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Research article

A Bhattacharyya Triangular intuitionistic fuzzy sets with a Owa operator-based decision making for optimal portfolio selection in Saudi exchange

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Abstract: The capital market in Saudi Arabia is fast growing. Assurance of an informed decision while investing in the Saudi Stock Exchange is critical. There has also been an increased quest for advanced decision-making tools due to complexities in selecting a given portfolio, which remains a critical issue of concern among investors in the face of modern investment environment challenges. The research paper offered shall deliver an innovative MCDM technique through which an MCDM model shall be developed in the Saudi Stock Exchange. This MCDM model uses BTIFS with an OWA operator. A novelty of the proposed study is identifying the optimal weight that will be obtained through a newly developed optimization technique known as TFOA. TFOA is a hybrid methodology that brings on board the strengths of DMOA, MPA, and EO for a more precise and efficient calculation of the ideal weights in the portfolio selection process. This would improve the adaptability and effectiveness of the suggested MCDM structure. The effectiveness of the approach is established by comparative analysis with the already existing methods of MCDM, which proves it superior for the optimization of investment portfolios. Sensitivity analysis also conducted to evaluate the strength and dependability of the suggested method. The ranking of weighted portfolios by the ELECTRE method is also, which more establishes the applicability of BTIFS-OWA in real life. The results indicate that the BTIFS-OWA approach along with the TFOA for determining optimal weights provides significant improvements in decision-making accuracy and portfolio optimization compared to traditional methods.

Keywords: multiple Criteria decision-making; Bhattacharyya distance; Triangular intuitionistic fuzzy sets; OWA operator; Saudi stock market
Mathematics Subject Classification: 03E72, 62C86

Nomenclature

Abbreviation	Description
ELECTRE	ELimination Et Choix Traduisant la REalité
OWA	Ordered Weighted Averaging
TIFS	Triangular Intuitionistic Fuzzy Sets
LICF	Linguistic Intuitionistic Cubic Fuzzy
IFS	Intuitionistic Fuzzy Sets
MCDM	Multi Criteria Decision Making
TIFN	Triangular Intuitionistic Fuzzy Number
DMOA	Dwarf Mongoose Optimization Algorithm
MPA	Marine Predators Algorithm
ROE	Return On Equality
ROA	Return On Assets
SONP	Single Objective Nonlinear Programming
MPMSEOA	Multi-Period Mean-Semi-Entropy Optimization Algorithm
CCS	Combined Compromise Solution
FL	Fuzzy Logic
IVIFS	Interval Valued Intuitionistic Fuzzy Sets
FS	Fuzzy Set
IFN	Intuitionistic Fuzzy Number
SSM	Saudi Stock Market
HMO	Heronian Mean Operator
GWHM	Generalized Weighted Heronian Mean
GGWHM	Generalized Geometric Weighted Heronian Mean
CV	Control Volume

1. Introduction

The intricacy of financial markets is always increasing. To maximize their profits, a variety of factors and market features need to be considered by investors [1]. Because of developments in financial engineering, a plethora of methods have been developed to study the behavior of financial markets [2]. Most investors link their cash to stock exchange markets since investing in individual stocks has inherent risks and prefer combinations of several stocks [3]. Thus, Selecting portfolio is an important topic for more investigation [4]. Many domains, including machine learning, artificial intelligence, and conventional and quantitative finance, have extensively studied the issue of portfolio selection [5]. The general goal of portfolio selection is to allocate money among a group of assets in order to meet specific long-term goals. Even while portfolio selection based on financial factors has been the subject of in-depth research in the past, it is still crucial to consider non-financial aspects [6]. Numerous direct and indirect factors might have an impact on selecting a portfolio much like any other decision-making (DM) problem [3]. Determining, evaluating, prioritizing, and putting into practice criteria for evaluating and choosing portfolios has proven to be challenging for researchers, executives, financiers, and practitioners [7].

Prior to choosing or ranking options, it is frequently necessary to simultaneously examine a number of criteria while solving decision-making (DM) problems. The way in which the information is conveyed has consequently remained a key difficulty and for the previous several years, has generated a great deal of attention among academics [8], since the data required to solve the DM issues is typically erratic, vague, and inconsistent. In addressing these concerns, Zadeh [9], who first proposed the idea of fuzzy set theory, has shown how these kinds of decision-making problems may be expressed using the fuzzy set (FS) notion. Nevertheless, in most situations, the membership function μA(x) the single function that characterizes the FS theory cannot be completely utilized to convey various types of complicated fuzzy information.

Atanassov [10] enhanced fuzzy set analysis by providing a fresh role known as "the non-membership function" to build the theory of the IFS. To circumvent these restrictions, Despi [11] has designed an IFS whose sum of the MF and NMF is greater than one, and either positive or negative disparities exist between them. The IFS has also been helpful for managing complex engineering problems and also have provided a flexible way of clarifying uncertainty [12]. A lot of extensions of the IFN [13–15] have been shown to increase the IFS.

Because of its proficiency with complex multi-criteria decision-making scenarios in general and portfolio selection on the Saudi Stock Exchange in particular, ELECTRE was selected for the study. In addition, one of the strengths of ELECTRE is the way it manages conflicting criteria in a pairwise comparison process, and thus it is perfect for financial decisions where there are relevant trade-offs between risk, return, and other factors. Unlike other methods, including MABAC, MAIRCA, VIKOR, and MARCOS, which are indeed effective, ELECTRE does not simplify or deal inadequately with related conflicts [14].

Another way ELECTRE stands out is by the flexibility to incorporate uncertainty and imprecision; it is able to model adequately the intrinsic uncertainties of financial markets. In addition, with outranking relations, it is very robust for decision-making purposes since it allows systematic elimination of the less-favored alternatives, even with criteria hardly quantifiable. Moreover, ELECTRE is quite appropriate in group decision-making since it permits various preferences among different stakeholders to be combined toward a commonly agreed-upon solution—an important characteristic in the context of investment [15]. The objectives of VIKOR, MARCOS, and the approximately ideal solutions of MAIRCA, respectively, are compromise solutions. These may not quantify the innumerable intricate trade-offs in the decision of portfolio selection. The number of objectives that ELECTRE can handle and the high rate of objective interactions between multiple objectives cements its position as the more complete system for the analysis of the portfolio.

1.1. Challenges in earlier studies

- **Case Study Perspective:** Previous research on portfolio selection on the Saudi Stock Exchange or other comparable markets frequently used traditional MCDM techniques, which were rigid and could not keep up with the fast-evolving financial landscape. These methods, which usually relied on simple fuzzy logic or crisp sets, are insufficient for capturing the ambiguity and uncertainty present in financial data. Consequently, the investing strategies that emerged from these techniques were sometimes too straightforward and neglected to include the intricate decision-making necessary in a turbulent market [2].
- **Limited Sensitivity to Market Conditions:** Investment plans that were less robust to market changes were produced by traditional methodologies, which frequently failed to take into account the whole range of possible market scenarios. Investors were unable to react to unforeseen market shifts with ease due to this lack of flexibility, which led to subpar portfolio performance.
- **Inadequate Handling of Investor Preferences:** Many previous models, especially those related to risk tolerance and investment objectives, did not take proper account of the numerous tastes of investors. This model's recommendation often lacked precision as it didn't measure up against the specific needs of individuals, making them useful only in theory.

1.2. Methodological perspective

From a methodological standpoint, the challenges of earlier studies can be attributed to the limitations of the tools and techniques employed in the decision-making process [16]. These include:

- **Simplistic Aggregation Methods:** Early aggregation methods were simple that used basic averaging or even very primitive fuzzy logic to all factors and had only superficially combined results, led to the loss of valuable information which in turn resulted in lower accuracy and reliability in investment decisions
- **Limited Use of Advanced Optimization Techniques:** Prior works often implemented traditional optimization techniques that were inappropriate for the intricate, high-dimensional portfolio selection scenarios. These algorithms are usually engineered to provide solutions for quite simple, single-objective optimization tasks and therefore face many theoretical obstacles in addressing the multi-criteria nature of portfolio selection with various conflicting objectives.
- **Insufficient Modeling of Uncertainty:** Previous research' use of simple fuzzy sets or crisp values failed to accurately capture the ambiguity and uncertainty inherent in financial data. Due to this restriction, it was challenging to appropriately represent the complexity of financial markets in the actual world, which resulted in choices that were either overly risky or cautious.

