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Research article

Finite-time decentralized event-triggered feedback control for generalized neural networks with mixed interval time-varying delays and cyber-attacks

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Abstract: This article investigates the finite-time decentralized event-triggered feedback control problem for generalized neural networks (GNNs) with mixed interval time-varying delays and cyber-attacks. A decentralized event-triggered method reduces the network transmission load and decides whether sensor measurements should be sent out. The cyber-attacks that occur at random are described employing Bernoulli distributed variables. By the Lyapunov-Krasovskii stability theory, we apply an integral inequality with an exponential function to estimate the derivative of the Lyapunov-Krasovskii functionals (LKFs). We present new sufficient conditions in the form of linear matrix inequalities. The main objective of this research is to investigate the stochastic finite-time boundedness of GNNs with mixed interval time-varying delays and cyber-attacks by providing a decentralized event-triggered method and feedback controller. Finally, a numerical example is constructed to demonstrate the effectiveness and advantages of the provided control scheme.

Keywords: generalized neural networks; finite-time stability; time-varying delays; feedback control; cyber-attacks; decentralized event-triggered scheme
Mathematics Subject Classification: 34D20, 37C75, 39A30

1. Introduction

Neural networks (NNs) are widely used in various fields, such as pattern recognition, combinatorial optimization, image processing, associate memory, signal processing, and fixed-point computations [1–4], due to their enormous capacity for information processing. The literatures [5–12] classifies neural networks into two categories. The first category consists of static neural networks (SNNs), which rely on the external states of neurons (neural states of neurons). The second category consists of local field neural networks (LFNNs), which depend on the internal states of neurons (local field states). In recent years, Zhang and Han [13] introduced a novel approach by combining SNNs and LFNNs to create a unified system of neural networks called generalized neural networks (GNNs). NNs may encounter a delay during execution caused by either the communication time of neurons or the finite switching speed of the neuron amplifiers. These time delays can have a detrimental effect on the performance of NNs, resulting in either instability, divergence, or oscillation. Consequently, there has been an increased focus on investigating the stability of GNNs under time delay [12–18]. Delayed GNNs are classified into various types: distributed delay, mixed delays, constant delay, interval time-varying delay and time-varying delay.

Event-triggered schemes are control strategies that minimize the amount of data transmitted or processed in computers and control systems. These schemes only transmit data when specific events occur, such as system state changes, errors, or specific time intervals. The event-triggered mechanism is a promising technique in networked control systems that aims to reduce communication costs and computational burden, while ensuring good control performance. This technique is necessary since the energy of sensors and network bandwidth is limited and needs to be used efficiently to accomplish other communication tasks. The event-triggered method only transmits signals when they meet the triggering threshold, conserving energy and improving communication bandwidth, as shown by the beneficial outcomes of event-triggered schemes. This success has led to its widespread usage in NNs [19–26]. For instance, Liu et al. [20] investigated a dissipativity-based synchronization method for Markovian jump NNs under the event-triggered framework. Later, Zha et al. [22] presented the problem of H_∞ control for NNs with time-varying delay and cyber-attacks under the event-triggered framework.

Cyber-attacks in networked control systems are malicious and unauthorized activities that target the communication network or control algorithms. These attacks aim to either disrupt normal system operations, steal data, or cause physical damage. Such attacks can result in significant losses, making the security and resilience of networked control systems against cyber-attacks a crucial area of research and practice. Over the past few years, several researchers [22, 25, 27–30] have placed a considerable emphasis on the problem of cyber-attacks in networked control systems. For example, Liu et al. [30] investigated the state estimation method for T-S fuzzy neural networks under stochastic cyber-attacks and an event-triggered scheme. Recently, a technique of the networked control for neural networks vulnerable to two different types of stochastic cyber-attacks has been proposed by Feng et al. [25]. This approach utilizes a decentralized event-triggered H_∞ control.

Stability analysis is a crucial concept in control theory, which usually concentrates on a system's asymptotic behavior over an infinite time interval. However, achieving faster convergence with a greater robustness is often favored in practical engineering applications. For instance, an industrial weight scale needs to attain its steady-state value within a specified threshold for a finite time. The system quickly achieves its equilibrium point using a magnetic force. The preceding situation is known as finite-time stability. The concept of finite-time stability was first introduced by Dorato in 1961 [31]. In 2001, Amato [32] presented finite-time boundedness by extending finite-time stability with external disturbance. While there has been considerable research on achieving finite-time stability for delayed NNs [9, 11, 12, 17, 26, 33, 34], the exploration of finite-time decentralized event-triggered feedback control for GNNs with cyber-attacks remains an unaddressed issue. This research gap serves as the primary motivation for the present study.

To address this research gap, this article introduces an innovative decentralized event-triggered method and feedback controller for GNNs with mixed interval time-varying delays and cyber-attacks.

The primary contributions of this article can be summarized as follows:

(1) We propose a novel decentralized event-triggered method and feedback controller for GNNs with mixed interval time-varying delays and cyber-attacks. This approach ensures finite-time boundedness while estimating the derivative of the Lyapunov-Krasovskii functionals using an integral inequality with an exponential function. By utilizing this method, we address the challenges associated with system stability and effectiveness in the presence of time-varying delays and cyber-attacks.

(2) The event-triggered approach introduced in this work leads to a reduction in network resource utilization. This reduction alleviates the transmission burden on the network by enabling each sensor to autonomously determine the optimal time for signal transmission. By minimizing unnecessary transmissions, the proposed approach enhances the overall efficiency and scalability of the system.

(3) We describe random cyber-attacks by utilizing Bernoulli-distributed variables and represent them through a nonlinear function that satisfies a specific condition. This modeling approach allows us to capture the realistic nature of cyber-attacks and effectively incorporate them into the control strategy.

(4) We provide an illustrative example accompanied by simulations to demonstrate the feasibility and effectiveness of the proposed control strategy. The results showcase the improved effectiveness achieved in terms of system stability, finite-time boundedness, and resilience against cyber-attacks.

The remainder of this article is structured as follows. In Section 2, we introduce GNNs and provide preliminaries. Section 3 uses a state feedback controller to examine the stochastic finite-time bounded conditions for delayed GNNs with cyber-attacks. Section 4 presents a numerical example to illustrate the effectiveness of the proposed methods. In Section 5, we conclude and discuss our article.

Notations: This article utilizes the following notations: I denotes the identity matrix; $\|\cdot\|$ represents the Euclidean vector norm of a matrix; $\mathbb{R}^n$ indicates the n-dimensional Euclidean space; Prob$\{X\}$ represents the probability of event X to occur; diag$\{\cdots\}$ refers a block-diagonal matrix; the notation P^T and P^{-1} stand for the transpose and inverse of matrix P, respectively; the expression $P < 0$ (or $P \leq 0$) signifies that the real symmetric matrix P is negative definite (or negative semi-definite); $\lambda_{\min}(P)$ (or $\lambda_{\max}(P)$) denotes the minimum (or maximum) eigenvalue of real symmetric matrix P; the term $\mathcal{L}_2[0,\infty)$ denotes a function space consisting of quadratically integrable functions over the interval $[0,\infty)$; the notation Sym$\{P\}$ represents the sum of P and its transpose, i.e., $P + P^T$; the symbol $*$ indicates the elements below the main diagonal in a symmetric matrix.

2. Problem formulation and preliminaries

In this article, we introduce a problem involving GNNs with mixed interval time-varying delays. The problem statement is described as follows:

$$\begin{cases} \dot{x}(t) = & \bar{A}x(t) + B_0 f(Wx(t)) + B_1 f(Wx(t-\tau(t))) + B_2 \int_{t-\eta_2(t)}^{t-\eta_1(t)} h(Wx(u))du + B_w\omega(t) + B_u u(t), \\ z(t) = & x(t), \end{cases} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ represents the state vector at time t; n is the number of neurals; $z(t) \in \mathbb{R}^n$ represents the output of the system; $\bar{A} = \text{diag}\{a_1, a_2, ..., a_n\}$ indicates a diagonal matrix; W, B_0, B_1 and B_2 refer connection weight matrices; the matrices B_w and B_u are real constant matrices with known values; $\omega(t)$ refers the external disturbance input; $u(t) \in \mathbb{R}^m$ denotes the control input; and $f(Wx(t)) = [f_1(Wx_1(t)), ..., f_n(Wx_n(t))]^T$ and $h(Wx(t)) = [h_1(Wx_1(t)), ..., h_n(Wx_n(t))]^T$ indicate the activation functions. The time delays in the system are represented by $\eta_i(t)(i = 1, 2)$ and $\tau(t)$, which correspond to interval distributed time-varying delays and interval time-varying delays, respectively.

The functions $\tau(t)$ and $\eta_i(t)(i = 1, 2)$ are continuous and satisfy the following conditions:

$$0 \leq \tau_m \leq \tau(t) \leq \tau_M \quad \text{and} \quad 0 \leq \eta_1 \leq \eta_1(t) \leq \eta_2(t) \leq \eta_2, t \in [0, T],$$

where $\tau_m, \tau_M, \eta_1, \eta_2 \in \mathbb{R}$ refer known real constants.

Additionally, we assume that the neuron activation function and communication network delays satisfy the following assumptions.

Assumption (A1). Each of the activation functions, $f_i(t)$ and $h_i(t)$, where $i = 1, 2, ..., n$, is assumed to be continuous and bounded, satisfies the following conditions: there exist constants F_i^-, F_i^+, H_i^-, and H_i^+ such that

$$F_i^- \leq \frac{f_i(W\alpha_1) - f_i(W\alpha_2)}{W\alpha_1 - W\alpha_2} \leq F_i^+, \qquad H_i^- \leq \frac{h_i(W\alpha_1) - h_i(W\alpha_2)}{W\alpha_1 - W\alpha_2} \leq H_i^+, \quad \forall \alpha_1, \alpha_2 \in \mathbb{R}, \alpha_1 \neq \alpha_2.$$

Assumption (A2). Let $\tau_{t_k^i}$ denote the communication network delay at the sampled instant t_k^i in the ith sensor. It is assumed that $0 < \tau_{t_k^i} < \bar{\tau}^i$, where $\bar{\tau} = \max_{i\in 1,2,...,n} \bar{\tau}^i$.

The diagram of the decentralized event-triggered control for GNNs with cyber-attacks is shown in Figure 1. This structure involves multiple sensors and controllers exchanging information over a communication network, which may experience time delays and potential cyber-attacks. To minimize network transmissions, event generators are employed at each sensor. When a new signal is sampled, it is promptly sent to the corresponding event generator. Additionally, the signal includes the disturbance input vector $\omega(t)$, representing either external factors or disturbances that influence the controlled system. The controlled output $z(t)$ represents the desired or targeted system output. Only signals that violate the event-triggered condition are transmitted over the network, thereby reducing communication bandwidth requirements. The primary objective of this decentralized event-triggered structure is to ensure the stability of the neural network while minimizing the impact of cyber-attacks. Furthermore, a three-line table summarizing the algorithm of the proposed method is provided below in Table 1.

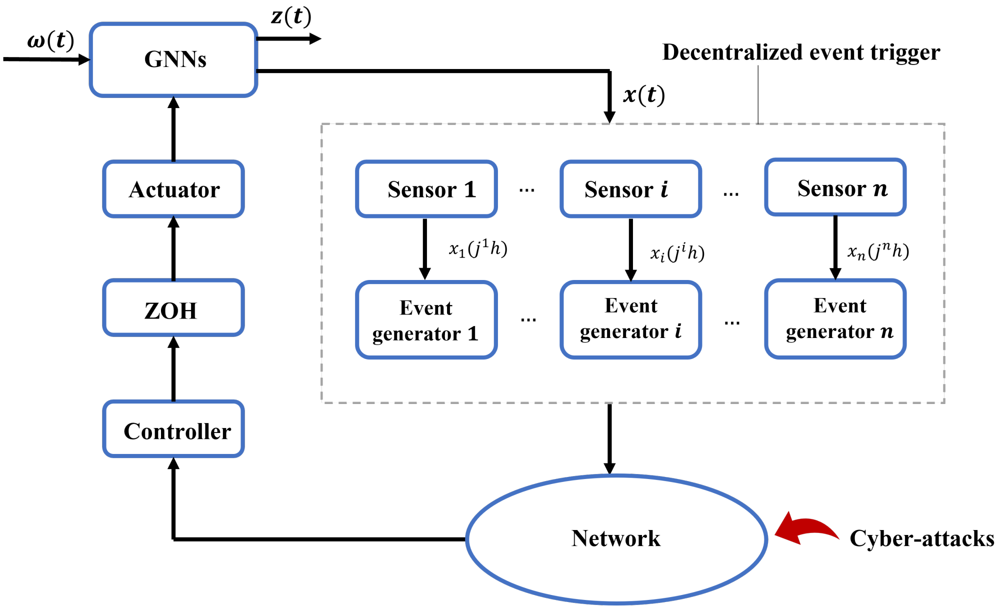

Figure 1. The diagram of decentralized event-triggered for GNNs with cyber-attacks.

