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Research article

Matrix measure-based exponential stability and synchronization of Markovian jumping QVNNs with time-varying delays and delayed impulses

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Abstract: This article explored the topics of global exponential stability and synchronization issues of a type of Markovian jumping quaternion-valued neural networks (QVNNs) that incorporate delayed impulses and time-varying delays. By utilizing the matrix measure strategy and delayed differential inequality techniques with an impulsive factor, several effective and practical criteria can be established to confirm that the impulsive QVNNs in question can achieve exponential synchronization with the given response system. Furthermore, the contained exponential convergence rate can be clearly presented. Notably, derived criteria are straightforward to verify and implement in real-world applications. In the end, to demonstrate the accuracy and effectiveness of achieved theoretical findings, one numerical example with an explanation was presented.

Keywords: quaternion-valued neural networks; Markovian jumping; synchronization; delayed impulses
Mathematics Subject Classification: 00A69

1. Introduction

Over the last couple of decades, famous neural networks have garnered special attention and have been extensively researched owing to their myriad of applications across various domains, as evidenced by numerous studies [1–5]. These applications encompass a wide spectrum, including but not limited to signal processing, image processing, and engineering optimization. Moreover, neural networks, including real-valued and complex-valued ones, have led to the presentation of numerous groundbreaking results in the field, as detailed in [6]. These results have significantly contributed to the advancement of neural network technologies and their practical applications. Furthermore, the quaternion, discovered by the British mathematician W. R. Hamilton, has found its way into various practical applications. It has been utilized in array processing, which involves the manipulation and analysis of data arrays in various contexts such as color image processing, which leverages the quaternion representation to capture and manipulate color information more accurately; and modeling of 3-D wind signals, which employs quaternion algebra to represent and analyze complex wind patterns in three-dimensional space. These applications, as referenced in [7–10], demonstrate the versatility and power of quaternion-based methods in addressing real-world problems.

It is worth noting that quaternion-valued neural networks (QVNNs) can be viewed as an extension of complex-valued neural networks (CVNNs), which exhibit far more intricate properties compared to CVNNs due to the non-commutative property of quaternion multiplication. These properties encompass quaternion-valued states, quaternion-valued connection weights, and quaternion-valued activation functions. In the realm of complex numbers, according to Liouville's theorem [11], every bounded entire function must be constant. From this point of view, choosing an appropriate activation function is very important. Fortunately, to date, numerous authors have delved into the analyticity problem [12, 13]. For instance, the existence issue and stability problem of stochastic delayed QVNNs have been addressed in [14]. Finite-time synchronization of fractional-order delayed QVNNs has been given in [15]. By resorting to the lexicographical order method, the exponential synchronization and state estimation problems of inertial quaternion-valued Cohen-Grossberg neural networks have been researched in [16]. Therefore, addressing the dynamics of neural networks with delays has risen as a vital issue.

It is well-established that numerous prior studies have primarily concentrated on deterministic neural models [17–19]. However, random phenomenons are ubiquitous in social life because of fluctuations in the environment. Therefore, they should be modeled using stochastic systems to accurately represent real-world situations [20]. With the help of the stochastic analysis technique, dissipativity results of delayed Markovian jumping CVNNs were investigated in [21]. Meanwhile, lots of dynamical works on stochastic Markovian switching models was reported [22–28]. For example, the asynchronous output feedback control has been addressed in [29] for semi-Markov systems with random delays. Additionally, time-varying gain controller synthesis was examined in [30] for nonhomogeneous semi-Markovian switching linear systems. The self-triggered control problem of Markovian jumping nonlinear systems with stochastic disturbances was researched in [31]. When talking about global exponential stability and synchronization in delayed Markovian jumping QVNNs, little research attention has been paid to these issues. This scarcity of research serves as the primary motivation behind our ongoing research efforts.

Apart from stochastic perturbations, impulsive effects and delayed impulses are also prevalent in neural systems [32–34]. Furthermore, impulse is also an effective method to implement in practical scenarios. For instance, combining the average impulsive interval with the Lyapunov method, p-th moment exponential stability was addressed in [35] of impulsive random delayed nonlinear systems with average-delay impulses. In [36], the almost sure exponential stability issue was demonstrated for the impulsive stochastic differential delay equation with bounded variable delays. By jointly introducing the matrix measure protocol, the global and exponential leader-following consensus issue was concentrated in [37] for nonlinear multi-agent systems with mixed delays. Despite the research on stability and synchronization of impulsive systems, there is limited literature focusing on

Markovian jumping QVNNs under delayed impulses. Therefore, with the help of the matrix measure method, our primary objective aims to propose the stability and synchronization results of delayed Markovian jumping QVNNs with delayed impulses.

Inspired by the preceding statements, this paper delves into exponential stability and synchronization of delayed Markovian jumping QVNNs with delayed impulses. The key highlights can be presented as follows. (1) With the consideration of a Markov chain, the investigated model becomes significantly more realistic in practical applications. Our findings reveal that impulses have a profound effect on the stability and synchronization of QVNNs. (2) By resorting to the advantage of the matrix measure approach, i.e., the measure value of the matrix can be positive or negative, the desired results can be derived. Meanwhile, the achieved criteria verify that it is a more practical and effective tool for dealing with the considered model. (3) This paper first takes into account not only time-varying delays but also delayed impulses on dynamical phenomenons of the proposed QVNNs.

Notations: The notations $\mathbb{R}^n$ and $\mathbb{Q}^n$ denote n-dimensional real vectors and n-dimensional quaternion value vectors, respectively. $\mathbb{R}^{m\times n}$ and $\mathbb{Q}^{m\times n}$ refer to $m\times n$-dimensional real matrices as well as $m\times n$-dimensional quaternion value matrices, respectively. The symbol "$\mathcal{I}$" denotes identity matrix, with its dimension determined by the context. $\tilde{A}^T$' signifies the transpose of matrix $\tilde{A}$, and "diag" is used to represent a diagonal matrix. The notation "$*$" is employed to indicate an element that is symmetrically implied in a given context, while $|x|$ refers to the magnitude of vector x. $(\hat{\Omega}, \hat{\mathcal{F}}, \{\hat{\mathcal{F}}\}_{t\geq 0}, \hat{\mathcal{P}})$ denotes the complete probability space. Moreover, $\lambda_{\max}(\tilde{A})$ ($\lambda_{\min}(\tilde{A})$) means the largest (smallest) eigenvalue of involved matrix $\tilde{A}$. $\mathbb{E}\{\cdot\}$ denotes mathematical expectation, and $\min\{a, b\}$ means the minimum value between a and b.

2. Problem formulation and preliminaries

Consider the delayed Markovian jumping QVNNs with delayed impulses below:

$$\begin{cases}\dot{\hbar}(t) = -\mathcal{R}_{\varsigma(t)}\hbar(t) + \mathcal{S}_{\varsigma(t)}\mathfrak{f}(\hbar(t)) + \mathcal{T}_{\varsigma(t)}\mathfrak{g}(\hbar(t-\mathfrak{I}(t))), & t\neq t_k,\\ \Delta\hbar(t_k) = \mathcal{L}_k\hbar(t_k^-) + \mathcal{M}_k\hbar(t_k^- - \vec{\omega}_k), & t = t_k\end{cases} \tag{2.1}$$

in which $k\in\mathbb{N}_+ = \{1, 2, \ldots\}$, $\hbar(t) = (\hbar_1(t), \hbar_2(t), \ldots, \hbar_n(t))^T \in \mathbb{Q}^n$ refers to the state value of QVNN (2.1). $\mathcal{R}_{\varsigma(t)} = \mathrm{diag}\{\tilde{c}_{1\varsigma(t)(t)}, \tilde{c}_{2\varsigma(t)(t)}, \ldots, \tilde{c}_{n\varsigma(t)(t)}\} \in \mathbb{R}^{n\times n}$ with $\tilde{c}_{\iota\varsigma(t)(t)} > 0$ denotes the self-feedback matrix, $\iota = 1, 2, \ldots, n$. $\mathcal{S}_{r(t)} = (a_{\iota\upsilon\varsigma(t)}) \in \mathbb{Q}^{n\times n}$ and $\mathcal{T}_{\varsigma(t)} = (b_{\iota\upsilon\varsigma(t)}) \in \mathbb{Q}^{n\times n}$ refer to connection matrix without delays and the delayed one, respectively. $\mathfrak{f}(\hbar(t))$ and $\mathfrak{g}(\hbar(t-\mathfrak{I}(t)))$ denote the quaternion-valued nonlinear activation functions without and with time delays, respectively. $\mathfrak{I}(t)$ stands for time-varying delay subject to $0 < \mathfrak{I}(t) < \vec{\mathfrak{I}}$, and $0 < \vec{\omega}_k \leq \vec{\mathfrak{I}}$ means the impulsive delay. t_k is the impulsive point satisfying $t_1 < t_2 < \ldots$ and $\lim_{k\to+\infty} t_k = +\infty$. $\Delta\hbar(t_k) \triangleq \hbar(t_k^+) - \hbar(t_k^-)$ means impulsive difference. Set $\hbar(t_k) = \hbar(t_k^+) \triangleq \lim_{t\to t_k^+}\hbar(t)$ and $\hbar(t_k^-) \triangleq \lim_{t\to t_k^-}\hbar(t)$. $\mathcal{L}_k$ and $\mathcal{M}_k$ refer to the strength of impulses. $\varsigma(t)$ denotes a Markov process, which is specified on space $(\hat{\Omega}, \hat{\mathcal{F}}, \{\hat{\mathcal{F}}\}_{t\geq 0}, \hat{\mathcal{P}})$ and takes its precise value in a set $\hat{\Upsilon} = \{1, 2, \ldots, \mathrm{N}\}$. Moreover, the corresponding generator is $\Theta = (\theta_{l\nu})_{\mathrm{N}\times\mathrm{N}}$, and the probability can be presented as

$$\hat{\mathcal{P}}\{\varsigma(t+\tilde{\Delta}t) = \nu \mid \varsigma(t) = l\} = \begin{cases}\vec{\theta}_{l\nu}\tilde{\Delta}t + o(\tilde{\Delta}t), & l\neq\nu,\\ 1 + \vec{\theta}_{l\nu}\tilde{\Delta}t + o(\tilde{\Delta}t), & l = \nu\end{cases} \tag{2.2}$$

in which $\tilde{\Delta}t > 0$ as well as $\lim_{\tilde{\Delta}t\to 0}(o(\tilde{\Delta}t)/\tilde{\Delta}t) = 0$. In addition, $\vec{\theta}_{l\nu} > 0$ $(l \neq \nu)$ refers to the jumping rate and $\vec{\theta}_{ll} \triangleq -\sum_{\nu=1,\nu\neq l}^{N} \vec{\theta}_{l\nu}$.

In order to achieve the desired outcomes, all involved nonlinear functions are presumed to meet the constraints below.

Assumption 2.1. *Set $\hbar = \hbar^R + \vec{i}\hbar^I + \vec{j}\hbar^J + \vec{k}\hbar^K$ with $\hbar^R, \hbar^I, \hbar^J, \hbar^K \in \mathbb{R}$, and the considered $\mathfrak{f}_\iota(\hbar)$ and $\mathfrak{g}_\iota(\hbar)$ can be expressed as follows:*

$$\mathfrak{f}_l(\hbar) = \mathfrak{f}_\iota^R(\hbar^R) + \vec{i}\mathfrak{f}_\iota^I(\hbar^I) + \vec{j}\mathfrak{f}_\iota^J(\hbar^J) + \vec{k}\mathfrak{f}_\iota^K(\hbar^K),$$
$$\mathfrak{g}_l(\hbar) = \mathfrak{g}_\iota^R(\hbar^R) + \vec{i}\mathfrak{g}_\iota^I(\hbar^I) + \vec{j}\mathfrak{g}_\iota^J(\hbar^J) + \vec{k}\mathfrak{g}_\iota^K(\hbar^K)$$

in which $\mathfrak{f}_\iota^r(\cdot), \mathfrak{g}_\iota^r(\cdot) : \mathbb{R} \to \mathbb{R}$ $(\iota = 1, 2, \cdots, n)$, $r =$ "R", "I", "J", "K" and satisfy

$$|\mathfrak{f}_\iota^r(\tilde{\hbar}) - \mathfrak{f}_\iota^r(\bar{\hbar})| \leq \mathfrak{m}_\iota^r|\tilde{\hbar} - \bar{\hbar}|, \quad |\mathfrak{g}_\iota^r(\tilde{\hbar}) - \mathfrak{g}_\iota^r(\bar{\hbar})| \leq \mathfrak{q}_\iota^r|\tilde{\hbar} - \bar{\hbar}|$$

in which $\mathfrak{m}_\iota^r$ and $\mathfrak{q}_\iota^r$ are known positive scalars, and $\tilde{\hbar}$, $\bar{\hbar}$ are scalars in $\mathbb{R}$. Moreover, $\mathfrak{f}_\iota(0) = \mathfrak{g}_\iota(0) = 0$.

Assumption 2.2. *For $\hbar = \hbar^R + \vec{i}\hbar^I + \vec{j}\hbar^J + \vec{k}\hbar^K$ with $\hbar^R, \hbar^I, \hbar^J, \hbar^K \in \mathbb{R}$, $\mathfrak{f}_\iota(\hbar)$ and $\mathfrak{g}_\iota(\hbar)$ can be expressed as follows:*

$$\mathfrak{f}_\iota(\hbar) = \mathfrak{f}_\iota^R(\hbar^R) + \vec{i}\mathfrak{f}_\iota^I(\hbar^I) + \vec{j}\mathfrak{f}_\iota^J(\hbar^J) + \vec{k}\mathfrak{f}_\iota^K(\hbar^K),$$
$$\mathfrak{g}_\iota(\hbar) = \mathfrak{g}_\iota^R(\hbar^R) + \vec{i}\mathfrak{g}_\iota^I(\hbar^I) + \vec{j}\mathfrak{g}_\iota^J(\hbar^J) + \vec{k}\mathfrak{g}_\iota^K(\hbar^K), \;\; \iota = 1, 2, \cdots, n,$$

where $r =$ "R", "I", "J", "K". Moreover, when $\tilde{\hbar} \neq \bar{\hbar}$, $\mathfrak{f}_\iota^r$ and $\mathfrak{g}_\iota^r$ satisfy the inequalities below:

$$0 \leq \frac{\mathfrak{f}_\iota^r(\tilde{\hbar}) - \mathfrak{f}_\iota^r(\bar{\hbar})}{\tilde{\hbar} - \bar{\hbar}} \leq \mathfrak{m}_\iota^r, \quad 0 \leq \frac{\mathfrak{g}_\iota^r(\tilde{\hbar}) - \}_\iota^r(\bar{\hbar})}{\tilde{\hbar} - \bar{\hbar}} \leq \mathfrak{q}_\iota^r.$$

Moreover, $\mathfrak{f}_\iota(0) = \mathfrak{g}_\iota(0) = 0$.

Next, set $\hbar(t) = \hbar^R(t) + \vec{i}\hbar^I(t) + \vec{j}\hbar^J(t) + \vec{k}\hbar^K(t)$, and then model (2.1) can also be converted to

$$\begin{cases} \dot{\aleph}(t) = -\tilde{\mathcal{R}}_{\varsigma(t)}\aleph(t) + \mathcal{S}_1^{\varsigma(t)}\mathfrak{f}_1(\aleph(t)) + \mathcal{S}_3^{\varsigma(t)}\mathfrak{f}_3(\aleph(t)) + \mathcal{S}_4^{\varsigma(t)}\mathfrak{f}_4(\aleph(t)) + \mathcal{T}_2^{\varsigma(t)}\mathfrak{g}_2(\aleph(t - \Im(t))) \\ \qquad + \mathcal{T}_1^{\varsigma(t)}\mathfrak{g}_1(\aleph(t - \Im(t))) + \mathcal{S}_2^{\varsigma(t)}\mathfrak{f}_2(\aleph(t)) + \mathcal{T}_3^{\varsigma(t)}\mathfrak{g}_3(\aleph(t - \Im(t))) + \mathcal{T}_4^{\varsigma(t)}\mathfrak{g}_4(\aleph(t - \Im(t))), \\ \Delta\aleph(t_k) = \tilde{\mathcal{L}}_k\hbar(t_k^-) + \tilde{\mathcal{M}}_k\aleph(t_k^- - \vec{\omega}_k), \end{cases} \tag{2.3}$$

in which $\aleph(t) = ((\hbar^R(t))^T, (\hbar^I(t))^T, (\hbar^J(t))^T, (\hbar^K(t))^T)^T$, and other symbols are showed as follows:

$$\tilde{\mathcal{R}}_{\varsigma(t)} = \mathrm{diag}\{\mathcal{R}_{\varsigma(t)}, \mathcal{R}_{\varsigma(t)}, \mathcal{R}_{\varsigma(t)}, \mathcal{R}_{\varsigma(t)}\},\; \tilde{\mathcal{M}}_k = \mathrm{diag}\{\mathcal{M}_k, \mathcal{M}_k, \mathcal{M}_k, \mathcal{M}_k\},\; \tilde{\mathcal{L}}_k = \mathrm{diag}\{\mathcal{L}_k, \mathcal{L}_k, \mathcal{L}_k, \mathcal{L}_k\},$$
$$\mathcal{S}_1^{\varsigma(t)} = \mathrm{diag}\{\mathcal{S}_{\varsigma(t)}^R, \mathcal{S}_{\varsigma(t)}^I, \mathcal{S}_{\varsigma(t)}^J, \mathcal{S}_{\varsigma(t)}^K\},\;\; \mathcal{S}_2^{\varsigma(t)} = \mathrm{diag}\{-\mathcal{S}_{\varsigma(t)}^I, \mathcal{S}_{\varsigma(t)}^R, \mathcal{S}_{\varsigma(t)}^K, -\mathcal{S}_{\varsigma(t)}^J\},$$
$$\mathcal{S}_3^{\varsigma(t)} = \mathrm{diag}\{-\mathcal{S}_{\varsigma(t)}^J, -\mathcal{S}_{\varsigma(t)}^K, \mathcal{S}_{\varsigma(t)}^R, \mathcal{S}_{\varsigma(t)}^I\},\;\; \mathcal{S}_4^{\varsigma(t)} = \mathrm{diag}\{-\mathcal{S}_{\varsigma(t)}^K, \mathcal{S}_{\varsigma(t)}^J, -\mathcal{S}_{\varsigma(t)}^I, \mathcal{S}_{\varsigma(t)}^R\},$$
$$\mathcal{T}_1^{\varsigma(t)} = \mathrm{diag}\{\mathcal{T}_{\varsigma(t)}^R, \mathcal{T}_{\varsigma(t)}^I, \mathcal{T}_{\varsigma(t)}^J, \mathcal{T}_{\varsigma(t)}^K\},\;\; \mathcal{T}_2^{r(t)} = \mathrm{diag}\{-\mathcal{T}_{\varsigma(t)}^I, \mathcal{T}_{\varsigma(t)}^R, \mathcal{T}_{\varsigma(t)}^K, -\mathcal{T}_{\varsigma(t)}^J\},$$
$$\mathcal{T}_3^{\varsigma(t)} = \mathrm{diag}\{-\mathcal{T}_{\varsigma(t)}^J, -\mathcal{T}_{\varsigma(t)}^K, \mathcal{T}_{\varsigma(t)}^R, \mathcal{T}_{\varsigma(t)}^I\},\;\; \mathcal{T}_4^{\varsigma(t)} = \mathrm{diag}\{-\mathcal{T}_{\varsigma(t)}^K, \mathcal{T}_{\varsigma(t)}^J, -\mathcal{T}_{\varsigma(t)}^I, \mathcal{T}_{\varsigma(t)}^R\},$$
$$\mathfrak{f}_1(\aleph(t)) = ((\mathfrak{f}^R(\hbar^R(t)))^T, (\mathfrak{f}^R(\hbar^R(t)))^T, (\mathfrak{f}^R(\hbar^R(t)))^T, (\mathfrak{f}^R(\hbar^R(t)))^T)^T,$$
$$\mathfrak{f}_2(\aleph(t)) = ((\mathfrak{f}^I(\hbar^I(t)))^T, (\mathfrak{f}^I(\hbar^I(t)))^T, (\mathfrak{f}^I(\hbar^I(t)))^T, (\mathfrak{f}^I(\hbar^I(t)))^T)^T,$$