This work focuses on Bhattacharyya triangular intuitionistic fuzzy sets OWA operator, BTIFOs for short. Such sets pertain to optimum portfolio selection and have not been examined in the reviewed literature. The use of BTIFOSs in DM from its capacity to quantify unknown quantities, represent assessment information in a more comprehensive way, and communicate decision information in many dimensions. In this work OWA operator and the Bhattacharyya distance measure is used to find the final decision matrix.

1.3. The significance of the ELECTRE model in portfolio selection

The ELECTRE model stands out as a well-known tool for Multiple Criteria Decision-Making (MCDM) [17,18]. People use it in many areas where they need to make choices such as in finance and when picking portfolios. This approach shines in situations where you have to pick between options with many competing factors. Here's why the ELECTRE model comes in handy when selecting a portfolio:

- **Handling Multiple Criteria:** Picking a portfolio means finding a balance between several things, like variety, profits, risks, and how you can cash out. The ELECTRE model shines when you need to weigh these factors against each other. It lets you take a deeper look at your investment options.
- **Outranking Capabilities:** The ELECTRE model functions by relying regarding the idea of outranking, which entails a comparison of the options in pairs to get the ranking via dominance relations. This comes very handy during portfolio selection, as one can select those portfolios that do better than others in the majority of the criteria, hence making the decisions better informed.
- **Robustness in Uncertainty:** The financial markets are characterized by volatility and uncertainty. Given the robustness of the ELECTRE model, it is very suitable for such contexts; it will allow the evaluation of different scenarios and sensitivity studies, all very useful in picking up the best possible portfolio.

1.4. The role of the OWA operator in portfolio selection

Another important aspect in portfolio selection is the OWA operator. The OWA operator enables a much more fine-grained and personalized process of decision-making by aggregating multiple criteria through weights assigned to them, showing their importance or preference [5]. This makes it important in portfolio selection for a variety of reasons:

- **Flexible Aggregation of Criteria:** Investors can customize how they want to make decisions using the OWA operator, which allows us to aggregate multiple criteria. This flexibility is essential theoretical work so investors who may emphasize different priorities with regards to their portfolios (for example achieving maximum return or minimum risk) can both compare notes.
- **Balancing Optimism and Pessimism:** The operator may be adapted to reflect a diverse range of risk behaviors, from more pessimistic (heavier on negative outcomes) or optimistic (more prone to positive events). When selecting portfolios, the ability to balance huge profit opportunities and potential financial losses is important.
- **Enhanced Decision Accuracy:** By incorporating the OWA operator, the portfolio selection process can achieve a higher degree of accuracy and relevance, as the operator ensures that the aggregated criteria reflect the investor's true preferences and priorities.

1.5. The necessity of implementing TIFSs in portfolio selection

Inserting TIFSs in the portfolio selection process is influential in the efficient management of the uncertainties and vagueness that come with financial markets. TIFSs propose a more flexible and accurate way of modeling that is much better than the traditional ones, better capturing the investor preferences and market conditions. Membership, non-membership, and Decision of hesitancy are perfect decisions by analyzing these means. These decisions can be close to be sure of the membership, not membership, or hesitancy. Even in this case, the sensitivity analysis can show it close to the statement that is true for the whole spectrum of investments, which can be seen as a certainty and to be well approached with this [1,13]. Furthermore, TIFSs can be used for the definitive theoretical models, the most deterministic of all of them for sure. This allows for examining the results of implementing different but balanced data. In turn, decision support models, such as the ELECTRE model, and the OWA operator, increase the process of choosing a portfolio's fidelity and accuracy through their use of Multi-Criteria Decision-Making/Soft Computing (MCDS).

1.6. Motivations

The research was initiated due to the viable market of stock exchanges like the Saudi Stock Exchange to increase finer Portfolio selection strategies. The traditional methods which are often do neither have the power to solve the existing uncertainties, opposing the criteria of the investors nor reveal their various types of preferences in a way that will satisfy the investors need. The changing financial environment is bringing areas like stock exchanges to the limelight and the conditions in the markets are getting more difficult as more complex information is being used to analyze them. Therefore, a wise investor will have to consider if such risky markets can be dealt with based on a more elaborate approach will grant them a well-tested and secure platform for their investment choice.

- **Need for Improved Decision-Making Tools:** The investors require the tools integrative in which the criteria are still there as well as the treatment of various forms of the humanly defined uncertainty that is more precise in comparison to the standard tools.
- **Adaptation to Market Volatility:** As markets fluctuate frequently, any decision framework that can incorporate and interpret these sudden increments, adding to resilience by considering the varied circumstances on the market is truly needed.

1.7. Contributions

This study makes several key contributions to the field of portfolio selection and decision-making in financial markets:

- **Introduction of BTIFS-OWA Methodology:** We present a novel approach by integrating Bhattacharyya Triangular Intuitionistic Fuzzy Sets (BTIFS) with the Ordered Weighted Averaging (OWA) operator. This combination provides a more nuanced and accurate aggregation of criteria, reflecting investor preferences more effectively than traditional methods.
- **Development of the Tri-Fusion Optimization Algorithm (TFOA):** A significant contribution is the creation of TFOA, a hybrid optimization algorithm that integrates the strengths of the Dwarf Mongoose Optimization Algorithm (DMOA), Marine Predators Algorithm (MPA), and Equilibrium Optimizer (EO). TFOA enhances the accuracy and efficiency of identifying optimal portfolio weights, offering a superior alternative to existing optimization techniques.
- **Comprehensive Comparative Analysis:** This provides a detailed comparative evaluation of the proposed BTIFS-OWA traditional methods, such as trapezoidal fuzzy numbers and intuitionistic fuzzy ordered weighted similarity (IVIFOWS). This analysis demonstrates the superiority of the suggested approach with regard to OWA scores and Bhattacharyya distances across various matrices, confirming its effectiveness in real-world applications.
- **Sensitivity Analysis and ELECTRE Integration:** The inclusion of sensitivity analysis and the integration of the ELECTRE method for ranking portfolios further validate the strength and practical applicability of the proposed approach in the Saudi Stock Exchange context.

1.8. Literature review

Qiyas et al. [16] created a number of LICF information using weighted aggregation operations. To show the effectiveness and possibility of the previously discussed innovative method, they develop a MCDM system within a LICF environment. Alamoudiand and Bafail [17] conducted a MCDM case study implementation approaches to investigate the applicability and efficiency of MCDM techniques in identifying and classifying the best equities for inclusion in a portfolio. The study’s conclusions about utilizing an integrated MCDM approach to determine the best securities advantageous for the banking industry on the substantial Saudi stock market. Nayagam, and Murugan [18] constructed the weighted triangular approximation process for IFN with several appropriate examples. Additionally, several helpful triangular approximation features on IFN have also been explored. provide the novel kind of IFNs in L–R triangular form, weighted triangular approximation using the distance. Hashemi et al. [19] employed an innovative approach to group DM, the ideal set of SIs was chosen to meet each of the three pillars of highway sustainability. To further differentiate the preferred order of SIs, new ranking scores and separation metrics are introduced. Finally, the approach’s applicability is assessed when it is utilized in a highway construction project case study. Remadi, and Frikha [20] developed an expansion of the CODAS methodology to handle uncertainty by employing TIF sets and to address issues involving numerous criteria in groups DM. Moreover, Remadi, and Frikha [21] created a ranking MCDM model to aid in the uncertain selection of green materials. It is predicated on examining CODAS, one of the MCDM techniques, to resolve group decision-making problems with numerous criteria in the framework of IFS. Each alternative’s rating is expressed in language words and transformed into triangular intuitionistic fuzzy numbers (TIFNs) to completely enhance material qualities and enhance environmental performance over time. Saeed et al. [22] evaluated each criterion’s weight and the alternatives' ratings using linguistic phrases within the framework of a TIFN. In this expanded TOPSIS model, a fresh fuzzy positive intuitionistic and negative ideal solution are provided. Prakash and Suresh [23] created a novel method for ranking IFN using Nagel points. Some outcomes and numerical illustrations supported the suggested new ranking. Babatunde et al. [24] employed a modified TIF combining and prioritizing function model to determine the most favored "end-of-life" management options for batteries. In order to evaluate the suggested updated TIFARF model, professional comments from the Nigerian renewable energy industry were gathered. Geng and Ma [25] created a few new n-IPFS aggregation operators and used them to solve MAGDM issues. First, the definition of the n-IPFSs' operating characteristics and scoring function. Next, three types of polygonal fuzzy aggregation operators that are n-intuitionistic are examined. Lastly, they provide a better method for the TOPSIS approach, which uses n-IPFSs and weights for unknown qualities. Biswas et al. [26] provided an intuitionistic fuzzy shortest path problem in directed graphs by introducing intuitionistic fuzzy numbers. In essence, it simply developed a technique based on a classical Dijkstra's algorithm, applied to graphs of crisp weights and arc weights. Such a method might have found certain applications in computer science, in communication networks, as well as in transportation systems.

