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Research article

A new study on the Newell-Whitehead-Segel equation with Caputo-Fabrizio fractional derivative

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Abstract: In this research, we propose a new numerical method that combines with the Caputo-Fabrizio Elzaki transform and the q-homotopy analysis transform method. This work aims to analyze the Caputo-Fabrizio fractional Newell-Whitehead-Segel (NWS) equation utilizing the Caputo-Fabrizio q-Elzaki homotopy analysis transform method. The Newell-Whitehead-Segel equation is a partial differential equation employed for modeling the dynamics of reaction-diffusion systems, specifically in the realm of pattern generation in biological and chemical systems. A convergence analysis of the proposed method was performed. Two-dimensional and three-dimensional graphs of the solutions have been drawn with the Maple software. It is seen that the resulting proposed method is more powerful and effective than the Aboodh transform homotopy perturbation method and conformable Laplace decomposition method in the results.

Keywords: Newell-Whitehead-Seqel equation; Caputo-Fabrizio q-Elzaki homotopy analysis transform method; Caputo-Fabrizio Elzaki transform
Mathematics Subject Classification: 35C05, 35R11, 65R10

1. Introduction

Differential equations with any real order greater than zero, denoted as n>0, are employed to represent a wide range of physical phenomena in diverse fields of science and engineering [1–5]. It is demonstrated how a first-order differential equation with a variable coefficient may be comparable to the initial value problem for a relaxation process controlled by a differential equation of non-integer order with a constant coefficient [6]. In [7], the intra-specific relationship between two predators and a food chain system that depends on prey is examined. There is a broad use for the generalized fractional reaction–diffusion equations, which are available in the form of noninteger order partial differential equations, to illustrate significant and practical physical phenomena, like subdiffusive and superdiffusive scenarios [8]. These mathematical models are used in several fields such as statistical mechanics, Brownian motion, visco-elasticity problems, continuum and quantum physics, biosciences, chemical engineering, and control theory [9–13].

The exponential growth in the popularity of the discipline referred to as fractional calculus (FC) has led to the emergence of multiple distinct approaches for defining fractional derivatives and fractional integrals [14]. Contrary to the Riemann-Liouville fractional integral, the fractional derivative has multiple definitions, some of which are only comparable to each other when certain conditions are placed on the function being differentiated [15,16]. Furthermore, there have been recent discussions regarding the categorization of the fractional operators, resulting in the proposal of three new classes [17,18].

Given the growing use of fractional derivatives in modeling physical problems, it is necessary to establish a clear and widely accepted definition for this type of derivative [19]. Additionally, it is crucial to develop a reliable and precise numerical approximation method to address problems that involve singularities and non-linearities in the systems [20]. Nevertheless, the non-local nature of these fractional operators has imposed constraints on the development of efficient codes, as it necessitates the inclusion of all previous information during simulation [21,22]. The presence of permanent memory, also known as non-volatile memory, leads to increased computational costs and slower performance [23]. There are several short memory principles discussed in recent literature that are commonly employed to minimize computational expenses and the accumulation of rounding-off errors when using numerical techniques [24]. As a result, these short memory principles are highly beneficial in solving fractional initial value problems [25–28].

The definition of the Caputo-Fabrizio operator, which is a fractional derivative operator without a singular kernel, is a direct consequence of the traditional Caputo derivative operator. The reason for this is that the later incorporates a unique mathematical expression known as the kernel in its formulation, which presents challenges in solving the related differential equations. In contrast, the kernel of the former does not have any singularity at $t = \tau$ [29].

Additional valuable characteristics and intriguing applications of this novel derivative operator, including the renowned Laplace transform approach, are detailed in [30]. The operator is also applied in recent publications that analyze the Korteweg-de Vries-Burgers equation in liquids and wave dynamics, the magnetohydrodynamic free convection flow of generalized Walters'-B fluid over a static vertical plate, and the nonlinear Fisher's reaction diffusion equation [31–33].

The advantages of the Caputo-Fabrizio fractional derivative include: It offers a method of regularizing the derivative, allowing for the smoothing of functions or signals and improving their behavior, even if they lack differentiability in the traditional sense. The Caputo-Fabrizio fractional derivative has demonstrated its physical significance in several applications, particularly in viscoelasticity, where it accurately represents the behavior of materials that possess both elastic and viscous characteristics. Its mathematical features, such as the semi-group property, make it ideal for both analytical and numerical analysis, enabling efficient numerical simulations. It is capable of simulating fractional diffusion processes, which are present in several natural and physical systems when traditional diffusion models are insufficient. The Caputo-Fabrizio fractional derivative is an extension of the Caputo fractional derivative and the Riemann-Liouville fractional derivative, offering a more adaptable framework for representing intricate systems [34].

The Elzaki transform and other transforms, including Laplace and Fourier transforms, are unified to a generalized integral transform [35].

The Elzaki transform, developed by Elzaki, is a variation of the existing Sumudu transform and is derived from the classical Fourier integral. The Elzaki transform's mathematical simplicity enables the efficient solution of ordinary and partial differential equations in the time domain [36–39].

Several powerful numerical methodologies were formed in the scientific literature, and various distinguished scientists made significant contributions to this subject. These approaches comprise the Adomian decomposition method (ADM) [40], the homotopy perturbation method (HPM) [41–43], the homotopy analysis method (HAM) [44], the fractional natural transform decomposition method (FNTDM) [45], the Elzaki transform differential transform method (ETDTM) [46], the homotopy perturbation Elzaki transform method [47–49], the Elzaki variational iteration method [50], the differential transform method [51,52], the He-Elzaki method [53], the Li-He's modified homotopy perturbation method [54], the Aboodh transform homotopy perturbation method (ATHPM) [55], and the conformable Laplace decomposition method (CLDM) [56].

