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Research article

kth powers in a generalization of Piatetski-Shapiro sequences

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Abstract: The article considers a generalization of Piatetski-Shapiro sequences in the sense of Beatty sequences. The sequence is defined by $(\lfloor \alpha n^c + \beta \rfloor)_{n=1}^{\infty}$, where $\alpha \geq 1$, $c > 1$, and β are real numbers. The focus of the study is on solving equations of the form $\lfloor \alpha n^c + \beta \rfloor = sm^k$, where m and n are positive integers, $1 \leq n \leq N$, and s is an integer. Bounds for the solutions are obtained for different values of the exponent k, and an average bound is derived over k-free numbers s in a given interval.

Keywords: kth power; Piatetski-Shapiro sequence; exponential sum; asymptotic formula
Mathematics Subject Classification: 11B83, 11L05

1. Introduction

The Piatetski-Shapiro sequences are defined by $\mathcal{N}^{(c)} = (\lfloor n^c \rfloor)_{n=1}^{\infty}$, where $c > 1$ and $c \notin \mathbb{N}$. In 1953, Piatetski-Shapiro [8] proved that $\mathcal{N}^{(c)}$ contains infinitely many primes if $c \in (1, \frac{12}{11})$. Moreover, he showed that the prime counting function $\pi^{(c)}(x)$, which counts the number of primes in $\mathcal{N}^{(c)}$ up to x, satisfies the asymptotic relation $\pi^{(c)}(x) \sim \frac{x^{1/c}}{\log x}$ as $x \to \infty$. Since then, the range of c for which it is known that $\mathcal{N}^{(c)}$ contains infinitely many primes has been extended several times, and it is now known that the above formula holds for all $c \in (1, \frac{2817}{2426})$ thanks to the work of Rivat and Sargos [10].

This study is related to the topic of Beatty sequences. A non-homogeneous Beatty sequence is a sequence of integers obtained from fixed real numbers $\alpha > 0$ and β, defined as $\mathcal{B}_{\alpha,\beta} = (\lfloor \alpha n + \beta \rfloor)_{n=1}^{\infty}$, which is sometimes referred to a generalized arithmetic progression. It is well-known that the sequence contains infinitely many prime numbers if α is irrational [4]. Moreover, it is possible to establish an asymptotic relation for the distribution of primes in such sequences, which is the subject of extensive research in number theory. We get that

$$\#\left\{\text{prime } p \leq x : p \in \mathcal{B}_{\alpha,\beta}\right\} \sim \alpha^{-1}\pi(x), \quad x \to \infty$$

holds, where $\pi(x)$ is the prime counting function.

The author of this study proposes a generalization of Piatetski-Shapiro sequences in the context of Beatty sequences, which similarly consists of infinitely many prime numbers. Specifically, let $\alpha \geq 1$ and β be real numbers. We investigate the following generalized Piatetski-Shapiro sequences:

$$\mathcal{N}_{\alpha,\beta}^{(c)} = (\lfloor \alpha n^c + \beta \rfloor)_{n=1}^{\infty} .$$

The Piatetski-Shapiro sequences have deep connections to several fundamental concepts in number theory, such as smooth numbers, square-free numbers and so on. Previous research by Liu, Shparlinski, and Zhang [5] focused on the distribution of squares in Piatetski-Shapiro sequences, while Qi, Guo, and Xu [9] investigated an intriguing equation related to these sequences. In this paper, we aim to extend their work by exploring the distribution of k-th powers in a generalization of Piatetski-Shapiro sequences, which can be viewed as an extension of their previous findings. To be precise, we define $Q_{c,k}^{\alpha,\beta}(s;N)$ as the number of solutions to the equation:

$$\lfloor \alpha n^c + \beta \rfloor = sm^k, \quad 1 \leq n \leq N, \quad m, n \in \mathbb{Z}.$$

We mention the trivial bound

$$Q_{c,k}^{\alpha,\beta}(s;N) \leq \min\left(N, s^{-\frac{1}{k}} (\alpha N^c + \beta)^{\frac{1}{k}}\right).$$

We prove the following theorem.

