma(ij(t_k))},\theta_{\sigma(ij(t_k))}\right),\left(\mathrm{K}_{\sigma(ij(t_k))},\varphi_{\sigma(ij(t_k))}\right)\right)$, we adopt the following two stages:
1) Obtain the score values of all π_j, corresponding to each alternative κ_i of each matrix R_{t_k} at time periods t_k by means of Definition 10.
2) Obtain the CIF permuted decision matrices by arranging the computed values from above stage of all criteria π_j, corresponding to each alternative κ_i of each matrix R_{t_k} at time periods t_k, in descending order.

Step 2. Utilize the CIFDOWG operator to obtain the collected CIF permuted decision matrix Ɗ as follows:

$$\begin{aligned}&CIFDOWG\left(\kappa_{\sigma(ij(t_1))},\kappa_{\sigma(ij(t_2))},\ldots,\kappa_{\sigma(ij(t_p))}\right)\\&=\begin{pmatrix}\left(\prod_{k=1}^{p}\left(\Gamma_{\sigma(ij(t_k))}\right)^{\lambda t_k},\prod_{k=1}^{p}\left(\theta_{\sigma(ij(t_k))}\right)^{\lambda t_k}\right),\\ \left(\left(1-\prod_{k=1}^{p}(1-\mathrm{K}_{\sigma(ij(t_k))})^{\lambda t_k},\ 1-\prod_{k=1}^{p}(1-\varphi_{\sigma(ij(t_k))})^{\lambda t_k}\right)\right)\end{pmatrix}.\end{aligned}$$

Step 3. Apply CIFWG operator to each row $Ɗ_i$ of the collective matrix Ɗ, to determine the preference value (κ_i) of each alternative κ_i, where $i=1,2,\ldots,m$ as follows:

$$\begin{aligned}CIFWG(\kappa_{i1},\kappa_{i2},\ldots,\kappa_{in})&=\begin{pmatrix}\left(\prod_{\gamma=1}^{n}\left(\Gamma_{i\gamma}\right)^{w_\gamma},\prod_{\gamma=1}^{n}\left(\theta_{i\gamma}\right)^{w_\gamma}\right),\\ \left(1-\prod_{\gamma=1}^{n}(1-\mathrm{K}_{i\gamma})^{w_\gamma},\ 1-\prod_{\gamma=1}^{n}(1-\varphi_{i\gamma})^{w_\gamma}\right)\end{pmatrix}\\&=\left((\Gamma_i,\theta_i),(\mathrm{K}_i,\varphi_i)\right)=(\kappa_i).\end{aligned}$$

Step 4. Calculate the score values $\acute{C}(\kappa_i)$ for each alternative κ_i using Definition 10.
Step 5. Determine the optimal option from the alternatives κ_i where $i=1,2,\ldots,m$ by ranking them according to their $\acute{C}(\kappa_i)$.

6. Application of proposed CIF dynamic aggregation operators in the MADM problem

In this section, we expand on the previously explained method by presenting a numerical illustration. Subsequently, we analyze the outcomes of this specific instance in relation to the findings of previous studies.

6.1. Case study

A solar cell consists of semiconducting materials, particularly silicon, that are meticulously designed into a p-n junction structure. The structural integrity is achieved by strategically including small amounts of boron and phosphorus, which induce p-type and n-type properties, respectively. This is exemplified in the case of silicon. Solar cells absorb incoming particles from the atmosphere that impact them during operation. Electrons undergo excitation and transition from the valence band to the conduction band upon absorption of photons. This can also occur in organic solar cells, where electrons move from occupied to vacant molecular orbitals. As a result, electron-hole pairs are formed. When these pairings appear along the boundary between p-type and n-type materials, the concentrated electric field helps to separate them towards opposite electrodes. As a result, an excess of electrons builds up on one side, while there is a large number of holes on the other side. Without external electrical connection or when subjected to high external load, the solar cell undergoes a process in which electrons and holes move towards equilibrium by diffusing back across the junction against the existing electric field. As a result, they recombine and release thermal energy. However, when the external load is not too heavy, the restoration of balance happens more easily by allowing extra electrons to flow through the external circuit, thus performing useful work during this phase of reestablishment.

A collection of solar cells converts incoming solar energy into a measurable amount of direct current electricity that can be used practically. In order to enhance its usefulness, an inverter plays a crucial role in transforming this electricity into alternating current. The most prominent solar cell design, widely acknowledged, is comprised of a large p-n junction made from silicon. Additional categorizations of solar cells include organic solar cells, dye-sensitized solar cells, perovskite solar cells, and quantum dot solar cells, among various others. The incident light on a solar cell typically interacts with a transparent conducting film on its exposed side, allowing the light to enter the active material and enabling the collection of generated charge carriers. This feature requires films that have a high level of transparency and strong electrical conductivity. These films usually use materials like indium tin oxide, conducting polymers, or conducting nanowire networks to meet this requirement.

6.1.1. Materials

Solar cells typically use naming conventions that reflect the semiconducting material they are made of. These materials require special characteristics in order to efficiently absorb solar light. Solar cell designs are customized to either adapt to solar radiation on Earth or to maximize efficiency in space environments. Solar cells can exist as either single-junction structures, consisting of a single light-absorbing material layer, or as multi-junction structures, which incorporate several physical configurations to optimize absorption and charge separation mechanisms.

Solar cells can be classified into three separate generations: first, second, and third. The first generation cells, also known as the initial iteration, refer to the widely used and dominant crystalline silicon solar cells. The conventional category, also known as wafer-based or traditional cells, is the most common type of cells used in commercial photovoltaic (PV) systems. These cells are made from materials such as polysilicon and monocrystalline silicon. On the other hand, the second generation of solar cells, known as thin-film solar cells, utilize materials like amorphous silicon, cadmium telluride and copper indium gallium selenide (CdTe and CIGS). These cells have gained significant commercial success, especially in large-scale photovoltaic power stations, integration into building structures (known as building integrated photovoltaics), and small, self-contained power systems. The third evolutionary phase of solar cell technology encompasses a spectrum of thin-film technologies, aptly designated as emerging photovoltaics. The majority within this classification remain situated in the realm of research and developmental stages, yet to witness broad commercial implementation. Many of these technologies employ organic materials, frequently in the form of organometallic compounds, and incorporate various inorganic substances. Historically, their efficiency levels have been modest, coupled with challenges pertaining to the stability of the absorber materials, rendering them less suitable for immediate commercial deployment. However, persistent investigation persists due to their potential to actualize the dual objective of low-cost production and elevated solar cell efficiency. Notably, as of 2016, prevailing prominence and efficiency in solar cell technology were still reserved for those constructed from thin silicon wafers, a technology tracing its origins to the earliest phases of solar cell development.

