Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 9(10): 27512–27534.
DOI: 10.3934/math.20241336
Received: 26 July 2024
Revised: 03 September 2024
Accepted: 13 September 2024
Published: 23 September 2024

Research article

An innovative algorithm based on weighted fuzzy soft multisets and its application in selecting optimal construction materials

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Abstract: Effective decision-making is critical across various domains, including technology, medicine, and engineering. To address the complexities of decision-making, particularly in scenarios involving both positive and negative parameters, this paper introduces an innovative algorithm based on weighted fuzzy soft multisets. This algorithm mitigates the issue of counterintuitive results often encountered in existing methods. By incorporating the concept of uniform fuzzy soft multisets and considering the conditional structure of these sets, our approach advances the theoretical framework of decision-making while providing a practical tool for complex scenarios. To demonstrate its practical applicability, we conduct a case study focused on selecting optimal construction materials for a building project, utilizing data from established engineering standards and a comprehensive wood properties database. The key findings of our sensitivity analysis highlight the algorithm's robustness to weight changes and adaptability to different decision sequences. These findings highlight the algorithm's potential to enhance decision support systems across various fields, such as engineering, healthcare, and environmental management. This potential is particularly valuable in complex, multi-criteria scenarios that demand nuanced, context-aware solutions.

Keywords: decision-making; uniform fuzzy soft multiset; weighted fuzzy soft multiset; construction material selection
Mathematics Subject Classification: 03E70, 03E72, 03E75

1. Introduction

Many problems across various fields, such as engineering, medical science, economics, machine learning, computer science, and information technology, involve a significant degree of uncertainty. One of the earliest theories for handling this vagueness is the fuzzy set theory, proposed by Zadeh [1] in 1965. Fuzzy set theory extends the concept of crisp sets, where an element either belongs or does not belong to a set, to fuzzy sets, where an element can have a degree of membership between 0 and 1.

This allows for a more nuanced representation of membership and has found applications in a wide range of fields, such as pattern recognition and medical diagnosis [2], cybersecurity [3], and visual monitoring [4]. Recent applications in decision-making include solutions for fuzzy systems of linear equations [5] and robot selection using interval-valued intuitionistic fuzzy information [6].

Similarly, the theory of soft sets [7] offers a mathematical framework for representing and manipulating vague information, distinguishing itself through its unique parameterization tools. This allows soft sets to be effectively applied in addressing problems. By enabling the selection of the desired form of parameter, the decision-making process is greatly simplified and becomes more efficient, even when faced with incomplete information.

Soft sets have various hybrid versions, including fuzzy soft sets [8], interval-valued soft sets [9], neutrosophic soft sets [10], and soft rough sets [11]. A fuzzy soft set is defined by combining soft sets and fuzzy sets. This entails attaching a degree of membership to the parameterization of the set. Fuzzy soft sets have applications in a broad spectrum of fields. Some of the most notable applications include supply chain management [12], ranking the order of software requirements [13], medical diagnosis [14], and classification of rock datasets in geology [15].

Roy and Maji [16] proposed a method for handling fuzzy soft set-based decision-making problems. However, Kong et al. [17] identified flaws in this approach and presented a revised algorithm. Similarly, Feng et al. [18] discussed the limitations of the Roy-Maji technique and offered an alternative method. In the same study, they also introduced the concept of weighted fuzzy soft sets, where values below the assigned weight have minimal influence on the optimal choice. Korkmaz et al. [19] developed a novel algorithm that incorporates both parameter weights and the presence of negative parameters in selection processes. They provided an illustrative example demonstrating how existing algorithms can yield counterintuitive results when membership values are close. To address this issue, they proposed a new approach that emphasizes dominant parameters.

Molodtsov [7] defines a soft set as a mapping that connects a set of parameters to a subset of a given universe. In other words, a soft set operates within a single universe and a single set of parameters. However, many practical scenarios involve multiple universes. To address this limitation, Alkhazaleh et al. [20] introduced the concept of a soft multiset as a solution for problems involving a multiset of universes. Subsequently, Salleh and Alkhazaleh [21] proposed an algorithm to solve decision-making problems based on this soft multiset framework.

Alkhazaleh et al. [22] defined fuzzy soft multisets as the combination of soft multisets and fuzzy sets. Furthermore, they proposed an algorithm for decision-making based on fuzzy soft multisets, where the score is calculated using Roy and Maji's [16] method. The theory of fuzzy soft multisets has attracted significant research interest, leading to various extensions and applications. For instance, Akın [23] explored the application of fuzzy soft multisets to algebraic structures, expanding the theoretical foundations of this approach. While Alkhazaleh et al. [22] proposed initial techniques for fuzzy soft multiset-based decision-making, Mukherjee and Das [24] argued that these techniques were inadequate and presented an improved algorithm based on Feng's method [18]. This highlights the ongoing refinement of decision-making approaches within this field. Further advancing the concept, Kandil et al. [25] investigated hesitant fuzzy soft multisets, introducing additional flexibility in decision-making processes. The theoretical aspects of these structures continue to evolve, as evidenced by Mukherjee and Das's [26] work on developing algebraic and topological structures on intuitionistic fuzzy soft multisets. Recognizing the importance of parameter weights in practical applications, Das [27] developed an adaptation of fuzzy soft multisets that incorporates these weights. The algorithm presented in [27] follows Feng's [18] method for calculating weighted choice values, utilizing the weight as a threshold.

1.1. Motivation and research gap

While these approaches are valuable, they each have limitations. Specifically, Das's algorithm, which follows Feng's method, can yield choices that are inconsistent with expectations in certain situations. Korkmaz et al. [19] provided an example of this, demonstrating how Feng's method may not align with intuitive outcomes, especially when membership values are clustered closely together. Furthermore, these algorithms do not adequately account for cases where a parameter of a fuzzy soft multiset contains both positive and negative components.

Consider a scenario in autonomous vehicle development where a team must select a sensor suite for an autonomous car. They consider three universes: the set of light detection and ranging (LIDAR) sensors including mechanical scanning LIDAR, solid-state LIDAR, and flash LIDAR; the set of radar sensors including Doppler radar, frequency-modulated continuous-wave (FMCW) radar, and phased-array radar; and the set of camera systems including monocular cameras, stereo cameras, and omnidirectional cameras. Each universe has its own set of parameters: range, resolution, and susceptibility to weather interference for LIDAR sensors; accuracy, angular coverage, and susceptibility to interference for radar sensors; and image resolution, field of view, and computational requirements for camera systems. Suppose the decision-making team creates criteria for evaluating sensor suites. These criteria consist of triples composed of components from each set of parameters, such as weather interference for LIDAR, range for radar, and computational requirements for cameras. Note that "susceptibility to weather interference" is a negative parameter since high susceptibility means sensors are more affected by adverse conditions, reducing their reliability. Thus, the criterion consisting of weather interference susceptibility, range, and computational requirements contains both positive and negative components.

The selection of optimal construction materials is a complex and critical decision-making process in the building industry, requiring a careful balance between multiple, often conflicting, criteria. Engineers and project managers must consider a wide range of materials, including various types of concrete, steel, and wood, each possessing its own unique set of properties and trade-offs. Quantifying and comparing these diverse attributes, which encompass both qualitative and quantitative aspects, presents a significant challenge. Suboptimal material selection can have severe consequences, potentially leading to increased costs, a reduced building lifespan, environmental degradation, or even structural failures. Therefore, the development of robust and flexible decision-making tools is crucial to ensure the safety, efficiency, and sustainability of building projects.