Theorem 1.1. *Let $k > 1$ be an integer. For any exponent pair (κ, λ), we have*

$$\begin{aligned} Q_{c,k}^{\alpha,\beta}(s;N) =& \gamma(k\gamma - k + 1)^{-1} s^{-\frac{1}{k}} N^{1-c+\frac{c}{k}} \\ &+ O\left(s^{-\frac{\lambda}{k(1+\kappa)}} N^{\frac{k\kappa + c\lambda}{k(1+\kappa)}+\varepsilon} + s^{\frac{\kappa-\lambda}{k}} N^{\frac{k\kappa + c(\lambda-\kappa)}{k}+\varepsilon} + s^{-\frac{1}{k}} N^{-c+\frac{c}{k}}\right). \end{aligned}$$

We also study $Q_{c,k}^{\alpha,\beta}(s;N)$ on average over positive k-free integers $s \leq S$. Recall that $\gamma = c^{-1}$and define that

$$\mathfrak{Q}_{c,k}^{\alpha,\beta}(S,N) = \sum_{\substack{s \leq S \\ \text{s is } k\text{-free}}} Q_{c,k}^{\alpha,\beta}(s;N).$$

We remark that only the case $S \leq \alpha N^c + \beta$ is meaningful, hence we always assume this. Liu, Shparlinski and Zhang [5] showed that

$$\begin{aligned} \mathfrak{Q}_{c,2}^{1,0}(S,N) =& \frac{12\gamma}{\pi^2(2\gamma-1)} S^{1-\frac{1}{k}} N^{1-\frac{c}{2}} \\ &+ O\left(S^{\frac{1}{5}} N^{\frac{1}{5}+\frac{2c}{5}} + S^{\frac{5}{8}} N^{\frac{3c}{8}} + S^{\frac{1}{8}} N^{\frac{1}{4}+\frac{3c}{8}} + SN^{1-c}\right). \end{aligned}$$

We obtain the following result.

Theorem 1.2. *For any $c > 1, c \notin \mathbb{N}$, we have*

$$\begin{aligned} \mathfrak{Q}_{c,k}^{\alpha,\beta}(S,N) =& \frac{k}{\zeta(k)(k-1)(k\gamma - k + 1)} S^{1-\frac{1}{k}} N^{1-c+\frac{c}{k}} \\ &+ O\left(S^{\frac{3}{5}-\frac{4}{5k}} N^{\frac{1}{5}+\frac{4c}{5k}} + S^{1-\frac{3}{4k}} N^{\frac{3c}{4k}} + S^{\frac{1}{2}-\frac{3}{4k}} N^{\frac{1}{4}+\frac{3c}{4k}} + SN^{1-c} + S^{1-\frac{1}{k}} N^{-c+\frac{c}{k}}\right). \end{aligned}$$

We remark that the topic is relative to harmonic numbers and their relationships which are interesting mathematical concepts, including number theory, calculus, and physics, as well as the study of degenerate versions or special cases.For further details, refer to [6, 7].

2. Preliminaries

2.1. Notation

We denote by $\lfloor t \rfloor$ and $\{t\}$ the greatest integer $\leqslant t$ and the fractional part of t, respectively. We also write $\mathbf{e}(t) = e^{2\pi i t}$ for all $t \in \mathbb{R}$, as usual. We make considerable use of the sawtooth function defined by

$$\psi(t) = t - \lfloor t \rfloor - \tfrac{1}{2} = \{t\} - \tfrac{1}{2} \qquad (t \in \mathbb{R}).$$

In this study, we consider the Piatetski-Shapiro sequence $(\lfloor n^c \rfloor)_{n=1}^{\infty}$, where $\lfloor \cdot \rfloor$ denotes the floor function, and γ is defined as the inverse of the constant c. The set of primes in the natural numbers is denoted by $\mathbb{P}$. We use the notation $m \sim M$ to indicate that m lies in the interval $(M, 2M]$.

