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Research article

An innovative decision-making framework for supplier selection based on a hybrid interval-valued neutrosophic soft expert set

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Abstract: The best way to achieve sustainable construction is to choose materials with a smaller environmental impact. In this regard, specialists and architects are advised to take these factors into account from the very beginning of the design process. This study offers a framework for selecting the optimal sustainable building material. The core goal of this article is to depict a novel structure of a neutrosophic soft expert set hybrid called an interval-valued neutrosophic soft expert set for utilization in construction supply chain management to select a suitable supplier for a construction project. This study applies two different techniques. One is an algorithmic technique, and the other is set-theoretic. The first one is applied for the structural characterization of an interval-valued neutrosophic expert set with its necessary operators like union and OR operations. The second one is applied for the construction of a decision-making system with the help of pre-described operators. The main purpose of the algorithm is to be used in supply chain management to select a suitable supplier for construction. This paper proposes a new model based on interval-valued, soft expert and neutrosophic settings. In addition to considering these settings jointly, this model is more flexible and reliable than existing ones because it overcomes the obstacles of existing studies on neutrosophic soft set-like models by considering interval-valued conditions, soft expert settings and neutrosophic settings. In addition, an example is presented to demonstrate how the decision support system would be implemented in practice. In the end, analysis, along with benefits, comparisons among existing studies and flexibility, show the efficacy of the proposed structure.

Keywords: fuzzy set; soft set; soft expert set; neutrosophic set; interval-valued neutrosophic set; decision-making
Mathematics Subject Classification: 03B52, 03E72, 03E75

1. Introduction

Neutrosophy is a new branch of philosophy conceptualized by Smarandache [1–3] and presented as an extension of the intuitionistic fuzzy set and its related logical models. Fuzzy sets [4] ($\mathcal{F}$-sets) and intuitionistic fuzzy sets [5] ($\mathcal{I}f$-sets) are characterized with membership and non-membership functions, respectively. In some situations, $\mathcal{F}$-sets and $\mathcal{I}f$-sets are not appropriate for the description of an object because of the inconsistent and indeterminate information. Smarandache initiated the neutrosophic set ($\mathcal{N}$-set). He applied this structure to handle the problems of inconsistent, indeterminate and incomplete information. The truth, indeterminacy and false memberships in an $\mathcal{N}$-set are totally independent. For real-life applications, the $\mathcal{N}$-set and its operators must be identified. In other ways, it becomes very tough to use in different practical examples. Consequently, Wang et al. [6] conceptualized a single-valued neutrosophic set ($\mathcal{S}vn$-set) and gave its operations some properties. The work on $\mathcal{S}vn$-sets and its related structures in different fields has been going forward promptly [7, 8]. To deal with uncertain data and uncleared objects as a parameterization tool, a soft set ($\mathcal{S}$-set) has been offered by Molodtsov [9]. Maji et al. [10] embedded the $\mathcal{F}$-set in the $\mathcal{S}$-set to form a fuzzy soft set ($\mathcal{F}s$-set) and talked about its very interesting applications. Çağman et al. [11] presented its aggregation operators that describe the best techniques for decision-making and applied them to an example with uncertain data. Çelik and Yamak [12] applied an $\mathcal{F}s$-set to medical diagnosis by applying the very famous Sanchez method. Jun et al. [13] successfully applied this concept in classical algebras and introduced its different notions, like closed and p-ideals, with certain characteristics. Çağman and Karataş [14] extended in $\mathcal{F}s$-set by introducing the concept of non-membership degree and giving the structure of an intuitionistic fuzzy soft set ($\mathcal{I}$fs-set). This theory made it easy for many researchers to solve decision-making problem (DMP) with different methods by using its operations and characteristics. Khalid and Abbas [15] defined distance measures and operations for $\mathcal{I}$fs-sets and applied them in medical diagnosis by giving their application. The $\mathcal{I}$fs-set can only handle the data in which membership and non-membership degrees are defined, but it fails when information or data is/are imprecise, incomplete, indeterminate or inconsistent. To overcome this situation, Maji [16] brought about the idea of the neutrosophic soft set ($\mathcal{N}$s-set), which helped the researchers handle the situation of indeterminacy. Later on, Deli and Broumi [17] defined the relations and functions of $\mathcal{N}$s-sets and applied them in decision-making. Gorzalczany [18] introduced the structure of an interval-valued fuzzy set in order to handle interval nature in data. Atanassov and Gargov [19] conceptualized interval-valued intuitionistic fuzzy sets, and Wang et al. [20] gave an interval-valued neutrosophic set. By combining an interval-valued fuzzy set and a soft set, Yang et al. [21] envisioned an interval-valued fuzzy soft set to address the interval nature of data. Jiang et al. [22] extended the structure of the intuitionistic fuzzy soft set by introducing the interval-valued intuitionistic fuzzy soft set. They also conceptualized certain operations. Deli [23] gave the idea of the interval-valued neutrosophic soft set to address the neutral degree in interval-type data. We see from the above analysis that interval-valued soft set-like structures are useful for a single expert's opinion. But, some practical situations demand opinions from more than one expert. To tackle this situation, Alkhazaleh and Salleh [24] suggested a soft expert set. Ihsan et al. [25, 26] conceptualized convexity on soft and fuzzy soft expert sets with certain properties. Then, Alkhazaleh and Salleh [27] developed a fuzzy soft expert set to handle such a situation. Broumi and Smarandache [28] extended the work of Alkhazaleh and conceptualized a new model of intuitionistic fuzzy soft expert sets with applications in decision-making. After this, Şahin et al. [29], working on an expert set, introduced the structure of the neutrosophic soft expert set and gave its new operations. Therefore, there is a demand for an interval-valued neutrosophic expert soft set-like structure for multi-decisive opinions in literature. Figure 1 presents a clear analysis of the soft set and soft expert set structures. It shows the visible choice of mobile phones by using parameters in the soft set and alternate numerical characteristics in the soft expert set.

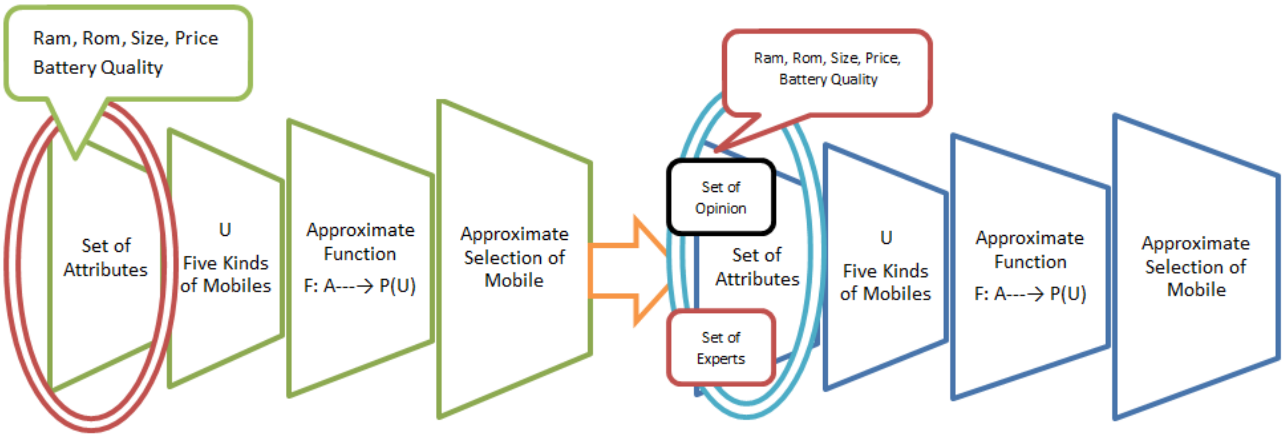

Figure 1. Comparison of soft set and soft expert set models.