6.1.2. Timeline of solar cells

During the 19th century, scientists made actual findings that showed when sunlight hits certain materials, it produces an electric current that can be detected. This phenomenon is called the photoelectric effect. This crucial breakthrough provided the essential foundation for the creation of solar cells. Solar cells have become widely used in numerous fields over time. Their first applications mostly focused on situations where there was no access to electricity from established power systems.

The progress of this technology has led to the increased importance of solar cells in generating power for satellites that travel in Earth's orbit. As a result, solar cells have become the leading method for capturing solar energy, effectively transforming sunlight into electrical power. Currently, solar cells are widely used in satellites, firmly confirming their essential function in modern technological systems.

I. First time period (1979–1993)

- During the late 1970s, amidst the energy crisis, there was a surge in public interest in solar energy utilization, encompassing photovoltaic systems and various solar applications in architectural designs, off-grid buildings, and residential settings.
- In the year 1980, the Institute of Energy Conversion at the University of Delaware achieved a significant milestone in solar cell technology by pioneering the development of the inaugural thin film solar cell surpassing a notable efficiency threshold of 10%, employing Cu2S/CdS technology.
- Concurrently, in 1981, the esteemed Fraunhofer Institute for Solar Energy Systems ISE was established under the guidance of Adolf Goetzberger in Freiburg, Germany.
- The subsequent year, 1982, witnessed a breakthrough with the unveiling of the inaugural amorphous silicon thin film solar cell, boasting an efficiency exceeding 10%.
- In the year 1983, the global photovoltaic production surpassed an aggregate capacity of 21.3 megawatts, accompanied by sales figures exceeding a substantial sum of $250 million.
- In 1984, a groundbreaking 30,000 sq. ft. Building-Integrated Photovoltaic (BI-PV) Roof emerged at Georgetown University. Eileen M. Smith, M.Arch., marked its 20th Anniversary in 2004

with an enlightening journey for Peace and Photovoltaics from the solar roof to Ground Zero. The BI-PV system displayed remarkable, sustained productivity in urban Washington, D.C.

- In 1985, the University of New South Wales' Centre for Photovoltaic Engineering achieved a breakthrough, yielding silicon cells boasting a remarkable 20% efficiency rate.
- Subsequently, in 1986, Lt. Colonel Richard T. Headrick of Irvine, California, secured a patent for the innovative "Solar-Voltaic DomeTM", an architecturally efficient design seamlessly integrating photovoltaics (BI-PV). This pioneering concept was realized in a field array in Hesperia, California.
- In 1988, Michael Grätzel and Brian O'Regan made history by introducing dye-sensitized solar cells, a photoelectrochemical variant distinguished by the presence of an organic dye compound within the cell. Notably cost-effective, these cells stood as a viable alternative, at approximately half the cost of traditional silicon solar cells.
- During the span of 1988 to 1991, AMOCO/Enron strategically employed Solarex patents in legal proceedings against ARCO Solar, resulting in the latter's exit from the amorphous silicon (a-Si) solar business (see Solarex Corp. (Enron/Amoco) v. Arco Solar, Inc. Ddel, 805 Fsupp 252 Fed Digest). Additionally, 1989 marked the advent of reflective solar concentrators in conjunction with solar cells, signifying a notable advancement in solar technology. Last, in 1990, the Magdeburg Cathedral achieved a historic feat by integrating solar cells onto its roof: a pioneering installation within East Germany's ecclesiastical landscape.
- In 1991, significant advancements were made in photoelectrochemical cell technology, aimed at elevating efficiency. Concurrently, President George H. W. Bush directed the U. S. Department of Energy to create the National Renewable Energy Laboratory, involving the transfer of the existing Solar Energy Research Institute.
- The year 1992 witnessed the inception of the PV Pioneer Program, led by the Sacramento Municipal Utility District. This initiative was groundbreaking, spearheading the commercialization of distributed, grid-connected PV systems-commonly denoted as "roof-top solar", and laid the foundation for the subsequent CA Million Solar Roofs Program. Additionally, in 1992, the University of South Florida achieved a noteworthy feat by producing a 15.89% efficient thin-film cell.
- In 1993, the National Renewable Energy Laboratory cemented its dedication to solar research and development with the establishment of the Solar Energy Research Facility.

II. Second time period (1994–2008)

- A seminal accomplishment occurred in 1994 when NREL engineered a GaInP/GaAs two-terminal concentrator cell operating at 180 suns, achieving a pioneering milestone as the first solar cell to surpass an impressive 30% conversion efficiency.
- In 1996, the National Center for Photovoltaics was developed. Professor Michael Grätzel achieved an 11% efficiency using dye-sensitized cells.
- By 1999, global installed photovoltaic power reached 1,000 megawatts, signifying a major leap in solar technology.
- In 2003, President George Bush installed a 9 KW photovoltaic (PV) system and solar thermal setups at the White House groundskeeping building.
- In 2004, Governor Schwarzenegger introduced the Solar rooftops Initiative, aiming for one million solar rooftops in California by 2017. In 2004, Kansas Governor Kathleen Sebelius issued Executive Order 04-05 to achieve 1,000 MWp of renewable power by 2015.
- By 2006, polysilicon use in photovoltaics outpaced all other applications. Also in 2006, the California Public Utilities Commission approved the extensive $2.8 billion California Solar Initiative (CSI) to incentivize solar development over 11 years. A transformative moment in 2006 saw solar cell technology breach the 40% efficiency barrier for sunlight-to-electricity conversion.

