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Research article

A group decision making approach based on the multi-dimensional Steiner point

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Abstract: The social division of labor has become increasingly specialized, and there are more and more group decision-making problems participated by multiple decision-makers. With respect to the multi-attribute group decision making problem, including two-tuple linguistic information, based on the theory and method of group decision making, Steiner point constraint and plant growth simulation algorithm, we establish a novel multi-attribute group decision making approach based on two-tuple linguistic information aggregation. We introduce Steiner points into group consensus decision making and use the PGSA algorithm to seek the global optimal point. The method seeks set points that are both mathematically and geometrically meaningful to reduce set bias. In this paper, to begin with, according to the constraints of multi-dimensional Steiner point, we map the evaluation vectors of the group experts over the alternatives into multi-dimensional space and then we propose a two-tuple linguistic information aggregation model. Moreover, we construct a comprehensive evaluation decision making approach and then design a plant growth simulation algorithm to select the optimal alternative. Finally, a case verifies the validity and rationality of the proposed model.

Keywords: two-tuple; plant growth simulation algorithm; multi-dimensional Steiner point; information aggregation
Mathematics Subject Classification: 03E72

1. Introduction

The issue of group decision-making widely exists in various fields such as society, economy and management and is mostly used in investment decision-making, project evaluation, quality evaluation, program selection, site selection, resource allocation, scientific research achievement evaluation, talent evaluation, industrial development order, comprehensive economic benefits and other aspects.

In the actual decision-making process, decision-making problems in the objective world are characterized by complexity and uncertainty. On the one hand, there are limitations in the knowledge and decision-making experience of decision-makers and ambiguity in human thinking. Jiang et al. [1] combined a new similarity calculation method for cloud model, the netting clustering and interval rough integrated cloud (IRIC) to solve large group decision-making (LGDM) in uncertain linguistic environments. In the process of large group emergency decision-making (LGEDM), Jiang et al. [2] proposed a decision-making method based on PHFS and cloud model. On the other hand, it is difficult for decision makers to express the preference information of decision problems in the form of precise numbers, resulting in group decision problems containing a large amount of linguistic information in reality. For example, when evaluating the overall quality of students, the performance of cars, etc. decision makers generally prefer to give it directly in the form of "excellent", "good", "medium", "poor" and other linguistic forms. The issue of how to quantify qualitative language evaluation information has attracted the attention of relevant scholars. Herrera-Viedma et al. [3] and Jiang et al. [4] transformed linguistic information into triangular fuzzy numbers using the affiliation function, Herrera et al. [5] proposed a binary semantic analysis method on linguistic information and Xu et al. [6] utilized the linguistic evaluation information itself for the processing, i.e., continuous virtual terminological indicators.

Existing information aggregation methods commonly use aggregation operators based on arithmetic average, geometric average or weighted average derivation, which are more traditional and difficult to guarantee the aggregation accuracy. To effectively process and extract linguistic information and make scientific decisions, it is necessary to develop an effective information aggregation method. In this paper, we extend Steiner points to the field of consensus decision making, and the Steiner point-based agglomerative method solves for the Steiner point that minimizes the sum of the distances from other points as the group preference by mapping the preference information into a planar coordinate system and using a plant growth simulation algorithm, so that the solved agglomerative point improves the accuracy of the agglomerative precision. Given this, we exploit a multi-dimensional Steiner point aggregation method to establish a group decision-making method, aiming at the language preference information of decision-makers in the form of two-tuple linguistic information.

The major research contents of this paper are as follows: Section 2 gives the related literatures on two-tuple linguistic information group decision making. In Section 3, we introduce some definitions and operation rules of the aggregation method of two-tuple linguistic information, plant growth simulation algorithm, Steiner-based aggregation method. In Section 4, we establish a group decision making approach based on multi-dimensional Steiner point. In Section 5, the selection of novel coronavirus vaccine verifies the effectiveness of the proposed method. Section 6 summarizes the research results and points out the future research direction.

2. Literature review

In the process of processing linguistic information, the aggregated results usually do not match any initial linguistic terms. Then an approximation process must be used, resulting in the lack of precision in the aggregated results of linguistic information. However, two-tuple linguistic information is an effective method to overcome this defect. In recent years, research on two-tuple linguistic information group decision making is mostly divided into the following two categories:

(1) Research on the aggregation method of two-tuple linguistic information. Spanish scholars Herrera and Martinez [7] first proposed a two-tuple linguistic information for language method information aggregation and also proposed an ordered weighted average (T-OWA) operator based on two-tuple linguistic information. Jiang and Fan [8] extended the ordered weighted geometry (OWG) operator in the traditional fuzzy set operator to a two-tuple linguistic information ordered weighted geometry (T-OWG) operator and further analyzed the T-OWA operator and T - Properties of the OWG operator. Herrera and Herrera-Viedma [9] proposed three weighted language information set calculators: LWD, LWC and LWA and proved their rationality through axiomatic research. Xu [10] proposed a fuzzy language preference matrix sorting method based on IOWA operator. Based on fuzzy language evaluation and GIOWA operator, Xu [11] proposed a multi-attribute group decision-making method. Xu [12] also proposed a multi-attribute group decision-making method based on fuzzy language evaluation and language OWA operator. Wei [13] extended the n-dimensional weighted harmonic average (WHA) operator and the OWHA operator to the two-tuple linguistic information environment, and proposed the two-tuple linguistic information weighted harmonic average (T-WHA) operator, two-tuple linguistic information Ordered Weighted Harmonic Average (T-OWHA) operator and two-tuple linguistic information Combination Weighted Harmonic Average (T-CWHA) operator. Faizi et al. [14] introduced a hesitant 2-tuple linguistic weighted average (H2TLWA) operator to solve multi-criteria group decision-making (MCGDM) problems. Furthermore, Akram et al. [15] proposed an integrated decision-making method based on 2-tuple linguistic m-polar fuzzy information. Then, Akram et al. [16] developed a new framework for group decision making based on pythagorean fuzzy N-Soft PROMETHEE approach. Based on the equivalent alpha - level type-1 OWA operator, He et al. [17] proposed the ELICIT type-1 OWA (ELICIT-t1-OWA) operator to simplify the CW processes using ELICIT trapezoidal fuzzy representations. Akram, Naz and Abbas [18] developed a method for solving multi-attribute group decision-making (MAGDM) problems with complex q-rung orthopair fuzzy 2-tuple linguistic sets. Then Akram et al. [19] proposed an extended CODAS method for MAGDM with 2-tuple linguistic T-spherical fuzzy sets. Akram et al. [20] develops an extended multi-attributive border approximation area comparison (MABAC) method for solving multiple attribute group decision-making problems in this study.

(2) Research on the application of two-tuple linguistic information group decision making. Herrera and Martinez [21] used a two-tuple linguistic approach to solve the multi-attribute decision-making problem with multi-granularity linguistic scales. Jiang and Fan [22] proposed a cluster knot and scheme selection method based on two-tuple linguistic information processing by using the T-OWA operator based on two-tuple linguistic information for group decision-making problems with preference information in the form of language judgment matrices with different granularities. Liu et al. [23] proposed an approach to probabilistic hesitant fuzzy risky multi-attribute decision making with unknown probability information. Aiming at the situation that both expert weight and attribute weight are unknown in group decision-making problem, a multi-attribute group decision-making method is

proposed based on two-dimensional two-tuple linguistic representation model [24]. Based on interval two-tuple linguistic information VIKOR, Dai and Qi [25] proposed a multi-attribute group decision-making method to solve the group decision-making problem in which expert weights and attribute weights are completely unknown and attribute values are interval two-tuple linguistic information. Zhang [26] defined the two-tuple linguistic information gray correlation coefficient and two-tuple linguistic information gray correlation degree and proposed a group decision method based on two-tuple linguistic information correlation analysis. Ge and Wei [27] defined the hesitant fuzzy two-tuple linguistic information set, the mean function, variance function and its set counter of hesitant fuzzy two-tuple linguistic information set and proposed a hesitant fuzzy language decision-making method based on two-tuple linguistic information.

In order to obtain the optimal solution and realize the optimal decision-making, scholars at home and abroad determine the optimal aggregation belief matrix based on the constructed information aggregation method and select the optimal decision-making solution. Among them, the WAA operator was used to calculate the final score of each scheme in the group decision matrix [28]; while simple weighted average was generally used to calculate each the comprehensive evaluation score of the scheme [29–32]; Cao et al. [33] used the relative closeness to rank the alternatives; and Liu et al. [34] ranked the schemes by calculating the interval-valued trust function. An interval-valued trust propagator for Uninform was proposed to obtain indirect trust relationships and generate personalized recommendations [35]. In addition to obtaining indirect trust relationships between decision makers through trust propagation operators, Gong et al. [36] established trust transfer models based on linear uncertain variables, and the single trust path constraint model is established by discussing the constraint relation between trust paths.