Several researchers have applied various decision-making techniques to address this challenge. Obradović and Pamučar [28] presented a novel multi-criteria model for predicting and evaluating building materials. Their model utilizes a fuzzy logic system where all input parameters are defined by fuzzy sets. Bhuiyan and Hammad [29] integrated technical aspects with the economic, social, and environmental pillars of sustainability. They employed fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), fuzzy VIKOR (VIseKriterijumska Optimizacija I Kompromisno Resenje — Multicriteria Optimization and Compromise Solution), and Shannon's entropy to select the most sustainable structural material. Al-Atesh et al. [30] developed an AHP (Analytic Hierarchy

Process) decision-making model to determine the optimal choice between steel and reinforced concrete building structures.

This study aims to advance the state-of-the-art in fuzzy soft multiset-based decision-making by addressing the limitations of existing approaches. We introduce the notion of a "uniform fuzzy soft multiset" to handle parameters containing both positive and negative components. Furthermore, we present a novel algorithm derived from the method discussed in [19] that facilitates a wide range of decision-making problems using fuzzy soft multisets. To demonstrate its practical applicability, we apply our algorithm to the selection of optimal construction materials, evaluating different types of concrete, steel, and wood to determine the most suitable materials for a building project. This application showcases the algorithm's versatility in real-world scenarios. Moreover, we conduct a sensitivity analysis, confirming the algorithm's robustness to weight changes and its adaptability to different decision sequences.

The remainder of this paper is structured as follows: Section 2 provides an overview of the theoretical background. Section 3 details the main contribution of this paper, introducing a novel algorithm for decision-making procedures that incorporates fuzzy soft multisets. Section 4 presents a practical application of the algorithm, highlighting its utility in real-world scenarios. Section 5 compares our proposed algorithm with an existing approach. Section 6 investigates the algorithm's sensitivity to variations in criteria weights and the order of decision-making. Section 7 summarizes the key contributions and advantages of the proposed decision-making algorithm. Finally, Section 8 concludes the paper and suggests directions for future research.

2. Preliminaries

This section lays the groundwork for subsequent sections by introducing key concepts and results. We begin by revisiting the fundamental notions of fuzzy sets, soft sets, fuzzy soft sets, and fuzzy soft multisets.

Definition 2.1. [1] *A fuzzy set $\tilde{A}$ on a crisp set X can be expressed as the set of pairs $\{x/\mu_{\tilde{A}}(x) : x \in X\}$, where $\mu_{\tilde{A}}(x)$ represents the membership degree assigned to each element $x \in X$.*

In the following discussion, let U denote the universe of discourse and E the set of relevant parameters.

Definition 2.2. [7] *A soft set is defined as a parameterized collection of subsets of a universe U. Formally, it is the set of ordered pairs $\{(e, F(e)) : e \in E\}$, where $F : E \to \mathscr{P}(U)$ is a function mapping each parameter $e \in E$ to a subset $F(e) \subseteq U$.*

Definition 2.3. [8] *Let $FS(U)$ represent the set of all possible fuzzy sets of U, and let $A \subseteq E$. A fuzzy soft set over U is a pair (F,A), where $F : A \to FS(U)$ is a function. For $e \in A$, $F(e) = \{(u, \mu(e,u)) : u \in U\}$ represents the fuzzy approximate values for parameter e, where $\mu(e,u)$ denotes the membership value of u associated with the parameter e.*

Definition 2.4. [18] *Given a fuzzy soft set (F,A), a weighted fuzzy soft set is a triple (F,A,w), where $w : A \to [0,1]$ is a weight function assigning a degree of importance to each attribute $e \in A$.*

When dealing with decision-making problems involving both negative and positive parameters, the initial step is to establish a uniform fuzzy soft set representation. This is achieved by applying the complement to membership values associated with negative parameters, as suggested by [31]. Subsequently, the decision-making procedure operates on this uniform representation. This paper adopts Zadeh's definition of the complement, denoted as $c(x) = 1 - x$.

Definition 2.5. [22] *Let $\{U_i : i \in I\}$ be a collection of disjoint universes (i.e., $\bigcap U_i = \emptyset$), and let $\{E_{U_i} : i \in I\}$ be the corresponding collection of parameter sets. Let $E = \Pi_{i\in I} E_{U_i}$ be the cartesian product of parameter sets, and let $U = \Pi_{i\in I} FS(U_i)$ be the cartesian product of the families of fuzzy subsets of each universe. Then, a fuzzy soft multiset over U is a pair (F,A), where $A \subseteq E$ and $F : A \to U$ is a mapping.*

Example 2.6. *Consider a senario where a city is trying to implement a smart waste management system using artificial intelligence (AI) technology. This system aims to optimize waste collection routes, improve recycling rates, and reduce overall environmental impact. They need to choose the best combination of an AI algorithm, a sensor type, and a data management platform. Let us define the universes:*

$U_1 = \{m_1 = \text{Machine Learning Algorithm}, m_2 = \text{Deep Learning Algorithm}, m_3 = \text{Reinforcement Learning Algorithm}\}$,

$U_2 = \{s_1 = \text{Ultrasonic Sensors}, s_2 = \text{Optical Sensors}, s_3 = \text{Weight Sensors}\}$,

$U_3 = \{d_1 = \text{Cloud-based Platform}, d_2 = \text{Edge Computing Platform}, d_3 = \text{Hybrid Platform}\}$,

and the corresponding parameter sets:

$E_{U_1} = \{e^1_{U_1} = \text{accuracy}, e^2_{U_1} = \text{energy efficiency}, e^3_{U_1} = \text{bias mitigation}\}$,

$E_{U_2} = \{e^1_{U_2} = \text{ reliability}, e^2_{U_2} = \text{cost}, e^3_{U_2} = \text{lifespan}\}$,

$E_{U_3} = \{e^1_{U_3} = \text{data security}, e^2_{U_3} = \text{scalability}, e^3_{U_3} = \text{latency}\}$.

Let $E = \Pi_{i\in I} E_{U_i}$ *and* $U = \Pi^3_{i=1} FS(U_i)$ *and,*

$A = \{a_1 = (e^1_{U_1}, e^1_{U_2}, e^1_{U_3}), a_2 = (e^2_{U_1}, e^2_{U_2}, e^2_{U_3}), a_3 = (e^3_{U_1}, e^2_{U_2}, e^3_{U_3}), a_4 = (e^1_{U_1}, e^2_{U_2}, e^3_{U_3})\}$.

Then the pair (F,A) is called a fuzzy soft multiset.

In fuzzy soft multiset theory, a weight function can be defined, similar to fuzzy soft set theory, to assign a degree of importance to each parameter.