Amplitude equations can be used to simulate various striped patterns, such as the fluctuations in sand and the lines on seashells. The Newell-Whitehead-Segel equation (NWSE) holds significant importance in the field of applied sciences as one of the most crucial amplitude equations. This demonstrates the manifestation of stripes in two-dimensional systems [56–60].

The classical NWSE is formed as [56–60]

$$u_t(x,\mathrm{t}) = \varrho \frac{\partial^2 \mathrm{u}(x,\mathrm{t})}{\partial t^2} + \mathrm{o}\mathrm{u}(x,\mathrm{t}) - \rho u^{\Im}(x,\mathrm{t}), \qquad \mathrm{t} \geq 0, x \in \mathbb{R}, \tag{1.1}$$

where $\Im \in \mathbb{Z}^+$, $\varrho, \mathrm{o}, \rho \in \mathbb{R}$. The initial condition of Eq (1.1) is $u(x,0) = \delta$.

The NWSE holds great importance in the examination of pattern creation in diverse physical and biological systems. The NWSE is significant in various application domains due to its importance. The NWSE is a mathematical model that can be used to describe how chemical concentrations change over time and space in reaction-diffusion systems. Understanding the creation of spatial patterns, such as chemical waves and Turing patterns, is crucial. The NWSE is employed to simulate biological pattern production, including the formation of animal coats, the distribution of animal spots and stripes, and the branching of nerve cells. These patterns frequently emerge as the result of reaction-diffusion processes that include chemical signaling. The NWSE is used in the fields of physics and material science to analyze pattern generation in systems such as solidification fronts, where the boundary between solid and liquid phases displays intricate patterns. The NWSE has been utilized to investigate pattern development in fluid dynamics, specifically the emergence of hexagonal convection cells in Rayleigh-Bénard convection. The NWSE is utilized in neuroscience to simulate the transmission of electrical signals in neural networks, a crucial aspect for comprehending brain function and information processing. The NWSE is crucial as it offers a mathematical framework to comprehend the emergence of intricate patterns from basic physical and chemical processes. It aids researchers in comprehending the mechanics underlying pattern development and can provide valuable insights into the self-organization of natural systems [60].

By adding the Caputo-Fabrizio fractional derivative to the NWSE, we incorporate a memory effect that reflects the system's history and non-local interactions. This is particularly useful in modeling systems where memory and non-local effects play a significant role in dynamics, such as in materials science, biology (for example, in describing anomalous transport in cells), and various physical processes involving complex interactions over time [61,62].

In this paper, we examine Caputo-Fabrizio NWSE:

$$ {}_{t_0}^{CF}D_t^{\mu}u(x,t) = \frac{\partial^2 \mathrm{u(x,t)}}{\partial \mathrm{x}^2} + 2\mathrm{u(x,t)} - 3u^2(x,t), \qquad \mu \in (0,1] \tag{1.2} $$

with the initial condition

$$ u(x,0) = \delta. $$

The primary objective of this study is to present a novel approach, known as the Caputo-Fabrizio q-Elzaki homotopy analysis transform method (CFq-EHATM), which is combined with the Caputo-Fabrizio Elzaki transform and q-homotopy analysis transform method. The secondary objective of this study is to acquire the novel numerical solutions for the Caputo-Fabrizio NWSE by the utilization of the CFq-EHATM.

The following portion of the research is provided in further detail. Section 2 of the document provides the basic concepts of fractional derivatives and introduces the Caputo-Fabrizio Elzaki transform (CFET) as a method for calculating these derivatives. Section 3 introduces the CFq-EHATM, which stands for the Caputo-Fabrizio q-Elzaki homotopy analysis transform method. In Section 4, we present the numerical solutions of the Caputo-Fabrizio Newell-Whitehead-Segel equation. Section 5 provides the concluding remarks or results.

2. Main definitions and theorems

In this section, the basic definitions and theorems will be given.

Definition 2.1. [62] Suppose that g is a differentiable function. The Caputo derivative of order $\mu \in (0,1)$ is defined by

$$ {}_{t_0}^{C}D_t^{\mu}[g(t)] = \frac{1}{\Gamma(n-\mu)} \int_0^t \frac{g^{(n)}(s)}{(t-s)^{1+\mu-n}} ds, \qquad n-1 < \mu \le n. \tag{2.1} $$

Definition 2.2. [63] The He's fractional derivative of order μ of the function $g(t)$ is defined as

$$ D_t^{\mu}[g(t)] = \frac{1}{\Gamma(n-\mu)} \frac{d^n}{dt^n} \int_{t_0}^t (s-t)^{n-\mu-1}[g_0(s) - g(s)]\, ds. \tag{2.2} $$

Definition 2.3. [64] The Atangana-Baleanu (AB) fractional derivative of order $\mu > 0$ of the function $g(t)$ is defined by

$$ {}_{t_0}^{AB}D_t^{\mu}[g(t)] = \frac{N(\mu)}{1-\mu} \int_0^t g'(\tau)\, E_\mu \left[-\frac{\mu(t-\tau)^{\mu}}{1-\mu} \right] d\tau, \qquad 0 < \mu \le 1, \tag{2.3} $$

where normalization function $N(\mu)$ is equal to 1 when $\mu = 0, \mu = 1$ is represented by $N(\mu)$, and E_μ represents the Mittag-Leffler function.

Definition 2.4. [61] The Caputo-Fabrizio (CF) fractional derivative of order $\mu > 0$ for the function

$g(t)$ is defined by

$$ {}^{CF}_{t_0}D^{\mu}_{0,t}[g(t)] = \frac{M(\mu)}{1-\mu}\int_0^t g'(\tau)\, exp\left[-\frac{\mu(t-\tau)}{1-\mu}\right] d\tau, \qquad \tau > 0, \tag{2.4} $$

where $M(\mu)$ is the normalization function such that $M(0) = M(1) = 1$, and $g \in H^1(a,b),\ b > a$.