In order to state our results, we introduce some notation. Throughout the paper, the symbol ε represents an arbitrarily small positive constant, which may vary from one occurrence to another. The implied constants in the symbols O, $\ll$, and $\gg$ may depend on the parameters c , ε , α, and β, but are absolute otherwise. For given functions F and G, the notations $F \ll G$, $G \gg F$, and $F = O(G)$ are all equivalent to the assertion that the inequality $|F| \leqslant C|G|$ holds with some constant $C > 0$.

2.2. Technical lemmas

Lemma 2.1. *Let*

$$L(Q) = \sum_{i=1}^{I} A_i Q^{a_i} + \sum_{j=1}^{J} B_j Q^{-b_j},$$

where $A_i, a_i, B_j, b_j > 0$. Then, (1) for any $Q_2 \geq Q_1 > 0$ there exists $Q \in [Q_1, Q_2]$ such that

$$L(Q) \ll \sum_{i=1}^{I} \sum_{j=1}^{J} \left(A_i^{b_j} B_j^{a_i}\right)^{\frac{1}{a_i+b_j}} + \sum_{i=1}^{I} A_i Q_1^{a_i} + \sum_{j=1}^{J} B_j Q_2^{-b_j}.$$

(2) For any $Q_1 > 0$ there exists $Q \in (0, Q_1]$ such that

$$L(Q) \ll \sum_{i=1}^{I} \sum_{j=1}^{J} \left(A_i^{b_j} B_j^{a_i}\right)^{\frac{1}{a_i+b_j}} + \sum_{j=1}^{J} B_j Q_1^{-b_j}.$$

Proof. See [3, Lemma 2.4] □

Lemma 2.2. *For any $J > 0$, there holds*

$$\psi(x) = \sum_{1 \leq |j| \leq J} a_j \mathbf{e}(jx) + O\left(\sum_{|j| \leq J} b_j \mathbf{e}(jx)\right),$$

where

$$a_j \ll |j|^{-1}, \quad b_j \ll J^{-1}.$$

Proof. This is the result of Vaaler [11]. □

Lemma 2.3. $\left(\frac{13}{84} + \varepsilon, \frac{55}{84} + \varepsilon\right)$ *is an exponent pair.*

Proof. See [1, Theorem 6]. □

Lemma 2.4. *Let $\alpha, \alpha_1, \alpha_2$ be real constants such that*

$$\alpha \notin 1 \quad \text{and} \quad \alpha\alpha_1\alpha_2 \notin 0.$$

Let $M, M_1, M_2, x \geq 1$ and let

$$\Phi = (\varphi_m)_{m\sim M} \quad \text{and} \quad \Psi = (\psi_{m_1,m_2})_{m_1\sim M_1, m_2\sim M_2}$$

be two sequences of complex numbers supported on $m \sim M, m_1 \sim M_1$ and $m_2 \sim M_2$ with $|\varphi_m| \leq 1$. Then, for the sum

$$S_{\Phi,\Psi}(x; M, M_1, M_2) = \sum_{m\sim M}\sum_{m_1\sim M_1}\sum_{m_2\sim M_2} \varphi_m \psi_{m_1,m_2} \mathbf{e}\left(x\frac{m^\alpha m_1^{\alpha_1} m_2^{\alpha_2}}{M^\alpha M_1^{\alpha_1} M_2^{\alpha_2}}\right)$$

we have

$$\begin{aligned} S_{\Phi,\Psi}(x; M, M_1, M_2) \ll \Big(& x^{\frac{1}{4}} M^{\frac{1}{2}} (M_1M_2)^{\frac{3}{4}} + M^{\frac{7}{10}} M_1 M_2 \\ &+ M (M_1M_2)^{\frac{3}{4}} + x^{-\frac{1}{4}} M^{\frac{11}{10}} M_1 M_2\Big) \log^2(2MM_1M_2). \end{aligned}$$