Table 1. A three-line table summarizing the algorithm of the proposed method.

Step	Description
1	Initialize network parameters, event-triggering thresholds and cyber-attack detection mechanisms
2	Sample input signals, check event-triggering conditions and detect potential cyber-attacks
3	Update network states, transmit relevant information among nodes and implement countermeasures against cyber-attacks

The following expression gives the predefined event-triggered criterion for the ith sensor:

$$e_i^T(t)\Omega_i e_i(t) < \sigma_i x_i^T(t_k^i h + j^i h)\Omega_i x_i(t_k^i h + j^i h), \tag{2.2}$$

where a weighting matrix is denoted by $\Omega_i > 0$, $\sigma_i \in [0,1)$, $i \in \{1,2,..,n\}$, h is the sampling period of sensor, $j^i h$ is the sampling instant for the ith sensor, and $e_i(t)$ is the difference between the latest transmitted signal $x_i(t_k^i h)$ and the current sampled signal $x(t_k^i h + j^i h)$ at time t.

Remark 2.1. In the event generator of the ith sensor, the most recently triggered instant is $t_k^i h$, and the current sampling instant is $t_k^i h + j^i h$. It is important to note that the set $\{t_1^i, t_2^i, ..., \} \subseteq \{h, 2h, ..., j^i h, ...\}$ for $i \in \{1,2,...,n\}$.

The zero holder order (ZOH) holding interval $[t_k^i h, t_{k+1}^i h)$ can be divided into several subsets denoted by $\cup_{j^i=0}^{j_M^i}\Upsilon_{j^i}$. Each subset $\cup_{j^i=0}^{j_M^i}\Upsilon_{j^i}$ is given by $\Upsilon_{j^i} = [t_k^i h + j^i h + \beta_{t_k^i + j^i}, t_k^i h + j^i h + h + \beta_{t_k^i + j^i + 1})$ for $j^i = 0, 1, ..., j_M^i$, $j_U^i = t_{k+1}^i - t_k^i - 1$.

Following a similar approach to [35], we establish a sequence of buffers on the actuator side, each with a unique timestamp to hold the controller outputs. This enables the actuators to update the controlled inputs by selecting the corresponding controller output from the buffers. Thus, the input update time set for the actuators is defined as $t_{k+1}h = t_kh + jh$, where $jh = \text{argmini}_{i\in\{1,2,...,n\}}\{j^ih\}$ can be obtained from (2.2).

Define $\beta(t) = t - t_kh - jh$. It is evident that $0 \leq \beta_{t_k} \leq \beta(t) \leq \bar{\beta}$, where $\bar{\beta} = h + \beta_{t_k+j+1}$. Based on (2.2), the condition for n channels can be derived as follows:

$$e^T(t)\Omega e(t) < \sigma x^T(t - \beta(t))\Omega x(t - \beta(t)), \tag{2.3}$$

where $e(t) = x(t_kh) - x(t_{k+1}h + jh)$, $\Omega = \text{diag}\{\Omega_1, ..., \Omega_n\}$ and $\sigma = \text{diag}\{\sigma_1, ..., \sigma_n\}$.

The following is a description of the proposed method for designing the controller model, which takes into account both the decentralized event-triggered scheme and cyber-attacks:

$$u(t) = \rho(t_k)\mathcal{K}x(t_kh) + (1 - \rho(t_k))\mathcal{K}g(x(t - d(t))), \tag{2.4}$$

where $\rho(t_k) \in \{0, 1\}$, $\text{Prob}\{\rho(t_k) = 1\} = \bar{\rho}$, $\text{Prob}\{\rho(t_k) = 0\} = 1 - \bar{\rho}$, the controller gain is denoted by $\mathcal{K}$, and the function of cyber-attacks is represented by $g(x(t - d(t)))$, where $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), ..., g_n(x_n(t))]^T$, $d(t) \in (0, \bar{d}]$ and $\bar{d}$ is a positive constant.

Remark 2.2. The dynamic event-triggered system (ETS) utilizes periodic sampling and a waiting period of h seconds to evaluate the event-triggering condition, allowing for ample decision-making time and preventing the occurrence of Zeno behavior. Zeno behavior refers to the destabilizing effect of rapid and continuous event triggering. The introduction of this temporal constraint ensures a balanced operation, averting undesirable system behavior and instability by triggering events at appropriate intervals.

Remark 2.3. This article considers the probability distribution of cyber-attacks, which is assumed to follow the Bernoulli distribution. The received sensor measurements are represented by $\rho(t_k)$, with $\rho(t_k) = 1$ indicating actual sensor measurements and $\rho(t_k) = 0$ indicating that the sensor measurements accessible through the communication network have been attacked.

Combining (2.1) and (2.4), we can derive that

$$\begin{aligned}\dot{x}(t) =& \bar{A}x(t) + B_0f(Wx(t)) + B_1f(Wx(t - \tau(t))) + B_2\int_{t-\eta_2(t)}^{t-\eta_1(t)} h(Wx(u))du + B_w\omega(t)\\ &+ \rho(t_k)B_u\mathcal{K}x(t_kh) + (1 - \rho(t_k))B_u\mathcal{K}g(x(t - d(t))), t \in [t_kh + \beta_{t_k}, t_{k+1}h + \beta_{t_{k+1}}].\end{aligned} \tag{2.5}$$

Taking the definition of $\beta(t)$ and the features of $\rho(t_k)$ into account, we can express (2.5) as the following:

$$\begin{aligned}\dot{x}(t) =& \bar{A}x(t) + \bar{\rho}B_u\mathcal{K}[x(t - \beta(t)) + e(t)] + (1 - \bar{\rho})B_u\mathcal{K}g(x(t - d(t))) + B_0f(Wx(t))\\ &+ B_1f(Wx(t - \tau(t))) + B_2\int_{t-\eta_2(t)}^{t-\eta_1(t)} h(Wx(s))ds + B_w\omega(t)\\ &+ (\rho(t_k) - \bar{\rho})B_u\mathcal{K}[x(t - \beta(t)) + e(t) - g(x(t - d(t)))], t \in [t_kh + \beta_{t_k}, t_{k+1}h + \beta_{t_{k+1}}].\end{aligned} \tag{2.6}$$

We present the following assumptions and lemmas instrumental to deriving our main results.

Assumption (A3). For each $g_i(t)$, $i = 1, 2, ..., n$ represents a cyber-attack function that is bounded. There exist constants G_i^- and G_i^+ such that

$$G_i^- \leq \frac{g_i(\alpha_1) - g_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq G_i^+, \quad \forall\alpha_1, \alpha_2 \in \mathbb{R}, \alpha_1 \neq \alpha_2.$$

Remark 2.4. Assumption (A3) allows for the analysis of the system's response to cyber-attacks, even in the absence of detailed information about the attack signals. It facilitates the design of resilient control strategies that can handle various attack scenarios and maintain system stability, despite bounded cyber-attacks.

Assumption (A4). The external disturbance $\omega(t)$ satisfies

$$\int_0^T \omega^T(t)\omega(t)dt \leq d_w, \quad d_w \geq 0,\ T \text{ is a time constant.}$$

Definition 2.1. [32] Given positive constants c_1, c_2 and T with $0 < c_1 < c_2$ and X is a symmetric positive definite matrix. The GNNs (2.1) is finite-time bounded with respect to (c_1, c_2, X, T), if $\forall t \in [0, T]$

$$\sup_{-\tau_M \leq s \leq 0}\{x^T(s)Xx(s), \dot{x}^T(s)X\dot{x}(s)\} \leq c_1 \Longrightarrow x^T(t)Xx(t) < c_2. \tag{2.7}$$

Lemma 2.2. *(Jensen's inequality [36]) For a symmetric positive-definite matrix, $M \in \mathbb{R}^{m\times m}$, and any given scalars d_1 and d_2, the following inequality holds:*

$$(d_2 - d_1)\int_{d_1}^{d_2} x^T(u)Mx(u)du \geq \left(\int_{d_1}^{d_2} x(u)du\right)^T M\left(\int_{d_1}^{d_2} x(u)du\right). \tag{2.8}$$

Lemma 2.3. *[35] Assume $\beta(t) \in [0, \bar{\beta}]$, for any matrices $R \in \mathbb{R}$ and L that satisfies $\begin{bmatrix} R & L \\ L^T & R \end{bmatrix} \geq 0$, the inequality holds as follows:*

$$-\bar{\beta}\int_{t-\bar{\beta}}^{t} \dot{x}^T(u)R\dot{x}(u)du \leq \begin{bmatrix} x(t) \\ x(t-\beta(t)) \\ x(t-\bar{\beta}) \end{bmatrix}^T \begin{bmatrix} -R & * & * \\ R^T - L^T & -2R + L + L^T & * \\ L^T & R^T - L^T & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\beta(t)) \\ x(t-\bar{\beta}) \end{bmatrix}. \tag{2.9}$$

Lemma 2.4. *[37] For any positive scalars $a, b > a$ and α, and any symmetric matrix $M = M^T > 0$ with dimension $n \times n$, the following inequality holds:*

$$\int_a^b e^{\alpha(t-u)}\dot{x}^T(u)M\dot{x}(u)du \geq \frac{1}{\Phi_0}\Sigma_0^T M\Sigma_0 + \frac{1}{\Phi_1}\Sigma_1^T M\Sigma_1, \tag{2.10}$$

where

$$\begin{aligned}
\Sigma_0 &= x(b) - x(a), \quad \Sigma_1 = \varepsilon_1 x(a) + \varepsilon_2 x(b) - \int_a^b x(u)du, \\
\varepsilon_1 &= \frac{(b-a)e^{-\alpha(t-b)}}{e^{-\alpha(t-b)} - e^{-\alpha(t-a)}} - \frac{1}{\alpha}, \quad \varepsilon_2 = \frac{1}{\alpha} - \frac{(b-a)e^{-\alpha(t-a)}}{e^{-\alpha(t-b)} - e^{-\alpha(t-a)}}, \\
\Phi_0 &= \int_a^b e^{-\alpha(t-u)}du = \frac{1}{\alpha}\left(e^{-\alpha(t-b)} - e^{-\alpha(t-a)}\right), \\
\Phi_1 &= \int_a^b e^{-\alpha(t-u)}l_1^2(u)du = \frac{e^{-2\alpha(t-a)} - (2 + \alpha^2(b-a)^2)e^{-\alpha(2t-a-b)} + e^{-2\alpha(t-b)}}{\alpha^3\left(e^{-\alpha(t-b)} - e^{-\alpha(t-a)}\right)}, \\
l_1(u) &= u - \left(\int_a^b e^{-\alpha(t-u)}du\right)^{-1}\left(\int_a^b e^{-\alpha(t-u)}udu\right).
\end{aligned}$$

Remark 2.5. When $\alpha = 0$, then the specific values of Φ_0, Φ_1, Σ_0, and Σ_1 are given by $\Phi_0 = b - a$, $\Phi_1 = \frac{(b-a)^3}{12}$, $\Sigma_0 = x(b) - x(a)$, and $\Sigma_1 = \frac{b-a}{2}\left[x(a) + x(b) - \frac{2}{b-a}\int_a^b x(u)du\right]$. This implies that Lemma 2.4 reduces to the well-known Wirtinger's inequality.