- In 2007, construction commenced on the 15 MW Nellis Solar Power Plant through a Power Purchase Agreement (PPA). Additionally, the Vatican planned solar panel installations on select buildings in 2007 as part of a comprehensive energy project for resource conservation. The University of Delaware claimed a pioneering solar cell technology achievement in the same year, allegedly reaching 42.8% efficiency, lacking independent confirmation. In another 2007 breakthrough, Nanosolar achieved a milestone by shipping the inaugural commercially printed Copper Indium Gallium Selenide (CIGS) solar modules, projecting future costs to fall below $1/watt, while withholding specific technical and pricing details.
- In 2008, NREL set a solar efficiency record at 40.8%, using a highly concentrated light of 326 suns on an inverted metamorphic triple-junction solar cell.

III. Third time period (2009–2023)

- In 2010, IKAROS achieved a historic feat, displaying solar sail prowess in interplanetary realms. Concurrently, President Barack Obama initiated solar enhancements at the White House.
- By 2011, Chinese factories accelerated solar module production, halving costs to $1.25 per watt, leading to a global surge in solar installations.
- In 2013, Completion of the White House solar upgrades came to fruition.
- In 2016, the University of New South Wales achieved a breakthrough, hitting a record 34.5% sunlight-to-electricity conversion. Their innovative four-junction mini-module, using a prism, split sunlight into four bands and maximized energy extraction. Concurrently, First Solar pioneered with a 22.1% efficiency using experimental cadmium telluride cells, a technology comprising 5% of global solar power.
- In 2018, Alta Devices set a verified solar cell efficiency record of 29.1% using gallium arsenide. Furthermore, Europe saw the inauguration of its inaugural dedicated solar panel recycling plant.
- In 2019, the National Renewable Energy Laboratory achieved a remarkable 47.1% efficiency in solar cell technology, surpassing the standard 37%.
- In 2020, Perovskite solar cells showed exceptional progress, soaring from 3.8% efficiency to an impressive 25.2% for single-junction designs. Moreover, silicon-based tandem cells reached an even higher 29.1% efficiency, surpassing the peak efficiency achieved by single-junction silicon solar cells.
- In 2021, researchers have successfully constructed a prototype and specified the essential design criteria for silicon solar cells incorporating contact on both sides, attaining outstanding conversion efficiencies approaching 26%. This achievement represents the highest level of efficiency in this specific category of solar cell technology on Earth.
- In 2022, Frank Dimroth and his team at Fraunhofer ISE have successfully developed a 4-junction solar cell, which has achieved an exceptional efficiency of 47.6% in converting solar energy. This achievement sets a remarkable world record in the field of solar energy conversion.

As of the year 2023, optimal laboratory instances of conventional crystalline silicon (c-Si) solar cells have exhibited efficiencies reaching 26.81%. In contrast, laboratory exemplars of multi-junction cells have showcased performance surpassing 46% when subjected to concentrated solar illumination.

6.2. Illustrated example

A company has determined the best location for a solar plant, it is time to select the type of cell that will optimize the installation from the numerous existing solar cells currently accessible. Based on this premise, a company is in search to identify a solar cell type that maximizes profits in grid-connected systems by increasing production or enhancing efficiency. Furthermore, there is a strong emphasis on the cell's ability to adapt to various climatic circumstances, while also prioritizing enhanced maturity and consistent reliability. For this purpose, the company has hired a group of physicists, who is an expert on photovoltaic technologies with more than 20 years of experience. The experts formulate a decision making problem in which the solar cells previously delineated are assumed to serve as its alternatives. Let $\{\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5\}$ be the set of alternatives:

i. κ_1: solar cells with advanced III–V thin layer with tracking systems for solar concentration;
ii. κ_2: solar cells with crystalline silicon (mono-crystalline and poly-crystalline);
iii. κ_3: solar cells with amorphous silicon;
iv. κ_4: solar cells with dye-sensitized;
v. κ_5: solar cells with inorganic thin layer (cadmium telluride and copper indium gallium selenide);
vi. Experts have assessed these alternatives by taking into account the subsequent attributes:
vii. π_1: Efficiency in energy conversion;
viii. π_2: Environmental impact;
ix. π_3: Reliability and durability;
x. π_4: Heat tolerance

These elements are classified into two characteristics to construct CIFNs.

- Higher efficiency solar cells produce more electricity for a given amount of sunlight and offer more flexibility in system design.
- Selecting solar cells with a lower environmental impact is in line with sustainability goals and the desire to reduce the carbon footprint associated with renewable energy production.
- Solar cells with higher reliability ensure consistent energy production over the system's lifespan and is essential for meeting energy requirements.
- Solar cells with good heat tolerance maintain higher efficiency levels even in hot climates and can reduce the need for additional cooling.

The experts are tasked with evaluating the five potential alternatives $\kappa_1, \kappa_2, \kappa_3, \kappa_4$ and κ_5 are to be evaluated using the CIF information for the four attributes π_1, π_2, π_3 and π_4 during the specified time intervals t_1, t_2, and t_3 corresponding to the years 1979 to 1993, 1994 to 2008, and 2009 to 2023, respectively. The weight vector of the time periods designated by the group of experts is denoted by $\lambda_t = [0.200,0.300,0.500]^T$. Similarly, the attribute weight vector is $w = [0.35,0.3,0.25,0.1]^T$. The group of physicists' expert opinion regarding the reliability of each alternative κ_i relating to each attribute π_j for the specified time periods t_k is concisely presented in the assessment matrices R_{t_k}, with entries in Tables 1–3, respectively, representing CIFNs.

Table 1. Decision matrix acquired from R_{t_1}.

	$\boldsymbol{\pi_1}$	$\boldsymbol{\pi_2}$	$\boldsymbol{\pi_3}$	$\boldsymbol{\pi_4}$
κ_1	((0.5,0.5),(0.4,0.3))	((0.6,0.6),(0.4,0.3))	((0.6,0.5),(0.4,0.4))	((0.6,0.7),(0.3,0.3))
κ_2	((0.6,0.7),(0.1,0.2))	((0.7,0.7),(0.1,0.1))	((0.4,0.6),(0.3,0.1))	((0.7,0.6),(0.1,0.2))
κ_3	((0.6,0.6),(0.2,0.2))	((0.5,0.8),(0.3,0.1))	((0.3,0.4),(0.5,0.6))	((0.7,0.7),(0.2,0.3))
κ_4	((0.8,0.1),(0.1,0.4))	((0.6,0.6),(0.3,0.4))	((0.4,0.9),(0.2,0.1))	((0.6,0.6),(0.3,0.1))
κ_5	((0.6,0.9),(0.1,0.1))	((0.7,0.6),(0.3,0.3))	((0.3,0.4),(0.6,0.4))	((0.6,0.6),(0.2,0.2))

Table 2. Decision matrix acquired from R_{t_2}.