Proof. See [2, Theorem 3] □

3. Proof of Theorem 1.1

Denote $\gamma = c^{-1}$ and $\theta = \alpha^{-\gamma}$. A kth power equals $\lfloor \alpha n^c + \beta \rfloor$ if and only if

$$sm^k \leq \alpha n^c + \beta < sm^k + 1,$$

which is equivalent to

$$\theta(sm^k - \beta)^\gamma \leq n < \theta(sm^k + 1 - \beta)^\gamma.$$

Let $M = s^{-\frac{1}{k}} (\alpha N^c + \beta)^{\frac{1}{k}}$, by a normal construction,

$$\begin{aligned} Q_{c,k}^{\alpha,\beta}(s; N) &= \sum_{m\leq M} \left(\left\lfloor -\theta\left(sm^k - \beta\right)^\gamma \right\rfloor - \left\lfloor -\theta\left(sm^k + 1 - \beta\right)^\gamma \right\rfloor\right) + O(1) \\ &= S_1 + S_2 + O(1), \end{aligned} \tag{3.1}$$

where

$$\begin{aligned} S_1 &= \sum_{m\leq M} \left(\theta\left(sm^k + 1 - \beta\right)^\gamma - \theta\left(sm^k - \beta\right)^\gamma\right) \\ &= \theta \sum_{m\leq M} \left(\gamma\left(sm^k - \beta\right)^{\gamma-1} + O\left(\left(sm^k - \beta\right)^{\gamma-2}\right)\right) \\ &= \theta \sum_{m\leq M} \left(\gamma s^{\gamma-1} m^{k(\gamma-1)} + O\left(s^{\gamma-2} m^{k(\gamma-2)}\right)\right) \\ &= \gamma(k\gamma - k + 1)^{-1} s^{-\frac{1}{k}} N^{1-c+\frac{c}{k}} + O(s^{-\frac{1}{k}} N^{-c+\frac{c}{k}}), \end{aligned} \tag{3.2}$$

and

$$S_2 = \sum_{m\le M}\left(\left\{-\theta\left(sm^k+1-\beta\right)^{\gamma}\right\}-\left\{-\theta\left(sm^k-\beta\right)^{\gamma}\right\}\right)$$
$$=\sum_{m\le M}\left(\psi\left(-\theta\left(sm^k+1-\beta\right)^{\gamma}\right)-\psi\left(-\theta\left(sm^k-\beta\right)^{\gamma}\right)\right).$$

Consider S_2. From Lemma 2.2 we have

$$S_2 = S_3 + O\left(S_4\right), \tag{3.3}$$

where

$$S_3 = \sum_{m\le M}\sum_{1\le|j|\le J} a_j\left(\mathbf{e}\left(-j\theta\left(sm^k+1-\beta\right)^{\gamma}\right)-\mathbf{e}\left(-j\theta\left(sm^k-\beta\right)^{\gamma}\right)\right),$$

and

$$S_4 = \sum_{m\le M}\sum_{|j|\le J} b_j\left(\mathbf{e}\left(-j\theta\left(sm^k+1-\beta\right)^{\gamma}\right)+\mathbf{e}\left(-j\theta\left(sm^k-\beta\right)^{\gamma}\right)\right),$$

for any $J\ge 1$. We begin with S_3. Remembering $a_j \ll |j|^{-1}$, we have

$$S_3 \ll \sum_{1\le|j|\le J}|j|^{-1}\left|\sum_{m\le M}\mathbf{e}\left(j\theta s^{\gamma}m^{k\gamma}\right)\right|.$$

Summing over m, we obtain

$$\sum_{m\le M}\mathbf{e}\left(j\theta s^{\gamma}m^{k\gamma}\right) \ll \log M \max_{1\le L\le M}\left|\sum_{L\le m\le 2L}\mathbf{e}\left(j\theta s^{\gamma}m^{k\gamma}\right)\right|.$$

Using the exponent pair (κ,λ) we get

$$\sum_{L\le m\le 2L}\mathbf{e}\left(j\theta s^{\gamma}m^{k\gamma}\right) \ll \left(j\theta s^{\gamma}L^{k\gamma-1}\right)^{\kappa}L^{\lambda}.$$

Then,

$$\sum_{m\le M}\mathbf{e}\left(j\theta s^{\gamma}m^{k\gamma}\right) \ll j^{\kappa}\theta^{\kappa}s^{\frac{\kappa-\lambda}{k}}N^{\frac{c(k\gamma\kappa-\kappa+\lambda)}{k}+\varepsilon},$$

which yields

$$S_3 \ll J^{\kappa}\theta^{\kappa}s^{\frac{\kappa-\lambda}{k}}N^{\frac{c(k\gamma\kappa-\kappa+\lambda)}{k}+\varepsilon}.$$