$$\mathfrak{f}_3(\aleph(t)) = ((\mathfrak{f}^J(\hbar^J(t)))^T, (\mathfrak{f}^J(\hbar^J(t)))^T, (\mathfrak{f}^J(\hbar^J(t)))^T, (\mathfrak{f}^J(\hbar^J(t)))^T)^T,$$
$$\mathfrak{f}_4(\aleph(t)) = ((\mathfrak{f}^K(\hbar^K(t)))^T, (\mathfrak{f}^K(\hbar^K(t)))^T, (\mathfrak{f}^K(\hbar^K(t)))^T, (\mathfrak{f}^K(\hbar^K(t)))^T)^T,$$
$$\mathfrak{g}_1(\aleph(t-\mathfrak{I}(t))) = \begin{pmatrix} \mathfrak{g}^R(\hbar^R(t-\mathfrak{I}(t))) \\ \mathfrak{g}^R(\hbar^R(t-\mathfrak{I}(t))) \\ \mathfrak{g}^R(\hbar^R(t-\mathfrak{I}(t))) \\ \mathfrak{g}^R(\hbar^R(t-\mathfrak{I}(t))) \end{pmatrix}, \quad \mathfrak{g}_2(\aleph(t-\mathfrak{I}(t))) = \begin{pmatrix} \mathfrak{g}^I(\hbar^I(t-\mathfrak{I}(t))) \\ \mathfrak{g}^I(\hbar^I(t-\mathfrak{I}(t))) \\ \mathfrak{g}^I(\hbar^I(t-\mathfrak{I}(t))) \\ \mathfrak{g}^I(\hbar^I(t-\mathfrak{I}(t))) \end{pmatrix},$$
$$\mathfrak{g}_3(\aleph(t-\mathfrak{I}(t))) = \begin{pmatrix} \mathfrak{g}^J(\hbar^J(t-\mathfrak{I}(t))) \\ \mathfrak{g}^J(\hbar^J(t-\mathfrak{I}(t))) \\ \mathfrak{g}^J(\hbar^J(t-\mathfrak{I}(t))) \\ \mathfrak{g}^J(\hbar^J(t-\mathfrak{I}(t))) \end{pmatrix}, \quad \mathfrak{g}_4(\aleph(t-\mathfrak{I}(t))) = \begin{pmatrix} \mathfrak{g}^K(\hbar^K(t-\mathfrak{I}(t))) \\ \mathfrak{g}^K(\hbar^K(t-\mathfrak{I}(t))) \\ \mathfrak{g}^K(\hbar^K(t-\mathfrak{I}(t))) \\ \mathfrak{g}^K(\hbar^K(t-\mathfrak{I}(t))) \end{pmatrix}.$$

Next, some important lemmas and definition are presented.

Definition 2.1. *[38] For a matrix $\tilde{\mathfrak{A}} = (\tilde{\mathfrak{a}}_{\theta\vartheta})_{n\times n}$, the homologous matrix measure value is set as*

$$\mu_\delta(\tilde{\mathfrak{A}}) = \lim_{\varrho\to 0^+} \frac{\|\mathcal{I} + \varrho\tilde{\mathfrak{A}}\|_\delta - 1}{\varrho} \tag{2.4}$$

in which δ takes its value in $\{1, 2, \infty\}$. The symbol $\|\cdot\|_\delta$ refers to the matrix δ-norm defined as

$$\|\tilde{\mathfrak{A}}\|_1 = \max_{1\le\vartheta\le n}\Big\{\sum_{\theta=1}^{n} |\tilde{\mathfrak{a}}_{\theta\vartheta}|\Big\}, \quad \|\tilde{\mathfrak{A}}\|_2 = \Big(\lambda_{\max}(\tilde{\mathfrak{A}}^T\tilde{\mathfrak{A}})\Big)^{\frac{1}{2}}, \quad \|\tilde{\mathfrak{A}}\|_\infty = \max_{1\le\theta\le n}\Big\{\sum_{\vartheta=1}^{n} |\tilde{\mathfrak{a}}_{\theta\vartheta}|\Big\},$$

and the corresponding matrix measurements can be acquired as outlined below:

$$\mu_1(\tilde{\mathfrak{A}}) = \max_{1\le\vartheta\le n}\Big\{\tilde{\mathfrak{a}}_{\vartheta\vartheta} + \sum_{\theta=1,\theta\ne\vartheta}^{n} |\tilde{\mathfrak{a}}_{\theta\vartheta}|\Big\}, \quad \mu_2(\tilde{\mathfrak{A}}) = \lambda_{\max}\Big(\frac{\tilde{\mathfrak{A}} + \tilde{\mathfrak{A}}^T}{2}\Big),$$
$$\mu_\infty(\tilde{\mathfrak{A}}) = \max_{1\le\theta\le n}\Big\{\tilde{\mathfrak{a}}_{\theta\theta} + \sum_{\vartheta=1,\theta\ne\vartheta}^{n} |\tilde{\mathfrak{a}}_{\theta\vartheta}|\Big\}.$$

Lemma 2.1. *[38] The involved matrix measure $\mu_\delta(\cdot)$ and it owns several properties: for a positive constant ζ, matrices $\tilde{\mathfrak{A}}$, $\tilde{\mathfrak{B}} \in \mathbb{R}^{n\times n}$ (1): $-\|\tilde{\mathfrak{A}}\|_\delta \le \mu_\delta(\tilde{\mathfrak{A}}) \le \|\tilde{\mathfrak{A}}\|_\delta$; (2): $\mu_\delta(\zeta\tilde{\mathfrak{A}}) = \zeta\mu_\delta(\tilde{\mathfrak{A}})$; (3): $\mu_\delta(\tilde{\mathfrak{A}} + \tilde{\mathfrak{B}}) \le \mu_\delta(\tilde{\mathfrak{A}}) + \mu_\delta(\tilde{\mathfrak{B}})$. From these properties, the matrix measure can be negative.*

Lemma 2.2. *[39] Take the differential inequalities as*

$$\begin{cases} D^+\vec{\mathcal{H}}(t) \le -\vec{\chi}_1\vec{\mathcal{H}}(t) + \vec{\chi}_2 \sup\limits_{t-\vec{\mathfrak{I}}\le s\le t} \vec{\mathcal{H}}(s), & t \ne t_k, \\ \vec{\mathcal{H}}(t_k) \le \vec{a}_k\vec{\mathcal{H}}(t_k^-) + \vec{b}_k \sup\limits_{t_k-\vec{\mathfrak{I}}\le s< t_k} \vec{\mathcal{H}}(s), & k \in \mathbb{N}_+ \end{cases} \tag{2.5}$$

in which $\vec{\mathcal{H}}(t)$ is nonnegative and continuous in $[t_0 - \vec{\mathfrak{I}}, t_0]$. Then, when $\vec{\chi}_1 > \vec{\chi}_2 \ge 0$, there owns a positive scalar $\vec{\varpi} > 1$ such that $t_k - t_{k-1} > \vec{\varpi}\vec{\mathfrak{I}}$, and then

$$\vec{\mathcal{H}}(t) \le \tilde{\eta}_1\tilde{\eta}_2\cdots\tilde{\eta}_{k+1}e^{k\vec{\zeta}\tau} \sup_{t_0-\vec{\mathfrak{I}}\le s\le t_0} \vec{\mathcal{H}}(s)e^{-\vec{\zeta}(t-t_0)} \tag{2.6}$$

in which $t \in [t_k, t_{k+1}]$ *and* $\tilde{\eta}_\gamma = \max\{1, \vec{a}_i + \vec{b}_i e^{\vec{\zeta}\vec{\mathfrak{I}}}\}$ *with* $\gamma = 1, 2, \ldots, k+1$*. Particularly, when* $\tilde{\xi} \triangleq \sup_{k\in\mathbb{N}_+}\{1, \vec{a}_k + \vec{b}_k e^{\vec{\zeta}\vec{\mathfrak{I}}}\}$*, one yields*

$$\vec{\mathcal{H}}(t) \le \tilde{\xi} \sup_{t_0-\vec{\mathfrak{I}}\le s\le t_0} \vec{\mathcal{H}}(s) e^{-(\vec{\zeta}-\frac{\ln(\tilde{\xi}e^{\vec{\zeta}\vec{\mathfrak{I}}})}{\vec{\varpi}\vec{\mathfrak{I}}})(t-t_0)}, \tag{2.7}$$

where $\vec{\zeta}$ *refers to the unique positive root of equation* $\vec{\zeta} = \vec{\chi}_1 - \vec{\chi}_2 e^{\vec{\zeta}\vec{\mathfrak{I}}}$.

Remark 2.1. *Compared to the existing relevant results [40, 41], they utilize the matrix norm method to handle the introduced matrix, resulting in a positive value. On the other hand, this paper employs the matrix measure method to deal with the introduced matrix, which can yield a negative value because matrix measures offer greater flexibility when applied to matrices. Therefore, the derived results in this paper, which are expressed in the format of a matrix measure, are more accurate than those achieved using the matrix norm method.*

3. Main results

In this section, by utilizing the matrix measure strategy, exponential stability criteria can be derived.

Theorem 3.1. *Under the premise of Assumption 2.1, system* (2.3) *can attain the provided global exponential stability that two constraints hold below for* δ*:*

¶$_1$ $\omega_1 > \omega_2 \ge 0$*, in which*

$$\omega_1 \triangleq \min_{1\le l\le \mathrm{N}}\left\{-\mu_\delta(-\tilde{\mathcal{R}}_l) - 4^{\frac{1}{\delta}}\left(\mathfrak{m}^R\|\mathcal{S}_1^l\|_\delta + \mathfrak{m}^I\|\mathcal{S}_2^l\|_\delta + \mathfrak{m}^J\|\mathcal{S}_3^l\|_\delta + \mathfrak{m}^K\|\mathcal{S}_4^l\|_\delta\right)\right\},$$
$$\omega_2 \triangleq \max_{1\le l\le \mathrm{N}}\left\{4^{\frac{1}{\delta}}\left(\mathfrak{q}^R\|\mathcal{T}_1^l\|_\delta + \mathfrak{q}^I\|\mathcal{T}_2^l\|_\delta + \mathfrak{q}^J\|\mathcal{T}_3^l\|_\delta + \mathfrak{q}^K\|\mathcal{T}_4^l\|_\delta\right)\right\},$$

with δ *belonging to* $\{1, 2, \infty\}$*, and*

$$\mathfrak{m}^R = \max_{1\le\iota\le n}\{\mathfrak{m}_\iota^R\}, \quad \mathfrak{m}^I = \max_{1\le\iota\le n}\{\mathfrak{m}_\iota^I\}, \quad \mathfrak{m}^J = \max_{1\le\iota\le n}\{\mathfrak{m}_\iota^J\}, \quad \mathfrak{m}^K = \max_{1\le\iota\le n}\{\mathfrak{m}_\iota^K\}.$$

¶$_2$ *There exists a constant* $\vec{\beta} > \frac{\ln(\sigma e^{\vec{\varrho}\vec{\mathfrak{I}}})}{\vec{\varrho}\vec{\mathfrak{I}}}$*, and one gets*

$$\inf_{k\in N_+}\{t_k - t_{k-1}\} > \vec{\beta}\vec{\mathfrak{I}}$$

in which $\sigma \triangleq \sup_{k\in N_+}\{1, a_k + b_k e^{\vec{\varrho}\vec{\mathfrak{I}}}\}$*,* $a_k \triangleq \|\tilde{\mathcal{L}}_k + \mathcal{I}\|_\delta$*,* $b_k \triangleq \|\tilde{\mathcal{M}}_k\|_\delta$*, and* $\vec{\varrho}$ *denotes the unique positive root of* $\vec{\varrho} = \omega_1 - \omega_2 e^{\vec{\varrho}\vec{\mathfrak{I}}}$.

Proof. In the case of $t \ne t_k$, take the differential of $\|\aleph(t)\|_\delta$, and we get

$$D^+\|\aleph(t)\|_\delta = \lim_{\varrho\to 0^+}\frac{\|\aleph(t+\varrho)\|_\delta - \|\aleph(t)\|_\delta}{\varrho} = \lim_{\varrho\to 0^+}\frac{\|\aleph(t) + \varrho\dot{\aleph}(t) + o(\varrho)\|_\delta - \|\aleph(t)\|_\delta}{\varrho} \tag{3.1}$$

in which

$$\|\aleph(t) + \varrho\dot{\aleph}(t) + o(\varrho)\|_\delta$$

$$
\begin{aligned}
&= \Big\| \aleph(t) + \varrho \Big[-\tilde{\mathcal{R}}_{\varsigma(t)}\aleph(t) + \mathcal{S}_2^{\varsigma(t)}\mathfrak{f}_2(\aleph(t)) + \mathcal{S}_1^{\varsigma(t)}\mathfrak{f}_1(\aleph(t)) + \mathcal{S}_3^{\varsigma(t)}\mathfrak{f}_3(\aleph(t)) \\
&\quad + \mathcal{S}_4^{\varsigma(t)}\mathfrak{f}_4(\aleph(t)) + \mathcal{T}_1^{\varsigma(t)}\mathfrak{g}_1(\aleph(t-\Im(t))) + \mathcal{T}_2^{\varsigma(t)}\mathfrak{g}_2(\aleph(t-\Im(t))) \\
&\quad + \mathcal{T}_3^{\varsigma(t)}\mathfrak{g}_3(\aleph(t-\Im(t))) + \mathcal{T}_4^{\varsigma(t)}\mathfrak{g}_4(\aleph(t-\Im(t)))\Big] + o(\varrho) \Big\|_\delta \\
&\le \left\| \mathcal{I} - \varrho\tilde{C}_{\varsigma(t)} \right\|_\delta \|\aleph(t)\|_\delta + \varrho \left\| \mathcal{S}_1^{\varsigma(t)} \right\|_\delta \|\mathfrak{f}_1(\aleph(t))\|_\delta + \varrho \left\| \mathcal{S}_2^{\varsigma(t)} \right\|_\delta \|\mathfrak{f}_2(\aleph(t))\|_\delta \\
&\quad + \varrho \left\| \mathcal{S}_3^{\varsigma(t)} \right\|_\delta \|\mathfrak{f}_3(\aleph(t))\|_\delta + \varrho \left\| \mathcal{S}_4^{\varsigma(t)} \right\|_\delta \|\mathfrak{f}_4(\aleph(t))\|_\delta + \varrho \left\| \mathcal{T}_1^{\varsigma(t)} \right\|_\delta \|\mathfrak{g}_1(\aleph(t-\Im(t)))\|_\delta \\
&\quad + \varrho \left\| \mathcal{T}_2^{\varsigma(t)} \right\|_\delta \|\mathfrak{g}_2(\aleph(t-\Im(t)))\|_\delta + \varrho \left\| \mathcal{T}_3^{\varsigma(t)} \right\|_\delta \|\mathfrak{g}_3(\aleph(t-\Im(t)))\|_\delta \\
&\quad + \varrho \left\| \mathcal{T}_4^{\varsigma(t)} \right\|_\delta \|\mathfrak{g}_4(\aleph(t-\Im(t)))\|_\delta + \|o(\varrho)\|_\delta.
\end{aligned}
$$

It follows from the expression of $\mathfrak{f}_1(\aleph(t))$ and Assumption 2.1 that

$$
\begin{aligned}
&\|\mathfrak{f}_1(u(t))\|_\delta \\
&\quad = \left\| \begin{matrix} \mathfrak{f}^R(\hbar^R(t)) - \mathfrak{f}^R(0) \\ \mathfrak{f}^R(\hbar^R(t)) - \mathfrak{f}^R(0) \\ \mathfrak{f}^R(\hbar^R(t)) - \mathfrak{f}^R(0) \\ \mathfrak{f}^R(\hbar^R(t)) - \mathfrak{f}^R(0) \end{matrix} \right\|_p = \left[4 \sum_{\iota=1}^n \left| \mathfrak{f}_l^R(\hbar_\iota^R(t)) - \mathfrak{f}_\iota^R(0) \right|^\delta \right]^{\frac{1}{\delta}} \\
&\quad \le \left[4 \sum_{\iota=1}^n \mathfrak{m}_l^R |\hbar_\iota^R(t)|^p \right]^{\frac{1}{\delta}} \le 4^{\frac{1}{\delta}} \mathfrak{m}^R \|\hbar^R(t)\|_\delta \le 4^{\frac{1}{\delta}} \mathfrak{m}^R \|\aleph(t)\|_\delta.
\end{aligned} \tag{3.2}
$$

Utilizing the similar method in (3.2), we have

$$
\|\mathfrak{f}_2(\aleph(t))\|_\delta \le 4^{\frac{1}{\delta}} \mathfrak{m}^I \|\aleph(t)\|_\delta, \quad \|\mathfrak{f}_3(\aleph(t))\|_\delta \le 4^{\frac{1}{\delta}} \mathfrak{m}^J \|\aleph(t)\|_\delta, \quad \|\mathfrak{f}_4(\aleph(t))\|_\delta \le 4^{\frac{1}{\delta}} \mathfrak{m}^K \|\aleph(t)\|_\delta, \tag{3.3}
$$

and the inequalities are satisfied below:

$$
\begin{aligned}
&\|\mathfrak{g}_1(\aleph(t-\Im(t)))\|_\delta \le 4^{\frac{1}{\delta}} \mathfrak{q}^R \|\aleph(t-\Im(t))\|_\delta, \quad \|\mathfrak{g}_3(\aleph(t-\Im(t)))\|_\delta \le 4^{\frac{1}{\delta}} \mathfrak{q}^J \|\aleph(t-\Im(t))\|_\delta, \\
&\|\mathfrak{g}_2(\aleph(t-\Im(t)))\|_\delta \le 4^{\frac{1}{\delta}} \mathfrak{q}^I \|\aleph(t-\Im(t))\|_\delta, \quad \|\mathfrak{g}_4(\aleph(t-\Im(t)))\|_\delta \le 4^{\frac{1}{\delta}} \mathfrak{q}^K \|\aleph(t-\Im(t))\|_\delta.
\end{aligned} \tag{3.4}
$$