1.1. Relevant literature

There are numerous recent studies in the literature using fuzzy and fuzzy-like structures for supplier selection (SS). The SS process in construction projects can be complex, and it often involves dealing with incomplete or uncertain information. In such situations, fuzzy logic can be used as a tool to help deal with this uncertainty and make more informed decisions. Fuzzy logic can be used to evaluate potential suppliers based on a set of criteria that may be imprecise or uncertain. For example, if the quality standards required for a project are not well defined, fuzzy logic can be used to evaluate potential suppliers based on their ability to meet a range of possible quality standards. By incorporating fuzzy logic into the SS process, project teams can make more informed decisions and reduce the risk of selecting a supplier that is not well suited to the project requirements. Hoseini et al. [30] developed a brand-new hybrid fuzzy best-worst method and fuzzy inference system (FBWM-FIS) model to improve the SS process in the construction industry. The ultimate weight of each aspect and criterion is determined in the first phase of this study by utilizing a fuzzy best-worst technique approach after criteria in three primary aspects have been determined. As compared to other models, the weighted fuzzy inference system's conclusions are more accurate in the next term, which is when the most environmentally friendly supplier is chosen. Lam et al. [31] explored a novel method called principal component analysis (PCA) to address the issue of choosing a material supplier for construction-related issues. The SS process in construction projects typically involves evaluating potential suppliers on factors such as cost, quality, delivery time and other factors that are important for the project's success. Multi-criteria decision-making (MCDM) techniques can be applied to help project teams make more informed decisions when selecting suppliers. In order to solve problems involving MCDM and organize the alternatives, Chen [32] proposed a technique that combined Grey Relational Analysis (GRA) techniques with the intuitionistic fuzzy entropy-based Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS) method. To determine a set of potential values for prioritizing SS, Rajesh and Ravi [33] employed GRA based on a linguistic evaluation of provider rating and attribute weightings. They used the six potential suppliers in the electronic supply chain as a case study and estimated the grey possibility values for SS before ranking the suppliers. To evaluate the effectiveness of suppliers at a Madrid-based construction company, Yazdani et al. [34] looked into an expanded variation of the combined compromise solution (CoCoSo-G) approach with grey numbers. In order to rank and pick the top green suppliers based on environmental and economic variables and then distribute the best order quantities among them, Kannan et al. [35] provided an integrated strategy that combines fuzzy multi-attribute utility theory with multi-objective programming. They initially used a fuzzy analytic hierarchy process (AHP) and a fuzzy technique for TOPSIS to identify the top green suppliers and analyze the significance of various criteria by taking opinions into account. They then took into account and developed other restrictions, including quality control, capacity and other objectives by using multi-objective linear programming. Through the creation and use of a suitable mathematical equation based on several criteria for the appropriate construction material suppliers, Aretoulis et al. [36] provided a standardized technique for choosing the best supplier. By utilizing the principles of virtual inventory management, workable materials management networks and a SS process, Safa et al. [37] developed an integrated construction material management (ICMM) model to address these issues. To address these issues, Yin et al. [38] suggested a novel dynamic MCDM approach for building projects with time constraints. They started by establishing the primary and supporting criteria for choosing green suppliers for building assignments. They then put forth a strategy that took into account the relationship between criteria, as well as the impact of the constructor's subjective choice and objective criteria information. The findings of the thorough evaluation of possible green suppliers were obtained by using a multi-target nonlinear programming (MTNP) model and operators for a fuzzy structure. Xiao et al. [39] made use of fuzzy soft set and fuzzy cognitive map (FCM) models in construction supply chain problems by using risk evaluation management. Chatterjee et al. [40] applied a decision-making approach to SS problems by using a rough approximation of a fuzzy soft set. Zhao et al. [41] suggested a new VIKOR technique and used it in SS. Rouyendegh et al. [42] conceptualized the intuitionistic fuzzy TOPSIS method and used it in green SS problems. A new picture fuzzy evaluation based on distance from average solution (EDAS) model for multiple-criterion group decision-making was presented by Zhang et al. [43] in 2019. To deal with competing qualities, they established the operational laws of picture fuzzy numbers and included them in the conventional EDAS model. Petrović et al. [44] contributed a discussion on supply chain management for the SS process and presented a fuzzy MCDM technique. For complex proportional assessment (COPRAS), Kumari and Mishra [45], in 2020, used an MCDM technique for SS. The issue of choosing green suppliers was then addressed by using the suggested strategy. Fuzzy sets and their linked models were used by Chen et al. [46] to investigate SS through MCDM. Chou and Chang [47] investigated SS using the MCDM method with a hybrid fuzzy set. For the purpose of choosing the most sustainable supplier, Chai et al. [48] created a novel fuzzy MCDM approach based on the combination of intuitionistic fuzzy sets, interval-valued fuzzy sets and cumulative prospect theory. To demonstrate the applicability of the suggested strategy, they looked at sustainable recycling SS for e-bike sharing. The validity and viability of the suggested strategy were confirmed by comparison with other approaches. Kaya [49] brought about the idea of new decision-making, called two-phase group decision-making. Actually, it was a combination of techniques called the picture fuzzy AHP (PF-AHP) and grey Measurement of Alternatives and Ranking According to Compromise Solution (MARCOS-G). By fusing manufacturing, administration and record management choices with the triple bottom line of sustainability and the circularity approach, Goodarzian et al. [50] looked into the citrus supply chain design issue. To construct a sustainable citrus closed-loop supply chain network (CLSCN), they consequently created a novel multi-objective mixed-integer linear programming (MILP) model. This model was used to formulate a multi-period, multi-echelon issue. They also employed the ϵ-constraint technique for simple issues to solve the proposed model. Additionally, they applied the Pareto envelope-based selection algorithm II and strength Pareto evolutionary algorithm II techniques to medium- and large-scale issues. After that, they effectively adjusted the algorithm's parameters using the Taguchi design technique. For a three-echelon air force satellite control network (AFSCN), Goodarzian et al. [51] suggested a mathematical model that simultaneously takes CO_2 emissions, coefficient of water, and time frame into account. The production-distribution-inventory-allocation problem was also given a bi-unbiased miscellaneous-number nonlinear indoctrination design. They simultaneously reduced the overall cost and CO_2 emissions by using the model. They also used an augmented-constraint method for small and medium-sized issues to explain the many-unbiased technique. Momenitabar et al. [52] designed a sustainable CLSCN to reduce the shortage that may develop at the time of the communication of manufactured items in the system while simultaneously taking the effects of standby dealers and resupply into account. To develop an effective SCLSCN resiliently, Momenitabar et al. [53] suggested a fuzzy multi-objective mixed-integer linear programming model. Then, in a fuzzy set's context, they used a robust fuzzy optimization strategy. To determine which of three well-known machine learning (ML) techniques would work best as an input for the suggested mathematical formula to determine the best standards for tactical, organizational and planned judgment rules, the researchers used these techniques. In order to identify Pareto-optimal solutions, they used a CPLEX solver for slight dimensions, along with two different systems. To create a sustainable bio-ethanol supply chain network, Momenitabar et al. [54] suggested a combined ML and computable optimization methodology. To forecast the demand for bioethanol, they used different ML techniques. By comparing the three approaches' performance metrics, they determined that the random forest method was the best one for predicting the demand for bioethanol as model inputs. Then, so as to fulfill the stability necessities established by the three objective functions, they suggested a mixed-integer linear programming technique. In order to illustrate the usefulness of the suggested strategy, they gave a case study. Table 1 represents a review of the existing papers.

Table 1. Review of existing papers on supplier selection.

Contributors	Structures	Models	Multi-decisive opinions
Huseini et al. [30]	$\mathfrak{F}$-set	FBWM-FIS	×
Lam et al. [31]	$\mathfrak{F}$-set	PCA	×
Chen [32]	$\mathfrak{I}$f-set	GRA and TOPSIS	×
Rajesh and Ravi [33]	$\mathfrak{F}$-set	GRA	×
Yazdani et al. [34]	$\mathfrak{F}$-set	CoCoSo-G	×
Kannan et al. [35]	$\mathfrak{F}$-set	Fuzzy AHP and TOPSIS	×
Artoulis et al. [36]	$\mathfrak{F}$-set	Multiple criteria	×
Safa et al. [37]	$\mathfrak{F}$-set	ICMM	×
Yin et al. [38]	$\mathfrak{I}$vif-set	MTNP and IVIFGWHM	×
Xiao et al. [39]	$\mathfrak{F}$s-set	FCM	×
Chatterjee et al. [40]	$\mathfrak{F}$s-set	Decision-making method	×
Zhao et al. [41]	$\mathfrak{I}$f-set	VIKOR	×
Rouyendegh et al. [42]	$\mathfrak{I}$f-set	TOPSIS	×
Zhang et al. [43]	$\mathfrak{P}$f-set	EDAS	×
Petrovic et al. [44]	$\mathfrak{F}$-set	MCDM	×
Kumari and Mishra et al. [45]	$\mathfrak{I}$f-set	COPRAS	×
Chen et al. [46]	$\mathfrak{F}$-set	MCDM	×
Kaya et al. [49]	$\mathfrak{P}$f-set	FP-AHP and MARCOS-G	×
Goodarzian et al. [50]	Classical approach	MILP and ϵ-constrain	×
Goodarzian et al. [51]	Classical approach	Augmented ϵ-constraint	×
Momenitaba et al. [52]	Classical approach	SCLSCN	×
Momenitaba et al. [53]	$\mathfrak{F}$-set	Fuzzy robust Optimization	×
Momenitaba et al. [54]	Classical approach	ML and quantitative optimization	×

1.2. Motivational aspects of proposed study

A careful examination of the SS literature mentioned above reveals that no model is presented that is capable of addressing all of the difficulties listed below as a single model:

1. **Ambiguousness of decision-makers**: In some cases, decision-making requires a more strategic approach than just selecting the best available goods or services. In these cases, what is best may depend on several parameters. One such case may occur when the decision-makers are unsure about the supplier's selection, and they give their opinion in the form of linguistic terms that need to be converted to neutrosophic form, i.e., membership, non-membership and neutral degrees, for approximating suppliers based on chosen parameters to deal with approximation-based vagueness.
2. **Rating of the alternatives**: Due to the fact that multi-attribute decision-making (MADM) plays a significant role in the SS process, using a thorough cost-benefit analysis and user preference survey, MADM determines the best option from a limited number of viable options based on multiple easily conflicting features. It has been used to solve a variety of real-world issues and including the evaluation of learning management systems, the choice of project portfolios, the planning of resources for electric utilities, economics and military affairs. Traditional MADM

approaches, however, are unable to resolve these types of issues because, in a real-world decision-making scenario, the information regarding the ranking of the alternatives with respect to the qualities cannot be assessed due to an unreliable source of information.