Definition 2.5. [61] The Caputo-Fabrizio Elzaki transform (ET) of the Caputo-Fabrizio derivative (CFD) for the function $g(t)$ is defined by

$$ {}^{CF}_{t_0}E_{\mu}\{{}^{CF}_{t_0}D^{\mu}_{t}[g(t)]\} = \frac{1}{1-\mu+\mu w}\left({}^{CF}_{t_0}E_{\mu}[g(t)] - w^2 g(0)\right), \qquad 0 < \mu \le 1. \tag{2.5} $$

2.1. q-Homotopy analysis transform method

Now, analyze the time-fractional partial differential equations [65]:

$$ D^{\mu}_{t} w(x,t) + Kw(x,t) + Sw(x,t) = \zeta(x,t), t \in (0,\infty), \qquad n-1 < \mu \le n, \tag{2.6} $$

where K and S are linear and nonlinear operators, $\zeta(x,t)$ is a nonhomogeneous function, and D^{μ}_{t} is Caputo fractional derivative of order μ.

Applying the Laplace transform to Eq (2.6) and using the initial condition, then Eq (2.7) is found by

$$ \begin{aligned} &\mathcal{L}[w(x,t)] - \frac{1}{s^{\mu}}\sum_{k=0}^{n-1} s^{\mu-k-1} w^{(k)}\, w(x,y,0) + \frac{1}{s^{\mu}}\mathcal{L}[Kw(x,t)] + \frac{1}{s^{\mu}}\mathcal{L}[Sw(x,t)] \\ &= \frac{1}{s^{\mu}}\mathcal{L}[\zeta(x,t)]. \end{aligned} \tag{2.7} $$

Via the homotopy analysis method, the nonlinear operator of $\psi(x,t;q)$ is described by

$$ \begin{aligned} N[\psi(x,t;q)] = \mathcal{L}[\psi(x,t;q)] &- \frac{1}{s^{\mu}}\sum_{k=0}^{n-1} s^{\mu-k-1}\psi^{(k)}(x,t;q)(0^+) \\ &+ \frac{1}{s^{\mu}}\mathcal{L}[K\psi(x,t;q)] + \frac{1}{s^{\mu}}\mathcal{L}[S\psi(x,t;q)] - \frac{1}{s^{\mu}}\mathcal{L}[\zeta(x,t)], \end{aligned} \tag{2.8} $$

where $q \epsilon \left[0, \frac{1}{n}\right]$.

A homotopy is generated by

$$ (1-nq)\mathcal{L}[\psi(x,t;q) - w_0(x,t)] = hqH^*(x,t)S[\psi(x,t;q)], \tag{2.9} $$

where $h \ne 0$ is an auxiliary parameter and $\mathcal{L}$ represents Laplace transform. For $q = 0$ and $q = \frac{1}{n}$, the outcomes of Eq (2.9) are found by

$$ \psi(x,t;0) = w_0(x,t), \psi\left(x,t;\frac{1}{n}\right) = w(x,t). \tag{2.10} $$

Therefore, when q increases from 0 to $1/n$, the solution $\varphi(x,t;q)$ converges from $w_0(x,t)$

to the solution $w(x,t)$. Using the Taylor theorem around q, then it is found by

$$\psi(x,t;q) = w_0(x,t) + \sum_{i=1}^{\infty} w_m(x,t)q^m, \tag{2.11}$$

where

$$w_m(x,t) = \frac{1}{m!}\frac{\partial^m \psi(x,t;q)}{\partial q^m}|_{q=0}. \tag{2.12}$$

Equation (2.11) converges at $q = \frac{1}{n}$ for the convenient $w_0(x,t)$, n and h. Therefore, the numerical solution of the nonlinear equation is obtained as

$$w(x,t) = w_0(x,t) + \sum_{m=1}^{\infty} w_m(x,t)\left(\frac{1}{n}\right)^m. \tag{2.13}$$

Differentiating the 0-th order deformation Eq (2.11) m-times with respect to q and dividing by $m!$, thus for $q = 0$, one obtains

$$\mathcal{L}[w_m(x,t) - k_m w_{m-1}(x,t)] = hH^*(x,t)\mathcal{R}_m(\vec{w}_{m-1}), \tag{2.14}$$

where the vectors are given as

$$\vec{w}_m = \{w_0(x,t), w_1(x,t), \dots, w_m(x,t)\}. \tag{2.15}$$

When inverse Laplace transform is used on Eq (2.14), the result is acquired as

$$w_m(x,t) = k_m w_{m-1}(x,t) + \mathcal{L}^{-1}[hH^*(x,t)\mathcal{R}_m(\vec{w}_{m-1})], \tag{2.16}$$

where

$$\begin{aligned}\mathcal{R}_m(\vec{w}_{m-1}) = \mathcal{L}[w_{m-1}(x,t)] - \left(1 - \frac{k_m}{n}\right)\frac{1}{s^\mu}\sum_{k=0}^{n-1} s^{\mu-k-1}w^{(k)}(x,0) \\ + \frac{1}{s^\mu}\mathcal{L}[Kw_{m-1}(x,t) + {H^*}_{m-1}(x,t) - \zeta(x,t)]\end{aligned} \tag{2.17}$$

and

$$k_m = \begin{cases} 0, & m \leq 1, \\ n, & m > 1, \end{cases} \tag{2.18}$$

where ${H^*}_m$ is homotopy polynomials and it is presented as

$${H^*}_{m-1} = \frac{1}{(m-1)!}\frac{\partial^{m-1}\psi(x,t;q)}{\partial q^{m-1}}|_{q=0}, \psi(x,t;q) = \psi_0 + q\psi_1 + q^2\psi_2 + \cdots. \tag{2.19}$$

Utilizing Eqs (2.16) and (2.17), one obtains

$$w_m(x,t) = (k_m + h)w_{m-1}(x,t) - \left(1 - \frac{k_m}{n}\right)\frac{1}{s^{\mu}}\sum_{k=0}^{n-1} s^{\mu-k-1} w^{(k)}(x,0)$$
$$+h(\mathcal{L})^{-1}\left[\left(\frac{1}{s^{\mu}}\mathcal{L}[Kw_{m-1}(x,t) + H^{*}{}_{m-1}(x,t) - \zeta(x,t)]\right)\right]. \tag{2.20}$$

Via q-homotopy analysis transform method, then it is obtained as

$$w(x,t) = \sum_{m=0}^{\infty} w_m(x,t). \tag{2.21}$$

3. The novel numerical method

In this part, we present the CFq-EHATM, which combines with Caputo-Fabrizio Elzaki transform and q-homotopy analysis transform method.