Definition 2.7. [27] *Given a fuzzy soft multiset (F,A), a weighted fuzzy soft multiset is defined as a triple (F,A,w), where $w : A \to [0,1]$ is a weight function and $w_i = w(a_i)$ represents the weight assigned to each parameter $a_i \in A$.*

Example 2.8. *Consider entering a cooking competition qualifier where you're challenged to create a dish using one fruit, one vegetable, and one protein source. The competition emphasizes sustainable and healthy cuisine, encouraging the use of locally sourced, seasonal ingredients. Judges will evaluate dishes based on nutritional value, flavor profile, and environmental impact.*

Let $U_1 = \{f_1, f_2, f_3\}$, $U_2 = \{v_1, v_2, v_3, v_4\}$ and $U_3 = \{p_1, p_2, p_3, p_4, p_5\}$ be the sets of fruits, vegetables and protein resources, respectively. The parameter sets associated with U_1, U_2, and U_3 are as follows:

$E_{U_1} = \{e^1_{U_1} = \text{allergenicity}, e^2_{U_1} = \text{nutritional value}, e^3_{U_1} = \text{glycemic index}, e^4_{U_1} = \text{versatility}\}$,

$E_{U_2} = \{e^1_{U_2} = \text{nutritional value}, e^2_{U_2} = \text{availability}, e^3_{U_2} = \text{price}\}$,

$E_{U_3} = \{e^1_{U_3} = \text{nutritional value}, e^2_{U_3} = \text{versatility}, e^3_{U_3} = \text{price}, e^4_{U_3} = \text{environmental impact}\}$.

Let $E = \Pi_{i\in I} E_{U_i}$ *and* $U = \Pi^3_{i=1} FS(U_i)$. *Consider the following subset of parameters:*

$A = \{a_1 = (e^1_{U_1}, e^1_{U_2}, e^1_{U_3}), a_2 = (e^2_{U_1}, e^1_{U_2}, e^2_{U_3}), a_3 = (e^3_{U_1}, e^3_{U_2}, e^3_{U_3}), a_4 = (e^4_{U_1}, e^2_{U_2}, e^4_{U_3})\}$.

Furthermore, assume that the judges assign the following weights to each parameter tuple in A: $w_1 = 0.3$, $w_2 = 0.4$, $w_3 = 0.5$, *and* $w_4 = 0.6$.

The tabular representation of the weighted fuzzy soft multiset (F,A,w) *is shown in Table 1.*

Table 1. Tabular representation of (F,A,w) given in Example 2.8.

U_i	$a_1, w_1 = 0.3$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.6$
U_1				
f_1	0.1	0.7	0.4	0.6
f_2	0.7	0.2	0.2	0.4
f_3	0.2	0.6	0.3	0.6
U_2				
v_1	0.2	0.8	0.4	0.3
v_2	0.6	0.1	0.2	0.4
v_3	0.1	0.6	0.3	0.9
v_4	0.3	0.5	0.3	0.6
U_3				
p_1	0.3	0.7	0.5	0.1
p_2	0.3	0.6	0.2	0.8
p_3	0.6	0.2	0.5	0.4
p_4	0.2	0.7	0.1	0.5
p_5	0.8	0.2	0.3	0.6

Importantly, membership values within a fuzzy soft multiset are conditionally related. This means that the characteristics of one material can influence preferences for characteristics of other materials within the same decision criteria.

Definition 2.9. [27] *Given a weighted fuzzy soft multiset* (F,A,w)*, the triple* $(e^j_{U_i}, F(e^j_{U_i}), w)$ *is called the* U_i*-part of* (F,A,w) *for all* $e^j_{U_i} \in a$*, where* $a \in A$*.*

Tables 2, 3, and 4 show the U_1, U_2, and U_3-parts, respectively, of the weighted fuzzy soft multiset presented in Example 2.8.

Das [27] introduced an adjustable decision-making method based on a weighted fuzzy soft multiset model. This algorithm utilizes Feng's method for each U_i-part of the weighted fuzzy soft multiset, allowing for the selection of an optimal object s_{k_i} for each U_i-part. The final decision is then represented as a tuple of objects $(s_{k_1}, s_{k_2}, \ldots, s_{k_n})$. However, it's important to note that the author only considered positive attributes in this decision-making process.

Table 2. U_1-part of (F,A,w) given in Example 2.8.

U_1	$a_1, w_1 = 0.3$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.6$
f_1	0.1	0.7	0.4	0.6
f_2	0.7	0.2	0.2	0.4
f_3	0.2	0.6	0.3	0.6

Table 3. U_2-part of (F,A,w) given in Example 2.8.

U_2	$a_1, w_1 = 0.3$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.6$
v_1	0.2	0.8	0.4	0.3
v_2	0.6	0.1	0.2	0.4
v_3	0.1	0.6	0.3	0.9
v_4	0.3	0.5	0.3	0.6

Table 4. U_3-part of (F,A,w) given in Example 2.8.

U_3	$a_1, w_1 = 0.3$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.6$
p_1	0.3	0.7	0.5	0.1
p_2	0.3	0.6	0.2	0.8
p_3	0.6	0.2	0.5	0.4
p_4	0.2	0.7	0.1	0.5
p_5	0.8	0.2	0.3	0.6

In contrast, Korkmaz et al. [19] demonstrated that Feng's algorithm can sometimes lead to counterintuitive choices. They addressed this limitation by considering both positive and negative parameters in their approach. Furthermore, they presented the algorithm illustrated in Figure 1, which we will refer to as Algorithm 1 throughout this paper.

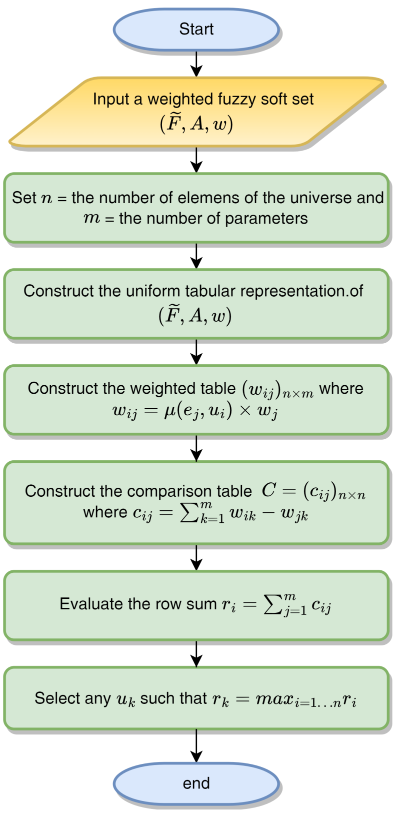

Figure 1. Flowchart of Algorithm 1.

Before presenting the proposed algorithm, it is essential to understand its theoretical foundation and its suitability for the construction material selection problem. The integration of fuzzy logic with soft set theory provides a powerful framework for modeling complex, real-world decision-making scenarios. Fuzzy soft sets, in particular, allow for the representation of both the parameterized nature of criteria and the inherent fuzziness in their evaluation. This combination is particularly advantageous for the multi-criteria decision-making problem at hand for several key reasons.

First, it enables the inclusion of both qualitative and quantitative criteria, crucial for capturing the diverse factors involved in construction material selection, such as material properties, costs, and environmental impacts. Second, the fuzzy aspect facilitates the handling of imprecise or subjective assessments, which are common in real-world evaluations. Third, the inherent flexibility of the soft set structure allows for the addition or removal of parameters without requiring a complete model overhaul, a valuable feature in dynamic decision environments. Furthermore, the extension to fuzzy soft multisets, as employed in this study, enhances the model's ability to handle multiple universes of discourse simultaneously, such as different types of construction materials.

This approach was chosen primarily because it effectively captures the multifaceted nature of the decision problem. This allows for a more nuanced and comprehensive evaluation that accurately reflects the complexities of real-world decision-making in fields like construction. This allows for a more informed and robust selection process, where multiple, often conflicting criteria can be effectively balanced to arrive at an optimal solution.

3. A novel approach to a fuzzy soft multiset-based decision-making problems

This section introduces a novel algorithm for solving fuzzy soft multiset-based decision-making problems. This algorithm builds upon and extends the approach presented in Algorithm 1.