Next, by substituting the inequalities from (3.2) to (3.4) into (3.1), we can derive that

$$
\begin{aligned}
&D^+\|\aleph(t)\|_\delta \\
&\le \lim_{\varrho\to 0^+} \frac{\|\mathcal{I} - \varrho\tilde{\mathcal{R}}_{\varsigma(t)}\|_\delta - 1}{\varrho} + 4^{\frac{1}{\delta}} \mathfrak{m}^I \|\mathcal{S}_2^{\varsigma(t)}\|_\delta \|\aleph(t)\|_\delta + 4^{\frac{1}{\delta}} \mathfrak{m}^R \|\mathcal{S}_3^{\varsigma(t)}\|_\delta \|\aleph(t)\|_\delta \\
&\quad + 4^{\frac{1}{\delta}} \mathfrak{m}^K \|\mathcal{S}_4^{\varsigma(t)}\|_\delta \|\aleph(t)\|_\delta + 4^{\frac{1}{\delta}} m^R \|A_1^{\varsigma(t)}\|_\delta \|\aleph(t)\|_\delta + 4^{\frac{1}{\delta}} \mathfrak{q}^R \|\mathcal{T}_1^{r(t)}\|_\delta \|\aleph(t-\Im(t))\|_\delta \\
&\quad + 4^{\frac{1}{\delta}} \mathfrak{q}^J \|\mathcal{T}_3^{\varsigma(t)}\|_\delta \|\aleph(t-\Im(t))\|_\delta + 4^{\frac{1}{\delta}} \mathfrak{q}^I \|\mathcal{T}_2^{\varsigma(t)}\|_\delta \|\aleph(t-\Im(t))\|_\delta \\
&\quad + 4^{\frac{1}{\delta}} \mathfrak{q}^K \|\mathcal{T}_4^{\varsigma(t)}\|_\delta \|\aleph(t-\Im(t))\|_\delta \\
&\le \Big(\mu_p(-\tilde{\mathcal{R}}_{\varsigma(t)}) + 4^{\frac{1}{\delta}} \mathfrak{m}^R \|\mathcal{S}_1^{\varsigma(t)}\|_\delta + 4^{\frac{1}{\delta}} \mathfrak{m}^I \|\mathcal{S}_2^{\varsigma(t)}\|_\delta + 4^{\frac{1}{\delta}} \mathfrak{m}^J \|\mathcal{S}_3^{\varsigma(t)}\|_\delta + 4^{\frac{1}{\delta}} \mathfrak{m}^K \|\mathcal{S}_4^{\varsigma(t)}\|_\delta \Big) \|\aleph(t)\|_\delta \\
&\quad + \Big(4^{\frac{1}{\delta}} \mathfrak{q}^R \|\mathcal{T}_1^{\varsigma(t)}\|_\delta + 4^{\frac{1}{\delta}} \mathfrak{q}^I \|\mathcal{T}_2^{\varsigma(t)}\|_\delta + 4^{\frac{1}{\delta}} \mathfrak{q}^J \|\mathcal{T}_3^{\varsigma(t)}\|_\delta + 4^{\frac{1}{\delta}} \mathfrak{q}^K \|\mathcal{T}_4^{\varsigma(t)}\|_\delta \Big) \sup_{t-\vec{\Im}\le s\le t} \|\aleph(s)\|_\delta,
\end{aligned} \tag{3.5}
$$

which further verifies that the following inequality is valid:

$$
D^+\|\aleph(t)\|_\delta \le -\omega_1 \|\aleph(t)\|_\delta + \omega_2 \sup_{t-\vec{\Im}\le s\le t} \|\aleph(s)\|_\delta. \tag{3.6}
$$

In addition, when $t = t_k$, one gets

$$\begin{aligned}\|\aleph(t_k)\|_\delta &= \left\|(\mathcal{I} + \tilde{\mathcal{L}}_k)\aleph(t_k^-) + \tilde{\mathcal{M}}_k\aleph(t_k^- - \vec{\omega}_k)\right\|_\delta \\ &\le \|\tilde{\mathcal{M}}_k\|_\delta\|\aleph(t_k^- - \vec{\omega}_k)\|_\delta + \|\mathcal{I} + \tilde{\mathcal{L}}_k\|_\delta\|\aleph(t_k^-)\|_\delta \\ &\le \|\tilde{\mathcal{M}}_k\|_\delta \sup_{t_k-\vec{\mathfrak{I}}\le s\le t_k} \|\aleph(s)\|_\delta + \|\mathcal{I} + \tilde{\mathcal{L}}_k\|_\delta\|\aleph(t_k^-)\|_\delta \\ &= a_k\|\aleph(t_k^-)\|_\delta + b_k \sup_{t_k-\vec{\mathfrak{I}}\le s\le t_k} \|\aleph(s)\|_\delta. \end{aligned} \tag{3.7}$$

Then, combine (3.6), (3.7), the conditions ¶$_1$ and ¶$_2$ in Theorem 3.1, as well as Lemma 2.2, and one can infer

$$\|\aleph(t)\|_\delta \le \sigma \sup_{t_0-\vec{\mathfrak{I}}\le s\le t_0} \|\aleph(s)\|_p e^{-(\vec{\varrho}-\frac{\ln(\sigma e^{\vec{\varrho}\vec{\mathfrak{I}}})}{\vec{\beta}\vec{\mathfrak{I}}})}(t-t_0), \quad t \ge t_0$$

in which $\vec{\varrho}$ denotes the unique positive root of $\vec{\varrho} = \omega_1 - \omega_2 e^{\vec{\varrho}\vec{\mathfrak{I}}}$, which further insures global exponential stability of system (2.3) under Assumption 2.1. The proof is now complete.

Theorem 3.2. *Under the premise of Assumption 2.2, considered system* (2.3) *can attain the provided global exponential stability of the given constraints below are valid for* $\delta = 1$ *or* ∞*:*

¶$_1$ $\omega_1 > \omega_2 \ge 0$*, in which*

$$\begin{aligned}\omega_1 &\triangleq \min_{1\le l\le \mathrm{N}}\Big\{ -\mu(-\tilde{\mathcal{R}}_l) - \mu(\tilde{\mathcal{S}}_l^*\mathcal{D}) - (\tilde{\mathfrak{m}} + 2\mathfrak{m}^R)\|\mathcal{S}_1^l\|_\delta - (\tilde{\mathfrak{m}} + 2\mathfrak{m}^I)\|\mathcal{S}_2^l\|_\delta \\ &\quad - (\tilde{\mathfrak{m}} + 2\mathfrak{m}^J)\|\mathcal{S}_3^l\|_\delta - (\tilde{\mathfrak{m}} + 2\mathfrak{m}^K)\|\mathcal{S}_4^l\|_\delta\Big\}, \\ \omega_2 &\triangleq \max_{1\le l\le \mathrm{N}}\left\{4^{\frac{1}{\delta}}\left(\mathfrak{q}^R\|\mathcal{T}_1^l\|_\delta + \mathfrak{q}^I\|\mathcal{T}_2^l\|_\delta + \mathfrak{q}^J\|\mathcal{T}_3^l\|_\delta + \mathfrak{q}^K\|\mathcal{T}_4^l\|_\delta\right)\right\}\end{aligned}$$

where $\delta \in \{1, 2, \infty\}$, $\mathcal{D} = \mathrm{diag}\left\{\mathfrak{m}_1^R, \cdots, \mathfrak{m}_n^R, \mathfrak{m}_1^I, \cdots, \mathfrak{m}_n^I, \mathfrak{m}_1^J, \cdots, \mathfrak{m}_n^J, \mathfrak{m}_1^K, \cdots, \mathfrak{m}_n^K\right\}$, $\tilde{\mathfrak{m}} = \mathfrak{m}^R + \mathfrak{m}^I + \mathfrak{m}^J + \mathfrak{m}^K$, $\mathcal{S}_1^l = (\bar{a}_{\iota\upsilon}^l)_{4n\times 4n}$, $\mathcal{S}_2^l = (\tilde{a}_{\iota\upsilon}^l)_{4n\times 4n}$, $\mathcal{S}_3^l = (\hat{a}_{\iota\upsilon}^l)_{4n\times 4n}$, $\mathcal{S}_4^l = (\check{a}_{\iota\upsilon}^l)_{4n\times 4n}$ *and* $\tilde{\mathcal{S}}_l^{\,*} = (a_{\iota\upsilon}^l)_{4n\times 4n}$ *with* $a_{\iota\upsilon}^l = \max\{0, \bar{a}_{\iota\upsilon}^l + \tilde{a}_{\iota\upsilon}^l + \hat{a}_{\iota\upsilon}^l + \check{a}_{\iota\upsilon}^l\}$ *while* $\iota = \upsilon$*. Otherwise* $a_{\iota\upsilon}^l = \bar{a}_{\iota\upsilon}^l + \tilde{a}_{\iota\upsilon}^l + \hat{a}_{\iota\upsilon}^l + \check{a}_{\iota\upsilon}^l$*, and all other symbols retain their definitions as stated in Theorem 3.1.*

¶$_2$ *For a constant* $\vec{\beta} > \frac{\ln(\sigma e^{\vec{\varrho}\vec{\mathfrak{I}}})}{\vec{\varrho}\vec{\mathfrak{I}}}$*, the inequality*

$$\inf_{k\in N_+}\{t_k - t_{k-1}\} > \vec{\beta}\vec{\mathfrak{I}}$$

holds, where $\sigma \triangleq \sup_{k\in N_+}\{1, a_k + b_k e^{\vec{\varrho}\vec{\mathfrak{I}}}\}$, $a_k \triangleq \|\tilde{\mathcal{L}}_k + \mathcal{I}\|_\delta$, $b_k \triangleq \|\tilde{\mathcal{M}}_k\|_\delta$*, and* $\vec{\varrho}$ *stands for the unique positive root of* $\vec{\varrho} = \omega_1 - \omega_2 e^{\vec{\varrho}\vec{\mathfrak{I}}}$.

Proof. In the case of $t \ne t_k$, analogous to the derivative process of Theorem 3.1, one can derive (3.1). Set

$$\Xi(\aleph(t)) = \mathrm{diag}\left\{\frac{\mathfrak{f}_1^R(\hbar_1^R(t)) - \mathfrak{f}_1^R(0)}{\hbar_1^R(t)}, \cdots, \frac{\mathfrak{f}_n^R(\hbar_n^R(t)) - \mathfrak{f}_n^R(0)}{\hbar_n^R(t)}, \frac{\mathfrak{f}_1^I(\hbar_1^I(t)) - \mathfrak{f}_1^I(0)}{\hbar_1^I(t)}, \cdots, \right.$$

$$\frac{\mathfrak{f}_n^I(\hbar_n^I(t))-\mathfrak{f}_n^I(0)}{\hbar_n^I(t)},\frac{\mathfrak{f}_1^J(\hbar_1^J(t))-\mathfrak{f}_1^J(0)}{\hbar_1^J(t)},\cdots,\frac{\mathfrak{f}_n^J(\hbar_n^J(t))-\mathfrak{f}_n^J(0)}{\hbar_n^J(t)},$$
$$\left.\frac{\mathfrak{f}_1^K(\hbar_1^K(t))-\mathfrak{f}_1^K(0)}{\hbar_1^K(t)},\cdots,\frac{\mathfrak{f}_n^K(\hbar_n^K(t))-\mathfrak{f}_n^K(0)}{\hbar_n^K(t)}\right\}.$$

Based on the explanation of $\mathfrak{f}_1(\aleph(t))$, one yields

$$\begin{aligned}\mathfrak{f}_1(\aleph(t))&=\begin{pmatrix}\mathfrak{f}^R(\hbar^R(t))-\mathfrak{f}^R(0)\\ \mathfrak{f}^R(\hbar^R(t))-\mathfrak{f}^R(0)\\ \mathfrak{f}^R(\hbar^R(t))-\mathfrak{f}^R(0)\\ \mathfrak{f}^R(\hbar^R(t))-\mathfrak{f}^R(0)\end{pmatrix}\\ &=\mathfrak{X}_1(t)-\mathfrak{X}_4(t)+\mathfrak{X}_5(t)-\mathfrak{X}_2(t)-\mathfrak{X}_3(t)+\mathfrak{X}_6(t)+\mathfrak{X}_7(t)\end{aligned}\tag{3.8}$$

in which

$$\mathfrak{X}_1(t)=\begin{pmatrix}\mathfrak{f}^R(\hbar^R(t))-\mathfrak{f}^R(0)\\ \mathfrak{f}^I(\hbar^I(t))-\mathfrak{f}^I(0)\\ \mathfrak{f}^J(\hbar^J(t))-\mathfrak{f}^J(0)\\ \mathfrak{f}^K(\hbar^K(t))-\mathfrak{f}^K(0)\end{pmatrix}=\Xi(\aleph(t))\aleph(t),$$

$$\mathfrak{X}_2(t)=\begin{pmatrix}O_{n\times1}\\ \mathfrak{f}^I(\hbar^I(t))-\mathfrak{f}^I(0)\\ O_{n\times1}\\ O_{n\times1}\end{pmatrix},\quad \mathfrak{X}_3(t)=\begin{pmatrix}O_{n\times1}\\ O_{n\times1}\\ \mathfrak{f}^J(\hbar^J(t))-\mathfrak{f}^J(0)\\ O_{n\times1}\end{pmatrix},$$

$$\mathfrak{X}_4(t)=\begin{pmatrix}O_{n\times1}\\ O_{n\times1}\\ O_{n\times1}\\ \mathfrak{f}^K(\hbar^K(t))-\mathfrak{f}^K(0)\end{pmatrix},\quad \mathfrak{X}_5(t)=\begin{pmatrix}O_{n\times1}\\ \mathfrak{f}^R(\hbar^R(t))-\mathfrak{f}^R(0)\\ O_{n\times1}\\ O_{n\times1}\end{pmatrix},$$

$$\mathfrak{X}_6(t)=\begin{pmatrix}O_{n\times1}\\ O_{n\times1}\\ \mathfrak{f}^R(\hbar^R(t))-\mathfrak{f}^R(0)\\ O_{n\times1}\end{pmatrix},\quad \mathfrak{X}_7(t)=\begin{pmatrix}O_{n\times1}\\ O_{n\times1}\\ O_{n\times1}\\ \mathfrak{f}^R(\hbar^R(t))-\mathfrak{f}^R(0)\end{pmatrix}.$$

Similarly, one can yield

$$\begin{aligned}\mathfrak{f}_2(\aleph(t))&=\begin{pmatrix}\mathfrak{f}^I(\hbar^I(t))-\mathfrak{f}^I(0)\\ \mathfrak{f}^I(\hbar^I(t))-\mathfrak{f}^I(0)\\ \mathfrak{f}^I(\hbar^I(t))-\mathfrak{f}^I(0)\\ \mathfrak{f}^I(\hbar^I(t))-\mathfrak{f}^I(0)\end{pmatrix}\\ &=\Xi(\aleph(t))\aleph(t)-\mathfrak{X}_4(t)-\mathfrak{X}_3(t)+\mathfrak{X}_9(t)-\mathfrak{X}_8(t)+\mathfrak{X}_{10}(t)+\mathfrak{X}_{11}(t)\end{aligned}\tag{3.9}$$

in which

$$\mathfrak{X}_8(t)=\begin{pmatrix}\mathfrak{f}^R(\hbar^R(t))-\mathfrak{f}^R(0)\\ O_{n\times1}\\ O_{n\times1}\\ O_{n\times1}\end{pmatrix},\quad \mathfrak{X}_9(t)=\begin{pmatrix}\mathfrak{f}^I(\hbar^I(t))-\mathfrak{f}^I(0)\\ O_{n\times1}\\ O_{n\times1}\\ O_{n\times1}\end{pmatrix},$$

$$\mathfrak{X}_{10}(t) = \begin{pmatrix} O_{n\times 1} \\ O_{n\times 1} \\ \mathfrak{f}^I(\hbar^I(t)) - \mathfrak{f}^I(0) \\ O_{n\times 1} \end{pmatrix}, \quad \mathfrak{X}_{11}(t) = \begin{pmatrix} O_{n\times 1} \\ O_{n\times 1} \\ O_{n\times 1} \\ \mathfrak{f}^I(\hbar^I(t)) - \mathfrak{f}^I(0) \end{pmatrix}.$$

$$\begin{aligned} \mathfrak{f}_3(\aleph(t)) &= \begin{pmatrix} \mathfrak{f}^J(\hbar^J(t)) - \mathfrak{f}^J(0) \\ \mathfrak{f}^J(\hbar^J(t)) - \mathfrak{f}^J(0) \\ \mathfrak{f}^J(\hbar^J(t)) - \mathfrak{f}^J(0) \\ \mathfrak{f}^J(\hbar^J(t)) - \mathfrak{f}^J(0) \end{pmatrix} \\ &= \Xi(\aleph(t))\aleph(t) + \mathfrak{X}_{12}(t) - \mathfrak{X}_2(t) - \mathfrak{X}_4(t) - \mathfrak{X}_8(t) + \mathfrak{X}_{13}(t) + \mathfrak{X}_{14}(t) \end{aligned} \tag{3.10}$$

in which

$$\mathfrak{X}_{12}(t) = \begin{pmatrix} \mathfrak{f}^J(\hbar^J(t)) - \mathfrak{f}^J(0) \\ O_{n\times 1} \\ O_{n\times 1} \\ O_{n\times 1} \end{pmatrix}, \quad \mathfrak{X}_{13}(t) = \begin{pmatrix} O_{n\times 1} \\ \mathfrak{f}^J(\hbar^J(t)) - \mathfrak{f}^J(0) \\ O_{n\times 1} \\ O_{n\times 1} \end{pmatrix},$$

$$\mathfrak{X}_{14}(t) = \begin{pmatrix} O_{n\times 1} \\ O_{n\times 1} \\ O_{n\times 1} \\ \mathfrak{f}^J(\hbar^J(t)) - \mathfrak{f}^J(0) \end{pmatrix}.$$

$$\begin{aligned} \mathfrak{f}_4(\aleph(t)) &= \begin{pmatrix} \mathfrak{f}^K(\hbar^K(t)) - \mathfrak{f}^K(0) \\ \mathfrak{f}^K(\hbar^K(t)) - \mathfrak{f}^K(0) \\ \mathfrak{f}^K(\hbar^K(t)) - \mathfrak{f}^K(0) \\ \mathfrak{f}^K(\hbar^K(t)) - \mathfrak{f}^K(0) \end{pmatrix} \\ &= \Xi(\aleph(t))\aleph(t) - \mathfrak{X}_3(t) - \mathfrak{X}_2(t) - \mathfrak{X}_8(t) + \mathfrak{X}_{15}(t) + \mathfrak{X}_{16}(t) + \mathfrak{X}_{17}(t) \end{aligned} \tag{3.11}$$

in which

$$\mathfrak{X}_{15}(t) = \begin{pmatrix} \mathfrak{f}^K(\hbar^K(t)) - \mathfrak{f}^K(0) \\ O_{n\times 1} \\ O_{n\times 1} \\ O_{n\times 1} \end{pmatrix}, \quad \mathfrak{X}_{16}(t) = \begin{pmatrix} O_{n\times 1} \\ \mathfrak{f}^K(\hbar^K(t)) - \mathfrak{f}^K(0) \\ O_{n\times 1} \\ O_{n\times 1} \end{pmatrix},$$

$$\mathfrak{X}_{17}(t) = \begin{pmatrix} O_{n\times 1} \\ O_{n\times 1} \\ \mathfrak{f}^K(\hbar^K(t)) - \mathfrak{f}^K(0) \\ O_{n\times 1} \end{pmatrix}.$$