Consider the Caputo-Fabrizio time-fractional order nonlinear partial differential equation (CFTFNPDE) to give the main idea of CFq-EHATM:

$${}_{t_0}^{CF}D_t^{\mu} u(\xi,\tau) + Au(\xi,\tau) + Hu(\xi,\tau) = \zeta(\xi,\tau), \qquad n-1 < \mu \leq n, \tag{3.1}$$

where A and H are linear and nonlinear operators, $\zeta(\xi,\tau)$ is a nonhomogeneous function, and ${}_{t_0}^{CF}D_t^{\mu}$ is CFD of order μ.

Step 1. By applying the CFET to Eq (3.1) and utilizing the initial condition, Eq (3.2) can be determined by

$$\frac{\left({}_{t_0}^{CF}E_{\mu}[u(\xi,\tau)] - w^2 u(\xi,0)\right)}{1-\mu+\mu w} + {}_{t_0}^{CF}E_{\mu}[Au(\xi,\tau) + Hu(\xi,\tau)] = {}_{t_0}^{CF}E_{\mu}[\zeta(\xi,\tau)]. \tag{3.2}$$

Rewriting the Eq (3.2), Eq (3.3) is obtained by

$${}_{t_0}^{CF}E_{\mu}[u(\xi,\tau)] - w^2 u(\xi,0) + (1-\mu+\mu w)\,{}_{t_0}^{CF}E_{\mu}[Au(\xi,\tau) + Hu(\xi,\tau)]$$
$$-(1-\mu+\mu w)\,{}_{t_0}^{CF}E_{\mu}[\zeta(\xi,\tau)] = 0. \tag{3.3}$$

Step 2. Via the HAM, the nonlinear operator of $\vec{u}(\xi,\tau;q)$ is described by

$$N^1[\vec{u}(\xi,\tau;q)] = {}_{t_0}^{CF}E_{\mu}[\vec{u}(\xi,\tau;q)] - w^2\vec{u}(\xi,\tau;q)(0^+)$$
$$+(1-\mu+\mu w)\left[{}_{t_0}^{CF}E_{\mu}\big(Au(\xi,\tau) + Hu(\xi,\tau) - \zeta(\xi,\tau)\big)\right], \tag{3.4}$$

where $q\epsilon\left[0,\frac{1}{n}\right]$.

A homotopy is generated by

$$(1-nq)\,{}^{CF}_{t_0}E_\mu[\vec{u}(\xi,\tau;q)-u_0(\xi,\tau)] = \hbar q H^*(\xi,\tau)\,{}^{CF}_{t_0}E_\mu[\vec{u}(\xi,\tau;q)], \tag{3.5}$$

where ${}^{CF}_{t_0}E_\mu$ signifies CFET and $\hbar \neq 0$ is an auxiliary parameter. The outcomes of Eq (3.5) are determined for $q=0$ and $q=\frac{1}{n}$ by

$$\vec{u}(\xi,\tau;0)=u_0(\xi,\tau), \vec{u}\left(\xi,\tau;\frac{1}{n}\right)=u(\xi,\tau). \tag{3.6}$$

With q increasing from 0 to $1/n$, $\vec{u}(\xi,\tau;q)$ converges to the solution $u(\xi,\tau)$ from $u_0(\xi,\tau)$.

When the Taylor theorem is applied to q, the result is found by

$$\vec{u}(\xi,\tau;q)=u_0(\xi,\tau)+\sum_{i=1}^{\infty}u_m(\xi,\tau)q^m, \tag{3.7}$$

where

$$u_m(\xi,\tau)=\frac{1}{m!}\frac{\partial^m\vec{u}(\xi,\tau;q)}{\partial q^m}|_{q=0}. \tag{3.8}$$

For $u_0(\xi,\tau)$, n and $\hbar$, which are convenient, Eq (3.7) converges at $\mathrm{q}=\frac{1}{n}$.

By dividing by $m!$ and differentiating the 0-th order deformation Eq (3.5) m times with regard to q, one obtains for $q=0$,

$${}^{CF}_{t_0}E_\mu[u_m(\xi,\tau)-\hbar_m u_{m-1}(\xi,\tau)] = \hbar H^{1^*}(\xi,\tau)\mathcal{R}_{1,m}(\vec{u}_{m-1}). \tag{3.9}$$

Step 3. When inverse CFET (ICFET) is used on Eq (3.9), the result is acquired as

$$u_m(\xi,\tau)=\hbar_m u_{m-1}(\xi,\tau)+\hbar\left({}^{CF}_{t_0}E_\mu\right)^{-1}\left[H^{1*}(\xi,\tau)\mathcal{R}_{1,m}(\vec{u}_{m-1})\right], \tag{3.10}$$

where

$$\begin{aligned}\mathcal{R}_{1,m}(\vec{u}_{m-1}) = {}&{}^{CF}_{t_0}E_\mu[u_{m-1}(\xi,\tau)]-w^2\left(1-\frac{\hbar_m}{n}\right)u_0(\xi,\tau)\\ &+(1-\mu+\mu w)\left[{}^{CF}_{t_0}E_\mu[Au(\xi,\tau)+Hu(\xi,\tau)-\zeta(\xi,\tau)]\right]\end{aligned} \tag{3.11}$$

and

$$\hbar_m=\begin{cases}0, & m\leq 1,\\ n, & m>1,\end{cases} \tag{3.12}$$

where H^{1*} is homotopy polynomials and it is presented as

$${}^{CF}_{t_0}E_\mu[u_m(\xi,\tau)-\hbar_m u_{m-1}(\xi,\tau)] = \hbar H^{1^*}(\xi,\tau)\mathcal{R}_{1,m}(\vec{u}_{m-1}) \tag{3.13}$$

and

$$H^{1*} = \frac{1}{m!}\frac{\partial^m \vec{u}(x,t;q)}{\partial q^m}|_{q=0}, \qquad \vec{u}(\xi,\tau;q) = \vec{u}_0 + q\vec{u}_1 + q^2\vec{u}_2 + \cdots \tag{3.14}$$