3. **Multi-decisive opinions**: The above-described soft set-like structures for the SS process are constructed for single expert opinions. So, these soft-like structures are unable to deal with multi-decisive opinions.

After combining the literature reviews from Part 1, Subsection 1.1 and this part, it is clear that the existing research has overlooked some issues that DMP may address when handling SS. These anticipated difficulties are listed below:

1. How can the ambiguous and uncertain features of the criteria and sub-criteria that are used to scrutinize suppliers for selection under the conditions of multi-decisive opinion be handled?
2. How can experts use information-based opinions of the interval and three-dimensional types to evaluate the criteria for SS in a trustworthy and consistent manner?
3. Consider a scenario in which experts request a specific order that enables them to express their thoughts on their own regarding the triplet of certainty, indeterminacy and untruth, while simulating the SS process. How can a SS procedure handle such a situation?
4. How can we handle the judgments of different experts using a single model?

The $\mathfrak{I}$vnse-set, i.e., the proposed model, is a brand-new structure that not only generalizes the pre-existing models, but also resolves the issues described above. The neutrosophic numbers of the $\mathfrak{I}$vnse-set are given an expert opinion in this structure to determine their degree of uncertainty. MCDM is a field that deals with the choice of the best substitute among a collection of available substitutes based on several standards. Fuzzy MCDM deals with choice-production difficulties in which the facts available are uncertain or vague. Fuzzy sets have been widely used in MCDM to deal with uncertainty, but they are not suitable for situations in which the information is incomplete or indeterminate. In such situations, interval-valued neutrosophic sets ($\mathfrak{I}$vns-sets) can be used to handle incomplete and indeterminate information. An $\mathfrak{I}$vnse-set is an extension of the $\mathfrak{I}$vns-set, which allows for the involvement of multiple experts in the decision-making process. It combines the concepts of interval-valued neutrosophic sets and soft computing to handle imperfect, unclassified and uncertain data in a more efficient way. The $\mathfrak{I}$vnse-set provides a more complete demonstration of the specialist‘s opinion than traditional methods by allowing for the incorporation of soft information in the form of linguistic terms, such as “very good” or “somewhat bad.” The $\mathfrak{I}$vnse-set approach is particularly useful when there is a lack of clear-cut data and experts’ opinions are required to make informed decisions. Given that complex decision-making situations often involve incomplete or ambiguous information, the application of an $\mathfrak{I}$vnse-set in MCDM offers a more adaptable and thorough approach to decision-making under uncertainty. This makes it a more flexible and all-encompassing paradigm for handling ambiguous data with care. In order to provide a more suitable parameterization tool that can represent the problem parameters in a more thorough and complete manner, the stated structure is therefore presented. Additionally, this has the benefit of letting customers access all expert opinions in a single model without requiring them to perform any additional laborious tasks. This research article involves the following valuable contributions:

1. The types of basic notions and elementary algebraic operations that are used in the construction of an $\mathfrak{I}$vnse-set are presented. Examples and their applications help to illustrate how these notions

and operations can be used to describe real-world phenomena.

2. An attempt has been made to understand the nature of attributes and sub-attributes and their effective use in supply chain management. In addition, the attributes used by the construction project owners are first analyzed by their operational role, and then their values are used with the help of suitable algebraic techniques.
3. To evaluate the strength of the suggested algorithm, a numerical problem-based scenario has been developed that is related to real-world business problems.
4. A structural comparison of different approaches is presented. In particular, a preliminary comparison of the proposed approach with some existing approaches is made. The advantages are judged through these comparisons.

The organization of the article is as follows. Section 2 is about the description of some healthy definitions, like a neutrosophic set, interval-valued neutrosophic set, neutrosophic soft set, etc. Section 3 is the main part of the paper, containing the definition and its relevant characteristics. Applications related to the DMP are discussed with numerical examples in Section 4. Comparative analysis is done in Section 5. Sections 6 and 7 have been constructed to respectively encompass the discussion and conclusion.

2. Preliminaries

This part shows the basic definitions from the literature. In this part, the set of parameters will be denoted by Γ and Ω as a universe of discourse, and the set of experts that is given by Λ and Υ will be a set of opinions, where $\ddot{\mathbb{M}} \subseteq \tau = \Gamma \times \Lambda \times \Upsilon$. $P(\Omega)$ will be used as a power set. An interval-valued neutrosophic set is a type of neutrosophic set that can similarly be used for scientific issues just as a neutrosophic set, and it can be regarded as follows:

Definition 2.1. [20] Let int$[0,1]$ be the collection of all closed subsets of $[0,1]$. Then, an $\mathcal{I}$vn-set is a set $I = \{\langle \dot{\varkappa} : \mu_I(\dot{\varkappa}), \nu_I(\dot{\varkappa}), \omega_I(\dot{\varkappa})\rangle, \dot{\varkappa} \in \Omega\}$ such that $\mu_I(\dot{\varkappa}) : \Omega \to [0,1]$, $\nu_I(\dot{\varkappa}) : \Omega \to [0,1]$, and $\omega_I(\dot{\varkappa}) : \Omega \to [0,1]$, satisfying $0 \leq sup\mu_I(\dot{\varkappa}) + sup\nu_I(\dot{\varkappa}) + sup\omega_I(\dot{\varkappa}) \leq 3\ \forall \dot{\varkappa} \in \Omega$. Here, $\mu_I(\dot{\varkappa}), \nu_I(\dot{\varkappa}), \omega_I(\dot{\varkappa})$ have the same meanings as described in the definition of a neutrosophic set. For simplicity, suppose that $\mu_I(\dot{\varkappa}) = [\mu_I^-(\dot{\varkappa}), \mu_I^+(\dot{\varkappa})]$, $\nu_I(\dot{\varkappa}) = [\nu_I^-(\dot{\varkappa}), \nu_I^+(\dot{\varkappa})]$ and $\omega_I(\dot{\varkappa}) = [\omega_I^-(\dot{\varkappa}), \omega_I^+(\dot{\varkappa})]$; then,
$I = \{\langle \dot{\varkappa} : [\mu_I^-(\dot{\varkappa}), \mu_I^+(\dot{\varkappa})], [\nu_I^-(\dot{\varkappa}), \nu_I^+(\dot{\varkappa})], [\omega_I^-(\dot{\varkappa}), \omega_I^+(\dot{\varkappa})]\rangle, \dot{\varkappa} \in \Omega\}$, satisfying $0 \leq sup\mu_I^+(\dot{\varkappa}) + sup\nu_I^+(\dot{\varkappa}) + sup\omega_I^+(\dot{\varkappa}) \leq 3\ \forall \dot{\varkappa} \in \Omega$.

Definition 2.2. [9] A soft set S is defined by an approximate function $\beta_S : \theta \to P(\Omega)$, which is defined by approximate elements $\beta_S(\hat{s})$ for all members $\hat{s}$ of θ, a subset of parameters. Actually, it is called mapping, which is considered from the parameters to $P(\Omega)$, and it is not considered an ordinary set because of consists of a parameterized family of subsets of Ω. If δ belongs to the parameter set, then $\beta_S(\delta)$ is taken as a set of δ, and it is called an approximate element of the soft set.

Definition 2.3. [23] Let the initial universal set be $\mathcal{R}$ and $\mathcal{D}$ be a subset of set parameters $\mathcal{P}$. Then, the $\mathcal{I}$vns-set can be treated as a pair $(N, \mathcal{D})$ with N as a mapping $N : \mathcal{D} \to IVNS(\mathcal{R})$, and IVNS$(\mathcal{R})$ is represented here as a collection of all interval-valued neutrosophic subsets of $\mathcal{R}$.

Alkhazaleh and Salleh [24] developed a new structure of soft expert sets by combining soft sets and expert sets. This is very useful in different fields of mathematics, especially in decision-making.

Definition 2.4. [24] A soft expert set ϖ is characterized by an approximate function $\Phi_{\varpi} : \ddot{\mathbb{M}} \to P(\Omega)$ which is defined by approximate elements $\Phi_{\varpi}(\hat{v})$ for all members $\hat{v}$ of $\ddot{\mathbb{M}}$, where $\ddot{\mathbb{M}} \subseteq \tau$.

3. Methodology

Since MADM is an important part of the SS process, an intelligent MADM approach has been used for SS. For this purpose, a new structure of interval-valued neutrosophic soft expert sets is applied as a methodology. So, for the purpose of describing the methodology, certain notions of these structures are discussed here. In Figure 2, the stages of the adopted approach are briefly displayed.

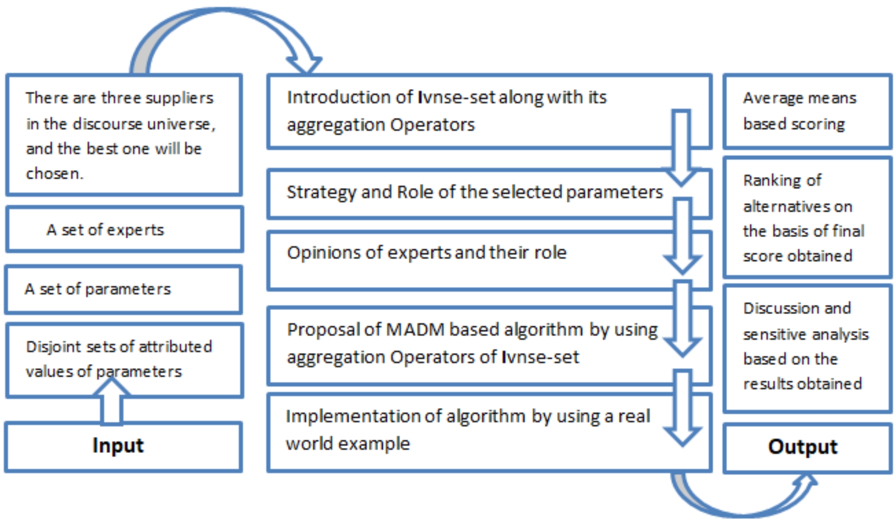

Figure 2. Graphical representation of suggested methodology.