Lemma 2.5. *[38] For a full column rank matrix $L \in \mathbb{R}^{n\times m}$, the singular decomposition is $L = U_1\Sigma U_2^T$, where U_1 and U_2 are orthogonal matrices, and $\Sigma \in \mathbb{R}^{n\times m}$ is a rectangular diagonal matrix with positive real numbers. Let M be a matrix of the form $M = U_1\begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}U_1^T$, then there exists $X \in \mathbb{R}^{m\times m}$ such that $ML = LX$.*

Lemma 2.6. *(Schur complement [39]) If matrices X, Y, and Z have appropriate dimensions and satisfy $X = X^T$ and $Y = Y^T > 0$, then the inequality $X + Z^T Y^{-1} Z < 0$ holds if and only if*

$$\begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} -Y & Z \\ Z^T & X \end{bmatrix} < 0. \tag{2.11}$$

3. Analysis of stochastic finite-time boundedness

In this section, we introduce new sufficient conditions for delayed GNNs that build on the main theorems. To start, we define the parameters as follows: $\tau_{Mm} = \tau_M - \tau_m, \tau_{Mm} \neq 0$,

$$\begin{aligned}
\theta_{1a} =& \frac{1}{\alpha}(1 - e^{-\alpha\tau_m}), \quad \theta_{1b} = \frac{e^{-2\alpha\tau_m} - (2 + \alpha^2\tau_m^2)e^{-\alpha\tau_m} + 1}{\alpha^3(1 - e^{-\alpha\tau_m})}, \\
\theta_{2a} =& \frac{1}{\alpha}(1 - e^{-\alpha\tau_M}), \quad \theta_{2b} = \frac{e^{-2\alpha\tau_M} - (2 + \alpha^2\tau_M^2)e^{-\alpha\tau_M} + 1}{\alpha^3(1 - e^{-\alpha\tau_M})}, \\
\theta_{3a} =& \frac{1}{\alpha}(e^{-\alpha\tau_m} - e^{-\alpha\tau(t)}), \\
\theta_{3b} =& \frac{1}{\alpha^3(e^{-\alpha\tau_m} - e^{-\alpha\tau(t)})}\left[e^{-2\alpha\tau(t)} - (2 + \alpha^2(\tau(t) - \tau_m)^2)e^{-\alpha(\tau(t)+\tau_m)} + e^{-2\alpha\tau_m}\right], \\
\theta_{4a} =& \frac{1}{\alpha}(e^{-\alpha\tau(t)} - e^{-\alpha\tau_M}), \\
\theta_{4b} =& \frac{1}{\alpha^3(e^{-\alpha\tau(t)} - e^{-\alpha\tau_M})}\left[e^{-2\alpha\tau_M} - (2 + \alpha^2(\tau_M - \tau(t))^2)e^{-\alpha(\tau(t)+\tau_M)} + e^{-2\alpha\tau(t)}\right], \\
\epsilon_{11} =& \frac{\tau_m}{1 - e^{-\alpha\tau_m}} - \frac{1}{\alpha}, \quad \epsilon_{12} = \frac{1}{\alpha} - \frac{\tau_m e^{-\alpha\tau_m}}{1 - e^{-\alpha\tau_m}}, \\
\epsilon_{21} =& \frac{\tau_M}{1 - e^{-\alpha\tau_M}} - \frac{1}{\alpha}, \quad \epsilon_{22} = \frac{1}{\alpha} - \frac{\tau_M e^{-\alpha\tau_M}}{1 - e^{-\alpha\tau_M}}, \\
\epsilon_{31} =& \frac{(\tau(t) - \tau_m)e^{-\alpha\tau_m}}{e^{-\alpha\tau_m} - e^{-\alpha\tau(t)}} - \frac{1}{\alpha}, \quad \epsilon_{32} = \frac{1}{\alpha} - \frac{(\tau(t) - \tau_m)e^{-\alpha\tau(t)}}{e^{-\alpha\tau_m} - e^{-\alpha\tau(t)}}, \\
\epsilon_{41} =& \frac{(\tau_M - \tau(t))e^{-\alpha\tau(t)}}{e^{-\alpha\tau(t)} - e^{-\alpha\tau_M}} - \frac{1}{\alpha}, \quad \epsilon_{42} = \frac{1}{\alpha} - \frac{(\tau_M - \tau(t))e^{-\alpha\tau_M}}{e^{-\alpha\tau(t)} - e^{-\alpha\tau_M}}, \\
\Gamma_1 =& \left[e_1^T - e_2^T\right]^T, \quad \Gamma_2 = [\epsilon_{11}e_2^T + \epsilon_{12}e_1^T - \tau_m e_{13}^T]^T, \\
\Gamma_3 =& [e_1^T - e_4^T]^T, \quad \Gamma_4 = [\epsilon_{21}e_4^T + \epsilon_{22}e_1^T - \tau_M e_{14}^T]^T, \\
\Gamma_5 =& [e_2^T - e_3^T]^T, \quad \Gamma_6 = [\epsilon_{31}e_3^T + \epsilon_{32}e_2^T - \tau_{Mm} e_{15}^T]^T,
\end{aligned}$$

$$
\begin{aligned}
\Gamma_7 =&[e_3^T - e_4^T]^T,\ \Gamma_8 = [\epsilon_{41} e_4^T + \epsilon_{42} e_3^T - \tau_{Mm} e_{16}^T]^T,\\
\Gamma_9 =&[e_9^T - e_1^T W^T F_M^T]^T,\ \Gamma_{10} = [F_P W e_1 - e_9],\\
\Gamma_{11} =&[e_{10}^T - e_3^T W^T F_M^T]^T, \quad \Gamma_{12} = [F_P W e_3 - e_{10}],\\
\Gamma_{13} =&[e_9^T - e_{10}^T - e_1^T W^T F_M^T + e_3^T W^T F_M^T]^T,\\
\Gamma_{14} =&[F_P W e_1 - F_P W e_3 - e_9 + e_{10}],\\
\Gamma_{15} =&[e_{11}^T - e_7^T G_M^T]^T,\ \Gamma_{16} = [G_P e_7 - e_{11}],\\
\Gamma_{17} =&[e_{12}^T - e_1^T W^T H_M^T]^T, \quad \Gamma_{18} = [H_P W e_1 - e_{12}],\\
F_M =&\text{diag}\{F_1^-, ..., F_n^-\}, \quad F_P = \text{diag}\{F_1^+, ..., F_n^+\},\\
G_M =&\text{diag}\{G_1^-, ..., G_n^-\}, \quad G_P = \text{diag}\{G_1^+, ..., G_n^+\},\\
H_M =&\text{diag}\{H_1^-, ..., H_n^-\}, \quad H_P = \text{diag}\{H_1^+, ..., H_n^+\},\\
\gamma_1 =&\frac{e^{\alpha\tau_m} - 1}{\alpha}, \quad \gamma_2 = \frac{e^{\alpha\tau_M} - 1}{\alpha}, \quad \gamma_3 = \frac{e^{\alpha\bar{\beta}} - 1}{\alpha}, \quad \gamma_4 = \frac{e^{\alpha\bar{\beta}} - 1}{\alpha}, \quad \gamma_5 = \frac{e^{\alpha\tau_m} - \alpha\tau_m - 1}{\alpha^2},\\
\gamma_6 =&\frac{e^{\alpha\tau_M} - \alpha\tau_M - 1}{\alpha^2}, \quad \gamma_7 = \frac{e^{\alpha\tau_M} - e^{\alpha\tau_m} - \alpha\tau_{Mm}}{\alpha^2}, \quad \gamma_8 = \frac{e^{\alpha\bar{\beta}} - \alpha\bar{\beta} - 1}{\alpha^2},\\
\gamma_9 =&\frac{e^{\alpha\bar{d}} - \alpha\bar{d} - 1}{\alpha^2}, \quad \gamma_{10} = \frac{e^{\alpha\eta_2} - e^{\alpha\eta_1} - \alpha\eta_{21}}{\alpha^2}, \quad \eta_{21} = \eta_2 - \eta_1,
\end{aligned}
$$

and we define vectors as follows:

$$
\begin{aligned}
\xi(t) = \Bigg[& x^T(t),\ x^T(t-\tau_m),\ x^T(t-\tau(t)),\ x^T(t-\tau_M),\ x^T(t-\beta(t)),\ x^T(t-\bar{\beta}),\ x^T(t-d(t)),\\
& x^T(t-\bar{d}),\ f^T(Wx(t)),\ f^T(Wx(t-\tau(t))),\ g^T(x(t-d(t))),\ h^T(Wx(t)),\\
& \frac{1}{\tau_m}\int_{t-\tau_m}^{t} x^T(u)du,\ \frac{1}{\tau_M}\int_{t-\tau_M}^{t} x^T(u)du,\ \frac{1}{\tau(t)-\tau_m}\int_{t-\tau(t)}^{t-\tau_m} x^T(u)du,\\
& \frac{1}{\tau_M-\tau(t)}\int_{t-\tau_M}^{t-\tau(t)} x^T(u)du,\ \int_{t-\eta_2(t)}^{t-\eta_1(t)} h^T(Wx(u))du,\ e^T(t),\ \omega^T(t)\Bigg]^T,
\end{aligned}
$$

$$
e_i = \begin{bmatrix} 0_{n\times(i-1)n} & I_n & 0_{n\times(19-i)n} \end{bmatrix}, i = 1, 2, ..., 19.
$$

First, we obtain new sufficient conditions of the finite-time decentralized event-triggered feedback control problem for GNNs with mixed interval time-varying delays and cyber-attacks as follows.

Theorem 3.1. *Assume that Assumptions (A1)-(A4) are satisfied. Then, for given scalars* $\bar{\beta}, \bar{d}, \bar{\rho}, \sigma, d_w, \tau_m, \tau_M, \eta_1, \eta_2, T, c_1, c_2$ *and* α*, the delayed GNNs with cyber-attacks (2.6) under the state feedback controller is stochastic finite-time bounded regarding* (c_1, c_2, T, X, d_w)*, if there exist symmetric positive definite matrices* P*,* $Q_i, R_j (i = 1, 2, 3, 4, j = 1, 2, 3, 4, 5)$*,* M, S *and positive diagonal matrices* G_1, G_2, G_3, G_4, G_5*, such that the following conditions hold:*

$$
\begin{bmatrix} \Pi & * & * \\ \Gamma_a & -\Lambda & * \\ \Gamma_b & 0 & -\Lambda \end{bmatrix} < 0, \tag{3.1}
$$

$$e^{\alpha T}\left[\Pi_\lambda c_1 + d_w\lambda_{13}(1-e^{-\alpha T})\right] < \lambda_1 c_2, \tag{3.2}$$

