AIMS Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 9(4): 7945–7954.
DOI:10.3934/math.2024386
Received: 20 December 2023
Revised: 01 February 2024
Accepted: 05 February 2024
Published: 26 February 2024

Research article

Fixed point theorems of contractive mappings on soft parametric metric space

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Abstract: The purpose of this study was to introduce soft topology generated by soft parametric metric space and prove Banach's fixed point theorem as an extension of soft complete parametric metric space. An illustrative example was given by using this fixed point theorem.

Keywords: parametric soft metric space; Banach's fixed point theorem; soft complete parametric metric space
Mathematics Subject Classification: 32H50, 47H10, 54H25

1. Introduction

Functional analysis is a significant branch of science that can be used to solve different kinds of problems including both linear and nonlinear types. It has a great deal of applications in physics, chemistry, biology, economics, etc. [3, 4, 11, 12, 18]. A difficulty arises in that one must establish existing proof by frequently applying the contractivity method, monotonicity theory, etc. This absolutely enhances the importance and significance of fixed point theory as a valuable topic in functional analysis. Fixed point theorems establish an important part for proving the existence of solutions of the different types of linear and nonlinear problems such as deformation of rod, melting process, heat radiation, diffusion processes in physics, chemistry, biology, quantum field theory and game theory in economics. Additionally, sophisticated problems could not be analyzed utilizing general methods because of incomplete information or uncertainty data that occurred in the soft set theory [19]. The literature has produced many results on soft sets, actively. Soft set theory's properties, operations and applications were studied by [17].

Some researchers obtained fixed point theorems by using soft set theory on different soft metric spaces, which are complete soft usual metric spaces [8], soft cone metric spaces [16], dislocated soft metric spaces [5], soft G-metric spaces [13], soft S-metric spaces [14], soft rectangular B-metric spaces [21], soft metric spaces [1, 2, 9, 15, 23, 24], soft parametric metric space [7] and parametric soft b-metric spaces [22].

In this paper, we introduce soft topology generated by soft parametric metric space after giving some preliminary results, then we extend Banach's fixed point theorem in a soft complete parametric metric space.

2. Preliminaries

We give a start with the most necessary basic definitions and concepts to follow up the results obtained in this manuscript. Additional explanations are contained in the cited references.

Definition 2.1. *[19] A pair $(\widetilde{\rho}, \Omega)$ is claimed to be soft set on the universe set U if and only if $\widetilde{\rho}$ is a set valued mapping on Ω taking values in $P(U)$, where $P(U)$ is the power set of U. A soft set $(\widetilde{\rho}, \Omega)$ can be accepted as a parametrized family of subsets of the universe set U. The set $\widetilde{\rho}(a)$ in U is called an-approximate element of the soft set $(\widetilde{\rho}, \Omega)$, for each a in Ω.*

Definition 2.2. *[17] A soft set $(\widetilde{\rho}, \Omega)$ over U is called a null soft set if $\widetilde{\rho}(a) = \emptyset$ for all $a \in \Omega$ and is denoted by $\widetilde{\emptyset}$, an absolute soft set if $\widetilde{\rho}(a) = U$ for all $a \in \Omega$ and is denoted by $\widetilde{U}$.*

Definition 2.3. *[20] A collection $\widetilde{\tau}$ of soft sets over U is a soft topology on U if*

(1) $\widetilde{\emptyset}$, $\widetilde{U}$ are included in $\widetilde{\tau}$;
(2) the union of any number of soft sets in $\widetilde{\tau}$ is included in $\widetilde{\tau}$;
(3) the intersection of any two soft sets in $\widetilde{\tau}$ is included in $\widetilde{\tau}$.

The triplet $(U, \widetilde{\tau}, \Omega)$ is claimed to be a soft topological space over U. Members of $\widetilde{\tau}$ are said to be soft open sets in U. A soft set $(\widetilde{\rho}, \Omega)$ over U is a soft closed set whenever its complement $(\widetilde{\rho}, \Omega)^c$ is included in $\widetilde{\tau}$.

Proposition 2.4. *[20] For each $a \in \Omega$, the collection $\widetilde{\tau}_a = \{\widetilde{\rho}(a) : (\widetilde{\rho}, \Omega) \in \widetilde{\tau}\}$ defines a topology on U, where $(U, \widetilde{\tau}, \Omega)$ is a soft topological space over U.*

Definition 2.5. *[6, 10] $\widetilde{u}_a$ is called a soft point on the soft set $(\widetilde{\rho}, \Omega)$ over U if $(\widetilde{\rho}, \Omega)$ is defined as*

$$\widetilde{\rho} : \Omega \to P(U), \widetilde{\rho}(a) = \begin{cases} \{u\}, & \text{if} \quad a \in \Omega, \\ \emptyset, & \text{if} \quad a' \in \Omega - \{a\}. \end{cases}$$

It is clear that each soft set can be denoted as a union of soft points. Therefore, when we have the family of all soft sets on U, it is suitable to give only soft points on U.

Definition 2.6. *[10] A soft set $(\widetilde{\rho}, \Omega)$ defined by $\widetilde{\rho} : \Omega \to B(\mathbb{R})$ is said to be a soft real set, where $\mathbb{R}$ is the set of all real numbers and $B(\mathbb{R})$ is the collection of all nonempty bounded subsets of $\mathbb{R}$. When a soft real set $(\widetilde{\rho}, \Omega)$ is a singleton soft set, it is called a soft real number and denoted by $\widetilde{r}, \widetilde{s}, \widetilde{t}$. It is clear that $\overline{0}(a) = 0$, $\overline{1}(a) = 1$, for all $a \in \Omega$.*

Definition 2.7. *[10] The collection of nonnegative soft real numbers is denoted by $\mathbb{R}(\Omega)^*$ and the pair $(\mathbb{R}(\Omega)^*, \leq)$ is a partially ordered set, such that for $\widetilde{r}, \widetilde{s}$ as two soft real numbers,*

(i) $\widetilde{r}\,\widetilde{\leq}\,\widetilde{s}$ if $\widetilde{r}(a) \leq \widetilde{s}(a)$ for all $a \in \Omega$,
(ii) $\widetilde{r}\,\widetilde{\geq}\,\widetilde{s}$ if $\widetilde{r}(a) \geq \widetilde{s}(a)$ for all $a \in \Omega$,
(iii) $\widetilde{r}\,\widetilde{<}\,\widetilde{s}$ if $\widetilde{r}(a) < \widetilde{s}(a)$ for all $a \in \Omega$,
(iv) $\widetilde{r}\,\widetilde{>}\,\widetilde{s}$ if $\widetilde{r}(a) > \widetilde{s}(a)$ for all $a \in \Omega$.