3.1. Interval valued neutrosophic soft expert set

This section defines the $\mathfrak{I}$vnse-set and provides examples of its aggregation methods.

Definition 3.1. (Interval-valued neutrosophic soft expert set)
A pair $(\mathfrak{I}, \mathbb{O})$ is called as an interval-valued neutrosophic soft expert set over $\mathfrak{R}$ if $\mathfrak{I} : \mathbb{O} \to IVNF^{\mathfrak{R}}$, where $\mathbb{O} \subseteq \mathfrak{I} = \mathcal{W} \times \mathcal{R} \times \mathbb{L}$, $\mathcal{W}$= set of attributes, $\mathcal{R}$ = set of specialists and $\mathbb{L}$ = set of conclusions. For understanding, we have used $\mathbb{L} = \{0 = disagree, 1 = agree\}$ as the conclusion set and $IVNF^{\mathfrak{R}}$ represents the collection of all $\mathfrak{I}$vn-subsets of $\mathfrak{R}$.

Example 3.2. Consider a multinational industrial business strategy to evaluate its mass-produced things through external assessors. Consider $\mathfrak{R} = \{m_1, m_2\}$ as a set of products and $\mathcal{W} = \{\mu_1, \mu_2, \mu_3\}$ as distinct attributes: μ_1 = humble to consume, μ_2 = nature, μ_3 = modest.
Now, $\mathcal{S} = \mathcal{W} \times \mathcal{R} \times \mathbb{L}$:

$$\mathcal{S} = \left\{ \begin{array}{l} (\mu_1, s, 0), (\mu_1, s, 1), (\mu_1, t, 0), (\mu_1, t, 1), (\mu_1, u, 0), (\mu_1, u, 1), (\mu_2, s, 0), (\mu_2, s, 1), (\mu_2, t, 0), \\ (\mu_2, t, 1), (\mu_2, u, 0), (\mu_2, u, 1)(\mu_3, s, 0), (\mu_3, s, 1), (\mu_3, t, 0), (\mu_3, t, 1), (\mu_3, u, 0), (\mu_3, u, 1), \end{array} \right\},$$

and $\mathcal{R} = \{s, t, u, \}$ is a set of specialists. The following choices have been made by the specialists:

$$\partial_1 = \partial(\mu_1, s, 1) = \left\{ \frac{m_1}{\langle [0.4, 0.5], [0.1, 0.2], [0.7, 0.8] \rangle}, \frac{m_2}{\langle [0.3, 0.4], [0.3, 0.5], [0.5, 0.6] \rangle} \right\},$$

$$\partial_2 = \partial(\mu_1, t, 1) = \left\{ \frac{m_1}{\langle [0.1, 0.2], [0.7, 0.9], [0.4, 0.5] \rangle}, \frac{m_2}{\langle [0.8, 0.9], [0.5, 0.6], [0.5, 0.7] \rangle} \right\},$$

$$\partial_3 = \partial(\mu_1, u, 1) = \left\{ \frac{m_1}{\langle [0.7, 0.8], [0.3, 0.4], [0.1, 0.3] \rangle}, \frac{m_2}{\langle [0.5, 0.6], [0.6, 0.7], [0.2, 0.6] \rangle} \right\},$$

$$\partial_4 = \partial(\mu_2, s, 1) = \left\{ \frac{m_1}{\langle [0.3, 0.4], [0.1, 0.2], [0.7, 0.8] \rangle}, \frac{m_2}{\langle [0.4, 0.5], [0.8, 0.9], [0.1, 0.3] \rangle} \right\},$$

$$\partial_5 = \partial(\mu_2, t, 1) = \left\{ \frac{m_1}{\langle [0.4, 0.5], [0.6, 0.7, [0.6, 0.7]] \rangle}, \frac{m_2}{\langle [0.8, 0.9], [0.7, 0.8], [0.2, 0.6] \rangle} \right\},$$

$$\partial_6 = \partial(\mu_2, u, 1) = \left\{ \frac{m_1}{\langle [0.5, 0.6], [0.7, 0.8], [0.2, 0.3] \rangle}, \frac{m_2}{\langle [0.3, 0.6], [0.6, 0.8], [0.2, 0.3] \rangle} \right\},$$

$$\partial_7 = \partial(\mu_3, s, 1) = \left\{ \frac{m_1}{\langle [0.6, 0.7], [0.4, 0.5], [0.4, 0.5] \rangle}, \frac{m_2}{\langle [0.7, 0.9], [0.4, 0.6], [0.2, 0.3] \rangle} \right\},$$

$$\partial_8 = \partial(\mu_3, t, 1) = \left\{ \frac{m_1}{\langle [0.2, 0.3], [0.4, 0.5], [0.3, 0.4] \rangle}, \frac{m_2}{\langle [0.6, 0.7], [0.1, 0.9], [0.5, 0.7] \rangle} \right\},$$

$$\partial_9 = \partial(\mu_3, u, 1) = \left\{ \frac{m_1}{\langle [0.7, 0.8], [0.6, 0.7], [0.1, 0.3] \rangle}, \frac{m_2}{\langle [0.3, 0.5], [0.2, 0.3], [0.2, 0.3] \rangle} \right\},$$

$$\partial_{10} = \partial(\mu_1, s, 0) = \left\{ \frac{m_1}{\langle [0.3, 0.4], [0.1, 0.2], [0.2, 0.6] \rangle}, \frac{m_2}{\langle [0.4, 0.5], [0.5, 0.6], [0.1, 0.3] \rangle} \right\},$$

$$\partial_{11} = \partial(\mu_1, t, 0) = \left\{ \frac{m_1}{\langle [0.8, 0.9], [0.4, 0.5], [0.3, 0.4] \rangle}, \frac{m_2}{\langle [0.7, 0.8], [0.2, 0.3], [0.2, 0.4] \rangle} \right\},$$

$$\partial_{12} = \partial(\mu_1, u, 0) = \left\{ \frac{m_1}{\langle [0.7, 0.8], [0.5, 0.7], [0.4, 0.5] \rangle}, \frac{m_2}{\langle [0.8, 0.9], [0.6, 0.7], [0.2, 0.7] \rangle} \right\},$$

$$\partial_{13} = \partial(\mu_2, s, 0) = \left\{ \frac{m_1}{\langle [0.8, 0.9], [0.4, 0.5], [0.5, 0.6] \rangle}, \frac{m_2}{\langle [0.4, 0.5], [0.7, 0.9], [0.2, 0.5] \rangle} \right\},$$

$$\partial_{14} = \partial(\mu_2, t, 0) = \left\{ \frac{m_1}{\langle [0.1, 0.2], [0.4, 0.5], [0.7, 0.8] \rangle}, \frac{m_2}{\langle [0.7, 0.8], [0.4, 0.5], [0.2, 0.3] \rangle} \right\},$$

$$\partial_{15} = \partial(\mu_2, u, 0) = \left\{ \frac{m_1}{\langle [0.1, 0.2], [0.6, 0.7], [0.4, 0.5] \rangle}, \frac{m_2}{\langle [0.6, 0.8], [0.4, 0.7], [0.1, 0.3] \rangle} \right\},$$

$$\partial_{16} = \partial(\mu_3, s, 0) = \left\{ \frac{m_1}{\langle [0.6, 0.7], [0.3, 0.4], [0.2, 0.3] \rangle}, \frac{m_2}{\langle [0.4, 0.5], [0.3, 0.4], [0.2, 0.7] \rangle} \right\},$$

$$\partial_{17} = \partial(\mu_3, t, 0) = \left\{ \frac{m_1}{\langle [0.2, 0.7], [0.7, 0.8], [0.2, 0.6] \rangle}, \frac{m_2}{\langle [0.1, 0.9], [0.6, 0.8], [0.2, 0.3] \rangle} \right\},$$

$$\partial_{18} = \partial(\mu_3, u, 0) = \left\{ \frac{m_1}{\langle [0.5, 0.6], [0.3, 0.7], [0.3, 0.4] \rangle}, \frac{m_2}{\langle [0.6, 0.7], [0.1, 0.5], [0.1, 0.3] \rangle} \right\}.$$