where

$$\begin{aligned}
\Pi =& \sum_{i=1}^{6}\Pi_i,\\
\Pi_1 =& 2e_1^T P\bar{A}e_1 + 2e_1^T\bar{\rho}PB_u\mathcal{K}[e_5+e_{18}] + 2(1-\bar{\rho})e_1^T PB_u\mathcal{K}e_{11} + 2e_1^T PB_0e_9\\
&+ 2e_1^T PB_1e_{10} + 2e_1^T PB_2e_{17} + 2e_1^T PB_we_{19} - \alpha e_1^T Pe_1,\\
\Pi_2 =& e_1^T(Q_1+Q_2+Q_3+Q_4)e_1 - e^{\alpha\tau_m}e_2^T Q_1e_2 - e^{\alpha\tau_M}e_4^T Q_2e_4 - e^{\alpha\bar{\beta}}e_6^T Q_3e_6 - e^{\alpha\bar{d}}e_8^T Q_4e_8,\\
\Pi_3 =& -\frac{\tau_m}{\theta_{1a}}\Gamma_1^T R_1\Gamma_1 - \frac{\tau_m}{\theta_{1b}}\Gamma_2^T R_1\Gamma_2 - \frac{\tau_M}{\theta_{2a}}\Gamma_3^T R_2\Gamma_3 - \frac{\tau_M}{\theta_{2b}}\Gamma_4^T R_2\Gamma_4 - \frac{\tau_{Mm}}{\theta_{3a}}\Gamma_5^T R_3\Gamma_5\\
&- \frac{\tau_{Mm}}{\theta_{3b}}\Gamma_6^T R_3\Gamma_6 - \frac{\tau_{Mm}}{\theta_{4a}}\Gamma_7^T R_3\Gamma_7 - \frac{\tau_{Mm}}{\theta_{4b}}\Gamma_8^T R_3\Gamma_8\\
&+ e^{\alpha\bar{\beta}}\Big[-e_1^T R_4e_1 + 2e_1^T(R_4-L_1)e_5 + 2e_1^T L_1e_6 + e_5^T(-2R_4+L_1+L_1^T)e_5\\
&\qquad + 2e_5^T(R_4-L_1)e_6 + e_6^T R_4e_6\Big]\\
&+ e^{\alpha\bar{d}}\Big[-e_1^T R_5e_1 + 2e_1^T(R_5-L_2)e_7 + 2e_1^T L_2e_8 + e_7^T(-2R_5+L_2+L_2^T)e_7\\
&\qquad + 2e_7^T(R_5-L_2)e_8 + e_8^T R_5e_8\Big],\\
\Pi_4 =& \eta_{21}^2 e_{12}^T Me_{12} - e^{\alpha\eta_2}e_{17}^T Me_{17},\\
\Pi_5 =& 2\Gamma_9^T G_1\Gamma_{10} + 2\Gamma_{11}^T G_2\Gamma_{12} + 2\Gamma_{13}^T G_3\Gamma_{14} + 2\Gamma_{15}^T G_4\Gamma_{16} + 2\Gamma_{17}^T G_5\Gamma_{18},\\
\Pi_6 =& -\alpha e_{19}^T Se_{19} - e_{18}^T\Omega e_{18} + \sigma e_5^T\Omega e_5,\\
\Pi_\lambda =& \lambda_2 + \gamma_1\lambda_3 + \gamma_2\lambda_4 + \gamma_3\lambda_5 + \gamma_4\lambda_6 + \tau_m\gamma_5\lambda_7 + \tau_M\gamma_6\lambda_8 + \tau_{Mm}\gamma_7\lambda_9 + \bar{\beta}\gamma_8\lambda_{10} + \bar{d}\gamma_9\lambda_{11}\\
&+ \eta_{21}\gamma_{10}\lambda_{12},\\
\lambda_1 =& \lambda_{\min}(\overline{P}), \lambda_2 = \lambda_{\max}(\overline{P}), \lambda_3 = \lambda_{\max}(\overline{Q}_1), \lambda_4 = \lambda_{\max}(\overline{Q}_2), \lambda_5 = \lambda_{\max}(\overline{Q}_3), \lambda_6 = \lambda_{\max}(\overline{Q}_4),\\
\lambda_7 =& \lambda_{\max}(\overline{R}_1), \lambda_8 = \lambda_{\max}(\overline{R}_2), \lambda_9 = \lambda_{\max}(\overline{R}_3), \lambda_{10} = \lambda_{\max}(\overline{R}_4), \lambda_{11} = \lambda_{\max}(\overline{R}_5), \lambda_{12} = \lambda_{\max}(\overline{M}),\\
\lambda_{13} =& \lambda_{\max}(\overline{S}),\\
\Lambda =& diag\{PR_1^{-1}P, PR_2^{-1}P, PR_3^{-1}P, PR_4^{-1}P, PR_5^{-1}P\},\\
\Gamma_a =& \begin{bmatrix}\tau_m\Gamma_{a1}^T & \tau_M\Gamma_{a1}^T & \tau_{Mm}\Gamma_{a1}^T & \bar{\beta}\Gamma_{a1}^T & \bar{d}\Gamma_{a1}^T\end{bmatrix}^T,\\
\Gamma_b =& \begin{bmatrix}\tau_m\Gamma_{b1}^T & \tau_M\Gamma_{b1}^T & \tau_{Mm}\Gamma_{b1}^T & \bar{\beta}\Gamma_{b1}^T & \bar{d}\Gamma_{b1}^T\end{bmatrix}^T,\\
\Gamma_{a1} =& \begin{bmatrix}\Gamma_{a11} & \Gamma_{a12}\end{bmatrix},\\
\Gamma_{a11} =& \begin{bmatrix}PA & 0_{1\times 3} & \bar{\rho}PB_u\mathcal{K} & 0_{1\times 3} & PB_0 & PB_1\end{bmatrix},\\
\Gamma_{a12} =& \begin{bmatrix}(1-\bar{\rho})PB_u\mathcal{K} & 0_{1\times 5} & PB_2 & \bar{\rho}PB_u\mathcal{K} & PB_w\end{bmatrix},\\
\Gamma_{b1} =& \begin{bmatrix}0_{1\times 4} & \delta PB_u\mathcal{K} & 0_{1\times 5} & \delta PB_u\mathcal{K} & 0_{1\times 6} & \delta PB_u\mathcal{K} & 0\end{bmatrix}, \quad \delta = \sqrt{\bar{\rho}(1-\bar{\rho})}.
\end{aligned}$$

Proof. We formulate the LKFs as follows:

$$V(x_t,t) = \sum_{i=1}^{3} V_i(x_t,t), \tag{3.3}$$

where

$$
\begin{aligned}
V_1(x_t,t) =& x(t)^T P x(t),\\
V_2(x_t,t) =& \int_{t-\tau_m}^{t} e^{\alpha(t-u)} x^T(u) Q_1 x(u)\, du + \int_{t-\tau_M}^{t} e^{\alpha(t-u)} x^T(u) Q_2 x(u)\, du\\
&+ \int_{t-\bar{\beta}}^{t} e^{\alpha(t-u)} x^T(u) Q_3 x(u)\, du + \int_{t-\bar{d}}^{t} e^{\alpha(t-u)} x^T(u) Q_4 x(u)\, du,\\
V_3(x_t,t) =& \tau_m \int_{t-\tau_m}^{t}\int_{u}^{t} e^{\alpha(t-s)} \dot{x}^T(s) R_1 \dot{x}(s)\, ds\, du + \tau_M \int_{t-\tau_M}^{t}\int_{u}^{t} e^{\alpha(t-s)} \dot{x}^T(s) R_2 \dot{x}(s)\, ds\, du\\
&+ \tau_{Mm} \int_{t-\tau_M}^{t-\tau_m}\int_{u}^{t} e^{\alpha(t-s)} \dot{x}^T(s) R_3 \dot{x}(s)\, ds\, du + \bar{\beta} \int_{t-\bar{\beta}}^{t}\int_{u}^{t} e^{\alpha(t-s)} \dot{x}^T(s) R_4 \dot{x}(s)\, ds\, du\\
&+ \bar{d} \int_{t-\bar{d}}^{t}\int_{u}^{t} e^{\alpha(t-s)} \dot{x}^T(s) R_5 \dot{x}(s) ds\, du\\
&+ (\eta_2-\eta_1) \int_{t-\eta_2}^{t-\eta_1}\int_{u}^{t} e^{\alpha(t-s)} h^T(Wx(s)) M h(Wx(s)) ds du.
\end{aligned}
$$

Taking the mathematical expectation of the derivative of (3.3) over the trajectory of the GNNs (2.6), we obtain

$$
\begin{aligned}
\mathbb{E}\{\dot{V}_1(x_t,t)\} =& 2x^T(t) P \dot{x}(t) - \alpha x^T(t) P x(t) + \alpha V_1(x_t,t)\\
=& \xi^T(t) \Pi_1 \xi(t) + \alpha V_1(x_t,t),
\end{aligned} \tag{3.4}
$$

$$
\begin{aligned}
\mathbb{E}\{\dot{V}_2(x_t,t)\} =& x^T(t)\Big\{Q_1+Q_2+Q_3+Q_4\Big\}x(t) - e^{\alpha\tau_m} x^T(t-\tau_m) Q_1 x(t-\tau_m)\\
&- e^{\alpha\tau_M} x^T(t-\tau_M) Q_2 x(t-\tau_M) - e^{\alpha\bar{\beta}} x^T(t-\bar{\beta}) Q_3 x(t-\bar{\beta})\\
&- e^{\alpha\bar{d}} x^T(t-\bar{d}) Q_4 x(t-\bar{d}) + \alpha V_2(x_t,t)\\
=& \xi^T(t) \Pi_2 \xi(t) + \alpha V_2(x_t,t),
\end{aligned} \tag{3.5}
$$

$$
\begin{aligned}
\mathbb{E}\{\dot{V}_3(x_t,t)\} =& \mathbb{E}\{\dot{x}^T(t) \widehat{\mathcal{R}} \dot{x}(t)\} - \tau_m \int_{t-\tau_m}^{t} e^{\alpha(t-u)} \dot{x}^T(u) R_1 \dot{x}(u)\, du\\
&- \tau_M \int_{t-\tau_M}^{t} e^{\alpha(t-u)} \dot{x}^T(u) R_2 \dot{x}(u)\, du - \tau_{Mm} \int_{t-\tau_M}^{t-\tau_m} e^{\alpha(t-u)} \dot{x}^T(u) R_3 \dot{x}(u)\, du\\
&- \bar{\beta} \int_{t-\bar{\beta}}^{t} e^{\alpha(t-u)} \dot{x}^T(u) R_4 \dot{x}(u)\, du - \bar{d} \int_{t-\bar{d}}^{t} e^{\alpha(t-u)} \dot{x}^T(u) R_4 \dot{x}(u)\, du + \alpha V_3(x_t,t),
\end{aligned} \tag{3.6}
$$

where $\widehat{\mathcal{R}} = \tau_m^2 R_1 + \tau_M^2 R_2 + \tau_{Mm}^2 R_3 + \bar{\beta}^2 R_4 + \bar{d}^2 R_5$,

$$
\begin{aligned}
\mathbb{E}\{\dot{V}_4(x_t,t)\} =& (\eta_2-\eta_1)^2 h^T(Wx(t)) M h(Wx(t)) - (\eta_2-\eta_1) \int_{t-\eta_2}^{t-\eta_1} e^{\alpha(t-u)} h^T(Wx(u)) M h(Wx(u)) du\\
&+ \alpha V_4(x_t,t)
\end{aligned}
$$

$$\leq(\eta_2-\eta_1)^2h^T(Wx(t))Mh(Wx(t))-(\eta_2(t)-\eta_1(t))e^{\alpha\eta_2}\int_{t-\eta_2(t)}^{t-\eta_1(t)}h^T(Wx(u))Mh(Wx(u))du$$
$$+\alpha V_4(x_t,t). \tag{3.7}$$

Applying Lemma 2.4 to the integral, we obtain

$$\begin{aligned}-\tau_m\int_{t-\tau_m}^{t}e^{\alpha(t-u)}\dot{x}^T(u)R_1\dot{x}(u)\,du \leq&\xi^T(t)\bigg\{-\frac{\tau_m}{\theta_{1a}}(e_1-e_2)^TR_1(e_1-e_2)-\frac{\tau_m}{\theta_{1b}}(\epsilon_{11}e_2+\epsilon_{12}e_1-\tau_me_{13})^T\\&\times R_1(\epsilon_{11}e_2+\epsilon_{12}e_1-\tau_me_{13})\bigg\}\xi(t)\\=&\xi^T(t)\left\{-\frac{\tau_m}{\theta_{1a}}\Gamma_1^TR_1\Gamma_1-\frac{\tau_m}{\theta_{1b}}\Gamma_2^TR_1\Gamma_2\right\}\xi(t),\end{aligned} \tag{3.8}$$

$$\begin{aligned}-\tau_M\int_{t-\tau_M}^{t}e^{\alpha(t-u)}\dot{x}^T(u)R_2\dot{x}(u)\,du \leq&\xi^T(t)\bigg\{-\frac{\tau_M}{\theta_{2a}}(e_1-e_4)^TR_2(e_1-e_4)-\frac{\tau_M}{\theta_{2b}}(\epsilon_{21}e_4+\epsilon_{22}e_1-\tau_Me_{14})^T\\&\times R_2(\epsilon_{21}e_4+\epsilon_{22}e_1-\tau_Me_{14})\bigg\}\xi(t)\\=&\xi^T(t)\bigg\{-\frac{\tau_M}{\theta_{2a}}\Gamma_3^TR_2\Gamma_3-\frac{\tau_M}{\theta_{2b}}\Gamma_4^TR_2\Gamma_4\bigg\}\xi(t),\end{aligned} \tag{3.9}$$

$$\begin{aligned}-\tau_{Mm}\int_{t-\tau_M}^{t-\tau_m}e^{\alpha(t-u)}\dot{x}^T(u)R_3\dot{x}(u)\,du =&-\tau_{Mm}\int_{t-\tau(t)}^{t-\tau_m}e^{\alpha(t-u)}\dot{x}^T(u)R_3\dot{x}(u)\,du\\&-\tau_{Mm}\int_{t-\tau_M}^{t-\tau(t)}e^{\alpha(t-u)}\dot{x}^T(u)R_3\dot{x}(u)\,du\\\leq&\xi^T(t)\bigg\{-\frac{\tau_{Mm}}{\theta_{3a}}(e_2-e_3)^TR_3(e_2-e_3)-\frac{\tau_{Mm}}{\theta_{3b}}(\epsilon_{31}e_3+\epsilon_{32}e_2-\tau_{Mm}e_{15})^T\\&\times R_3(\epsilon_{31}e_3+\epsilon_{32}e_2-\tau_{Mm}e_{15})\\&-\frac{\tau_{Mm}}{\theta_{4a}}(e_3-e_4)^TR_3(e_3-e_4)-\frac{\tau_{Mm}}{\theta_{4b}}(\epsilon_{41}e_4+\epsilon_{42}e_3-\tau_{Mm}e_{16})^T\\&\times R_3(\epsilon_{41}e_4+\epsilon_{42}e_3-\tau_{Mm}e_{16})\bigg\}\xi(t)\\=&\xi^T(t)\bigg\{-\frac{\tau_{Mm}}{\theta_{3a}}\Gamma_5^TR_3\Gamma_5-\frac{\tau_{Mm}}{\theta_{3b}}\Gamma_6^TR_3\Gamma_6\\&-\frac{\tau_{Mm}}{\theta_{4a}}\Gamma_7^TR_3\Gamma_7-\frac{\tau_{Mm}}{\theta_{4b}}\Gamma_8^TR_3\Gamma_8\bigg\}\xi(t).\end{aligned} \tag{3.10}$$

