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Research article

Obesity treatment applying effective fuzzy soft multiset-based decision-making process

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Abstract: Nowadays, obesity is recognized as a worldwide epidemic that has become a major cause of death and comorbidities. Recommending appropriate treatment is critical in the global health environment. For obesity treatment to be effective, the person must be able to follow a specific diet that meets his needs so that he can follow it for a long time or forever to maintain fitness. This research aims to determine the best diet among the trusted diets for every person based on his needs and circumstances. This occurs when applying a decision-making technique based on the effective fuzzy soft multiset concept. For this purpose, the definition of the effective fuzzy soft multiset as well as its types, operations, and properties are introduced. Furthermore, a decision-making method is proposed based on the effective fuzzy soft multiset environment. Using matrices operations, one can easily apply the decision-making process based on this new extension of sets to choose the optimal diet for everyone. Finally, an extensive comparative analysis of the previous methods is undertaken and also summarized in a chart to attract focus on the benefits of the suggested algorithm and to demonstrate how they differ from the current one.

Keywords: decision-making; effective set; fuzzy set; multiset; soft set
Mathematics Subject Classification: 03E72, 03E05, 92C50

1. Introduction

Obesity, as well as its consequences, are major global problems that are continuously increasing. Obesity leads to the development of a wide range of noncontagious diseases, including high blood pressure, type 2 diabetes, and stroke, in addition to cardiovascular, as well as several other types of cancer, and mental health concerns. In addition to obesity, an imbalance between energy expenditure (physical activity), as well as energy intake (diet) leads to being overweight. Obesity or even being overweight happens when the energy intake is greater than the energy expenditure. The problem that many obese patients face when they try to lose weight is that they follow a diet that may be approved and reliable, but it does not suit the needs,requirements, and circumstances of this person. Some prefer to reduce the amount of food, but without depriving it of any type. This is what is called the Luqaimat diet [2], in which satiety of any kind does not occur, but deprivation of any type of food does not occur. Some prefer to eat certain types of food and abstain from eating other types, and in return, they can feel completely satisfied. This is called the keto diet [7]. Others prefer to fast for long hours and then break their fast with one large meal consisting of all types of food and containing all the nutrients. This is called the warrior diet [20], which is one of the fasting diets. Other types of fasting diets including alternate day fasting (ADF) and the 2-5 fast diet, can be found in [22], in addition to [26]. Based on the above, the optimal diet must be determined for each individual based on his answers to several questions as well as his subjection to some tests. Upon obtaining the results of this individual along with some expert opinions, we can apply the proposed methodology to determine the optimal option that he can adhere to for a long time. This long time may be up to six months in cases of overweight and up to one or two years in cases of severe obesity. Some may even follow a diet that is comfortable for them from a physical, psychological, and organizational perspective for life because they are simply comfortable with it and do not want to change it, and this is the most important thing.

Pharmacotherapy, bariatric surgery, and lifestyle changes are the traditional methods of treating obesity. Diet, exercise, and behavior modification-focused behavioral therapies have demonstrated the potential to foster long-term weight loss, as well as enhance metabolic health. However, individual differences in the efficacy of these interventions underscore the necessity of individualized treatment plans catered to the particular requirements, in addition to circumstances, of each patient. The body mass index (BMI) is calculated by dividing a person's weight in kilograms by his or her height in meters squared. BMI is a cheap, as well as simple way to screen for weight categories such as being underweight, having a healthy weight, being overweight, and being obese. When the BMI value is below 18.5, then it indicates the individual is underweight. If the value of BMI is between 18.5 and 24.9, then it shows a healthy, normal, or ideal weight, but if the BMI value ranges between 25 and 29.9, it means the individual is overweight. Furthermore, obesity is proved, when the value of the BMI is between 30 and 34.9. Of course, if the BMI is greater than 35, then it is extreme obesity. Figure 1 sums up the above information about BMI.

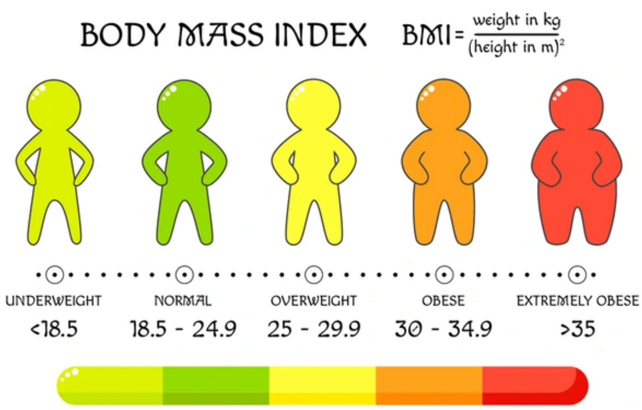

Figure 1. Body mass index.

One can say that the procedure of identifying and picking possibilities based on individual preferences is called decision-making. Officially speaking, the circumstances that surround a decision can be considered as a set of information, and options, in addition to substitutes, along with additional things that are easily accessible at the time a decision needs to be made. We can frame any conclusions we reach within this framework since the effort, as well as time, needed to get information or investigate alternatives, restricts knowledge or substitutes for it. In fact, because decision-making is closely linked to efficiency and success, it has become more and more significant in recent years. Successful people use productive, effective decision-making skills to achieve their professional goals, along with their personal goals. Decision-makers frequently use their viewpoints, beliefs, attitudes, concepts, and ideas to guide their choices.

In 1965, Zadeh [33] invented the concept of fuzzy sets, an alternative viewpoint to the crisp set theory that addresses uncertainty. The fuzzy set can be seen as an extension of the crisp set. A fuzzy set over a domain X, like a crisp set over a universal set X, can be defined by the characteristic (membership) function that maps from X to the interval [0,1].

Alperin, as well as Berzosa [6], in 2011, used the fuzzy sets concept to estimate the prevalence of being overweight in Luxembourg. This approach makes it possible to distinguish between moderate, in addition to being seriously overweight, along with identifying the categories of people who are contributing the most to being overweight worldwide. In addition, in 2019, Ibne Farhad et al. [11] applied the fuzzy sets concept to balance human weight to be ideal, not overweight or underweight, since both of them have serious effects on human health.

If one relied on ordinary or fuzzy knowledge only, the lack of a parameterized tool might be confusing when dealing with issues that demand it. Molodtsov [25] developed the unique concept of soft set theory in 1999. The newly developed concept of soft set theory, as provided through the softness definition framework, is a fresh and practical mathematical instrument that is free of the aforementioned difficulties. In 2016, Ahmed et al. [1] used the soft computing technique to remove or minimize the wrong measurement of BMI or wrong diagnosis by medical practitioners to determine the accurate correct category of the patient. After that, Maji et al. eventually performed a detailed investigation, in addition to doing a systematic review of Molodtsov soft set's unfamiliar surroundings in 2002 (see [23], as well as [24] for documentation). They were trying to clear any points that weren't clear in Molodtsov's work about soft sets. They also explored several concepts related to the new concept of the soft set, in addition to developing a comprehensive conceptual framework for this information, along with applying the soft set theory to a practical decision-making setting and environment.

Then, in 2001, Maji et al. developed the innovative concept of fuzzy soft settings by combining the previously presented idea of the fuzzy set with the soft set idea. Moreover, Roy and Maji et al. created a decision-making methodology based on the fuzzy soft set theory to aid in any necessary selection of the best option from a variety of options. Furthermore, based on the primary fuzzy soft set settings, Yang et al. [32] developed a matrix notation for the fuzzy soft set theory. In addition, Çağman et al. [9] carried out a thorough investigation of fuzzy soft matrices, in addition to carrying out many algebraic operations, along with theoretical research in the relatively unexplored area of fuzzy soft set surroundings. Kumar and Kaur et al. [21], and Basu et al. [8] presented new ideas in addition to their associated operations in that study on fuzzy soft matrices. To learn more about the broadened fuzzy soft environments, along with their newly acquired characteristics, one can access an extensive number of results, theorems, and helpful illustrations by consulting [12–19].

In 2023, El-Atik et al. [10] used fuzzy soft sets to promote healthy, in addition to balanced diets by selecting a burning problem for the nutrition of students, as well as successfully applying the fuzzy soft set theory in decision-making. Alkhazaleh et al. [5] introduced the definition of soft multiset, along with establishing its properties, operations, and applications. After that, Alkhazaleh and Salleh [4] gave the definition of fuzzy soft multiset as a combination of the soft multiset, in addition to the fuzzy set, and studied its properties, operations, and applications. The multiset (or mset) is a development of the idea of the ordinary set. In fact, unlike the ordinary set, the multiset lets you have many repetitions of each element. The multiplicity of an element in a multiset is the number of repetitions assigned to it.

Later, Alkhazaleh noted a limitation in the fuzzy soft set environment in 2022. According to this idea, in certain scenarios involving decision-making, the ultimate choice can typically be made based only on traditional characteristics, without taking into account external factors. Alkhazaleh developed a brand new concept, known as the "effective parameter set", for expressing these external parameters to overcome this problem. Furthermore, he came up with another entirely novel notion, known as "effective fuzzy soft sets," which is predicated on the inventive notion of effective sets. Alkhazaleh also gave an example of how to apply the useful fuzzy soft setting to specific decision-making issues. Alkhazaleh's last example demonstrated the applicability of this novel theory to medical diagnostics and included a fictitious case study to highlight the process.

In 2022, for risky multi-criteria decision-making (MCDM) issues with the interval type-2 fuzzy (IT2F) truth degrees problems, in which the criteria have heterogeneous relationships and the decision-makers act following bound rationality, Tang et al. [28] suggested a novel IT2F programming technique. In addition, in the same year, Tang et al. [29] proposed a q-rung orthopair fuzzy (q-ROF) multiple attribute decision-making (MADM) strategy to assist in tackling issues with decision-making while evaluating medical apps. After that, Tang et al. [30] established a new multi-objective q-ROF programming technique for heterogeneous group decision-making in 2023. Moreover, in the same year, for multiple attribute group decision-making (MAGDM) problems, Tang et al. [31] created an R-mathematical programming methodology that uses R-sets to describe the truth degrees of pairwise alternative comparisons, as well as assessment values of alternatives, all while the decision maker maintains subjective bounded rationality. Furthermore, in 2024, Saqlain and Saeed [27] gave a comprehensive understanding of similarity measures in the surroundings of multi-polar interval-valued intuitionistic fuzzy soft sets.

Work motivation

We have outlined four distinct previously discussed types of sets as follows: the fuzzy set, as well as the multiset, in addition to the soft set, along with discussing the effective set, as was already mentioned in the discussion above. Furthermore, there exist other combined sets that fuse the previously described concepts; like the fuzzy soft set, along with the recently developed notion of the effective fuzzy soft set. However, to address some concerns, it is still necessary to broaden the definition of the unique idea of the effective fuzzy soft set to include the effective fuzzy soft multisets, which is a bigger domain.

Recent studies indicate high rates of people with obesity, and who are overweight in the Kingdom of Saudi Arabia, where it ranks third in the Arab world, coming after both Kuwait and Qatar. To eliminate being overweight, obesity, and the diseases that result from them that may lead to death in the Kingdom of Saudi Arabia, we must apply a decision-making strategy based on the effective fuzzy soft multiset to select the optimal diet suitable for each person so that he can follow it permanently to lose weight, then reach the ideal weight and not go back to obesity again.

The choice of using fuzzy soft multisets in the study over other generalizations of fuzzy sets, such as rough sets, intuitionistic fuzzy sets (IFS), or q-ROFs, likely stems from the specific characteristics and advantages that fuzzy soft multisets offer in the context of obesity treatment and decision-making.

The motivation for this research comes from the fact that the integration of the effective set concept, and fuzzy soft set theory with the multiset approach to make decisions for obesity treatment holds promise for improving patient outcomes, enhancing clinical decision support, and advancing personalized healthcare delivery. This occurs when we have some external factors impacting the decision, which are called effective parameters, along with the usual parameters represented in some preferences and circumstances of the patients. Of course, the fuzzy soft environment is the best one that can represent these values because the fuzzy value expresses a number from 0 to 1 which is suitable. When we talk about obesity, we have three categories; male, female, and child, so we need to use a multiset concept to classify the three categories. Combining these concepts, the effective set, the fuzzy set, the soft set, and the multiset leads to the effective fuzzy soft multiset which is the most suitable outline for this proposed decision-making environment.

Work contribution

The innovative value of the contribution proposed by the authors in this study lies in its originality because of its novel application of the effective fuzzy soft multisets to a challenging and crucial healthcare problem. The study has the potential to revolutionize personalized medicine and clinical decision-making by building a new decision-making method that can handle the complexities of obesity treatment. This could result in more effective treatment techniques, better patient outcomes, and a wider use of the methodology in other complicated decision-making situations.

As a consequence, the effective fuzzy soft multiset is covered in this article together with its different features, as well as its classifications, along with giving fully operational guidelines, in addition to its applications in practical situations. The following is an outline of the sections that follow in this work: The essential definitions, as well as underlying concepts, are covered in Section (2). Moreover, Section (3) seeks to explore the notion of the effective fuzzy soft multiset, as well as clarify its different varieties, in addition to presenting a few novel concepts that are connected to it. Besides this, the procedures of the union, in addition to the intersection related to these sets are explained in Section (4). Furthermore, Section (5) goes on to combine other relevant qualities such as distributive laws, as well as absorption properties, in addition to commutative properties, and associative properties, along with De Morgan's laws. Finally, the primary focus of Section (6) is to create a decision-making mechanism based on the efficient fuzzy soft multiset construction.

How to arrive at the best way to make a decision is the goal. In this way, determining the optimal diet for any obese or overweight patient is possible. This diet must be suitable for him, allowing him to follow it for a very long time or even for life. This is an attempt to get rid of obesity in the Kingdom of Saudi Arabia, return to fitness, and be freed from the prison of fat. Finally, using the *Wolfram Mathematica*® program allows us to do addition of matrices, as well as multiplication of matrices, which makes it easier to derive effective sets and speeds up computations. To highlight the differences between the current method and the old ones, a thorough comparison with the latter is carried out and summarized in a chart to make the picture complete in Section (6). At last, Section (7) is devoted to summarizing key takeaways and outlining possible directions for further research. Figure 2 illustrates the structural arrangement of the paper's material and provides an overview of its contents.

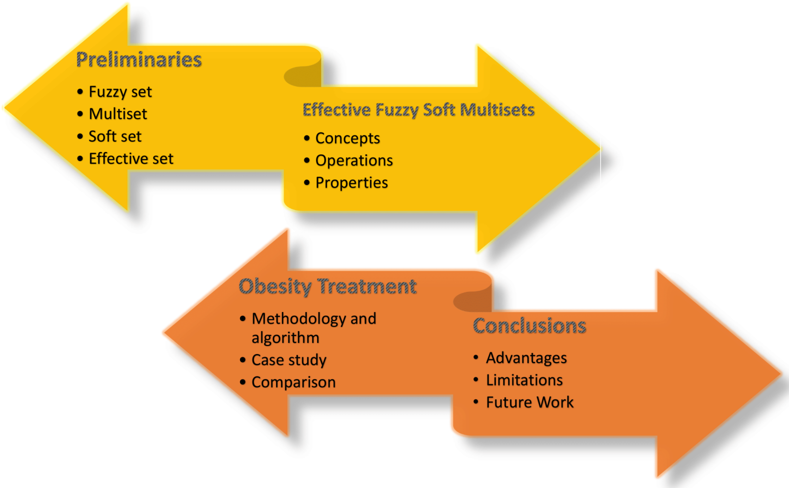

Figure 2. Paper content.

2. Preliminaries

This section goes over the fundamental terminology needed to understand the results that come next. This part includes concepts for the fuzzy set, in addition to the soft set, as well as the soft multiset, and the fuzzy soft set, along with the effective fuzzy set, in addition to the effective fuzzy soft set. Refer to [3, 5, 25, 33] for more explanations, and examples, in addition to more detailed results about the above ideas.

Definition 2.1. ***(Fuzzy set)*** *[33]*

Let us assume that we have an initial universal set Ψ. A fuzzy class (or a fuzzy set) F over Ψ can be constructed in the following way: a set distinguished by a membership function, namely χ_F, taking into account that $\chi_F : \Psi \to [0, 1]$. For the fuzzy set F, the characteristic function or the indicator function can also be considered as other names for the concept χ_F of the membership function. Furthermore, the membership grade value or the degree of membership of an element ψ in Ψ throughout the fuzzy set F is denoted as $\chi_F(u)$. In such a scenario, one of the two sequel forms: $F = \{(\psi, \chi_F(\psi)) : \psi \in \Psi,\ \chi_F(\psi) \in [0, 1]\}$, or $F = \{(\chi_F(\psi)/\psi) : \psi \in \Psi,\ \chi_F(\psi) \in [0, 1]\}$ can be used to represent the fuzzy set F over the original universal set Ψ.