The $\mathfrak{I}$vnse-set is formed as

$$(\partial, \mathcal{S}) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{ \frac{m_1}{\langle [0.4,0.5],[0.1,0.2],[0.7,0.8]\rangle}, \frac{m_2}{\langle [0.3,0.4],[0.3,0.5],[0.5,0.6]\rangle} \right\}\right), \\ \left((\mu_1, t, 1), \left\{ \frac{m_1}{\langle [0.1,0.2],[0.7,0.9],[0.4,0.5]\rangle}, \frac{m_2}{\langle [0.8,0.9],[0.5,0.6],[0.5,0.7]\rangle} \right\}\right), \\ \left((\mu_1, u, 1), \left\{ \frac{m_1}{\langle [0.7,0.8],[0.3,0.4],[0.1,0.3]\rangle}, \frac{m_2}{\langle [0.5,0.6],[0.6,0.7],[0.2,0.6]\rangle} \right\}\right), \\ \left((\mu_2, s, 1), \left\{ \frac{m_1}{\langle [0.3,0.4],[0.1,0.2],[0.7,0.8]\rangle}, \frac{m_2}{\langle [0.4,0.5],[0.6,0.9],[0.1,0.3]\rangle} \right\}\right), \\ \left((\mu_2, t, 1), \left\{ \frac{m_1}{\langle [0.4,0.5],[0.6,0.7],[0.6,0.7]\rangle}, \frac{m_2}{\langle [0.8,0.9],[0.7,0.8],[0.2,0.6]\rangle} \right\}\right), \\ \left((\mu_2, u, 1), \left\{ \frac{m_1}{\langle [0.5,0.6],[0.7,0.8],[0.2,0.3]\rangle}, \frac{m_2}{\langle [0.3,0.6],[0.4,0.5],[0.8,0.9]\rangle} \right\}\right), \\ \left((\mu_3, s, 1), \left\{ \frac{m_1}{\langle [0.2,0.7],[0.8,0.9],[0.4,0.5]\rangle}, \frac{m_2}{\langle [0.7,0.9],[0.1,0.2],[0.2,0.3]\rangle} \right\}\right), \\ \left((\mu_3, t, 1), \left\{ \frac{m_1}{\langle [0.2,0.3],[0.4,0.5],[0.7,0.9]\rangle}, \frac{m_2}{\langle [0.6,0.7],[0.1,0.9],[0.5,0.7]\rangle} \right\}\right), \\ \left((\mu_3, u, 1), \left\{ \frac{m_1}{\langle [0.7,0.8],[0.6,0.7],[0.1,0.3]\rangle}, \frac{m_2}{\langle [0.3,0.5],[0.2,0.3],[0.2,0.3]\rangle} \right\}\right), \\ \left((\mu_1, s, 0), \left\{ \frac{m_1}{\langle [0.3,0.4],[0.7,0.9],[0.5,0.6]\rangle}, \frac{m_2}{\langle [0.4,0.5],[0.5,0.6],[0.1,0.3]\rangle} \right\}\right), \\ \left((\mu_1, t, 0), \left\{ \frac{m_1}{\langle [0.8,0.9],[0.4,0.5],[0.3,0.4]\rangle}, \frac{m_2}{\langle [0.7,0.8],[0.2,0.3],[0.2,0.4]\rangle} \right\}\right), \\ \left((\mu_1, u, 0), \left\{ \frac{m_1}{\langle [0.7,0.8],[0.5,0.7],[0.4,0.5]\rangle}, \frac{m_2}{\langle [0.8,0.9],[0.6,0.7],[0.2,0.7]\rangle} \right\}\right), \\ \left((\mu_2, s, 0), \left\{ \frac{m_1}{\langle [0.8,0.9],[0.4,0.5],[0.5,0.6]\rangle}, \frac{m_2}{\langle [0.4,0.5],[0.7,0.9],[0.2,0.5]\rangle} \right\}\right), \\ \left((\mu_2, t, 0), \left\{ \frac{m_1}{\langle [0.1,0.2],[0.4,0.5],[0.7,0.8]\rangle}, \frac{m_2}{\langle [0.7,0.8],[0.4,0.5],[0.2,0.3]\rangle} \right\}\right), \\ \left((\mu_2, u, 0), \left\{ \frac{m_1}{\langle [0.1,0.2],[0.6,0.7],[0.4,0.5]\rangle}, \frac{m_2}{\langle [0.6,0.8],[0.4,0.7],[0.1,0.3]\rangle} \right\}\right), \\ \left((\mu_3, s, 0), \left\{ \frac{m_1}{\langle [0.6,0.7],[0.3,0.4],[0.2,0.3]\rangle}, \frac{m_2}{\langle [0.4,0.5],[0.3,0.4],[0.2,0.7]\rangle} \right\}\right), \\ \left((\mu_3, t, 0), \left\{ \frac{m_1}{\langle [0.2,0.7],[0.7,0.8],[0.2,0.6]\rangle}, \frac{m_2}{\langle [0.1,0.9],[0.6,0.8],[0.2,0.3]\rangle} \right\}\right), \\ \left((\mu_3, u, 0), \left\{ \frac{m_1}{\langle [0.5,0.6],[0.3,0.7],[0.3,0.4]\rangle}, \frac{m_2}{\langle [0.6,0.7],[0.3,0.5],[0.2,0.3]\rangle} \right\}\right), \end{array} \right\}.$$

Definition 3.3. (Interval-valued neutrosophic soft expert subset)
An $\mathfrak{I}$vnse-set $(\partial_1, \mathbb{O}) \subseteq (\partial_2, \mathbb{P})$ on $\mathfrak{R}$ if $\mathbb{O} \subseteq \mathbb{P}$ and $\forall\ n \in \mathbb{O}, \partial_1(n)$ is $\mathfrak{I}$vnse-subset $\partial_2(n)$, i.e., $infT_{\mathbb{O}}(n) \leq infT_{\mathbb{P}}(n)$,$supT_{\mathbb{O}}(n) \leq supT_{\mathbb{P}}(n)$, $infI_{\mathbb{O}}(n) \geq infI_{\mathbb{P}}(n)$, $supI_{\mathbb{O}}(n) \geq supT_{\mathbb{P}}(n)$, $infF_{\mathbb{O}}(n) \geq infF_{\mathbb{P}}(n)$, $supF_{\mathbb{O}}(n) \geq supF_{\mathbb{P}}(n)$.

Definition 3.4. The union of $(\Xi_1, \mathbb{O})$ and $(\Xi_2, \mathbb{P})$ over $\mathfrak{R}$ is $(\Xi_3, \mathbb{L})$ with $\mathbb{L} = \mathbb{O} \cup \mathbb{P}$, defined as

$$\Xi_3(\ddot{o}) = \begin{cases} \Xi_1(\ddot{o}) & ; \ddot{o} \in \mathbb{O} - \mathbb{P}, \\ \Xi_2(\ddot{o}) & ; \ddot{o} \in \mathbb{P} - \mathbb{O}, \\ \cup(\Xi_1(\ddot{o}), \Xi_2(\ddot{o})) & ; \ddot{o} \in \mathbb{O} \cap \mathbb{P}, \end{cases}$$

where $\cup(\Xi_1(\ddot{o}), \Xi_2(\ddot{o})) =$
$\{\langle \ddot{o},\ \max\ \{\mu_1(\ddot{o}), \mu_2(\ddot{o})\},\ \min\ \{\nu_1(\ddot{o}), \nu_2(\ddot{o})\},\ \min\ \{\omega_1(\ddot{o}), \omega_2(\ddot{o})\}\rangle\ :\ \ddot{o} \in \mathfrak{R}\}$. In other words, $\cup(\Xi_1(\ddot{o}), \Xi_2(\ddot{o})) =\{\langle[max(infT_{\mathbb{O}}(\ddot{o}), infT_{\mathbb{P}}(\ddot{o})), max(supT_{\mathbb{O}}(\ddot{o}), supT_{\mathbb{P}}(\ddot{o}))],$

$[\min(\inf I_{\mathbb{O}}(\ddot{o}), \inf I_{\mathbb{P}}(\ddot{o})), \min(\sup I_{\mathbb{O}}(\ddot{o}), \sup I_{\mathbb{P}}(\ddot{o}))]$,
$[\min(\inf F_{\mathbb{O}}(\ddot{o}), \inf F_{\mathbb{P}}(\ddot{o})), \min(\sup F_{\mathbb{O}}(\ddot{o}), \sup F_{\mathbb{P}}(\ddot{o}))]\}$.

Example 3.5. Consider Example 3.2 with two sets:
$A_1 = \{(\mu_1, s, 1), (\mu_1, t, 0)\}$, $A_2 = \{(\mu_1, s, 1), (\mu_1, t, 1), (\mu_1, t, 0)\}$.
Suppose that (∂_1, A_1) and (∂_2, A_2) over $\mathfrak{R}$ are two Jvnse-sets such that

$$(\partial_1, A_1) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{\langle[0.4,0.5],[0.1,0.2],[0.7,0.8]\rangle}, \frac{m_2}{\langle[0.3,0.4],[0.3,0.5],[0.5,0.6]\rangle}\right\}\right), \\ \left((\mu_1, t, 0), \left\{\frac{m_1}{\langle[0.8,0.9],[0.4,0.5],[0.3,0.4]\rangle}, \frac{m_2}{\langle[0.7,0.8],[0.2,0.3],[0.2,0.4]\rangle}\right\}\right), \end{array} \right\}.$$

$$(\partial_2, A_2) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{\langle[0.5,0.6],[0.2,0.3],[0.8,0.9]\rangle}, \frac{m_2}{\langle[0.4,0.5],[0.4,0.6],[0.6,0.7]\rangle}\right\}\right), \\ \left((\mu_1, t, 1), \left\{\frac{m_1}{\langle[0.2,0.3],[0.2,0.4],[0.3,0.6]\rangle}, \frac{m_2}{\langle[0.9,0.9],[0.6,0.7],[0.6,0.8]\rangle}\right\}\right), \\ \left((\mu_1, t, 0), \left\{\frac{m_1}{\langle[0.9,0.9],[0.5,0.6],[0.4,0.5]\rangle}, \frac{m_2}{\langle[0.8,0.9],[0.3,0.4],[0.3,0.5]\rangle}\right\}\right), \end{array} \right\}.$$