Utilizing Eqs (3.10) and (3.11), one obtains

$$\begin{aligned} u_m(\xi,\tau) &= (\hbar_m + \hbar)u_{m-1}(\xi,\tau) - w^2\left(1 - \frac{\hbar_m}{n}\right)u_0(\xi,\tau) \\ &\quad + \hbar\left({}^{CF}_{t_0}E_\mu\right)^{-1}\left[(1-\mu+\mu w)\,{}^{CF}_{t_0}E_\mu[Au_{m-1}(\xi,\tau) + H^{1*}_{m-1}(\xi,\tau) - \zeta(\xi,\tau)]\right]. \end{aligned} \tag{3.15}$$

Step 4. It is then acquired via CFq-EHATM as

$$u(\xi,\tau) = u_0(\xi,\tau) + \sum_{c=1}^{\infty} u_c(\xi,\tau)\left(\frac{1}{n}\right)^c. \tag{3.16}$$

4. Convergence analysis

Theorem 4.1. (Uniqueness theorem) [66,67] The solution for the nonlinear Caputo-Fabrizio fractional partial differential Eq (3.1) acquired by CFq-EHATM is unique for $\forall \mu \epsilon (0,1)$, where $\mu = (n + \hbar) + \hbar(\varpi + \upsilon)\Upsilon$.

Proof. The solution of nonlinear Caputo-Fabrizio FPDEs Eq (3.1) is presented as

$$\varphi(\xi,\tau) = \sum_{c=1}^{\infty} \varphi_c(\xi,\tau)\left(\frac{1}{n}\right)^c, \tag{4.1}$$

where

$$\begin{aligned} \varphi_m(\xi,\tau) &= (\hbar_m + \hbar)\varphi_{m-1}(\xi,\tau) - w^2\left(1 - \frac{\hbar_m}{n}\right)\varphi_0(\xi,\tau) \\ &\quad + \hbar\left({}^{CF}_{t_0}E_\mu\right)^{-1}\left[(1-\mu+\mu w)\,{}^{CF}_{t_0}E_\mu[Au_{m-1}(\xi,\tau) + H^{1*}{}_{m-1}(\xi,\tau) - \zeta(\xi,\tau)]\right]. \end{aligned} \tag{4.2}$$

Let φ and ϕ are two distinct solutions of Eq (3.1), then it is obtained as

$$\begin{aligned} |\varphi - \phi| &= \Big|(n+h)(\varphi-\phi) + h\left({}^{CF}_{t_0}E_\mu\right)^{-1}[(1-\mu+\mu w)\,{}^{CF}_{t_0}E_\mu[A(\varphi-\phi) \\ &\qquad + H(\varphi-\phi)]]\Big|. \end{aligned} \tag{4.3}$$

Via the convolution theorem for the CFET, we have

$$|\varphi - \phi| \le (n+h)|\varphi-\phi| + h\int_0^t (|A(\varphi-\phi)| + |H(\varphi-\phi)|)\frac{(t-\tau)^\mu}{\Gamma(1+\mu)}d\tau \tag{4.4}$$

$$\leq (n+h)|\varphi-\phi| + h\int_0^t (\varpi|\varphi-\phi|+\upsilon|\varphi-\phi|)\frac{(t-\tau)^{\mu}}{\Gamma(1+\mu)}d\tau.$$

Utilizing the integral mean-value theorem (IMVT), then we obtain

$$|\varphi-\phi| \leq (n+h)|\varphi-\phi| + h(\varpi|\varphi-\phi|+\upsilon|\varphi-\phi|)\Upsilon \leq \alpha|\varphi-\phi|. \tag{4.5}$$

Thus, we have $(1-\tau)|\varphi-\phi| \leq 0$. For $\tau \in (0,1)$, thus, $|\varphi-\phi| = 0$. For $\mu \in (0,1)$, that is $\varphi = \phi$. Therefore, the solution is unique.

Theorem 4.2. (Convergence theorem) [66,67] Let X is a Banach space (BS) and $G: X \to X$ is a nonlinear mapping. Assume that the inequality

$$\|G(a) - G(h)\| \leq \gamma\|a-h\|, \qquad \forall a, b \in X \tag{4.6}$$

holds, then G has a fixed point in view of Banach fixed point theory. Also, for the arbitrary selection of $a_0, b_0 \in X$, the sequence created by the CFq-EHATM converges to a fixed point of G and

$$\|\phi_m - \phi_n\| \leq \frac{\gamma^n}{1-\gamma}\|\phi_1 - \phi_0\|, \qquad \forall a, b \in X. \tag{4.7}$$

Proof. Let a BS $(C[J], \|.\|)$ of all continuous functions on J via the norm expressed as $\|g(t\| = \max_{t\in J}|g(t)|$.