Definition 2.8. *[6, 10] Two soft points $\widetilde{u}_a, \widetilde{v}_{a'}$ are called equal when $a = a'$ and $u = v$. Thus, $\widetilde{u}_a \neq \widetilde{v}_{a'}$ if and only if $u \neq v$ or $a \neq a'$.*

The family of all soft points of the set $\widetilde{U}$ is denoted by $SP\left(\widetilde{U}\right)$.

Definition 2.9. *[7] Let $\widetilde{d} : SP\left(\widetilde{U}\right) \times SP\left(\widetilde{U}\right) \times (0,\infty) \to \mathbb{R}(\Omega)^*$ be a function satisfying the following conditions for all $\widetilde{u}_{a_1}, \widetilde{v}_{a_2}, \widetilde{z}_{a_3} \in \widetilde{U}, t > 0$,*

(1) $\widetilde{d}(\widetilde{u}_{a_1}, \widetilde{v}_{a_2}, t) = \overline{0}$ if and only if $\widetilde{u}_{a_1} = \widetilde{v}_{a_2}$ for all $t > 0$,
(2) $\widetilde{d}(\widetilde{u}_{a_1}, \widetilde{v}_{a_2}, t) = \widetilde{d}(\widetilde{v}_{a_2}, \widetilde{u}_{a_1}, t)$ for all $\widetilde{u}_{a_1}, \widetilde{v}_{a_2} \in \widetilde{U}, t > 0$,
(3) $\widetilde{d}(\widetilde{u}_{a_1}, \widetilde{z}_{a_3}, t)\,\widetilde{<}\,\widetilde{d}(\widetilde{u}_{a_1}, \widetilde{v}_{a_2}, t) + \widetilde{d}(\widetilde{v}_{a_2}, \widetilde{z}_{a_3}, t)$ for all $\widetilde{u}_{a_1}, \widetilde{v}_{a_2}, \widetilde{z}_{a_3} \in \widetilde{U}, t > 0$.
Therefore, $\widetilde{d}$ is called a soft parametric metric on $\widetilde{U}$ and the pair $\left(\widetilde{U}, \widetilde{d}, \Omega\right)$ is called a soft parametric metric space.

Example 2.10. *Let $U = \mathbb{R}^+ \cup \{0\}$ be a universe set, $\Omega = \mathbb{N} \cup \{0\}$ be a parameter set, $g : (0,\infty) \to (0,\infty)$ be a continous positive function. Let $\widetilde{d} : SP\left(\widetilde{U}\right) \times SP\left(\widetilde{U}\right) \times (0,\infty) \to \mathbb{R}(\Omega)^*$ be denoted by*

$$\widetilde{d}(\widetilde{u}_{a_1}, \widetilde{v}_{a_2}, t) = \begin{cases} \overline{0}, & \text{if} \quad \widetilde{u}_{a_1} = \widetilde{v}_{a_2}, \\ g(t)(\max\{u, v\} + \max\{a_1, a_2\}), & \text{if} \quad \widetilde{u}_{a_1} \neq \widetilde{v}_{a_2}, \end{cases}$$

for all $\widetilde{u}_{a_1}, \widetilde{v}_{a_2} \in \widetilde{U}$, then $\widetilde{d}$ is a soft parametric metric on $\widetilde{U}$ and $\left(\widetilde{U}, \widetilde{d}, \Omega\right)$ is a soft parametric metric space.

Definition 2.11. *[7] A sequence $\left\{\widetilde{u}^n_{a_n}\right\}$ in soft parametric metric space $\left(\widetilde{U}, \widetilde{d}, \Omega\right)$ is convergent if there is a soft point $\widetilde{u}^0_{a_0}$ in $\widetilde{U}$, such that $\lim_{n\to\infty} \widetilde{d}\left(\widetilde{u}^n_{a_n}, \widetilde{u}^0_{a_0}, t\right) = \overline{0}$ denoted by $\lim_{n\to\infty} \widetilde{u}^n_{a_n} = \widetilde{u}^0_{a_0}$ for all $t > 0$.*

Definition 2.12. *[7] A sequence $\left\{\widetilde{u}^n_{a_n}\right\}$ in soft parametric metric space $\left(\widetilde{U}, \widetilde{d}, \Omega\right)$ is a Cauchy sequence if $\lim_{n,m\to\infty} \widetilde{d}\left(\widetilde{u}^n_{a_n}, \widetilde{u}^m_{a_m}, t\right) = \overline{0}$ for all $t > 0$.*

Definition 2.13. *[7] If every Cauchy sequence is a convergent in soft parametric metric space $\left(\widetilde{U}, \widetilde{d}, \Omega\right)$, then $\left(\widetilde{U}, \widetilde{d}, \Omega\right)$ is called a soft complete parametric metric space.*

3. Soft topology generated by soft parametric metric spaces

This section confidently presents the significant findings of soft parametric metric spaces.

Let $\{(U, \tau_t)\}_{t\in T}$, $T \subset \mathbb{R}$ be a family of a topological space corresponding to $t-$parameter. A mapping between $\{(U, \tau_t)\}_{t\in T}$ and $\{(V, \widetilde{\tau}_{t'})\}_{t'\in T'}$ consists of the mappings $f : U \to V$ and $\psi : T \to T'$ such that $f : (U, \tau_t) \to \left(V, \widetilde{\tau}_{\psi(t)}\right)$ is a mapping of topological spaces.