In 2022, Bisht and Kumar [27] have proposed a framework that integrates the insights of novice investors and experts to simplify stock selection. The paper illustrated a fuzzy Base-Criterion for weighing stock selection criteria and Dempster-Shafer theory for categorizing securities. Ranking of top ten securities is optimized through consensus-based ranking with a deep recurrent neural network having LSTM. In 2021, Jiang and Qing [28] a fuzzy rough set model based on decision theory was presented which is put forth for MADM with hesitant fuzzy information systems. In 2021, Zhou and Li [29] proposed the use of semi-entropy as a successful method of controlling downside risk management in multi-period portfolio optimization in fuzzy environments. MPMSEOA is designed that takes bankruptcy occurrences and transaction costs into account. Further, with the help of a risk-aversion factor, the program is changed to a crisp SONP model, and the solution is obtained through a genetic algorithm. In 2022, Narang et al. [30] have proposed an integrated method for choosing stocks in a portfolio in a two-stage framework: first, suggested a fresh approach to stock selection decision-making by combining the HMO, in particular, the enhanced GWHM and the enhanced GGWHM with the traditional CCS method; second determined the respective ideal weights of the given selection criteria using the Base-Criterion technique. The CoCoSo-H model proposed herein takes care of the vagueness, variability, and anomalies in the data and increases the flexibility of difficult decisions. A case study on choosing stocks for a portfolio listed on the National Stock Exchange (NSE) with different portfolios constructed using PSO validated the applicability of the model.

The studies by Jagtap and Karande [31], Mishra et al. [32], Isabels et al. [33], and Wang et al. [34] highlight the diverse and evolving applications of FL and MCDM methods. Jagtap and Karande [31] enhance the ELECTRE-I method using m-polar fuzzy sets and revised weight calculations through Simos' and AHP, focusing on the complex choice of unconventional machining techniques. Mishra et al. apply IVIFS with MAIRCA method to assess sustainable wastewater treatment technologies, emphasizing the handling of imprecise data. Isabels et al. incorporate intuitionistic trapezoidal fuzzy sets into the VIKOR approach to evaluate and rank Metaverse platforms, addressing uncertainties in emerging digital technologies. Wang et al. use advanced fuzzy aggregation operators for green supplier selection, demonstrating the importance of nuanced fuzzy methods in resilient supply chain management. Collectively, these studies underscore the significance of integrating sophisticated fuzzy logic and MCDM techniques to effectively manage complex, uncertain, and multi-dimensional decision problems across fields.

1.9. Research gap

Despite significant advances In the area of choosing a portfolio, several research lacunae remain unmet, which this study has set out to be the solution of:

- **Limited Handling of Uncertainty and Vagueness:** In most cases, methodologies depend more on simplified models that fail to properly account for uncertainty and vagueness in financial data [35]. Hence, TIFSs are introduced to handle this issue, giving a more flexible and more realistic framework than other models.
- **Insufficient Integration of Advanced Optimization Techniques:** In the case of most research works, advanced optimization algorithms are not optimally used, primarily in multicriteria decision-making. The development of TFOA does away with this deficiency as it is a powerful tool integrated with multiple optimization techniques that improve the selection process's precision and effectiveness.
- **Lack of Comprehensive Comparative Studies:** Despite the existence of a lot of methods that can be used for portfolio selection, very few have fully conducted a deep analysis of them, proving newer methods are more effective than traditional ones. Hence, through this research, we can fill this gap by providing a bright comparison of the BTIFS-OWA approach to the recognized ones that show the best performance among them.
- **Need for Robust Decision-Making Frameworks in Emerging Markets:** The Saudi Stock Exchange is an area of the market that is continuously changing and where trading is so volatile that decisions are hard to make and implement in a timely manner. Through this work, we make an effort to address this problem by bringing a flexible but also solid way of decision-making to these settings, which are peculiar in their demand.

2. Preliminaries

The basic definitions and ideas of IFS and TIFN are explained in this section.

Definition 1. *IFS [21]*

The definition of the intuitionistic FS is Within the debate X universe, intuitionistic FS H is termed as per Eq (1).

$$H = \{\langle e, \mu_H(e), v_H(e)\rangle | e \in E\}, \tag{1}$$

where $\mu_H: E \to [0,1]$ and $v_H: E \to [0,1]$.

Next, the various operators and relations for IFS are provided by per Eqs (2)–(7), respectively. The graph of IFS is manifested in Figure 1.

$$H.N = \{\langle e, \mu_H(e) \cdot \mu_N(e), v_H(e) + v_N(e) - v_H(e) \cdot v_N(e)\rangle | e \in E\}. \tag{2}$$

$$H + N = \{\langle e, \mu_H(e) + \mu_N(e) - \mu_H(e) \cdot \mu_N(e), v_H(e) \cdot v_N(e)\rangle | e \in E\}. \tag{3}$$

$$\lambda A = \left\{\langle e, 1 - \left(1 - \mu_H(e)\right)^{\lambda}, \left(v_N(e)\right)^{\lambda}\rangle | e \in E\right\}, \lambda > 0. \tag{4}$$

$$A^{\lambda} = \left\{\langle e, \left(\mu_H(e)\right)^{\lambda}, 1 - \left(1 - v_N(e)\right)^{\lambda}\rangle | e \in E\right\}, \lambda > 0. \tag{5}$$

$$H = N \; iff \, \mu_H(e) = \mu_N(e) \; and \; v_H(e) = v_N(e) \; \forall e \in E. \tag{6}$$

$$H \leq N \; iff \, \mu_H(e) \leq \mu_N(e) \; and \; v_H(e) \geq v_N(e) \; \forall e \in E. \tag{7}$$

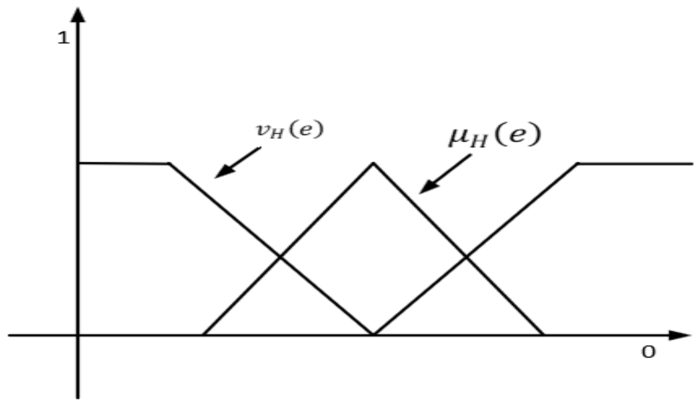

Figure 1. Graph of IFS.

TIFN

TIFN is the word for the IFN that is based on the TFN. The TIFN is the usage of the classic TFN to describe the MF, $\mu_H(e)$,and NMF $v_N(e)$. The following introduces the fundamental ideas around the TIFN:

Definition 2 [36].