We show that the sequence $\{\phi_n\}$ is a Cauchy sequence in the BS:

$$\begin{aligned}
\|\phi_m - \phi_n\| &= \max_{t\in J}|\phi_m - \phi_n| \\
&= \max_{t\in J}\Big|(n+h)(\phi_{m-1}-\phi_{n-1}) + h\left({}_{t_0}^{CF}E_\mu\right)^{-1}[(1-\mu+\mu w) \\
&\quad \times {}_{t_0}^{CF}E_\mu[A(\phi_{m-1}-\phi_{n-1}) + H(\phi_{m-1}-\phi_{n-1})]\Big| \\
&\leq \max_{t\in J}\Big[(n+h)|\phi_{m-1}-\phi_{n-1}| + h\left({}_{t_0}^{CF}E_\mu\right)^{-1}[(1-\mu+\mu w) \\
&\quad \times \left({}_{t_0}^{CF}E_\mu\right)^{-1}(A|\phi_{m-1}-\phi_{n-1}| + H|\phi_{m-1}-\phi_{n-1}|)]\Big].
\end{aligned} \tag{4.8}$$

Via the convolution theorem for the CFET, then we obtain

$$\begin{aligned}
\|\phi_m - \phi_n\| &\leq \max_{t\in J}[(n+h)|\phi_{m-1}-\phi_{n-1}| + (n+h)|\phi_{m-1}-\phi_{n-1}| \\
&\quad + h\int_0^t (A|\phi_{m-1}-\phi_{n-1}| + H|\phi_{m-1}-\phi_{n-1}|)\frac{(t-\tau)^{\boldsymbol{\mu}}}{\Gamma(1+\mu)} \\
&\leq \max_{t\in J}[(n+h)|\phi_{m-1}-\phi_{n-1}| + (n+h)|\phi_{m-1}-\phi_{n-1}| \\
&\quad + h\int_0^t (\varpi|\phi_{m-1}-\phi_{n-1}| + \upsilon|\phi_{m-1}-\phi_{n-1}|)\frac{(t-\tau)^{\boldsymbol{\mu}}}{\Gamma(1+\mu)}d\tau].
\end{aligned} \tag{4.9}$$

Using the IMVT, then we have

$$\|\phi_m - \phi_n\| \le \max_{t\in J}[(n+h)|\phi_{m-1} - \phi_{n-1}| \\ +h(\rho|\phi_{m-1} - \phi_{n-1}| + \delta|\phi_{m-1} - \phi_{n-1}|)\Upsilon] \le \tau\|\phi_{m-1} - \phi_{n-1}\|. \tag{4.10}$$

Let $m = n + 1$, then we obtain

$$\|\phi_{n+1} - \phi_n\| \le \tau\|\phi_n - \phi_{n-1}\| \le \tau^2\|\phi_{n-1} - \phi_{n-2}\| \le \cdots \le \tau^n\|\phi_1 - \phi_0\|. \tag{4.11}$$

Via the triangular inequality, we obtain

$$\begin{aligned} \|\phi_m - \phi_n\| &\le \|\phi_{n+1} - \phi_n\| + \|\phi_{n+2} - \phi_{n+1}\| + \cdots + \|\phi_m - \phi_{m-1}\| \\ &\le [\tau^n + \tau^{n+1} + \cdots + \tau^{m-1}]\|\phi_1 - \phi_0\| \\ &\le \tau^n[1 + \tau + \tau^2 + \cdots + \tau^{m-n-1}]\|\phi_1 - \phi_0\| \\ &\le \tau^n\left[\frac{1-\tau^{m-n-1}}{1-\tau}\right]\|\phi_1 - \phi_0\|. \end{aligned} \tag{4.12}$$

Since $\tau \in (0,1)$, $1 - \tau^{m-n-1} < 1$, then we obtain

$$\|\phi_m - \phi_n\| \le \frac{\tau^n}{1-\tau}\|\phi_1 - \phi_0\|. \tag{4.13}$$

For $\|\phi_1 - \phi_0\| < \infty$, so as $m \to \infty$, then $\|\phi_m - \phi_n\| \to 0$. Thus, the sequence $\{\phi_n\}$ is Cauchy sequence in $C[J]$, and so the sequence is convergent.

5. Application

The part aims to present visual representations of the time-fractional coupled Newell-Whitehead-Segel equation in a Caputo-Fabrizio sense.

Example 5.1. Consider the Caputo-Fabrizio time-fractional NWSE (CFTFNWSE)

$$^{CF}_{t_0}D^{\mu}_{t}u(x,t) = \frac{\partial^2 \mathrm{u(x,t)}}{\partial \mathrm{x}^2} + 2\mathrm{u(x,t)} - 3u^2(x,t), \qquad \mu \in (0,1] \tag{5.1}$$

with the initial condition

$$u(x,0) = \delta. \tag{5.2}$$

CFET is applied to Eq (5.1), and by employing Eq (5.2), then we obtain

$$^{CF}_{t_0}E_{\mu}[u(x,t)] - w^2u(x,0) - (1-\mu+\mu w)\,^{CF}_{t_0}E_{\mu}\left[\frac{\partial^2 u(x,t)}{\partial x^2} + 2u(x,t) - 3u^2(x,t)\right] = 0. \tag{5.3}$$

The nonlinear operator is defined by employing Eq (5.3):

$$N^1[\varphi(x,t;q)\,] = {}^{CF}_{t_0}E_\mu[\varphi(x,t;q)] - \delta w^2 - (1-\mu+\mu w)$$
$$\times\, {}^{CF}_{t_0}E_\mu\left[\frac{\partial^2\varphi(x,t;q)}{\partial x^2} + 2\varphi(x,t;q) - 3\varphi^2(x,t;q)\right]. \tag{5.4}$$

The m-th order deformation equation is defined by the application of the proposed algorithm:

$${}^{CF}_{t_0}E_\mu[u_m(x,t) - \hbar_m u_{m-1}(x,t)] = h\mathcal{R}_{1,m}[\vec{u}_{m-1}], \tag{5.5}$$

where

$$\mathcal{R}_{1,m}[\vec{u}_{m-1}(x,t)] = {}^{CF}_{t_0}E_\mu[\vec{u}_{m-1}\,(x,t)] - \delta w^2\left(1-\frac{k_m}{n}\right)$$
$$-(1-\mu+\mu w)\,{}^{CF}_{t_0}E_\mu\left[\frac{\partial^2 u_{m-1}\,(x,t)}{\partial x^2} + 2u_{m-1}(x,t) - 3\sum_{r=0}^{m-1} u_r\frac{\partial u_{m-1-r}}{\partial x}\right]. \tag{5.6}$$