	π_1	π_2	π_3	π_4
κ_1	((0.6,0.8),(0.3,0.2))	((0.5,0.7),(0.4,0.2))	((0.6,0.3),(0.3,0.6))	((0.6,0.8),(0.2,0.2))
κ_2	((0.4,0.9),(0.5,0.1))	((0.1,0.3),(0.6,0.5))	((0.5,0.5),(0.3,0.3))	((0.7,0.7),(0.3,0.2))
κ_3	((0.7,0.6),(0.2,0.2))	((0.8,0.3),(0.1,0.5))	((0.2,0.7),(0.8,0.2))	((0.6,0.6),(0.3,0.2))
κ_4	((0.8,0.3),(0.1,0.6))	((0.4,0.8),(0.5,0.2))	((0.3,0.1),(0.6,0.3))	((0.5,0.5),(0.2,0.1))
κ_5	((0.8,0.6),(0.1,0.2))	((0.6,0.4),(0.1,0.5))	((0.3,0.6),(0.3,0.1))	((0.5,0.3),(0.4,0.6))

Table 3. Decision matrix acquired from R_{t_3}.

	π_1	π_2	π_3	π_4
κ_1	((0.8,0.6),(0.2,0.3))	((0.7,0.7),(0.2,0.2))	((0.7,0.8),(0.3,0.1))	((0.70.4),(0.2,0.5))
κ_2	((0.8,0.5),(0.1,0.4))	((0.6,0.5),(0.3,0.4))	((0.6,0.3),(0.3,0.5))	((0.7,0.6),(0.2,0.3))
κ_3	((0.7,0.3),(0.1,0.5))	((0.6,0.4),(0.4,0.4))	((0.3,0.1),(0.6,0.3))	((0.6,0.5),(0.1,0.3))
κ_4	((0.7,0.9),(0.1,0.1))	((0.6,0.5),(0.1,0.1))	((0.4,0.3),(0.2,0.5))	((0.7,0.3),(0.3,0.3))
κ_5	((0.5,0.5),(0.3,0.4))	((0.5,0.3),(0.4,0.6))	((0.2,0.8),(0.5,0.1))	((0.4,0.8),(0.5,0.1))

Step 1. In order to obtain the CIF permuted decision matrices $\mathrm{R}_{\sigma(\mathrm{t_k})} = \left(\kappa_{\sigma(\mathrm{ij}(\mathrm{t_k}))}\right)_{\mathrm{m}\times\mathrm{n}} = \left(\left(\Gamma_{\sigma(\mathrm{ij}(\mathrm{t_k}))}, \theta_{\sigma(\mathrm{ij}(\mathrm{t_k}))}\right), \left(\mathrm{K}_{\sigma(\mathrm{ij}(\mathrm{t_k}))}, \varphi_{\sigma(\mathrm{ij}(\mathrm{t_k}))}\right)\right)$, we adopt the following two stages:

1) Obtain the score values of all $\pi_j(j = 1,2,3,4)$, corresponding to each alternative $\kappa_i(i = 1,2,3,4,5)$ of each matrix R_{t_k} at time periods $t_k(k = 1,2,3)$ by means of Definition 10.

2) Obtain the CIF permuted decision matrices (see Tables 4–6) by arranging the computed values from above stage of all criteria π_j, corresponding to each alternative κ_i of each matrix R_{t_k} at time periods t_k, in descending order.

Table 4. CIF permuted decision matrix acquired from $R_{\sigma(t_1)}$.

	π_1	π_2	π_3	π_4
κ_1	((0.6,0.7),(0.3,0.3))	((0.6,0.5),(0.4,0.4))	((0.6,0.6),(0.4,0.3))	((0.5,0.5),(0.4,0.3))
κ_2	((0.6,0.7),(0.1,0.2))	((0.7,0.6),(0.1,0.2))	((0.7,0.7),(0.1,0.1))	((0.4,0.6),(0.3,0.1))
κ_3	((0.7,0.7),(0.2,0.3))	((0.6,0.6),(0.2,0.2))	((0.5,0.8),(0.3,0.1))	((0.3,0.4),(0.5,0.6))
κ_4	((0.6,0.6),(0.3,0.4))	((0.4,0.9),(0.2,0.1))	((0.8,0.1),(0.1,0.4))	((0.6,0.6),(0.3,0.1))
κ_5	((0.6,0.9),(0.1,0.1))	((0.7,0.6),(0.3,0.3))	((0.6,0.6),(0.2,0.2))	((0.3,0.4),(0.6,0.4))

Table 5. CIF permuted decision matrix acquired from $R_{\sigma(t_2)}$.

	π_1	π_2	π_3	π_4
κ_1	((0.6,0.8),(0.2,0.2))	((0.6,0.8),(0.3,0.2))	((0.6,0.3),(0.3,0.6))	((0.5,0.7),(0.4,0.2))
κ_2	((0.7,0.7),(0.3,0.2))	((0.5,0.5),(0.3,0.3))	((0.4,0.9),(0.5,0.1))	((0.1,0.3),(0.6,0.5))
κ_3	((0.8,0.3),(0.1,0.5))	((0.7,0.6),(0.2,0.2))	((0.6,0.6),(0.3,0.2))	((0.2,0.7),(0.8,0.2))
κ_4	((0.8,0.3),(0.1,0.6))	(((0.4,0.8),(0.5,0.2))	((0.5,0.5),(0.2,0.1))	((0.3,0.1),(0.6,0.3))
κ_5	((0.8,0.6),(0.1,0.2))	((0.6,0.4),(0.1,0.5))	((0.5,0.3),(0.4,0.6))	((0.3,0.6),(0.3,0.1))

Table 6. CIF permuted decision matrix acquired from $R_{\sigma(t_3)}$.