Let γ be the TIFN, with the following definitions for γ's MF-$\mu_\gamma(e)$ and NMF-$v_\gamma(e)$. This is shown mathematically in Eqs (8) and (9), respectively. The graph showing TIFN is shown in Figure 2.

$$\mu_\gamma(e) = \{\frac{(e-h_1)\mu_\gamma}{h_2-h_1} \;\; for \;\; h_1 \leq e < h_2 \; \mu_\gamma \;\; for \;\; e = h_2 \frac{(h_3-e)\mu_\gamma}{h_3-h_2} \;\; for \;\; h_2 < e \leq h_3 \; 0, \tag{8}$$

otherwise,

$$v_\gamma(e) = \{\frac{\left(h_2-e+v_\gamma(e-h_1')\right)}{h_2-h_1'} \;\; for \;\; h_1' \leq e \leq h_2 \; v_\gamma \;\; for \;\; e = h_2 \frac{\left(e-h_2+v_\gamma(h_3'-e)\right)}{h_3'-h_2} \;\; for \;\; h_2 \leq e \leq h_3' \; 0, \tag{9}$$

otherwise, where $0 \leq \mu_\gamma \leq 1; 0 \leq v_\gamma \leq 1; 0 \leq \mu_\gamma + v_\gamma \leq 1, h_1, h_2, h_3, h_1', h_3' \in R$

TIFN can be represented as $\gamma' = \langle([h_1, h_2, h_3]; \mu_\gamma), ([h_1, h_2, h_3]; v_\gamma)\rangle$ when $\mu_\gamma = 1$ and $v_\gamma = 0$ and γ'convert into traditional TFN.

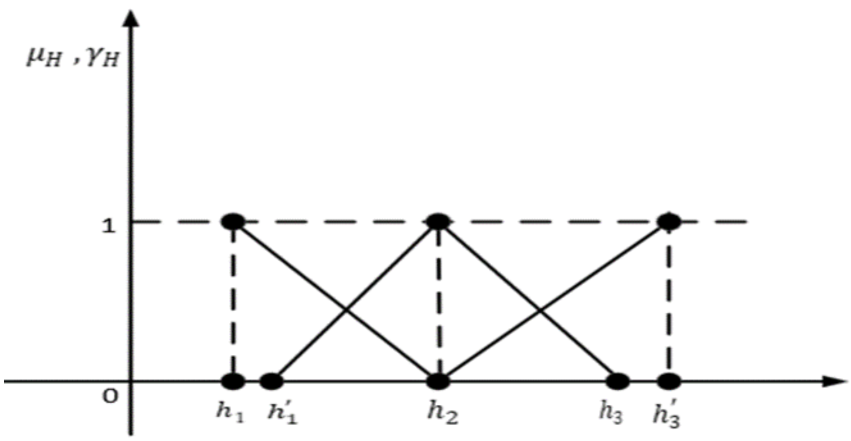

Figure 2. Graph of TIFN.

Definition 3 (Operational rules of two TIFNs). *It is shown mathematically in Eqs (10)–(13), respectively.*
$\varphi_1 = \left([h_1, i_1, j_1]; \mu_{\gamma_1,} v_{\gamma_1}\right)$ and $\varphi_2 = \left([h_2, i_2, j_2]; \mu_{\gamma_2,} v_{\gamma_2}\right)$ represents two TIFNs also $\lambda \leq 0$.

$$\varphi_1 + \varphi_2 = \left([h_1 + h_2, i_1 + i_2, j_1 + j_2]; \mu_{\gamma_1} + \mu_{\gamma_2,} - \mu_{\gamma_1}\mu_{\gamma_2}, v_{\gamma_1 v_{\gamma_2}}\right). \tag{10}$$

$$\varphi_1\varphi_2 = \left([h_1h_2, i_1i_2, j_1j_2]; \mu_{\gamma_1}\mu_{\gamma_2,} v_{\gamma_1 +} v_{\gamma_2} - v_{\gamma_1}v_{\gamma_2}\right). \tag{11}$$

$$\lambda\varphi = \left([\lambda h, \lambda i, \lambda j]; 1 - \left(1 - \mu_\gamma\right)^\lambda, v_\gamma\right), \lambda \geq 0. \tag{12}$$

$$\gamma^\lambda = \left(\left[h^\lambda, i^\lambda, j^\lambda\right]; \left(\mu_\gamma\right)^\lambda, 1 - \left(1 - v_\gamma\right)^\lambda\right), \lambda \geq 0. \tag{13}$$

The following operations (shown in Eqs (14)–(19)) provide the operational outcomes for the two TIFNs' rules listed in Definition 3:

$$\varphi_1 + \varphi_2 = \varphi_2 + \varphi_1. \tag{14}$$

$$\varphi_1 \otimes \varphi_2 = \varphi_2 \otimes \varphi_1. \tag{15}$$

$$\lambda(\varphi_1 + \varphi_2) = \lambda\varphi_2 + \lambda\varphi_1, \lambda \geq 0. \tag{16}$$

$$\lambda_1\varphi + \lambda_2\varphi = (\lambda_1 + \lambda_2)\varphi, \lambda_1, \lambda_2 \geq 0. \tag{17}$$

$$\varphi^{\lambda_1} \otimes \varphi^{\lambda_2} = \varphi^{\lambda_1 + \lambda_2}, \lambda_1, \lambda_2 \geq 0. \tag{18}$$

$$\varphi_1{}^\lambda \otimes \varphi_2{}^\lambda = (\varphi_1 \otimes \varphi_2)^\lambda, \lambda \geq 0. \tag{19}$$

3. Methodology for Bhattacharyya Triangular intuitionistic fuzzy sets with OWA operator-based decision making and Tri-Fusion optimization algorithm

In this section, we introduce the Bhattacharyya Triangular Intuitionistic Fuzzy Sets MCDM issue, which we rank using the ELECTRE technique. The assessment data from each decision maker must be combined for the Triangular Intuitionistic Fuzzy Linguistic multi-attribute decision procedure. All of the decision makers' evaluation data is combined using the OWA operators. Let m viable alternatives be A_λ, $\lambda = 1, 2, \cdots, m$, that are evaluated by k decision-makers (DM), D_ν (for $\nu = 1, 2, \dots, k$), based on n criteria that are incompatible and non-commensurable, C_μ $\mu = 1, 2, \cdots, n$. Using the decision-evaluation makers of each alternative, A_λ in relation to each criterion, C_μ create a decision matrix, shown by $TB = \left[tb_{\lambda\mu}\right]_{m \times n}$. The following describes the procedure of the BTIFO based decision making.

Step 1: Decision matrix creation

Create a decision matrix $K = \left(l_{ij}\right)_{n \times m}$. A normalized decision matrix $K = \left(\varphi_{ij}\right)_{n \times m}$ can be created from this matrix.

Step 2: Construction of the optimized weighted matrix

(a) Decision matrix as well as the weight vector are initialized.

- The element φ_{ij} of the initial decision and the j^{th} weight of the criterion w_j play a major role in the normalized decision matrix μ_{ij} computation. The mathematical formula for μ_{ij} is shown in Eq (22).

$$\mu_{ij} = \varphi_{ij}w_j, i = 1, 2, .., n; j = 1, 2, .., m, \tag{20}$$

where w_j denotes the weight of jth criterion.

- To ensure that the sum of weight is equal to 1. Mathematically, this is shown in Eq (21).

$$\sum_{j=}^{n} w_j = 1. \tag{21}$$

- To optimize the weight w_j iteratively using the new tri-fusion optimizer. The proposed tri-fusion optimizer is the combination of the Dwarf Mongoose Optimization Algorithm (DMOA), Marine Predators Algorithm (MPA), and Equilibrium Optimizer (EO).

The objective function for TFOA model is manifested in Eq (22).

$$Obj_{function} = \sum_{k=1}^{n} \sum_{i=k+1}^{n} w_k w_l . B(l_i, l_k). \tag{22}$$

The count of fuzzy set is denoted as n, and the Bhattacharyya distance between $l-th\ and\ k-th$ criterion is denoted as $B(l_i, l_k)$. In addition, $B(l_i, l_k)$ and $w_k w_l$represents the $l-th\ and\ k-th$ fuzzy sets and its corresponding weights, respectively. The optimized weight function acquired from TFOA model is denoted as w_{opt}.

Step 3: Calculation of concordance set and discordance set of subscripts

The following is a representation of the concordance set of subscripts that should meet the constraint $l_{ij} J l_{kj}$.

$$B_{ik} = \{b | l_{ij} J l_{kj}\}, (i, k = 1, 2, \ldots, n), \tag{23}$$

$l_{ij} J l_{kj}$ denotes $l_{ij} \succ_s l_{kj}$ or $l_{ij} \succ_w l_{kj}$ or $l_{ij} \succ_I l_{kj}$.