In addition, substituting (3.8)–(3.11) into $\|\aleph(t) + \varrho\dot{\aleph}(t) + o(\varrho)\|_\delta$, one can derive that

$$\begin{aligned} &\|\aleph(t) + \varrho\dot{\aleph}(t) + o(\varrho)\|_\delta \\ &= \Big\|\aleph(t) + \varrho(-\tilde{\mathcal{R}}_{\varsigma(t)})\aleph(t) - \mathfrak{X}_2(t) - \mathfrak{X}_3(t) - \mathfrak{X}_4(t) + \mathfrak{X}_5(t) + \varrho\mathcal{S}_1^{\varsigma(t)}(\Xi(\aleph(t))\aleph(t) \end{aligned}$$

$$
\begin{aligned}
&+ \mathfrak{X}_6(t) + \mathfrak{X}_7(t)) + \varrho\mathcal{S}_2^{\varsigma(t)}(\Xi(\aleph(t))\aleph(t) - \mathfrak{X}_3(t) - \mathfrak{X}_4(t) - \mathfrak{X}_8(t) + \mathfrak{X}_9(t)\\
&+ \mathfrak{X}_{10}(t) + \mathfrak{X}_{11}(t)) + \varrho\mathcal{S}_3^{r(t)}(\Xi(\aleph(t))\aleph(t) - \mathfrak{X}_2(t) - \mathfrak{X}_4(t) - \mathfrak{X}_8(t) + \mathfrak{X}_{12}(t)\\
&+ \mathfrak{X}_{13}(t) + \mathfrak{X}_{14}(t)) + \varrho\mathcal{S}_4^{\varsigma(t)}(\Xi(\aleph(t))\aleph(t) - \mathfrak{X}_2(t) - \mathfrak{X}_3(t) - \mathfrak{X}_8(t) + \mathfrak{X}_{15}(t)\\
&+ \mathfrak{X}_{16}(t) + \mathfrak{X}_{17}(t)) + \varrho\mathcal{T}_2^{\varsigma(t)}\mathfrak{g}_2(\aleph(t - \mathfrak{I}(t))) + \varrho\mathcal{T}_1^{\varsigma(t)}\mathfrak{g}_1(\aleph(t - \mathfrak{I}(t)))\\
&+ \varrho\mathcal{T}_3^{\varsigma(t)}\mathfrak{g}_3(\aleph(t - \mathfrak{I}(t))) + \varrho\mathcal{T}_4^{\varsigma(t)}\mathfrak{g}_4(\aleph(t - \mathfrak{I}(t))) + o(\varrho)\Big\|_\delta\\
\leq& \left\|\mathcal{I} + \varrho\left[-\tilde{\mathcal{R}}_{\varsigma(t)} + \left(\mathcal{S}_1^{\varsigma(t)} + \mathcal{S}_2^{\varsigma(t)} + \mathcal{S}_3^{\varsigma(t)} + \mathcal{S}_4^{\varsigma(t)}\right)\Xi(\aleph(t))\right]\right\|_\delta \|\aleph(t)\|_\delta\\
&+ \varrho\|\mathcal{S}_2^{\varsigma(t)}\|_\delta(\|\mathfrak{X}_3(t)\|_\delta + \|\mathfrak{X}_4(t)\|_\delta + \|\mathfrak{X}_8(t)\|_\delta + \|\mathfrak{X}_9(t)\|_\delta + \|\mathfrak{X}_{10}(t)\|_\delta + \|\mathfrak{X}_{11}(t)\|_\delta)\\
&+ \varrho\|\mathcal{S}_1^{r(t)}\|_\delta(\|\mathfrak{X}_2(t)\|_\delta + \|\mathfrak{X}_3(t)\|_\delta + \|\mathfrak{X}_4(t)\|_\delta + \|\mathfrak{X}_5(t)\|_\delta + \|\mathfrak{X}_6(t)\|_\delta + \|\mathfrak{X}_7(t)\|_\delta)\\
&+ \varrho\|\mathcal{S}_4^{r(t)}\|_\delta(\|\mathfrak{X}_2(t)\|_\delta + \|\mathfrak{X}_3(t)\|_\delta + \|\mathfrak{X}_8(t)\|_\delta + \|\mathfrak{X}_{15}(t)\|_\delta + \|\mathfrak{X}_{16}(t)\|_\delta + \|\mathfrak{X}_{17}(t)\|_\delta)\\
&+ \varrho\|\mathcal{S}_1^{r(t)}\|_\delta(\|\mathfrak{X}_2(t)\|_\delta + \|\mathfrak{X}_3(t)\|_\delta + \|\mathfrak{X}_4(t)\|_\delta + \|\mathfrak{X}_5(t)\|_\delta + \|\mathfrak{X}_6(t)\|_\delta + \|\mathfrak{X}_7(t)\|_\delta)\\
&+ \varrho\|\mathcal{T}_3^{r(t)}\|_\delta\|\mathfrak{g}_3(\aleph(t - \mathfrak{I}(t)))\|_\delta + \varrho\|\mathcal{T}_4^{\varsigma(t)}\|_\delta\|\mathfrak{g}_4(\aleph(t - \mathfrak{I}(t)))\|_\delta\\
&+ \varrho\|\mathcal{T}_2^{\varsigma(t)}\|_\delta\|\mathfrak{g}_2(\aleph(t - \mathfrak{I}(t)))\|_\delta + \varrho\|\mathcal{T}_1^{\varsigma(t)}\|_\delta\|\mathfrak{g}_1(\aleph(t - \mathfrak{I}(t)))\|_\delta + \|o(\varrho)\|_\delta\\
\leq& \left\|\mathcal{I} + \varrho\left[-\tilde{\mathcal{R}}_{\varsigma(t)} + \left(\mathcal{S}_1^{\varsigma(t)} + \mathcal{S}_2^{\varsigma(t)} + \mathcal{S}_3^{\varsigma(t)} + \mathcal{S}_4^{\varsigma(t)}\right)\Xi(\aleph(t))\right]\right\|_\delta \|\aleph(t)\|_\delta\\
&+ \varrho(\tilde{\mathfrak{m}} + 2\mathfrak{m}^R)\|\mathcal{S}_1^{\varsigma(t)}\|_\delta\|\aleph(t)\|_\delta + \varrho(\tilde{\mathfrak{m}} + 2\mathfrak{m}^I)\|\mathcal{S}_2^{\varsigma(t)}\|_\delta\|\aleph(t)\|_\delta\\
&+ \varrho(\tilde{\mathfrak{m}} + 2\mathfrak{m}^J)\|\mathcal{S}_3^{\varsigma(t)}\|_\delta\|\aleph(t)\|_\delta + \varrho(\tilde{\mathfrak{m}} + 2\mathfrak{m}^K)\|\mathcal{S}_4^{\varsigma(t)}\|_\delta\|\aleph(t)\|_\delta\\
&+ 4^{\frac{1}{\delta}}\mathfrak{q}^R\varrho\|\mathcal{T}_1^{\varsigma(t)}\|_\delta\|\aleph(t - \mathfrak{I}(t))\|_\delta + 4^{\frac{1}{\delta}}\mathfrak{q}^I\varrho\|\mathcal{T}_2^{\varsigma(t)}\|_\delta\|\aleph(t - \mathfrak{I}(t))\|_\delta\\
&+ 4^{\frac{1}{\delta}}\mathfrak{q}^J\varrho\|\mathcal{T}_3^{\varsigma(t)}\|_\delta\|\aleph(t - \mathfrak{I}(t))\|_\delta + 4^{\frac{1}{\delta}}\mathfrak{q}^K\varrho\|\mathcal{T}_4^{\varsigma(t)}\|_\delta\|\aleph(t - \mathfrak{I}(t))\|_\delta + \|o(\varrho)\|_\delta.
\end{aligned}
\tag{3.12}
$$

Hence, the result can eventually be reached that

$$
\begin{aligned}
&D^+\|\aleph(t)\|_\delta\\
&= \lim_{\varrho\to 0^+}\frac{\|\aleph(t+\varrho)\|_\delta - \|\aleph(t)\|_\delta}{\varrho}\\
&\leq \lim_{\varrho\to 0^+}\frac{\|\mathcal{I} + \varrho[-\tilde{\mathcal{R}}_{\varsigma(t)} + (\mathcal{S}_1^{\varsigma(t)} + \mathcal{S}_2^{\varsigma(t)} + \mathcal{S}_3^{\varsigma(t)} + \mathcal{S}_4^{\varsigma(t)})]\|_\delta - 1}{\varrho}\|\aleph(t)\|_\delta\\
&\quad + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^R)\|\mathcal{S}_1^{\varsigma(t)}\|_\delta\|\aleph(t)\|_\delta + \varrho(\tilde{\mathfrak{m}} + 2\mathfrak{m}^I)\|\mathcal{S}_2^{\varsigma(t)}\|_\delta\|\aleph(t)\|_\delta + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^I)\|\mathcal{S}_2^{\varsigma(t)}\|_\delta\|\aleph(t)\|_\delta\\
&\quad + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^J)\|\mathcal{S}_3^{\varsigma(t)}\|_\delta\|\aleph(t)\|_\delta + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^K)\|\mathcal{S}_4^{\varsigma(t)}\|_\delta\|\aleph(t)\|_\delta\\
&\quad + 4^{\frac{1}{\delta}}\left(\mathfrak{q}^R\|\mathcal{T}_1^{\varsigma(t)}\|_\delta + \mathfrak{q}^I\|\mathcal{T}_2^{\varsigma(t)}\|_\delta + \mathfrak{q}^J\|\mathcal{T}_3^{\varsigma(t)}\|_\delta + \mathfrak{q}^K\|\mathcal{T}_4^{\varsigma(t)}\|_\delta\right)\|\aleph(t - \mathfrak{I}(t))\|_\delta\\
&= \Big[\mu\left(-\tilde{\mathcal{R}}_{\varsigma(t)} + (\mathcal{S}_3^{\varsigma(t)} + \mathcal{S}_1^{\varsigma(t)} + \mathcal{S}_2^{\varsigma(t)} + \mathcal{S}_4^{\varsigma(t)})\Xi(\aleph(t))\right) + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^R)\|\mathcal{S}_1^{\varsigma(t)}\|_\delta\\
&\quad + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^I)\|\mathcal{S}_2^{\varsigma(t)}\|_\delta + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^J)\|\mathcal{S}_3^{\varsigma(t)}\|_\delta + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^K)\|\mathcal{S}_4^{\varsigma(t)}\|_\delta\Big]\|\aleph(t)\|_\delta\\
&\quad + 4^{\frac{1}{\delta}}\left(\mathfrak{q}^R\|\mathcal{T}_1^{\varsigma(t)}\|_\delta + \mathfrak{q}^I\|\mathcal{T}_2^{\varsigma(t)}\|_\delta + \mathfrak{q}^J\|\mathcal{T}_3^{\varsigma(t)}\|_\delta + \mathfrak{q}^K\|\mathcal{T}_4^{\varsigma(t)}\|_\delta\right)\|\aleph(t - \mathfrak{I}(t))\|_\delta\\
&\leq \Big[\mu\left(-\tilde{\mathcal{R}}_{\varsigma(t)} + (\mathcal{S}_3^{\varsigma(t)} + \mathcal{S}_1^{\varsigma(t)} + \mathcal{S}_2^{\varsigma(t)} + \mathcal{S}_4^{\varsigma(t)})\Xi(\aleph(t))\right) + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^R)\|\mathcal{S}_1^{\varsigma(t)}\|_\delta\\
&\quad + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^I)\|\mathcal{S}_2^{\varsigma(t)}\|_\delta + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^J)\|\mathcal{S}_3^{\varsigma(t)}\|_\delta + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^K)\|\mathcal{S}_4^{\varsigma(t)}\|_\delta\Big]\|\aleph(t)\|_\delta\\
&\quad + 4^{\frac{1}{\delta}}\left(\mathfrak{q}^R\|\mathcal{T}_1^{\varsigma(t)}\|_\delta + \mathfrak{q}^I\|\mathcal{T}_2^{\varsigma(t)}\|_\delta + \mathfrak{q}^J\|\mathcal{T}_3^{\varsigma(t)}\|_\delta + \mathfrak{q}^K\|\mathcal{T}_4^{\varsigma(t)}\|_\delta\right)\sup_{t-\vec{\mathfrak{I}}\leq s\leq t}\|\aleph(s)\|_\delta
\end{aligned}
$$

$$
\begin{aligned}
&\leq \Big[\mu(-\tilde{\mathcal{R}}_{\varsigma(t)}) + \mu(\tilde{\mathcal{S}}^{*}_{\varsigma(t)}\mathcal{D}) + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^{R})\|\mathcal{S}_1^{\varsigma(t)}\|_\delta + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^{I})\|\mathcal{S}_2^{\varsigma(t)}\|_\delta + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^{J})\|\mathcal{S}_3^{\varsigma(t)}\|_\delta \\
&\quad + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^{K})\|\mathcal{S}_4^{\varsigma(t)}\|_\delta\Big] \|\aleph(t)\|_\delta + 4^{\frac{1}{\delta}}\left(\mathfrak{q}^R\|\mathcal{T}_1^{\varsigma(t)}\|_\delta + q^I\|\mathcal{T}_2^{\varsigma(t)}\|_\delta + \mathfrak{q}^J\|\mathcal{T}_3^{\varsigma(t)}\|_\delta\right. \\
&\quad \left. + \mathfrak{q}^K\|\mathcal{T}_4^{\varsigma(t)}\|_\delta\right) \sup_{t-\vec{\mathfrak{I}}\leq s\leq t}\|\aleph(s)\|_\delta \\
&\leq -\omega_1\|\aleph(t)\|_\delta + \omega_2 \sup_{t-\vec{\mathfrak{I}}\leq s\leq t}\|\aleph(s)\|_\delta. \qquad (3.13)
\end{aligned}
$$

Based on this, in the case of $t = t_k$, the conclusion is immediately available that

$$
\|\aleph(t)\|_\delta \leq \sigma \sup_{t_0-\vec{\mathfrak{I}}\leq s\leq t_0}\|\aleph(s)\|_p e^{-(\vec{\varrho}-\frac{\ln(\sigma e^{\vec{\varrho}\vec{\mathfrak{I}}})}{\vec{\beta}\vec{\mathfrak{I}}})}(t-t_0), \quad t \geq t_0 \qquad (3.14)
$$

in which $\vec{\varrho}$ denotes the unique positive root of $\vec{\varrho} = \omega_1 - \omega_2 e^{\vec{\varrho}\vec{\mathfrak{I}}}$, and this solution further ensures the desired exponential stability under Assumption 2.2. The detailed proof is thus concluded.

In the subsequent section, we will be devoted to exploring the synchronization issue of considered system. If the drive system is (2.1), then the corresponding response system can be shown as

$$
\begin{cases}
\dot{\mathfrak{R}}(t) = -\mathcal{R}_{\varsigma(t)}z(t) + \mathcal{S}_{\varsigma(t)}\mathfrak{f}(\mathfrak{R}(t)) + \mathcal{T}_{\varsigma(t)}\mathfrak{g}(\mathfrak{R}(t-\mathfrak{I}(t))) + \mathfrak{U}_{\varsigma(t)}(t), \quad t \neq t_k, \\
\Delta\mathfrak{R}(t_k) = \mathcal{L}_k\mathfrak{R}(t_k^-) + \mathcal{M}_k\mathfrak{R}(t_k^- - \vec{\omega}_k), \quad t = t_k,
\end{cases} \qquad (3.15)
$$

where $\mathfrak{R}(t) = \mathfrak{R}^R(t) + \vec{i}\mathfrak{R}^I(t) + \vec{j}\mathfrak{R}^J(t) + \vec{k}\mathfrak{R}^K(t)$ refers to the corresponding status value with $\mathfrak{R}^R(t), \mathfrak{R}^I(t), \mathfrak{R}^J(t), \mathfrak{R}^K(t) \in \mathbb{R}^n$, and $\mathfrak{U}_{r(t)}(t)$ stands for the designed control input as

$$
\mathfrak{U}_{\varsigma(t)}(t) = \mathfrak{P}_{\varsigma(t)}\wp(t), \qquad (3.16)
$$

where $\mathfrak{P}_{\varsigma(t)} \in \mathbb{R}^{n\times n}$ represents designed the gain matrix and $\wp(t)$ is given later.

Take the initial value of (3.15) as

$$
\mathfrak{R}(s) = \vartheta(s), \quad s \in [t_0 - \vec{\mathfrak{I}}, t_0],
$$

where $\vartheta(s) = \vartheta^R(s) + \vec{i}\vartheta^I(s) + \vec{j}\vartheta^J(s) + \vec{k}\vartheta^K(s)$ with $\vartheta^R(s), \vartheta^I(s), \vartheta^J(s), \vartheta^K(s) \in \mathbf{C}([t_0 - \vec{\mathfrak{I}}, t_0], \mathbb{R}^n)$.