Definition 2.2. ***(Soft set)*** *[25]*

Take into consideration that we have an initial universal set Ψ, in addition to a set of parameters (or attributes) Θ, as well as a subset Λ of Θ. Moreover, keep in mind that the power set of Ψ is obtained by calculating $P(\Psi) = 2^{\Psi}$. Furthermore, given that $\daleth$ is a mapping represented as $\daleth : \Lambda \to P(\Psi)$, a pair indicated as $(\daleth, \Lambda)$, or even $\daleth_\Lambda$, is described as a soft set over the universal set Ψ in this context. It is also possible to express $\daleth_\Lambda$ as an ordered pair set as the following: $\daleth_\Lambda = (\lambda, \daleth_\Lambda(\lambda)) : \lambda \in \Lambda, \daleth_\Lambda(\lambda) \in P(\Psi)$. In such a way, the support of $\daleth_\Lambda$ can be recognized by Λ. In addition, it is noteworthy that, for every $\lambda \in \Lambda$, we have $\daleth_\Lambda(\lambda) \neq \phi$, as well as, for any $\lambda \notin \Lambda$, we have $\daleth_\Lambda(\lambda) = \phi$. This scenario, then, suggests that one can think of a parameterized collection of subsets of the universal set Ψ as the concept of the soft set $(\daleth, \Lambda)$.

Definition 2.3. ***(Soft multiset)*** *[5]*

Assume that $\{\Psi_i, i \in I\}$ *is a collection of universal sets, taking into account that* $\bigcap_{i\in I}\Psi_i = \phi$. *In addition, suppose that* $\{\Theta_{\Psi_i}, i \in I\}$ *is a collection of sets of parameters. Furthermore, consider that* $\Psi = \prod_{i\in I} P(\Psi_i)$, *taking into consideration that* $P(\Psi_i)$ *represents the power set of* Ψ_i, $\Theta = \prod_{i\in I}\Theta_{\Psi_i}$, *as well as letting that* $\Lambda \subseteq \Theta$. *In such a scenario, a pair symbolized as* $(\daleth, \Lambda)$, *or even* $\daleth_\Lambda$, *is described as a soft multiset over the universal set* Ψ, *in which* $\daleth$ *is a mapping with the notation* $\daleth : \Lambda \to \Psi$.

Definition 2.4. ***(Effective fuzzy set)*** *[3]*

A fuzzy set Υ *constructed over the initial universal set* Ω *is designated as an effective fuzzy set, taking into account that* Υ *can be illustrated by the mapping* $\Upsilon : \Omega \to [0, 1]$. *In this case,* Ω *essentially refers to the set of all effective parameters or attributes that have an impact on each element's membership value. When applied to an element, these effective parameters have a positive effect on its membership value. It is important to note that some membership values don't change even after the application of the effective parameters. In this context, the following formulation:* $\Upsilon = \{(\omega, \varrho_\Upsilon(\omega)), \omega \in \Omega\}$ *can be used to represent the effective fuzzy set. Note that* $\varrho_\Upsilon(\omega)$ *expresses the effective membership value for a certain* $\omega \in \Omega$ *in this formulation.*

Definition 2.5. ***(Effective fuzzy soft set)*** *[3]*

Assume that we have the collection of all fuzzy subsets of an initial universal set Ψ *identified as* $\mathfrak{F}(\Psi)$. *Moreover, consider that the standard parameters are* $\theta_i \in \Theta$. *Additionally, take into account that the set of effective parameters serves as* Ω, *in addition to letting the effective set over* Ω *emerge as* Υ. *Referred to an effective fuzzy soft set over* Ψ *in this context, we mean the pair* $(\Delta_\Upsilon, \Theta)$. *It is understood that the mapping* $\Delta : \Omega \to \mathfrak{F}(\Psi)$ *can be determined by the following given expression:* $\Delta(\omega_i)\Upsilon = \{(\psi j, \chi_\Delta(\psi_j)\Upsilon), \psi j \in \Psi, \omega_i \in \Omega\}$. *The sequel equation is applicable, for each* $\omega_k \in \Omega$, *as given below:*

$$\chi_\Delta(\psi_j, \theta_i)_\Upsilon = \begin{cases} \chi_\Delta(\psi_j, \theta_i) + \frac{(1-\chi_\Delta(\psi_j,\theta_i))\sum_k \varrho_{\Upsilon_{\psi_j}}(\omega_k)}{|\Omega|}, & if\ \chi_\Delta(\psi_j, \theta_i) \in (0, 1), \\ \chi_\Delta(\psi_j), & otherwise. \end{cases} \tag{2.1}$$

In the overall picture described above, note that $|\Omega|$ *denotes the number of items in the provided effective parameter set* Ω, *as well as* $\chi_\Delta(\psi_j, \theta_i)$ *signifies the element* ψ_j*'s membership degree value for the given parameter* θ_i, *in addition to* $\sum_k \varrho_{\Upsilon_{\psi_j}}(\omega_k)$ *indicates the overall number of all effective parameter values corresponding to* ψ_j *(specific item).*

Example 2.1. *Assume that one can begin with an initial universal set* Ψ *that includes the following items:* ψ_1, ψ_2, *and* ψ_3, *in addition to a parameter set* Θ *that consists of the following parameters:* θ_1, θ_2, *and* θ_3. *Furthermore, for the parameter* θ_1, *suppose that the fuzzy soft set is as follows:* $(\Delta, \Theta)(\theta_1) = \{(\psi_1, 0.6), (\psi_2, 0.2), (\psi_3, 0.9)\}$. *For the first element* ψ_1 *possessing* 0.3 *as a membership value for the first parameter* θ_1, *we must take into account the provided effective set* $\Upsilon(\psi_1) = \{(\omega_1, 0.7), (\omega_2, 0), (\omega_3, 0.5), (\omega_4, 1)\}$, *for* ψ_1 *to calculate its effective membership value. The effective parameters given here are* ω_1, ω_2, ω_3, *and* ω_4. *With Formula 2.1 stated in Definition (2.5), one can determine the effective membership value as the following:*

$$\chi_\Delta(\psi_1, \theta_1)_\Upsilon = 0.6 + \frac{(1 - 0.6)(0.7 + 0 + 0.5 + 1)}{4} = 0.6 + \frac{0.4 \times 2.2}{4} = 0.6 + 0.22 = 0.82.$$

Likewise, for the other two parameters, θ_2, as well as θ_3 of ψ_1, as well as for the other two elements, ψ_2, along with ψ_3, one can easily calculate the effective membership values. Please see [3], page 3, for a more thorough example that demonstrates this term.

Remark 2.1. *To make things less complicated, rather than going over the whole complicated Formula 2.1 that is mentioned in Definition 2.5, we can write χ_Υ instead of $\chi_\Delta(\psi_j, \theta_i)_\Upsilon$ that shows the effective membership value that corresponds to the membership value $\chi_\Delta(\psi_j, \theta_i)$ of a specific item ψ_j for a specific parameter θ_i. This acronym is particularly useful when discussing the fuzzy soft set Δ exclusively. Moreover, $\varrho_{\mu_{\psi_j}}$ can be streamlined to ϱ_μ. In contrast, we should use the full formulas like $\chi_{\Delta_1}(\psi_j, \theta_i)$, as well as $\chi_{\Delta_2}(\psi_j, \theta_i)$ for Δ_1, in addition to Δ_2, respectively, to distinguish between two or more fuzzy soft sets.*

3. Effective fuzzy soft multisets

In this section, the main purpose is to define the effective fuzzy soft multiset, as well as to make it clearer with an illustrative example. Moreover, several kinds of effective fuzzy soft multisets, in addition to some related concepts to them are established.

Definition 3.1. ***(Effective fuzzy soft multiset)***

Suppose that $\{\Psi_i, i \in I\}$ is a collection of universal sets taking into consideration that $\bigcap_{i\in I}\Psi_i = \phi$, along with $\Psi = \prod_{i\in I}\mathfrak{F}(\Psi_i)$, in which $\mathfrak{F}(\Psi_i)$ represents the collection of all fuzzy subsets on Ψ. Moreover, let $\{\Theta_{\Psi_i}, i \in I\}$ be a collection of sets of parameters such that $\Theta = \prod_{i\in I}\Theta_{\Psi_i}$. Furthermore, consider that Ω is the effective parameter set, as well as μ signifies the effective set defined over Ω. Consequently, in such a particular scenario, the effective fuzzy soft multiset over Ψ can be symbolized as the pair (Δ_μ, Θ), in addition to the designation recognizing that the following formula can be used to create the mapping $\Delta : \Omega \to \mathfrak{F}(\Psi)$:

$$\Delta(\omega_i)_\mu = \{(\psi_j, \gamma_\Delta(\psi_j)_\mu), \psi_j \in \Psi, \omega_i \in \Omega\},$$

understanding that the following is true for any $\omega_k \in \Omega$:

$$\gamma_\mu = \begin{cases} \gamma + \frac{(1-\gamma)\sum_k \eta_\mu(\omega_k)}{|\Omega|}, & if\ \gamma \in (0,1), \\ \gamma, & otherwise, \end{cases} \tag{3.1}$$

considering the fact that $|\Omega|$ denotes the number of items contained in the supplied set of the effective parameters Ω.

Example 3.1. *Assume that there are three initial universal sets Ψ_1, Ψ_2, and Ψ_3 representing three different categories of objects, respectively, with their three associated parameter sets Θ_{Ψ_1}, Θ_{Ψ_2}, and Θ_{Ψ_3}. Let $\Psi_1 = \{m_1, m_2, m_3\}$ represent three available cars, $\Psi_2 = \{s_1, s_2\}$ represent two available houses, and $\Psi_3 = \{f_1, f_2, f_3, f_4\}$ represent four available wedding dresses. Suppose that a woman wants to choose the best car, the best house, and the best wedding dress for her to buy. The parameter sets that determine the attributes for each universal set are as follows: $\Theta_{\Psi_1} = \{a_1, a_2, a_3\}$ associated with the universal set of cars Ψ_1, where a_1 = expensive, a_2 = comfortable, and a_3 = modern, $\Theta_{\Psi_2} = \{b_1, b_2, b_3\}$ associated with the universal set of houses Ψ_2, where b_1 = expensive, b_2 = near city center, and b_3 = wide living space, and $\Theta_{\Psi_3} = \{c_1, c_2, c_3\}$ associated with the universal set of wedding dresses Ψ_3, where*

c_1 = beautiful, c_2 = comfortable, and c_3 = fashionable. Then, the following fuzzy soft multiset (Δ, Θ) describes the attractiveness of all above items:

$$\begin{aligned}(\Delta, \Theta) = \{&(a_1, \{(m_1, 0.4), (m_2, 0.8), (m_3, 0.6)\}), (a_2, \{(m_1, 1), (m_2, 0.3), (m_3, 0)\}),\\ &(a_3, \{(m_1, 0.2), (m_2, 0.9), (m_3, 0)\}), (b_1, \{(s_1, 0.8), (s_2, 0.5)\}), (b_2, \{(s_1, 1), (s_2, 0.6)\}),\\ &(b_3, \{(s_1, 0.1), (s_2, 0)\}), (c_1, \{(f_1, 0.7), (f_2, 0.4), (f_3, 1), (f_4, 0.5)\}),\\ &(c_2, \{(f_1, 1), (f_2, 0.3), (f_3, 0.9), (f_4, 0)\}), (c_3, \{(f_1, 0.2), (f_2, 0.7), (f_3, 1), (f_4, 0.6)\})\}.\end{aligned}$$

In addition, if $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ is the set of effective parameters, where ω_1 = there is previous ownership, ω_2 = installment payment system available, ω_3 = warranty coverage for maintenance, and ω_4 = there are additional costs like taxes, as well as insurance, then the effective set μ over Ω can be as follows:

$$\begin{aligned}\mu(m_1) &= \{(\omega_1, 0.1), (\omega_2, 0.3), (\omega_3, 1), (\omega_4, 0)\},\\ \mu(m_2) &= \{(\omega_1, 0.8), (\omega_2, 0.2), (\omega_3, 0), (\omega_4, 0.3)\},\\ \mu(m_3) &= \{(\omega_1, 1), (\omega_2, 0.1), (\omega_3, 0.9), (\omega_4, 0)\},\\ \mu(s_1) &= \{(\omega_1, 0.3), (\omega_2, 0.7), (\omega_3, 0), (\omega_4, 1)\},\\ \mu(s_2) &= \{(\omega_1, 1), (\omega_2, 0.5), (\omega_3, 0.1), (\omega_4, 0)\}.\\ \mu(f_1) &= \{(\omega_1, 0), (\omega_2, 0.2), (\omega_3, 0.4), (\omega_4, 0.1)\}.\\ \mu(f_2) &= \{(\omega_1, 0.1), (\omega_2, 0.6), (\omega_3, 0.7), (\omega_4, 1)\}.\\ \mu(f_3) &= \{(\omega_1, 0), (\omega_2, 0), (\omega_3, 0.2), (\omega_4, 0.7)\}.\\ \mu(f_4) &= \{(\omega_1, 0.9), (\omega_2, 0), (\omega_3, 1), (\omega_4, 0)\}.\end{aligned}$$

After that, for the first object m_1, having 0.4 as a membership value for the first parameter a_1, one can use the abovementioned effective set μ for m_1 to calculate its effective membership value using Formula 3.1 given in Definition (3.1) as below:

$$\begin{aligned}\gamma_\Delta(m_1, a_1)_\mu &= 0.4 + \frac{(1-0.4)(0.1+0.3+1+0)}{4}\\ &= 0.4 + \frac{0.6 \times 1.4}{4} = 0.4 + \frac{0.84}{4} = 0.4 + 0.21 = 0.61.\end{aligned}$$

Similarly, for the other membership values of m_1 associated with the two other parameters, a_2, as well as a_3, one can determine the effective membership values.

By repeating this process, we, then, have the effective fuzzy soft multiset (Δ_μ, Θ) on Ψ, effectively illustrating the attraction of the previously mentioned items as shown below:

$$\begin{aligned}(\Delta_\mu, \Theta) = \{&(a_1, \{(m_1, 0.61), (m_2, 0.865), (m_3, 0.8)\}), (a_2, \{(m_1, 1), (m_2, 0.5275), (m_3, 0)\}),\\ &(a_3, \{(m_1, 0.48), (m_2, 0.9325), (m_3, 0)\}), (b_1, \{(s_1, 0.9), (s_2, 0.5)\}), (b_2, \{(s_1, 1),\\ &(s_2, 0.76)\}), (b_3, \{(s_1, 0.5), (s_2, 0)\}), (c_1, \{(f_1, 0.7525), (f_2, 0.76), (f_3, 1),\\ &(f_4, 0.7375)\}), (c_2, \{(f_1, 1), (f_2, 0.72), (f_3, 0.9225), (f_4, 0)\}), (c_3, \{(f_1, 0.34),\\ &(f_2, 0.88), (f_3, 1), (f_4, 0.79)\})\}.\end{aligned}$$

The description that (Δ_μ, Θ) presents can help the purchasers decide which products are best for their needs, as well as desires.

Definition 3.2. ***(Complete effective fuzzy soft multiset)***

Suppose that $\{\Psi_i, i \in I\}$ is a collection of universal sets in which $\bigcap_{i\in I}\Psi_i = \phi$ as well as $\Psi = \prod_{i\in I}\mathfrak{F}(\Psi_i)$, taking into account that $\mathfrak{F}(\Psi_i)$ represents the collection of all fuzzy subsets on Ψ. Moreover, assume that $\{\Theta_{\Psi_i}, i \in I\}$ is a collection of parameters sets along with $\Theta = \prod_{i\in I}\Theta_{\Psi_i}$. Consider that μ is the effective set defined over Ω, and Ω represents the effective parameter set. Therefore, any effective fuzzy soft multiset $(C\mu, \Theta)$ that is specified over Ψ and created with the help of an effective set μ can be referred to as complete (or absolute). The definition of this absolute set is as follows: It maintains that $\gamma\Delta_\Theta(\theta)(\psi)\mu = 1$ for every $\theta \in \Theta$ for each $\theta \in \Theta$. Put otherwise, the representation of the absolute set as $(C\mu, \Theta) = \{(\theta, \psi, 1) : \theta \in \Theta, \psi \in \Psi\}$, for each $\theta \in \Theta$,, in addition to each $\psi \in \Psi$.