$f : (U, \tau_t) \to \left(V, \widetilde{\tau}_{\psi(t)}\right)$ is a mapping on family of topological spaces generated by $f : U \to V$ and $\psi : T \to T'$.

Definition 3.1. *Let* $\{(U,\tau_t)\}_{t\in T}$ *and* $\{(V,\widetilde{\tau}_{t'})\}_{t'\in T'}$ *be two families of topological spaces. If for each* $t \in T$, $f : (U,\tau_t) \to \left(V,\widetilde{\tau}_{\psi(t)}\right)$ *is a continuous mapping on topological spaces, then* $(f,\psi) : \{(U,\tau_t)\}_{t\in T} \to \{(V,\widetilde{\tau}_{t'})\}_{t'\in T'}$ *is continuous as a mapping of families of soft topological spaces.*

These concepts will be extended in a confident way to soft topological spaces. Let $\left\{\left(\widetilde{U},\widetilde{\tau}_t,\Omega\right)\right\}_{t\in T}$ be a family of a soft topological space corresponding to $t-$parameter. A mapping between soft topological spaces $(f,\psi,\varphi) : \left\{\left(\widetilde{U},\widetilde{\tau}_t,\Omega\right)\right\}_{t\in T} \to \left\{\left(\widetilde{V},\widetilde{\tau}'_{t'},\Omega'\right)\right\}_{t'\in T'}$ consists of the mappings $f : U \to V$, $\psi : T \to T'$, and $\varphi : \Omega \to \Omega'$ such that $(f,\varphi) : \left(\widetilde{U},\widetilde{\tau}_t,\Omega\right) \to \left(\widetilde{V},\widetilde{\tau}'_{\psi(t)},\Omega'\right)$ is a mapping of soft topological spaces for each $t \in T$.

Definition 3.2. *If for each* $t \in T$, $(f,\varphi) : \left(\widetilde{U},\widetilde{\tau}_t,\Omega\right) \to \left(\widetilde{V},\widetilde{\tau}'_{\psi(t)},\Omega'\right)$ *is a soft continuous mapping on soft topological spaces, then* $(f,\psi,\varphi) : \left\{\left(\widetilde{U},\widetilde{\tau}_t,\Omega\right)\right\}_{t\in T} \to \left\{\left(\widetilde{V},\widetilde{\tau}'_{t'},\Omega'\right)\right\}_{t'\in T'}$ *is a continuous mapping of families of soft topological spaces.*

Note 3.3. *Let* (U,d,t) *be a parametric metric space. Corresponding to each parameter* t*, we have a metric space* (U,d_t)*. If* τ_{d_t} *induced by* d_t *is a topology, then* (U,τ_{d_t}) *is a topological space. Thus,* (U,d,t) *is a parametric metric space, which gives a parameterized family of metric space* $\{(U,d_t)\}_{t\in T}$*. Similarly,* $\left(\widetilde{U},\widetilde{d},\Omega,T\right)$ *is a soft parametric metric space, which gives a parameterized family of soft metric space* $\left\{\left(\widetilde{U},\widetilde{d}_t,\Omega\right)\right\}_{t\in T}$.

Let $\left(\widetilde{U},\widetilde{d},\Omega,T\right)$ and $\left(\widetilde{V},\widetilde{d}',\Omega',T'\right)$ be two soft parametric metric spaces. A mapping on soft parametric metric spaces is given by $(f,\varphi,\psi) : \left(\widetilde{U},\widetilde{d},\Omega,T\right) \to \left(\widetilde{V},\widetilde{d}',\Omega',T'\right)$, where $f : U \to V$, $\varphi : \Omega \to \Omega'$, and $\psi : T \to T'$.

Assuming $T = T'$ and $\psi = I_T$, we can confidently define a mapping on soft parametric metric spaces as $(f,\varphi) : \left(\widetilde{U},\widetilde{d},\Omega\right) \to \left(\widetilde{V},\widetilde{d}',\Omega'\right)$.

Definition 3.4. *[7] Let* $(f,\varphi) : \left(\widetilde{U},\widetilde{d},\Omega\right) \to \left(\widetilde{V},\widetilde{d}',\Omega'\right)$ *be a soft mapping on soft parametric metric spaces. If* $\lim_{n\to\infty}(f,\varphi)\left(\widetilde{u}^n_{a_n}\right) = (f,\varphi)(\widetilde{u}_a)$ *for any sequence of soft points* $\left\{\widetilde{u}^n_{a_n}\right\}$ *in* $\left(\widetilde{U},\widetilde{d},\Omega\right)$ *and all* $t > 0$ *satisfying* $\lim_{n\to\infty}\widetilde{u}^n_{a_n} = \widetilde{u}_a$*, then* (f,φ) *is said to be a soft continuous mapping at* $\widetilde{u}_a$.

Definition 3.5. *Let* $\left(\widetilde{U},\widetilde{d},\Omega\right)$ *be a soft parametric metric space. A mapping* $(f,\varphi) : \left(\widetilde{U},\widetilde{d},\Omega\right) \to \left(\widetilde{U},\widetilde{d},\Omega\right)$ *is called a soft contraction mapping if there exists a soft real number* $\overline{0}\widetilde{\leq}\widetilde{\alpha}\widetilde{<}\overline{1}$ *such that*

$$\widetilde{d}\left((f,\varphi)(\widetilde{u}_a),(f,\varphi)(\widetilde{v}_{a'}),t\right)\widetilde{\leq}\widetilde{\alpha}\widetilde{d}\left(\widetilde{u}_a,\widetilde{v}_{a'},t\right)$$

for all $t > 0, \widetilde{u}_a, \widetilde{v}_{a'} \in SP\left(\widetilde{U}\right)$.

Proposition 3.6. *[24]* $(f,\varphi)(\widetilde{u}_a)$ *is a soft point* $\widetilde{V}$*, for all soft points* $\widetilde{u}_a$ *in* $\widetilde{U}$.