Considering factors such as the behavior of decision makers and the relationship between decision makers, researchers constructed many social network group consensus models. The behavior of decision makers may affect decision efficiency and decision result. Wu et al. [37] defined individual logarithmic square compatibility measures and group logarithmic square compatibility measures and then constructed an optimal group selection model to distinguish between redundant preferences and optimal groups. Chu et al. [38] considered decision-maker prestige factors and used importance-induced ordered weighted average operators to assemble all individual fuzzy preference relationships with social exponents. Lu et al. [39] proposed a minimal-cost model based on robust optimization to solve the consensus problem in social networks. Wu et al. [40] established a minimal-adjustment consensus model in social network group decision making and gave a feedback mechanism framework for decision makers who require modifying opinions at three consensus levels. In order to explore the impact of trust on consensus, Wu et al. [41] proposed a minimal-cost consensus model based on invisible trust between individuals and regulators. Access control trust was introduced in the field of information technology and established subjective and objective trust to evaluate the trust level [42]. Wu et al. [43] constructed calibrated individual semantic model to achieve linguistic calibration for different DMs. In the above linguistic representations, PLTS describes the evaluation information using several possible linguistic terms with corresponding probabilities, which is suitable to explain the probability distribution information in both individual context and group context [44]. To achieve this goal, consensus reaching process (CRP) is applied to GDM in many real-world problems, such as the medical equipment selection [45], hydropower development project assessments [46], International Exchange candidate selection [47], etc. Based on the similarity degree and knowledge degree, similarity degree-based social network (SDSN) and trust relationship-based social network (TRSN) are constructed successively. According to the constructed SDSN and TRSN, the proposed CRP method consists of two stages [48].

According to the above discussion and analysis, the existing research has the following two problems: (1) For the aggregation problem of two-tuple linguistic information, most scholars at home and abroad use the aggregation method based on arithmetic, geometric or weighted average derivation, but its aggregation accuracy is not high, and the resulting rally point is not optimal. (2) In order to screen the optimal decision-making scheme, most scholars use the arithmetic average or its derivative calculation formula to calculate the scheme score and then sort them. However, its aggregation accuracy is not high, and the resulting rally point is not optimal. Aiming at the information aggregation problem of multi-attribute group decision making with two-tuple linguistic information, we use the theory and method of group decision making, Steiner point constraint and plant growth simulation algorithm to construct a group decision method based on multi-dimensional Steiner point aggregation. Furthermore, based on the multi-dimensional Steiner point aggregation, a comprehensive evaluation method for population decision-making is established, and a plant growth simulation algorithm is intended to solve the optimal decision-making scheme.

3. Preliminary knowledge

Assume that $A=\{A_1,A_2,\cdots,A_n\}$ is a finite scheme set, $C=\{C_1,C_2,\cdots,C_m\}$ is an attribute set and $E=\{E_1,E_2,\cdots,E_q\}$ is a set of judging experts. Let $W=(w_1,w_2,\cdots,w_m)$ be attribute weight and let $V=(v_1,v_2,\cdots,v_q)$ be judging expert's weight. Among them, $0\le w_j,v_l\le 1$, $\sum_{j=1}^{m}w_j=\sum_{l=1}^{q}v_l=1$, $j=1,2,\cdots,m$, $l=1,2,\cdots,q$. In reaching decisions, each expert must evaluate each attribute of each option. Experts use a natural linguistic evaluation set S to represent their evaluations. The evaluation value of the j-th attribute of the i-th scheme by the lth expert is x_{ij}^l. In this way, the following judgment matrix can represent the evaluation matrix of the lth expert:

$$X^l=[x_{ij}^l]_{n\times m}=\begin{bmatrix} x_{11}^l & x_{12}^l & \cdots & x_{1m}^l \\ x_{21}^l & x_{22}^l & \cdots & x_{2m}^l \\ \vdots & \vdots & x_{ij}^l & \vdots \\ x_{n1}^l & x_{n2}^l & \cdots & x_{nm}^l \end{bmatrix}.$$

Among them $x_{ij}^l\in S$, $i=1,2,\ldots n$, $j=1,2,\ldots m$, $l=1,2,\cdots,q$.

The natural linguistic evaluation set $S=\{s_0,s_1,\cdots,s_g\}$ usually contains an odd number of linguistic terms, and $g+1$ is the number of elements in the natural linguistic evaluation set. Generally, S is required to have the following properties:

(1) Orderliness: If $i>j$, then $s_i>s_j$;

(2) Containing inverse operators: $Neg(s_i)=s_j$ to make $j=g-i$;

(3) Maximization operator: If $s_i\ge s_j$, then $\max(s_i,s_j)=s_i$;

(4) Minimization operator: If $s_i\le s_j$, then $\min(s_i,s_j)=s_i$.

For example: For a classical set of seven terms, S can be given as follows:

$S=\{s_0=Very\ Poor, s_1=Poor, s_2=Slightly\ Poor, s_3=Fair, s_4=Slightly\ Good, s_5=Good, s_6=Verr\ Good\}$

To perform subsequent processing on the expert's linguistic preference matrix obtained above, it is necessary to unify the language evaluation information. The method of processing information consistency adopted is based on the concept of two-tuple linguistic information proposed by Herrera and Martinez [49].