Definition 3.3. ***(Null effective fuzzy soft multiset)***

Assume that $\{\Psi_i, i \in I\}$ serves as a collection of universal sets satisfying $\bigcap_{i\in I}\Psi_i = \phi$, as well as $\Psi = \prod_{i\in I}\mathfrak{F}(\Psi_i)$, where $\mathfrak{F}(\Psi_i)$ is the collection of all fuzzy subsets on Ψ. In addition, assume that $\{\Theta_{\Psi_i}, i \in I\}$ represents a collection of parameters sets in addition to $\Theta = \prod_{i\in I}\Theta_{\Psi_i}$. Furthermore, suppose that Ω signifies the effective parameter set, as well as μ is the effective set defined over Ω. Then, any effective fuzzy soft multiset created using an effective set μ in Ψ denoted as (Δ_μ, Θ), is considered null (or empty), indicated as (Φ_μ, Θ). This property is what distinguishes this empty set: For any $\theta \in \Theta$, it can be determined that $\gamma\Delta_\Theta(\theta)(\psi)\mu = 0$. This means that the empty set is $(\Phi\mu, \Theta) = \{(\theta, \psi, 0) : \theta \in \Theta, \psi \in \Psi\}$ for all $\theta \in \Theta$, in addition to for all $\psi \in \Psi$.

4. Operations on effective fuzzy soft multi sets

This section's main objective is to introduce operations on effective fuzzy soft multisets. Numerous operations are defined, including union, and intersection, as well as complement, and subset. Moreover, an example of how to carry out each operation is given for each one.

First, consider that $\{\Psi_i, i \in I\}$ serves as a collection of universal sets, keeping into consideration that $\bigcap_{i\in I}\Psi_i = \phi$, in addition to $\Psi = \prod_{i\in I}\mathfrak{F}(\Psi_i)$, in which $\mathfrak{F}(\Psi_i)$ represents the collection of all fuzzy subsets on Ψ. Moreover, let $\{\Theta^1_{\Psi_i}, i \in I\}$, as well as $\{\Theta^2_{\Psi_i}, i \in I\}$ be two collections of parameters sets, satisfying that $\Theta^1 = \prod_{i\in I}\Theta^1_{\Psi_i}$, in addition to $\Theta^2 = \prod_{i\in I}\Theta^2_{\Psi_i}$, respectively. Furthermore, consider μ_1, as well as μ_2 are two effective-parameter sets defined over Ω.

Definition 4.1. ***(Union operation of the two effective fuzzy soft multisets)***

The union operation of two effective fuzzy soft multisets, symbolized by $(\Delta_{1\mu_1}, \Theta^1)$, as well as $(\Delta_{2\mu_2}, \Theta^2)$, over Ψ can be displayed as a newly created, effective fuzzy soft multiset that is designated as $(\Delta^U\mu^U, \Theta^U)$. We have the notation $\{(\omega, \eta^U(\omega)), \omega \in \Omega\}$ providing the effective set μ^U. In addition, we have $\eta^U = \eta_1\widetilde{\cup}\eta_2$ yielding the effective membership union value, η^U, in which η_1, as well as η_2 denote the effective membership values related, respectively, to μ_1, as well as μ_2. Furthermore, we establish that $(\Delta, \Theta)^U = (\Delta^U, \Theta^U) = (\Delta_1, \Theta^1)\widetilde{\cup}(\Delta_2, \Theta^2)$ for (Δ_1, Θ^1), along with (Δ_2, Θ^2). It could be important to maintain in mind that this approach considers $\Theta^U = \Theta^1 \cup \Theta^2$. The computation of η^U for each $\omega \in \Omega$, as well as for each $\psi \in \Psi$ can be done using the following formula:

$$\eta_{\mu^U_\psi}(\omega)=\begin{cases}\eta_{\mu_{1\psi}}(\omega), & if\ \omega\in\mu_1-\mu_2,\\ \eta_{\mu_{2\psi}}(\omega), & if\ \omega\in\mu_2-\mu_1,\\ \max\{\eta_{\mu_{1\psi}}(\omega),\eta_{\mu_{2\psi}}(\omega)\}, & if\ \omega\in\mu_1\cap\mu_2.\end{cases} \tag{4.1}$$

On top of that, the formula using to calculate $(\Delta^U,\Theta^U)=(\Delta_1,\Theta^1)\widetilde{\cup}(\Delta_2,\Theta^2)$ can be investigated, for each $\psi\in\Psi$, as the following:

$$(\Delta^U,\Theta^U)=\begin{cases}\{(\theta,\{\psi,\gamma_{\Delta_1(\theta)}(\psi)_\mu\}),\psi\in\Psi\},\ if\ \theta\in\Theta^1-\Theta^2,\\ \{(\theta,\{\psi,\gamma_{\Delta_2(\theta)}(\psi)_\mu\}),\psi\in\Psi\},\ if\ \theta\in\Theta^2-\Theta^1,\\ \{(\theta,\{\psi,\max\{\gamma_{\Delta_1(\theta)}(\psi)_\mu,\gamma_{\Delta_2(\theta)}(\psi)_\mu\}\}),\psi\in\Psi\}, if\ \theta\in\Theta^1\cap\Theta^2,\end{cases} \tag{4.2}$$

for each $\theta\in\Theta^U$.

Example 4.1. *Considering the information contained in Example (3.1), we can construct two effective sets μ_1, as well as μ_2 over $\Omega=\{\omega_1,\omega_2,\omega_3,\omega_4\}$, for $m_1,m_2\in\Psi_1$, as well as $s_1,s_2\in\Psi_2$, in addition to $f_1,f_2\in\Psi_3$, as given below:*

$$\mu_1(m_1)=\{(\omega_1,0.5),(\omega_2,0.4),(\omega_3,0.9),(\omega_4,1)\},\mu_2(m_1)=\{(\omega_1,0.2),(\omega_2,0.6),(\omega_3,0.8)\},$$

$$\mu_1(m_2)=\{(\omega_1,0.2),(\omega_2,0.4),(\omega_3,0.7)\},\mu_2(m_2)=\{(\omega_1,0.1),(\omega_2,0.4),(\omega_4,0.5)\},$$

$$\mu_1(s_1)=\{(\omega_1,0.2),(\omega_2,0.6),(\omega_3,0.8),(\omega_4,1)\},\mu_2(s_1)=\{(\omega_2,0.7),(\omega_3,0.9),(\omega_4,1)\},$$

$$\mu_1(s_2)=\{(\omega_1,0.2),(\omega_2,0.1),(\omega_4,0.2)\},\mu_2(s_2)=\{(\omega_1,0.1),(\omega_2,0.2),(\omega_4,0.2)\},$$

$$\mu_1(f_1)=\{(\omega_1,0.3),(\omega_3,0.5),(\omega_4,0.2)\},\mu_2(f_1)=\{(\omega_1,0.4),(\omega_3,0.4),(\omega_4,0.1)\},$$

$$\mu_1(f_2)=\{(\omega_1,0.1),(\omega_2,0.5),(\omega_4,0.8)\},\mu_2(f_2)=\{(\omega_1,0.1),(\omega_2,0.4),(\omega_3,0.1),(\omega_4,0.7)\},$$

related, respectively, to the two following fuzzy soft multisets (Δ_1,Θ^1), as well as (Δ_2,Θ^2) defined on Ψ:

$$\begin{aligned}(\Delta_1,\Theta^1)=\{&(a_1,\{(m_1,0.2)\}),(a_2,\{(m_1,0.9),(m_2,0.4)\}),(a_3,\{(m_1,0.4),(m_2,0.9)\}),\\&(b_1,\{(s_1,0.8),(s_2,0.2)\}),(b_2,\{(s_1,0.3),(s_2,1)\}),(b_3,\{(s_1,0.6),(s_2,0.1)\}),\\&(c_1,\{(f_2,0.7)\}),(c_2,\{(f_1,1),(f_2,0.6)\}),(c_3,\{(f_1,0.2),(f_2,0.4)\})\},\end{aligned}$$

$$\begin{aligned}(\Delta_2,\Theta^2)=\{&(a_1,\{(m_1,0.3),(m_2,0.1)\}),(a_2,\{(m_1,0.6),(m_2,0.5)\}),(a_3,\{(m_2,1)\}),\\&(b_1,\{(s_1,0.4),(s_2,0.2)\}),(b_2,\{(s_1,0.6),(s_2,0.7)\}),(b_3,\{(s_1,0.2),(s_2,0.1)\}),\\&(c_1,\{(f_1,0.3),(f_2,0.5)\}),(c_2,\{(f_1,0.9),(f_2,0.8)\}),(c_3,\{(f_1,0.9)\})\}.\end{aligned}$$

Then, the union μ^U of the two effective sets for each component of the multiset can be calculated, as shown below, by applying Formula 4.1 stated in Definition (4.1):

$$\mu^U(m_1)=\{(\omega_1,0.5),(\omega_2,0.6),(\omega_3,0.9),(\omega_4,1)\},\mu^U(m_2)=\{(\omega_1,0.2),(\omega_2,0.4),(\omega_3,0.7),(\omega_4,0.5)\},$$

$$\mu^U(s_1)=\{(\omega_1,0.2),(\omega_2,0.7),(\omega_3,0.9),(\omega_4,1)\},\mu^U(s_2)=\{(\omega_1,0.2),(\omega_2,0.2),(\omega_4,0.2)\},$$

$$\mu^U(f_1)=\{(\omega_1,0.4),(\omega_3,0.5),(\omega_4,0.2)\},\mu^U(f_2)=\{(\omega_1,0.1),(\omega_2,0.5),(\omega_3,0.1),(\omega_4,0.8)\}.$$

In addition, the union of the two fuzzy soft multisets (Δ_1, Θ^1)*, along with* (Δ_2, Θ^2)*, namely* $(\Delta^U, \Theta^U) = (\Delta, \Theta)^U$*, can be computed, in which* $\Theta^U = \Theta^1 \cup \Theta^2$*, with the help of Formula 4.2 stated in Definition (4.1), as the following:*

$$\begin{aligned}(\Delta^U, \Theta^U) = \{&(a_1, \{(m_1, 0.3), (m_2, 0.1)\}), (a_2, \{(m_1, 0.9), (m_2, 0.5)\}), (a_3, \{(m_1, 0.4), (m_2, 1)\}),\\ &(b_1, \{(s_1, 0.8), (s_2, 0.2)\}), (b_2, \{(s_1, 0.6), (s_2, 1)\}), (b_3, \{(s_1, 0.6), (s_2, 0.1)\}),\\ &(c_1, \{(f_1, 0.3), (f_2, 0.7)\}), (c_2, \{(f_1, 1), (f_2, 0.8)\}), (c_3, \{(f_1, 0.9), (f_2, 0.4)\})\}.\end{aligned}$$

In the end, one can obtain the effective union of fuzzy soft multisets, namely $(\Delta^U_{\mu^U}, \Theta^U)$*, by applying calculations stated in Formula 3.1 given in Definition (3.1), as follows:*

$$\begin{aligned}(\Delta^U_{\mu^U}, \Theta^U) = \{&(a_1, \{(m_1, 0.825), (m_2, 0.505)\}), (a_2, \{(m_1, 0.975), (m_2, 0.725)\}), (a_3, \{(m_1, 0.85),\\ &(m_2, 1)\}), (b_1, \{(s_1, 0.94), (s_2, 0.36)\}), (b_2, \{(s_1, 0.88), (s_2, 1)\}), (b_3, \{(s_1, 0.88),\\ &(s_2, 0.28)\}), (c_1, \{(f_1, 0.5566), (f_2, 0.8125)\}), (c_2, \{(f_1, 1), (f_2, 0.875)\}),\\ &(c_3, \{(f_1, 0.9366), (f_2, 0.625)\})\}.\end{aligned}$$

Second, suppose that $\{\Psi_i, i \in I\}$ is a collection of universal sets, in which $\bigcap_{i \in I} \Psi_i = \phi$, along with $\Psi = \prod_{i \in I} \mathfrak{F}(\Psi_i)$, taking into account that $\mathfrak{F}(\Psi_i)$ represents the collection of all fuzzy subsets on Ψ. Furthermore, consider that $\{\Theta^1_{\Psi_i}, i \in I\}$, in addition to $\{\Theta^2_{\Psi_i}, i \in I\}$ are two collections of parameters sets, fulfilling that $\Theta^1 = \prod_{i \in I} \Theta^1_{\Psi_i}$, as well as $\Theta^2 = \prod_{i \in I} \Theta^2_{\Psi_i}$, respectively. Moreover, let us consider having two effective-parameter sets, say, μ_1, in addition to μ_2 defined over Ω.

Definition 4.2. ***(Restricted union operation of the two effective fuzzy soft multisets)***

The restricted union operation of two effective fuzzy soft multisets $(\Delta_{1\mu_1}, \Theta^1)$*, along with* $(\Delta_{2\mu_2}, \Theta^2)$ Ψ *can be obtained as a new resulting effective fuzzy soft multiset* $(\Delta^{U_R}_{\mu^{U_R}}, \Theta^{U_R})$*, in which* $\mu^{U_R} = \{(\omega, \eta^{U_R}(\omega)), \omega \in \Omega\}$*. Therefore, we have that the value of the effective membership union* $\eta^{U_R} : \Omega \to [0, 1]$ *can be characterized by* $\eta^{U_R} = \eta_1 \widetilde{\cup_R} \eta_2$*, taking into consideration that* η_1*, as well as* η_2 *are, respectively, the values of the effective membership associated with* μ_1*, as well as* μ_2*. On top of that, if we have* (Δ_1, Θ^1)*, in addition to* (Δ_2, Θ^2)*, then, we can obtain the following formula:* $(\Delta, \Theta)^{U_R} = (\Delta^{U_R}, \Theta^{U_R}) = (\Delta_1, \Theta^1)\widetilde{\cup_R}(\Delta_2, \Theta^2)$*, where* $\Theta^{U_R} = \Theta^1 \cap \Theta^2 \neq \phi$*, along with* $\mu_1 \cap \mu_2 \neq \phi$*. In addition, for every* $\omega \in \Omega$*, as well as for every* $\psi \in \Psi$*, the value* η^{U_R} *can be determined by using the following formula:* $\eta_{\mu_\psi^{U_R}}(\omega) = \max\{\eta_{\mu_{1\psi}}(\omega), \eta_{\mu_{2\psi}}(\omega)\}$*. Moreover, we can investigate the formula that computes* $(\Delta^{U_R}, \Theta^{U_R}) = (\Delta_1, \Theta^1)\widetilde{\cup_R}(\Delta_2, \Theta^2)$ *as follows:*
$(\Delta^{U_R}, \Theta^{U_R}) = \{(\theta, \{\psi, \max\{\gamma_{\Delta_1(\theta)}(\psi)_\mu, \gamma_{\Delta_2(\theta)}(\psi)_\mu\}\}), \psi \in \Psi\}$*, for every* $\psi \in \Psi$*, as well as for every* $\theta \in \Theta^{U_R}$*.*

Third, given $\{\Psi_i, i \in I\}$ is a collection of universal sets, in which $\bigcap_{i \in I} \Psi_i = \phi$, as well as $\Psi = \prod_{i \in I} \mathfrak{F}(\Psi_i)$, keeping into account that $\mathfrak{F}(\Psi_i)$ represents the collection of all fuzzy subsets on Ψ. In addition, suppose that $\{\Theta^1_{\Psi_i}, i \in I\}$, along with $\{\Theta^2_{\Psi_i}, i \in I\}$ are two collections of parameters sets. In such a scenario, let $\Theta^1 = \prod_{i \in I} \Theta^1_{\Psi_i}$, in addition to letting $\Theta^2 = \prod_{i \in I} \Theta^2_{\Psi_i}$. Furthermore, assume that μ_1, as well as μ_2 represent two effective-parameter sets over Ω.