As highlighted by Alcandut et al. [31], real-world problems often involve both positive and negative parameters. However, it's crucial to recognize that a given parameter tuple $a \in A$ may not consist solely of negative or positive components. This makes it challenging to definitively categorize the parameter tuple as strictly negative or positive. For instance, in Example 2.8, $e^1_{U_1}$, $e^3_{U_1}$, $e^3_{U_2}$, $e^3_{U_3}$, and $e^4_{U_3}$ represent negative parameters, while the remaining parameters are positive. Notably, the parameter tuples a_1, a_3, and a_4 contain both positive and negative attributes as components. Consequently, the complement of a fuzzy soft multiset, as defined in [22], is insufficient to address this scenario. This challenge necessitates the introduction of a new concept—the uniform fuzzy soft multiset.

Uniform fuzzy soft multiset: A uniform fuzzy soft multiset is derived from a weighted fuzzy soft multiset by applying the complement to the portions of the U_i-parts associated with negative parameters. It is denoted by (F_u, A, w).

Table 5 illustrates the uniform tabular representation of the fuzzy soft multiset presented in Example 2.8.

It is crucial to recognize that the evaluation of choices for each set within a fuzzy soft multiset requires an interconnected rather than independent approach due to the conditional relationships between membership values. In other words, determining the optimal choice for each part of the fuzzy soft multiset in isolation is not accurate. The optimal choice made within one universe set should influence the choices within other universe sets.

Considering these factors, we now present a novel algorithm for fuzzy soft multiset-based decision-making problems, as illustrated in Figure 2. This algorithm, which we will refer to as Algorithm 2 throughout this paper, addresses the limitations of previous approaches by incorporating the interconnectedness of choices within a fuzzy soft multiset.

Table 5. Uniform tabular representation of (F, A, w) given in Example 2.8.

U_i	$a_1, w_1 = 0.3$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.6$
U_1				
f_1	0.9	0.7	0.6	0.6
f_2	0.3	0.2	0.8	0.4
f_3	0.8	0.6	0.7	0.6
U_2				
v_1	0.2	0.8	0.6	0.3
v_2	0.6	0.1	0.8	0.4
v_3	0.1	0.6	0.7	0.9
v_4	0.3	0.5	0.7	0.6
U_3				
p_1	0.3	0.7	0.5	0.9
p_2	0.3	0.6	0.8	0.2
p_3	0.6	0.2	0.5	0.6
p_4	0.2	0.7	0.9	0.5
p_5	0.8	0.2	0.7	0.4

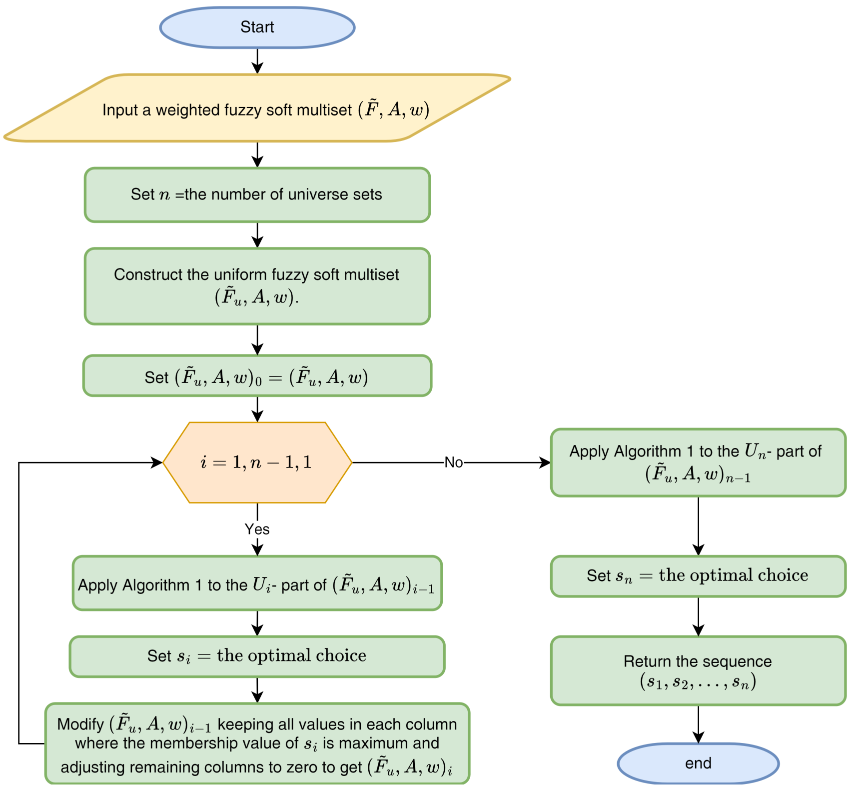

Figure 2. Flowchart of Algorithm 2.

This novel algorithm serves as a generalization of the approach outlined in [19], enabling the solution of a wider range of problems in fuzzy soft multiset-based decision-making. Notably, it effectively handles scenarios involving both positive and negative parameters of varying significance, addressing the issue of counterintuitive results often encountered when using existing methods. Furthermore, this algorithm explicitly considers the conditional structure of fuzzy soft multisets, ensuring that choices made from the universal sets are interdependent rather than independent.

4. Application of the algorithm

To demonstrate the practical application of Algorithm 2, we will now apply it to a numerical example.

A construction company aims to evaluate different types of concrete, steel, and wood to select the most suitable materials for an upcoming building project. They identify three potential universes of discourse:
$U_1 = \{c_1 = C30, c_2 = C35, c_3 = C40\}$,
$U_2 = \{s_1 = S420, s_2 = S500, s_3 = S600\}$, and
$U_3 = \{m_1 = \text{European beech}, m_2 = \text{sessile oak}, m_3 = \text{Scots pine}, m_4 = \text{Eastern red cedar}\}$.

Each universe has its own set of parameters denoted by E_{U_i}.
$E_{U_1} = \{e^1_{U_1} = \text{mean tensile strength}, e^2_{U_1} = \text{elastic modulus}, e^3_{U_1} = \text{price}\}$,
$E_{U_2} = \{e^1_{U_2} = \text{yield strength}, e^2_{U_2} = \text{ultimate strength}\}$,
$E_{U_3} = \{e^1_{U_3} = \text{modulus of elasticity}, e^2_{U_3} = \text{volumetric shrinkage}, e^3_{U_3} = \text{Janka hardness}\}$.

Note that the price for concrete $(e^3_{U_1})$ and volumetric shrinkage for wood $(e^2_{U_3})$ are negative parameters, while all other parameters are positive.

Let $A = \{a_1, a_2, a_3, a_4, a_5\}$, where $a_1 = (e^1_{U_1}, e^1_{U_2}, e^1_{U_3})$, $a_2 = (e^1_{U_1}, e^2_{U_2}, e^3_{U_3})$, $a_3 = (e^3_{U_1}, e^1_{U_2}, e^1_{U_3})$, $a_4 = (e^1_{U_1}, e^1_{U_2}, e^2_{U_3})$, $a_5 = (e^3_{U_1}, e^2_{U_2}, e^3_{U_3})$, and $a_6 = (e^3_{U_1}, e^2_{U_2}, e^2_{U_3})$.