	π_1	π_2	π_3	π_4
κ_1	((0.8,0.6),(0.2,0.3))	((0.7,0.4),(0.2,0.5))	((0.7,0.7),(0.2,0.2))	((0.7,0.8),(0.3,0.1))
κ_2	((0.8,0.5),(0.1,0.4))	((0.7,0.6),(0.2,0.3))	((0.6,0.5),(0.3,0.4))	((0.6,0.3),(0.3,0.5))
κ_3	((0.7,0.3),(0.1,0.5))	((0.6,0.5),(0.1,0.3))	((0.6,0.4),(0.4,0.4))	((0.3,0.1),(0.6,0.3))
κ_4	((0.7,0.9),(0.1,0.1))	((0.6,0.5),(0.1,0.1))	((0.4,0.3),(0.2,0.5))	((0.7,0.3),(0.3,0.3))
κ_5	((0.5,0.5),(0.3,0.4))	((0.5,0.3),(0.4,0.6))	((0.4,0.8),(0.5,0.1))	((0.2,0.8),(0.5,0.1))

Step 2. Use the CIFDOWA operator to obtain the collected CIF combined decision matrix Ɖ displayed in Table 7.

Table 7. Collective CIF decision matrix Ɖ under CIFDOWA operator.

	π_1	π_2	π_3	π_4
κ_1	$\begin{pmatrix}(0.717,0.693),\\(0.217,0.266)\end{pmatrix}$	$\begin{pmatrix}(0.654,0.548),\\(0.259,0.363)\end{pmatrix}$	$\begin{pmatrix}(0.654,0.590),\\(0.259,0.302)\end{pmatrix}$	$\begin{pmatrix}(0.612,0.729),\\(0.346,0.153)\end{pmatrix}$
κ_2	$\begin{pmatrix}(0.741,0.613),\\(0.139,0.283)\end{pmatrix}$	$\begin{pmatrix}(0.650,0.572),\\(0.197,0.277)\end{pmatrix}$	$\begin{pmatrix}(0.574,0.721),\\(0.281,0.200)\end{pmatrix}$	$\begin{pmatrix}(0.447,0.374),\\(0.369,0.362)\end{pmatrix}$
κ_3	$\begin{pmatrix}(0.734,0.409),\\(0.115,0.451)\end{pmatrix}$	$\begin{pmatrix}(0.633,0.553),\\(0.141,0.245)\end{pmatrix}$	$\begin{pmatrix}(0.582,0.574),\\(0.346,0.246)\end{pmatrix}$	$\begin{pmatrix}(0.271,0.403),\\(0.631,0.305)\end{pmatrix}$
κ_4	$\begin{pmatrix}(0.719,0.763),\\(0.125,0.226)\end{pmatrix}$	$\begin{pmatrix}(0.510,0.725),\\(0.186,0.123)\end{pmatrix}$	$\begin{pmatrix}(0.544,0.335),\\(0.174,0.295)\end{pmatrix}$	$\begin{pmatrix}(0.590,0.325),\\(0.369,0.241)\end{pmatrix}$
κ_5	$\begin{pmatrix}(0.638,0.661),\\(0.173,0.246)\end{pmatrix}$	$\begin{pmatrix}(0.578,0.402),\\(0.249,0.495)\end{pmatrix}$	$\begin{pmatrix}(0.476,0.665),\\(0.389,0.197)\end{pmatrix}$	$\begin{pmatrix}(0.252,0.693),\\(0.445,0.132)\end{pmatrix}$

Step 3. Apply CIFWA operator to each row $Ɖ_i$ of the collective matrix Ɖ, to determine the preference values (κ_i) of the alternative κ_i, where $i = 1, 2, \dots, m$. The collective matrix Ɖ is presented in Table 8.

Table 8. Preference values of alternatives under CIFWA operator.

	κ_i
κ_1	((0.674,0.643),(0.251,0.285))
κ_2	((0.654,0.614),(0.203,0.264))
κ_3	((0.637,0.499),(0.191,0.310))
κ_4	((0.611,0.644),(0.170,0.203))
κ_5	((0.553,0.603),(0.260,0.270))

Step 4. Calculate the scores $Ć(\kappa_i)$, for all $i = 1,2,3,4,5$, of the overall CIF preference values κ_i in order to rank all the alternatives κ_i.

$$Ć(\kappa_1) = 0.683,$$

$$Ć(\kappa_2) = 0.679,$$

$$Ć(\kappa_3) = 0.644,$$

$$Ć(\kappa_4) = 0.657,$$

$$Ć(\kappa_5) = 0.593.$$

Step 5. Since $Ć(\kappa_1) > Ć(\kappa_2) > Ć(\kappa_4) > Ć(\kappa_3) > Ć(\kappa_5)$, therefore, the alternatives are ranked as follows: $\kappa_1 > \kappa_2 > \kappa_4 > \kappa_3 > \kappa_5$.

Similarly, the aforementioned MADM problem, within the context of CIFDOWG operator, is resolved in the following manner:

Step 1. In order to obtain the CIF permuted decision matrices $R_{\sigma(t_k)} = \left(\kappa_{\sigma(ij(t_k))}\right)_{m\times n} = \left(\left(\Gamma_{\sigma(ij(t_k))}, \theta_{\sigma(ij(t_k))}\right), \left(\mathrm{K}_{\sigma(ij(t_k))}, \varphi_{\sigma(ij(t_k))}\right)\right)$, we adopt the following two stages:

1) Obtain the score values of all $\pi_j (j = 1,2,3,4)$, corresponding to each alternative $\kappa_i (i = 1,2,3,4,5)$ of each matrix R_{t_k} at time periods $t_k (k = 1,2,3)$ by means of Definition 10.

2) Obtain the CIF permuted decision matrices (see Tables 9–11) by arranging the computed values from above stage of all criteria π_j corresponding to each alternative κ_i of each matrix R_{t_k} at time periods t_k.

Table 9. CIF permuted decision matrix acquired from $R_{\sigma(t_1)}$.