Definition 4.3. ***(Intersection operation of the two effective fuzzy soft multisets)***

The two effective fuzzy soft multisets $(\Delta_{1\mu_1}, \Theta^1)$*, as well as* $(\Delta_{2\mu_2}, \Theta^2)$ *on* Ψ *have their intersection operation available to us as a third consequent effective fuzzy soft multiset, say* $(\Delta^I_{\mu^I}, \Theta^I)$*, taking into consideration that* $\mu^I = \{(\omega, \eta^I(\omega)), \omega \in \Omega\}$*. Additionally, we can give the value of the effective membership intersection, namely* $\eta^I : \Omega \to [0,1]$ *by* $\eta^I = \eta_1 \widetilde{\cap} \eta_2$*, in which* η_1*, in addition to* η_2 *serve, respectively, as the values of the effective membership related to* μ_1*, as well as* μ_2*. On top of that, the result that* $(\Delta, \Theta)^I = (\Delta^I, \Theta^I) = (\Delta_1, \Theta^1) \widetilde{\cap} (\Delta_2, \Theta^2)$ *is satisfied for* (Δ_1, Θ^1)*, along with* (Δ_2, Θ^2)*, such that* $\Theta^I = \Theta^1 \cup \Theta^2$*. Moreover, to calculate* η^I*, for all* $\omega \in \Omega$*, the following formula can be used, for each* $\psi \in \Psi$*:*

$$\eta_{\mu^I_\psi}(\omega) = \begin{cases} \eta_{\mu_{1\psi}}(\omega), & if\ \omega \in \mu_1 - \mu_2, \\ \eta_{\mu_{2\psi}}(\omega), & if\ \omega \in \mu_2 - \mu_1, \\ \min\{\eta_{\mu_{1\psi}}(\omega), \eta_{\mu_{2\psi}}(\omega)\}, & if\ \omega \in \mu_1 \cap \mu_2. \end{cases} \tag{4.3}$$

Furthermore, the formula utilized to calculate $(\Delta^I, \Theta^I) = (\Delta_1, \Theta^1) \widetilde{\cap} (\Delta_2, \Theta^2)$*, for each* $\psi \in \Psi$*, as well as for each* $\theta \in \Theta^I$*, can be created as shown below:*

$$(\Delta^I, \Theta^I) = \begin{cases} \{(\theta, \{\psi, \gamma_{\Delta_1(\theta)}(\psi)_\mu\}), \psi \in \Psi\},\ if\ \theta \in \Theta^1 - \Theta^2, \\ \{(\theta, \{\psi, \gamma_{\Delta_2(\theta)}(\psi)_\mu\}), \psi \in \Psi\},\ if\ \theta \in \Theta^2 - \Theta^1, \\ \{(\theta, \{\psi, \min\{\gamma_{\Delta_1(\theta)}(\psi)_\mu, \gamma_{\Delta_2(\theta)}(\psi)_\mu\}\}), \psi \in \Psi\}, if\ \theta \in \Theta^1 \cap \Theta^2. \end{cases} \tag{4.4}$$

Example 4.2. *To obtain the intersection* $\mu^I = \mu_1 \widetilde{\cap} \mu_2$ *of the two effective sets* μ_1*, as well as* μ_2 *given in Example (4.1), one can seek help from Definition (4.3) by using Formula 4.3, as given below:*

$$\mu^I(m_1) = \{(\omega_1, 0.2), (\omega_2, 0.4), (\omega_3, 0.8), (\omega_4, 1)\}, \mu^I(m_2) = \{(\omega_1, 0.1), (\omega_2, 0.4), (\omega_3, 0.7), (\omega_4, 0.5)\},$$

$$\mu^I(s_1) = \{(\omega_1, 0.2), (\omega_2, 0.6), (\omega_3, 0.8), (\omega_4, 1)\}, \mu^I(s_2) = \{(\omega_1, 0.1), (\omega_2, 0.1), (\omega_4, 0.2)\},$$

$$\mu^I(f_1) = \{(\omega_1, 0.3), (\omega_3, 0.4), (\omega_4, 0.1)\}, \mu^I(f_2) = \{(\omega_1, 0.1), (\omega_2, 0.4), (\omega_3, 0.1), (\omega_4, 0.7)\}.$$

Additionally, to find the intersection $\Theta^I = \Theta^1 \cup \Theta^2$ *of the two fuzzy soft sets* (Δ_1, Θ^1)*, in addition to* (Δ_2, Θ^2) *that have been defined in Example 4.1, apply Formula 4.4 given in Definition (4.3), as follows:*

$$\begin{aligned}(\Delta^I, \Theta^I) = \{&(a_1, \{(m_1, 0.2), (m_2, 0.1)\}), (a_2, \{(m_1, 0.6), (m_2, 0.4)\}), (a_3, \{(m_1, 0.4), (m_2, 0.9)\}), \\ &(b_1, \{(s_1, 0.4), (s_2, 0.2)\}), (b_2, \{(s_1, 0.3), (s_2, 0.7)\}), (b_3, \{(s_1, 0.2), (s_2, 0.1)\}), \\ &(c_1, \{(f_1, 0.3), (f_2, 0.5)\}), (c_2, \{(f_1, 0.9), (f_2, 0.6)\}), (c_3, \{(f_1, 0.2), (f_2, 0.4)\})\}.\end{aligned}$$

Therefore, to compute the effective intersection operation for these two fuzzy soft sets, say $(\Delta^I_{\mu^I}, \Theta^I)$*, make use of Formula 3.1 existing in Definition (3.1), as the following:*

$$\begin{aligned}(\Delta^I_{\mu^I}, \Theta^I) = \{&(a_1, \{(m_1, 0.68), (m_2, 0.4825)\}), (a_2, \{(m_1, 0.84), (m_2, 0.655)\}), (a_3, \{(m_1, 0.76), \\ &(m_2, 0.9425)\}), (b_1, \{(s_1, 0.79), (s_2, 0.3066)\}), (b_2, \{(s_1, 0.755), (s_2, 0.74)\}), \\ &(b_3, \{(s_1, 0.72), (s_2, 0.22)\}), (c_1, \{(f_1, 0.4866), (f_2, 0.6625)\}), (c_2, \{(f_1, 0.9266), \\ &(f_2, 0.73)\}), (c_3, \{(f_1, 0.4133), (f_2, 0.595)\})\}.\end{aligned}$$

Fourth, assume that $\{\Psi_i, i \in I\}$ expresses a collection of universal sets, in which $\bigcap_{i \in I} \Psi_i = \phi$, in addition to $\Psi = \prod_{i \in I} \mathfrak{F}(\Psi_i)$, taking into consideration that $\mathfrak{F}(\Psi_i)$ represents the collection of all fuzzy subsets on

Ψ. In addition, suppose that $\{\Theta^1_{\Psi_i}, i \in I\}$, as well as $\{\Theta^2_{\Psi_i}, i \in I\}$ are two collections of parameters sets, respectively, satisfying that $\Theta^1 = \prod_{i \in I} \Theta^1_{\Psi_i}$, as well as $\Theta^2 = \prod_{i \in I} \Theta^2_{\Psi_i}$. Furthermore, consider that μ_1, along with μ_2 are two effective-parameter sets over Ω.

Definition 4.4. ***(Restricted intersection operation of the two effective fuzzy soft multisets)***

The restricted intersection operation of the two effective fuzzy soft multisets $(\Delta_{1\mu_1}, \Theta^1)$*, as well as* $(\Delta_{2\mu_2}, \Theta^2)$ *on* Ψ *can be investigated as another resulting effective fuzzy soft multiset* $(\Delta^{I_R}_{\mu^{I_R}}, \Theta^{I_R})$*, in which* $\mu^{I_R} = \{(\omega, \eta^{I_R}(\omega)), \omega \in \Omega\}$. *Moreover, one can determine the value of the effective membership union, namely,* $\eta^{I_R} : \Omega \to [0, 1]$ *by* $\eta^{I_R} = \eta_1 \widetilde{\cap}_R \eta_2$*, keeping into account that* η_1*, as well as* η_2 *serve as the values of the effective membership connected to* μ_1*, as well as* μ_2*, respectively. On top of that, given that* (Δ_1, Θ^1)*, along with* (Δ_2, Θ^2)*, one has* $(\Delta, \Theta)^{I_R} = (\Delta^{I_R}, \Theta^{I_R}) = (\Delta_1, \Theta^1) \widetilde{\cap}_R (\Delta_2, \Theta^2)$*, keeping in mind that* $\Theta^{I_R} = \Theta^1 \cap \Theta^2 \neq \phi$*, in addition to* $\mu_1 \cap \mu_2 \neq \phi$*. Additionally, the formula used to compute* η^{I_R} *can be introduced as the following, for all* $\psi \in \Psi$*, and for all* $\omega \in \Omega$*:* $\eta_{\mu^{I_R}_{\psi}}(\omega) = \min\{\eta_{\mu_{1\psi}}(\omega), \eta_{\mu_{2\psi}}(\omega)\}$*. In addition, we can establish the formulation to calculate* $(\Delta^{I_R}, \Theta^{I_R}) = (\Delta_1, \Theta^1) \widetilde{\cap}_R (\Delta_2, \Theta^2)$ *as follows:* $(\Delta^{I_R}, \Theta^{I_R}) = \{(\theta, \{\psi, \min\{\gamma_{\Delta_1(\theta)}(\psi)_\mu, \gamma_{\Delta_2(\theta)}(\psi)_\mu\}\}), \psi \in \Psi\}$*, for all* $\psi \in \Psi$*, as well as for all* $\theta \in \Theta^{I_R}$.

Fifth, let $\{\Psi_i, i \in I\}$ be a collection of universal sets, in which $\bigcap_{i \in I} \Psi_i = \phi$, along with $\Psi = \prod_{i \in I} \mathfrak{F}(\Psi_i)$, where $\mathfrak{F}(\Psi_i)$ serves as the collection of all fuzzy subsets on Ψ. On top of that, let $\{\Theta^1_{\Psi_i}, i \in I\}$, as well as $\{\Theta^2_{\Psi_i}, i \in I\}$ be two collections of parameters sets, fulfilling that $\Theta^1 = \prod_{i \in I} \Theta^1_{\Psi_i}$, in addition to $\Theta^2 = \prod_{i \in I} \Theta^2_{\Psi_i}$. Additionally, suppose that μ_1, as well as μ_2 represent two effective-parameter sets over Ω. Furthermore, let $(\Delta_{1\mu_1}, \Theta^1)$, along with $(\Delta_{2\mu_2}, \Theta^2)$ be two effective fuzzy soft multisets on Ψ.

Definition 4.5. ***(Subset operation of the effective fuzzy soft multiset)***

One can call $(\Delta_{1\mu_1}, \Theta^1)$ *an effective fuzzy soft multi subset of* $(\Delta_{2\mu_2}, \Theta^2)$*, provided that the conditions below are satisfied, for every* $\psi \in \Psi$*:*

(1) $\mu_1 \subseteq \mu_2$*, i.e.,* $\eta_{\mu_{1\psi}}(\omega) \leq \eta_{\mu_{2\psi}}(\omega)$*, for all* $\omega \in \Omega$*,*

(2) $\Theta^1 \subseteq \Theta^2$*, in other words, the usual subset (the normal inclusion) is provided,*

(3) $\Delta_1(\theta) \subseteq \Delta_2(\theta)$*, i.e.,* $\gamma_{\Delta_1(\theta)}(\psi) \leq \gamma_{\Delta_2(\theta)}(\psi)$*, for each* $\theta \in \Theta_1$.

In such a scenario, $(\Delta_{1\mu_1}, \Theta^1) \widetilde{\subseteq} (\Delta_{2\mu_2}, \Theta^2)$ *can be written to express the effective fuzzy soft multi subset operation. Furthermore, if we write* $(\Delta_{2\mu_2}, \Theta^2) \widetilde{\supseteq} (\Delta_{1\mu_1}, \Theta^1)$*, then we mean that* $(\Delta_{2\mu_2}, \Theta^2)$ *is an effective fuzzy soft multi superset of* $(\Delta_{1\mu_1}, \Theta^1)$*.*

Sixth, let's suppose that $\{\Psi_i, i \in I\}$ is a collection of universal sets satisfying $\bigcap_{i \in I} \Psi_i = \phi$, along with $\Psi = \prod_{i \in I} \mathfrak{F}(\Psi_i)$, where $\mathfrak{F}(\Psi_i)$ represents the collection of all fuzzy subsets on Ψ. Furthermore, assume that $\{\Theta^1_{\Psi_i}, i \in I\}$, in addition to $\{\Theta^2_{\Psi_i}, i \in I\}$ serve as two collections of parameters sets, in which $\Theta^1 = \prod_{i \in I} \Theta^1_{\Psi_i}$ as well as $\Theta^2 = \prod_{i \in I} \Theta^2_{\Psi_i}$. Moreover, give that μ_1, as well as μ_2 are two effective-parameter sets over Ω.

Definition 4.6. ***(Equality operation of the two effective fuzzy soft multisets)***

The two effective fuzzy soft multisets $(\Delta_{1\mu_1}, \Theta^1)$*, in addition to* $(\Delta_{2\mu_2}, \Theta^2)$ *on* Ψ *are said to be effective fuzzy soft multi equal under the condition that each one of them satisfies the effective fuzzy soft multi* *subset operation for the other, as given in Definition (4.5). In symbols, if we have* $(\Delta_{1\mu_1}, \Theta^1)\widetilde{\subseteq}(\Delta_{2\mu_2}, \Theta^2)$, *along with* $(\Delta_{2\mu_2}, \Theta^2)\widetilde{\subseteq}(\Delta_{1\mu_1}, \Theta^1)$*, then this implies that* $(\Delta_{1\mu_1}, \Theta^1)\widetilde{=}(\Delta_{2\mu_2}, \Theta^2)$.

Seventh, given that $\{\Psi_i, i \in I\}$ serves as a collection of universal sets, fulfilling that $\bigcap_{i\in I}\Psi_i = \phi$, in addition to $\Psi = \prod_{i\in I}\mathfrak{F}(\Psi_i)$, in which $\mathfrak{F}(\Psi_i)$ represents the collection of all fuzzy subsets on Ψ, as well as $\{\Theta_{\Psi_i}, i \in I\}$ is a collection of parameters sets, where $\Theta = \prod_{i\in I}\Theta_{\Psi_i}$.

Definition 4.7. ***(Complement operation of the effective fuzzy soft multiset)***
The complement operation of any effective fuzzy soft multiset (Δ_μ, Θ) *on an initial universal set* Ψ *denoted as* $(\Delta_\mu, \Theta)^c = (\Delta^c_{\mu^c}, \Theta)$ *can be defined as follows:* $\mu^c = \{(\omega, \eta^c(\omega)), \omega \in \Omega\}$, *can be established, taking into consideration that* $\eta^c : \Omega \to [0, 1]$. *The following formula can determine* η^c *as the following:* $\eta_{\mu^c_\psi}(\omega) = 1 - \eta_{\mu_\psi}(\omega)$, *for each* $\omega \in \Omega$, *as well as for each* $\psi \in \Psi$. *Consequently, it concludes that* $\mu^c = \{(\omega, 1 - \eta(\omega)), \omega \in \Omega\}$. *In addition,* $\Delta^c : \Theta \to \Psi$ *can be investigated by the formula given below:* $\gamma_{\Delta^c_\Theta(\theta)}(\psi)_{\mu^c} = 1 - \gamma_{\Delta_\Theta(\theta)}(\psi)_{\mu^c}$, *for every* $\theta \in \Theta$, *along with for every* $\psi \in \Psi$. *On top of that,* $(\Delta_\mu, \Theta)^c = \{(\theta, \{\psi, 1 - \gamma_{\Delta_\Theta(\theta)}(\psi)_{\mu^c}\}) : \theta \in \Theta, \psi \in \Psi\}$ *can be considered as the full formula of the complement operation of the effective fuzzy soft multiset.*

Remark 4.1. *Beyond the case of two sets, the previously given concepts can be broadened to a more general case including a family of sets. It is easy to deduce these more general formulations that can explain those definitions, as well as each of them can have an example given.*

5. Properties for effective fuzzy soft multi sets

This section covers a wide range of significant properties of effective fuzzy soft multisets, such as absorption properties, in addition to distributive properties, as well as commutative properties, and associative properties, along with De Morgan's laws. Using the operations, as well as formulas from Definition (3.2), as well as Definition (3.3) given in Section (3), in addition to Definitions (4.1), (4.3), (4.4), (4.5), as well as (4.7) stated in Section (4), any of the following theorems can be proved easily.

Theorem 5.1. *Suppose that* $\{\Psi_i, i \in I\}$ *is a collection of universal sets, fulfilling that* $\bigcap_{i\in I}\Psi_i = \phi$, *along with* $\Psi = \prod_{i\in I}\mathfrak{F}(\Psi_i)$, *in which* $\mathfrak{F}(\Psi_i)$ *serves as the collection of all fuzzy subsets on* Ψ. *On top of that, assume that* $\{\Theta_{\Psi_i}, i \in I\}$ *is a collection of parameters sets, keeping into account that* $\Theta = \prod_{i\in I}\Theta_{\Psi_i}$. *In addition, it is given that* (Δ_μ, Θ) *is an effective fuzzy soft multiset on* Ψ *created through an effective set* μ. *Moreover, consider that* (Φ_μ, Θ), *as well as* (C_μ, Θ) *behave, respectively, as the null effective fuzzy soft multiset, as well as the absolute effective fuzzy soft multiset on* Ψ. *Then, the following are true:*

(1) $(\Delta_\mu, \Theta)\tilde{\cup}(\Delta_\mu, \Theta) = (\Delta_\mu, \Theta)\tilde{\cap}(\Delta_\mu, \Theta) = (\Delta_\mu, \Theta)$.