Theorem 3.7. *Let* $\left(\widetilde{U},\widetilde{d},\Omega\right)$ *be a soft complete parametric metric space. If* $(f,\varphi) : \left(\widetilde{U},\widetilde{d},\Omega\right) \to \left(\widetilde{U},\widetilde{d},\Omega\right)$ *is a soft contraction mapping, then there exists a unique soft point* $\widetilde{u}^*_a \in SP\left(\widetilde{U}\right)$ *such that* $(f,\varphi)(\widetilde{u}^*_a) = \widetilde{u}^*_a$.

Proof. Let $\widetilde{u}^0_a$ be an arbitrary soft point in $SP\left(\widetilde{U}\right)$. Let us set

$$\widetilde{u}^1_{a_1} = (f,\varphi)\left(\widetilde{u}^0_a\right) = \left(f\left(\widetilde{u}^0_a\right)\right)_{\varphi(a)},$$

$$\begin{aligned}
\widetilde{u}^2_{a_2} &= (f,\varphi)\left(\widetilde{u}^1_{a_1}\right) = \left(f^2\left(\widetilde{u}^0_a\right)\right)_{\varphi^2(a)}, \\
&\cdots \\
\widetilde{u}^{n+1}_{a_{n+1}} &= (f,\varphi)\left(\widetilde{u}^n_{a_n}\right) = \left(f^{n+1}\left(\widetilde{u}^0_a\right)\right)_{\varphi^{n+1}(a)}.
\end{aligned}$$

We have

$$\begin{aligned}
\widetilde{d}\left(\widetilde{u}^{n+1}_{a_{n+1}}, \widetilde{u}^n_{a_n}, t\right) &= \widetilde{d}\left((f,\varphi)\left(\widetilde{u}^n_{a_n}\right), (f,\varphi)\left(\widetilde{u}^{n-1}_{a_{n-1}}\right), t\right) \widetilde{\leq} \widetilde{\alpha}\widetilde{d}\left(\widetilde{u}^n_{a_n}, \widetilde{u}^{n-1}_{a_{n-1}}, t\right) \\
&\widetilde{\leq} \widetilde{\alpha}^2\widetilde{d}\left(\widetilde{u}^{n-1}_{a_{n-1}}, \widetilde{u}^{n-2}_{a_{n-2}}, t\right) \widetilde{\leq} \cdots \widetilde{\leq} \widetilde{\alpha}^n\widetilde{d}\left(\widetilde{u}^1_{a_1}, \widetilde{u}^0_{a_0}, t\right).
\end{aligned}$$

So, for $n > m$,

$$\begin{aligned}
\widetilde{d}\left(\widetilde{u}^n_{a_n}, \widetilde{u}^m_{a_m}, t\right) &\widetilde{\leq} \widetilde{d}\left(\widetilde{u}^n_{a_n}, \widetilde{u}^{n-1}_{a_{n-1}}, t\right) + \widetilde{d}\left(\widetilde{u}^{n-1}_{a_{n-1}}, \widetilde{u}^{n-2}_{a_{n-2}}, t\right) + \cdots + \widetilde{d}\left(\widetilde{u}^{m+1}_{a_{m+1}}, \widetilde{u}^m_{a_m}, t\right) \\
&\widetilde{\leq} \left(\widetilde{\alpha}^{n-1} + \widetilde{\alpha}^{n-2} + \cdots + \widetilde{\alpha}^m\right)\widetilde{d}\left(\widetilde{u}^1_{a_1}, \widetilde{u}^0_{a_0}, t\right) \\
&\widetilde{\leq} \frac{\widetilde{\alpha}^m\left(1-\widetilde{\alpha}^{n-m}\right)}{1-\widetilde{\alpha}}\widetilde{d}\left(\widetilde{u}^1_{a_1}, \widetilde{u}^0_{a_0}, t\right).
\end{aligned}$$

Hence, we get $\widetilde{d}\left(\widetilde{u}^n_{a_n}, \widetilde{u}^m_{a_m}, t\right) \widetilde{\leq} \left(\frac{\widetilde{\alpha}^m}{1-\widetilde{\alpha}} - \frac{\widetilde{\alpha}^n}{1-\widetilde{\alpha}}\right)\widetilde{d}\left(\widetilde{u}^1_{a_1}, \widetilde{u}^0_{a_0}, t\right)$. This implies $\widetilde{d}\left(\widetilde{u}^n_{a_n}, \widetilde{u}^m_{a_m}, t\right) \to \overline{0}$ as $n, m \to \infty$. Hence, $\left\{\widetilde{u}^n_{a_n}\right\}$ is a soft Cauchy sequence. By the completeness of $\widetilde{U}$, there is a soft point $\widetilde{u}^*_a \in SP\left(\widetilde{U}\right)$ such that $\widetilde{u}^n_{a_n} \to \widetilde{u}^*_a$ as $n \to \infty$. Since $\widetilde{u}^n_{a_n} \to \widetilde{u}^*_a$ as $n \to \infty$,

$$\begin{aligned}
\widetilde{d}\left((f,\varphi)\left(\widetilde{u}^*_a\right), \widetilde{u}^*_a, t\right) &\widetilde{\leq} \widetilde{d}\left((f,\varphi)\left(\widetilde{u}^*_a\right), (f,\varphi)\left(\widetilde{u}^n_{a_n}\right), t\right) + \widetilde{d}\left((f,\varphi)\left(\widetilde{u}^n_{a_n}\right), \widetilde{u}^*_a, t\right) \\
&\widetilde{\leq} \widetilde{\alpha}\widetilde{d}\left(\widetilde{u}^*_a, \widetilde{u}^n_{a_n}, t\right) + \widetilde{d}\left(\widetilde{u}^{n+1}_{a_{n+1}}, \widetilde{u}^*_a, t\right) \to \overline{0}, n \to \infty.
\end{aligned}$$