Assume that the company determines the weights for a_1, a_2, a_3, a_4, a_5, and a_6 as $w_1 = 0.4$, $w_2 = 0.4$, $w_3 = 0.5$, $w_4 = 0.4$, $w_5 = 0.2$, and $w_6 = 0.3$, respectively. Figure 3 illustrates the methodology employed in this study. The process begins with data collection from established engineering standards and material property databases. The collected data is then processed and normalized, and a fuzzy soft multiset is constructed to represent the alternatives and parameters. Algorithm 2 is subsequently applied to this fuzzy soft multiset to determine the optimal material selection. Finally, sensitivity and feasibility analyses are conducted to evaluate the practicality and robustness of the proposed algorithm.

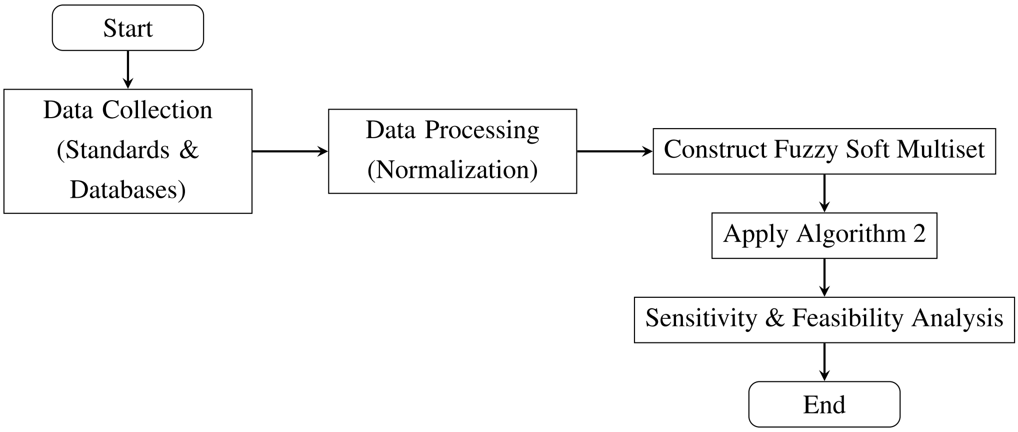

Figure 3. Flowchart dof methodology.

In this study, we normalize data values to the interval $[0,1]$ using the formula $\mu(e_j,u_i) = \dfrac{c-a}{b-a}$, where a is the minimum value of the parameter under consideration, b is the maximum value of the parameter under consideration, and c is the actual value of the parameter for a given alternative. This formula essentially scales the data values linearly so that the minimum value maps to 0 and the maximum value maps to 1.

The maximum and minimum values used for normalization are determined based on the highest and lowest values of the parameters utilized in the study, as specified by relevant standards and a comprehensive material property database.

For calculating membership values for elements in U_1, we refer to EN 1992-1-1:2023, which provides specifications for concrete design properties [32]. For mean tensile strength, the maximum and minimum values are 5.04 megapascal (MPa) and 1.57 MPa, respectively. The mean tensile strengths for C30, C35, and C40 are 2.9 MPa, 3.21 MPa, and 3.51 MPa. For elastic modulus, the maximum and minimum values are 43,631 MPa and 27,085 MPa, respectively. The corresponding values for C30, C35, and C40 are 32,837 MPa, 34,077 MPa, and 35,220 MPa. Finally, for price per cubic meter, the maximum and minimum values are 2500 Turkish Liras (TRY) and 1440 TRY (based on 2023 prices). The corresponding prices for C30, C35, and C40 are 1670 TRY, 1790 TRY, and 2000 TRY. The membership values of C30, C35, and C40 with respect to each parameter are tabulated in Table 6.

Table 6. Membership values for concrete.

U_2	$e^1_{U_1}$	$e^2_{U_1}$	$e^3_{U_1}$
c_1	0.38	0.35	0.22
c_2	0.47	0.42	0.33
c_3	0.56	0.49	0.53

Following a similar procedure for steel, we refer to the EN 1993-1-1:2022 standard for specifications on hot rolled non-alloy structural steels [33]. This standard provides valuable information regarding yield strength and ultimate tensile strength for structural steels with nominal thicknesses less than 40 mm for various steel grades. According to the standard, the minimum and maximum yield strength values are 235 MPa and 700 MPa, respectively. With these bounds, the membership values for S420, S500, and S600 are calculated as 0.40, 0.57, and 0.78, respectively, based on their respective yield strengths of 420 MPa, 500 MPa, and 600 MPa.

Similarly, for ultimate tensile strength, the maximum value is 770 MPa, and the minimum value is 360 MPa. Using these values, the membership values for S420, S500, and S600, with ultimate strengths of 510 MPa, 580 MPa, and 650 MPa, are calculated as 0.37, 0.54, and 0.71, respectively.

To determine the values for relevant parameters of preferred wood species, we utilize a comprehensive database containing the mechanical and physical properties of over 600 wood species [34]. We consider three key parameters for our evaluation: modulus of elasticity, volumetric shrinkage, and Janka hardness. The modulus of elasticity values for European beech, sessile oak, Scots pine, and Eastern red cedar were 14.31 gigapascal (GPa), 10.47 GPa, 10.08 GPa, and 6.07 GPa, respectively. Their corresponding volumetric shrinkage values are 17.9%, 14.2%, 13.6%, and 7.8%, while their Janka hardness values are 6460 Newton (N), 4990 N, 2420 N, and 4000 N, respectively. Moreover, the maximum and minimum values for modulus of elasticity are 31.14 GPa and 3.71 GPa, respectively. For volumetric shrinkage, the maximum and minimum values are 21% and 5.4%, while for Janka hardness, they are 20595 N and 298 N. The resulting normalized membership values are presented in Table 7.

Table 7. Membership values for wood.

U_2	$e_{U_3}^1$	$e_{U_3}^2$	$e_{U_3}^3$
m_1	0.39	0.80	0.30
m_2	0.25	0.56	0.23
m_3	0.23	0.53	0.10
m_3	0.09	0.15	0.18

Table 8 presents the fuzzy soft multiset (F,A,w) derived from the membership values of all objects.

Table 8. Tabular representation of (F,A,w).

U_i	$a_1, w_1 = 0.4$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.4$	$a_5, w_5 = 0.2$	$a_6, w_6 = 0.3$
U_1						
c_1	0.38	0.38	0.22	0.35	0.22	0.22
c_2	0.47	0.47	0.33	0.42	0.33	0.33
c_3	0.56	0.56	0.53	0.49	0.53	0.53
U_2						
s_1	0.40	0.37	0.40	0.40	0.37	0.37
s_2	0.57	0.54	0.57	0.57	0.54	0.54
s_3	0.78	0.71	0.78	0.78	0.71	0.71
U_3						
m_1	0.39	0.30	0.39	0.80	0.30	0.80
m_2	0.25	0.23	0.25	0.56	0.23	0.56
m_3	0.23	0.10	0.23	0.53	0.10	0.53
m_4	0.09	0.18	0.09	0.15	0.18	0.15

By applying the complement to the portions of the U_i-parts associated with negative parameters, we can now obtain a uniform fuzzy soft multiset. Table 9 presents the tabular representation of (F_u, A, w). Specifically, for concrete (U_1), the price parameter ($e^3_{U_1}$) is transformed, affecting the parameter tuples a_3, a_5, and a_6. For wood (U_3), the volumetric shrinkage parameter ($e^2_{U_3}$) is transformed, impacting the parameter tuples a_4 and a_6. No transformations are required for the steel parameters (U_2), as they are all positive.

Table 9. Tabular representation of (F_u, A, w).