Then, $(\partial_1, A_1) \cup (\partial_2, A_2) = (\partial_3, A_3)$:

$$(\partial_3, A_3) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{\langle[0.5,0.6],[0.1,0.2],[0.7,0.8]\rangle}, \frac{m_2}{\langle[0.4,0.5],[0.3,0.5],[0.5,0.6]\rangle}\right\}\right), \\ \left((\mu_1, t, 1), \left\{\frac{m_1}{\langle[0.2,0.3],[0.1,0.3],[0.2,0.5]\rangle}, \frac{m_2}{\langle[0.9,0.9],[0.5,0.6],[0.5,0.7]\rangle}\right\}\right), \\ \left((\mu_1, t, 0), \left\{\frac{m_1}{\langle[0.9,0.9],[0.4,0.5],[0.3,0.4]\rangle}, \frac{m_2}{\langle[0.8,0.9],[0.2,0.3],[0.2,0.4]\rangle}\right\}\right), \end{array} \right\}.$$

Definition 3.6. The intersection of $(\partial_1, \mathbb{O})$ and $(\partial_2, \mathbb{P})$ over $\mathfrak{R}$ is $(\partial_3, \pounds)$ with $\pounds = \mathbb{O} \cap \mathbb{P}$, defined as

$$\partial_3(\ddot{\sigma}) = \left\{ \begin{array}{ll} \partial_1(\ddot{\sigma}) & ; \ddot{\sigma} \in \mathbb{O} - \mathbb{P}, \\ \partial_2(\ddot{\sigma}) & ; \ddot{\sigma} \in \mathbb{P} - \mathbb{O}, \\ \cap(\partial_1(\ddot{\sigma}), \partial_2(\ddot{\sigma})) & ; \ddot{\sigma} \in \mathbb{O} \cap \mathbb{P}, \end{array} \right.$$

where $\cap(\partial_1(\ddot{\sigma}), \partial_2(\ddot{\sigma}))$
$= \{\langle \ddot{\sigma}, \min\{\mu_1(\ddot{\sigma}), \mu_2(\ddot{\sigma})\}, \max\{\nu_1(\ddot{\sigma}), \nu_2(\ddot{\sigma})\}, \max\{\omega_1(\ddot{\sigma}), \omega_2(\ddot{\sigma})\}\rangle : \ddot{\sigma} \in \mathfrak{R}\}$. In other words, $\cup(\partial_1(\ddot{\sigma}), \partial_2(\ddot{\sigma})) = \{\langle[\min(\inf T_{\mathbb{O}}(\ddot{\sigma}), \inf T_{\mathbb{P}}(\ddot{\sigma})), \min(\sup T_{\mathbb{O}}(\ddot{\sigma}), \sup T_{\mathbb{P}}(\ddot{\sigma}))]$,
$[\max(\inf I_{\mathbb{O}}(\ddot{\sigma}), \inf I_{\mathbb{P}}(\ddot{\sigma})), \max(\sup I_{\mathbb{O}}(\ddot{\sigma}), \sup I_{\mathbb{P}}(\ddot{\sigma}))]$,
$[\max(\inf F_{\mathbb{O}}(\ddot{\sigma}), \inf F_{\mathbb{P}}(\ddot{\sigma})), \max(\sup F_{\mathbb{O}}(\ddot{\sigma}), \sup F_{\mathbb{P}}(\ddot{\sigma}))]$.

Example 3.7. Following Example 3.2, we have two sets:
$O_1 = \{(\mu_1, s, 1), (\mu_1, u, 0)\}$, $O_2 = \{(\mu_1, s, 1), (\mu_3, s, 0), (\mu_1, u, 0)\}$.
Suppose that (∂_1, O_1) and (∂_2, O_2) over $\mathfrak{R}$ are two Jvnse-sets such that

$$(\partial_1, O_1) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{\langle[0.4,0.5],[0.1,0.2],[0.7,0.8]\rangle}, \frac{m_2}{\langle[0.3,0.4],[0.3,0.5],[0.5,0.6]\rangle}\right\}\right), \\ \left((\mu_1, u, 0), \left\{\frac{m_1}{\langle[0.7,0.8],[0.5,0.7],[0.4,0.5]\rangle}, \frac{m_2}{\langle[0.8,0.9],[0.6,0.7],[0.2,0.7]\rangle}\right\}\right), \end{array} \right\},$$

$$(\partial_2, O_2) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{\langle[0.5,0.6],[0.2,0.3],[0.8,0.9]\rangle}, \frac{m_2}{\langle[0.4,0.5],[0.4,0.6],[0.6,0.7]\rangle}\right\}\right), \\ \left((\mu_1, u, 0), \left\{\frac{m_1}{\langle[0.8,0.9],[0.6,0.8],[0.5,0.6]\rangle}, \frac{m_2}{\langle[0.9,0.9],[0.7,0.8],[0.3,0.8]\rangle}\right\}\right), \\ \left((\mu_3, s, 0), \left\{\frac{m_1}{\langle[0.6,0.7],[0.4,0.8],[0.4,0.5]\rangle}, \frac{m_2}{\langle[0.7,0.8],[0.2,0.6],[0.2,0.4]\rangle}\right\}\right), \end{array} \right\}.$$

Then, $(\partial_1, O_1) \cap (\partial_2, O_2) = (\partial_3, O_3)$:

$$\left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{\langle[0.4,0.5],[0.2,0.3],[0.8,0.9]\rangle}, \frac{m_2}{\langle[0.3,0.4],[0.4,0.6],[0.6,0.7]\rangle}\right\}\right), \\ \left((\mu_1, u, 0), \left\{\frac{m_1}{\langle[0.7,0.8],[0.6,0.8],[0.5,0.6]\rangle}, \frac{m_2}{\langle[0.8,0.9],[0.7,0.8],[0.3,0.8]\rangle}\right\}\right), \end{array} \right\}.$$

Definition 3.8. If $(\partial_1, \top_1)$ and $(\partial_2, \top_2)$ are two $\mathfrak{J}$vnse-sets over $\mathfrak{R}$, then $(\partial_1, \top_1)$ AND $(\partial_2, \top_2)$ is defined by $(\partial_1, \top_1) \wedge (\partial_2, \top_2) = (\partial_3, \top_1 \times \top_2)$, while $\partial_3(\tilde{o}, \bar{o}) = \partial_1(\tilde{o}) \cap \partial_2(\bar{o}), \forall(\tilde{o}, \bar{o}) \in \top_1 \times \top_2$.

Example 3.9. Taking Example 3.2, let
$\top_1 = \{(\mu_1, s, 1), (\mu_1, t, 1)\}$, $\top_2 = \{(\mu_1, s, 0), (\mu_3, s, 1)\}$ be two $\mathfrak{J}$vnse-sets such that

$$(\partial_1, \top_1) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{\langle[0.4,0.5],[0.1,0.2],[0.7,0.8]\rangle}, \frac{m_2}{\langle[0.3,0.4],[0.3,0.5],[0.5,0.6]\rangle}\right\}\right), \\ \left((\mu_1, t, 1), \left\{\frac{m_1}{\langle[0.1,0.2],[0.1,0.3],[0.2,0.5]\rangle}, \frac{m_2}{\langle[0.8,0.9],[0.5,0.6],[0.5,0.7]\rangle}\right\}\right), \end{array} \right\},$$

$$(\partial_2, \top_2) = \left\{ \begin{array}{l} \left((\mu_1, s, 0), \left\{\frac{m_1}{\langle[0.3,0.4],[0.1,0.2],[0.2,0.6]\rangle}, \frac{m_2}{\langle[0.4,0.5],[0.5,0.6],[0.1,0.3]\rangle}\right\}\right), \\ \left((\mu_3, s, 1), \left\{\frac{m_1}{\langle[0.2,0.7],[0.2,0.5],[0.4,0.5]\rangle}, \frac{m_2}{\langle[0.0,0.1],[0.1,0.2],[0.2,0.3]\rangle}\right\}\right), \end{array} \right\}.$$

Then, $(\partial_3, \top_3) \wedge (\partial_2, \top_2) = (\partial_3, \top_1 \times \top_2)$:

$$\left\{ \begin{array}{l} \left(((\mu_1, s, 1), (\mu_1, s, 0)), \left\{\frac{m_1}{\langle[0.3,0.4],[0.1,0.2],[0.7,0.8]\rangle}, \frac{m_2}{\langle[0.3,0.4],[0.5,0.6],[0.5,0.6]\rangle}\right\}\right), \\ \left(((\mu_1, t, 1), (\mu_1, s, 0)), \left\{\frac{m_1}{\langle[0.1,0.2],[0.1,0.3],[0.2,0.6]\rangle}, \frac{m_2}{\langle[0.4,0.5],[0.5,0.6],[0.5,0.6]\rangle}\right\}\right), \\ \left(((\mu_1, t, 1), (\mu_3, s, 1)), \left\{\frac{m_1}{\langle[0.1,0.2],[0.2,0.5],[0.4,0.5]\rangle}, \frac{m_2}{\langle[0.0,0.1],[0.5,0.6],[0.5,0.7]\rangle}\right\}\right), \\ \left(((\mu_1, s, 1), (\mu_3, s, 1)), \left\{\frac{m_1}{\langle[0.2,0.5],[0.2,0.5],[0.7,0.8]\rangle}, \frac{m_2}{\langle[0.0,0.1],[0.3,0.5],[0.5,0.6]\rangle}\right\}\right), \end{array} \right\}.$$

Definition 3.10. If $(\partial_1, ¥_1)$ and $(\partial_2, ¥_2)$ are two $\mathfrak{J}$vnse-sets over $\mathfrak{R}$, then $(\partial_1, ¥_1)$ OR $(\partial_2, ¥_2)$ denoted by $(\partial_1, ¥_1) \vee (\partial_2, ¥_2)$ is defined by $(\partial_1, ¥_1) \vee (\partial_2, ¥_2) = (\partial_3, ¥_1 \times ¥_2)$, and $\partial_3(\ddot{o}, \bar{o}) = \partial_1(\ddot{o}) \cup \partial_2(\bar{o}), \forall(\ddot{o}, \bar{o}) \in ¥_1 \times ¥_2$.