Set the error system $\wp(t) \triangleq \mathfrak{R}(t) - \hbar(t)$ with $\wp(t) = ((\wp^R(t))^T, (\wp^I(t))^T, (\wp^J(t))^T, (\wp^K(t))^T)^T$ with $\wp^R(t) = \mathfrak{R}^R(t) - \hbar^R(t), \wp^J(t) = \mathfrak{R}^J(t) - \hbar^J(t), \wp^I(t) = \mathfrak{R}^I(t) - \hbar^I(t), \wp^K(t) = \mathfrak{R}^K(t) - \hbar^K(t)$, and it follows from systems (2.1) and (3.15), as well as controller (3.16), that one obtains

$$
\begin{cases}
\begin{aligned}
\dot{\wp}^R(t) = &-\mathcal{R}_{\varsigma(t)}\wp^R(t) - \mathcal{S}^I_{\varsigma(t)}\tilde{\mathfrak{f}}^I(\wp^I(t)) + \mathcal{S}^R_{\varsigma(t)}\tilde{\mathfrak{f}}^R(\wp^R(t)) - \mathcal{S}^J_{\varsigma(t)}\tilde{\mathfrak{f}}^J(\wp^J(t)) \\
&+ \mathcal{T}^R_{\varsigma(t)}\tilde{\mathfrak{g}}^R(\wp^R(t-\mathfrak{J}(t))) - \mathcal{S}^K_{\varsigma(t)}\tilde{\mathfrak{f}}^K(\wp^K(t)) - \mathcal{T}^I_{\varsigma(t)}\tilde{\mathfrak{g}}^I(\wp^I(t-\mathfrak{J}(t))) \\
&- \mathcal{T}^K_{\varsigma(t)}\tilde{\mathfrak{g}}^K(\wp^K(t-\mathfrak{J}(t))) - \mathcal{T}^J_{\varsigma(t)}\tilde{\mathfrak{g}}^J(\wp^J(t-\mathfrak{J}(t))) + \mathfrak{P}_{\varsigma(t)}\wp^R(t), \qquad t \neq t_k,
\end{aligned} \\
\begin{aligned}
\dot{\wp}^I(t) = &-\mathcal{R}_{\varsigma(t)}\wp^I(t) + \mathcal{S}^I_{\varsigma(t)}\tilde{\mathfrak{f}}^R(\wp^R(t)) + \mathcal{S}^R_{\varsigma(t)}\tilde{\mathfrak{f}}^I(\wp^I(t)) + \mathcal{S}^J_{\varsigma(t)}\tilde{\mathfrak{f}}^K(\wp^K(t)) \\
&+ \mathcal{T}^R_{\varsigma(t)}\tilde{\mathfrak{g}}^I(\wp^I(t-\mathfrak{J}(t))) - \mathcal{S}^K_{\varsigma(t)}\tilde{\mathfrak{f}}^J(\wp^J(t)) + \mathcal{T}^I_{\varsigma(t)}\tilde{\mathfrak{g}}^R(\wp^R(t-\mathfrak{J}(t))) \\
&- \mathcal{T}^K_{\varsigma(t)}\tilde{\mathfrak{g}}^J(\wp^J(t-\mathfrak{J}(t))) + \mathcal{T}^J_{\varsigma(t)}\tilde{\mathfrak{g}}^K(\wp^K(t-\mathfrak{J}(t))) + \mathfrak{P}_{\varsigma(t)}\wp^I(t), \qquad t \neq t_k,
\end{aligned} \\
\begin{aligned}
\dot{\wp}^J(t) = &-\mathcal{R}_{\varsigma(t)}\wp^J(t) - \mathcal{S}^I_{\varsigma(t)}\tilde{\mathfrak{f}}^K(\wp^K(t)) + \mathcal{S}^R_{\varsigma(t)}\tilde{\mathfrak{f}}^J(\wp^J(t)) + \mathcal{S}^J_{\varsigma(t)}\tilde{\mathfrak{f}}^R(\wp^R(t)) \\
&+ \mathcal{T}^R_{\varsigma(t)}\tilde{\mathfrak{g}}^J(\wp^J(t-\mathfrak{J}(t))) + \mathcal{S}^K_{\varsigma(t)}\tilde{\mathfrak{f}}^I(\wp^I(t)) - \mathcal{T}^I_{\varsigma(t)}\tilde{\mathfrak{g}}^K(\wp^K(t-\mathfrak{J}(t))) \\
&+ \mathcal{T}^K_{\varsigma(t)}\tilde{\mathfrak{g}}^I(\wp^I(t-\mathfrak{J}(t))) + \mathcal{T}^J_{\varsigma(t)}\tilde{\mathfrak{g}}^R(\wp^R(t-\mathfrak{J}(t))) + \mathfrak{P}_{\varsigma(t)}\wp^J(t), \qquad t \neq t_k,
\end{aligned} \\
\begin{aligned}
\dot{\wp}^K(t) = &-\mathcal{R}_{\varsigma(t)}\wp^K(t) + \mathcal{S}^I_{\varsigma(t)}\tilde{\mathfrak{f}}^J(\wp^J(t)) + \mathcal{S}^R_{\varsigma(t)}\tilde{\mathfrak{f}}^K(\wp^K(t)) - \mathcal{S}^J_{\varsigma(t)}\tilde{\mathfrak{f}}^I(\wp^I(t)) \\
&+ \mathcal{T}^R_{\varsigma(t)}\tilde{\mathfrak{g}}^K(\wp^K(t-\mathfrak{J}(t))) + \mathcal{S}^K_{\varsigma(t)}\tilde{\mathfrak{f}}^R(\wp^R(t)) + \mathcal{T}^I_{\varsigma(t)}\tilde{\mathfrak{g}}^J(\wp^J(t-\mathfrak{J}(t))) \\
&+ \mathcal{T}^K_{\varsigma(t)}\tilde{\mathfrak{g}}^R(\wp^R(t-\mathfrak{J}(t))) - \mathcal{T}^J_{\varsigma(t)}\tilde{\mathfrak{g}}^I(\wp^I(t-\mathfrak{J}(t))) + \mathfrak{P}_{\varsigma(t)}\wp^K(t), \qquad t \neq t_k,
\end{aligned} \\
\Delta\wp^R(t_k) = \mathcal{L}_k\wp^R(t_k^-) + \mathcal{M}_k\wp^R(t_k^- - \vec{\omega}_k), \qquad t = t_k, \\
\Delta\wp^I(t_k) = \mathcal{L}_k\wp^I(t_k^-) + \mathcal{M}_k\wp^I(t_k^- - \vec{\omega}_k), \qquad t = t_k, \\
\Delta\wp^J(t_k) = \mathcal{L}_k\wp^J(t_k^-) + \mathcal{M}_k\wp^J(t_k^- - \vec{\omega}_k), \qquad t = t_k, \\
\Delta\wp^K(t_k) = \mathcal{L}_k\wp^K(t_k^-) + \mathcal{M}_k\wp^K(t_k^- - \vec{\omega}_k), \qquad t = t_k,
\end{cases}
\tag{3.17}
$$

in which $\tilde{\mathfrak{f}}^R(\wp^R(t)) = \mathfrak{f}^R(\mathfrak{R}^R(t)) - \mathfrak{f}^R(\hbar^R(t))$, $\tilde{\mathfrak{g}}^R(\wp^R(t-\mathfrak{J}(t))) = \mathfrak{g}^R(\mathfrak{R}^R(t-\mathfrak{J}(t))) - \mathfrak{g}^R(\hbar^R(t-\mathfrak{J}(t)))$, $\tilde{\mathfrak{f}}^I(\wp^I(t)) = \mathfrak{f}^I(\mathfrak{R}^I(t)) - \mathfrak{f}^I(\hbar^I(t))$, $\tilde{\mathfrak{g}}^I(\wp^I(t-\mathfrak{J}(t))) = \mathfrak{g}^I(\mathfrak{R}^I(t-\mathfrak{J}(t))) - \mathfrak{g}^I(\hbar^I(t-\mathfrak{J}(t)))$, $\tilde{\mathfrak{f}}^J(\wp^J(t)) = \mathfrak{f}^J(\mathfrak{R}^J(t)) - \mathfrak{f}^J(\hbar^J(t))$, $\tilde{\mathfrak{f}}^K(\wp^K(t)) = \mathfrak{f}^K(\mathfrak{R}^K(t)) - \mathfrak{f}^K(\hbar^K(t))$, $\tilde{\mathfrak{g}}^J(\wp^J(t-\mathfrak{J}(t))) = \mathfrak{g}^J(\mathfrak{R}^J(t-\mathfrak{J}(t))) - \mathfrak{g}^J(\hbar^J(t-\mathfrak{J}(t)))$, $\tilde{\mathfrak{g}}^K(\wp^K(t-\mathfrak{J}(t))) = \mathfrak{g}^K(\mathfrak{R}^K(t-\mathfrak{J}(t))) - \mathfrak{g}^K(\hbar^K(t-\mathfrak{J}(t)))$. To facilitate the subsequent discussions, one useful definition can be introduced.

Definition 3.1. *Systems* (2.1) *and* (3.15) *are said to realize exponential synchronization if the impulsive switched error system* (3.17) *reaches the desired exponential stability, i.e., there exist positive scalars* ζ *and* ξ*, and one can derive*

$$
\begin{aligned}
&\|\wp^R(t)\|_\delta + \|\wp^I(t)\|_\delta + \|\wp^J(t)\|_\delta + \|\wp^K(t)\|_\delta \\
&\leq \zeta \sup_{t_0-\vec{\mathfrak{J}}\leq s\leq t_0} (\|\wp^R(s)\|_\delta + \|\wp^I(s)\|_\delta + \|\wp^J(s)\|_\delta + \|\wp^K(s)\|_\delta) e^{-\xi(t-t_0)}, \quad t \geq t_0.
\end{aligned}
$$

Theorem 3.3. *Under the premise of Assumption 2.1, the considered system* (2.1) *can attain globally exponential synchronization with system* (3.15) *if the given constraints below are valid for* δ*:*

$\P_1$ $\omega_1 > \omega_2 \geq 0$*, in which*

$$
\begin{aligned}
\omega_1 &\triangleq \min_{1\leq l\leq \mathrm{N}} \left\{-\mu_\delta(-\mathcal{R}_l + \mathfrak{P}_l) - \widetilde{\mathfrak{m}}\left(\|\mathcal{S}^R_l\|_\delta + \|\mathcal{S}^I_l\|_\delta + \|\mathcal{S}^J_l\|_\delta + \|\mathcal{S}^K_l\|_\delta\right)\right\}, \\
\omega_2 &\triangleq \max_{1\leq l\leq \mathrm{N}} \left\{\widetilde{q}\left(\|\mathcal{T}^R_l\|_\delta + \|\mathcal{T}^I_l\|_\delta + \|\mathcal{T}^J_l\|_\delta + \|B^K_l\|_\delta\right)\right\}
\end{aligned}
$$

in which $\delta \in \{1, 2, \infty\}$*, and* $\widetilde{\mathfrak{m}} = \max\{\mathfrak{m}^R, \mathfrak{m}^I, \mathfrak{m}^J, \mathfrak{m}^K\}$, $\mathfrak{m}^R = \max_{1\leq\iota\leq n}\{\mathfrak{m}^R_\iota\}$, $\mathfrak{m}^I = \max_{1\leq\iota\leq n}\{\mathfrak{m}^I_\iota\}$, $\mathfrak{m}^J = \max_{1\leq\iota\leq n}\{\mathfrak{m}^J_\iota\}$, $\mathfrak{m}^K = \max_{1\leq\iota\leq n}\{\mathfrak{m}^K_\iota\}$, $\widetilde{\mathfrak{q}} = \max\{\mathfrak{q}^R, \mathfrak{q}^I, \mathfrak{q}^J, \mathfrak{q}^K\}$, $\mathfrak{q}^R = \max_{1\leq\iota\leq n}\{\mathfrak{q}^R_\iota\}$, $\mathfrak{q}^I = \max_{1\leq\iota\leq n}\{\mathfrak{q}^I_\iota\}$, $\mathfrak{q}^J = \max_{1\leq\iota\leq n}\{\mathfrak{q}^J_\iota\}$, *and* $\mathfrak{q}^K = \max_{1\leq\iota\leq n}\{\mathfrak{q}^K_\iota\}$.

¶$_2$ *There exists a constant* $\vec{\beta} > \frac{\ln(\sigma e^{\vec{\varrho}\vec{\Im}})}{\vec{\varrho}\vec{\Im}}$, *where*

$$\inf_{k\in N_+}\{t_k - t_{k-1}\} > \vec{\beta}\vec{\Im},$$

in which $\sigma \triangleq \sup_{k\in N_+}\{1, a_k + b_k e^{\vec{\varrho}\vec{\Im}}\}$, $a_k \triangleq \|\mathcal{L}_k + \mathcal{I}\|_\delta$, $b_k \triangleq \|\mathcal{M}_k\|_\delta$, *and* $\vec{\varrho}$ *stands for the unique positive root of* $\vec{\varrho} = \omega_1 - \omega_2 e^{\vec{\varrho}\vec{\Im}}$.

Proof. For the case of $t \neq t_k$, choose the Lyapunov functional factor as

$$V(\wp(t)) = \|\wp^R(t)\|_\delta + \|\wp^I(t)\|_\delta + \|\wp^J(t)\|_\delta + \|\wp^K(t)\|_\delta, \tag{3.18}$$

and then, take the differential of $V(\wp(t))$, and one gets

$$\begin{aligned}
&\lim_{\varrho\to0^+}\Big(\|\wp^R(t+\varrho)\|_\delta - \|\wp^R(t)\|_\delta + \|\wp^I(t+\varrho)\|_\delta - \|\wp^I(t)\|_\delta \\
&\quad + \|\wp^J(t+\varrho)\|_\delta - \|\wp^J(t)\|_\delta + \|\wp^K(t+\varrho)\|_\delta - \|\wp^K(t)\|_\delta\Big)/\varrho \\
=& \lim_{\varrho\to0^+}\Big(\|\wp^I(t) + \varrho\dot{\wp}^I(t) + o(\varrho)\|_\delta - \|\wp^I(t)\|_\delta + \|\wp^J(t) + \varrho\dot{\wp}^J(t) + o(\varrho)\|_\delta - \|\wp^J(t)\|_\delta \\
&+ \|\wp^R(t) + \varrho\dot{\wp}^R(t) + o(\varrho)\|_\delta - \|\wp^R(t)\|_\delta + \|\wp^K(t) + \varrho\dot{\wp}^K(t) + o(\varrho)\|_\delta - \|\wp^K(t)\|_\delta\Big)/\varrho.
\end{aligned} \tag{3.19}$$

Combining the expression of $\|\tilde{\mathfrak{f}}^R(\wp^R(t))\|_\delta$ with Assumption 2.1, one gets

$$\begin{aligned}
\|\tilde{\mathfrak{f}}^R(\wp^R(t))\|_\delta &= \|\mathfrak{f}^R(\mathfrak{R}^R(t)) - \mathfrak{f}^R(\hbar^R(t))\|_\delta = \left[\sum_{\iota=1}^{n}\left|\mathfrak{f}_l^R(\mathfrak{R}_\iota^R(t)) - \mathfrak{f}_\iota^R(\hbar_\iota^R(t))\right|^p\right]^{\frac{1}{\delta}} \\
&\le \left[\sum_{\iota=1}^{n}\left(\mathfrak{m}_l^R\left|\mathfrak{R}_\iota^R(t) - \hbar_\iota^R(t)\right|\right)^p\right]^{\frac{1}{\delta}} \le \mathfrak{m}^R\|\wp^R(t)\|_\delta \le \widetilde{\mathfrak{m}}\|\wp^R(t)\|_\delta.
\end{aligned} \tag{3.20}$$

Using a similar approach, the following inequalities can be derived:

$$\|\tilde{\mathfrak{f}}^I(\wp^I(t))\|_\delta \le \mathfrak{m}^I\|\wp^I(t)\|_\delta \le \widetilde{\mathfrak{m}}\|\wp^I(t)\|_\delta, \tag{3.21}$$

$$\|\tilde{\mathfrak{f}}^J(\wp^J(t))\|_\delta \le \mathfrak{m}^J\|\wp^J(t)\|_\delta \le \widetilde{\mathfrak{m}}\|\wp^J(t)\|_\delta, \tag{3.22}$$

$$\|\tilde{\mathfrak{f}}^K(\wp^K(t))\|_\delta \le \mathfrak{m}^K\|\wp^K(t)\|_\delta \le \widetilde{\mathfrak{m}}\|\wp^K(t)\|_\delta, \tag{3.23}$$

and one also obtains

$$\|\tilde{\mathfrak{g}}^R(\wp^R(t-\Im(t)))\|_\delta \le \mathfrak{q}^R\|\wp^R(t-\Im(t))\|_\delta \le \widetilde{\mathfrak{q}}\|\wp^R(t-\Im(t))\|_\delta, \tag{3.24}$$

$$\|\tilde{\mathfrak{g}}^I(\wp^I(t-\Im(t)))\|_\delta \le \mathfrak{q}^I\|\wp^I(t-\Im(t))\|_\delta \le \widetilde{\mathfrak{q}}\|\wp^I(t-\Im(t))\|_\delta, \tag{3.25}$$

$$\|\tilde{\mathfrak{g}}^J(\wp^J(t-\Im(t)))\|_\delta \le \mathfrak{q}^J\|\wp^J(t-\Im(t))\|_\delta \le \widetilde{\mathfrak{q}}\|\wp^J(t-\Im(t))\|_\delta, \tag{3.26}$$

$$\|\tilde{\mathfrak{g}}^K(\wp^K(t-\Im(t)))\|_\delta \le \mathfrak{q}^K\|\wp^K(t-\Im(t))\|_\delta \le \widetilde{\mathfrak{q}}\|e^K(t-\Im(t))\|_\delta. \tag{3.27}$$

By resorting to the inequalities (3.21)–(3.27), one can calculate

$$\|\wp^R(t+\varrho)\|_\delta = \|\wp^R(t) + \varrho\dot{\wp}^R(t) + o(\varrho)\|_\delta$$

$$\le \left\|\mathcal{I}+\varrho(-\mathcal{R}_{\varsigma(t)}+\mathfrak{P}_{\varsigma(t)})\right\|_\delta \|\wp^R(t)\|_\delta+\varrho\widetilde{\mathfrak{m}}\Big(\|\mathcal{S}^I_{\varsigma(t)}\|_\delta\|\wp^I(t)\|_\delta+\|\mathcal{S}^R_{\varsigma(t)}\|_\delta\|\wp^R(t)\|_\delta+\|\mathcal{S}^J_{\varsigma(t)}\|_\delta \times\|\wp^J(t)\|_\delta+\|\mathcal{S}^K_{\varsigma(t)}\|_\delta\|\wp^K(t)\|_\delta\Big)+\varrho\widetilde{\mathfrak{q}}\Big(\|\mathcal{T}^I_{\varsigma(t)}\|_\delta\|\wp^I(t-\mathfrak{I}(t))\|_\delta+\|\mathcal{T}^R_{\varsigma(t)}\|_\delta\|\wp^R(t-\mathfrak{I}(t))\|_\delta +\|\mathcal{T}^K_{\varsigma(t)}\|_\delta\|\wp^K(t-\mathfrak{I}(t))\|_\delta+\|\mathcal{T}^J_{\varsigma(t)}\|_\delta\|\wp^J(t-\mathfrak{I}(t))\|_\delta\Big)+\|o(\varrho)\|_\delta. \tag{3.28}$$