Applying Lemma 2.2, we obtain

$$-(\eta_2(t)-\eta_1(t))e^{\alpha\eta_2}\int_{t-\eta_2(t)}^{t-\eta_1(t)}h^T(Wx(u))Mh(Wx(u))du\leq-e^{\alpha\eta_2}\left(\int_{t-\eta_2(t)}^{t-\eta_1(t)}h^T(Wx(u))du\right)^T$$

$$\times M\Bigg(\int_{t-\eta_2(t)}^{t-\eta_1(t)} h^T(Wx(u))du\Bigg)$$
$$= \xi^T(t)\Bigg\{-e_{17}^T(e^{\alpha\eta_2}M)e_{17}\Bigg\}\xi(t). \tag{3.11}$$

Applying Lemma 2.3, we obtain

$$\begin{aligned}
-\bar{\beta}\int_{t-\bar{\beta}}^{t} e^{\alpha(t-u)}\dot{x}^T(u)R_4\dot{x}(u)\,du \le & e^{\alpha\bar{\beta}}\begin{bmatrix} x(t) \\ x(t-\beta(t)) \\ x(t-\bar{\beta}) \end{bmatrix}^T \begin{bmatrix} -R_4 & * & * \\ R_4^T-L_1^T & -2R_4+L_1+L_1^T & * \\ L_1^T & R_4^T-L_1^T & -R_4 \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\beta(t)) \\ x(t-\bar{\beta}) \end{bmatrix} \\
= & e^{\alpha\bar{\beta}}\xi^T(t)\Big\{-e_1^TR_4e_1+2e_1^T(R_4-L_1)e_5+2e_1^TL_1e_6 \\
& +e_5^T(-2R_4+L_1+L_1^T)e_5 \\
& +2e_5^T(R_4-L_1)e_6+e_6^TR_4e_6\Big\}\xi(t),
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
-\bar{d}\int_{t-\bar{d}}^{t} e^{\alpha(t-u)}\dot{x}^T(u)R_5\dot{x}(u)\,du \le & e^{\alpha\bar{d}}\begin{bmatrix} x(t) \\ x(t-d(t)) \\ x(t-\bar{d}) \end{bmatrix}^T \begin{bmatrix} -R_5 & * & * \\ R_5^T-L_2^T & -2R_5+L_2+L_2^T & * \\ L_2^T & R_5^T-L_2^T & -R_5 \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-d(t)) \\ x(t-\bar{d}) \end{bmatrix} \\
= & e^{\alpha\bar{d}}\xi^T(t)\Big\{-e_1^TR_5e_1+2e_1^T(R_5-L_2)e_7+2e_1^TL_2e_8 \\
& +e_7^T(-2R_5+L_2+L_2^T)e_7 \\
& +2e_7^T(R_5-L_2)e_8+e_8^TR_5e_8\Big\}\xi(t).
\end{aligned} \tag{3.13}$$

Noting that $\mathbb{E}\{\dot{x}^T(t)\widehat{\mathcal{R}}\dot{x}(t)\} = \mathcal{A}^T\widehat{\mathcal{R}}\mathcal{A} + \bar{\rho}(1-\bar{\rho})\mathcal{B}^T\widehat{\mathcal{R}}\mathcal{B}$, where $\mathcal{A} = \bar{A}x(t) + \bar{\rho}B_u\mathcal{K}[x(t-\beta(t)) + e(t)] + (1-\bar{\rho})B_u\mathcal{K}g(x(t-d(t))) + B_0f(Wx(t)) + B_1f(Wx(t-\tau(t))) + B_2\int_{t-\eta_2(t)}^{t-\eta_1(t)} h(Wx(s))ds + B_w\omega(t)$, $\mathcal{B} = B_u\mathcal{K}[x(t-\beta(t)) + e(t) - g(x(t-d(t)))]$.

One can deduce from Assumption (A1) that if $\beta_{1i},\beta_{2i},\beta_{3i},\beta_{4i},\beta_{5i} > 0$ for $i = 1,2,...,n$, then

$$2[f_i(W_ix(t)) - F_i^-W_ix(t)]\beta_{1i}[F_i^+W_ix(t) - f_i(W_ix(t))] \ge 0, \tag{3.14}$$
$$2[f_i(W_ix(t-\tau(t))) - F_i^-W_ix(t-\tau(t))]\beta_{2i}[F_i^+W_ix(t-\tau(t)) - f_i(W_ix(t-\tau(t)))] \ge 0, \tag{3.15}$$
$$\begin{aligned}&2[f_i(W_ix(t)) - f_i(W_ix(t-\tau(t))) - F_i^-(W_ix(t) - W_ix(t-\tau(t)))]\beta_{3i}[F_i^+(W_ix(t) - W_ix(t-\tau(t))) \\ &\quad - f_i(W_ix(t)) + f_i(W_ix(t-\tau(t)))] \ge 0,\end{aligned} \tag{3.16}$$
$$2[g_i(W_ix(t-d(t))) - G_i^-W_ix(t-d(t))]\beta_{4i}[G_i^+W_ix(t-d(t)) - g_i(W_ix(t-d(t)))] \ge 0, \tag{3.17}$$
$$2[h_i(W_ix(t)) - H_i^-W_ix(t)]\beta_{5i}[H_i^+W_ix(t) - h_i(W_ix(t))] \ge 0, \tag{3.18}$$

which implies that

$$\begin{aligned}
2\xi^T(t)\Gamma_9^TG_1\Gamma_{10}\xi(t) &\ge 0, \\
2\xi^T(t)\Gamma_{11}^TG_2\Gamma_{12}\xi(t) &\ge 0, \\
2\xi^T(t)\Gamma_{13}^TG_3\Gamma_{14}\xi(t) &\ge 0, \\
2\xi^T(t)\Gamma_{15}^TG_4\Gamma_{16}\xi(t) &\ge 0, \\
2\xi^T(t)\Gamma_{17}^TG_5\Gamma_{18}\xi(t) &\ge 0.
\end{aligned}$$

Thus, we have

$$\begin{aligned}0 \leq& \xi^T(t)\Big\{2\Gamma_9^T G_1\Gamma_{10} + 2\Gamma_{11}^T G_2\Gamma_{12} + 2\Gamma_{13}^T G_3\Gamma_{14} + 2\Gamma_{15}^T G_4\Gamma_{16} + 2\Gamma_{17}^T G_5\Gamma_{18}\Big\}\xi(t)\\ =&\xi^T(t)\Pi_5\xi(t),\end{aligned} \tag{3.19}$$

where $G_1 = \text{diag}\{\beta_{11}, ..., \beta_{1n}\}$, $G_2 = \text{diag}\{\beta_{21}, ..., \beta_{2n}\}$, $G_3 = \text{diag}\{\beta_{31}, ..., \beta_{3n}\}$, $G_4 = \text{diag}\{\beta_{41}, ..., \beta_{4n}\}$ and $G_5 = \text{diag}\{\beta_{51}, ..., \beta_{5n}\}$.

Combining (3.4)–(3.19) with (2.3), we obtain

$$\begin{aligned}\mathbb{E}\{\dot{V}(x_t, t) - \alpha V(x_t, t)\} - \alpha\omega^T(t)S\,\omega(t) \leq& \xi^T(t)\Big\{\Pi_1 + \Pi_2 + \Pi_3 + \Pi_4 + \Pi_5\Big\}\xi(t)\\ &- \alpha\omega^T(t)S\,\omega(t) - e^T(t)\Omega e(t)\\ &+ \sigma x^T(t-\beta(t))\Omega x(t-\beta(t)) + \mathcal{A}^T\widehat{\mathcal{R}}\mathcal{A}\\ &+ \bar{\rho}(1-\bar{\rho})\mathcal{B}^T\widehat{\mathcal{R}}\mathcal{B}\\ =&\xi^T(t)\overline{\Pi}\xi(t) + \mathcal{A}^T\widehat{\mathcal{R}}\mathcal{A} + \bar{\rho}(1-\bar{\rho})\mathcal{B}^T\widehat{\mathcal{R}}\mathcal{B}.\end{aligned} \tag{3.20}$$

Applying the Schur complement [39], we can derive that (3.1) is equal to:

$$\mathbb{E}\{\dot{V}(x_t, t) - \alpha V(x_t, t)\} - \alpha\omega^T(t)S\,\omega(t) < 0. \tag{3.21}$$

Then, multiplying (3.21) by $e^{-\alpha t}$, we can be written as

$$\mathbb{E}\left\{\frac{d}{dt}\left(e^{-\alpha t}V(x_t, t)\right)\right\} < \alpha e^{-\alpha t}\omega^T(t)S\,\omega(t). \tag{3.22}$$

Assumption (A2) and integrating (3.22) from 0 to t with $t \in [0, T]$, we obtain

$$\begin{aligned}\mathbb{E}\{V(x_t, t)\} <& e^{\alpha T}\Big[V(x_0, 0) + \alpha\int_0^T e^{-\alpha u}\omega^T(u)S\,\omega(u)du\Big]\\ <& e^{\alpha T}\Big[V(x_0, 0) + d_w\lambda_{13}(1 - e^{-\alpha T})\Big].\end{aligned} \tag{3.23}$$

Considering $V(x_0, 0)$, we can derive that

$$\begin{aligned}\mathbb{E}\{V(x_0, 0)\} =&\mathbb{E}\Big\{x^T(0)Px(0) + \int_{-\tau_m}^0 e^{-\alpha u}x^T(u)Q_1x(u)\,du + \int_{-\tau_M}^0 e^{-\alpha u}x^T(u)Q_2x(u)\,du\\ &+ \int_{-\bar{\beta}}^0 e^{-\alpha u}x^T(u)Q_3x(u)\,du + \int_{-\bar{d}}^0 e^{-\alpha u}x^T(u)Q_4x(u)\,du\\ &+ \tau_m\int_{-\tau_m}^0\int_u^0 e^{-\alpha s}\dot{x}(s)R_1\dot{x}(s)\,dsdu + \tau_M\int_{-\tau_M}^0\int_u^0 e^{-\alpha s}\dot{x}(s)R_2\dot{x}(s)\,dsdu\\ &+ \tau_{Mm}\int_{-\tau_M}^{-\tau_m}\int_u^0 e^{-\alpha s}\dot{x}(s)R_3\dot{x}(s)\,dsdu + \bar{\beta}\int_{-\bar{\beta}}^0\int_u^0 e^{-\alpha s}\dot{x}(s)R_4\dot{x}(s)\,dsdu\\ &+ \bar{d}\int_{-\bar{d}}^0\int_u^0 e^{-\alpha s}\dot{x}(s)R_5\dot{x}(s)\,dsdu + \eta_{21}\int_{-\eta_2}^{-\eta_1}\int_u^0 e^{-\alpha s}h^T(Wx(t))Mh(Wx(t))\,dsdu\Big\}\end{aligned}$$

$$
\begin{aligned}
\leq & \mathbb{E}\Big\{x^T(0)Px(0) + \int_{-\tau_m}^{0} e^{-\alpha u} x^T(u) Q_1 x(u)\, du + \int_{-\tau_M}^{0} e^{-\alpha u} x^T(u) Q_2 x(u)\, du \\
& + \int_{-\bar{\beta}}^{0} e^{-\alpha u} x^T(u) Q_3 x(u)\, du + \int_{-\bar{d}}^{0} e^{-\alpha u} x^T(u) Q_4 x(u)\, du \\
& + \tau_m \int_{-\tau_m}^{0}\int_{u}^{0} e^{-\alpha s} \dot{x}(s) R_1 \dot{x}(s)\, dsdu + \tau_M \int_{-\tau_M}^{0}\int_{u}^{0} e^{-\alpha s} \dot{x}(s) R_2 \dot{x}(s)\, dsdu \\
& + \tau_{Mm} \int_{-\tau_M}^{-\tau_m}\int_{u}^{0} e^{-\alpha s} \dot{x}(s) R_3 \dot{x}(s)\, dsdu + \bar{\beta} \int_{-\bar{\beta}}^{0}\int_{u}^{0} e^{-\alpha s} \dot{x}(s) R_4 \dot{x}(s)\, dsdu \\
& + \bar{d} \int_{-\bar{d}}^{0}\int_{u}^{0} e^{-\alpha s} \dot{x}(s) R_5 \dot{x}(s)\, dsdu + \eta_{21} \int_{-\eta_2}^{-\eta_1}\int_{u}^{0} e^{-\alpha s} x^T(s) F_w M F_w x(s)\, dsdu\Big\},
\end{aligned}
$$

where $F_w = \text{diag}\left\{F_1^+, F_2^+, ..., F_n^+\right\} W$.