By utilizing the ICFET to Eq (5.5), we obtain

$$u_m(x,t) = \hbar_m u_{m-1}(x,t) + h\left({}^{CF}_{t_0}E_\mu\right)^{-1}\left\{\mathcal{R}_{1,m}[\vec{u}_{m-1}\,(x,t)]\right\}. \tag{5.7}$$

By employing initial conditions, we obtain

$$u_0(x,t) = \delta. \tag{5.8}$$

To get the value of $u_1(x,t)$, we substitute $m=1$ into Eq (5.7), resulting in the following expression:

$$u_1(x,t) = -h(2\delta - 3\delta^2)(1-\mu+\mu t). \tag{5.9}$$

In a similar vein, by substituting $m=2$ into Eq (5.7), then we obtain the value for $u_2(x,t)$:

$$u_2(x,t) = (n+h)\big(-h(2\delta-3\delta^2)(1-\mu+\mu t)\big)$$
$$+h^2(2\delta-3\delta^2)(2-6\delta)[(1-\mu)^2 + 2\mu t - 2\mu^2 t + \mu^2 t^2)]. \tag{5.10}$$

By employing this approach, it is possible to identify the remaining terms. The solution of the CFTFNWSE is determined via the CFq-EHATM

$$u(x,t) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t)\left(\frac{1}{n}\right)^m. \tag{5.11}$$

By substituting $\mu=1,\ n=1,\ h=-1$ into Eq (5.11), we have that the resulting outcomes, denoted as $\sum_{m=1}^{M} u_m(x,t)\left(\frac{1}{n}\right)^m$ converge to the exact solutions $u(x,t) = \frac{-\frac{2}{3}\delta exp\,(2t)}{-\frac{2}{3}+\delta-\delta exp\,(2t)}$ of the CFTFNWSE when $M\to\infty$.

Figure 1 displays the 3D graphical representations of CFq-EHATM, the exact solution, and the absolute error for $u(x,t)$.

Figure 2 shows the two-dimensional graphical representations of CFq-EHATM for $u(x,t)$ solution and the exact solution for different μ values.

Table 1 shows the numerical solution of $u(x,t)$ obtained from the solution of CFTFNWSE with CFq-EHATM for different x, t and μ values.