The discordance set of subscripts is the complementary subset of the concordance set of subscripts, and the following is its definition:

$$N_{ik} = U - B_{ik}. \tag{24}$$

Step 4: Calculate the matrix for concordance discordance

The concordance index $B(l_i, l_k)$ is denoted by the weight vector e linked with the criterion as follows:

$$B(l_i, l_k) = \sum_{j \in B_{ik}}^{n} e_j. \tag{25}$$

Hence, the concordance matrix B is $B = (- \ b_{12} \ \cdots \ b_{1n} \ b_{21} \ - \ \cdots \ \cdots \ \cdots \ \cdots \ - \ \cdots \ b_{n1} \ b_{n2} \ \cdots \ -)$

The discordance index $N(l_i, l_k)$ is denoted as per Eq (26)

$$N_{ik} = \frac{\{d(l_{ij}, l_{kj})\}}{\{d(l_{ij}, l_{kj})\}}. \tag{26}$$

Here, the normalized Euclidean distance between l_{ij} and l_{kj}, is denoted by the symbol $d(l_{ij}, l_{kj})$.

The discrepancy matrix N is shown in Eq (27)

$$N = (- \ n_{12} \ \cdots \ n_{1n} \ n_{21} \ - \ \cdots \ \cdots \ \cdots \ \cdots \ - \ \cdots \ n_{n1} \ n_{n2} \ \cdots \ -). \tag{27}$$

Step 5: Bhattacharyya distance calculation

In this step the Bhattacharyya distance is calculated for Eqs (28)–(30) using the following steps:

Define the Bhattacharyya coefficient for membership, non-membership, and hesitant function as follows:

$$Bh_{co}(\mu_A, \mu_B) = \sum_x \sqrt{\mu_A(x) \mu_B(x)}. \tag{28}$$

$$Bh_{co}(\vartheta_A, \vartheta_B) = \sum_x \sqrt{\vartheta_A(x) \vartheta_B(x)}. \tag{29}$$

$$Bh_{co}(\pi_A, \pi_B) = \sum_x \sqrt{\pi_A(x) \pi_B(x)}. \tag{30}$$

Where μ, ϑ, π represents the MF, NMF, and hesitancy function respectively of triangular intuitionistic fuzzy numbers

Aggregation of the Bhattacharyya coefficient is shown in Eq (31).

$$Bh_{co}(A,B) = \frac{1}{3}\left(Bh_{co}(\mu_A,\mu_B) + Bh_{co}(\vartheta_A,\vartheta_B) + Bh_{co}(\pi_A,\pi_B)\right). \quad (31)$$

Calculate the Bhattacharyya distance [37] using Eq (32)

$$Bh_d(A,B) = -\ln\ln Bh_{co}(A,B). \quad (32)$$

Hence, the resulting matrix is the Bhattacharyya distance [37,38]-based decision matrix (BDDM).

Step 6: OWA operator-based aggregation to get the final decision matrix.

We will get a range of aggregation operators from the OWA operator that are set between the lowest and the maximum. The parameters can be rearranged with this operator according to their values [39]. A mapping OWA is an OWA operator with dimensions n with $R^n \to R$ that it has an associated weighting vector $W_{opt} = \left[w_{opt1}, w_{opt2}, \dots, w_{optn}\right]^T$ with $w_{optj} \in [0,1]$ and $\sum_{j=1}^{n} w_{optj} = 1$ such that $OWA(a_1, a_2, \dots, a_n) = \sum_{i,j=1}^{n} w_{optj} BDDM = 1$, where BDDM is the Bhattacharyya distance-based decision matrix.

Step 7: Ranking.

To find the best alternative we use the final decision matrix to rank the alternatives according to the criteria.

4. Tri-Fusion Optimization Algorithm (TFOA) based optimal weight assignment

The architecture of the TFOA model is illustrated in Figure 3.

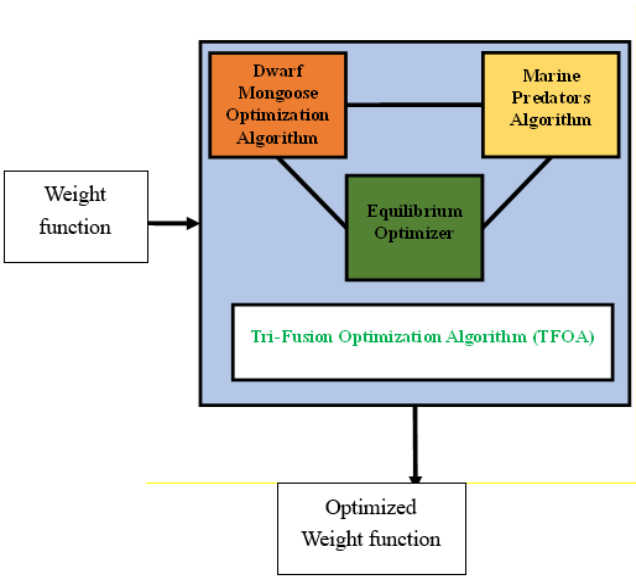

Figure 3. Architecture of proposed TFOA.

4.1. DMOA

An innovative optimization method called DMOA is aimed at solving the major challenge of identifying the optimal weight function.

Figure 4 describes the format of the Dwarf Mongoose Optimization Algorithm. In this structure, mongooses are searching for food in their natural habitat. This shows the initial stages of foraging, after which there is a group of scouts that survey the area for safety and potential predators. As other members do other things, some babysitters are appointed to take care of the weakest ones. The last arrow indicates a cycle where the members who are young or hurt are taken care of by the alpha group. This graph expresses teamwork, division of duties, and caring about the weaker members. Like this way, the Dwarf Mongoose Optimization Algorithm will work. So, to elaborate further on the application of the Dwarf Mongoose Optimization Algorithm (DMOA) for addressing issues in optimal weight identification, it integrates several equations that illustrate the optimization processes.

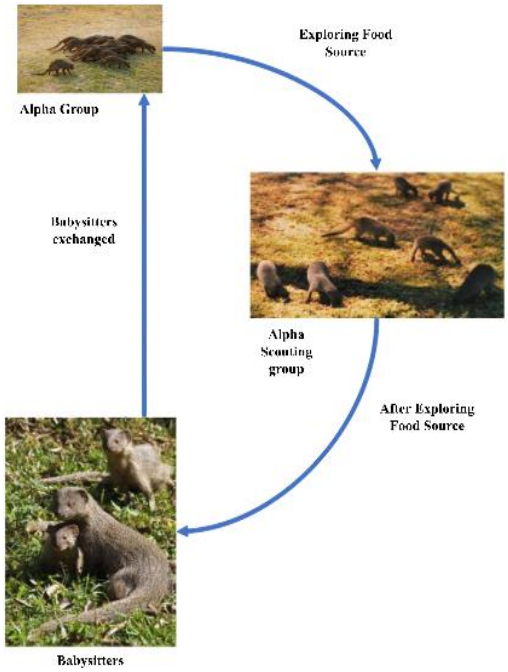

Figure 4. Structure of DMOA.

The update equations for the positions (weight) of dwarf mongooses (search for optimal solutions) in the search space describe the optimization dynamics of DMOA as follows in Eq (33):

$$w_t^{k+1} = w_t^k + \alpha\left(w_b^k - w_t^k\right) + \gamma(w_r^k - w_t^k). \tag{33}$$

- w_t^k is the current position of the t^{th} mongoose at iteration k.
- w_b^k represents the best position found by the mongoose swarm up to iteration k (global best position).
- w_r^k denotes the position of a randomly selected mongoose in the swarm at iteration k (local best position).
- α and γ are the coefficients that control the influence of the global and local best positions, respectively.

Such a mechanism for updating enables DMOA to effectively equal the exploration and exploitation, facilitating effective optimization over complicated search regions. It is possible that adjusting its search strategy dynamically through these types of updates improves a system's performance on various optimization tasks.

4.2. MPA

MPA is a cutting-edge optimization method which imitates the strategic hunting habits of marine predators. It tackles the major problems faced in the identification of the optimal weights of the matrix.