(2) $(\Delta_\mu, \Theta)\tilde{\cap}(C_\mu, \Theta) = (\Delta_\mu, \Theta)\tilde{\cup}(\Phi_\mu, \Theta) = (\Delta_\mu, \Theta)$.

(3) $(\Delta_\mu, \Theta)\tilde{\cup}(C_\mu, \Theta) = (C_\mu, \Theta)\tilde{\cup}(\Phi_\mu, \Theta) = (C_\mu, \Theta)$.

(4) $(\Delta_\mu, \Theta)\tilde{\cap}(\Phi_\mu, \Theta) = (C_\mu, \Theta)\tilde{\cap}(\Phi_\mu, \Theta) = (\Phi_\mu, \Theta)$.

Proof. The same procedure can be used to prove (1)–(4). We now wish to establish $(C_\mu, \Theta)\tilde{\cap}(\Phi_\mu, \Theta) = (\Phi_\mu, \Theta)$ for (4). Furthermore, the same method may be used to conclude $(\Delta_\mu, \Theta)\tilde{\cap}(\Phi_\mu, \Theta) = (\Phi_\mu, \Theta)$.

We have, respectively, $(\Phi_\mu, \Theta) = \{(\theta, \{\psi, 0\}) : \theta \in \Theta, \psi \in \Psi\}$, in addition to $(C_\mu, \Theta) = \{(\theta, \{\psi, 1\}) : \theta \in \Theta, \psi \in \Psi\}$, depending on Definitions (3.3), in addition to (3.2). Considering that $\Theta = \Theta \cup \Theta = \Theta$, we can obtain

$$\begin{aligned}(C_\mu, \Theta)\tilde{\cap}(\Phi_\mu, \Theta) = (\Delta_\mu, \Theta) &= \{(\theta, \{\psi, \gamma_{\Delta(\theta)}(\psi)_\mu\}) : \theta \in \Theta, \psi \in \Psi\}\\ &= \{(\theta, \{\psi, \min\{1, 0\}_\mu\}) : \theta \in \Theta, \psi \in \Psi\}\\ &= \{(\theta, \{\psi, 0\}) : \theta \in \Theta, \psi \in \Psi\} = (\Phi_\mu, \Theta).\end{aligned}$$

Consequently, the third item stated in Definition (4.3) holds if $\theta \in \Theta \cap \Theta = \Theta$ is satisfied. However, we have an absence of parameters for both the first case, in addition to the second case due to $\theta \in \Theta - \Theta = \phi$. □

Theorem 5.2. *Consider that $\{\Psi_i, i \in I\}$ is a collection of universal sets satisfying that $\bigcap_{i\in I}\Psi_i = \phi$, in addition to $\Psi = \prod_{i\in I}\mathfrak{F}(\Psi_i)$, where $\mathfrak{F}(\Psi_i)$ represents the collection of all fuzzy subsets on Ψ. In addition, let $\{\Theta^1_{\Psi_i}, i \in I\}$, as well as $\{\Theta^2_{\Psi_i}, i \in I\}$ be two collections of sets of parameters, fulfilling that $\Theta^1 = \prod_{i\in I}\Theta^1_{\Psi_i}$, along with $\Theta^2 = \prod_{i\in I}\Theta^2_{\Psi_i}$. Furthermore, assume that $(\Delta_{1\mu}, \Theta_1)$, in addition to $(\Delta_{2\mu}, \Theta_2)$ behave as two effective fuzzy soft multisets on Ψ, for a commonly effective set μ, then we get that the properties of absorption hold as indicated below:*

(1) $(\Delta_{1\mu}, \Theta^1)\tilde{\cup}((\Delta_{1\mu}, \Theta^1)\tilde{\cap}_R(\Delta_{2\mu}, \Theta^2)) = (\Delta_{1\mu}, \Theta^1)$.

(2) $(\Delta_{1\mu}, \Theta^1)\tilde{\cap}_R((\Delta_{1\mu}, \Theta^1)\tilde{\cup}(\Delta_{2\mu}, \Theta^2)) = (\Delta_{1\mu}, \Theta^1)$.

Proof. To arrive at (1), first consider that

$$(\Delta_{1\mu}, \Theta^1) = \{(\theta, \{\psi, \gamma_{\Delta_1(\theta)}(\psi)_\mu\}) : \theta \in \Theta^1, \psi \in \Psi\},$$

$$(\Delta_{2\mu}, \Theta^2) = \{(\theta, \{\psi, \gamma_{\Delta_2(\theta)}(\psi)_\mu\}) : \theta \in \Theta^2, \psi \in \Psi\}.$$

As given in Definition (4.1), (1) must be shown to be true for each one of the following three cases:
(i) When $\theta \in \Theta^1 - \Theta^2$, then, as stated in Definition (4.4), one can obtain:

$$(\Delta_{3\mu}, \Theta^3) = (\Delta_{1\mu}, \Theta^1)\tilde{\cap}_R(\Delta_{2\mu}, \Theta^2) = \{(\theta, \{\psi, \gamma_{\Delta_3(\theta)}(\psi)_\mu\}) : \theta \in \Theta^1 - \Theta^2, \psi \in \Psi\} = \phi.$$

(ii) In case that $\theta \in \Theta^2 - \Theta^1$, one can conclude using Definition (4.4) that:

$$(\Delta_{3\mu}, \Theta^3) = (\Delta_{1\mu}, \Theta^1)\tilde{\cap}_R(\Delta_{2\mu}, \Theta^2) = \{(\theta, \{\psi, \gamma_{\Delta_3(\theta)}(\psi)_\mu\}) : \theta \in \Theta^2 - \Theta^1, \psi \in \Psi\} = \phi.$$

Hence, for the two cases (i) and (ii), by making use of (3) stated in Theorem (5.1), the following is obtained:

$$(\Delta_{4\mu}, \Theta^4) = (\Delta_{1\mu}, \Theta^1)\tilde{\cup}(\Delta_{3\mu}, \Theta^3) = (\Delta_{1\mu}, \Theta^1)\tilde{\cup}\phi = (\Delta_{1\mu}, \Theta^1).$$

(iii) If we have $\theta \in \Theta^1 \cap \Theta^2$, the following is established by applying Definition (4.4):

$$\begin{aligned}(\Delta_{3\mu}, \Theta^3) &= (\Delta_{1\mu}, \Theta^1)\tilde{\cap}_R(\Delta_{2\mu}, \Theta^2)\\ &= \{(\theta, \{\psi, \gamma_{\Delta_3(\theta)}(\psi)_\mu\}) : \theta \in \Theta^1 \cap \Theta^2, \psi \in \Psi\}\\ &= \{(\theta, \{\psi, \min\{\gamma_{\Delta_1(\theta)}(\psi)_\mu, \gamma_{\Delta_2(\theta)}(\psi)_\mu\}\}) : \theta \in \Theta^1 \cap \Theta^2, \psi \in \Psi\}.\end{aligned}$$

Finally, the following is investigated by applying Definition (4.1):

$$\begin{aligned}(\Delta_{4\mu},\Theta^4) &= (\Delta_{1\mu},\Theta^1)\tilde{\cup}(\Delta_{3\mu},\Theta^3)\\ &= \{(\theta,\{\psi,\max\{\gamma_{\Delta_1(\theta)}(\psi)_\mu,\min\{\gamma_{\Delta_1(\theta)}(\psi)_\mu,\gamma_{\Delta_2(\theta)}(\psi)_\mu\}\}\}):\theta\in\Theta^1\cap\Theta^2,\psi\in\Psi\}\\ &= (\Delta_{1\mu},\Theta^1).\end{aligned}$$

Like (1), we can prove (2) easily by following the same steps. □

Corollary 5.1. *Given that $\{\Psi_i, i\in I\}$ is a collection of universal sets, in which $\bigcap_{i\in I}\Psi_i=\phi$, as well as $\Psi=\prod_{i\in I}\mathfrak{F}(\Psi_i)$, where $\mathfrak{F}(\Psi_i)$ represents the collection of all fuzzy subsets on Ψ. In addition, let $\{\Theta^1_{\Psi_i}, i\in I\}$, along with $\{\Theta^2_{\Psi_i}, i\in I\}$ be two collections of parameters sets. Hence, for two effective fuzzy soft multisets $(\Delta_{1\mu},\Theta^1)$, in addition to $(\Delta_{2\mu},\Theta^2)$ on Ψ generated by a common effective set μ, we have that:*

$$(\Delta_{1\mu},\Theta^1)\tilde{\cup}((\Delta_{1\mu},\Theta^1)\tilde{\cap}_R(\Delta_{2\mu},\Theta^2)) = (\Delta_{1\mu},\Theta^1)\tilde{\cap}_R((\Delta_{1\mu},\Theta^1)\tilde{\cup}(\Delta_{2\mu},\Theta^2)) = (\Delta_{1\mu},\Theta^1).$$

Proof. By following the same techniques used to prove the Theorem (5.2), it can be simply proven. □

Theorem 5.3. *Assume that $\{\Psi_i, i\in I\}$ indicates a collection of universal sets such that $\bigcap_{i\in I}\Psi_i=\phi$, in addition to $\Psi=\prod_{i\in I}\mathfrak{F}(\Psi_i)$, where $\mathfrak{F}(\Psi_i)$ is the collection of all fuzzy subsets on Ψ. Moreover, assume that $\{\Theta^1_{\Psi_i}, i\in I\}$, as well as $\{\Theta^2_{\Psi_i}, i\in I\}$ serve as two collections of parameters sets, taking into consideration that $\Theta^1=\prod_{i\in I}\Theta^1_{\Psi_i}$. along with $\Theta^2=\prod_{i\in I}\Theta^2_{\Psi_i}$. On top of that, consider that we have a commonly effective set μ, associated with two effective fuzzy soft multisets, say $(\Delta_{1\mu},\Theta^1)$, as well as $(\Delta_{2\mu},\Theta^2)$, then the commutative property is true as below:*

(1) $(\Delta_{1\mu},\Theta^1)\tilde{\cap}(\Delta_{2\mu},\Theta^2)=(\Delta_{2\mu},\Theta^2)\tilde{\cap}(\Delta_{1\mu},\Theta^1)$.

(2) $(\Delta_{1\mu},\Theta^1)\tilde{\cup}(\Delta_{2\mu},\Theta^2)=(\Delta_{2\mu},\Theta^2)\tilde{\cup}(\Delta_{1\mu},\Theta^1)$.

Proof. Similar to Theorem (5.2), using Definitions (4.1), along with (4.3) makes the proof simple. □

Proposition 5.1. *Suppose that $\{\Psi_i, i\in I\}$ represents a collection of universal sets, in which $\bigcap_{i\in I}\Psi_i=\phi$, along with $\Psi=\prod_{i\in I}\mathfrak{F}(\Psi_i)$, satisfying that $\mathfrak{F}(\Psi_i)$ serves as the collection of all fuzzy subsets on Ψ. Furthermore, assume that $\{\Theta^1_{\Psi_i}, i\in I\}$, as well as $\{\Theta^2_{\Psi_i}, i\in I\}$ are two collections of sets of parameters, keeping into account that $\Theta^1=\prod_{i\in I}\Theta^1_{\Psi_i}$, in addition to $\Theta^2=\prod_{i\in I}\Theta^2_{\Psi_i}$. Additionally, let's have a commonly effective set μ, associated with two effective fuzzy soft multisets, say $(\Delta_{1\mu},\Theta^1)$, as well as $(\Delta_{2\mu},\Theta^2)$, and in case that $(\Delta_{1\mu},\Theta^1)\tilde{\subseteq}(\Delta_{2\mu},\Theta^2)$, the following are true:*

(1) $(\Delta_{1\mu},\Theta^1)\tilde{\cap}_R(\Delta_{2\mu},\Theta^2)=(\Delta_{1\mu},\Theta^1)$.

(2) $(\Delta_{1\mu},\Theta_1)\tilde{\cup}(\Delta_{2\mu},\Theta^2)=(\Delta_{2\mu},\Theta^2)$.

Proof. When applying Definitions (4.2) and (4.4), this proposition is easily verified directly as Theorem (5.2). □

Theorem 5.4. *Consider that $\{\Psi_i, i \in I\}$ serves as a collection of universal sets, keeping into consideration that $\bigcap_{i\in I}\Psi_i = \phi$, in addition to $\Psi = \prod_{i\in I}\mathfrak{F}(\Psi_i)$, in which $\mathfrak{F}(\Psi_i)$ represents the collection of all fuzzy subsets on Ψ. Moreover, assume that $\{\Theta^1_{\Psi_i}, i \in I\}$, and $\{\Theta^2_{\Psi_i}, i \in I\}$, as well as $\{\Theta^3_{\Psi_i}, i \in I\}$ are three collections of sets of parameters, satisfying that $\Theta^1 = \prod_{i\in I}\Theta^1_{\Psi_i}$, and $\Theta^2 = \prod_{i\in I}\Theta^2_{\Psi_i}$, as well as $\Theta^3 = \prod_{i\in I}\Theta^3_{\Psi_i}$ for a common effective set μ, respectively, on Ψ. Therefore, the associative laws, in addition to the distributive laws can be obtained as the following, respectively:*

(1) $(\Delta_{1\mu}, \Theta^1)\tilde{\cap}((\Delta_{2\mu}, \Theta^2)\tilde{\cap}(\Delta_{3\mu}, \Theta^3)) = ((\Delta_{1\mu}, \Theta^1)\tilde{\cap}(\Delta_{2\mu}, \Theta^2))\tilde{\cap}(\Delta_{3\mu}, \Theta^3)$.

(2) $(\Delta_{1\mu}, \Theta^1)\tilde{\cup}((\Delta_{2\mu}, \Theta^2)\tilde{\cup}(\Delta_{3\mu}, \Theta^3)) = ((\Delta_{1\mu}, \Theta^1)\tilde{\cup}(\Delta_{2\mu}, \Theta^2))\tilde{\cup}(\Delta_{3\mu}, \Theta^3)$.

(3) $(\Delta_{1\mu}, \Theta^1)\tilde{\cap}((\Delta_{2\mu}, \Theta^2)\tilde{\cup}(\Delta_{3\mu}, \Theta^3)) = ((\Delta_{1\mu}, \Theta^1)\tilde{\cap}(\Delta_{2\mu}, \Theta^2))\tilde{\cup}((\Delta_{1\mu}, \Theta^1)\tilde{\cap}(\Delta_{3\mu}, \Theta^3))$.

(4) $(\Delta_{1\mu}, \Theta^1)\tilde{\cup}((\Delta_{2\mu}, \Theta^2)\tilde{\cap}(\Delta_{3\mu}, \Theta^3)) = ((\Delta_{1\mu}, \Theta^1)\tilde{\cup}(\Delta_{2\mu}, \Theta^2))\tilde{\cap}((\Delta_{1\mu}, \Theta^1)\tilde{\cup}(\Delta_{3\mu}, \Theta^3))$.

Proof. It may be shown using Definitions (4.1) and (4.3), similarly to Theorem (5.2). □

Theorem 5.5. *Given that $\{\Psi_i, i \in I\}$ represents a collection of universal sets, taking into consideration that $\bigcap_{i\in I}\Psi_i = \phi$, as well as $\Psi = \prod_{i\in I}\mathfrak{F}(\Psi_i)$, in which $\mathfrak{F}(\Psi_i)$ serves as the collection of all fuzzy subsets on Ψ. In addition, assume that $\{\Theta^1_{\Psi_i}, i \in I\}$, along with $\{\Theta^2_{\Psi_i}, i \in I\}$ indicate two collections of parameters sets, fulfilling, respectively, that $\Theta^1 = \prod_{i\in I}\Theta^1_{\Psi_i}$, in addition to $\Theta^2 = \prod_{i\in I}\Theta^2_{\Psi_i}$. Then, we get that De Morgan's laws are true as indicated below, for any two effective fuzzy soft multisets $(\Delta_{1\mu}, \Theta^1)$, as well as $(\Delta_{2\mu}, \Theta^2)$, through a commonly effective set μ, on Ψ:*

(1) $((\Delta_{1\mu}, \Theta^1)\tilde{\cup}(\Delta_{2\mu}, \Theta^2))^c = (\Delta_{1\mu}, \Theta^1)^c\tilde{\cap}(\Delta_{2\mu}, \Theta^2)^c$.