Symbol model is a linguistic calculation method that uses linguistic category indicators to calculate. Set up a natural linguistic evaluation set $S=\{s_0,s_1,\dots s_g\}$, which is an ordered set of evaluations when $i<j$, $s_i<s_j$. The result of the calculation in the operation is a numerical value β, $\beta\in[0,g]$. Through approximate function (approximate) $app_2:[0,g]\to\{0,\cdots\in,g\}$, each calculation result can be an integer value in $[0,g]$ as much as possible and then it can represent the categories of linguistic evaluation terms, namely $s_{app_2(\beta)}\in S$. As follows: $S^n\xrightarrow{C}[0,g]\xrightarrow{app_2(\cdot)}\{0,\cdots,g\}\to S$.

Among them, C is a symbolic linguistic aggregation operator, and $app_2(\bullet)$ is an approximate function. A category value in $\{0,\cdots,g\}$ can be obtained through a value between $[0,g]$, corresponding to a linguistic term in the language evaluation set S.

Based on the above-mentioned symbolic model, the two-tuple linguistic model introduces the concept of symbol transfer to represent language information in the form of (s,α), that is, binary semantics, where s is the linguistic term and α is the symbol transition.

Definition 1. [37,38] *Suppose $s_i\in S$ is a linguistic term, then its corresponding two-tuple linguistic information can be obtained through the following transformation function θ:*

$$\theta:S\to S\times[-0.5,0.5),\theta(s_i)=(s_i,0),s_i\in S. \tag{1}$$

Definition 2. [45,46] *Suppose $S=\{s_0,s_1,\dots s_g\}$ is a linguistic evaluation set, $\beta\in[0,g]$ is the initial result after the aggregation operation, then the linguistic information equivalent to β can be represented in the form of the following two-tuple linguistic:*

$$\Delta:[0,g]\to S\times[-0.5,0.5), \tag{2}$$

$$\Delta(\beta)=\begin{cases}s_i, & i=round(\beta)\\ \alpha=\beta-i, & \alpha\in[-0.5,0.5)\end{cases}. \tag{3}$$

Among them, $round(\bullet)$ is the rounding operation for "rounding", s_i is the linguistic term category closest to β and α is the sign transfer value.

Definition 3. [5,7] *Supposing that $S=\{s_0,s_1,\dots s_g\}$ is a linguistic evaluation set, (s_i,α) is a two-tuple linguistic, there is always such a function Δ^{-1}, which can convert binary semantics into numerical values $\beta\in[0,g]\in R$:*

$$\Delta^{-1}:S\times[-0.5,0.5)\to[0,g], \tag{4}$$

$$\Delta^{-1}(s_i,\alpha)=i+\alpha=\beta. \tag{5}$$

In particular, for the initial linguistic term, by introducing the value of 0 as a symbolic transfer, it is converted into two-tuple linguistic, and then the two-tuple linguistics are converted into numerical values β:

$$s_i\in S\Rightarrow(i,0)\Rightarrow i. \tag{6}$$

Supposing that (s_k,α_1) and (s_l,α_2) are two 2-tuples. they should have the following properties [50,51]:

(1) Orderliness:

If $k < l$, (s_k, α_1) is smaller than (s_l, α_2);

If $k = l$,

1) when $\alpha_1 = \alpha_2$, (s_k, α_1), (s_l, α_2) represents the same information;

2) when $\alpha_1 < \alpha_2$, (s_k, α_1) is smaller than (s_l, α_2);

3) when $\alpha_1 > \alpha_2$, (s_k, α_1) is bigger than (s_l, α_2).

(2) Negation Operator:

We define the negation operator over 2-tuple as

$$Neg((s_i, \alpha)) = \Delta(g - (\Delta^{-1}(s_i, \alpha))), \tag{7}$$

where $g+1$ is the cardinality of S, $S = \{s_0, s_1, \ldots s_g\}$.

(3) Maximize operation:

If $(s_k, \alpha_1) > (s_l, \alpha_2)$, $Max\{(s_k, \alpha_1), (s_l, \alpha_2)\} = (s_k, \alpha_1)$.

(4) Minimization operation:

If $(s_k, \alpha_1) > (s_l, \alpha_2)$, $Min\{(s_k, \alpha_1), (s_l, \alpha_2)\} = (s_l, \alpha_2)$.

Among them, $round(\bullet)$ is the rounding operation for "rounding", s_i is the linguistic–term category closest to β and α is the sign transfer value.

4. A multi-attribute group decision making method

4.1. Steiner point

There are three points, A, B and C, on the plane that are not on the same straight line. If there is a point P such that the sum $D = |PA| + |PB| + |PC|$ of the distance from P to the three points is the smallest, the point P is called "Fermat point". This problem is a Steiner problem, and the points solved are called generalized Fermat points or Steiner points [52,53]. Then Steiner introduced the weight parameter in the above problem. Given that there are n points on a given plane $P_1, P_2, \cdots, P_n (n \geq 2)$, if there is a point P such that the sum $D = \sum_{i=1}^{n} w_i |PP_i|$ of the distances from P to n points on the plane is the smallest, point P is called a Steiner point. Here, $w_1, w_2, \cdots, w_n (n \geq 2)$ is the weight of each point. It is this Steiner point can best represent the comprehensive opinions of group experts. Based on this, we consider mapping expert opinions into multi-dimensional point sets, and uses the improved plant growth simulation algorithm to solve multi-dimensional Steiner points.