Setting $\overline{P} = X^{-\frac{1}{2}} P X^{-\frac{1}{2}}$, $\overline{Q}_i = X^{-\frac{1}{2}} Q_i X^{-\frac{1}{2}}$, $\overline{R}_j = X^{-\frac{1}{2}} R_j X^{-\frac{1}{2}}$, $\overline{M} = X^{-\frac{1}{2}} F_w M F_w X^{-\frac{1}{2}}$, $i = 1, 2, 3, 4$, $j = 1, 2, 3, 4, 5$, we have

$$
\begin{aligned}
\mathbb{E}\{V(x_0, 0)\} \leq & \mathbb{E}\Big\{x^T(0) X^{\frac{1}{2}} \overline{P} X^{\frac{1}{2}} x(0) + \int_{-\tau_m}^{0} e^{-\alpha u} x^T(u) X^{\frac{1}{2}} \overline{Q}_1 X^{\frac{1}{2}} x(u)\, du \\
& + \int_{-\tau_M}^{0} e^{-\alpha u} x^T(u) X^{\frac{1}{2}} \overline{Q}_2 X^{\frac{1}{2}} x(u)\, du + \int_{-\bar{\beta}}^{0} e^{-\alpha u} x^T(u) X^{\frac{1}{2}} \overline{Q}_3 X^{\frac{1}{2}} x(u)\, du \\
& + \int_{-\bar{d}}^{0} e^{-\alpha u} x^T(u) X^{\frac{1}{2}} \overline{Q}_4 X^{\frac{1}{2}} x(u)\, du + \tau_m \int_{-\tau_m}^{0}\int_{u}^{0} e^{-\alpha s} \dot{x}(s) X^{\frac{1}{2}} \overline{R}_1 X^{\frac{1}{2}} \dot{x}(s)\, dsdu \\
& + \tau_M \int_{-\tau_M}^{0}\int_{u}^{0} e^{-\alpha s} \dot{x}(s) X^{\frac{1}{2}} \overline{R}_2 X^{\frac{1}{2}} \dot{x}(s)\, dsdu \\
& + \tau_{Mm} \int_{-\tau_M}^{-\tau_m}\int_{u}^{0} e^{-\alpha s} \dot{x}(s) X^{\frac{1}{2}} \overline{R}_3 X^{\frac{1}{2}} \dot{x}(s)\, dsdu \\
& + \bar{\beta} \int_{-\bar{\beta}}^{0}\int_{u}^{0} e^{-\alpha s} \dot{x}(s) X^{\frac{1}{2}} \overline{R}_4 X^{\frac{1}{2}} \dot{x}(s)\, dsdu \\
& + \bar{d} \int_{-\bar{d}}^{0}\int_{u}^{0} e^{-\alpha s} \dot{x}(s) X^{\frac{1}{2}} \overline{R}_5 X^{\frac{1}{2}} \dot{x}(s)\, dsdu \\
& + \eta_{21} \int_{-\eta_2}^{-\eta_1}\int_{u}^{0} e^{-\alpha s} x^T(s) X^{\frac{1}{2}} \overline{M} X^{\frac{1}{2}} x(s)\, dsdu\Big\} \\
\leq & \Big\{\lambda_2 + \gamma_1\lambda_3 + \gamma_2\lambda_4 + \gamma_3\lambda_5 + \gamma_4\lambda_6 + \tau_m\gamma_5\lambda_7 + \tau_M\gamma_6\lambda_8 + \tau_{Mm}\gamma_7\lambda_9 + \bar{\beta}\gamma_8\lambda_{10} \\
& + \bar{d}\gamma_9\lambda_{11} + \eta_{21}\gamma_{10}\lambda_{12}\Big\} c_1 \\
= & \Pi_\lambda c_1.
\end{aligned}
$$

Furthermore, by referring Eq (3.3), we obtain the following:

$$
\mathbb{E}\{V(x_t, t)\} \geq \mathbb{E}\{x^T(t) P x(t)\} \geq \lambda_{\min}(\overline{P}) \mathbb{E}\{x^T(t) X x(t)\} = \lambda_1 \mathbb{E}\{x^T(t) X x(t)\}. \tag{3.24}
$$

Subsequently, by utilizing Eqs (3.23) to (3.24), we obtain the following:

$$\mathbb{E}\{x^T(t)Xx(t)\} \leq \frac{e^{\alpha T}}{\lambda_1}[\Pi_\lambda c_1 + d_w\lambda_{13}(1 - e^{-\alpha T})].$$

Thus, if the relation in (3.2) holds true, then it implies that $\mathbb{E}\{x^T(t)Xx(t)\} < c_2, \forall t \in [0, T]$. Consequently, it can be concluded that the delayed GNNs (2.6) with cyber-attacks are stochastically finite-time bounded with respect to (c_1, c_2, T, X, d_w). This completes the proof. □

Remark 3.1. The activation functions play a crucial role in determining the existence and uniqueness of solutions in GNNs. Specifically, the activation functions in GNNs need to satisfy the Lipschitz condition, which ensures that the functions possess certain properties that guarantee the existence and uniqueness of solutions. The Lipschitz condition requires the functions to have a bounded derivative, thereby controlling the rate of change of the functions. In this article, the activation function is assumed to satisfy Assumptions (A1) and (A3). In this case, it is important to note that the activation function does not necessarily need to be non-monotonic and differentiable. The constants F_i^-, F_i^+, H_i^-, H_i^+, G_i^- and G_i^+, where $i = 1, 2, \ldots, n$, can take either positive, zero, or negative values. Assumptions (A1) and (A3), as considered in Eqs (3.14)–(3.18) of this article, ensuring not only $F_i^- \leq \frac{f_i(Wx(t))}{Wx(t)} \leq F_i^+$, $F_i^- \leq \frac{f_i(Wx(t-\tau(t)))}{Wx(t-\tau(t))} \leq F_i^+$, $H_i^- \leq \frac{h_i(Wx(t))}{Wx(t)} \leq H_i^+$, $H_i^- \leq \frac{h_i(Wx(t-\tau(t)))}{Wx(t-\tau(t))} \leq H_i^+$, $G_i^- \leq \frac{g_i(x(t))}{x(t)} \leq G_i^+$ and $G_i^- \leq \frac{g_i(x(t-\tau(t)))}{x(t-\tau(t))} \leq G_i^+$, but also $F_i^- \leq \frac{f_i(Wx_1)-f_i(Wx_2)}{Wx_1-Wx_2} \leq F_i^+$, $H_i^- \leq \frac{h_i(Wx_1)-h_i(Wx_2)}{Wx_1-Wx_2} \leq H_i^+$ and $G_i^- \leq \frac{g_i(x_1)-g_i(x_2)}{x_1-x_2} \leq G_i^+$. Therefore, this assumption is weaker and more general than the usual Lipschitz condition. In conclusion, the activation functions in GNNs play a critical role in ensuring the existence and uniqueness of solutions. While the assumption considered in this article relaxes the requirement for non-monotonicity and differentiability, it provides a more general condition that still guarantees the desired properties.

Based on the results of Theorem 3.1, we can now propose a controller design approach for the delayed GNNs (2.6) as follows.

Assumption (A5) To handle the nonlinear terms in Theorem 3.1, we adopt a similar assumption to [38], B is assumed to be full column rank, and the singular decomposition for B as $B = L_1 \begin{bmatrix} B_{u_0} \\ 0 \end{bmatrix} L_2$, where B_{u_0} is the first u_0 columns of B, L_1 and L_2 are appropriate matrices with compatible dimensions.

Theorem 3.2. *Assume that Assumptions (A1)–(A5) are satisfied. Then, for the given scalars* $\bar{\beta}, \bar{d}, \bar{\rho}, \sigma, d_w, \tau_m, \tau_M, \eta_1, \eta_2, T, c_1, c_2$ *and* α, *the delayed GNNs with cyber attacks (2.6) under the state feedback controller is stochastic finite-time bounded regarding* (c_1, c_2, T, X, d_w), *if there exist symmetric positive definite matrices* P, $Q_i, R_j (i = 1, 2, 3, 4, j = 1, 2, 3, 4, 5)$, M, S, *and positive diagonal matrices* G_1, G_2, G_3, G_4, G_5, *such that the conditions hold as follows:*

$$\begin{bmatrix} \Pi_u & * & * \\ \overline{\Gamma}_{a_u} & -\Lambda_u & * \\ \overline{\Gamma}_{b_u} & 0 & -\Lambda_u \end{bmatrix} < 0, \tag{3.25}$$

$$e^{\alpha T}\left[\Pi_\lambda c_1 + d_w\lambda_{13}(1 - e^{-\alpha T})\right] < \lambda_1 c_2, \tag{3.26}$$

where

$$
\begin{aligned}
\Pi_u =&\overline{\Pi}_1 + \sum_{i=2}^{6} \Pi_i,\\
\overline{\Pi}_1 =&2e_1^T P\bar{A}e_1 + 2e_1^T \bar{\rho} B_u Y[e_5 + e_{18}] + 2(1-\bar{\rho})e_1^T B_u Y e_{11} + 2e_1^T PB_0 e_9\\
&+ 2e_1^T PB_1 e_{10} + 2e_1^T PB_2 e_{17} + 2e_1^T PB_w e_{19} - \alpha e_1^T Pe_1,\\
\Lambda_u =&diag\{-2\kappa_1 P + \kappa_1^2 R_1, -2\kappa_2 P + \kappa_2^2 R_2, -2\kappa_3 P + \kappa_3^2 R_3, -2\kappa_4 P + \kappa_4^2 R_4, -2\kappa_5 P + \kappa_5^2 R_5\},\\
\overline{\Gamma}_{a_u} =&\begin{bmatrix}\tau_m \overline{\Gamma}_{a1}^T & \tau_M \overline{\Gamma}_{a1}^T & \tau_{Mm} \overline{\Gamma}_{a1}^T & \bar{\beta}\overline{\Gamma}_{a1}^T & \bar{d}\overline{\Gamma}_{a1}^T\end{bmatrix}^T,\\
\overline{\Gamma}_{b_u} =&\begin{bmatrix}\tau_m \overline{\Gamma}_{b1}^T & \tau_M \overline{\Gamma}_{b1}^T & \tau_{Mm} \overline{\Gamma}_{b1}^T & \bar{\beta}\overline{\Gamma}_{b1}^T & \bar{d}\overline{\Gamma}_{b1}^T\end{bmatrix}^T,\\
\overline{\Gamma}_{a1} =&\begin{bmatrix}\overline{\Gamma}_{a11} & \overline{\Gamma}_{a12}\end{bmatrix},\\
\overline{\Gamma}_{a11} =&\begin{bmatrix}PA & 0_{1\times 3} & \bar{\rho}B_u Y & 0_{1\times 3} & PB_0 & PB_1\end{bmatrix},\\
\overline{\Gamma}_{a12} =&\begin{bmatrix}(1-\bar{\rho})B_u Y & 0_{1\times 5} & PB_2 & \bar{\rho}B_u Y & PB_w\end{bmatrix},\\
\overline{\Gamma}_{b1} =&\begin{bmatrix}0_{1\times 4} & \delta B_u Y & 0_{1\times 5} & \delta B_u Y & 0_{1\times 6} & B_u Y & 0\end{bmatrix}, \quad \delta = \sqrt{\bar{\rho}(1-\bar{\rho})},
\end{aligned}
$$

where $P = L_1 diag\{P_1, P_2\} L_1^T$, $P_1 \in \mathbb{R}^{m\times m}, P_2 \in \mathbb{R}^{(n-m)\times(n-m)}$. *The other matrices are given in Theorem 3.1. The gain of the controller can be determined as follows:*

$$
\mathcal{K} = P_k^{-1} Y, P_k = (B_{u_0} L_2)^{-1} P_1 B_{u_0} L_2, \tag{3.27}
$$

in which B_{u_0} *and* L_2 *are defined in Assumption (A5).*

Proof. Let P and B_u be defined as follows based on Assumption (A5):

$$
P = L_1 \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} L_1^T \quad \text{and} \quad B_u = L_1 \begin{bmatrix} B_{u_0} \\ 0 \end{bmatrix} L_2.
$$