U_i	$a_1, w_1 = 0.4$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.4$	$a_5, w_5 = 0.2$	$a_6, w_6 = 0.3$
U_1						
c_1	0.38	0.38	0.78	0.35	0.78	0.78
c_2	0.47	0.47	0.67	0.42	0.67	0.67
c_3	0.56	0.56	0.47	0.49	0.47	0.47
U_2						
s_1	0.40	0.37	0.40	0.40	0.37	0.37
s_2	0.57	0.54	0.57	0.57	0.54	0.54
s_3	0.78	0.71	0.78	0.78	0.71	0.71
U_3						
m_1	0.39	0.30	0.39	0.20	0.30	0.20
m_2	0.25	0.23	0.25	0.44	0.23	0.44
m_3	0.23	0.10	0.23	0.47	0.10	0.47
m_4	0.09	0.18	0.09	0.85	0.18	0.85

Next, we apply Algorithm 1 to the U_1-part of (F_u, A, w) to determine the optimal choice for concrete. The weighted and comparison tables for the U_1-part are presented in Tables 10 and 11, respectively. The weighted table is constructed by multiplying the membership values of each concrete type with the corresponding weights assigned to each parameter combination, providing a quantitative representation of the weighted preferences for each concrete type. Subsequently, the comparison table is generated based on the weighted table. Each entry c_{ij} in the comparison table reflects the degree to which the ith concrete type is preferred over the jth concrete type. This preference degree is calculated by summing the differences in weighted values for each parameter combination between the two concrete types being compared.

Table 10. Weighted table of U_1-part of (F_u, A, w).

U_i	$a_1, w_1 = 0.4$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.4$	$a_5, w_5 = 0.2$	$a_6, w_6 = 0.3$
c_1	0.15	0.15	0.39	0.14	0.16	0.23
c_2	0.19	0.19	0.34	0.17	0.13	0.20
c_3	0.22	0.22	0.24	0.20	0.09	0.14

Table 11. Comparison table of U_1-part of (F_u, A, w).

U_1	c_1	c_2	c_3
c_1	0	0.01	0.11
c_2	-0.01	0	0.10
c_3	-0.11	-0.10	0

Following the construction of the comparison table, we evaluate the row sums. For each concrete type c_i, the sum of the entries in its corresponding row (r_i) is calculated. This row sum (r_i) serves as a measure of the overall preference for c_i compared to the other options. The concrete type with the highest row sum is then selected as the optimal choice within the U_1 universe, indicating the most preferred option based on the weighted evaluation across all relevant parameters. In this case, c_1 is chosen as the optimal choice, as it exhibits the highest row sum of 0.12, while c_2 and c_3 have choice values of 0.09 and -0.21, respectively.

Next, we modify the uniform fuzzy soft multiset (F_u, A, w) to reflect the selection of c_1 as the optimal choice for concrete. We denote the initial uniform fuzzy soft multiset as $(F_u, A, w)_0$. In the modified version, denoted by $(F_u, A, w)_1$, we retain all values in each column where the membership value of c_1 is maximum. Simultaneously, we set the values in the remaining columns to zero. This modified fuzzy soft multiset is shown in Table 12.

Table 12. Tabular representation of $(F_u, A, w)_1$.

U_i	$a_1, w_1 = 0.4$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.4$	$a_5, w_5 = 0.2$	$a_6, w_6 = 0.3$
U_1						
c_1	0.38	0.38	0.78	0.35	0.78	0.78
c_2	0.47	0.47	0.67	0.42	0.67	0.67
c_3	0.56	0.56	0.47	0.49	0.47	0.47
U_2						
s_1	0	0	0.40	0	0.37	0.37
s_2	0	0	0.57	0	0.54	0.54
s_3	0	0	0.78	0	0.71	0.71
U_3						
m_1	0	0	0.39	0	0.30	0.20
m_2	0	0	0.25	0	0.23	0.44
m_3	0	0	0.23	0	0.10	0.47
m_4	0	0	0.09	0	0.18	0.85

Having determined the optimal choice for concrete, we now shift our attention to the steel universe. We apply Algorithm 1 to the U_2-part of $(F_u, A, w)_1$, as shown in Table 13.

Table 13. U_2-part of $(F_u, A, w)_1$.

U_2	$a_1, w_1 = 0.4$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.4$	$a_5, w_5 = 0.2$	$a_6, w_6 = 0.3$
s_1	0	0	0.40	0	0.37	0.37
s_2	0	0	0.57	0	0.54	0.54
s_3	0	0	0.78	0	0.71	0.71

Applying Algorithm 1 to the U_2-part results in the weighted and comparison tables shown in Tables 14 and 15, respectively. These tables provide a structured framework for evaluating the steel types based on their weighted performance across the relevant parameters.

Table 14. Weighted table of U_2-part of $(F_u, A, w)_1$.

U_2	$a_1, w_1 = 0.4$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.4$	$a_5, w_5 = 0.2$	$a_6, w_6 = 0.3$
s_1	0	0	0.20	0	0.07	0.11
s_2	0	0	0.29	0	0.11	0.16
s_3	0	0	0.39	0	0.14	0.21

Table 15. Comparison table of U_2-part of $(F_u, A, w)_1$.

U_2	s_1	s_2	s_3
s_1	0	-0.17	-0.36
s_2	0.17	0	-0.19
s_3	0.36	0.19	0

Based on the calculated row sums ($r_1 = -0.53$, $r_2 = -0.02$, and $r_3 = 0.55$), we select s_3 as the optimal choice for steel. To proceed, we further modify the fuzzy soft multiset. We derive $(F_u, A, w)_2$ from $(F_u, A, w)_1$ by retaining the membership values associated with the selected optimal elements from the U_1 and U_2 parts and setting the remaining column values to zero. The tabular representation of $(F_u, A, w)_2$ is shown in Table 16. The resulting U_3-part of $(F_u, A, w)_2$ is shown in Table 17.

Table 16. Tabular representation of $(F_u, A, w)_2$.

U_i	$a_1, w_1 = 0.4$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.4$	$a_5, w_5 = 0.2$	$a_6, w_6 = 0.3$
U_1						
c_1	0.38	0.38	0.78	0.35	0.78	0.78
c_2	0.47	0.47	0.67	0.42	0.67	0.67
c_3	0.56	0.56	0.47	0.49	0.47	0.47
U_2						
s_1	0	0	0.40	0	0.37	0.37
s_2	0	0	0.57	0	0.54	0.54
s_3	0	0	0.78	0	0.71	0.71
U_3						
m_1	0	0	0.39	0	0.30	0.20
m_2	0	0	0.25	0	0.23	0.44
m_3	0	0	0.23	0	0.10	0.47
m_4	0	0	0.09	0	0.18	0.85

Table 17. U_3-part of $(F_u, A, w)_2$.

U_3	$a_1, w_1 = 0.4$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.4$	$a_5, w_5 = 0.2$	$a_6, w_6 = 0.3$
m_1	0	0	0.39	0	0.30	0.20
m_2	0	0	0.25	0	0.23	0.44
m_3	0	0	0.23	0	0.10	0.47
m_4	0	0	0.09	0	0.18	0.85

Subsequently, we construct the weighted and comparison tables for the U_3-part of $(F_u, A, w)_2$, presented in Tables 18 and 19, respectively.

Table 18. Weighted table of U_3-part of $(F_u, A, w)_2$.