4.2. Expert judgment matrix mapping to multidimensional Steiner point aggregation

Let $A = \{A_1, A_2, \cdots, A_n\}$ be a finite set of solutions, $C = \{C_1, C_2, \cdots, C_m\}$ be an attribute set and $E = \{E_1, E_2, \cdots, E_q\}$ be the set of evaluation experts. When making decisions, each expert must evaluate each attribute of each solution. For example, in two-tuple linguistic multi-attribute group decision-making, the evaluation vector of expert E_l for scheme A_i is $\vec{x}_i^l = [x_{ij}^l]_{1\times m} = [x_{i1}^l, x_{i2}^l, \cdots, x_{im}^l], j = 1, 2, \cdots, m$. Each of the attributes represents a dimension, that is, m attributes represent m dimensions. So the evaluation vector $\vec{x}_i^l$ of the expert E_l on the plan A_i can be mapped to a point x_i^l in the m-dimensional multi-dimensional through the relevant transformation rules, where the coordinates of the point x_i^l is $(x_{i1}^l, x_{i2}^l, \cdots, x_{im}^l)$, that is:

$$[x_{ij}^l]_{1\times m} \to (x_{i1}^l, x_{i2}^l, \cdots, x_{im}^l) \in R^m . \tag{8}$$

Among them, R^m is the m-dimensional Euclidean space, $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$, $l = 1, 2, \cdots, q$.

4.3. An aggregation method based on multidimensional Steiner points

For the same program A_i, $i = 1, 2, \cdots, n$, each of the q experts has a corresponding evaluation vector. The mapping rules in 4.2 correspond to q points $x_i^1, x_i^2, \cdots, x_i^q$ in m-dimensional multi-dimensional. The purpose of a multidimensional Steiner assembly is to find a point x_i^* that minimizes $D = \sum_{l=1}^{q} v_l \mid x_i^l x_i^* \mid$. Point x_i^* is the aggregation of q experts' evaluation vectors for scheme A_i, which can reflect the overall opinions of q experts on the same scheme. Similarly, for n schemes, n multi-dimensional Steiner rendezvous points $x_1^*, x_2^*, \cdots, x_n^*$ can be aggregated, namely the optimal aggregation matrix $R^* = (x_1^*, x_2^*, \cdots, x_n^*)'$.

Definition 4. *Let* $X_i = \{x_i^1, x_i^2, \cdots, x_i^q\}$ *,* $(i = 1, 2, \cdots, n)$ *be the set of m-dimensional space points representing the opinion preferences of all experts on option* A_i *. Weight vector of q experts is* $v = (v_1, v_2, \cdots, v_q)$*. Among them,* $0 \le v_l \le 1$ *,* $(l = 1, 2, \cdots q)$ *and* $\sum_{v=1}^{q} v_l = 1$*. If the optimal rally point* x_i^* *exists, the sum of the Euclidean distances from* x_i^* *to all other points should satisfy the following conditions:*

$$D_i = \min \sum_{l=1}^{q} v_l \mid x_i^l x_i^* \mid = \min \sum_{l=1}^{q} v_l \sqrt{(x_{i1}^l - x_{i1}^*)^2 + (x_{i2}^l - x_{i2}^*)^2 + \cdots + (x_{im}^l - x_{im}^*)^2} . \tag{9}$$

The optimal rally point satisfies Pareto optimality. In the study of aggregated group preference, the preference of experts is mapped as a set of points in the m-dimensional space, so the distance between two points reflects the difference in experts' preference. Then, the experts' preferences can be aggregated by calculating the optimal rendezvous point to construct a rendezvous matrix.

Definition 5. *Let the coordinate of each scheme assembly point in the optimal assembly matrix* $R^* = (x_1^*, x_2^*, \cdots, x_n^*)'$ *be* $x_i^* = (x_{i1}^*, x_{i2}^*, \cdots, x_{im}^*)$ *, and the corresponding attribute weight be* $W = (w_1, w_2, \cdots, w_m)$*,* $j = 1, 2, \cdots, m$*. Map each attribute preference value* x_{ij}^* *of scheme* x_i^* *into a one-dimensional coordinate system to construct a one-dimensional Steiner aggregation model. Similarly, for n schemes, n one-dimensional Steiner gathering points* $x_1^{**}, x_2^{**}, \cdots, x_n^{**}$ *can be assembled, that is, the comprehensive evaluation score vector:*