In addition, by the similar method as in (3.28), one also has

$$\|\wp^I(t+\varrho)\|_\delta=\|\wp^I(t)+\varrho\dot{e}^I(t)+o(\varrho)\|_\delta \le \left\|\mathcal{I}+\varrho(-\mathcal{R}_{\varsigma(t)}+\mathfrak{P}_{\varsigma(t)})\right\|_\delta \|\wp^I(t)\|_\delta+\varrho\widetilde{\mathfrak{m}}\Big(\|\mathcal{S}^I_{\varsigma(t)}\|_\delta\|\wp^R(t)\|_\delta+\|\mathcal{S}^R_{\varsigma(t)}\|_\delta\|\wp^I(t)\|_\delta+\|\mathcal{S}^J_{\varsigma(t)}\|_\delta \times\|\wp^K(t)\|_\delta+\|\mathcal{S}^K_{\varsigma(t)}\|_\delta\|\wp^J(t)\|_\delta\Big)+\varrho\widetilde{\mathfrak{q}}\Big(\|\mathcal{T}^I_{\varsigma(t)}\|_\delta\|\wp^R(t-\mathfrak{I}(t))\|_\delta+\|\mathcal{T}^R_{\varsigma(t)}\|_\delta\|\wp^I(t-\mathfrak{I}(t))\|_\delta + \|\mathcal{T}^J_{\varsigma(t)}\|_\delta\|\wp^K(t-\mathfrak{I}(t))\|_\delta+\|\mathcal{T}^K_{\varsigma(t)}\|_\delta\|\wp^J(t-\mathfrak{I}(t))\|_\delta\Big)+\|o(\varrho)\|_\delta, \tag{3.29}$$

$$\|\wp^J(t+\varrho)\|_\delta=\|\wp^J(t)+\varrho\dot{\wp}^J(t)+o(\varrho)\|_\delta \le \left\|\mathcal{I}+\varrho(-\mathcal{R}_{\varsigma(t)}+\mathfrak{P}_{\varsigma(t)})\right\|_\delta \|\wp^J(t)\|_\delta+\varrho\widetilde{\mathfrak{m}}\Big(\|\mathcal{S}^R_{\varsigma(t)}\|_\delta\|\wp^J(t)\|_\delta+\|\mathcal{S}^I_{\varsigma(t)}\|_\delta\|\wp^K(t)\|_\delta+\|\mathcal{S}^J_{\varsigma(t)}\|_\delta \times\|\wp^R(t)\|_\delta+\|\mathcal{S}^K_{\varsigma(t)}\|_\delta\|\wp^I(t)\|_\delta\Big)+\varrho\widetilde{\mathfrak{q}}\Big(\|\mathcal{T}^R_{\varsigma(t)}\|_\delta\|\wp^J(t-\mathfrak{I}(t))\|_\delta+\|\mathcal{T}^I_{\varsigma(t)}\|_\delta\|\wp^K(t-\mathfrak{I}(t))\|_\delta + \|\mathcal{T}^J_{\varsigma(t)}\|_\delta\|\wp^R(t-\mathfrak{I}(t))\|_\delta+\|\mathcal{T}^K_{\varsigma(t)}\|_\delta\|\wp^I(t-\mathfrak{I}(t))\|_\delta\Big)+\|o(\varrho)\|_\delta, \tag{3.30}$$

$$\|\wp^K(t+\varrho)\|_\delta=\|\wp^K(t)+\varrho\dot{\wp}^K(t)+o(\varrho)\|_\delta \le \left\|\mathcal{I}+\varrho(-\mathcal{R}_{\varsigma(t)}+\mathfrak{P}_{\varsigma(t)})\right\|_\delta \|\wp^K(t)\|_\delta+\varrho\widetilde{\mathfrak{m}}\Big(\|\mathcal{S}^I_{\varsigma(t)}\|_\delta\|\wp^J(t)\|_\delta+\|\mathcal{S}^R_{\varsigma(t)}\|_\delta\|\wp^K(t)\|_\delta+\|\mathcal{S}^J_{\varsigma(t)}\|_\delta \times\|\wp^I(t)\|_\delta+\|\mathcal{S}^K_{\varsigma(t)}\|_\delta\|\wp^R(t)\|_\delta\Big)+\varrho\widetilde{\mathfrak{q}}\Big(+\|\mathfrak{B}^I_{\varsigma(t)}\|_\delta\|\wp^J(t-\mathfrak{I}(t))\|_\delta+\|\mathcal{T}^R_{\varsigma(t)}\|_\delta\|\wp^K(t-\mathfrak{I}(t))\|_\delta + \|\mathcal{T}^K_{\varsigma(t)}\|_\delta\|\wp^R(t-\mathfrak{I}(t))\|_\delta+\|\mathcal{T}^J_{\varsigma(t)}\|_\delta\|\wp^I(t-\mathfrak{I}(t))\|_\delta\Big)+\|o(\varrho)\Big\|_\delta. \tag{3.31}$$

By substituting equations (3.28) to (3.31) into Eq (3.19), one can have

$$\begin{aligned} D^+V(e(t)) &\le \Big[\mu_\delta(-\mathcal{R}_{\varsigma(t)}+\mathfrak{P}_{\varsigma(t)})+\widetilde{\mathfrak{m}}\Big(\|\mathcal{S}^I_{\varsigma(t)}\|_\delta+\|\mathcal{S}^R_{\varsigma(t)}\|_\delta+\|\mathcal{S}^K_{\varsigma(t)}\|_\delta+\|\mathcal{S}^J_{\varsigma(t)}\|_\delta\Big)\Big]V(\wp(t)) \\ &\quad+\widetilde{\mathfrak{q}}\Big(\|\mathfrak{B}^I_{\varsigma(t)}\|_\delta+\|\mathcal{T}^R_{\varsigma(t)}\|_\delta+\|\mathcal{T}^J_{\varsigma(t)}\|_\delta+\|\mathcal{T}^K_{\varsigma(t)}\|_\delta\Big)\sup_{t-\vec{\mathfrak{I}}\le s\le t}V(\wp(s)) \\ &=-\omega_1V(\wp(t))+\omega_2\sup_{t-\vec{\mathfrak{I}}\le s\le t}V(\wp(s)). \end{aligned} \tag{3.32}$$

Likewise, when $t=t_k$, one gets

$$\begin{aligned} V(\wp(t_k)) &= \|\wp^R(t_k)\|_\delta+\|\wp^J(t_k)\|_\delta+\|\wp^I(t_k)\|_\delta+\|\wp^K(t_k)\|_\delta \\ &\le \|\mathcal{L}_k+\mathcal{I}\|_\delta(\|\|\wp^J(t_k^-)\|_\delta+\wp^R(t_k^-)\|_\delta+\|\wp^I(t_k^-)\|_\delta+\|\wp^K(t_k^-)\|_\delta) \\ &\quad+\|\mathcal{M}_k\|_\delta(\|\wp^R(t_k^--\vec{\omega}_k)\|_\delta+\|\wp^J(t_k^--\vec{\omega}_k)\|_\delta+\|\wp^K(t_k^--\vec{\omega}_k)\|_\delta+\|\wp^I(t_k^--\vec{\omega}_k)\|_\delta) \\ &\le \|\mathcal{L}_k+\mathcal{I}\|_pV(\wp(t_k^-))+\|\mathcal{M}_k\|_\delta\sup_{t_k-\vec{\mathfrak{I}}\le s<t_k}V(e(s)) \\ &= a_kV(\wp(t_k^-))+b_k\sup_{t_k-\vec{\mathfrak{I}}\le s<t_k}V(\wp(s)). \end{aligned} \tag{3.33}$$

Combine (3.32) and (3.33) with Lemma 2.2 as well as conditions $\P_1$ and $\P_2$ stated in this theorem, one can yield

$$V(\wp(t))\le\sigma\sup_{t_0-\vec{\mathfrak{I}}\le s\le t_0}V(\wp(s))e^{-(\vec{\varrho}-\frac{\ln(\sigma_\wp\vec{\varrho}\vec{\mathfrak{I}})}{\vec{\beta}\vec{\mathfrak{I}}})(t-t_0)},\quad t\ge t_0 \tag{3.34}$$

is right, in which $\vec{\varrho}$ represents the unique positive root of $\vec{\varrho} = \omega_1 - \omega_2 e^{\vec{\varrho}\vec{\Im}}$. Consequently, in accordance with Definition 3.1, we can deduce that systems (2.1) and (3.15) achieve global exponential synchronization. Thus, the proof has been fully established.

Theorem 3.4. *Under the premise of Assumption 2.2, the system* (2.1) *can attain global exponential synchronization with the system* (3.15) *if the given constraints below are valid for δ:*

¶$_1$ $\bar{\omega}_1 > \bar{\omega}_2 \geq 0$, *in which*

$$\bar{\omega}_1 \triangleq \min_{1\leq l\leq \mathrm{N}}\Big\{-\mu_\delta\left(-\tilde{\mathcal{R}}_l + \tilde{\mathfrak{P}}_l\right) - \mu_\delta\left(\tilde{\mathcal{S}}_l^{*}\mathcal{D}\right) - (\tilde{\mathfrak{m}} + 2\mathfrak{m}^R)\|\mathcal{S}_1^l\|_\delta - (\tilde{\mathfrak{m}} + 2\mathfrak{m}^I)\|\mathcal{S}_2^l\|_\delta$$
$$- (\tilde{\mathfrak{m}} + 2\mathfrak{m}^J)\|\mathcal{S}_3^l\|_\delta - (\tilde{\mathfrak{m}} + 2\mathfrak{m}^K)\|\mathcal{S}_4^l\|_\delta\Big\},$$
$$\bar{\omega}_2 \triangleq \max_{1\leq l\leq \mathrm{N}}\left\{4^{\frac{1}{\delta}}\left(\mathfrak{q}^R\|\mathcal{T}_1^l\|_\delta + \mathfrak{q}^I\|\mathcal{T}_2^l\|_\delta + \mathfrak{q}^J\|\mathcal{T}_3^l\|_\delta + \mathfrak{q}^K\|\mathcal{T}_4^l\|_\delta\right)\right\}$$

with $\mathcal{D} = \mathrm{diag}\left\{\mathfrak{m}_1^R, \cdots, \mathfrak{m}_n^R, \mathfrak{m}_1^I, \cdots, \mathfrak{m}_n^I, \mathfrak{m}_1^J, \cdots, \mathfrak{m}_n^J, \mathfrak{m}_1^K, \cdots, \mathfrak{m}_n^K\right\}$.

¶$_2$ *There exists a constant* $\vec{\beta} > \frac{\ln(\sigma e^{\vec{\varrho}\vec{\Im}})}{\vec{\varrho}\vec{\Im}}$, *where*

$$\inf_{k\in N_+}\{t_k - t_{k-1}\} > \vec{\beta}\vec{\Im},$$

in which $\delta = 1$ *or* ∞, $\sigma \triangleq \sup_{k\in N_+}\{1, a_k + b_k e^{\vec{\varrho}\vec{\Im}}\}$, $a_k \triangleq \|\mathcal{I} + \mathcal{L}_k\|_\delta$, $b_k \triangleq \|\mathcal{M}_k\|_\delta$, *and* $\vec{\varrho}$ *denotes unique positive root of* $\vec{\varrho} = \bar{\omega}_1 - \bar{\omega}_2 e^{\vec{\varrho}\tau}$. *The remaining symbols retain their definitions as stated in Theorem 3.2.*

Proof. In the case of $t \neq t_k$, based on the specific expression of $\tilde{\mathfrak{f}}_1(\wp(t))$, one can infer

$$\tilde{\mathfrak{f}}_1(\wp(t)) = \begin{pmatrix} \tilde{\mathfrak{f}}^R(\wp^R(t))) \\ \tilde{\mathfrak{f}}^R(\wp^R(t))) \\ \tilde{\mathfrak{f}}^R(\wp^R(t))) \\ \tilde{\mathfrak{f}}^R(\wp^R(t))) \end{pmatrix} = \begin{pmatrix} \mathfrak{f}^R(\Re^R(t)) - \mathfrak{f}^R(\hbar^R(t)) \\ \mathfrak{f}^R(\Re^R(t)) - \mathfrak{f}^R(\hbar^R(t)) \\ \mathfrak{f}^R(\Re^R(t)) - \mathfrak{f}^R(\hbar^R(t)) \\ \mathfrak{f}^R(\Re^R(t)) - \mathfrak{f}^R(\hbar^R(t)) \end{pmatrix}$$
$$= \mathfrak{Y}_5(t) + \mathfrak{Y}_1(t) - \mathfrak{Y}_3(t) - \mathfrak{Y}_4(t) + \mathfrak{Y}_6(t) - \mathfrak{Y}_2(t) + \mathfrak{Y}_7(t) \tag{3.35}$$

in which

$$\mathfrak{Y}_2(t) = \begin{pmatrix} O_{n\times 1} \\ \mathfrak{f}^I(\Re^I(t)) - \mathfrak{f}^I(\hbar^I(t)) \\ O_{n\times 1} \\ O_{n\times 1} \end{pmatrix}, \quad \mathfrak{Y}_1(t) = \begin{pmatrix} \mathfrak{f}^R(\Re^R(t)) - \mathfrak{f}^R(\hbar^R(t)) \\ \mathfrak{f}^I(\Re^I(t)) - \mathfrak{f}^I(\hbar^I(t)) \\ \mathfrak{f}^J(\Re^J(t)) - \mathfrak{f}^J(\hbar^J(t)) \\ \mathfrak{f}^I(\Re^K(t)) - \mathfrak{f}^I(\hbar^K(t)) \end{pmatrix},$$

$$\mathfrak{Y}_3(t) = \begin{pmatrix} O_{n\times 1} \\ O_{n\times 1} \\ \mathfrak{f}^J(\Re^J(t)) - \mathfrak{f}^J(\hbar^J(t)) \\ O_{n\times 1} \end{pmatrix}, \quad \mathfrak{Y}_4(t) = \begin{pmatrix} O_{n\times 1} \\ O_{n\times 1} \\ O_{n\times 1} \\ \mathfrak{f}^K(\Re^K(t)) - \mathfrak{f}^K(\hbar^K(t)) \end{pmatrix},$$

$$\mathfrak{Y}_5(t) = \begin{pmatrix} O_{n\times 1} \\ \mathfrak{f}^R(\Re^R(t)) - \mathfrak{f}^R(\hbar^R(t)) \\ O_{n\times 1} \\ O_{n\times 1} \end{pmatrix}, \quad \mathfrak{Y}_6(t) = \begin{pmatrix} O_{n\times 1} \\ O_{n\times 1} \\ \mathfrak{f}^R(\Re^R(t)) - \mathfrak{f}^R(\hbar^R(t)) \\ O_{n\times 1} \end{pmatrix},$$

$$\mathfrak{Y}_7(t) = \begin{pmatrix} O_{n\times 1} \\ O_{n\times 1} \\ O_{n\times 1} \\ \mathfrak{f}^R(\mathfrak{R}^R(t)) - \mathfrak{f}^R(\hbar^R(t)) \end{pmatrix}.$$

Utilizing an analogous approach employed for Eq (3.35), we can likewise derive that

$$\begin{aligned} \tilde{\mathfrak{f}}_2(e(t)) &= \begin{pmatrix} \mathfrak{f}^I(\mathfrak{R}^I(t)) - \mathfrak{f}^I(\hbar^I(t)) \\ \mathfrak{f}^I(\mathfrak{R}^I(t)) - \mathfrak{f}^I(\hbar^I(t)) \\ \mathfrak{f}^I(\mathfrak{R}^I(t)) - \mathfrak{f}^I(\hbar^I(t)) \\ \mathfrak{f}^I(\mathfrak{R}^I(t)) - \mathfrak{f}^I(\hbar^I(t)) \end{pmatrix} \\ &= \mathfrak{Y}_1(t) - \mathfrak{Y}_3(t) - \mathfrak{Y}_4(t) - \mathfrak{Y}_8(t) + \mathfrak{Y}_9(t) + \mathfrak{Y}_{10}(t) + \mathfrak{Y}_{11}(t) \end{aligned} \tag{3.36}$$

in which

$$Y_8(t) = \begin{pmatrix} \mathfrak{f}^R(\mathfrak{R}^R(t)) - \mathfrak{f}^R(\hbar^R(t)) \\ O_{n\times 1} \\ O_{n\times 1} \\ O_{n\times 1} \end{pmatrix}, \quad \mathfrak{Y}_9(t) = \begin{pmatrix} \mathfrak{f}^I(\mathfrak{R}^I(t)) - \mathfrak{f}^I(\hbar^I(t)) \\ O_{n\times 1} \\ O_{n\times 1} \\ O_{n\times 1} \end{pmatrix},$$

$$\mathfrak{Y}_{10}(t) = \begin{pmatrix} O_{n\times 1} \\ O_{n\times 1} \\ \mathfrak{f}^I(\mathfrak{R}^I(t)) - \mathfrak{f}^I(\hbar^I(t)) \\ O_{n\times 1} \end{pmatrix}, \quad \mathfrak{Y}_{11}(t) = \begin{pmatrix} O_{n\times 1} \\ O_{n\times 1} \\ O_{n\times 1} \\ \mathfrak{f}^I(\mathfrak{R}^I(t)) - \mathfrak{f}^I(\hbar^I(t)) \end{pmatrix}.$$

$$\begin{aligned} \tilde{\mathfrak{f}}_3(\wp(t)) &= \begin{pmatrix} \mathfrak{f}^J(\mathfrak{R}^J(t)) - \mathfrak{f}^J(\hbar^J(t)) \\ \mathfrak{f}^J(\mathfrak{R}^J(t)) - \mathfrak{f}^J(\hbar^J(t)) \\ \mathfrak{f}^J(\mathfrak{R}^J(t)) - \mathfrak{f}^J(\hbar^J(t)) \\ \mathfrak{f}^J(\mathfrak{R}^J(t)) - \mathfrak{f}^J(\hbar^J(t)) \end{pmatrix} \\ &= \mathfrak{Y}_1(t) - \mathfrak{Y}_2(t) - \mathfrak{Y}_4(t) - \mathfrak{Y}_8(t) + \mathfrak{Y}_{12}(t) + \mathfrak{Y}_{13}(t) + \mathfrak{Y}_{14}(t) \end{aligned} \tag{3.37}$$

in which

$$\mathfrak{Y}_{12}(t) = \begin{pmatrix} \mathfrak{f}^J(\mathfrak{R}^J(t)) - \mathfrak{f}^J(\hbar^J(t)) \\ O_{n\times 1} \\ O_{n\times 1} \\ O_{n\times 1} \end{pmatrix}, \quad \mathfrak{Y}_{13}(t) = \begin{pmatrix} O_{n\times 1} \\ \mathfrak{f}^J(\mathfrak{R}^J(t)) - \mathfrak{f}^J(\hbar^J(t)) \\ O_{n\times 1} \\ O_{n\times 1} \end{pmatrix},$$