Thus, we have $(f,\varphi)\left(\widetilde{u}^*_a\right) = \widetilde{u}^*_a$, which is the proof that the mapping (f,φ) has a fixed soft point. For the uniqueness, we suppose the converse, which means there could be two different fixed points $\widetilde{u}^*_a, \widetilde{v}^*_{a'}$ of the mapping (f,φ), then

$$\widetilde{d}\left(\widetilde{u}^*_a, \widetilde{v}^*_{a'}, t\right) = \widetilde{d}\left((f,\varphi)\left(\widetilde{u}^*_a\right), (f,\varphi)\left(\widetilde{v}^*_{a'}\right), t\right) \;\widetilde{\leq}\; \widetilde{\alpha}\widetilde{d}\left(\widetilde{u}^*_a, \widetilde{v}^*_{a'}, t\right),$$

for $\overline{0}\widetilde{\leq}\widetilde{\alpha}\widetilde{<}\overline{1}$. This contradicts the soft real number of $\widetilde{\alpha}$, that is, $\widetilde{u}^*_a$ is a unique soft fixed point of the mapping (f,φ).

Theorem 3.8. *Let $\left(\widetilde{U}, \widetilde{d}, \Omega\right)$ be a soft complete parametric metric space. If a mapping (f,φ) : $\left(\widetilde{U}, \widetilde{d}, \Omega\right) \to \left(\widetilde{U}, \widetilde{d}, \Omega\right)$ satisfies the following soft contractive condition*

$$\widetilde{d}\left((f,\varphi)\left(\widetilde{u}_a\right), (f,\varphi)\left(\widetilde{v}_{a'}\right), t\right) \;\widetilde{\leq}\; \widetilde{\alpha}\left[\widetilde{d}\left((f,\varphi)\left(\widetilde{u}_a\right), \widetilde{u}_a, t\right) + \widetilde{d}\left((f,\varphi)\left(\widetilde{v}_{a'}\right), \widetilde{v}_{a'}, t\right)\right]$$

for all $t > 0, \widetilde{u}_a, \widetilde{v}_{a'} \in SP\left(\widetilde{U}\right), \overline{0}\widetilde{\leq}\widetilde{\alpha}\widetilde{<}\widetilde{\left(\frac{1}{2}\right)}$, then there exists a unique soft point $\widetilde{u}^*_a \in SP\left(\widetilde{U}\right)$ such that $(f,\varphi)\left(\widetilde{u}^*_a\right) = \widetilde{u}^*_a$.

Proof. Let $\widetilde{u}^0_a$ be an arbitrary soft point in $SP\left(\widetilde{U}\right)$. Let us set

$$\begin{aligned}
\widetilde{u}^1_{a_1} &= (f,\varphi)\left(\widetilde{u}^0_a\right) = \left(f\left(\widetilde{u}^0_a\right)\right)_{\varphi(a)}, \\
\widetilde{u}^2_{a_2} &= (f,\varphi)\left(\widetilde{u}^1_{a_1}\right) = \left(f^2\left(\widetilde{u}^0_a\right)\right)_{\varphi^2(a)}, \\
&\cdots
\end{aligned}$$

$$\widetilde{u}^{n+1}_{a_{n+1}} = (f,\varphi)\left(\widetilde{u}^{n}_{a_n}\right) = \left(f^{n+1}\left(\widetilde{u}^{0}_{a}\right)\right)_{\varphi^{n+1}(a)}.$$

We have

$$\begin{aligned}\widetilde{d}\left(\widetilde{u}^{n+1}_{a_{n+1}},\widetilde{u}^{n}_{a_n},t\right) &= \widetilde{d}\left((f,\varphi)\left(\widetilde{u}^{n}_{a_n}\right),(f,\varphi)\left(\widetilde{u}^{n-1}_{a_{n-1}}\right),t\right)\\ &\widetilde{\leq} \widetilde{\alpha}\left[\widetilde{d}\left((f,\varphi)\left(\widetilde{u}^{n}_{a_n}\right),\widetilde{u}^{n}_{a_n},t\right)+\widetilde{d}\left((f,\varphi)\left(\widetilde{u}^{n-1}_{a_{n-1}}\right),\widetilde{u}^{n-1}_{a_{n-1}},t\right)\right]\\ &\widetilde{\leq} \widetilde{\alpha}\left[\widetilde{d}\left(\widetilde{u}^{n+1}_{a_{n+1}},\widetilde{u}^{n}_{a_n},t\right)+\widetilde{d}\left(\widetilde{u}^{n}_{a_n},\widetilde{u}^{n-1}_{a_{n-1}},t\right)\right],\end{aligned}$$

which gives

$$\widetilde{d}\left(\widetilde{u}^{n+1}_{a_{n+1}},\widetilde{u}^{n}_{a_n},t\right) \;\widetilde{\leq}\; \widetilde{\beta}\widetilde{d}\left(\widetilde{u}^{n}_{a_n},\widetilde{u}^{n-1}_{a_{n-1}},t\right),$$

where $\widetilde{\beta}=\frac{\widetilde{\alpha}}{1-\widetilde{\alpha}}$. So, for $n>m$,

$$\begin{aligned}\widetilde{d}\left(\widetilde{u}^{n}_{a_n},\widetilde{u}^{m}_{a_m},t\right) &\widetilde{\leq} \widetilde{d}\left(\widetilde{u}^{n}_{a_n},\widetilde{u}^{n-1}_{a_{n-1}},t\right)+\widetilde{d}\left(\widetilde{u}^{n-1}_{a_{n-1}},\widetilde{u}^{n-2}_{a_{n-2}},t\right)+\cdots+\widetilde{d}\left(\widetilde{u}^{m+1}_{a_{m+1}},\widetilde{u}^{m}_{a_m},t\right)\\ &\widetilde{\leq} \left(\widetilde{\beta}^{n-1}+\widetilde{\beta}^{n-2}+\cdots+\widetilde{\beta}^{m}\right)\widetilde{d}\left(\widetilde{u}^{1}_{a_1},\widetilde{u}^{0}_{a_0},t\right)\\ &\widetilde{\leq} \frac{\widetilde{\beta}^{m}\left(1-\widetilde{\beta}^{n-m}\right)}{1-\widetilde{\beta}}\widetilde{d}\left(\widetilde{u}^{1}_{a_1},\widetilde{u}^{0}_{a_0},t\right),\end{aligned}$$