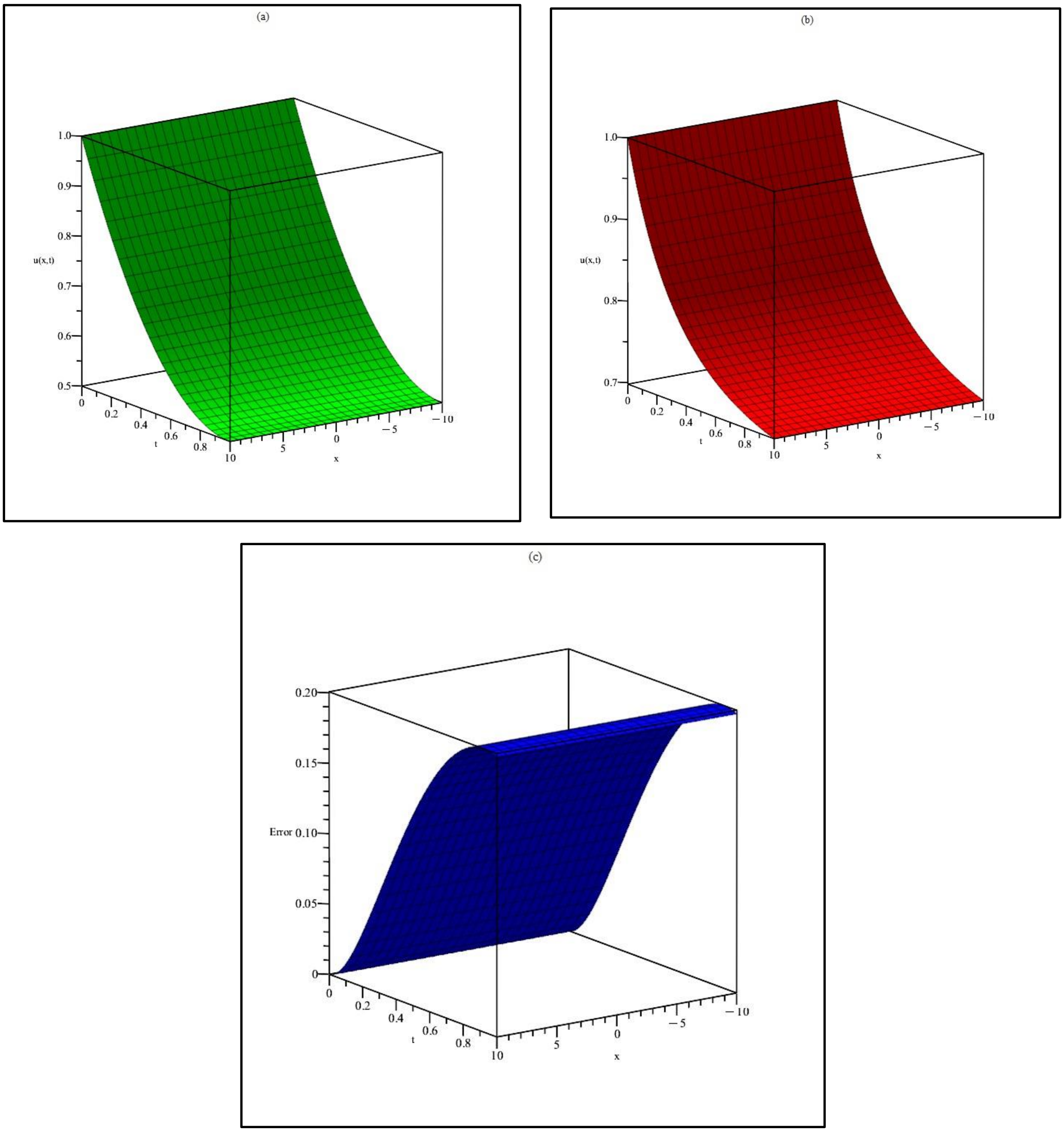

Figure 1. (a) Nature of CFq-EHATM solution $u(x,t)$, (b) Nature of exact solution $u(x,t)$, (c) Nature of absolute error=$|u_{exact} - u_{CFq-EHATM}|$ at $\delta = 1,\ h = -1,\ n = 1,\ \mu = 1$.

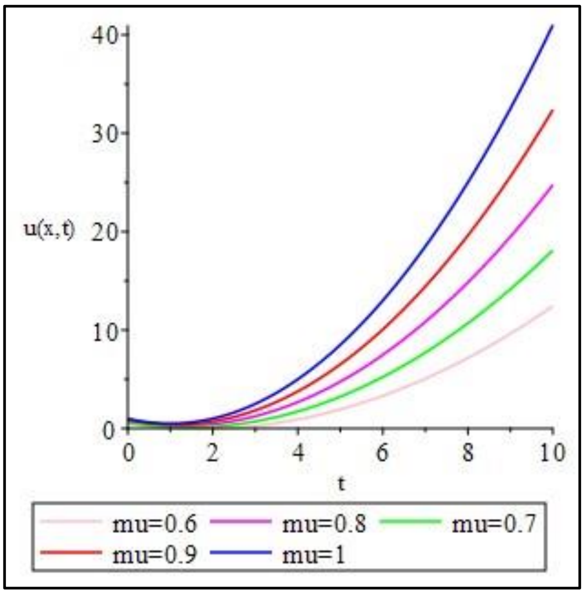

Figure 2. The comparison of the CFq-EHATM solutions for $u(x,t)$ when $\forall x \in \mathbb{R}, \delta = 1,\ h = -1, n = 1$ with different μ.

Table 1. Comparision ATHPM [55], CLDM [56] and CFq-EHATM solutions $u(x,t)$ for CFTFNWSE at $\delta = 1,\ \mu = 1,\ h = -1,\ n = 1$ with values of t.

t	$\lvert u_{exact} - u_{ATHPM} \rvert$	$\lvert u_{exact} - u_{CLDM} \rvert$	$\lvert u_{exact} - u_{CFq-EHATM} \rvert$
0.001	8.0×10^{-7}	7.5×10^{-7}	2.8×10^{-8}
0.002	1.8×10^{-6}	1.5×10^{-6}	1.1×10^{-7}
0.003	2.5×10^{-6}	2.2×10^{-6}	2.6×10^{-7}
0.004	3.5×10^{-6}	3.0×10^{-6}	4.6×10^{-7}
0.005	3.9×10^{-6}	3.7×10^{-6}	7.2×10^{-7}
0.006	4.8×10^{-6}	4.5×10^{-6}	1.0×10^{-7}
0.007	5.5×10^{-6}	5.2×10^{-6}	1.4×10^{-7}
0.008	6.3×10^{-6}	6.0×10^{-6}	1.8×10^{-7}
0.009	6.3×10^{-6}	6.0×10^{-6}	2.3×10^{-7}
0.010	7.8×10^{-6}	7.0×10^{-6}	2.9×10^{-7}

6. Results and discussion

Figure 1 shows the 3D graphs of the numerical solution obtained using CFq-EHATM, together with the exact solution and the absolute error between the Cq-FHATM solutions and the exact solution for the CFTFNWSE. Figure 2 shows the 2D graphs of the solutions $u(x,t)$ of the CFTFNWSE which derived using the CFq-EHATM for different μ values.

Figure 2 illustrates that as the μ value approaches one, the temperature $u(x,t)$ reaches a state of convergence. The absolute errors of the third-order CFq-EHATM solution were found and are presented in Table 1. According to Table 1, the absolute error experiences a substantial increase when both the values of the space variable x and the value of time t increases. Table 1 illustrates that the CFq-EHATM produces significantly more reliable outcomes compared to ATHPM and CLDM.

7. Conclusions

In this article, numerical solutions of the Caputo-Fabrizio fractional Newell-Whitehead-Segel equation were obtained for the first time with a new method, CF-qEHATM. It is essential to illustrate the influence of the fractional operator incorporated in the model being examined. Furthermore, the MAPLE software has been utilized to generate 2D and 3D graphs that visually represent the solutions to this system for different values of μ. The Maple software exhibits a significant range of variations in the fundamental structure of surface graphs produced for Eq (5.10). In addition, the MAPLE software was used to provide visual depictions of the numerical solutions of this system for $\mu = 1$. Upon evaluating CFTFNWSE, it becomes apparent that the overall configuration of surface graphs produced in Maple software differs. The numerical solutions for CFTFNWSE have been promptly and efficiently obtained. It is seen in Table 1 that the numerical results obtained with CFq-EHATM are better than the results found with ATHPM and CLDM in the current literature. It is concluded that the proposed CFq-EHATM is a more effective method than ATHPM and CLDM. Therefore, the proposed method can be used to obtain the new numerical solutions of Caputo-Fabrizio fractional partial differential equations.

Author contributions

Aslı Alkan: Conceptualization, Formal Analysis, Methodology, Software, Resources, Writing–original draft; Halil Anaç: Funding acquisition, Investigation, Supervision, Validation, Visualization, Writing–review & editing. All authors have read and agreed to the published version of the manuscript.

Conflict of interest

The authors declare no conflicts of interest.

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