U_3	$a_1, w_1 = 0.4$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.4$	$a_5, w_5 = 0.2$	$a_6, w_6 = 0.3$
m_1	0	0	0.20	0	0.06	0.06
m_2	0	0	0.13	0	0.05	0.13
m_3	0	0	0.12	0	0.02	0.14
m_4	0	0	0.05	0	0.04	0.26

Table 19. Comparison table of U_3-part of $(F_u, A, w)_2$.

U_2	m_1	m_2	m_3	m_4
m_1	0	0.01	0.04	-0.02
m_2	-0.01	0	0.03	-0.03
m_3	-0.04	-0.03	0	-0.06
m_4	0.02	0.03	0.06	0

The final step in determining the optimal combination involves evaluating the U_3-part. We find that m_4 emerges as the preferred choice, achieving the highest choice value of $r_4 = 0.11$. For comparison, the other alternatives yield choice values of $r_1 = 0.03$, $r_2 = -0.02$, and $r_3 = -0.13$, respectively.

Therefore, the optimal combination selected by the algorithm is (c_1, s_3, m_4). This combination represents a balanced solution that considers structural integrity, cost-effectiveness, and sustainability, as reflected in the weighted fuzzy soft multiset. Figure 4 provides a visual representation of the choice values for the concrete, steel, and wood alternatives.

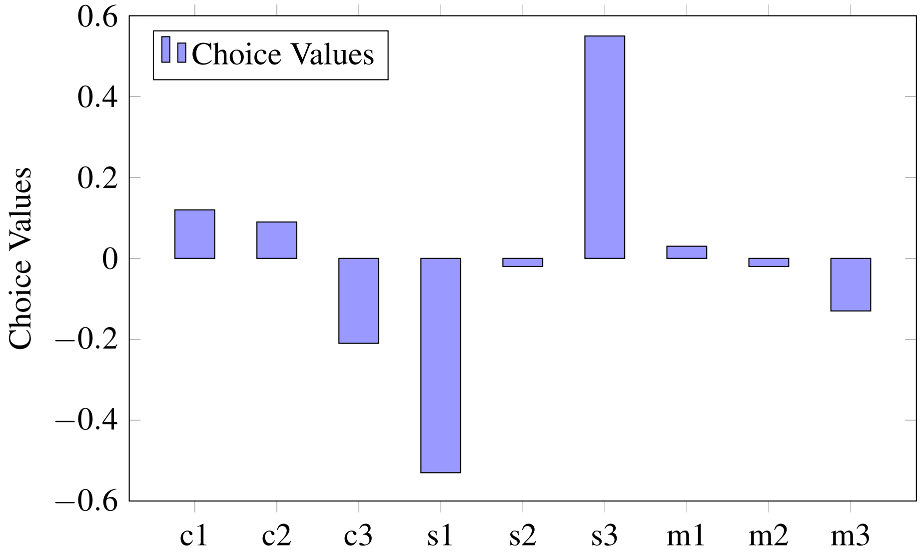

Figure 4. Choice values for concrete, steel, and wood alternatives.

5. Comparing the algorithm with existing method

As a way to illustrate the benefits of the proposed model, we compare it with the existing model, i.e., Das' algorithm [27]. This algorithm differs from the one defined in this study in that it uses Feng's algorithm to calculate the choice values for U_i-parts in a weighted fuzzy soft multiset. As shown in [18], the tabular representation $T = (t_{ij})$ of the level soft sets of a weighted fuzzy soft set can be constructed by setting:

$$t_{ij} = \begin{cases} 1 & \text{if } \mu(e_j, u_i) \geq w_j, \\ 0 & \text{otherwise.} \end{cases}$$

Based on this tabular representation, the weighted choice value r_i for each object u_i is then calculated by the formula $r_i = \sum_{j=1}^{m} t_{ij} \times w_j$.

We begin by applying Feng's algorithm to the U_1-part of (F_u, A, w). It's important to note that Das's algorithm assumes all parameters are positive. Therefore, we use the uniform weighted fuzzy soft multiset, (F_u, A, w), instead of the original weighted fuzzy soft multiset. This procedure yields the weighted choice values (r_i) shown in Table 20. Since c_2 has the highest weighted choice value of 2.2, it is selected as the optimal choice for concrete.

Table 20. Tabular representation of the level soft sets of U_1-part of (F_u, A, w) with weighted choice values.

U_1	$a_1, w_1 = 0.4$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.4$	$a_5, w_5 = 0.2$	$a_6, w_6 = 0.3$	r_i
c_1	0	0	1	0	1	1	1
c_2	1	1	1	1	1	1	2.2
c_3	1	1	0	1	1	1	1.7

However, upon considering the weights of the parameters, it becomes clear that c_1 is a more logical choice than the algorithm's selection of c_2. Since fuzzy soft multisets possess a conditional structure, the decision at this point holds considerable significance. Choosing c_2 at this stage has significant consequences for subsequent decisions, as it influences the selection of other objects from the remaining universes.

Furthermore, if we were to modify (F_u, A, w) by replacing all the values in the columns where c_2 is not maximum by zero, all columns of $(F_u, A, w)_1$ becomes zero. This would render the selection of optimal options for steel and wood impossible.

Let us now apply Das' algorithm to Example 2.8. Applying Feng's algorithm to the U_1-part of the uniform fuzzy soft multiset shown in Table 5, we obtain the weighted choice values of 1.8, 0.8, and 1.8 for f_1, f_2, and f_3, respectively. As noted in [18], when multiple objects have the same highest weighted choice value, any of these objects can be chosen. Therefore, we will consider the following scenarios:

- Case 1: If we choose f_1, applying Feng's algorithm to the U_2-part of $(F_u, A, w)_1$ yields the weighted choice values shown in Table 21. Thus, v_4 is the optimal choice. However, if we modify Table 5 by replacing all values in the columns where v_4 does not have a maximum membership value with zero, all columns of the resulting fuzzy soft multiset become zero. This renders the selection of an optimal protein source impossible.
- Case 2: If we choose f_3, the weighted choice values obtained by applying Feng's Algorithm to the U_2-part of $(F_u, A, w)_1$ are as in the Table 22. If we select v_3 from the objects with equal choice values and modify Table 5 accordingly, we obtain p_1 and p_3 as the optimal protein sources. However, selecting v_4 does not lead to an optimal choice for protein sources, as all columns of the U_3-part of $(F_u, A, w)_2$ become zero.

Thus, Das' algorithm does not provide a definitive optimal choice in this scenario. In contrast, Algorithm 2, when applied to Example 2.8, yields a clear optimal choice of (f_1, v_3, p_1). We leave it to the reader to verify this result using Algorithm 2.

Table 21. Tabular representation of the level soft sets of U_2-part of $(F_u, A, w)_1$ in Example 2.8-Case 1.

U_2	$a_1, w_1 = 0.3$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.6$	r_i
v_1	0	1	0	0	0.4
v_2	1	0	0	0	0.3
v_3	0	1	0	1	1
v_4	1	1	0	1	1.3

Table 22. Tabular representation of the level soft sets of U_2-part of $(F_u, A, w)_1$ in Example 2.8-Case 2.

U_2	$a_1, w_1 = 0.3$	$a_2, w_2 = 0.4$	$a_3, w_3 = 0.5$	$a_4, w_4 = 0.6$	r_i
v_1	0	0	0	0	0
v_2	0	0	0	0	0
v_3	0	0	0	1	0.6
v_4	0	0	0	1	0.6

6. Sensitivity analysis and feasibility

To assess the robustness and reliability of our proposed algorithm, we conduct a sensitivity analysis to examine how variations in input parameters influence the final decision outcomes. This analysis involves two key investigations.