We get $\widetilde{d}\left(\widetilde{u}^{n}_{a_n},\widetilde{u}^{m}_{a_m},t\right)\widetilde{\leq}\left(\frac{\widetilde{\beta}^{m}}{1-\widetilde{\beta}}-\frac{\widetilde{\beta}^{n}}{1-\widetilde{\beta}}\right)\widetilde{d}\left(\widetilde{u}^{1}_{a_1},\widetilde{u}^{0}_{a_0},t\right)$. This implies $\widetilde{d}\left(\widetilde{u}^{n}_{a_n},\widetilde{u}^{m}_{a_m},t\right)\to\overline{0}$ as $n,m\to\infty$. Hence, $\left\{\widetilde{u}^{n}_{a_n}\right\}$ is a soft Cauchy sequence. By the completeness of $\widetilde{U}$, there is a soft point $\widetilde{u}^{*}_{a}\in SP\left(\widetilde{U}\right)$ such that $\widetilde{u}^{n}_{a_n}\to\widetilde{u}^{*}_{a}$ as $n\to\infty$. Since $\widetilde{u}^{n}_{a_n}\to\widetilde{u}^{*}_{a}$ as $n\to\infty$,

$$\begin{aligned}\widetilde{d}\left((f,\varphi)\left(\widetilde{u}^{*}_{a}\right),\widetilde{u}^{*}_{a},t\right) &\widetilde{\leq} \widetilde{d}\left((f,\varphi)\left(\widetilde{u}^{*}_{a}\right),(f,\varphi)\left(\widetilde{u}^{n}_{a_n}\right),t\right)+\widetilde{d}\left((f,\varphi)\left(\widetilde{u}^{n}_{a_n}\right),\widetilde{u}^{*}_{a},t\right)\\ &\widetilde{\leq} \widetilde{\alpha}\left[\widetilde{d}\left((f,\varphi)\left(\widetilde{u}^{*}_{a}\right),\widetilde{u}^{*}_{a},t\right)+\widetilde{d}\left((f,\varphi)\left(\widetilde{u}^{n}_{a_n}\right),\widetilde{u}^{n}_{a_n},t\right)\right]+\widetilde{d}\left(\widetilde{u}^{n+1}_{a_{n+1}},\widetilde{u}^{*}_{a},t\right)\\ &= \widetilde{\alpha}\widetilde{d}\left((f,\varphi)\left(\widetilde{u}^{*}_{a}\right),\widetilde{u}^{*}_{a},t\right)+\widetilde{\alpha}\widetilde{d}\left(\widetilde{u}^{n+1}_{a_{n+1}},\widetilde{u}^{n}_{a_n},t\right)+\widetilde{d}\left(\widetilde{u}^{n+1}_{a_{n+1}},\widetilde{u}^{*}_{a},t\right),\end{aligned}$$

which gives

$$\widetilde{d}\left((f,\varphi)\left(\widetilde{u}^{*}_{a}\right),\widetilde{u}^{*}_{a},t\right) \;\widetilde{\leq}\; \frac{\widetilde{\alpha}}{1-\widetilde{\alpha}}\widetilde{d}\left(\widetilde{u}^{n+1}_{a_{n+1}},\widetilde{u}^{n}_{a_n},t\right)+\frac{\widetilde{1}}{1-\widetilde{\alpha}}\widetilde{d}\left(\widetilde{u}^{n+1}_{a_{n+1}},\widetilde{u}^{*}_{a},t\right)\to\overline{0},\ \text{as } n\to\infty.$$

Thus, we have $(f,\varphi)\left(\widetilde{u}^{*}_{a}\right)=\widetilde{u}^{*}_{a}$, which is the proof that the mapping (f,φ) has a fixed soft point. For the uniqueness, we suppose the converse, which means there could be two different fixed points $\widetilde{u}^{*}_{a},\widetilde{v}^{*}_{a'}$ of the mapping (f,φ), then

$$\begin{aligned}\widetilde{d}\left(\widetilde{u}^{*}_{a},\widetilde{v}^{*}_{a'},t\right) &= \widetilde{d}\left((f,\varphi)\left(\widetilde{u}^{*}_{a}\right),(f,\varphi)\left(\widetilde{v}^{*}_{a'}\right),t\right)\\ &\widetilde{\leq} \widetilde{\alpha}\left[\widetilde{d}\left((f,\varphi)\left(\widetilde{u}^{*}_{a}\right),\widetilde{u}^{*}_{a},t\right)+\widetilde{d}\left((f,\varphi)\left(\widetilde{v}^{*}_{a'}\right),\widetilde{v}^{*}_{a'},t\right)\right]=\overline{0},\end{aligned}$$

for $\overline{0}\widetilde{\leq}\widetilde{\alpha}\widetilde{<}\widetilde{\left(\frac{1}{2}\right)}$. That is, $\widetilde{u}^{*}_{a}$ is a unique soft fixed point of the mapping (f,φ).

Theorem 3.9. *Let* $\left(\widetilde{U},\widetilde{d},\Omega\right)$ *be a soft complete parametric metric space. If a mapping* $(f,\varphi):\left(\widetilde{U},\widetilde{d},\Omega\right)\to\left(\widetilde{U},\widetilde{d},\Omega\right)$ *satisfies the following soft contractive condition*

$$\widetilde{d}\left((f,\varphi)\left(\widetilde{u}_{a}\right),(f,\varphi)\left(\widetilde{v}_{a'}\right),t\right) \;\widetilde{\leq}\; \widetilde{\alpha}\left[\widetilde{d}\left((f,\varphi)\left(\widetilde{u}_{a}\right),\widetilde{v}_{a'},t\right)+\widetilde{d}\left((f,\varphi)\left(\widetilde{v}_{a'}\right),\widetilde{u}_{a},t\right)\right]$$

for all $t>0,\widetilde{u}_{a},\widetilde{v}_{a'}\in SP\left(\widetilde{U}\right),\overline{0}\widetilde{\leq}\widetilde{\alpha}\widetilde{<}\widetilde{\left(\frac{1}{2}\right)}$, *then there exists a unique soft point* $\widetilde{u}^{*}_{a}\in SP\left(\widetilde{U}\right)$ *such that* $(f,\varphi)\left(\widetilde{u}^{*}_{a}\right)=\widetilde{u}^{*}_{a}$.