By applying Lemma 2.5, we can find a new variable P_k that satisfies $PB_u = B_u P_k$, from

$$
L_1 \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} L_1^T L_1 \begin{bmatrix} B_{u_0} \\ 0 \end{bmatrix} L_2 = L_1 \begin{bmatrix} B_{u_0} \\ 0 \end{bmatrix} L_2 P_k,
$$

we get

$$
P_k = (B_{u_0} L_2)^{-1} P_1 B_{u_0} L_2.
$$

From $PB_u = B_u P_k$, then we substitute $PB_u\mathcal{K} = B_u P_k \mathcal{K}$ and define $Y = P_k \mathcal{K}$, it follows from (3.1) that

$$
\begin{bmatrix} \Pi_u & * & * \\ \overline{\Gamma}_{a_u} & -\Lambda & * \\ \overline{\Gamma}_{b_u} & 0 & -\Lambda \end{bmatrix} < 0. \tag{3.28}
$$

In terms of the following inequality:

$$
(R_j - \kappa_j^{-1} P) R_j^{-1} (R_j - \kappa_j^{-1} P) \geq 0, (j = 1, 2, 3, 4, 5).
$$

One has that

$$-PR_j^{-1}P \leq -2\kappa_j P + \kappa_j^2 R_j.$$

By replacing $-PR_j^{-1}P$ with $-2\kappa_j P + \kappa_j^2 R_j$ in (3.28), it becomes evident that (3.25) is obtained. Thus, the proof is complete. □

The delayed GNNs (2.6) take on the following forms under the decentralized event-triggered scheme without cyber-attacks:

$$\begin{aligned}\dot{x}(t) =& \bar{A}x(t) + B_u\mathcal{K}[x(t-\beta(t)) + e(t)] + B_0 f(Wx(t)) + B_1 f(Wx(t-\tau(t))) \\ &+ B_2\int_{t-\eta_2(t)}^{t-\eta_1(t)} h(Wx(s))ds + B_w\omega(t), \quad \forall t \in [t_k h + \beta_{t_k}, t_{k+1}h + \beta_{t_{k+1}}].\end{aligned} \tag{3.29}$$

Following the same approach as the proof for Theorem 3.2, we can derive the following corollary. Furthermore, we also define the vectors as follows:

$$\begin{aligned}\xi(t) = \Bigg[& x^T(t),\ x^T(t-\tau_m),\ x^T(t-\tau(t)),\ x^T(t-\tau_M),\ x^T(t-\beta(t)),\ x^T(t-\bar{\beta}),\ f^T(Wx(t)), \\ & f^T(Wx(t-\tau(t))),\ h^T(Wx(t)),\ \frac{1}{\tau_m}\int_{t-\tau_m}^{t} x^T(u)du,\ \frac{1}{\tau_M}\int_{t-\tau_M}^{t} x^T(u)du,\ \frac{1}{\tau(t)-\tau_m}\int_{t-\tau(t)}^{t-\tau_m} x^T(u)du, \\ & \frac{1}{\tau_M-\tau(t)}\int_{t-\tau_M}^{t-\tau(t)} x^T(u)du,\ \int_{t-\eta_2(t)}^{t-\eta_1(t)} h^T(Wx(u))du,\ e^T(t),\ \omega^T(t)\Bigg]^T,\end{aligned}$$

$$e_i = \left[0_{n\times(i-1)n} \quad I_n \quad 0_{n\times(16-i)n}\right], i = 1, 2, ..., 16.$$

Corollary 3.3. *Assume that Assumptions (A1), (A2), (A4) and (A5) are satisfied. Then, for the given scalars* $\bar{\beta}, \bar{\rho}, \sigma, d_w, \tau_m, \tau_M, \eta_1, \eta_2, T, c_1, c_2$ *and* α*, the delayed GNNs without cyber attacks (3.29) under the state feedback controller are stochastic finite-time bounded regarding* (c_1, c_2, T, X, d_w)*, if there exist symmetric positive definite matrices* P*,* $Q_i, R_j (i = 1, 2, 3, j = 1, 2, 3, 4)$*,* M, S*, and positive diagonal matrices* G_1, G_2, G_3, G_5*, such that the conditions hold as follows:*

$$\begin{bmatrix}\widehat{\Pi}_u & * \\ \widehat{\Gamma}_{a_u} & -\widehat{\Lambda}_u\end{bmatrix} < 0, \tag{3.30}$$

$$e^{\alpha T}\left[\widehat{\Pi}_\lambda c_1 + d_w\lambda_{13}(1-e^{-\alpha T})\right] < \lambda_1 c_2, \tag{3.31}$$

where

$$\widehat{\Pi}_u = \sum_{i=1}^{6}\widehat{\Pi}_{ui},$$

$$\widehat{\Pi}_{u1} = 2e_1^T P\bar{A}e_1 + 2e_1^T B_u Y[e_5 + e_{15}] + 2e_1^T PB_0 e_7 + 2e_1^T PB_1 e_8 + 2e_1^T PB_2 e_{14} + 2e_1^T PB_w e_{16} - \alpha e_1^T Pe_1,$$

$$\widehat{\Pi}_{u2} = e_1^T(Q_1 + Q_2 + Q_3)e_1 - e^{\alpha\tau_m}e_2^T Q_1 e_2 - e^{\alpha\tau_M}e_4^T Q_2 e_4 - e^{\alpha\bar{\beta}}e_6^T Q_3 e_6,$$

$$\begin{aligned}
\widehat{\Pi}_{u3} =& -\frac{\tau_m}{\theta_{1a}}\Gamma_1^T R_1 \Gamma_1 - \frac{\tau_m}{\theta_{1b}}\widehat{\Gamma}_2^T R_1 \widehat{\Gamma}_2 - \frac{\tau_M}{\theta_{2a}}\Gamma_3^T R_2 \Gamma_3 - \frac{\tau_M}{\theta_{2b}}\widehat{\Gamma}_4^T R_2 \widehat{\Gamma}_4 - \frac{\tau_{Mm}}{\theta_{3a}}\Gamma_5^T R_3 \Gamma_5 - \frac{\tau_{Mm}}{\theta_{3b}}\widehat{\Gamma}_6^T R_3 \widehat{\Gamma}_6 \\
& - \frac{\tau_{Mm}}{\theta_{4a}}\Gamma_7^T R_3 \Gamma_7 - \frac{\tau_{Mm}}{\theta_{4b}}\widehat{\Gamma}_8^T R_3 \widehat{\Gamma}_8 \quad + e^{\alpha\bar{\beta}}\Big[- e_1^T R_4 e_1 + 2e_1^T (R_4 - L_1) e_5 + 2e_1^T L_1 e_6 \\
& + e_5^T(-2R_4 + L_1 + L_1^T) e_5 + 2e_5^T (R_4 - L_1) e_6 + e_6^T R_4 e_6 \Big], \\
\widehat{\Pi}_{u4} =& \eta_{21}^2 e_9^T M e_9 - e^{\alpha\eta_2} e_{14}^T M e_{14}, \\
\widehat{\Pi}_{u5} =& 2\widehat{\Gamma}_9^T G_1 \widehat{\Gamma}_{10} + \widehat{\Gamma}_{11}^T G_2 \widehat{\Gamma}_{12} + \widehat{\Gamma}_{13}^T G_3 \widehat{\Gamma}_{14} + \widehat{\Gamma}_{17}^T G_5 \widehat{\Gamma}_{18}, \\
\widehat{\Pi}_{u6} =& -\alpha e_{16}^T S e_{16} - e_{15}^T \Omega e_{15} + \sigma e_5^T \Omega e_5, \\
\widehat{\Pi}_{\lambda} =& \lambda_2 + \gamma_1\lambda_3 + \gamma_2\lambda_4 + \gamma_3\lambda_5 + \tau_m\gamma_5\lambda_7 + \tau_M\gamma_6\lambda_8 + \tau_{Mm}\gamma_7\lambda_9 + \bar{\beta}\gamma_8\lambda_{10} + \eta_{21}\gamma_{10}\lambda_{12}, \\
\widehat{\Lambda}_u =& diag\{-2\kappa_1 P + \kappa_1^2 R_1, -2\kappa_2 P + \kappa_2^2 R_2, -2\kappa_3 P + \kappa_3^2 R_3, -2\kappa_4 P + \kappa_4^2 R_4\}, \\
\widehat{\Gamma}_{a_u} =& \begin{bmatrix} \tau_m \widehat{\Gamma}_{a1}^T & \tau_M \widehat{\Gamma}_{a1}^T & \tau_{Mm} \widehat{\Gamma}_{a1}^T & \bar{\beta}\widehat{\Gamma}_{a1}^T \end{bmatrix}^T, \widehat{\Gamma}_{a1} = \begin{bmatrix} \widehat{\Gamma}_{a11} & \widehat{\Gamma}_{a12} \end{bmatrix}, \\
\widehat{\Gamma}_{a11} =& \begin{bmatrix} PA & 0_{1\times 3} & B_u Y & 0_{1\times 3} & PB_0 & PB_1 \end{bmatrix}, \quad \widehat{\Gamma}_{a12} = \begin{bmatrix} 0_{1\times 3} & PB_2 & B_u Y & PB_w \end{bmatrix}, \\
\widehat{\Gamma}_2 =& [\epsilon_{11} e_2^T + \epsilon_{12} e_1^T - \tau_m e_{10}^T]^T, \quad \widehat{\Gamma}_4 = [\epsilon_{21} e_4^T + \epsilon_{22} e_1^T - \tau_M e_{11}^T]^T, \\
\widehat{\Gamma}_6 =& [\epsilon_{31} e_3^T + \epsilon_{32} e_2^T - \tau_{Mm} e_{12}^T]^T, \quad \widehat{\Gamma}_8 = [\epsilon_{41} e_4^T + \epsilon_{42} e_3^T - \tau_{Mm} e_{13}^T]^T, \\
\widehat{\Gamma}_9 =& [e_7^T - e_1^T W^T F_M^T]^T, \quad \widehat{\Gamma}_{10} = [F_P W e_1 - e_7], \quad \widehat{\Gamma}_{11} = [e_8^T - e_3^T W^T F_M^T]^T, \\
\widehat{\Gamma}_{12} =& [F_P W e_3 - e_8], \quad \widehat{\Gamma}_{13} = [e_7^T - e_8^T - e_1^T W^T F_M^T + e_3^T W^T F_M^T]^T, \\
\widehat{\Gamma}_{14} =& [F_P W e_1 - F_P W e_3 - e_7 + e_8], \quad \widehat{\Gamma}_{17} = [e_9^T - e_1^T W^T H_M^T]^T, \quad \widehat{\Gamma}_{18} = [H_P W e_1 - e_9],
\end{aligned}$$

where $P = L_1 diag\{P_1, P_2\} L_1^T$, $P_1 \in \mathbb{R}^{m\times m}, P_2 \in \mathbb{R}^{(n-m)\times(n-m)}$. *The other matrices are given in Theorem 3.1. The gain of the controller can be determined as follows:*

$$\mathcal{K} = P_k^{-1} Y, P_k = (B_{u_0} L_2)^{-1} P_1 B_{u_0} L_2, \tag{3.32}$$

in which B_{u_0} *and* L_2 *are defined in Assumption (A5).*

Remark 3.2. The problem of NNs with a time-varying delay and cyber-attacks under the event-triggered framework using H_∞ control was investigated by Zha et al. in [22]. In [30], Liu et al. presented a state estimation method for T-S fuzzy neural networks under stochastic cyber-attacks and an event-triggered scheme. However, previous works mainly focused on the asymptotic stability of NNs and did not address finite-time stability for GNNs. Asymptotic stability refers to the long-term behavior of a system, whereas finite-time stability is concerned with short-term behavior. Thus, this article aims to tackle the finite-time bounded problem of GNNs with decentralized event-triggered communication and cyber-attacks. The proposed decentralized event-triggered scheme enhances system effectiveness and minimizes network transmissions, resulting in a more efficient and secure system.