(2) $((\Delta_{1\mu}, \Theta^1)\tilde{\cap}(\Delta_{2\mu}, \Theta^2))^c = (\Delta_{1\mu}, \Theta^1)^c\tilde{\cup}(\Delta_{2\mu}, \Theta^2)^c$.

Proof. Using Definitions (4.3), (4.1), and (4.7), as well as Theorem (5.2), can help to prove it. □

6. Obesity treatment

This section aims to deal with a diagnostic-related practical issue. We establish a diagnosis technique built on the effective fuzzy soft multiset. To accurately determine the best decision, this method can be used with matrix operations, as well as the attributes that go along with it. In addition, we offer a thorough case study that illustrates the decision-making procedure in how to determine the best diet for some obese patients. To ease the computational components of this method, we have arranged the phases inside a matrix operations framework. We also use the *Wolfram Mathematica®* program to speed up, in addition to improving the accuracy of activities like matrix multiplication, as well as effective membership calculations. This selection of tools contributes to making these calculations easier to do, more accurate, and faster. Furthermore, we establish a detailed comparative analysis to demonstrate the rationality, effectiveness, and advantages of the proposed method.

6.1. Methodology and algorithm

The main variables participating in the study are categories types involved in the study (multiset), all patients' preferences, as well as circumstances (fuzzy soft set), external factors (effective parameters), and outcomes to diagnose (the greatest number found in every row within the diagnosis matrix). Assume that we have n universal sets each has a set of m obese or overweight patients according to specified categories, along with a set of k characteristics. These characteristics relate to a well-defined set of r different diets. Moreover, suppose that we have a set of q effective parameters (external factors). In fact, the effective set is built based on the circumstances of the patients. In addition, by questioning patients, as well as putting them through expert evaluations or exams, one fuzzy soft multiset is established. Furthermore, another fuzzy soft multiset is generated based on the description of the various diets, in addition to the associated features based on expert estimations.

Under the above assumptions, the following algorithm determines which diet is the best for every patient: We obtain the effective fuzzy soft multiset by using the effective set, in addition to the first fuzzy soft multiset is the first step of the algorithm. After that, the second step is to extract the matrices corresponding to the resulting effective fuzzy soft multiset, as well as the second fuzzy soft multiset. These two matrices are called the patient-characteristic matrix, as well as the diet-characteristic matrix, respectively. By taking the transpose of the diet-characteristic matrix, one has the characteristic-diet matrix. Then, the third step is to multiply the patient-characteristic matrix by the characteristic-diet matrix to eliminate the characteristics, in addition to obtaining the patient-diet matrix, called the diagnosis matrix. As a final step, the fourth step is to identify the greatest number in every row of the diagnosis matrix. In such a scenario, we have the most suitable diet for every patient being the one corresponding to this maximum value in his row. We can explain the algorithm steps for the proposed methodology more clearly step-by-step as indicated below.

Algorithm steps:

(1) Using the effective set and the first fuzzy soft multiset, construct the effective fuzzy soft multiset.

(2) Extract the matrices representing the fuzzy membership values for the effective fuzzy soft multiset components. Do the same for the second fuzzy soft multiset.

(3) To generate the diagnosis matrix, multiply the two matrices obtained in Step (2). According to the circumstances of the problem, we may need sometimes to take the transpose for matrices representing the second fuzzy soft multiset before multiplication.

(4) Pick out the greatest score in each row of the diagnosis matrix to determine the most appropriate diet for every patient, which corresponds to this value.

Finally, one can find a brief visual illustration, for simplicity, of the algorithm's steps contained in Figure 3.

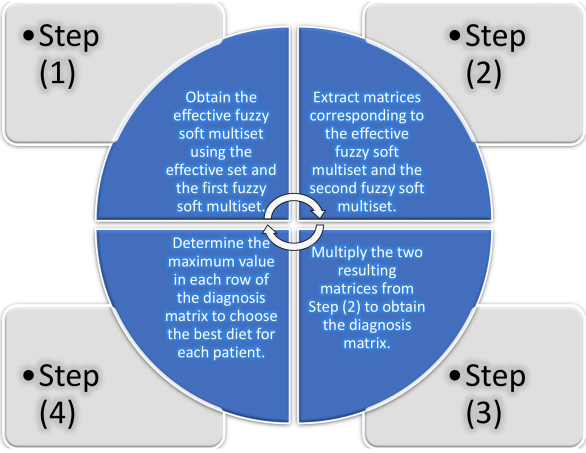

Figure 3. The proposed algorithm's steps.

6.2. Case study

In this section, we provide an extensive case study that demonstrates the process of making decisions regarding the optimal diet for some obese patients.

Example 6.1. *Assume that $\{\Pi_1, \Pi_2, \Pi_3\}$ is a collection of three universal sets representing obese patients. Each universal set represents one category of obese patients. The first universal set $\Pi_1 = \{m_1, m_2, m_3\}$ represents the male obese patients. In addition, the second universal set $\Pi_2 = \{f_1, f_2, f_3, f_4\}$ represents the female obese patients. Moreover, the third universal set $\Pi_3 = \{ch_1, ch_2\}$ represents the child obese patients. Each obese patient needs to determine the best diet that meets his/her needs to lose weight among three provided diets that represent another universal set $\Psi = \{\psi_1, \psi_2, \psi_3\}$, in which ψ_1 = the Luqaimat diet, ψ_2 = the keto diet, ψ_3 = the fast diet (2-5).*

Furthermore, suppose that $\{\Theta_{\Pi_1}, \Theta_{\Pi_2}, \Theta_{\Pi_3}\}$ is a collection of three parameter sets related to the above three universal sets. Θ_{Π_1}, Θ_{Π_2}, and Θ_{Π_3}, respectively, represent some preferences, as well as circumstances of male obese patients, female obese patients, and child obese patients. These preferences and circumstances can be the following: $\Theta_{\Pi_1} = \{\theta^1_{\Pi_1}, \theta^2_{\Pi_1}, \theta^3_{\Pi_1}, \theta^4_{\Pi_1}, \theta^5_{\Pi_1}\}$, where $\theta^1_{\Pi_1}$ = able to fast, $\theta^2_{\Pi_1}$ = suffers from constipation, $\theta^3_{\Pi_1}$ = fat lover, $\theta^4_{\Pi_1}$ = wants to build muscles, and $\theta^5_{\Pi_1}$ = financially capable. $\Theta_{\Pi_2} = \{\theta^1_{\Pi_2}, \theta^2_{\Pi_2}, \theta^3_{\Pi_2}, \theta^4_{\Pi_2}, \theta^5_{\Pi_2}\}$, where $\theta^1_{\Pi_2}$ = able to fast, $\theta^2_{\Pi_2}$ = suffers from constipation, $\theta^3_{\Pi_2}$ = sweets lover, $\theta^4_{\Pi_2}$ = suffering from irregular menstruation, and $\theta^5_{\Pi_2}$ = pregnant or breastfeeding. $\Theta_{\Pi_3} = \{\theta^1_{\Pi_3}, \theta^2_{\Pi_3}, \theta^3_{\Pi_3}, \theta^4_{\Pi_3}, \theta^5_{\Pi_3}\}$, where $\theta^1_{\Pi_3}$ = able to fast, $\theta^2_{\Pi_3}$ = suffers from constipation, $\theta^3_{\Pi_3}$ = sweets lover, $\theta^4_{\Pi_3}$ = appears to be showing signs of puberty, and $\theta^5_{\Pi_3}$ = suffering from anemia.

Then, after the obese patients' answers to a questionnaire (giving responses from 0 to 10 for each question), the fuzzy soft multiset $(\Delta, \Theta) = \{(\Delta_1, \Theta_{\Pi_1}), (\Delta_2, \Theta_{\Pi_2}), (\Delta_3, \Theta_{\Pi_3})\}$ represents the degree of each preference or circumstance for each obese patient. Its three components $(\Delta_1, \Theta_{\Pi_1})$, $(\Delta_2, \Theta_{\Pi_2})$, and $(\Delta_3, \Theta_{\Pi_3})$ representing the degree of each preference or circumstance for male obese patients, female obese patients, and child obese patients, respectively, are obtained as the following:

$$\begin{aligned}(\Delta_1, \Theta_{\Pi_1}) = \{&(\theta^1_{\Pi_1}, \{(m_1, 0.9), (m_2, 0.1), (m_3, 0.5)\}), (\theta^2_{\Pi_1}, \{(m_1, 0), (m_2, 0.5), (m_3, 0.3)\}),\\ &(\theta^3_{\Pi_1}, \{(m_1, 0), (m_2, 0.4), (m_3, 0.6)\}), (\theta^4_{\Pi_1}, \{(m_1, 0.5), (m_2, 0.3), (m_3, 0.4)\}),\\ &(\theta^5_{\Pi_1}, \{(m_1, 0.6), (m_2, 0.8), (m_3, 0.5)\})\},\end{aligned}$$

$$\begin{aligned}(\Delta_2, \Theta_{\Pi_2}) = \{&(\theta^1_{\Pi_2}, \{(f_1, 0.7), (f_2, 0), (f_3, 0.9), (f_4, 0.4)\}), (\theta^2_{\Pi_2}, \{(f_1, 0.4), (f_2, 0.6),\\ &(f_3, 0), (f_4, 0.2)\}), (\theta^3_{\Pi_2}, \{(f_1, 0.8), (f_2, 0.4), (f_3, 0.3), (f_4, 0.7)\}), (\theta^4_{\Pi_2}, \{(f_1, 0.2),\\ &(f_2, 0.7), (f_3, 0.5), (f_4, 0)\}), (\theta^5_{\Pi_2}, \{(f_1, 0), (f_2, 0.1), (f_3, 0.2), (f_4, 1)\})\},\end{aligned}$$

$$\begin{aligned}(\Delta_3, \Theta_{\Pi_3}) = \{&(\theta^1_{\Pi_3}, \{(ch_1, 0.3), (ch_2, 0.4)\}), (\theta^2_{\Pi_3}, \{(ch_1, 0.6), (ch_2, 0.7)\}), (\theta^3_{\Pi_3}, \{(ch_1, 0),\\ &(ch_2, 1)\}), (\theta^4_{\Pi_3}, \{(ch_1, 0.7), (ch_2, 0.1)\}), (\theta^5_{\Pi_3}, \{(ch_1, 0.2), (ch_2, 0.5)\})\}.\end{aligned}$$

Moreover, depending on responses extracted from another questionnaire made for nutrition experts, as well as doctors, another fuzzy soft multiset $(\gimel, \Theta) = \{(\gimel_1, \Theta_{\Pi_1}), (\gimel_2, \Theta_{\Pi_2}), (\gimel_3, \Theta_{\Pi_3})\}$ *that gives the relation between the diets, in addition to the preferences or circumstances.*

Its three components $(\gimel_1, \Theta_{\Pi_1})$*,* $(\gimel_2, \Theta_{\Pi_2})$*, and* $(\gimel_3, \Theta_{\Pi_3})$ *showing the relation degree between each one of the three diets, along with each preference or circumstance for male obese patients, female obese patients, and child obese patients, respectively, are as follows:*

$$\begin{aligned}(\gimel_1, \Theta_{\Pi_1}) = \{&(\theta^1_{\Pi_1}, \{(\psi_1, 0.1), (\psi_2, 0.5), (\psi_3, 0.9)\}), (\theta^2_{\Pi_1}, \{(\psi_1, 0.3), (\psi_2, 0.6), (\psi_3, 0.1)\}),\\ &(\theta^3_{\Pi_1}, \{(\psi_1, 0.6), (\psi_2, 0.8), (\psi_3, 0.7)\}), (\theta^4_{\Pi_1}, \{(\psi_1, 0.5), (\psi_2, 0.9), (\psi_3, 0.8)\}),\\ &(\theta^5_{\Pi_1}, \{(\psi_1, 0.5), (\psi_2, 0.7), (\psi_3, 0.6)\})\},\end{aligned}$$

$$\begin{aligned}(\gimel_2, \Theta_{\Pi_2}) = \{&(\theta^1_{\Pi_2}, \{(\psi_1, 0.1), (\psi_2, 0.5), (\psi_3, 0.9)\}), (\theta^2_{\Pi_2}, \{(\psi_1, 0.3), (\psi_2, 0.6), (\psi_3, 0.1)\}),\\ &(\theta^3_{\Pi_2}, \{(\psi_1, 0.8), (\psi_2, 0), (\psi_3, 0.6)\}), (\theta^4_{\Pi_2}, \{(\psi_1, 0.2), (\psi_2, 0.5), (\psi_3, 0.7)\}),\\ &(\theta^5_{\Pi_2}, \{(\psi_1, 0.9), (\psi_2, 0.1), (\psi_3, 0.5)\})\},\end{aligned}$$

$$\begin{aligned}(\gimel_3, \Theta_{\Pi_3}) = \{&(\theta^1_{\Pi_3}, \{(\psi_1, 0.1), (\psi_2, 0.5), (\psi_3, 0.9)\}), (\theta^2_{\Pi_3}, \{(\psi_1, 0.3), (\psi_2, 0.6), (\psi_3, 0.1)\}),\\ &(\theta^3_{\Pi_3}, \{(\psi_1, 0.8), (\psi_2, 0), (\psi_3, 0.6)\}), (\theta^4_{\Pi_3}, \{(\psi_1, 0.7), (\psi_2, 0.6), (\psi_3, 0.5)\}),\\ &(\theta^5_{\Pi_3}, \{(\psi_1, 0), (\psi_2, 0.7), (\psi_3, 0.6)\})\}.\end{aligned}$$

Furthermore, suppose that $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ *is a set of effective parameters, where* ω_1 *= he/she needs to lose weight quickly because of suffering from diseases due to obesity,* ω_2 *= he/she needs to perform some exercises regularly so that his/her skin doesn't become saggy,* ω_3 *= his/her metabolism isn't normal, and* ω_4 *= he/she doesn't sleep well.*

Doctors can talk to the obese patients, as well as make some tests for them to determine the effective set μ *over* Ω*, for the three given categories; male obese patients* m_1, m_2, m_3*, female obese patients* f_1, f_2, f_3, f_4*, and child obese patients* ch_1, ch_2*, as follows:*

$$\mu(m_1) = \{(\omega_1, 0.5), (\omega_2, 0.8), (\omega_3, 0.2), (\omega_4, 0.1)\},$$

$$\mu(m_2) = \{(\omega_1, 0.3), (\omega_2, 0.7), (\omega_3, 0.4), (\omega_4, 0.2)\},$$

$$\mu(m_3) = \{(\omega_1, 0.6), (\omega_2, 0.8), (\omega_3, 0.3), (\omega_4, 0.4)\},$$

$$\mu(f_1) = \{(\omega_1, 0.2), (\omega_2, 0.3), (\omega_3, 0.5), (\omega_4, 0.1)\},$$

$$\mu(f_2) = \{(\omega_1, 0.1), (\omega_2, 0.2), (\omega_3, 0.8), (\omega_4, 0.9)\}.$$

$$\mu(f_3) = \{(\omega_1, 0.8), (\omega_2, 0.9), (\omega_3, 0.1), (\omega_4, 0.3)\}.$$

$$\mu(f_4) = \{(\omega_1, 0.3), (\omega_2, 0.5), (\omega_3, 0.6), (\omega_4, 0.7)\}.$$

$$\mu(ch_1) = \{(\omega_1, 0.9), (\omega_2, 0.8), (\omega_3, 0.4), (\omega_4, 0.6)\}.$$

$$\mu(ch_2) = \{(\omega_1, 0.1), (\omega_2, 0.3), (\omega_3, 0.2), (\omega_4, 0.1)\}.$$

Can you determine the best diet for each obese patient?