Proof. Let $\widetilde{u}_a^0$ be an arbitrary soft point in $SP\left(\widetilde{U}\right)$. Let us set

$$\begin{aligned}
\widetilde{u}_{a_1}^1 &= (f,\varphi)\left(\widetilde{u}_a^0\right) = \left(f\left(\widetilde{u}_a^0\right)\right)_{\varphi(a)},\\
\widetilde{u}_{a_2}^2 &= (f,\varphi)\left(\widetilde{u}_{a_1}^1\right) = \left(f^2\left(\widetilde{u}_a^0\right)\right)_{\varphi^2(a)},\\
&\cdots\\
\widetilde{u}_{a_{n+1}}^{n+1} &= (f,\varphi)\left(\widetilde{u}_{a_n}^n\right) = \left(f^{n+1}\left(\widetilde{u}_a^0\right)\right)_{\varphi^{n+1}(a)}.
\end{aligned}$$

We have

$$\begin{aligned}
\widetilde{d}\left(\widetilde{u}_{a_{n+1}}^{n+1}, \widetilde{u}_{a_n}^n, t\right) &= \widetilde{d}\left((f,\varphi)\left(\widetilde{u}_{a_n}^n\right), (f,\varphi)\left(\widetilde{u}_{a_{n-1}}^{n-1}\right), t\right)\\
&\widetilde{\leq} \widetilde{\alpha}\left[\widetilde{d}\left((f,\varphi)\left(\widetilde{u}_{a_n}^n\right), \widetilde{u}_{a_{n-1}}^{n-1}, t\right) + \widetilde{d}\left((f,\varphi)\left(\widetilde{u}_{a_{n-1}}^{n-1}\right), \widetilde{u}_{a_n}^n, t\right)\right]\\
&\widetilde{\leq} \widetilde{\alpha}\left[\widetilde{d}\left(\widetilde{u}_{a_{n+1}}^{n+1}, \widetilde{u}_{a_{n-1}}^{n-1}, t\right) + \widetilde{d}\left(\widetilde{u}_{a_n}^n, \widetilde{u}_{a_n}^n, t\right)\right],
\end{aligned}$$

which gives

$$\widetilde{d}\left(\widetilde{u}_{a_{n+1}}^{n+1}, \widetilde{u}_{a_n}^n, t\right) \;\widetilde{\leq}\; \widetilde{\beta}\widetilde{d}\left(\widetilde{u}_{a_n}^n, \widetilde{u}_{a_{n-1}}^{n-1}, t\right),$$

where $\widetilde{\beta} = \frac{\widetilde{\alpha}}{1-\widetilde{\alpha}}$. So, for $n > m$,

$$\begin{aligned}
\widetilde{d}\left(\widetilde{u}_{a_n}^n, \widetilde{u}_{a_m}^m, t\right) &\widetilde{\leq} \widetilde{d}\left(\widetilde{u}_{a_n}^n, \widetilde{u}_{a_{n-1}}^{n-1}, t\right) + \widetilde{d}\left(\widetilde{u}_{a_{n-1}}^{n-1}, \widetilde{u}_{a_{n-2}}^{n-2}, t\right) + \cdots + \widetilde{d}\left(\widetilde{u}_{a_{m+1}}^{m+1}, \widetilde{u}_{a_m}^m, t\right)\\
&\widetilde{\leq} \left(\widetilde{\beta}^{n-1} + \widetilde{\beta}^{n-2} + \cdots + \widetilde{\beta}^m\right)\widetilde{d}\left(\widetilde{u}_{a_1}^1, \widetilde{u}_{a_0}^0, t\right)\\
&\widetilde{\leq} \frac{\widetilde{\beta}^m\left(1-\widetilde{\beta}^{n-m}\right)}{1-\widetilde{\beta}}\widetilde{d}\left(\widetilde{u}_{a_1}^1, \widetilde{u}_{a_0}^0, t\right).
\end{aligned}$$

This implies $\widetilde{d}\left(\widetilde{u}_{a_n}^n, \widetilde{u}_{a_m}^m, t\right) \to \overline{0}$ as $n, m \to \infty$. Hence, $\left\{\widetilde{u}_{a_n}^n\right\}$ is a soft Cauchy sequence. By the completeness of $\widetilde{U}$, there is a soft point $\widetilde{u}_a^* \in SP\left(\widetilde{U}\right)$ such that $\widetilde{u}_{a_n}^n \to \widetilde{u}_a^*$ as $n \to \infty$. Since $\widetilde{u}_{a_n}^n \to \widetilde{u}_a^*$ as $n \to \infty$,