MPA also changes normalization parameters in order to maintain uniformity in different datasets through its exploratory and exploitation phases. The MPA is a novel mean for optimizing preprocessing tasks consequently leading to the enhancement of accuracy and reliability of seizure prediction models significantly.

Figure 5 illustrates the structure of MPA. It is bio-inspired optimization, taken from the foraging behaviors of ocean predators. It has borrowed concepts about their movements in Levy flights and Brownian motions. MPA is a rather parameter less, user-friendly, flexible technique that is appropriate for many optimization problems. Imitating the predator-prey interactions, it efficiently surveys the search space; therefore, it becomes very powerful in computational intelligence.

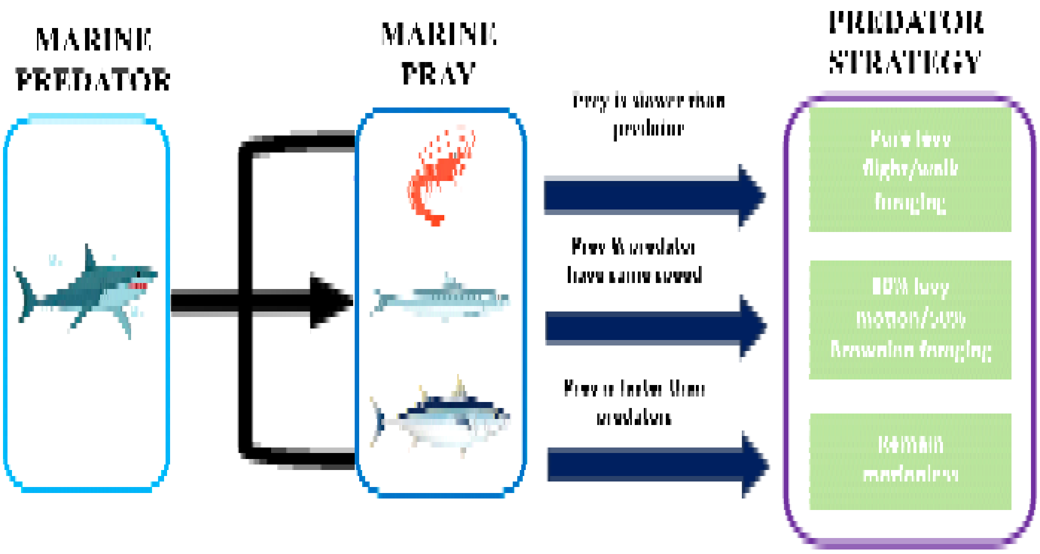

Figure 5. Structure marine predators algorithm (MPA).

4.3. Equilibrium Optimizer (EO)

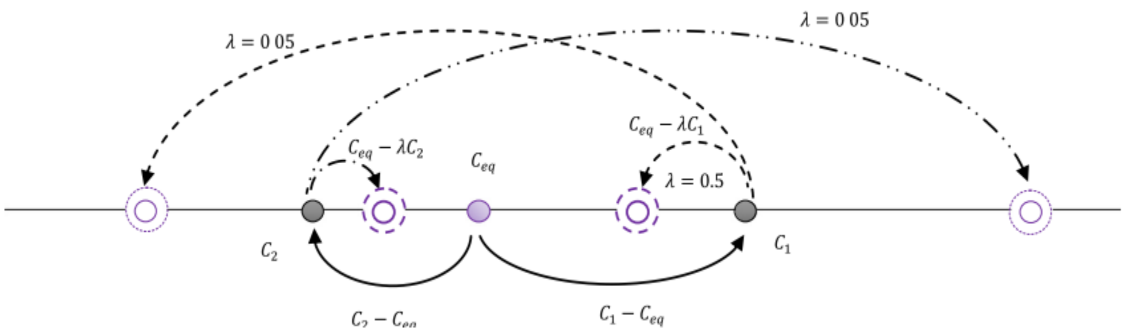

Figure 6. Structure of equilibrium optimizer (EO).

Figure 6 shows the structure of EO, which describes the working process. Equation (34) describes the fundamental mass-balance problem expressed as a first-order ordinary differential equation, where the total mass entering the system plus the mass produced within the system minus the total mass leaving the system represents the change in mass over time.

$$V\frac{dC}{dt} = QC_{eq} - QC - G. \tag{34}$$

C_{eq} denotes the concentration in a state of equilibrium where there is no generation inside the CV, Inside the CV, G is the mass generation rate, and C is the concentration inside the CV (V). $V\frac{dC}{dt}$ is the rate of change of mass in the CV. Q is the volumetric flow rate into and out of the CV. There is a state of steady equilibrium when $V\frac{dC}{dt}$ approaches zero. Equation (5) are rearranged to solve for $\frac{dC}{dt}$ as a function of $\frac{Q}{V}$, where $\frac{Q}{V}$ is the turnover rate (i.e., $\lambda = \frac{Q}{V}$) or the inverse of the residence period, here denoted as λ. Consequently, the concentration in the CV (C) as a function of time (t) also be found by rearranging Eq (34) as follows in Eq (35)

$$\frac{dC}{\lambda C_{eq}-\lambda C+\frac{G}{V}} = dt. \tag{35}$$

Equation (36) showcase the integration of Eq (35) over time:

$$\int_{C_0}^{C}\frac{dC}{\lambda C_{eq}-\lambda C+\frac{G}{V}} = \int_{t_0}^{t} dt. \tag{36}$$

The results shown in Eq (37):

$$C = C_{eq} + \left(C_0 - C_{eq}\right)F + \frac{G}{\lambda V}(1 - F). \tag{37}$$

In the Eq (37), the F is calculated as follows in Eq (38):

$$F = exp[-\lambda(t - t_0)], \tag{38}$$

where, depending on the integration interval, t_0 and C_0 represent the initial start time and concentration respectively. The balance between searching for new answers and taking advantage of the best options is what explains why the updates equation moves locations of solutions about the balance condition.

5. Case study

5.1. Step-by-step computation

A. Criteria weights determination

Methodology: The values of the criteria weights are calculated through a set methodology that utilizes AHP and the TFOA – the Tri-Fusion Optimization Algorithm, which is composed of DMOA, MPA, as well as, EO.

Step-by-Step Calculation:

Step 1-Initial Criteria Weight Assignment: Some of the parameter ratings will be pre-determined while the initial weights can be arrived at by using AHP for pairwise comparison.

Step 2-Standardise the weights in a manner that their sum is equal to 1.

B. TFOA optimization

Step 1- Random creation of the initial generation with potential solutions to the technical problem.

Step 2- Check the efficiency of each of the solutions by the objective function (<|reserved_special_token_272|>, for example, portfolio return rate and risk minimization).

Step 3- Apply DMOA, MPA, and EO to iteratively improve the solution: o Step 3: Apply DMOA, MPA, and EO to iteratively improve the solution:

- DMOA: In accordance to the movement of the prey and the predator, update the positions.

- MPA: Fine-tune the solutions in relation to the strategies employed by marine predators.
- EO: Further improve solutions through the concepts of equilibrium strategies.

Step 4- Choose the obtained solution with the highest value of the corresponding criteria set as the final set of criteria weights.

C. Portfolio selection using BTIFS and OWA operator

Methodology: The process of diversification of portfolio is conducted based on the combination of the BTIFS with the Order Weighted Averaging operator to provide decision making.

Step-by-Step Calculation:

Step 1- Define TIFS [40].

Step 2- For each portfolio, depending on the performance characteristics, specify the values of the criterion BTIFS.

Step 3- Example: Portfolio A can have the measures of BTIFS ranging between 0. 3 and 0. 5 on return and between 0. 4 and 0. 4 on risk.

D. Calculate OWA scores

Step 1- Arrange the performance metrics of each portfolio from the highest to the lowest order.

Step 2- Perform the operation of OWA using the weights determined to combine the metrics.

E. Apply the ELECTRE method

Step 1- Compare the portfolios in pairs in accordance with the obtained OWA scores.

Step 2- Construct the outranking matrix so as to identify the dominance relations.

Step 3- Sort the portfolios according to the outranking relations and determine the best portfolio.