	π_1	π_2	π_3	π_4
κ_1	((0.6,0.7),(0.3,0.3))	((0.6,0.5),(0.4,0.4))	((0.6,0.6),(0.4,0.3))	((0.5,0.5),(0.4,0.3))
κ_2	((0.6,0.7),(0.1,0.2))	((0.7,0.6),(0.1,0.2))	((0.7,0.7),(0.1,0.1))	((0.4,0.6),(0.3,0.1))
κ_3	((0.7,0.7),(0.2,0.3))	((0.6,0.6),(0.2,0.2))	((0.5,0.8),(0.3,0.1))	((0.3,0.4),(0.5,0.6))
κ_4	((0.6,0.6),(0.3,0.4))	((0.4,0.9),(0.2,0.1))	((0.8,0.1),(0.1,0.4))	((0.6,0.6),(0.3,0.1))
κ_5	((0.6,0.9),(0.1,0.1))	((0.7,0.6),(0.3,0.3))	((0.6,0.6),(0.2,0.2))	((0.3,0.4),(0.6,0.4))

Table 10. CIF permuted decision matrix acquired from $R_{\sigma(t_2)}$.

	π_1	π_2	π_3	π_4
κ_1	((0.6,0.8),(0.2,0.2))	((0.6,0.8),(0.3,0.2))	((0.6,0.3),(0.3,0.6))	((0.5,0.7),(0.4,0.2))
κ_2	((0.7,0.7),(0.3,0.2))	((0.5,0.5),(0.3,0.3))	((0.4,0.9),(0.5,0.1))	((0.1,0.3),(0.6,0.5))
κ_3	((0.8,0.3),(0.1,0.5))	((0.7,0.6),(0.2,0.2))	((0.6,0.6),(0.3,0.2))	((0.2,0.7),(0.8,0.2))
κ_4	((0.8,0.3),(0.1,0.6))	(((0.4,0.8),(0.5,0.2))	((0.5,0.5),(0.2,0.1))	((0.3,0.1),(0.6,0.3))
κ_5	((0.8,0.6),(0.1,0.2))	((0.6,0.4),(0.1,0.5))	((0.5,0.3),(0.4,0.6))	((0.3,0.6),(0.3,0.1))

Table 11. CIF permuted decision matrix acquired from $R_{\sigma(t_3)}$.

	π_1	π_2	π_3	π_4
κ_1	((0.8,0.6),(0.2,0.3))	((0.7,0.4),(0.2,0.5))	((0.7,0.7),(0.2,0.2))	((0.7,0.8),(0.3,0.1))
κ_2	((0.8,0.5),(0.1,0.4))	((0.7,0.6),(0.2,0.3))	((0.6,0.5),(0.3,0.4))	((0.6,0.3),(0.3,0.5))
κ_3	((0.7,0.3),(0.1,0.5))	((0.6,0.5),(0.1,0.3))	((0.6,0.4),(0.4,0.4))	((0.3,0.1),(0.6,0.3))
κ_4	((0.7,0.9),(0.1,0.1))	((0.6,0.5),(0.1,0.1))	((0.4,0.3),(0.2,0.5))	((0.7,0.3),(0.3,0.3))
κ_5	((0.5,0.5),(0.3,0.4))	((0.5,0.3),(0.4,0.6))	((0.4,0.8),(0.5,0.1))	((0.2,0.8),(0.5,0.1))

Step 2. Employ the CIFDOWG operator to obtain the collected CIF permuted decision matrix Ɗ given in Table 12.

Table 12. Collective CIF decision matrix Ɗ under CIFDOWG operator.

	π_1	π_2	π_3	π_4
κ_1	$\begin{pmatrix}(0.693,0.675),\\(0,221,0.271)\end{pmatrix}$	$\begin{pmatrix}(0.648,0.515),\\(0.274,0.403)\end{pmatrix}$	$\begin{pmatrix}(0.648,0.526),\\(0.274,0.367)\end{pmatrix}$	$\begin{pmatrix}(0.592,0.700),\\(0.352,0.174)\end{pmatrix}$
κ_2	$\begin{pmatrix}(0.628,0.592),\\(0.165,0.307)\end{pmatrix}$	$\begin{pmatrix}(0.633,0.568),\\(0.213,0.281)\end{pmatrix}$	$\begin{pmatrix}(0.548,0.638),\\(0.335,0.265)\end{pmatrix}$	$\begin{pmatrix}(0.323,0.345),\\(0.408,0.438)\end{pmatrix}$
κ_3	$\begin{pmatrix}(0.729,0.355),\\(0.121,0.465)\end{pmatrix}$	$\begin{pmatrix}(0.628,0.548),\\(0.151,0.252)\end{pmatrix}$	$\begin{pmatrix}(0.579,0.519),\\(0.352,0.291)\end{pmatrix}$	$\begin{pmatrix}(0.266,0.237),\\(0.660,0.349)\end{pmatrix}$
κ_4	$\begin{pmatrix}(0.706,0.597),\\(0.144,0.349)\end{pmatrix}$	$\begin{pmatrix}(0.490,0.648),\\(0.263,0.131)\end{pmatrix}$	$\begin{pmatrix}(0.491,0.281),\\(0.181,0.381)\end{pmatrix}$	$\begin{pmatrix}(0.526,0.248),\\(0.408,0.264)\end{pmatrix}$
κ_5	$\begin{pmatrix}(0.597,0.594),\\(0.206,0.291)\end{pmatrix}$	$\begin{pmatrix}(0.565,0.376),\\(0.301,0.522)\end{pmatrix}$	$(0.464,0.563),$ $(0.420,0.311)$	$\begin{pmatrix}(0.245,0.639),\\(0.471,0.170)\end{pmatrix}$

Step 3. Apply CIFWG operator to each row $Ɗ_i$ of the collective matrix Ɗ, to determine the preference values (κ_i) of the alternative κ_i, where $i = 1, 2,3,4,5$. The collective matrix Ɗ is presented in Table 13.

Table 13. Permuted values of alternatives under CIFWG operator.