$$\phi(A_i) = (x_1^{**}, x_2^{**}, \cdots, x_n^{**})' . \tag{10}$$

Definition 6. *Let the coordinate of each scheme assembly point in the optimal assembly matrix* $R^* = (x_1^*, x_2^*, \cdots, x_n^*)'$ *be* $x_i^* = (x_{i1}^*, x_{i2}^*, \cdots, x_{im}^*)$*, and the corresponding attribute weight* $W = (w_1, w_2, \cdots, w_m)$*, among which* $0 \le w_j \le 1$*,* $(j = 1, 2, \cdots, m)$ *and* $\sum_{j=1}^{m} w_j = 1$*. If the optimal rally point* x_i^{**} *exists, the sum of the Euclidean distances from* x_i^* *to all other points should satisfy the following conditions:*

$$d_i = \min \sum_{j=1}^{m} w_j \mid x_{ij}^* x_i^{**} \mid = \min \sum_{i=1}^{m} w_j \mid x_{ij}^* - x_i^{**} \mid . \tag{11}$$

The optimal solution is selected according to the ranking of the comprehensive evaluation scores.

4.4. Group decision-making algorithm based on multidimensional Steiner point aggregation

According to the characteristics of the Steiner problem, a solution algorithm is designed for this model based on the principle of plant growth simulation algorithm [54–59].

The self-similar structure of artificial plant growth is defined as: Growing in four directions of east, west, north and south at the growth point and producing new branches, and the rotation angle between the new branches $\alpha = 90°$. The branch length is generally set to l/1000 (l is the length of the bounded closed box). For n points $A_1, A_2, \cdots, A_n$, their respective weights are $w_1, w_2, \cdots, w_n$, and a point P is found to minimize $\sum_{i=1}^{n} w_j |PA_j|$. The iterative steps of the plant growth simulation algorithm for this problem are shown in Algorithm 1.

Algorithm 1. Group Decision-Making Algorithm.

Input: The finite set of solutions $A = \{A_1, A_2, \cdots, A_n\}$, the attribute set $C = \{C_1, C_2, \cdots, C_m\}$, the set of evaluation experts $E = \{E_1, E_2, \cdots, E_q\}$, the attribute weight $W = (w_1, w_2, \cdots, w_m)$, the judging experts weight $V = (v_1, v_2, \cdots, v_q)$.

Output: The ranking of alternative.

Step 1: Select arbitrarily the initial point $a_i \in X$, among which X is the given set of points to be assembled. Set the upper limit of the number of iterations to 10000.

Step 2: Find the probability value of each growth point according to the formula:

$$p_i = \frac{\sum_{j=1}^{n} \left(\frac{1}{|a_i A_j|} \right)}{\sum_{i=1}^{v} \sum_{j=1}^{n} \left(\frac{1}{|a_i A_j|} \right)}. \tag{12}$$

Step 3: Construct the probability space of each growth point in 0-1 according to the probability obtained above, and then select the growth point a_i of this iteration according to the random number.

Step 4:

(1) Set the step size to 0.001, and the growth point a_i is grown according to the L-system of $\alpha = 90°$.

(2) Calculate the sum of the Euclidean distances from all growth points to all the points to be assembled.

(3) Record the growth point minimizing sum of distances as the optimal growth point.

(4) Replace a_i with the optimal point in the new growth point;

Step 5: If there is no new growth point, and the iteration number of optimization is the same as the preset one, the solution can be regarded as the global optimal solution, and the calculation can be stopped. If the above conditions are not met, go to the step 2.

Step 6: After stopping the calculation, connect the global optimal growth point and $A_1, A_2, \cdots, A_n$ respectively to draw a graph (two-dimensional and three-dimensional scatter plots are supported).

Step 7: Compute the evaluation values. Output the ranking of alternatives.

For example, when solving a three-dimensional Steiner point, the results of obtaining the three-dimensional Steiner point by using the plant growth simulation algorithm are shown in Figure 1:

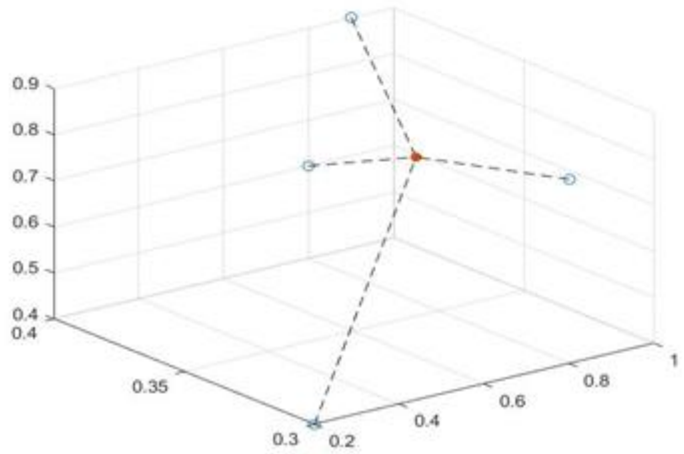

Figure 1. PGSA solves Steiner points in three-dimensional space.