Solution. $Step(1)$: Compute the effective fuzzy soft multiset $(\Delta_\mu, \Theta) = \{(\Delta_{1\mu}, \Theta_{\Pi_1}), (\Delta_{2\mu}, \Theta_{\Pi_2}), (\Delta_{3\mu}, \Theta_{\Pi_3})\}$, which determines the cases of the above obese patients. Using Formula 3.1 from Definition (3.1), one can calculate the three components of the effective fuzzy soft multiset $(\Delta_{1\mu}, \Theta_{\Pi_1})$, $(\Delta_{2\mu}, \Theta_{\Pi_2})$, and $(\Delta_{3\mu}, \Theta_{\Pi_3})$, respectively, as the following:

$$\begin{aligned}(\Delta_{1\mu}, \Theta_{\Pi_1}) = \{&(\theta^1_{\Pi_1}, \{(m_1, 0.94), (m_2, 0.46), (m_3, 0.76)\}), (\theta^2_{\Pi_1}, \{(m_1, 0), (m_2, 0.7), (m_3, 0.66)\}),\\ &(\theta^3_{\Pi_1}, \{(m_1, 0), (m_2, 0.64), (m_3, 0.81)\}), (\theta^4_{\Pi_1}, \{(m_1, 0.7), (m_2, 0.58), (m_3, 0.71)\}),\\ &(\theta^5_{\Pi_1}, \{(m_1, 0.76), (m_2, 0.88), (m_3, 0.76)\})\},\end{aligned}$$

$$\begin{aligned}(\Delta_{2\mu}, \Theta_{\Pi_2}) = \{&(\theta^1_{\Pi_2}, \{(f_1, 0.78), (f_2, 0), (f_3, 0.95)(f_4, 0.71)\}), (\theta^2_{\Pi_2}, \{(f_1, 0.56), (f_2, 0.8), (f_3, 0),\\ &(f_4, 0.62)\}), (\theta^3_{\Pi_2}, \{(f_1, 0.85), (f_2, 0.7), (f_3, 0.65), (f_4, 0.85)\}), (\theta^4_{\Pi_2}, \{(f_1, 0.42),\\ &(f_2, 0.85), (f_3, 0.75), (f_4, 0)\}), (\theta^5_{\Pi_2}, \{(f_1, 0), (f_2, 0.55), (f_3, 0.6), (f_4, 1)\})\},\end{aligned}$$

$$\begin{aligned}(\Delta_{3\mu}, \Theta_{\Pi_3}) = \{&(\theta^1_{\Pi_3}, \{(ch_1, 0.77), (ch_2, 0.5)\}), (\theta^2_{\Pi_3}, \{(ch_1, 0.87), (ch_2, 0.75)\}), (\theta^3_{\Pi_3}, \{(ch_1, 0),\\ &(ch_2, 1)\}), (\theta^4_{\Pi_3}, \{(ch_1, 0.9), (ch_2, 0.25)\}), (\theta^5_{\Pi_3}, \{(ch_1, 0.74), (ch_2, 0.58)\})\}.\end{aligned}$$

$Step(2)$: Extract the matrix corresponding to each component of the effective fuzzy soft multiset $(\Delta_\mu, \Theta) = \{(\Delta_{1\mu}, \Theta_{\Pi_1}), (\Delta_{2\mu}, \Theta_{\Pi_2}), (\Delta_{3\mu}, \Theta_{\Pi_3})\}$, namely $\widetilde{\Delta_{1\mu}}$, $\widetilde{\Delta_{2\mu}}$, and $\widetilde{\Delta_{3\mu}}$. The matrices $\widetilde{\Delta_{1\mu}}$, $\widetilde{\Delta_{2\mu}}$, and $\widetilde{\Delta_{3\mu}}$ represent the obese male, female, and child patients' needs, respectively, according to the three components $(\Delta_{1\mu}, \Theta_{\Pi_1})$, $(\Delta_{2\mu}, \Theta_{\Pi_2})$, and $(\Delta_{3\mu}, \Theta_{\Pi_3})$ of the effective fuzzy soft multiset (Δ_μ, Θ), as the following:

$$\widetilde{\Delta_{1\mu}} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccccc} \theta^1_{\Pi_1} & \theta^2_{\Pi_1} & \theta^3_{\Pi_1} & \theta^4_{\Pi_1} & \theta^5_{\Pi_1} \end{array} \\ \begin{pmatrix} 0.94 & 0 & 0 & 0.7 & 0.76 \\ 0.46 & 0.7 & 0.64 & 0.58 & 0.88 \\ 0.76 & 0.66 & 0.81 & 0.71 & 0.76 \end{pmatrix} \end{array},$$

$$\widetilde{\Delta_{2\mu}} = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \\ f_4 \end{array} \begin{array}{c} \begin{array}{ccccc} \theta^1_{\Pi_2} & \theta^2_{\Pi_2} & \theta^3_{\Pi_2} & \theta^4_{\Pi_2} & \theta^5_{\Pi_2} \end{array} \\ \begin{pmatrix} 0.78 & 0.56 & 0.85 & 0.42 & 0 \\ 0 & 0.8 & 0.7 & 0.85 & 0.55 \\ 0.95 & 0 & 0.65 & 0.75 & 0.6 \\ 0.71 & 0.62 & 0.85 & 0 & 1 \end{pmatrix} \end{array},$$

$$\widetilde{\Delta_{3\mu}} = \begin{array}{c} \\ ch_1 \\ ch_2 \end{array} \begin{array}{c} \begin{array}{ccccc} \theta^1_{\Pi_3} & \theta^2_{\Pi_3} & \theta^3_{\Pi_3} & \theta^4_{\Pi_3} & \theta^5_{\Pi_3} \end{array} \\ \begin{pmatrix} 0.77 & 0.87 & 0 & 0.9 & 0.74 \\ 0.5 & 0.75 & 1 & 0.25 & 0.58 \end{pmatrix} \end{array}.$$

Furthermore, extract the matrix corresponding to each component of the fuzzy soft multiset $(\gimel, \Theta) = \{(\gimel_1, \Theta_{\Pi_1}), (\gimel_2, \Theta_{\Pi_2}), (\gimel_3, \Theta_{\Pi_3})\}$, namely $\widetilde{\gimel_1}$, $\widetilde{\gimel_2}$, and $\widetilde{\gimel_3}$. The matrices $\widetilde{\gimel_1}$, $\widetilde{\gimel_2}$, and $\widetilde{\gimel_3}$ represent the relations between the diets, as well as the preferences or circumstances for each obese male, female, and child patient, respectively, according to the three components $(\gimel_1, \Theta_{\Pi_1})$, $(\gimel_2, \Theta_{\Pi_2})$, and $(\gimel_3, \Theta_{\Pi_3})$ of the fuzzy soft multiset $(\gimel, \Theta)$, as the following:

$$\widetilde{\gimel_1} = \begin{array}{c} \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{array} \begin{array}{c} \begin{array}{ccccc} \theta^1_{\Pi_1} & \theta^2_{\Pi_1} & \theta^3_{\Pi_1} & \theta^4_{\Pi_1} & \theta^5_{\Pi_1} \end{array} \\ \begin{pmatrix} 0.1 & 0.3 & 0.6 & 0.5 & 0.5 \\ 0.5 & 0.6 & 0.8 & 0.9 & 0.7 \\ 0.9 & 0.1 & 0.7 & 0.8 & 0.6 \end{pmatrix} \end{array},$$

$$\widetilde{\gimel_2} = \begin{array}{c} \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{array} \begin{array}{c} \begin{array}{ccccc} \theta^1_{\Pi_2} & \theta^2_{\Pi_2} & \theta^3_{\Pi_2} & \theta^4_{\Pi_2} & \theta^5_{\Pi_2} \end{array} \\ \begin{pmatrix} 0.1 & 0.3 & 0.8 & 0.2 & 0.9 \\ 0.5 & 0.6 & 0 & 0.5 & 0.1 \\ 0.9 & 0.1 & 0.6 & 0.7 & 0.5 \end{pmatrix} \end{array},$$

$$\widetilde{\gimel_3} = \begin{array}{c} \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{array} \begin{array}{c} \begin{array}{ccccc} \theta^1_{\Pi_3} & \theta^2_{\Pi_3} & \theta^3_{\Pi_3} & \theta^4_{\Pi_3} & \theta^5_{\Pi_3} \end{array} \\ \begin{pmatrix} 0.1 & 0.3 & 0.8 & 0.7 & 0 \\ 0.5 & 0.6 & 0 & 0.6 & 0.7 \\ 0.9 & 0.1 & 0.6 & 0.5 & 0.6 \end{pmatrix} \end{array}.$$

Step(3): To arrive at the patient-diet matrix (the diagnosis matrix) for each category of obese patients, one can take the transpose for the three matrices $\widetilde{\gimel_1}$, $\widetilde{\gimel_2}$, and $\widetilde{\gimel_3}$, resulting from the previous step. Then, one can obtain the male obese patient-diet matrix, the female obese patient-diet matrix, and the child obese patient-diet matrix, respectively, by calculating the three products $\widetilde{D_1} = \widetilde{\Delta_{1\mu}} \times \widetilde{\gimel_1}^T$, $\widetilde{D_2} = \widetilde{\Delta_{2\mu}} \times \widetilde{\gimel_2}^T$, and $\widetilde{D_3} = \widetilde{\Delta_{3\mu}} \times \widetilde{\gimel_3}^T$ as follows:

$$\widetilde{D_1} = \widetilde{\Delta_{1\mu}} \times \widetilde{\gimel_1}^T = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} \psi_1 & \psi_2 & \psi_3 \end{array} \\ \begin{pmatrix} 0.82 & 1.63 & \mathbf{1.86} \\ 1.37 & \mathbf{2.3} & 1.92 \\ 1.49 & \mathbf{2.59} & 2.34 \end{pmatrix} \end{array},$$

$$\widetilde{D_2} = \widetilde{\Delta_{2\mu}} \times \widetilde{\Im_2}^T = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \\ f_4 \end{array} \!\!\begin{array}{c} \begin{array}{ccc} \psi_1 & \psi_2 & \psi_3 \end{array} \\ \begin{pmatrix} 1.01 & 0.93 & \mathbf{1.56} \\ \mathbf{1.46} & 0.96 & 1.37 \\ 1.3 & 0.91 & \mathbf{2.07} \\ \mathbf{1.83} & 0.82 & 1.71 \end{pmatrix} \end{array},$$

$$\widetilde{D_3} = \widetilde{\Delta_{3\mu}} \times \widetilde{\Im_3}^T = \begin{array}{c} \\ ch_1 \\ ch_2 \end{array} \!\!\begin{array}{c} \begin{array}{ccc} \psi_1 & \psi_2 & \psi_3 \end{array} \\ \begin{pmatrix} 0.96 & \mathbf{1.96} & 1.67 \\ 1.25 & 1.25 & \mathbf{1.59} \end{pmatrix} \end{array}.$$

In addition, for simplicity, the above three diagnosis matrices $\widetilde{D_1}$, $\widetilde{D_2}$, and $\widetilde{D_3}$ could be merged into one diagnosis matrix, namely $\widetilde{D}$ as given below:

$$\widetilde{D} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ ch_1 \\ ch_2 \end{array} \!\!\begin{array}{c} \begin{array}{ccc} \psi_1 & \psi_2 & \psi_3 \end{array} \\ \begin{pmatrix} 0.82 & 1.63 & \mathbf{1.86} \\ 1.37 & \mathbf{2.3} & 1.92 \\ 1.49 & \mathbf{2.59} & 2.34 \\ 1.01 & 0.93 & \mathbf{1.56} \\ \mathbf{1.46} & 0.96 & 1.37 \\ 1.3 & 0.91 & \mathbf{2.07} \\ \mathbf{1.83} & 0.82 & 1.71 \\ 0.96 & \mathbf{1.96} & 1.67 \\ 1.25 & 1.26 & \mathbf{1.59} \end{pmatrix} \end{array}.$$

$Step(4)$: Finally, to determine the best diet for each obese patient (the final diagnosis) from the above resulting final diagnosis matrix $\widetilde{D}$, the maximum value in each row must be specified. It is evident that for the first male obese patient m_1, the first and the third female obese patients f_1, f_3, as well as the second child obese patient ch_2, the third value is the greatest one. That is to say that the values 1.86, 1.56, 2.07, and 1.59 are the maximum values for obese patients m_1, f_1, f_3, and ch_2, respectively, corresponding to the third diet ψ_3 (the fast diet 2-5). Therefore, the third diet ψ_3 (the fast diet 2-5) is the most suitable diet for obese patients m_1, f_1, f_3, and ch_2. In addition, for the second and the third male obese patients m_2, m_3, as well as the first child obese patient ch_1, the second value is the maximum value. That is to say that the values 2.3, 2.59, and 1.96 are the maximum values for the obese patients m_2, m_3, and ch_1, respectively, corresponding to the second diet ψ_2, which is the keto diet. This means that the second diet ψ_2 (the keto diet) is the most suitable diet for obese patients m_2, m_3, and ch_1. Moreover, for the second and the fourth female obese patients f_2, f_4, the first value is the maximum value. Then, the values 1.46, and 1.83 are the maximum values for the obese patients f_2, and f_4, respectively, corresponding to the first diet ψ_1, representing the Luqaimat diet. That is, the first diet ψ_2 (the Luqaimat diet) is the most suitable diet for obese patients f_2, and f_4.

At the end, the ranking of the diets (as alternatives) for each patient, with priority is $\psi_3 > \psi_2 > \psi_1$ for m_1, and ch_2, $\psi_2 > \psi_3 > \psi_1$ for m_2, m_3, and ch_1, $\psi_3 > \psi_1 > \psi_2$ for f_1, and f_3, as well as $\psi_1 > \psi_3 > \psi_2$ for f_2, and f_4. Also, the ranking of the obese patients, according to priority in general is: $m_3 > m_2 > f_3 > ch_1 > m_1 > f_4 > ch_2 > f_1 > f_2$.

6.3. Comparative analysis

In this section, a comparison is performed to evaluate decision-making in the effective fuzzy soft multiset environment with previous methods or techniques. We apply the previously defined variants of the same Example (6.1). An outline of the results of the comparative analysis is provided below:

(1) The outcomes are displayed below if we use Yang et al.'s [32] fuzzy soft set and their algorithm phases of development for the final determination. The following is how the final diagnosis matrix $\widetilde{D}$ looks:

$$\widetilde{D} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ ch_1 \\ ch_2 \end{array} \begin{array}{c} \begin{array}{ccc} \psi_1 & \psi_2 & \psi_3 \end{array} \\ \begin{pmatrix} 0.6 & 1.08 & \mathbf{1.48} \\ 0.96 & \mathbf{1.05} & 1.04 \\ 1 & 1.1 & \mathbf{1.4} \\ 0.86 & 0.93 & \mathbf{1.3} \\ 0.84 & \mathbf{0.98} & 0.83 \\ 0.63 & 0.96 & \mathbf{1.4} \\ 1.08 & 1 & \mathbf{1.39} \\ 0.63 & \mathbf{1.07} & 0.91 \\ 1.26 & 1.2 & \mathbf{1.41} \end{pmatrix} \end{array}.$$

It is clear using the final resulting diagnosis matrix $\widetilde{D}$ that the second value in the row for each of the obese patients m_2, f_2, and ch_1 is the maximum number (i.e., 1.05, 0.98, and 1.07, respectively), and each of these values is corresponding to the second diet ψ_2, which is the keto diet. Moreover, one can find from $\widetilde{D}$ that the greatest value in the row for each of the obese patients m_1, m_3, f_1, f_3, f_4, and ch_2, which are 1.48, 1.4, 1.3, 1.4, 1.39, and 1.41, respectively, is the third one that corresponds to the third diet ψ_3, which is the fast diet (2-5). Furthermore, one can give the order of the diets (as alternatives) for each patient, by the following priority: $\psi_3 > \psi_2 > \psi_1$ for m_1, m_3, f_1, and f_3, $\psi_2 > \psi_3 > \psi_1$ for m_2, and ch_1, $\psi_2 > \psi_1 > \psi_3$ for f_2, as well as $\psi_3 > \psi_1 > \psi_2$ for f_4, and ch_2. In addition, we have the priority ranking of obese patients, in general, as follows: $m_1 > ch_2 > m_3 = f_3 > f_4 > f_1 > ch_1 > m_2 > f_2$.