We can readily eliminate the contribution of S_4. By utilizing the fact that b_j is bounded by J^{-1}, we obtain the following result:

$$S_4 \ll J^{-1}\sum_{|j|\le J}\left|\sum_{m\le M}\mathbf{e}\left(j\theta s^{\gamma}m^{k\gamma}\right)\right|$$
$$\ll J^{-1}s^{-\frac{1}{k}}N^{\frac{c}{k}} + J^{-1}\sum_{1\le|j|\le J}\left|\sum_{m\le M}\mathbf{e}\left(j\theta s^{\gamma}m^{k\gamma}\right)\right|.$$

By a similar argument, we have

$$S_4 \ll J^{-1}s^{-\frac{1}{k}}N^{\frac{c}{k}} + J^{\kappa}s^{\frac{\kappa-\lambda}{k}}N^{\frac{c(k\gamma\kappa-\kappa+\lambda)}{k}+\varepsilon}.$$

It yields that

$$J = s^{\frac{\lambda-\kappa-1}{k+k\kappa}}N^{\frac{c(1+\kappa-k\gamma\kappa-\lambda)}{k+k\kappa}}.$$

Applying Lemma 2.1 to the bounds on terms in (3.3), we have

$$S_2 \ll s^{-\frac{\lambda}{k(1+\kappa)}}N^{\frac{k\kappa+c\lambda}{k(1+\kappa)}+\varepsilon} + s^{\frac{\kappa-\lambda}{k}}N^{\frac{k\kappa+c(\lambda-\kappa)}{k}+\varepsilon}.$$

Now the result follows from (3.3) and (3.1). Applying the exponent pair $\left(\frac{13}{84}+\varepsilon, \frac{55}{84}+\varepsilon\right)$ from Lemma 2.3 by Bourgain [1], people can get the asymptotic formula

$$Q_{c,k}^{\alpha,\beta}(s;N) = \gamma(k\gamma-k+1)^{-1}s^{-\frac{1}{k}}N^{1-c+\frac{c}{k}} + O\left(s^{-\frac{55}{97k}}N^{\frac{13}{97}+\frac{55c}{97k}+\varepsilon} + s^{-\frac{1}{2k}}N^{\frac{13}{84}+\frac{c}{2k}+\varepsilon}\right).$$

4. Proof of Theorem 1.2

This proof is almost identical to the proof given in [5, Theorem 2.3], so we will provide only a brief outline. We start that

$$\Phi_k(S) = \sum_{\substack{s\le S\\ s \text{ is } k\text{-free}}} s^{-\frac{1}{k}}.$$

Applying a commonly known result, (see [11, p. 181]),

$$\sum_{\substack{s\le S\\ s \text{ is } k\text{-free}}} 1 = \frac{S}{\zeta(k)} + O\left(S^{\frac{1}{k}}\right),$$

and a partial summation, we obtain

$$\Phi_k(S) = \frac{k}{\zeta(k)(k-1)}S^{1-\frac{1}{k}} + O(\log S). \tag{4.1}$$

We proceed as in the proof of Theorem 1.1, so set $\theta = \alpha^{-\gamma}$, $T = (\alpha N^c + \beta)^{\gamma}$, and

$$\mathfrak{Q}_{c,k}^{\alpha,\beta}(S;N) = K_0 + K_1 + O(1),$$

where

$$K_0 = \sum_{\substack{sm^k\le\alpha N^c+\beta\\ s\le S\\ s \text{ is } k\text{-free}}} \left(\theta\left(sm^k+1-\beta\right)^{\gamma} - \theta\left(sm^k-\beta\right)^{\gamma}\right),$$

and

$$K_1 = \sum_{\substack{sm^k\le\alpha N^c+\beta\\ s\le S\\ s \text{ is } k\text{-free}}} \left(\psi\left(-\theta\left(sm^k+1-\beta\right)^{\gamma}\right) - \psi\left(-\theta\left(sm^k-\beta\right)^{\gamma}\right)\right).$$