Example 3.11. Taking Example 3.2, suppose that the sets
$A_1 = \{(\mu_1, s, 1), (\mu_1, t, 1)\}$, $A_2 = \{(\mu_1, s, 0), (\mu_3, s, 1)\}$ are two $\mathfrak{J}$vnse-sets such that

$$(\partial_1, A_1) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{\langle[0.4,0.5],[0.1,0.2],[0.7,0.8]\rangle}, \frac{m_2}{\langle[0.3,0.4],[0.3,0.5],[0.5,0.6]\rangle}\right\}\right), \\ \left((\mu_1, t, 1), \left\{\frac{m_1}{\langle[0.1,0.2],[0.1,0.3],[0.2,0.5]\rangle}, \frac{m_2}{\langle[0.8,0.9],[0.5,0.6],[0.5,0.7]\rangle}\right\}\right), \end{array} \right\},$$

$$(\partial_2, A_2) = \left\{ \begin{array}{l} \left((\mu_1, s, 0), \left\{\frac{m_1}{\langle[0.3,0.4],[0.1,0.2],[0.2,0.6]\rangle}, \frac{m_2}{\langle[0.4,0.5],[0.5,0.6],[0.1,0.3]\rangle}\right\}\right), \\ \left((\mu_3, s, 1), \left\{\frac{m_1}{\langle[0.2,0.7],[0.2,0.5],[0.4,0.5]\rangle}, \frac{m_2}{\langle[0.0,0.1],[0.1,0.2],[0.2,0.3]\rangle}\right\}\right), \end{array} \right\}.$$

Then, $(\partial_3, A_3) \vee (\partial_2, A_2) = (\partial_3, A_1 \times A_2)$:

$$\left\{ \begin{array}{l} \left(((\mu_1, s, 1), (\mu_1, s, 0)), \left\{\frac{m_1}{\langle[0.3,0.4],[0.1,0.2],[0.7,0.8]\rangle}, \frac{m_2}{\langle[0.3,0.4],[0.5,0.6],[0.5,0.6]\rangle}\right\}\right), \\ \left(((\mu_1, t, 1), (\mu_1, s, 0)), \left\{\frac{m_1}{\langle[0.1,0.2],[0.1,0.3],[0.2,0.6]\rangle}, \frac{m_2}{\langle[0.4,0.5],[0.5,0.6],[0.5,0.6]\rangle}\right\}\right), \\ \left(((\mu_1, t, 1), (\mu_3, s, 1)), \left\{\frac{m_1}{\langle[0.1,0.2],[0.2,0.5],[0.4,0.5]\rangle}, \frac{m_2}{\langle[0.0,0.1],[0.5,0.6],[0.5,0.7]\rangle}\right\}\right), \\ \left(((\mu_1, s, 1), (\mu_3, s, 1)), \left\{\frac{m_1}{\langle[0.2,0.5],[0.2,0.5],[0.7,0.8]\rangle}, \frac{m_2}{\langle[0.0,0.1],[0.3,0.5],[0.5,0.6]\rangle}\right\}\right), \end{array} \right\}.$$

3.2. Strategy for the selection of parameters

The choice of suppliers is influenced by a number of parameters. Criteria (parameters) can be of decisive importance for the decision-making process. Therefore, careful selection and calculation are required. It is a long-standing practice that contractors often select suppliers based on price. The criteria (parameters) discussed in this paper are found to be much more relevant and significant to construction supply chain management than the criteria previously discussed by many researchers in different papers [30, 36]. The suggested study is concerned with the neutrosophic soft expert environment, which means that only criteria can be used that may fruitfully fulfill the requirements of the environment.

3.3. Operational role of opted parameters

1. **Strength**: Strength is the property of a material that determines its ability to withstand forces and deformations without breaking. It is a critical parameter for pleasant management within the construction process and the most crucial standard for constructing materials. Strength indicates the componential ability of a cloth to face up to the failure below the motion of stresses due to hazards, which include compression, tension, bending and/or effect, that may be precipitated because of the forces of nature or human activity. See Figure 3 for types of construction materials.
2. **Cost of the Material**: The unit cost of a product is the cost to produce one unit of that product. Determining the appropriate manufacturing cost to assign to each unit is important, as it affects decisions such as how many units to yield, in what way accounts should be conveyed and how much inventory should be manufactured in advance of customer orders. The objective of this article is to explain how to calculate the unit cost of a manufactured item.
3. **Handling and Storage of the Material**: This is the application of material selection to the action, safety, storage and regulation of things and goods through the development, supply and feeding processes. Inventory management and control are also supported by this process, as are customer delivery services. After-sales support includes equipment maintenance which may include the repair or replacement of broken parts for added reliability.
4. **Climate Compatibility**: The basic concept is to make sure that the design of the house is in accordance with the climate. For example, being extremely cold or hot during the winter or summer months, respectively, can affect the air conditioning and heating systems, as well as heat preservation systems like insulation, which are very important. If a homeowner chooses materials for construction according to the site conditions, then there will be no problem with achieving comfort inside houses.
5. **Skills Required and Their Availability**: When choosing a fabric for a structure, it is crucial to recognize the volume of ability required to apply that material. Although one will not be capable of deciding the ability required based on the cost, it could boost the price of the development due to the fact you need to appoint or rent a professional character (labor) to apply that fabric, and if the professional character or labor is not always available, especially in far-off regions, then it will put off the work.

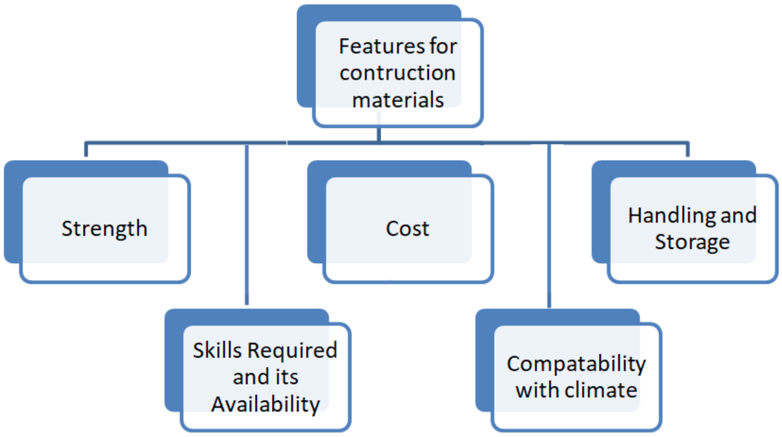

Figure 3. Pictographic view of features for construction materials.

3.4. *Operational role of expert's opinions*

Decisions are an important part of daily life. Decisions require target setting and solution evaluation. Business requires decision-makers, i.e., people who can make decisions based on variables, including time constraints, resources available, the amount and type of information available and the number of stakeholders involved. Decision-makers are the people who make most of the decisions that affect the operation of a business. They must have the ability to communicate effectively with other employees, customers and suppliers in order to ensure that all aspects of the company operate smoothly. Decision-makers are the most responsible members of an organization and are the key component in ensuring the smooth running of a business. They take on larger company decisions and work to keep it efficiently running. Decision-makers usually hold a position in management, but they may also be members of a financial department or operations. In construction supply chain management, the procurement department manages the contracting of goods, services and materials for all areas of construction. It is responsible for the employment of experts for decision-making, the advertisement of bids, the scrutiny of suppliers and other functions related to ensuring that projects are completed on time and within budget. In a large business, the procurement department can be headed by a procurement manager. The manager leads a team of procurement agents and specialists to procure goods and services for the business.

4. The $\mathfrak{J}$vnse-set-based decision-support system

The main target of this study is to introduce a generalized algorithm that could be used to solve any DMP. To demonstrate the power of the algorithm, we applied it to clear up a specific DMP, namely, the $\mathfrak{J}$vnse-set.