(2) If the ultimate decision is made using the process steps and the fuzzy soft multiset, as suggested by Alkhazaleh and Salleh [4], the results are as follows. The following is how the final diagnosis matrix $\widetilde{D}$ is obtained:

$$\widetilde{D} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ ch_1 \\ ch_2 \end{array} \begin{array}{c} \begin{array}{ccc} \psi_1 & \psi_2 & \psi_3 \end{array} \\ \begin{pmatrix} 0.64 & 1.32 & \mathbf{1.57} \\ 0.95 & \mathbf{1.5} & 1.14 \\ 0.95 & \mathbf{1.62} & 1.52 \\ 0.87 & 0.69 & \mathbf{1.29} \\ 0.73 & 0.72 & \mathbf{0.84} \\ 0.61 & 0.72 & \mathbf{1.44} \\ \mathbf{1.56} & 0.42 & 1.3 \\ 0.7 & \mathbf{1.07} & 0.8 \\ 1.12 & 1.03 & \mathbf{1.38} \end{pmatrix} \end{array}.$$

Then, extracting from the above final diagnosis matrix $\widetilde{D}$, the first value in the 7^{th} row, which is 1.56 is the greatest number for the obese patient f_4, and this value is corresponding to the first diet ψ_1, which is the Luqaimat diet. In addition, the maximum value in the row for each of the obese patients m_2, m_3, and ch_1, which are 1.5, 1.62, and 1.07, respectively, is the second one that corresponds to the second diet ψ_2, which is the keto diet. Furthermore, for the obese patients m_1, f_1, f_2, f_3, and ch_2, it is clear that the greatest value in each row is the third value (respectively, 1.57, 1.29, 0.84, 1.44, and 1.38) corresponding to the third diet ψ_3, which is the fast diet (2-5). On top of that, the diets (as alternatives) can be ordered for each patient, with the following order of priority: $\psi_3 > \psi_2 > \psi_1$ for m_1, and f_3, $\psi_2 > \psi_3 > \psi_1$ for m_2, m_3, and ch_1, $\psi_3 > \psi_1 > \psi_2$ for f_1, f_2, and ch_2, as well as $\psi_1 > \psi_3 > \psi_2$ for f_4. Additionally, in general, one can provide the following priority ranking of obese patients: $m_3 > m_1 > f_4 > m_2 > f_3 > ch_2 > f_1 > ch_1 > f_2$.

(3) The following results are obtained when the method steps are applied to the effective fuzzy soft set given by Alkhazaleh [3] and the conclusion is reached under it. The source of the final diagnosis matrix $\widetilde{D}$ is:

$$\widetilde{D} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ ch_1 \\ ch_2 \end{array} \begin{array}{c} \begin{array}{ccc} \psi_1 & \psi_2 & \psi_3 \end{array} \\ \begin{pmatrix} 0.77 & 1.31 & \mathbf{1.74} \\ 1.4 & 1.64 & \mathbf{1.77} \\ 1.55 & 1.84 & \mathbf{2.16} \\ 1.06 & 1.23 & \mathbf{1.57} \\ 1.4 & \mathbf{1.5} & 1.4 \\ 1.2 & 1.44 & \mathbf{2.1} \\ 1.34 & 1.45 & \mathbf{1.8} \\ 1.1 & \mathbf{1.87} & 1.79 \\ 1.39 & 1.42 & \mathbf{1.65} \end{pmatrix} \end{array}.$$

From the final diagnosis matrix $\widetilde{D}$, it is clear that the greatest value in each row for the obese patients f_2, and ch_1 is the second value (1.5 and 1.87, respectively), and these values are corresponding to the second diet ψ_2, which is the keto diet. On top of that, the maximum number in the row for each of the obese patients m_1, m_2, m_3, f_1, f_3, f_4, and ch_2, which are 1.74, 1.77, 2.16, 1.57, 2.1, 1.8, and 1.65, respectively, is the third value that corresponds to the third diet ψ_3, which is the fast diet (2-5). Moreover, we can introduce the order of priority of the diets (as alternatives) for each patient as follows: $\psi_3 > \psi_2 > \psi_1$ for m_1, m_2, m_3, f_1, f_3, f_4, and ch_2, $\psi_2 > \psi_1 = \psi_3$ for f_2, as well as $\psi_2 > \psi_3 > \psi_1$ for ch_1. Also, the priority ranking of the obese patients, in general, can be investigated as the following: $m_3 > f_3 > ch_1 > f_4 > m_2 > m_1 > ch_2 > f_1 > f_2$.

For Example (6.1), Tables 1, 3, and 5 give the different models' diagnosis values for the Luqaimat diet (ψ_1), the keto diet (ψ_2), and the fast diet (2-5) (ψ_3), respectively. Moreover, Tables 2, 4, and 6 show the patients' priority order by different models for the Luqaimat diet (ψ_1), the keto diet (ψ_2), and the fast diet (2-5) (ψ_3), respectively. Furthermore, Tables 7 and 8 summarize the final decision, as well as the diets' priority order, for each patient by different models, respectively. Finally, Table 8 describes the general priority order of the patients by different models.

Table 1. Different models' diagnosis values for the Luqaimat diet (ψ_1).

Patients	Yang et al. [32]	Alkhazaleh & Salleh [4]	Alkhazaleh [3]	Proposed model
m_1	0.6	0.64	0.77	0.82
m_2	0.96	0.95	1.4	1.37
m_3	1	0.95	1.55	1.49
f_1	0.86	0.87	1.06	1.01
f_2	0.84	0.73	1.4	1.46
f_3	0.63	0.61	1.2	1.3
f_4	1.08	1.56	1.34	1.83
ch_1	0.63	0.7	1.1	0.96
ch_2	1.26	1.12	1.39	1.25

Table 2. Patients' priority order for the Luqaimat diet (ψ_1) by different models.

Models	The Luqaimat diet (ψ_1)
Yang et al. [32]	$ch_2 > f_4 > m_3 > m_2 > f_1 > f_2 > f_3 = ch_1 > m_1$
Alkhazaleh & Salleh [4]	$f_4 > ch_2 > m_2 = m_3 > f_1 > f_2 > ch_1 > m_1 > f_3$
Alkhazaleh [3]	$m_3 > m_2 > f_2 > ch_2 > f_4 > f_3 > ch_1 = f_1 > m_1$
Proposed model	$f_4 > m_3 > f_2 > m_2 > f_3 > ch_2 > f_1 > ch_1 > m_1$

Table 3. Different models' diagnosis values for the keto diet (ψ_2).

Patients	Yang et al. [32]	Alkhazaleh & Salleh [4]	Alkhazaleh [3]	Proposed model
m_1	1.08	1.32	1.31	1.63
m_2	1.05	1.5	1.64	2.3
m_3	1.1	1.62	1.84	2.59
f_1	0.93	0.69	1.23	0.93
f_2	0.98	0.72	1.5	0.96
f_3	0.96	0.72	1.44	0.91
f_4	1	0.42	1.45	0.82
ch_1	1.07	1.07	1.87	1.96
ch_2	1.2	1.03	1.42	1.26

Table 4. Patients' priority order for the keto diet (ψ_2) by different models.

Models	The keto diet (ψ_2)
Yang et al. [32]	$ch_2 > m_3 > m_1 > ch_1 > m_2 > f_4 > f_2 > f_3 > f_1$
Alkhazaleh & Salleh [4]	$m_3 > m_2 > m_1 > ch_1 > ch_2 > f_2 = f_3 > f_1 > f_4$
Alkhazaleh [3]	$ch_1 > m_3 > m_2 > f_2 > f_4 > f_3 > ch_2 > m_1 > f_1$
Proposed model	$m_3 > m_2 > ch_1 > m_1 > ch_2 > f_2 > f_1 > f_3 > f_4$

Table 5. Different models' diagnosis values for the fast diet (2-5) (ψ_3).

Patients	Yang et al. [32]	Alkhazaleh & Salleh [4]	Alkhazaleh [3]	Proposed model
m_1	1.48	1.57	1.74	1.86
m_2	1.04	1.14	1.77	1.92
m_3	1.4	1.52	2.16	2.34
f_1	1.3	1.29	1.57	1.56
f_2	0.83	0.84	1.4	1.37
f_3	1.4	1.44	2.1	2.07
f_4	1.39	1.3	1.8	1.71
ch_1	0.91	0.8	1.79	1.67
ch_2	1.41	1.38	1.65	1.59

Table 6. Patients' priority order for the fast diet (2-5) (ψ_3) by different models.

Models	The fast diet (2-5) (ψ_3)
Yang et al. [32]	$m_1 > ch_2 > m_3 = f_3 > f_4 > f_1 > m_2 = ch_1 > f_2$
Alkhazaleh & Salleh [4]	$m_1 > m_3 > f_3 = ch_2 > f_4 > f_1 > m_2 > f_2 > ch_1$
Alkhazaleh [3]	$m_3 > f_3 > f_4 > ch_1 > m_2 > m_1 > ch_2 > f_1 > f_2$
Proposed model	$m_3 > f_3 > m_2 > m_1 > f_4 > ch_1 > ch_2 > f_1 > f_2$

Table 7. The final decision for each patient by different models.

Decision (the best diet)	Yang et al. [32]	Alkhazaleh & Salleh [4]	Alkhazaleh [3]	Proposed model
ψ_1 (Luqaimat diet)	−	f_4	−	f_2, f_4
ψ_2 (Keto diet)	m_2, f_2, ch_1	m_2, m_3, ch_1	f_2, ch_1	m_2, m_3, ch_1
ψ_3 (Fast diet) (2-5)	$m_1, m_3, f_1, f_3, f_4, ch_2$	m_1, f_1, f_2, f_3, ch_2	$m_1, m_2, m_3, f_1, f_3, f_4, ch_2$	m_1, f_1, f_3, ch_2

Table 8. Diets' priority order for each patient by different models.

Diets' priority order	Yang et al. [32]	Alkhazaleh & Salleh [4]	Alkhazaleh [3]	Proposed model
$\psi_1 > \psi_3 > \psi_2$	−	f_4	−	f_2, f_4
$\psi_2 > \psi_1 > \psi_3$	−	−	−	−
$\psi_2 > \psi_1 = \psi_3$	f_2	−	f_2	−
$\psi_2 > \psi_3 > \psi_1$	m_2, ch_1	m_2, m_3, ch_1	ch_1	m_2, m_3, ch_1
$\psi_3 > \psi_1 > \psi_2$	f_4, ch_2	f_1, f_2, ch_2	−	f_1, f_3
$\psi_3 > \psi_2 > \psi_1$	m_1, m_3, f_1, f_3	m_1, f_3	$m_1, m_2, m_3, f_1, f_3, f_4, ch_2$	m_1, ch_2

Table 9. General priority order of the patients by different models.

Models	Patients' priority order in general
Yang et al. [32]	$m_1 > ch_2 > m_3 = f_3 > f_4 > f_1 > ch_1 > m_2 > f_2$
Alkhazaleh & Salleh [4]	$m_3 > m_1 > f_4 > m_2 > f_3 > ch_2 > f_1 > ch_1 > f_2$
Alkhazaleh [3]	$m_3 > f_3 > ch_1 > f_4 > m_2 > m_1 > ch_2 > f_1 > f_2$
Proposed model	$m_3 > m_2 > f_3 > ch_1 > m_1 > f_4 > ch_2 > f_1 > f_2$

To sum up, Figure 4 gives a brief visualization of the diagnosis values of Tables 1, 3, and 5 represented with a simple chart.

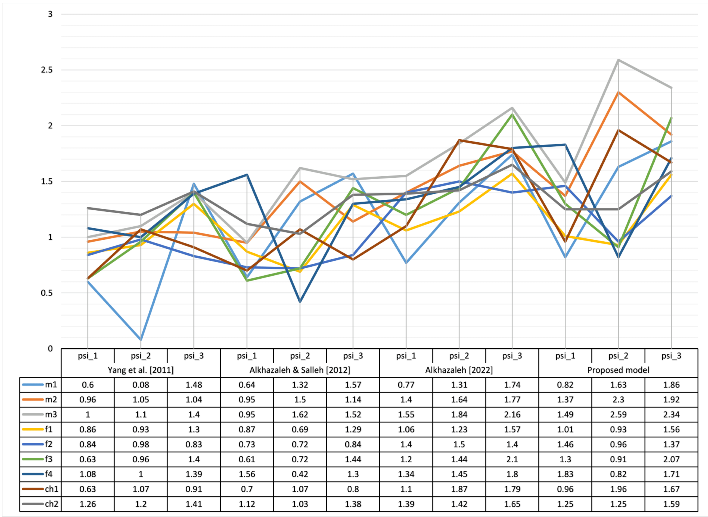

Figure 4. Different models' comparative results.

Validation and discussion: From the above tables and Figure 3, one can find it clear that the results of the proposed model are more accurate than those obtained from the other existing models. From Table 2, we notice that in all the previous models, there is one equality in the patients' priority order, which can confuse us when dealing with these cases, but the proposed model has no equality. The same note exists in Tables 4, 6, and 9. In addition, Table 8 shows that there is one order of priority for diets represented in $\psi_2 > \psi_1 = \psi_3$ appearing for the obese patient f_2 when applying Yang's model or Alkhazaleh's model. This order has one equality, which means that if this obese patient follows the most nominated diet, which is the second one (the keto diet), then after a period of time, she wants to change her diet, will she follow the first diet (the Luqaimat diet) or the third diet (the fast diet (2-5))? The proposed model out of all applied models in this comparison is the only one that hasn't any sign of equality.

Furthermore, from Table 7, one can easily see that, for the proposed model, the decisions are distributed normally; two obese patients obtain the first diet (the Luqaimat diet) as the best one, three obese patients are expected to follow the second diet (the keto diet) as their optimal diet, and four obese patients get the most suitable diet as the third diet, which is the fast diet (2-5), whereas the decisions have some biased distribution for the other previous models. For instance, one can notice that when applying Yang's model or Alkhazaleh's model, there aren't any obese patients expected to follow the first diet as the most suitable diet which is the Luqaimat diet. In addition, when applying Alkhazaleh's model, seven out of nine obese patients obtain their most suitable diet as the third diet, which is the fast diet (2-5). Moreover, it is clear that, for Yang's model or Alkhazaleh & Salleh's model, the effective parameters (the external factors), which can impact the decision on various problems, have been neglected.

7. Conclusions and future studies

The main purpose of this study has been to create an effective fuzzy soft multiset, which is a novel hybrid extension of the basic crisp set notion. It has included an explanation of the numerous kinds as well as recently included crucial ideas, as well as procedures. It has also investigated distributive laws, in addition to De Morgan's laws. Along with associative properties, it has also exhibited absorption, in addition to commutative properties. The paper has also offered a decision-making methodology and algorithm built on fuzzy soft multisets that work well.

We have had a new effective method to determine the optimal diet for each obese patient, which he can follow for a very long time without getting bored until he reaches the ideal desired weight because the choice of this diet came according to the requirements, as well as circumstances of this person so that he can continue with it. We should never specify only one diet that all people follow without considering the differences between them. We also have taken into consideration many other external factors that affect the validity of our decision, such as age, and health, as well as social conditions, in addition to environmental conditions, and sleep schedules, along with the number of hours of sleep, etc. Hence, anyone who wants to lose weight can apply the aforementioned method based on this new extension of sets to choose the best diet suitable for him. Additionally, to illustrate the positive aspects of the suggested method, the paper compares it to various currently available methods.

The importance of this paper lies in the fact that it has been concerned with treating a very serious issue, which is obesity, its spread, the difficulty of confronting it, and the highly dangerous diseases that result from it. In this research, the new strategy developed is effective and more general than previous ones to determine the optimal choice of reliable diets that suit the patient's needs, along with living conditions. This strategy is built on the new extension of sets, which is the effective fuzzy soft multiset. In addition, this strategy can treat problems containing multi-universal sets, as well as multi-attribute sets. Furthermore, this method, although the given environment is complex, is seen to be easier to apply than other methods.

One of the advantages of the proposed method is seen in the above comparison that it is more accurate in the final decisions than the other previous methods. Moreover, the proposed technique is more general than the other existing techniques since it takes into consideration the effective parameters (external factors), along with the multiset concept when we have some different categories. In addition, the proposed model is more appropriate than the other previous models or the other generalizations for the circumstances of the problem.

The study's adoption of the effective fuzzy soft multisets was most likely motivated by their capacity to successfully model the complexity, uncertainty, and multiplicity of aspects involved in the obesity treatment decision-making outline. While different generalizations of fuzzy sets; like IFS, q-ROF, etc., have their own advantages, the effective fuzzy soft multisets provide a complete and adaptable framework that is well-suited to the study's specific objectives.

It's crucial to recognize that the recommended strategy could have intrinsic flaws or limitations, just like any other technique or structure. In particular, it might have trouble processing a large number of attributes or items, which would need a lot of calculations. Mathematical programs like *Wolfram Mathematica*® or *MATLAB*® can be used to process enormous quantities of data efficiently to overcome this constraint. The paper also notes that scenarios involving bipolarity with fuzzy soft data, which are frequent in real-world circumstances, would not be appropriate for the effective fuzzy soft multiset. Future research could explore expanding the study's concepts to include bipolar-valued fuzzy soft multisets.

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Conflicts of interest

There is no conflict of interest confirmed by the author.

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