	κ_i
κ_1	((0.657,0.587),(0.264,0.329))
κ_2	((0.569,0.564),(0.251,0.304))
κ_3	((0.595,0.427),(0.267,0.353))
κ_4	((0.561,0.464),(0.220,0.290))
κ_5	((0.504,0.515),(0.322,0.365))

Step 4. Calculate the scores $Ć(\kappa_i)$ of the overall CIF preference values κ_i, to rank all the alternatives κ_i.

$$Ć(\kappa_1) = 0.664,$$

$$Ć(\kappa_2) = 0.607,$$

$$Ć(\kappa_3) = 0.550,$$

$$Ć(\kappa_4) = 0.553,$$

$$Ć(\kappa_5) = 0.544.$$

Step 5. Since $Ć(\kappa_1) > Ć(\kappa_2) > Ć(\kappa_4) > Ć(\kappa_3) > Ć(\kappa_5)$, therefore, the alternatives are ranked as follows: $\kappa_1 > \kappa_2 > \kappa_4 > \kappa_3 > \kappa_5$.

The above discussion shows that technology of advanced III–V thin layer with tracking systems for solar concentration is the best approach for the selection of solar cells.

6.3. *Sensitivity analysis*

In order to ascertain the effects of modifications in decision-making data on the outcomes of the rankings, it is necessary to conduct a sensitivity analysis. This analysis offers valuable insights into the outcomes of various possible circumstances and ascertains the impact of potential input data alterations on the desirability of the proposed solutions. A sensitivity analysis shows how outcomes might change as a result of external factors, in addition to giving decision-makers deeper insights and a wider perspective on the problem at hand.

Table 14 displays the rankings obtained by the CIFDOWA and CIFDOWG operators, after systematically removing one attribute at a time from the given situation. Upon removing attribute π_1, which had the highest weight in the problem, the ranking order of the alternatives underwent moderate deviations. The new order of demand for the decision alternatives is $\kappa_2 > \kappa_1 > \kappa_4 > \kappa_5 > \kappa_3$, whereas the original ranking obtained through the proposed method was $\kappa_1 > \kappa_2 > \kappa_4 > \kappa_3 > \kappa_5$. It is important to mention that the removal of attribute π_4 did not impact the ranking of the choice possibilities. The elimination of attributes π_2 and π_3 resulted in noticeable minor alterations.

Table 14. Impact of attribute exclusion on CIFDOWA and CIFDOWG rankings.

Framework	Ranking
CIFDOWA-Original	$\kappa_1 > \kappa_2 > \kappa_4 > \kappa_3 > \kappa_5$
CIFDOWA-Removing π_1	$\kappa_2 > \kappa_1 > \kappa_4 > \kappa_5 > \kappa_3$
CIFDOWA-Removing π_2	$\kappa_1 > \kappa_4 > \kappa_2 > \kappa_3 > \kappa_5$
CIFDOWA-Removing π_3	$\kappa_1 > \kappa_2 > \kappa_3 > \kappa_4 > \kappa_5$
CIFDOWA-Removing π_4	$\kappa_1 > \kappa_2 > \kappa_4 > \kappa_3 > \kappa_5$
CIFDOWG-Original	$\kappa_1 > \kappa_2 > \kappa_4 > \kappa_3 > \kappa_5$
CIFDOWG-Removing π_1	$\kappa_2 > \kappa_1 > \kappa_4 > \kappa_5 > \kappa_3$
CIFDOWG-Removing π_2	$\kappa_1 > \kappa_4 > \kappa_2 > \kappa_3 > \kappa_5$
CIFDOWG-Removing π_3	$\kappa_1 > \kappa_2 > \kappa_3 > \kappa_4 > \kappa_5$
CIFDOWG-Removing π_4	$\kappa_1 > \kappa_2 > \kappa_4 > \kappa_3 > \kappa_5$

The sensitivity study demonstrates that the decision-making process displays both robustness and sensitivity, with some features leading to moderate shifts in rankings. This highlights the need of carefully considering important factors when formulating decisions.

6.4. *Comparative analysis*

In this section, we consider various developed dynamic operators in IF and CIF environments to investigate the validity and authenticity of our proposed operators. The Tables 15 and 16 depicting the aggregated values obtained from different operators and the ranking of the alternatives show the comparison between our techniques and IFDWA, IFDWG, CIFDWA, and CIFDWG strategies.

Table 15. Aggregated values obtained from various dynamic aggregation operators.

	IFDWA [55]	IFDWG [56]	CIFDWA [38]	CIFDWG [38]
κ_1	(0.602,0.332)	(0.522,0.388)	((0.666,0.661),(0.275,0.246))	((0.644,0.613),(0.291,0.289))
κ_2	(0.606,0.231)	(0.510,0.298)	((0.606,0.595),(0.231,0.276))	((0.509,0.514),(0.298,0.349))
κ_3	(0.604,0.236)	(0.530,0.367)	((0.594,0.475),(0.246,0.310))	((0.521,0.376),(0.365,0.359))
κ_4	(0.612,0.176)	(0.554,0.241)	((0.612,0.623),(0.176,0.216))	((0.554,0.415),(0.241,0.301))
κ_5	(0.528,0.265)	(0.458,0.331)	((0.528,0.608),(0.265,0.254))	((0.458,0.525),(0.331,0.347))

Table 16. Score values and ranking of alternatives under existing and newly proposed dynamic aggregation operators.

Operators	κ_1	κ_2	κ_3	κ_4	κ_5	Ranking
IFDWA [55]	0.272	0.393	0.368	0.436	0.296	$\kappa_4 > \kappa_2 > \kappa_3 > \kappa_1 > \kappa_5$
IFDWG [56]	0.134	0.212	0,163	0.313	0.127	$\kappa_4 > \kappa_2 > \kappa_3 > \kappa_1 > \kappa_5$
CIFDWA [38]	0.494	0.417	0.329	0.475	0.378	$\kappa_1 > \kappa_4 > \kappa_2 > \kappa_5 > \kappa_3$
CIFDWG [38]	0.432	0.264	0.182	0.286	0.228	$\kappa_1 > \kappa_4 > \kappa_2 > \kappa_5 > \kappa_3$
CIFDOWA	0.683	0.679	0.644	0.657	0.593	$\kappa_1 > \kappa_2 > \kappa_4 > \kappa_3 > \kappa_5$
CIFDOWG	0.664	0.607	0.550	0.553	0.544	$\kappa_1 > \kappa_2 > \kappa_4 > \kappa_3 > \kappa_5$

Comparison 1. The strategies outlined in this article exhibit a greater degree of generalizability in comparison to other established methodologies, as evidenced by the preceding discourse. As a result, when information is lost within the current IF framework, the optimal preference varies. Nevertheless, this issue is effectively resolved by the CIF dynamic aggregation operators. Notably, Zeshui Xu and G. W. WEI proposed operators [55,56] that are special cases of these operators by disregarding the permuted factor and holding the second dimension constant.