Using (3.2) and (4.1), we compute K_0 directly as follows

$$K_0 = \gamma(k\gamma-k+1)^{-1}N^{1-c+\frac{c}{k}}\Phi_k(S) + O\left(SN^{1-c} + S^{1-\frac{1}{k}}N^{-c+\frac{c}{k}}\right).$$

By Lemma 2.2

$$K_1 \ll K_{11} + K_{12},$$

where

$$K_{11} = \sum_{\substack{sm^k \le \alpha N^c + \beta \\ s \le S \\ s \text{ is } k\text{-free}}} \sum_{0<|j|<J} a_j \left(\psi\left(-\theta\left(sm^k + 1 - \beta\right)^{\gamma}\right) - \psi\left(-\theta\left(sm^k - \beta\right)^{\gamma}\right)\right),$$

and

$$K_{12} = \sum_{\substack{sm^k \le \alpha N^c + \beta \\ s \le S \\ s \text{ is } k\text{-free}}} \sum_{0\le|j|<J} b_j \left(\psi\left(-\theta\left(sm^k + 1 - \beta\right)^{\gamma}\right) - \psi\left(-\theta\left(sm^k - \beta\right)^{\gamma}\right)\right).$$

Using a similar approach in [5, p. 250] we have

$$K_{11} \ll \sum_{0<|j|<J} j^{-1} |S(R, D, M; j)|,$$

where

$$S(R, D, M; j) = \sum_{\substack{r\sim R, d\sim D, m\sim M \\ rd^k m^k \le \alpha N^c + \beta, rd^k \le S}} \mu(d) \mathbf{e}\left(-j\theta r^{\gamma} d^{k\gamma} m^{k\gamma}\right).$$

From Lemma 2.4, we have

$$\begin{aligned} S(R, D, M; j) = &\left(jR^{\gamma} D^{k\gamma} M^{k\gamma}\right)^{\frac{1}{4}} R^{\frac{1}{2}} (DM)^{\frac{3}{4}} + R^{\frac{7}{10}} DM \\ &+ R(DM)^{\frac{3}{4}} + \left(jR^{\gamma} D^{k\gamma} M^{k\gamma}\right)^{-\frac{1}{4}} R^{\frac{11}{10}} DM. \end{aligned}$$

Noting that $\gamma > \frac{1}{2}$, it can be easily verified that the fourth term can be combined with the third term on the right-hand side. We can obtain

$$K_{11} \ll j^{\frac{1}{4}} N^{\frac{1}{4} + \frac{3c}{4k}} S^{\frac{1}{2} - \frac{3}{4k}} + N^{\frac{c}{k}} S^{\frac{7}{10} - \frac{1}{k}} + N^{\frac{3c}{4k}} S^{1 - \frac{3}{4k}}.$$

Hence,

$$\begin{aligned} |K_{11}| + |K_{12}| \ll &J^{\frac{1}{4}} N^{\frac{1}{4} + \frac{3c}{4k}} S^{\frac{1}{2} - \frac{3}{4k}} + J^{-1} N^{\frac{c}{k}} S^{1 - \frac{1}{k}} \\ &+ N^{\frac{c}{k}} S^{\frac{7}{10} - \frac{1}{k}} + N^{\frac{3c}{4k}} S^{1 - \frac{3}{4k}}, \end{aligned}$$

where the term $J^{-1} N^{\frac{c}{k}} S^{1-\frac{1}{k}}$ results from the choice $j = 0$ in the summation on the right-hand side in Lemma 2.2. Now Lemma 2.1 gives

$$|K_{11}| + |K_{12}| \ll N^{\frac{1}{5} + \frac{4c}{5k}} S^{\frac{3}{5} - \frac{4}{5k}} + N^{\frac{3c}{4k}} S^{1 - \frac{3}{4k}} + N^{\frac{1}{4} + \frac{3c}{4k}} S^{\frac{1}{2} - \frac{3}{4k}}.$$

We have the final result.

5. Conclusions

We have proved Theorems 1.1 and 1.2.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

We declare no conflict of interest.

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