5. Case analysis

5.1. The application of the proposed methodology

In order to deal with some unexpected events, as a result of the large number of mild patients and strong mobility, to make shift hospitals treat these patients to a large extent, some governments often construct some designated hospitals to expand the treatment capacity. Assume that a local government plan to set up five makeshift hospitals with respect to some unexpected event, which can be denoted as $A=\{A_1,A_2,A_3,A_4,A_5\}$, respectively. However, due to limited manpower and material resources, the five makeshift hospitals can only be built one by one. In order to maximize its effectiveness, it is necessary to comprehensively consider and sort the cabin hospitals. The four major factors $C=\{C_1,C_2,C_3,C_4\}$ affecting the efficiency of makeshift hospitals are comprehensively considered: C_1 = patient distribution, C_2 = regional medical and health resource allocation, C_3 = traffic conditions, C_4 = municipal pipe network communication facilities, and assign corresponding weights $W=(0.3,0.3,0.2,0.2)$ for these factors. Now invite 3 experts $E_k(k=1,2,3)$ to conduct a comprehensive linguistic evaluation of the five schemes, the experts' weight vector is $V=(0.3,0.4,0.3)$. The natural linguistic evaluation set of experts to the scheme is set:

$$S=\{s_0=Very\ \text{Poor}, s_1=Poor, s_2=Slightly\ Poor, s_3=Fair, s_4=Slightly\ Good, s_5=Good, s_6=Verr\ Good\}.$$

According to the above–linguistic evaluation set, the linguistic evaluation matrix of the three experts is as follows:

$$R_1=\begin{pmatrix} s_3 & s_4 & s_2 & s_2 \\ s_2 & s_1 & s_3 & s_2 \\ s_4 & s_3 & s_4 & s_0 \\ s_5 & s_2 & s_2 & s_4 \\ s_6 & s_0 & s_1 & s_3 \end{pmatrix},\ R_2=\begin{pmatrix} s_2 & s_3 & s_1 & s_1 \\ s_1 & s_0 & s_4 & s_4 \\ s_3 & s_4 & s_2 & s_6 \\ s_6 & s_1 & s_1 & s_3 \\ s_2 & s_1 & s_3 & s_1 \end{pmatrix},\ R_3=\begin{pmatrix} s_4 & s_2 & s_1 & s_5 \\ s_1 & s_4 & s_2 & s_4 \\ s_5 & s_1 & s_4 & s_2 \\ s_4 & s_5 & s_6 & s_1 \\ s_3 & s_1 & s_3 & s_4 \end{pmatrix}.$$

According to Definitions 2 and 3, transform the linguistic evaluation matrix of 3 experts into two-tuple linguistic information:

$$R_1=\begin{pmatrix}(s_3,0)&(s_4,0)&(s_2,0)&(s_2,0)\\(s_2,0)&(s_1,0)&(s_3,0)&(s_2,0)\\(s_4,0)&(s_3,0)&(s_4,0)&(s_0,0)\\(s_5,0)&(s_2,0)&(s_2,0)&(s_4,0)\\(s_6,0)&(s_0,0)&(s_1,0)&(s_3,0)\end{pmatrix},\ R_2=\begin{pmatrix}(s_2,0)&(s_3,0)&(s_1,0)&(s_1,0)\\(s_1,0)&(s_0,0)&(s_4,0)&(s_4,0)\\(s_3,0)&(s_4,0)&(s_2,0)&(s_6,0)\\(s_6,0)&(s_1,0)&(s_1,0)&(s_3,0)\\(s_2,0)&(s_1,0)&(s_3,0)&(s_1,0)\end{pmatrix},\ R_3=\begin{pmatrix}(s_4,0)&(s_2,0)&(s_1,0)&(s_5,0)\\(s_1,0)&(s_4,0)&(s_2,0)&(s_4,0)\\(s_5,0)&(s_1,0)&(s_4,0)&(s_2,0)\\(s_4,0)&(s_5,0)&(s_6,0)&(s_1,0)\\(s_3,0)&(s_1,0)&(s_3,0)&(s_4,0)\end{pmatrix}.$$

According to formulas (3) and (4), converting the preference value in the two-tuple linguistic evaluation matrix into the form of numerical value β:

$$R_1=\begin{pmatrix}3&4&2&2\\2&1&3&2\\4&3&4&0\\5&2&2&4\\6&0&1&3\end{pmatrix},\ R_2=\begin{pmatrix}2&3&1&1\\1&0&4&4\\3&4&2&6\\6&1&1&3\\2&1&3&1\end{pmatrix},\ R_3=\begin{pmatrix}4&2&1&5\\1&4&2&4\\5&1&4&2\\4&5&6&1\\3&1&3&4\end{pmatrix}.$$