Comparison 2. These operators can represent the relative relevance of items better than CIF dynamic weighted aggregation approaches [38] by letting us define the weighting order. Additionally, in circumstances where the relative significance of values fluctuates depending on the context, CIF dynamic ordered weighted aggregation operators exhibit a notable advantage over CIF dynamic aggregation operators by virtue of their capacity to assign weights to values in accordance with their order or rank.

Comparison 3. The aggregation operators proposed by Garg and Rani [35], which are based on CIFS, do not incorporate time intervals and permuted factor, leading to significant data loss. The suggested operators effectively manage both dynamic and permuted aspects of the MADM problems. For instance, the data presented in Tables 1–3 was gathered from three distinct time intervals, and the CIF dynamic ordered weighted aggregation operators possess the ability to manage this specific kind of data, whereas the CIFS-based aggregation operators proposed in [35] are unable to deal the information shown in Tables 1–3.

Comparison 4. The approaches presented in [36] lack dynamic features, which means they cannot be used in situations such as those in our investigation where the initial decision data is gathered over three separate time intervals. As a result, the suggested CIF dynamic ordered weighted aggregation operators are more flexible to handle such scenarios, while others fail.

Comparison 5. The geometric operators described in reference [37] are unable to analyze the decision data presented in Tables 1–3, which is obtained from three distinct time intervals. These operators cannot handle the challenge of data of a dynamic nature. This drawback leads to a loss of information. However, the recently proposed strategies become more significant as they improve this deficiency in the existing literature.

The Figure 1 illustrates the diagrammatic representation of the rating of different alternatives.

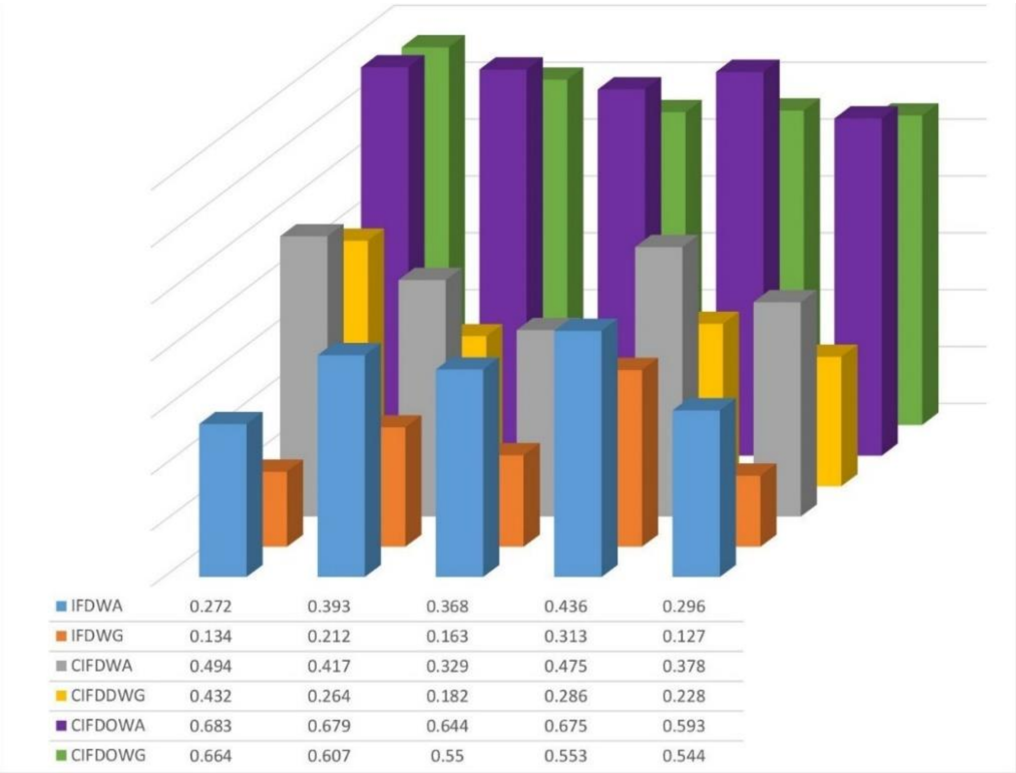

Figure 1. Ranking of alternatives using different operators.

7. Conclusions

In this article, we have endeavored to innovate novel strategies for resolving decision-making challenges within dynamic CIF settings. Despite the extensive development of useful operators in the existing literature, none have explicitly addressed time periods within the context of CIF dynamic ordered weighted aggregation knowledge. Consequently, utilizing a dynamic CIF environment model proves to be a more efficacious method for representing time-periodic difficulties, given its ability to handle two-dimensional data within a singular set. Considering these factors, we have introduced a distinct set of dynamic ordered weighted averaging and geometric aggregation operators within the CIF framework, enriching its functionality and applicability. We have proceeded to thoroughly examine the properties of these operators. Additionally, we have introduced an innovative score function designed for the assessment and identification of the most favorable alternative. In the scope of our research, we have presented an original methodology for addressing dynamic and intricate decision-making challenges involving multiple attributes in the CIF domain. Moreover, our decision-making process incorporates data gathered from distinct time periods. Additionally, we have successfully employed these innovative strategies to select efficient solar cells in a CIF dynamic environment. Last, we have carried out a comparative analysis aiming to underscore the importance and credibility of these novel methodologies in contrast to established techniques.

The main limitation of the current study is that the suggested operators cannot handle situations that are outside the scope of CIF dynamic information. In order to rectify this limitation, it is necessary to explore the validity of recently developed approaches in more generalized frameworks, such as complex Pythagorean fuzzy sets, complex Fermatean fuzzy sets, and complex q-rung orthopair fuzzy sets. Our future study will focus to extend the domain of dynamic aggregation operators in various directions, specifically the CIF dynamic Dombi aggregation operators and the CIF Dombi exponential aggregation operators.

Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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