$$\mathfrak{Y}_{14}(t) = \begin{pmatrix} O_{n\times 1} \\ O_{n\times 1} \\ O_{n\times 1} \\ \mathfrak{f}^J(\mathfrak{R}^J(t)) - \mathfrak{f}^J(\hbar^J(t)) \end{pmatrix}.$$

$$\tilde{\mathfrak{f}}_4(e(t)) = \begin{pmatrix} \mathfrak{f}^K(\mathfrak{R}^K(t)) - \mathfrak{f}^K(\hbar^K(t)) \\ \mathfrak{f}^K(\mathfrak{R}^K(t)) - \mathfrak{f}^K(\hbar^K(t)) \\ \mathfrak{f}^K(\mathfrak{R}^K(t)) - \mathfrak{f}^K(\hbar^K(t)) \\ \mathfrak{f}^K(\mathfrak{R}^K(t)) - \mathfrak{f}^K(\hbar^K(t)) \end{pmatrix}$$

$$= \mathfrak{Y}_1(t) - \mathfrak{Y}_2(t) - \mathfrak{Y}_3(t) - \mathfrak{Y}_8(t) + \mathfrak{Y}_{15}(t) + \mathfrak{Y}_{16}(t) + \mathfrak{Y}_{17}(t) \tag{3.38}$$

in which

$$\mathfrak{Y}_{15}(t) = \begin{pmatrix} \mathfrak{f}^K(\mathfrak{R}^K(t)) - \mathfrak{f}^K(\hbar^K(t)) \\ O_{n\times 1} \\ O_{n\times 1} \\ O_{n\times 1} \end{pmatrix}, \quad \mathfrak{Y}_{16}(t) = \begin{pmatrix} O_{n\times 1} \\ \mathfrak{f}^K(\mathfrak{R}^K(t)) - \mathfrak{f}^K(\hbar^K(t)) \\ O_{n\times 1} \\ O_{n\times 1} \end{pmatrix},$$

$$\mathfrak{Y}_{17}(t) = \begin{pmatrix} O_{n\times 1} \\ O_{n\times 1} \\ \mathfrak{f}^K(\mathfrak{R}^K(t)) - \mathfrak{f}^K(\hbar^K(t)) \\ O_{n\times 1} \end{pmatrix}, \quad \mathfrak{Y}_1(t) = \begin{pmatrix} \mathfrak{f}^R(\mathfrak{R}^R(t)) - \mathfrak{f}^R(\hbar^R(t)) \\ \mathfrak{f}^I(\mathfrak{R}^I(t)) - \mathfrak{f}^I(\hbar^I(t)) \\ \mathfrak{f}^J(\mathfrak{R}^J(t)) - \mathfrak{f}^J(\hbar^J(t)) \\ \mathfrak{f}^I(\mathfrak{R}^K(t)) - \mathfrak{f}^I(\hbar^K(t)) \end{pmatrix} = \Xi(\wp(t))\wp(t)$$

with

$$\Xi(\wp(t)) = \operatorname{diag}\Bigg\{ \frac{\mathfrak{f}_1^R(\mathfrak{R}_1^R(t)) - \mathfrak{f}_1^R(\hbar_1^R(t))}{\mathfrak{R}_1^R(t) - \hbar_1^R(t)}, \cdots, \frac{\mathfrak{f}_n^R(\mathfrak{R}_n^R(t)) - \mathfrak{f}_n^R(\hbar_n^R(t))}{\mathfrak{R}_n^R(t) - \hbar_n^R(t)}, \frac{\mathfrak{f}_1^I(\mathfrak{R}_1^I(t)) - \mathfrak{f}_1^I(\mathfrak{R}_1^I(t))}{\mathfrak{R}_1^I(t) - \hbar_1^I(t)}, \cdots,$$
$$\frac{\mathfrak{f}_n^I(\mathfrak{R}_n^I(t)) - \mathfrak{f}_n^I(\hbar_n^I(t))}{\mathfrak{R}_n^I(t) - \hbar_n^I(t)}, \frac{\mathfrak{f}_1^J(\mathfrak{R}_1^J(t)) - \mathfrak{f}_1^J(\hbar_1^J(t))}{\mathfrak{R}_1^J(t) - \hbar_1^J(t)}, \cdots, \frac{\mathfrak{f}_n^J(\mathfrak{R}_n^J(t)) - \mathfrak{f}_n^J(\hbar_n^J(t))}{\mathfrak{R}_n^J(t) - \hbar_n^J(t)},$$
$$\frac{\mathfrak{f}_1^K(\mathfrak{R}_1^K(t)) - \mathfrak{f}_1^K(\hbar_1^K(t))}{\mathfrak{R}_1^K(t) - \hbar_1^K(t)}, \cdots, \frac{\mathfrak{f}_n^K(\mathfrak{R}_n^K(t)) - \mathfrak{f}_n^K(\hbar_n^K(t))}{\mathfrak{R}_n^K(t) - \hbar_n^K(t)}\Bigg\}.$$

By incorporating the inequalities (3.35)–(3.38) into $\|\wp(t) + \varrho\dot{\wp}(t) + o(\varrho)\|_\delta$, one further derives

$$\begin{aligned}
&\|\wp(t) + \varrho\dot{\wp}(t) + o(\varrho)\|_\delta \\
&\le \left\| \mathcal{I} + \varrho\left[-\tilde{C}_{\varsigma(t)} + \left(\mathcal{S}_1^{\varsigma(t)} + \mathcal{S}_3^{\varsigma(t)} + \mathcal{S}_2^{\varsigma(t)} + \mathcal{S}_4^{\varsigma(t)}\right)\Xi(\wp(t))\right]\right\|_\delta \|\wp(t)\|_\delta \\
&\quad + \varrho(\tilde{\mathfrak{m}} + 2\mathfrak{m}^R)\|\mathcal{S}_1^{\varsigma(t)}\|_\delta\|\wp(t)\|_\delta + \varrho(\tilde{\mathfrak{m}} + 2\mathfrak{m}^I)\|\mathcal{S}_2^{\varsigma(t)}\|_\delta\|\wp(t)\|_\delta \\
&\quad + \varrho(\tilde{\mathfrak{m}} + 2\mathfrak{m}^J)\|\mathcal{S}_3^{\varsigma(t)}\|_\delta\|\wp(t)\|_\delta + \varrho(\tilde{\mathfrak{m}} + 2\mathfrak{m}^K)\|\mathcal{S}_4^{\varsigma(t)}\|_\delta\|\wp(t)\|_\delta \\
&\quad + 4^{\frac{1}{\delta}} q^I\varrho\|\mathcal{T}_2^{\varsigma(t)}\|_\delta\|\wp(t - \mathfrak{I}(t))\|_\delta + 4^{\frac{1}{\delta}}\mathfrak{q}^R\varrho\|\mathcal{T}_1^{\varsigma(t)}\|_\delta\|\wp(t - \mathfrak{I}(t))\|_\delta \\
&\quad + 4^{\frac{1}{\delta}}\mathfrak{q}^K\varrho\|\mathcal{T}_4^{\varsigma(t)}\|_\delta\|\wp(t - \mathfrak{I}(t))\|_\delta + 4^{\frac{1}{\delta}}\mathfrak{q}^J\varrho\|\mathcal{T}_3^{\varsigma(t)}\|_\delta\|\wp(t - \mathfrak{I}(t))\|_\delta + \|o(\varrho)\|_\delta.
\end{aligned} \tag{3.39}$$

Hence, it follows from the presented first condition in Theorem 3.4 that one has

$$\begin{aligned}
&D^+\|\wp(t)\|_\delta \\
&\le \Big[\mu_p\left(-\tilde{C}_{\varsigma(t)} + \tilde{\mathcal{P}}_{\varsigma(t)} + (\mathcal{S}_1^{\varsigma(t)} + \mathcal{S}_3^{\varsigma(t)} + \mathcal{S}_2^{\varsigma(t)} + \mathcal{S}_4^{\varsigma(t)})\Xi(\wp(t))\right) + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^R)\|\mathcal{S}_1^{\varsigma(t)}\|_\delta \\
&\quad + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^I)\|\mathcal{S}_2^{\varsigma(t)}\|_\delta + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^J)\|\mathcal{S}_3^{\varsigma(t)}\|_\delta + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^K)\|\mathcal{S}_4^{\varsigma(t)}\|_\delta\Big]\|\wp(t)\|_\delta \\
&\quad + 4^{\frac{1}{\delta}}\left(\mathfrak{q}^R\|\mathcal{T}_1^{\varsigma(t)}\|_\delta + \mathfrak{q}^I\|\mathcal{T}_2^{\varsigma(t)}\|_\delta + \mathfrak{q}^J\|\mathcal{T}_3^{\varsigma(t)}\|_\delta + \mathfrak{q}^K\|\mathcal{T}_4^{\varsigma(t)}\|_\delta\right)\|\wp(t - \mathfrak{I}(t))\|_\delta \\
&\le \Big[\mu_p\left(-\tilde{\mathcal{R}}_{\varsigma(t)} + \tilde{\mathcal{P}}_{\varsigma(t)}\right) + \mu_p\left(\tilde{\mathcal{S}}^*_{\varsigma(t)}\mathcal{D}\right) + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^I)\|\mathcal{S}_2^{\varsigma(t)}\|_\delta + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^R)\|\mathcal{S}_1^{\varsigma(t)}\|_\delta \\
&\quad + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^K)\|\mathcal{S}_4^{\varsigma(t)}\|_\delta + (\tilde{\mathfrak{m}} + 2\mathfrak{m}^J)\|\mathcal{S}_3^{\varsigma(t)}\|_\delta\Big]\|\wp(t)\|_\delta
\end{aligned}$$

$$
\begin{aligned}
&+ 4^{\frac{1}{\delta}}(\mathrm{q}^R\|\mathcal{T}_1^{\varsigma(t)}\|_\delta + \mathrm{q}^I\|\mathcal{T}_2^{\varsigma(t)}\|_\delta \\
&+ \mathrm{q}^J\|\mathcal{T}_3^{\varsigma(t)}\|_\delta + \mathrm{q}^K\|\mathcal{T}_4^{\varsigma(t)}\|_\delta) \sup_{t-\vec{\Im}\le s\le t}\|\wp(s)\|_\delta \\
\le & -\bar{\omega}_1\|\wp(t)\|_\delta + \bar{\omega}_2 \sup_{t-\vec{\Im}\le s\le t}\|\wp(s)\|_\delta.
\end{aligned} \tag{3.40}
$$

When $t = t_k$, combine (3.39) and (3.40) with Lemma 2.2, and one can conclude

$$
V(\wp(t)) \le \sigma \sup_{t_0-\vec{\Im}\le s\le t_0} V(\wp(s))e^{-(\vec{\varrho}-\frac{\ln(\sigma e^{\vec{\varrho}\vec{\Im}})}{\vec{\beta}\vec{\Im}})(t-t_0)}, \quad t \ge t_0 \tag{3.41}
$$

in which $\vec{\varrho}$ denotes the unique positive root of $\vec{\varrho} = \bar{\omega}_1 - \bar{\omega}_2 e^{\vec{\varrho}\vec{\Im}}$. Hence, according to Definition 3.1, we obtain that systems (2.1) and (3.15) can reach global exponential synchronization. The specific proof has been completed.

Remark 3.1. *The sufficient and effective criteria for achieving exponential synchronization between systems* (2.1) *and* (3.15) *are contingent upon several factors, including the time delay, network parameters, intensity of the impulses, pulse delay, and the minimum duration between impulsive intervals. Furthermore, these criteria build upon previous studies outlined in references [29–37], offering new insights and expanding our understanding of synchronization in such systems.*

Remark 3.2. *It is crucial to acknowledge that time delays are inherently prevalent in information processing because signals travel through various links at a limited speed. Deservedly, the time delay also exists in pulse signals, which are usually called delay impulses. In comparison to the existing literatures [32, 33, 36, 39], this paper first addresses the exponential stability and synchronization control issues of QVNNs with delayed impulses, which is more in line with practical engineering.*

Remark 3.3. *The connection between a Markov process and general stochastic process can be described as follows. A Markov process is a specific subset of a stochastic process. While a stochastic process represents a broad framework for modeling random variables evolving over time, a Markov process imposes additional structure through its memoryless property, making them particularly suitable for modeling scenarios. This distinction allows Markov processes to serve as foundational models in areas such as queueing theory, population dynamics, and financial modeling [42, 43].*

4. Numerical example

Example 4.1. *Consider a two-neuron QVNN given in system* (2.1) *with two jumping modes as the master system, and the involved parameters can be taken as* $\mathcal{R}_1 = \mathrm{diag}\{0.4, 0.5\}$, $\mathcal{R}_2 = \mathrm{diag}\{0.4, 0.6\}$, $\mathcal{L}_1 = \mathcal{L}_2 = \mathrm{diag}\{0.2, 0.4\}$, $\mathcal{M}_1 = \mathcal{M}_2 = \mathrm{diag}\{0.2, 0.3\}$, $\Im(t) = 0.2 + 0.2\sin 2t$, $\vec{\omega}_k = 0.4$ *with* $\vec{\Im} = 0.4$, *and*

$$
\mathcal{S}_1 = \begin{bmatrix} 1.1 - 1.1\vec{i} + 1.2\vec{j} + 1.3\vec{k} & 0.9 + 0.8\vec{i} + 0.9\vec{j} + 0.7\vec{k} \\ 1.0 + 0.8\vec{i} + 1.1\vec{j} + 1.3\vec{k} & 1.2 + 0.9\vec{i} + 1.2\vec{j} + 0.7\vec{k} \end{bmatrix},
$$

$$
\mathcal{S}_2 = \begin{bmatrix} 1.2 - 1.4\vec{i} + 1.5\vec{j} + 1.2\vec{k} & 1.4 + 1.6\vec{i} + 1.2\vec{j} + 1.1\vec{k} \\ 1.2 + 1.4\vec{i} + 1.3\vec{j} + 1.4\vec{k} & 1.4 + 0.1\vec{i} + 1.2\vec{j} + 1.6\vec{k} \end{bmatrix},
$$

$$\mathcal{T}_1 = \begin{bmatrix} 0.8 - 1.9\vec{i} + 0.7\vec{j} - 0.5\vec{k} & 1.1 - 1.2\vec{i} - 1.1\vec{j} + 1.2\vec{k} \\ 0.8 + 0.9\vec{i} - 1.1\vec{j} + 0.9\vec{k} & 0.6 - 1.1\vec{i} + 0.8\vec{j} + 1.1\vec{k} \end{bmatrix},$$
$$\mathcal{T}_2 = \begin{bmatrix} 0.9 - 1.3\vec{i} + 0.9\vec{j} - 1.5\vec{k} & 2.1 - 1.6\vec{i} - 2.1\vec{j} + 1.5\vec{k} \\ 0.9 + 1.4\vec{i} - 1.3\vec{j} + 1.2\vec{k} & 1.2 - 1.4\vec{i} + 0.9\vec{j} + 1.3\vec{k} \end{bmatrix}.$$

Besides, the involved nonlinear activation functions take the form

$$\mathfrak{f}_\iota^R(\hbar) = \mathfrak{f}_\iota^J(\hbar) = \mathfrak{g}_\iota^R(\hbar) = \mathfrak{g}_\iota^J(\hbar) = \frac{1}{4}\sin(\hbar),$$
$$\mathfrak{f}_\iota^I(\hbar) = \mathfrak{f}_\iota^K(\hbar) = \mathfrak{g}_\iota^I(\hbar) = \mathfrak{g}_\iota^K(\hbar) = \frac{1}{4}\cos(\hbar).$$

Based on this, it can be computed according to Assumption 2.1 that $\mathfrak{m}_\iota^r = \mathfrak{q}_\iota^r = 0.5$. *In addition, the considered response system can be described as* (3.15) *with mode-dependent controller* (3.16)*, and the controller gain matrices are set as follows:*

$$\mathfrak{P}_1 = \begin{bmatrix} -16 & 0.4 \\ 0.3 & -17 \end{bmatrix}, \quad \mathfrak{P}_2 = \begin{bmatrix} -18 & 0.1 \\ 0.2 & -17 \end{bmatrix}.$$

Besides, the considered Markov process with initial modal value $\varsigma(0) = 1$ *follows exponential distribution, and the corresponding jumping trajectory can be presented in Figure 1. Moreover, the infinitesimal generator matrix is set as*

$$\Theta = \begin{bmatrix} -3 & 3 \\ 4 & -4 \end{bmatrix}.$$

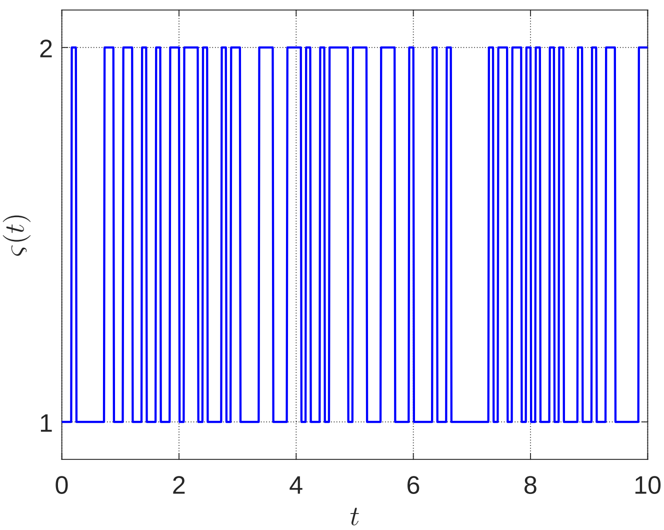

Figure 1. Evolution of Markov process $\varsigma(t)$ with two modes.

Combine the system parameters given above, and we can get the following pleasing results. When $\delta = 1$ *and* $\varsigma(t) = 1$, *one can get* $-\mu_1(-\mathcal{R}_1 + \mathfrak{P}_1) - \widetilde{\mathfrak{m}}\left(\|\mathcal{S}_1^R\|_1 + \|\mathcal{S}_1^I\|_1 + \|\mathcal{S}_1^J\|_1 + \|\mathcal{S}_1^K\|_1\right) = 9.649$, *and*

$\widetilde{\mathfrak{q}}\Big(\|\mathcal{T}_1^R\|_1 + \|\mathcal{T}_1^I\|_1 + \|\mathcal{T}_1^J\|_1 + \|\mathcal{T}_1^K\|_1\Big) = 4.10$. *While* $r(t) = 2$, *it also gets* $-\mu_1(-\mathcal{R}_2 + \mathfrak{P}_2) - \widetilde{\mathfrak{m}}\Big(\|\mathcal{S}_2^R\|_1 + \|\mathcal{S}_2^I\|_1 + \|\mathcal{S}_2^J\|_1 + \|\mathcal{S}_2^K\|_1\Big) = 9.650$, *and* $\widetilde{\mathfrak{q}}\Big(\|\mathcal{T}_2^R\|_1 + \|\mathcal{T}_2^I\|_1 + \|\mathcal{T}_2^J\|_1 + \|\mathcal{T}_2^K\|_1\Big) = 6.05$. *In this case, one has* $\omega_1 = \min\{9.649, 9.650\} = 9.649 > \omega_2 = \max\{6.05, 4.10\} = 6.05$, *which means that the first condition in Theorem 3.3 is valid. Besides, it is easy to have* $a_k = 1.4$, $b_k = 0.3$, $\sigma = 1.8330$, *and* $\vec{\varrho} = 0.9173$. *According to Theorem 3.3, it can be inferred that systems* (2.1) *and* (3.15) *can attain global exponential synchronization if there exists a scalar* $\vec{\beta}$ *satisfying the condition* $\vec{\beta} \triangleq 2.66 > 2.6515$ *such that the involved impulsive interval conforms to* $\inf_{k\in N_+}\{t_k - t_{k-1}\} > \vec{\beta}\vec{\mathfrak{I}} = 1.0640$.