According to Sections 4.2 and 4.3, map expert preferences into four-dimensional spatial point sets, including 15 elements. Then, a multi-dimensional Steiner point-based aggregation model is constructed. Substitute the expert weight vector $V=(0.3,0.4,0.3)$ into the model, and use PGSA to solve the group optimal aggregation matrix:

$$R^*=\begin{pmatrix}2.67&3.33&1.44&1.90\\1.35&0.96&3.34&3.30\\4.27&2.25&3.57&2.24\\5.23&1.89&1.95&3.53\\3.13&0.82&2.65&2.61\end{pmatrix}.$$

Map each attribute preference value x_i^* of scheme x_{ij}^* into a one-dimensional coordinate system to construct a one-dimensional Steiner aggregation model. Substitute the attribute weight $W=(0.3,0.3,0.2,0.2)$ into it and use PGSA to obtain the comprehensive evaluation score vector ϕ: $\phi=(2.67,1.35,2.32,1.95,2.65)$. Then the 5 available alternatives can be ranked as: $A_1\succ A_5\succ A_3\succ A_4\succ A_2$. Therefore, the construction sequence of the makeshift hospital is as follows: A_1, A_5, A_3, A_4, A_2.

5.2. Comparison between methods

To verify the proposed method, with respect to the multi-attribute decision-making problem of binary semantic information, based the methods of IOWA operator [10], GIOWA operator [11], WGA operator [21] and GDM method with PFPRs [47], denoted as method (1), (2), (3) and (4), respectively, we can determine the results shown in the Table 1. According to Table 1, based on the method (1) and method (2), the operation results are similar. The result based on the method (3) is similar to that based on the proposed method in this paper. On the contrary, the ranking results based on the method (1) and method (2) are completely different with that based on the proposed method. Based on the method (4) and the proposed method, A_1 and A_5 has the same ranking results, but the other ranking is completely different. The results of the difference are produced by the way of the gathered information and the mining precision. In this paper, comparing the four methods, by extending two-dimensional aggregation to multi-dimensional aggregation, the proposed method can reflect the comprehensive level of the alternatives and well mine the decision information and then give a much more accurate, reasonable and realistic results.

Table 1. Results based on different methods.

Techniques / Hospitals	Ranks				
	This paper	Method (1)	Method (2)	Method (3)	Method (4)
A_1	1	2	2	1	1
A_2	5	5	5	4	3
A_3	3	3	3	2	4
A_4	4	4	4	5	5
A_5	2	1	1	3	2

According to the above results and model analysis, based on the principle of PGSA algorithm and multi-dimensional aggregation, the proposed method can improve the aggregation accuracy and better reflect the comprehensive level of the alternatives.

6. Conclusions

Aiming at the current widely used paradigm of solving binary semantic information aggregation, we propose a new aggregation method from a new angle, namely multidimensional mapping. The major innovations of this method are as follows:

(1) The multi-dimensional Steiner point is applied to the process of 2-tuple linguistic processing of linguistic information, and a multi-dimensional Steiner point-based aggregation model is constructed, and then the optimal aggregation point is obtained by the PGSA algorithm. The obtained assembly point has both mathematical and geometric significance, which provides a new paradigm reference for solving decision-making and evaluation problems. In this study, we extend the Steiner point to the field of consensus decision-making, enrich the system of group decision-making methods and provide a possible research idea and perspective for information aggregation.

(2) A new scheme sorting method in the selection process is proposed based on the idea of Steiner point, compared with the weighted method that is widely used and can more comprehensively reflect the value of the alternatives.

(3) Aiming at the decision-making model based on the multi-dimensional Steiner aggregation method constructed in this paper, the model solving algorithm is designed based on the idea of the PGSA algorithm, and the traditional plant growth simulation algorithm is improved as follows: Extending plane aggregation to multidimensional spatial aggregation and adding a drawing module.

In conclusion, the method utilizes the PGSA algorithm to seek global set points that are both mathematically and geometrically meaningful, reducing the set bias. On this basis, a group interaction consensus model based on a trust relationship is proposed. The model obtains different consensus results by setting different values of the compromise degree parameter to facilitate the inconsistent decision-making individuals to independently choose to adjust the parameter and maximally retain the initial judgment of decision-making individuals. Finally, it is validated and analyzed with numerical examples to illustrate its effectiveness. Moreover, the method proposed in this paper has the characteristics of high reliability, easy programming and high-speed operation on the computer. Therefore, the efficiency of linguistic information group decision-making is improved. For those evaluation problems whose indicators are objective measurements, such as environmental evaluation, water resources evaluation and nutritional value evaluation, this method also has a large application space, which is also the direction that needs further research in the future. The scheme ranking method based on Steiner points proposed in this paper in the selection process provides a new idea for scholars. Then, research on decision-making with fuzzy numbers such as intuitionistic fuzzy numbers, triangular fuzzy numbers and Pythagorean fuzzy numbers as expert preference values will be extended in the future.

Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflicts of interest.

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