F. Sensitivity analysis

Methodology: In addition to this, it is appropriate to conduct sensitivity analysis which will help the firm to determine the strength and reliability of the portfolio selection results given different weights of the criteria [40].

Step-by-Step Calculation:

Vary Criteria Weights:

- Determine values that belong to a reasonable set of weight values (e. g., 10% from the measured weight) and recalculate OWA scores. Example: If original weight is 0. 3, test weights 0. 27 and 0. 33.
- Recalculate OWA Scores and Rankings: Recalculate OWA Scores and Rankings: Whenever adjusting the weight, renew OWA scores and again undertake the procedure of ELECTRE method.
- Compare the obtained portfolios ranking with the ranking before adjustment.

G. Assess stability

o Compare if the new weights fit the desirable portfolio or not.

o Assess consequences of marked shifts in rankings on the decision making.

5.2. Main criteria and sub criteria used in this study

The rating of banks that are listed on the Saudi stock exchange is the main objective. Financial observers typically consider the banking industry to be the most significant and rapidly expanding more than 22 industries listed on the SSM. We consider 10 banks as alternatives: RIBL (FB1), BJAZ (FB2), SAIB (FB 3), BSFR (FB 4), SABB (FB 5), ANB (FB 6), ALRAJHI (FB 7), ALBILAD (FB 8), ALINMA (FB 9), SNB (FB 10). The aggregated Criteria Weights are 0.156, 0.093, 0.062, 0.125, 0.032, 0.064, 0.042, 0.042, 0.032, 0.096, 0.187, 0.021, 0.021, 0.011, 0.006.

The detailed criteria and sub-criteria for this study are shown in Tables 1 and 2, and the linguistic variables for TIFN are also shown. These criteria are important assessments of the investment options and the TIFN linguistic variables assist in portraying the vagueness and subjectiveness of the financial information. Table 2 defines the linguistic variables for TIFN used in the model. Each term (Extremely Low, Low, Moderate, High, and Extremely High) is represented by a TIFN with specified MF, NMF, and hesitancy values. These linguistic variables help in expressing the uncertainty and subjective assessments of financial asset performance.

Table 1. Main criteria and sub criteria used in this study.

Main criteria	Sub criteria
Profitability (MC1)	(ROE) (SuCr1) (ROA) (SuCr2) Margin of net profit (SuCr3)
Liquidity (MC2)	Current ratio (SuCr4)
Market (MC3)	Ratio of price to sales (P/S) (SuCr5) Capital to Debt Ratio (D/C) (SuCr6) Price-to-Book (P/B) (SuCr7) Dividend yield (SuCr8) ratio of price to earnings (SuCr9) Earnings per share (SuCr10)
Valuation (MC4)	Asset Turnover (SuCr11)
Others (MC5)	Volume (SuCr12) Mean % (SuCr13) Std. dev. % (SuCr14) Beta (SuCr15)

Table 2. Linguistic variables for TIFN.

Linguistic terms	TIFN
Extremely Low	(0.1,0.2,0.3); 0.9, 0.05
Low	(0.2,0.35,0.5); 0.8, 0.15
Moderate	(0.4,0.6,0.8); 0.7, 0.2
High	(0.6,0.8,0.9); 0.6, 0.3
Extremely High	(0.8,0.9,1); 0.5, 0.1

Table 3 presents the aggregated decision matrix, encompassing 15 sub-criteria evaluated across 10 investment alternatives. Each cell in the matrix represents the aggregated fuzzy value derived from the TIFN for a specific sub-criterion and alternative. This matrix serves as the foundational data for making informed and optimal portfolio selections in the Saudi Exchange. Table 3 presents the overall decision matrix in which each cell shows the fuzzy value of a sub-criterion and a particular alternative. This matrix is important in establishing how each choice fares in terms of the defined criteria. For instance, MC1 appears to be fairly acceptable at the sub-criteria SuCr5 (Ratio of price to sales) and SuCr6 (Capital to Debt Ratio) these deal with specific market attributes.

Table 3. Aggregated decision matrix.

	SuCr1	SuCr2	SuCr3	SuCr4	SuCr5	SuCr6	SuCr7
MC1	0.02028	0.00186	0.03038	0.04	0.2592	3.3632	0.08568
MC2	0.01248	0.00093	0.01612	0.02	0.16064	2.99456	0.063
MC3	0.01092	0.00093	0.02294	0.02375	0.19616	3.92832	0.05124
MC4	0.01404	0.00186	0.02852	0.0225	0.24288	2.46656	0.06258
MC5	0.00936	0.00093	0.02108	0.0175	0.27328	1.77856	0.06342
MC6	0.01092	0.00093	0.02418	0.02	0.24864	2.3424	0.0588
MC7	0.03588	0.00279	0.03596	0.015	0.4288	1.40224	0.19194
MC8	0.0234	0.00186	0.02356	0.01875	0.32352	3.23904	0.16338
MC9	0.0156	0.00186	0.02418	0.015	0.3072	2.84864	0.0945
MC10	0.01716	0.00186	0.02728	0.0225	0.30624	3.04128	0.07644

The regularized matrix (Table 4) of alternatives is calculated using the criteria weight by Step 2. The normalized matrix of alternatives is shown in the Table 4, which contains the values of criteria after their aggregation with the corresponding weights. This normalization is useful when evaluating alternatives on one or the other scale. For instance, MC = 1 has the largest normalized value of SuCr1 (Return-on-Equity); this means that this investment option has better profitability as compared and contrasted with other options.

Table 4. Normalised matrix.

	SuCr1	SuCr2	SuCr3	SuCr4	SuCr5	SuCr6	SuCr7
MC1	0.565217	0.666667	0.844828	1	0.604478	0.856142	0.446389
MC2	0.347826	0.333333	0.448276	0.5	0.374627	0.7623	0.328228
MC3	0.304348	0.333333	0.637931	0.59375	0.457463	1	0.266958
MC4	0.391304	0.666667	0.793103	0.5625	0.566418	0.627892	0.326039
MC5	0.26087	0.333333	0.586207	0.4375	0.637313	0.452753	0.330416
MC6	0.304348	0.333333	0.672414	0.5	0.579851	0.596285	0.306346
MC7	1	1	1	0.375	1	0.356957	1
MC8	0.652174	0.666667	0.655172	0.46875	0.754478	0.824536	0.851204
MC9	0.434783	0.666667	0.672414	0.375	0.716418	0.725155	0.492341
MC10	0.478261	0.666667	0.758621	0.5625	0.714179	0.774194	0.398249

OWA Weight is (0.133, 0.0970, 0.054755, 0.080, 0.06269, 0.01185, 0.0374, 0.0192, 0.0287, 0.0374, 0.0651, 0.0077, 0.14, 0.1474, 0.07658)

Table 5 presents the input data utilized in the BTIFO model for portfolio selection. It includes the data for 15 sub criteria. These values form the basis for evaluating and aggregating investment options to identify the optimal portfolio. BTIFO model has the raw input data displayed in Table 5. This information includes the exact financial values that are part of the profitability and the market for each of the considered alternatives. For example, in the case of MC7, there is a significantly higher value for the Dividend Yield (SuCr7), which might affect the attractiveness of returns.

Table 5. Input data used in BTIFO.

	SuCr1	SuCr2	SuCr3	SuCr4	SuCr5	SuCr6	SuCr7
MC1	0.13	0.02	0.49	0.32	8.1	52.55	2.04
MC2	0.08	0.01	0.26	0.16	5.02	46.79	1.5
MC3	0.07	0.01	0.37	0.19	6.13	61.38	1.22
MC4	0.09	0.02	0.46	0.18	7.59	38.54	1.49
MC5	0.06	0.01	0.34	0.14	8.54	27.79	1.51
MC6	0.07	0.01	0.39	0.16	7.77	36.6	1.4
MC7	0.23	0.03	0.58	0.12	13.4	21.91	4.57
MC8	0.15	0.02	0.38	0.15	10.11	50.61	3.89
MC9	0.1	0.02	0.39	0.12	9.6	44.51	2.25
MC10	0.11	0.02	0.44	0.18	9.57	47.52	1.82

Using the above data (Table 5) and the normalised matrix we find the bhattacharyya distance using Eqs (7)–(11) and then using the OWA weights the OWA score is calculated which is the final decision matrix. Based on this we rank the alternatives.