4. Numerical examples

A simulation example is presented in this section to showcase the effectiveness and superiority of the theoretical method on the finite-time decentralized event-triggered feedback control problem. The parameters of the system (2.1) are given as follows:

$$\bar{A} = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & -0.1 & 0 \\ 0 & 0 & -0.1 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0.2 & 0.2 & 0.3 \\ -0.2 & 0.3 & 0.1 \\ -0.3 & -0.15 & -0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.1 & 0.2 & 0 \\ -0.1 & 0.2 & 0.1 \\ -0.2 & 0 & -0.3 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.15 & 0.3 & 0.15 \\ -0.15 & 0.25 & 0.15 \\ -0.3 & 0.1 & -0.2 \end{bmatrix}, \quad B_w = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad B_u = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},$$

$$W = \text{diag}\{1,1,1\},\ F_m = \text{diag}\{0,0,0\},\ F_P = \text{diag}\{0.03,0.06,0.03\},\ G_m = F_m,$$
$$G_p = \text{diag}\{0.03,0.06,0.03\},\ H_m = F_m,\ H_p = \text{diag}\{0.04,0.04,0.04\},$$
$$f(x) = \begin{bmatrix} \tanh(0.03x_1(t)) \\ \tanh(0.06x_2(t)) \\ \tanh(0.03x_3(t)) \end{bmatrix} \text{ and } h(x) = \begin{bmatrix} \tanh(0.04x_1(t)) \\ \tanh(0.04x_2(t)) \\ \tanh(0.04x_3(t)) \end{bmatrix}.$$

The function of cyber-attacks is given by $g(x) = \begin{bmatrix} \tanh(0.03x_1(t)) \\ \tanh(0.06x_2(t)) \\ \tanh(0.03x_3(t)) \end{bmatrix}$. Furthermore, the selected initial condition for generating the simulation results is given by $x(0) = \begin{bmatrix} -1.1 & -0.5 & 0.8 \end{bmatrix}^T$.

Two cases will be analyzed for delayed GNNs. The first case considers both the event-triggered scheme and the cyber-attacks in delayed GNNs (2.6). The second case, on the other hand, looks at the event-triggered scheme used in delayed GNNs without cyber-attacks (3.29).

Case I: In this case, we investigate the effects of both the event-triggered method and cyber-attacks on delayed GNNs (2.6). Assuming that the following parameter values are used: $c_1 = 2.1, T = 50, \alpha = 0.3, d_w = 0.5, \bar{\beta} = 0.5, \bar{d} = 0.5, \bar{\rho} = 0.1, \sigma = 0.1, \kappa_i = 1$ (for $i = 1,2,3,...,5$), $\tau_m = 0.25$, $\tau_M = 0.9$, $\tau(t) = 0.7|\sin(t)| + 0.25$, $\eta_1 = 0.1$, $\eta_2 = 0.7$, $\eta_1(t) = 0.4 + 0.3\sin(t)$, $\eta_2(t) = 0.5 + 0.2\sin(t)$, and an external disturbance given by $0.5e^{-0.5t}\sin(t)$. We solve the LMIs in Theorem 3.2 to obtain a feasible solution that guarantees finite-time boundedness with respect to (c_1, c_2, T, X, d_w).

Applying (3.25), (3.26) and (3.27), we derive the corresponding minimum allowable lower bounds (MALBs) of $c_2 = 5.1788$ and control gain as follows:

$$\mathcal{K} = \begin{bmatrix} -0.0235 & -0.0116 & 0.0236 \\ -0.0092 & -0.9716 & -0.0048 \\ 0.0142 & -0.0007 & -0.2283 \end{bmatrix}.$$

We show the effectiveness of our results in Case I. Figure 2 illustrates the state responses of $x(t)$ for the delayed GNNs (2.6) without $u(t)$. The responses of the state variable $x(t)$ and the time history of $x^T(t)Xx(t)$ in Case I of the delayed GNNs with cyber-attacks (2.6) are depicted in Figures 3 and 4, respectively. Furthermore, the computed MALBs of c_2 for different c_1 are presented in Table 2. From the table, we can see that the MALBs of c_2 from our results increase as c_1 increases. Based on the simulation results presented above, it has been determined that the event-triggered state feedback control scheme for delayed GNNs (2.6) is stochastically finite-time bounded within a specified time interval. This conclusion remains valid even in the presence of cyber-attacks and gains fluctuations.

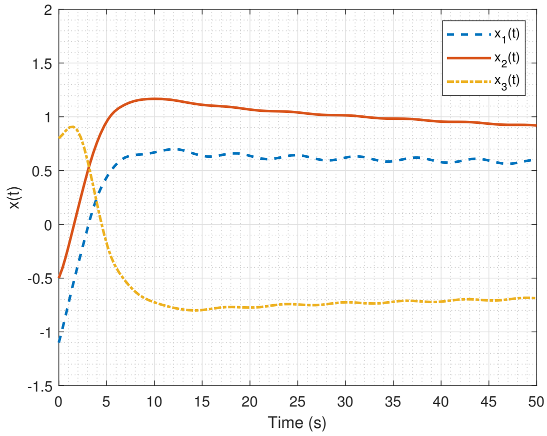

Figure 2. State responses of $x(t)$ for the GNNs (2.6) without $u(t)$.

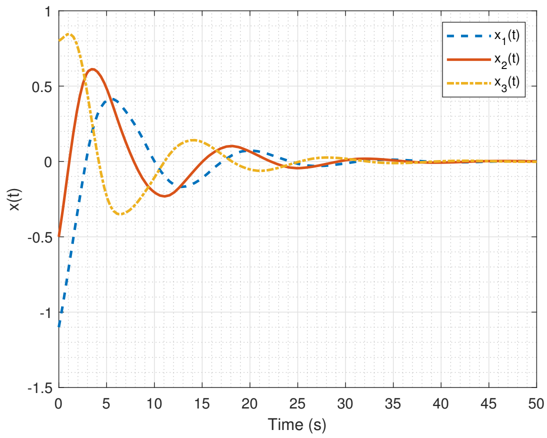

Figure 3. State responses of $x(t)$ for the GNNs with cyber-attacks (2.6) in Case I.

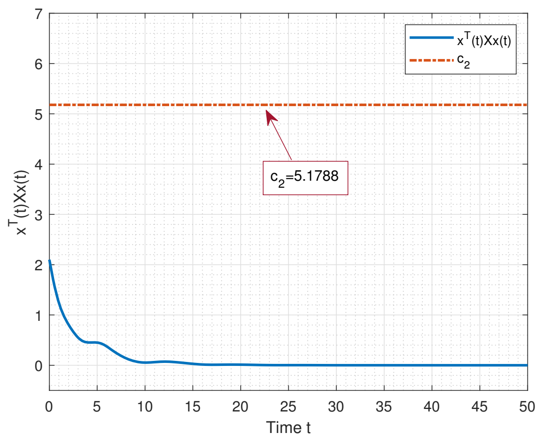

Figure 4. Time history of $x^T(t)Xx(t)$ for the GNNs with cyber-attacks (2.6) in Case I.

Table 2. MALBs of c_2 for different c_1.

c_1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1
c_2	3.1058	3.3361	3.5674	3.7970	4.0288	4.2617	4.4885	4.7190	4.9508	5.1788

Case II: This case focuses on delayed GNNs (3.29) under the event-triggered scheme without considering cyber-attacks. Assuming that the following parameter values are used: $c_1 = 2.1, T = 50, \alpha = 0.3, d_w = 0.5, \bar{\beta} = 0.5, \bar{d} = 0.5, \bar{\rho} = 0.1, \sigma = 0.1, \kappa_i = 1$ (for $i = 1, 2, 3, ..., 5$), $\tau_m = 0.25$, $\tau_M = 0.9$, $\tau(t) = 0.7|\sin(t)| + 0.25$, $\eta_1 = 0.1$, $\eta_2 = 0.7$, $\eta_1(t) = 0.4 + 0.3\sin(t)$, $\eta_2(t) = 0.5 + 0.2\sin(t)$, and an external disturbance given by $0.5e^{-0.5t}\sin(t)$. We solve the LMIs in Corollary 3.3 to obtain a feasible solution that guarantees finite-time boundedness with respect to (c_1, c_2, T, X, d_w).

Applying (3.30), (3.31) and (3.32), we derive the corresponding MALBs of $c_2 = 3.2378$ and control gain as follows

$$\mathcal{K} = \begin{bmatrix} -0.2941 & -0.1437 & -0.0008 \\ 0.1203 & -1.4131 & -0.2661 \\ 0.1263 & -0.0001 & -1.2606 \end{bmatrix}.$$

We present the effectiveness of our findings in Case II. The response of the state variable $x(t)$ and the time history of $x^T(t)Xx(t)$ in Case II of the delayed GNNs without cyber-attacks (3.29) are illustrated in Figures 5 and 6, respectively. Additionally, we provide the computed MALBs of c_2 for different c_1 in Table 3. The table reveals that as c_1 increases, the MALB of c_2 also increases according to our findings. The simulation results above confirm that the event-triggered state feedback control scheme for delayed GNNs without cyber-attacks (3.29) is stochastically finite-time bounded within a specified time interval.

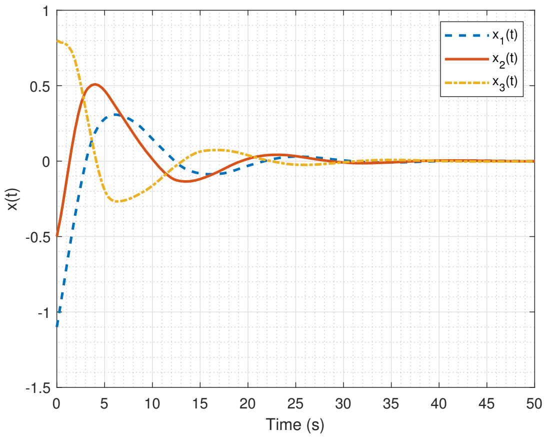

Figure 5. State responses of $x(t)$ for the GNNs without cyber-attacks (3.29) in Case II.

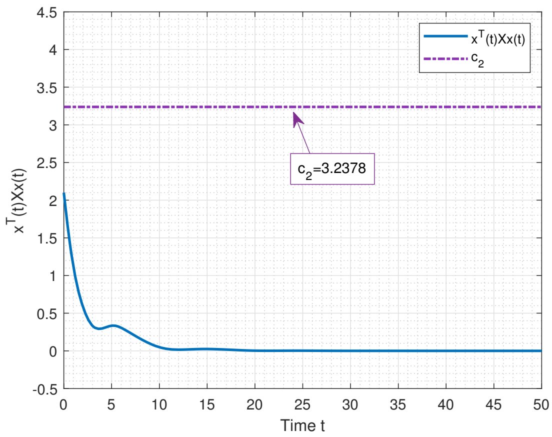

Figure 6. Time history of $x^T(t)Xx(t)$ for the GNNs without cyber-attacks (3.29) in Case II.

Table 3. MALBs of c_2 for different c_1.

c_1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1
c_2	1.9536	2.0963	2.2390	2.3817	2.5344	2.6670	2.8097	2.9524	3.0951	3.2378

Remark 4.1. Figure 5 illustrates the state responses of the GNNs, represented by the variable $x(t)$, in the absence of cyber-attacks. The graph displays three distinct curves, each corresponding to a specific experimental condition. The first curve represents the state response of the GNNs under normal operating conditions. The second curve represents the state response of the GNNs when subjected to external disturbances. The third curve demonstrates the state response of the GNNs when variations are introduced into the system. By examining these three curves, we can gain a comprehensive understanding of the GNNs' behavior under different experimental conditions without the presence of cyber-attacks.

5. Conclusions

This article proposes the decentralized event-triggered method and feedback controller to ensure the finite-time boundedness of GNNs with mixed interval time-varying delays and stochastic cyber-attacks. By the Lyapunov-Krasovskii stability theory, we apply the integral inequality with the exponential function to estimate the derivative of the LKFs. We also present new sufficient conditions in the form of linear matrix inequalities. The event-triggered approach reduces the network's resource utilization and transmission burden, while the random cyber-attacks are described applying Bernoulli distributed variables. A numerical example is provided to demonstrate the effectiveness and advantages of the proposed control scheme. Additionally, this research can be expanded in the future to include various dynamic systems, such as uncertain NNs [40], complex networks [41], neutral high-order Hopfield NNs [42], neutral-type NNs [43], quaternion-valued neural networks [44], spiking NNs [45], memristive NNs [46], stochastic memristive NNs [47] and synchronization of Lur'e Systems [48]. By focusing on these research directions, the proposed method can be further enhanced to achieve improved performance such as passivity [8], dissipativity [9], H_∞ [5, 22, 25, 26], and extended dissipative performances [12, 15, 16], even in the presence of cyber-attacks. These advancements will greatly contribute to the development of more robust and high-performing control systems specifically designed for networked applications.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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