$$\begin{aligned}
\widetilde{d}\left((f,\varphi)\left(\widetilde{u}_a^*\right), \widetilde{u}_a^*, t\right) &\widetilde{\leq} \widetilde{d}\left((f,\varphi)\left(\widetilde{u}_a^*\right), (f,\varphi)\left(\widetilde{u}_{a_n}^n\right), t\right) + \widetilde{d}\left((f,\varphi)\left(\widetilde{u}_{a_n}^n\right), \widetilde{u}_a^*, t\right)\\
&\widetilde{\leq} \widetilde{\alpha}\left[\widetilde{d}\left((f,\varphi)\left(\widetilde{u}_a^*\right), \widetilde{u}_{a_n}^n, t\right) + \widetilde{d}\left((f,\varphi)\left(\widetilde{u}_{a_n}^n\right), \widetilde{u}_a^*, t\right)\right] + \widetilde{d}\left(\widetilde{u}_{a_{n+1}}^{n+1}, \widetilde{u}_a^*, t\right)\\
&\widetilde{\leq} \widetilde{\alpha}\left[\widetilde{d}\left((f,\varphi)\left(\widetilde{u}_a^*\right), \widetilde{u}_a^*, t\right) + \widetilde{d}\left(\widetilde{u}_a^*, \widetilde{u}_{a_n}^n, t\right) + \widetilde{d}\left((f,\varphi)\left(\widetilde{u}_{a_n}^n\right), \widetilde{u}_a^*, t\right)\right] + \widetilde{d}\left(\widetilde{u}_{a_{n+1}}^{n+1}, \widetilde{u}_a^*, t\right),
\end{aligned}$$

which gives

$$\widetilde{d}\left((f,\varphi)\left(\widetilde{u}_a^*\right), \widetilde{u}_a^*, t\right) \widetilde{\leq} \frac{\widetilde{\alpha}}{1-\widetilde{\alpha}}\widetilde{d}\left(\widetilde{u}_a^*, \widetilde{u}_{a_n}^n, t\right) + \frac{\widetilde{\alpha}+\overline{1}}{1-\widetilde{\alpha}}\widetilde{d}\left(\widetilde{u}_{a_{n+1}}^{n+1}, \widetilde{u}_a^*, t\right) \to \overline{0}, \text{ as } n \to \infty.$$

Thus, we have $(f,\varphi)\left(\widetilde{u}_a^*\right) = \widetilde{u}_a^*$, which is the proof that the mapping (f,φ) has a fixed soft point. For the uniqueness, we suppose the converse, which means there could be two different fixed points $\widetilde{u}_a^*, \widetilde{v}_{a'}^*$ of the mapping (f,φ), then

$$\begin{aligned}
\widetilde{d}\left(\widetilde{u}_a^*, \widetilde{v}_{a'}^*, t\right) &= \widetilde{d}\left((f,\varphi)\left(\widetilde{u}_a^*\right), (f,\varphi)\left(\widetilde{v}_{a'}^*\right), t\right)\\
&\widetilde{\leq} \widetilde{\alpha}\left[\widetilde{d}\left((f,\varphi)\left(\widetilde{u}_a^*\right), \widetilde{v}_{a'}^*, t\right) + \widetilde{d}\left((f,\varphi)\left(\widetilde{v}_{a'}^*\right), \widetilde{u}_a^*, t\right)\right] = \overline{0},
\end{aligned}$$

for $\overline{0}\widetilde{\leq}\widetilde{\alpha}\widetilde{<}\widetilde{\left(\frac{1}{2}\right)}$. That is, $\widetilde{u}_a^*$ is a unique soft fixed point of the mapping (f,φ).

Example 3.10. *Let* $U = \mathbb{R}, \Omega = \mathbb{R}^{+}, t > 0,$ *and* $\widetilde{d} : SP\left(\widetilde{U}\right) \times SP\left(\widetilde{U}\right) \times (0,\infty) \to \mathbb{R}(\Omega)^{*}$ *be a soft parametric metric space on* $\widetilde{U}$ *defined by*

$$\widetilde{d}\left(\widetilde{u}_{a}, \widetilde{v}_{a'}, t\right) = t^{2}\left(|u-v| + |a-a'|\right).$$

Thus, $\left(\widetilde{U}, \widetilde{d}, \Omega\right)$ is a soft complete parametric metric space. Consider $(f,\varphi) : \left(\widetilde{U}, \widetilde{d}, \Omega\right) \to \left(\widetilde{U}, \widetilde{d}, \Omega\right)$ as a soft continuous mapping where $f : U \to U, \varphi : \Omega \to \Omega$ is defined by $f(u) = \frac{1}{4}\sin u$, $\varphi(a) = \frac{a}{3}$, then

$$\begin{aligned}
\widetilde{d}\left((f,\varphi)(\widetilde{u}_{a}), (f,\varphi)(\widetilde{v}_{a'}), t\right) &= \widetilde{d}\left(\left(\tfrac{1}{4}\sin u\right)_{\frac{a}{3}}, \left(\tfrac{1}{4}\sin v\right)_{\frac{a'}{3}}, t\right) \\
&= t^{2}\left(\left|\tfrac{1}{4}\sin u - \tfrac{1}{4}\sin v\right| + \left|\tfrac{a}{3} - \tfrac{a'}{3}\right|\right) \\
&\leq t^{2}\left(\tfrac{1}{4}|u-v| + \tfrac{1}{3}|a-a'|\right) \\
&\leq \tfrac{1}{3}t^{2}\left(|u-v| + |a-a'|\right) = \tfrac{1}{3}\widetilde{d}\left(\widetilde{u}_{a}, \widetilde{v}_{a'}, t\right).
\end{aligned}$$

Since $\widetilde{\alpha} = \widetilde{\left(\frac{1}{3}\right)}$, we conclude that $(f,\varphi) : \left(\widetilde{U}, \widetilde{d}, \Omega\right) \to \left(\widetilde{U}, \widetilde{d}, \Omega\right)$ is a soft contraction mapping. Using the Theorem 3.7, we have a unique soft point $\widetilde{u}_{a}^{*} \in SP\left(\widetilde{U}\right)$ such that $(f,\varphi)\left(\widetilde{u}_{a}^{*}\right) = \widetilde{u}_{a}^{*}$.

4. Conclusions

The role of fixed point theory is considered crucial in surveys conducted in both metric and topological spaces. As this theory has been applied by numerous authors in various metric spaces and applied sciences, this study has been conducted on fixed point theorems in parametric soft metric spaces. The article confidently introduces the soft topology generated by a parametric soft metric space. It has been shown that Banach's fixed point theorem can be extended to a soft complete parametric metric space. Moreover, an illustrative example has been provided using this fixed point theorem.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare no conflict of interest.

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