Next, in order to facilitate the simulation, set the impulsive interval to 1.08, that is, $t_{k+1} - t_k = 1.08$. *In addition, let the initial values of drive-response systems* (2.1) *and* (3.15) *be* $\hbar(t) = (\hbar_1(t), \hbar_2(t))^T$, $\mathfrak{R}(t) = (\mathfrak{R}_1(t), \mathfrak{R}_2(t))^T$ *with* $\hbar_1(t) = -4.4 + 2.6\vec{i} - 2.1\vec{j} + 3.4\vec{k}$, $\hbar_2(t) = -2.6 - 3.4\vec{i} - 3.6\vec{j} - 1.4\vec{k}$, $\mathfrak{R}_1(t) = 3.4 - 1.6\vec{i} + 2.4\vec{j} - 1.4\vec{k}$, *and* $\mathfrak{R}_2(t) = 2.1 + 3.2\vec{i} + 2.1\vec{j} + 2.5\vec{k}$ *for* $t \in [-0.4, 0]$. *Under the influence of the mode-dependent linear feedback controller* $\mathfrak{U}_{\varsigma(t)}(t)$, *the response time trajectories of states* $\mathfrak{R}^R(t) = (\mathfrak{R}_1^R(t), \mathfrak{R}_2^R(t))^T$ *and* $\hbar^R(t) = (\hbar_1^R(t), \hbar_2^R(t))^T$ *as well as their synchronization error* $\wp^R(t) = (\wp_1^R(t), \wp_2^R(t))^T$ *are shown in the upper-left corner of Figure 2. Similarly, the trajectory plots for the states* $\hbar^I(t)$, $\mathfrak{R}^I(t)$ *and* $\wp^I(t)$, *the states* $\hbar^J(t)$, $\mathfrak{R}^J(t)$ *and their synchronization error* $\wp^J(t)$, *as well as states* $\hbar^K(t)$, $\mathfrak{R}^K(t)$ *and error* $\wp^K(t)$, *are illustrated in the remaining positions of Figure 2.*

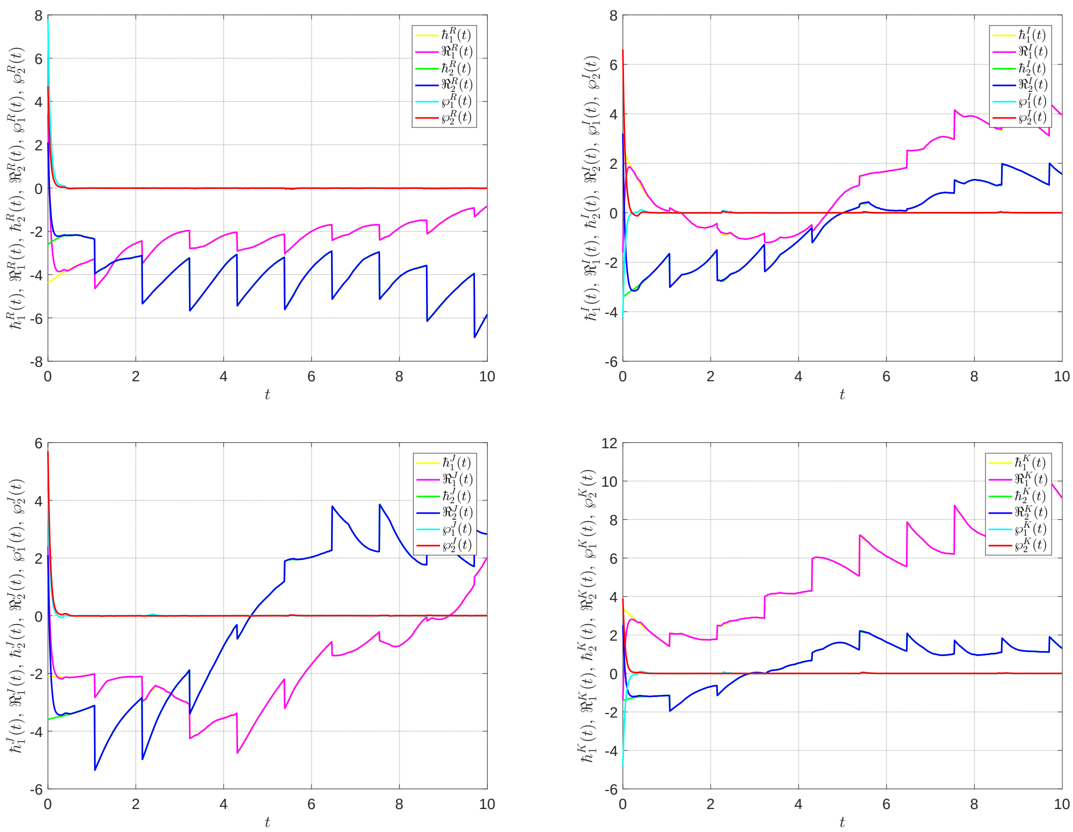

Figure 2. The response time trajectories of states $\hbar(t)$, $\mathfrak{R}(t)$ and their synchronization error $\wp(t)$.

5. Conclusions

This article delves into global exponential stability and synchronization problems for a specific type of delayed Markovian jumping QVNNs that incorporate delayed impulses. By employing the matrix measure strategy and delayed differential inequality methods that account for impulsive factors, we establish several practical and effective criteria to ascertain that the impulsive QVNNs in question can attain exponential synchronization with a given response system. Additionally, the explicit exponential convergence rate is provided. Notably, the derived criteria are straightforward to verify and can be readily applied in real-world scenarios. Finally, to underscore the precision and efficacy of our theoretical findings, we present one numerical example accompanied by an explanation. Moreover, inspired by the existing research findings in references [44, 45], our further focus will be on the event-triggered synchronization problem of semi-Markovian jumping QVNNs with time-varying delay, where the sojourn time follows a more general distribution.

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Author contributions

Miao Zhnag: Formal analysis, and original draft preparation; Bole Li: Writing-review and editing; Weiqiang Gong: Conceptualization, resources, writing-review and editing. Shuo Ma: Writing-review and editing; Qiang Li. Investigation, methodology and supervision. All authors have read and approved the final version of the manuscript for publication

Conflict of interest

The authors declare no conflict of interest.

References

1. T. Liu, S. Boumaiza, F. M. Ghannouchi, Dynamic behavioral modeling of 3G power amplifiers using real-valued time-delay neural networks, *IEEE T. Microw. Theory*, **52** (2004), 1025–1033. https://doi.org/10.1109/TMTT.2004.823583
2. H. Dong, F. Shen, J. Zhao, X. Mu, Understanding neural network through neuron level visualization, *Neural Networks*, **168** (2023), 484–495. https://doi.org/10.1016/j.neunet.2023.09.030

3. H. Zhang, Q. Yang, C. Zhang, Y. Li, Y. Chen, Magnetic properties simulation of electrical steel sheet based on recurrent neural network, *IEEE T. Magn.*, **60** (2024), 7300304. https://doi.org/10.1109/TMAG.2023.3316753

4. T. Zhang, Z. Li, Switching clusters' synchronization for discrete space-time complex dynamical networks via boundary feedback controls, *Pattern Recogn.*, **143** (2023), 109763. https://doi.org/10.1016/j.patcog.2023.109763

5. C. Zhang, C. Zhang, X. Zhang, F. Wang, Y. Liang, Dynamic event-triggered control for intra/inter-layer synchronization in multi-layer networks, *Commun. Nonlinear Sci.*, **119** (2023), 107124. https://doi.org/10.1016/j.cnsns.2023.107124

6. J. Hu, J. Wang, Global stability of complex-valued recurrent neural networks with time-delays, *IEEE T. Neur. Net. Lear.*, **23** (2012), 853–865. https://doi.org/10.1109/TNNLS.2012.2195028

7. X. Chen, Q. Song, State estimation for quaternion-valued neural networks with multiple time delays, *IEEE T. Syst. Man Cy. Syst.*, **49** (2019), 2278–2287. https://doi.org/10.1109/TSMC.2017.2776940

8. X. Qin, H. Jiang, J. Qiu, C. Hu, Fixed/prescribed-time synchronization of quaternion-valued fuzzy BAM neural networks under aperiodic intermittent pinning control: A non-separation approach, *Neurocomputing*, **549** (2023), 126460. https://doi.org/10.1016/j.neucom.2023.126460

9. W. Wei, J. Yu, L. Wang, C. Hu, H. Jiang, Fixed/preassigned-time synchronization of quaternion-valued neural networks via pure power-law control, *Neural Networks*, **146** (2022), 341–349. https://doi.org/10.1016/j.neunet.2021.11.023

10. N. Ohtani, N. Nagai, M. Suzuki, N. Miki, Formulation of quantum effects by using a complex-valued equivalent circuit, *Electr. Commun. Japan II*, **74** (1991), 11–19. https://doi.org/10.1002/ecjb.4420740702

11. D. P. Mandic, V. S. L. Goh, *Complex valued nonlinear adaptive filters: Noncircularity, widely linear and neural models*, John Wiley & Sons, 2009.

12. E. N. Sanchez, J. P. Perez, Input-to-state stability (ISS) analysis for dynamic neural networks, *IEEE T. Ind. Inform.*, **46** (1999), 1395–1398. https://doi.org/10.1109/81.802844

13. A. Cariow, G. Cariowa, Fast algorithms for quaternion-valued convolutional neural networks, *IEEE T. Neur. Net. Lear.*, **32** (2021), 457–462. https://doi.org/10.1109/TNNLS.2020.2979682

14. J. Xiang, M. Tan, Existence and stability of Stepanov-almost periodic solution in distribution for quaternion-valued memristor-based stochastic neural networks with delays, *Nolinear Dyn.*, **111** (2023), 1715–1732. https://doi.org/10.1007/s11071-022-07877-7

15. W. Zhang, H. Zhao, C. Sha, Y. Wang, Finite time synchronization of delayed quaternion valued neural networks with fractional order, *Neural Process. Lett.*, **53** (2021), 3607–3618. https://doi.org/10.1007/s11063-021-10551-5

16. H. Wei, B. Wu, Z. Tu. Exponential synchronization and state estimation of inertial quaternion-valued Cohen-Grossberg neural networks: Lexicographical order method, *Int. J. Robust Nonlin.*, **30** (2020), 2171–2185. https://doi.org/10.1002/rnc.4871

17. Y. Liu, D. Zhang, J. Lu, Global exponential stability for quaternion-valued recurrent neural networks with time-varying delays, *Nonlinear Dyn.*, **87** (2017), 553–565. https://doi.org/10.1007/s11071-016-3060-2
18. C. Zhang, H. Wu, X. Han, X. Zhang, Exponential synchronization of reaction-diffusion neural networks via switched event-triggered control, *Inform. Sci.*, **648** (2023), 119599. https://doi.org/10.1016/j.ins.2023.119599
19. R. Liu, C. Zhang, Y. Guo, X. Zhang, Intermittent event-triggered control for exponential synchronization of delayed neural networks on time scales, *Commun. Nonlinear Sci.*, **137** (2024), 108158. https://doi.org/10.1016/j.cnsns.2024.108158
20. J. Ran, Y. Zhou, H. Pu, Global stability and synchronization of stochastic discrete-time variable-order fractional-order delayed quaternion-valued neural networks, *Math. Comput. Simulat.*, **226** (2024), 413–437. https://doi.org/10.1016/j.matcom.2024.07.017
21. Q. Li, J. Liang, Dissipativity of the stochastic Markovian switching CVNNs with randomly occurring uncertainties and general uncertain transition rates, *Int. J. Syst. Sci.*, **51** (2020), 1102–1118. https://doi.org/10.1080/00207721.2020.1752418
22. H. Xu, Q. Zhu, W. X. Zheng, Exponential stability of stochastic nonlinear delay systems subject to multiple periodic impulses, *IEEE T. Automat. Contr.*, **69** (2024), 2621–2628. https://doi.org/10.1109/TAC.2023.3335005
23. B. Wang, Q. Zhu, S. Li. Stability analysis of discrete-time semi-Markov jump linear systems with time delay, *IEEE T. Automat. Contr.*, **68** (2023), 6758–6765. https://doi.org/10.1109/TAC.2023.3240926
24. B. Wang, Q. Zhu, S. Li, Stabilization of discrete-time hidden semi-Markov jump linear systems with partly unknown emission probability matrix, *IEEE T. Automat. Contr.*, **69** (2024), 1952–1959. https://doi.org/10.1109/TAC.2023.3272190
25. X. Yang, H. Wang, Q. Zhu, Event-triggered predictive control of nonlinear stochastic systems with output delay, *Automatica*, **140** (2022), 110230. https://doi.org/10.1016/j.automatica.2022.110230
26. Q. Zhu, Stabilization of stochastic nonlinear delay systems with exogenous disturbances and the event-triggered feedback control, *IEEE T. Automat. Contr.*, **64** (2019), 3764–3771. https://doi.org/10.1109/TAC.2018.2882067
27. Q. Li, J. Liang, W. Gong, K. Wang, J. Wang, Nonfragile state estimation for semi-Markovian switching CVNs with general uncertain transition rates: An event-triggered scheme, *Math. Comput. Simulat.*, **218** (2024), 204–222. https://doi.org/10.1016/j.matcom.2023.11.028
28. Q. Li, H. Wei, D. Hua, J. Wang, J. Yang, Stabilization of semi-Markovian jumping uncertain complex-valued networks with time-varying delay: A sliding-mode control approach, *Neural Process. Lett.*, **56** (2024), 111. https://doi.org/10.1007/s11063-024-11585-1
29. Y. Tian, H. Yan, H. Zhang, J. Cheng, H. Shen, Asynchronous output feedback control of hidden semi-Markov jump systems with random mode-dependent delays, *IEEE T. Automat. Contr.*, **67** (2022), 4107–4114. https://doi.org/10.1109/TAC.2021.3110006

30. Y. Tian, H. Yan, H. Zhang, M. Wang, J. Yi, Time-varying gain controller synthesis of piecewise homogeneous semi-Markov jump linear systems, *Automatica*, **146** (2022), 110594. https://doi.org/10.1016/j.automatica.2022.110594

31. W. Xie, Q. Zhu, Self-triggered state-feedback control for stochastic nonlinear systems with Markovian switching, *IEEE T. Syst. Man Cy. Syst.*, **50** (2020), 3200–3209. https://doi.org/10.1109/TSMC.2018.2870494

32. B. Li, Q. Huang, Synchronization of time-delay systems with impulsive delay via an average impulsive estimation approach, *Math. Biosci. Eng.*, **21** (2024), 4501–4520. https://doi.org/10.3934/mbe.2024199

33. Z. Xu, D. Peng, X. Li, Synchronization of chaotic neural networks with time delay via distributed delayed impulsive control, *Neural Networks*, **118** (2019), 332–337. https://doi.org/10.1016/j.neunet.2019.07.002

34. Y. Huang, A. Wu, J. E. Zhang, Exponential stability of stochastic time-delay neural networks with random delayed impulses, *Neural Process. Lett.*, **56** (2024), 38. https://doi.org/10.1007/s11063-024-11521-3

35. Y. Lu, Q. Zhu, Exponential stability of impulsive random delayed nonlinear systems with average-delay impulses, *J. Franklin I.*, **361** (2024), 106813. https://doi.org/10.1016/j.jfranklin.2024.106813

36. X. Liu, P. Cheng, Y. Cui, Almost sure exponential stability and stochastic stabilization of impulsive stochastic differential delay equations, *Syst. Control Lett.*, **187** (2024), 105786. https://doi.org/10.1016/j.sysconle.2024.105786

37. B. Chen, Z. Tang, J. Feng, Matrix measure-based distributed impulsive consensus on nonlinear multi-agent systems with mixed time-varying delays, *ISA T.*, **145** (2024), 104–111. https://doi.org/10.1016/j.isatra.2023.11.025

38. M. Vidyasagar, *Nonlinear system analysis*, Englewood Cliffs: Prentice Hall; 1993.

39. D. W. C. Ho, J. Liang, J. Lam, Global exponential stability of impulsive high-order BAM neural networks with time-varying delays, *Neural Networks*, **19** (2006), 1581–1590. https://doi.org/10.1016/j.neunet.2006.02.006

40. B. Gan, H. Chen, B. Xu, W. Kang, A norm stability condition of neutral-type Cohen-Grossberg neural networks with multiple time delays, *Chaos Solition. Fract.*, **175** (2023), 113958. https://doi.org/10.1016/j.chaos.2023.113958

41. V. Singh, New global robust stability results for delayed cellular neural networks based on norm-bounded uncertainties, *Chaos Solition. Fract.*, **30** (2006), 1165–1171. https://doi.org/10.1016/j.chaos.2005.08.183

42. M. Augustyniak, L. Bauwens, A. Dufays, A new approach to volatility modeling: The factorial hidden Markov volatility model, *J. Bus. Econ. Stat.*, **37** (2019), 696–709. https://doi.org/10.1080/07350015.2017.1415910

43. A. Eriksson, B. Mahjani, B. Mehlig, Sequential Markov coalescent algorithms for population models with demographic structure, *Theor. Popul. Biol.*, **76** (2009), 84–91. https://doi.org/10.1016/j.tpb.2009.05.002

44. Q. Li, H. Wei, W. Gong, J. Wang, H_∞ synchronization of semi-Markovian switching complex-valued networks with time-varying delay: A delay-dependent event-triggered mechanism, *Int. J. Robust Nonlin.*, 2024. https://doi.org/10.1002/rnc.7739

45. H. Zhang, Z. Qiu, J. Cao, M. Abdel-Aty, L. Xiong, Event-triggered synchronization for neutral-type semi-Markovian neural networks with partial mode-dependent time-varying delays, *IEEE T. Neural Net. Lear.*, **31** (2020), 4437–4450. https://doi.org/10.1109/TNNLS.2019.2955287

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