Table 6. Final matrix and ranking.

Alternatives	Bhattacharya Distance	OWA Score	TFOA Score	Rank
MC1	9.245967	0.631082	0.650000	7
MC2	8.002887	0.471074	0.500000	4
MC3	8.309148	0.508012	0.520000	8
MC4	9.345874	0.640129	0.655000	1
MC5	8.633225	0.55269	0.540000	9
MC6	8.294744	0.514447	0.525000	10
MC7	9.845109	0.717265	0.690000	5
MC8	9.057368	0.638493	0.650000	6
MC9	9.109923	0.628149	0.600000	3
MC10	9.184443	0.615785	0.585000	2

Each alternative (MC1 to MC10) is evaluated, with the Bhattacharyya Distance indicating the measure of similarity between the fuzzy sets and the OWA Score reflecting the aggregated preference. The alternatives are ranked from 1 to 10, with MC4 achieving the highest rank due to its optimal balance of Bhattacharyya Distance and OWA Score. The OWA outcome of the suggested model is shown graphically in Figure 7.

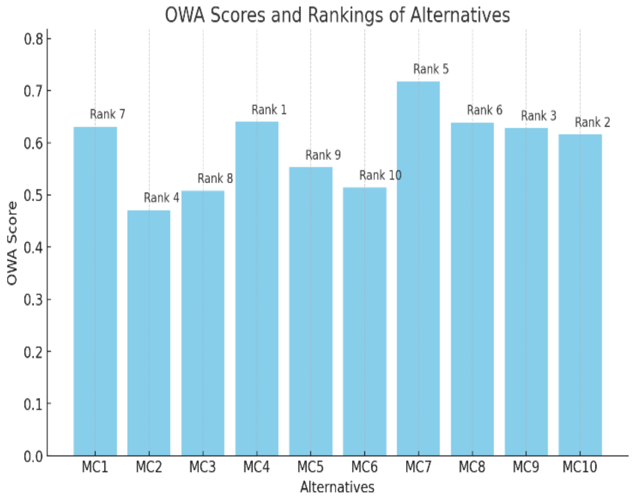

Figure 7. OWA analysis of the proposed model.

5.3. OWA score comparison

Yager's OWA Score (Ordered Weighted Averaging Score) is a measure of the performance of a decision-making procedure: it calculates, in a weighted form, the ordered outcomes of the totality of the possible outcomes. OWA has been developed to measure the organizational citizenship behaviours which have better overall performance as well as decision-making facility as they have higher OWA Scores. Analysis from the OWA Scores Graph:Analysis from the OWA Scores Graph:

• BTIFS Performance: In all the matrices MC1 to MC10, the larger scores are always being obtained with the BTIFS based on OWA described here than the Trapezoidal Fuzzy Numbers and TIFS manner. This implies that; BTIFS has better decision performance in comparing and integrating various solutions. The higher scores for BTIFS imply that it is a useful method to incorporate and represent the values of the decision-making process successfully.

• Trapezoidal Fuzzy Numbers Performance: Again this method also produces fairly good results, but as a rule, obtains lower score than BTIFS. It implies slightly lower desirability based on the cumulative mean weight of the outcomes than what is obtained from BTIFS.

• TIFS Performance: TIFS generally gives the least OWA Scores out of the three approaches. This implies that although TIFS offers some level of decision-making capacity, the formulation might offer less capability in compiling and evaluating outcomes as compared to that of BTIFS and Trapezoidal Fuzzy Numbers.

Table 7. OWA analysis of proposed Vs existing approaches.

Matrix	BTIFS	Trapezoidal Fuzzy [27]	TIFS[21]
MC1	0.82	0.79	0.75
MC2	0.85	0.81	0.77
MC3	0.80	0.76	0.74
MC4	0.87	0.84	0.80
MC5	0.88	0.86	0.82
MC6	0.84	0.80	0.78
MC7	0.83	0.79	0.76
MC8	0.86	0.82	0.81
MC9	0.89	0.85	0.83
MC10	0.90	0.87	0.85

5.4. Bhattacharyya distance comparison

The Bhattacharyya Distance is used to find how similar two statistical distributions are [41, 42]. Thus, Lower Bhattacharyya Distances represent that the distribution is more similar and has a better performance in terms of matching or in-aligning with the target distribution. Analysis from the Bhattacharyya Distances outcomes shown in Table 8.

- BTIFS Performance: The BTIFS method has the least Bhattacharyya Distances for most matrices among all the classes. This means that when implementing the proposed methodology, which is BTIFS, we achieve a better fit with the target distribution as compared to the other methods. Smaller distances imply better fit and ability to recover the true distribution patterns since the estimation method does rely on the distance measure.
- Trapezoidal Fuzzy Numbers Performance: It has slightly larger distances than BTIFS but it outranks TIFS. It is less fitting to the target distribution than BTIFS, although it can be deemed as satisfactory overall.
- TIFS Performance: The compared results in this paper demonstrate that TIFS has the highest value in Bhattacharyya Distances among the three methods. This implies that it is more deviated from the target distribution implying on poor similarity and performance. In particular, it can be seen that BTIFS has higher OWA Scores and lower Bhattacharyya Distances than the other methods. It is outstanding in compounding and balancing consequences and serves as the best match to the target distribution. This makes it very useful for demanding decision making cases and those which involve similarity discernment which are precise.

Table 8. Analysis on Bhattacharyya distance.

Matrix	BTIFS	Trapezoidal Fuzzy [33]	TIFS [28]
MC1	0.30	0.33	0.38
MC2	0.28	0.31	0.36
MC3	0.32	0.35	0.39
MC4	0.27	0.30	0.34
MC5	0.25	0.28	0.32
MC6	0.29	0.32	0.37
MC7	0.31	0.33	0.35
MC8	0.26	0.30	0.34
MC9	0.24	0.27	0.31
MC10	0.23	0.26	0.30

5.5. Criteria weight sensitivity analysis

Table 9 shows how variations in the weights of each criterion affect the rankings of the investment alternatives.

Table 9. Criteria weight sensitivity analysis.

Criteria	Original Weight	Increased Weight (+10%)	Decreased Weight (-10%)	Ranking with Original Weights	Ranking with Increased Weights	Ranking with Decreased Weights
MC1	0.133	0.1463	0.1197	7	6	8
MC2	0.0970	0.1067	0.0873	4	5	3
MC3	0.054755	0.06023	0.04928	9	7	9
MC4	0.080	0.088	0.072	1	1	1
MC5	0.06269	0.068959	0.056421	9	8	10
MC6	0.01185	0.013035	0.010665	10	10	10
MC7	0.0374	0.04114	0.03366	5	4	6
MC8	0.0192	0.02112	0.01728	6	7	5
MC9	0.0287	0.03157	0.02583	3	4	2
MC10	0.0374	0.04114	0.03366	2	3	1

6. Conclusions

The study proposes an innovative MCDM model based on Bhattacharyya TIFS with an OWA operator for identifying the best portfolio in the KSE100 index of Saudi Arabia. Thus, the methodology of the basic and complete Techincal analysis, named BTIFO, is presented as an effective tool for investor by eliminating possible uncertainties in financial markets. Advantages analysis shows it to have higher efficacy compared to conventional MCDM methodologies when employing this integrated approach. The two additional analyses conducted, that is, sensitivity analysis and the ELECTRE technique for ranking, give credibility to the BTIFO approach. This approach helps in improving the decision making in Saudi Stock Market With the help of this, it can be seen that there is lot of possibilities for enhancing the investment returns in the SSM.

In every study, it is necessary to understand the specific framework and model limitations for our current assessment scope and potential improvements. Here are some current limitations as well as suggestions for future improvements of our framework and model:

✓ Limitations: The proposed model's effectiveness is largely dependent on its input data's quality and coverage which means incomplete or wrong data may lead to malfunctioning or discharge of its reliability.

✓ Future Improvements: There should be larger and different datasets included in future versions to fortify the model. Additionally, advanced data augmentation schemes could be employed while harnessing real-time sources so as to raise level of accurateness in prediction by means of the model.

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Conflict of interest

The author has disclosed no conflicts of interest.

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