To begin, we explore the impact of altering the order in which the universes are considered in the decision-making process. Initial observations suggest a consistent selection of materials c_1, s_3, and m_4 across all permutations of the universes. However, further investigation reveals this consistency to be coincidental rather than a fundamental property of the algorithm.

In contrast, when applying Algorithm 2 to Example 2.8, we observe that changing the order of decision-making can indeed alter the optimal solution. For instance, considering the universes in the order U_1, U_2, U_3 leads to the selection of (f_1, v_3, p_1), while the order U_2, U_1, U_3 results in two optimal choices: (v_3, f_1, p_1) and (v_3, f_3, p_1).

This sensitivity to decision order highlights the inherent interconnectedness of multi-criteria problems, where the influence of one factor can depend on decisions made about others. Rather than viewing this sensitivity as a limitation, we consider it a valuable feature. It enables the algorithm to adapt to diverse decision-making contexts and provides users with a more comprehensive understanding of the solution space. By exploring multiple decision sequences, users can gain insights into potential trade-offs and make more informed decisions that better align with their specific priorities and constraints.

To further evaluate the algorithm's robustness, we analyzed the impact of varying the weights assigned to each criterion. We adjusted the weights by $\pm 10\%$ and $\pm 20\%$ and observed that the optimal solution in Example 2.8 remained consistent across all these variations. This result highlights the algorithm's robustness to moderate changes in criteria importance, a particularly valuable characteristic in real-world scenarios where precise weight determination can be challenging.

The feasibility of our algorithm is evident through the analysis of its computational complexity and observed performance in practical applications. The algorithm exhibits a time complexity of $O(nm^2p)$, where n represents the number of universe sets, m the maximum number of elements in any universe, and p the number of parameters. This complexity indicates that the algorithm scales linearly with the number of universe sets and parameters but quadratically with the size of the largest universe. In our case study involving construction material selection, the algorithm demonstrated efficient execution, enabling real-time decision support. This performance suggests the method's computational feasibility for practical use in real-world scenarios of moderate scale, where quick and reliable decisions are paramount.

In conclusion, our sensitivity analysis demonstrates the robustness and computational feasibility of the proposed algorithm, strengthening its suitability for real-world multi-criteria decision-making problems.

7. Contributions of the proposed method

The proposed algorithm for weighted fuzzy soft multiset-based decision-making offers several significant advantages and makes notable contributions to the field. Unlike many existing algorithms, our method effectively manages both positive and negative parameters simultaneously, enabling a more realistic representation of complex decision-making scenarios where attributes can have both beneficial and detrimental aspects. Furthermore, the algorithm addresses the limitation of existing methods that can produce counterintuitive results, particularly when dealing with closely clustered membership values. By explicitly considering the conditional structure of fuzzy soft multisets, our approach generates outcomes that are more aligned with intuitive expectations, enhancing trust and confidence in the decision-making process.

By explicitly considering the conditional relationships inherent in fuzzy soft multisets, the algorithm provides a more nuanced and accurate representation of the interdependencies between different factors, leading to more robust and reliable decisions, especially in real-world applications where precise weight determination can be challenging.

The algorithm's versatility is evident in its potential applications across diverse scenarios. In scientific research, it can optimize experimental conditions for biological processes or climate modeling, enabling researchers to navigate complex parameter spaces more effectively. In healthcare, the algorithm can enhance the design of clinical trials for new drug candidates, potentially streamlining the drug development process and improving resource allocation. As demonstrated in our case study, the algorithm can guide the selection of optimal materials for complex engineering and construction projects, effectively balancing factors such as performance, cost, and environmental impact. This flexibility, coupled with its ability to bridge the gap between theoretical advancements in fuzzy soft set theory and practical applications, underscores its significant value for addressing real-world, multi-criteria decision problems across multiple industries.

The proposed method provides a comprehensive framework that addresses multiple challenges within fuzzy soft multiset theory, offering a more complete solution compared to existing methods, which often focus on individual aspects of the problem. By providing insights into how different decision sequences and weight distributions affect outcomes, the algorithm offers richer decision support, allowing users to understand the robustness and sensitivity of their choices. The wide-ranging applicability of the algorithm—from scientific research to healthcare and engineering—underscores its potential to make a significant impact in diverse fields where complex decision-making is crucial.

8. Conclusions and future work

This study introduces a novel algorithm for weighted fuzzy soft multiset-based decision-making, addressing several key challenges in the field. The proposed approach successfully handles both positive and negative parameters, considers the conditional structure of fuzzy soft multisets, and provides more intuitive results than existing methods. A case study on construction material selection demonstrates the algorithm's practical applicability in real-world scenarios. Sensitivity analysis revealed interesting characteristics, particularly robustness to moderate changes in criteria weights and adaptability to different decision sequences. These findings not only validate the reliability of the approach but also provide valuable insights into its behavior under various conditions.

While efficient for typical construction material selection problems, our algorithm does have limitations. Its time complexity of $O(nm^2p)$ indicates quadratic scaling with the size of the largest universe, which could potentially hinder performance in scenarios involving substantially larger datasets. This scaling behavior may impact real-time decision support in large-scale applications requiring the evaluation of numerous alternatives. To address these limitations, future research will explore several avenues for improvement, including implementing parallel processing to distribute computations across multiple cores, investigating approximation methods such as clustering similar alternatives to reduce the effective size of the largest universe, and developing specialized optimization techniques tailored to fuzzy soft multisets.

This research has made substantial contributions but also opens up several promising avenues for future work. One important direction is expanding the application domains of the algorithm. Future research should focus on applying it to a wider range of real-world problems across different industries, including healthcare, such as medical treatment selection; finance, such as portfolio optimization; and environmental management, such as sustainable resource allocation. Additionally, investigating how deep learning techniques could be applied to automatically extract and refine fuzzy set memberships from complex, high-dimensional data could significantly advance the field.

To bridge the gap between theory and practice, future work could focus on creating user-friendly software with intuitive interfaces, enabling nonexpert users to apply the algorithm in various industries. This could involve developing plugins or extensions for existing decision support systems to incorporate fuzzy soft multiset capabilities. Adapting the algorithm to accommodate group decision-making scenarios is another important direction. This would involve incorporating multiple decision-makers' preferences and expertise into the fuzzy soft multiset framework.

Future research could also aim to deepen the theoretical underpinnings of fuzzy soft multiset-based decision-making. This could involve exploring the integration of other uncertainty theories, such as rough sets or intuitionistic fuzzy sets, with fuzzy soft multisets to handle different types of uncertainty. Finally, as the complexity of decision-making problems grows, enhancing the algorithm's computational efficiency becomes crucial. Future work could focus on developing parallel computing strategies to handle large-scale fuzzy soft multiset problems more efficiently.

In conclusion, this study significantly advances fuzzy soft multiset-based decision-making and opens new avenues for tackling intricate decision problems across diverse disciplines. By providing a more nuanced, flexible, and intuitive approach, this work contributes to the ongoing evolution of decision support systems designed to navigate the complexities of our increasingly interconnected world.

Acknowledgments

The author extends sincere gratitude to the anonymous referees for their thorough review and invaluable feedback. Their insightful comments and constructive suggestions significantly improved the quality and clarity of this manuscript.

Conflict of interest

The author declares that there is no conflict of interest regarding the publication of this paper.

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