4.1. *Problem declaration*

The management of a reputable real estate construction company called CONSCO (a fictitious name) is looking for the best supplier to supply all different types of construction materials (bricks, cement, sand, etc.) for their building projects in various parts of the city. For this reason, the CONSCO posts a request for bids, and some businesses respond. Following the publication of a request for bids, the company has formed three committees. Two experts, $\mathbb{P}_1$ and $\mathbb{P}_2$, who have relevant experience reviewing the proposals submitted in response to an advertisement, make up Committee A. The primary responsibility of the committee members is to implicitly and collectively shortlist the ideas in accordance with the firm's scrutiny policy, taking into account their collective and individual experiences. The proposals that made the shortlist are subsequently sent to Committees B and C. Two specialists from Committee B, $\mathbb{P}_3$ and $\mathbb{P}_4$, are domestic workers for the company. This committee's primary responsibility is to gather general information in the form of linguistic phrases while taking into account some useful parameters. Three experts, $\mathbb{P}_5$, $\mathbb{P}_6$ and $\mathbb{P}_7$, make up Committee C. They were engaged to examine Committee B's findings and offer their opinions in linguistic terms that reflect their level of approval. Due to the skewed market conditions, the government views this evaluation procedure as vital; hence, a thorough and dependable methodology may aid them in making the right decision.

4.2. Proposed algorithm

Algorithm 1. Construction descriptions are presented through the following algorithm.
Step-wise procedure for SS based on $\mathcal{J}$vnse-set.

▷ **Start**	
▷ **Input**	(i) Considering $\{\mathfrak{l}_1, \mathfrak{l}_2, \mathfrak{l}_3\}$ as a initial universe. (ii) Taking set of attributes $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5\}$. (iii) Expert set $\widetilde{Y} = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3\}$.
▷ **Construction**	(iv) Construct $\mathcal{J}$vnse-set $(\bigvee, \mathsf{M})$ based on the expert set, expert's opinions and set of attributes.
▷ **Computation**	(v) Determine Agree-$\mathcal{J}$vnse-set and Disagree-$\mathcal{J}$vnse-set depending upon the opinions of experts. For agree opinions, develop Agree-$\mathcal{J}$vnse-set and disagree opinions, determine Disagree-$\mathcal{J}$vnse-set. (vi) Determination of Interval Truth/Indeterminacy/Falsity Membership Function Components. (vii) Formation of Induced Interval Truth/Indeterminacy/Falsity Membership Function Components. (viii) Computation of Scores of Interval Truth/Indeterminacy/Falsity Membership Function Components.
▷ **Output**	(vii) Determination of Final Decision.
▷ **End**	

The flowchart of Algorithm 1 is given in Figure 4.

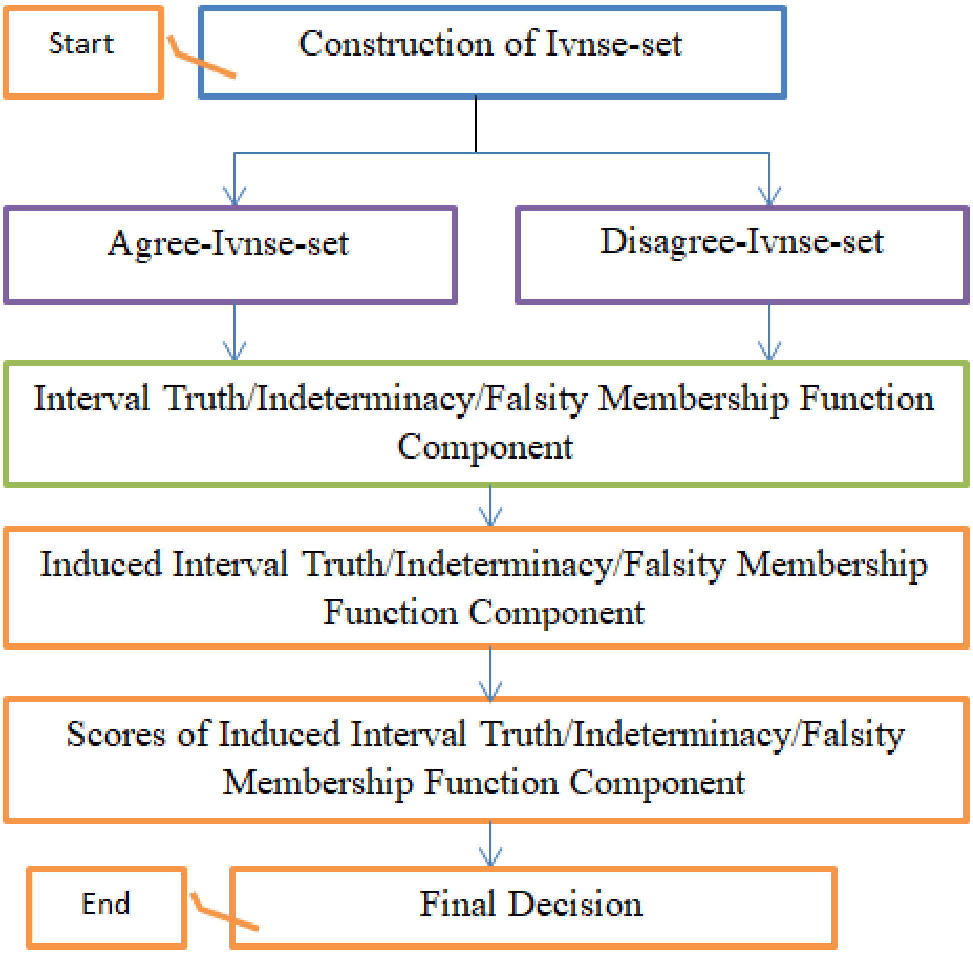

Figure 4. Graphical representation of algorithm.

4.3. Explanation of proposed algorithm

Example 4.1. Take into account that three suppliers, who make up the initial universe $Z = \{l_1, l_2, l_3\}$, are first examined by Committee A for further evaluation, and that the members of Committee B have decided on a set of efficient parameters $O = \{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5\}$, where $\tau_1, \tau_2, \tau_3, \tau_4$ and τ_5 respectively represent the strength, cost of material, handling and storage of material, climate and skills required and their availability, and Committee C is $\mathbb{Y} = \{\mathbb{P}_5 = \mathtt{Tait}, \mathbb{P}_6 = \mathtt{John}, \mathbb{P}_7 = \mathtt{Tieson}\}$.

Now, we have $\mathbb{I}$vnse-set $(\bigvee, \mathbf{M})_1 =$

$$\left\{\begin{array}{l}
\left((o_1,\mathcal{P}_1,1),\left\{\frac{l_1}{\langle[0.4,0.5],[0.1,0.2],[0.7,0.8]\rangle},\frac{l_2}{\langle[0.4,0.6],[0.1,0.3],[0.7,0.9]\rangle},\frac{l_3}{\langle[0.3,0.4],[0.3,0.5],[0.5,0.6]\rangle}\right\}\right),\\
\left((o_1,\mathcal{P}_2,1),\left\{\frac{l_1}{\langle[0.1,0.2],[0.1,0.3],[0.2,0.5]\rangle},\frac{l_2}{\langle[0.1,0.5],[0.2,0.3],[0.3,0.5]\rangle},\frac{l_3}{\langle[0.8,0.9],[0.5,0.6],[0.5,0.7]\rangle}\right\}\right),\\
\left((o_1,\mathcal{P}_3,1),\left\{\frac{l_1}{\langle[0.7,0.8],[0.3,0.4],[0.1,0.3]\rangle},\frac{l_2}{\langle[0.3,0.8],[0.3,0.5],[0.1,0.7]\rangle},\frac{l_2}{\langle[0.5,0.6],[0.6,0.7],[0.2,0.6]\rangle}\right\}\right),\\
\left((o_2,\mathcal{P}_1,1),\left\{\frac{l_1}{\langle[0.6,0.7],[0.8,0.9],[0.7,0.8]\rangle},\frac{l_2}{\langle[0.2,0.7],[0.3,0.6],[0.3,0.8]\rangle},\frac{l_3}{\langle[0.4,0.7],[0.1,0.1],[0.6,0.9]\rangle}\right\}\right),\\
\left((o_2,\mathcal{P}_2,1),\left\{\frac{l_1}{\langle[0.5,0.6],[0.3,0.7],[0.7,0.8]\rangle},\frac{l_2}{\langle[0.1,0.6],[0.3,0.6],[0.5,0.8]\rangle},\frac{l_3}{\langle[0.6,0.7],[0.5,0.8],[0.1,0.1]\rangle}\right\}\right),\\
\left((o_2,\mathcal{P}_3,1),\left\{\frac{l_1}{\langle[0.4,0.8],[0.1,0.1],[0.3,0.6]\rangle},\frac{l_1}{\langle[0.3,0.8],[0.1,0.5],[0.3,0.7]\rangle},\frac{l_3}{\langle[0.3,0.6],[0.4,0.8],[0.5,0.8]\rangle}\right\}\right),\\
\left((o_3,\mathcal{P}_1,1),\left\{\frac{l_1}{\langle[0.5,0.7],[0.1,0.9],[0.6,0.8]\rangle},\frac{l_2}{\langle[0.3,0.7],[0.1,0.8],[0.6,0.8]\rangle},\frac{l_3}{\langle[0.5,0.7],[0.6,0.9],[0.7,0.8]\rangle}\right\}\right),\\
\left((o_3,\mathcal{P}_2,1),\left\{\frac{l_1}{\langle[0.2,0.3],[0.4,0.6],[0.6,0.7]\rangle},\frac{l_2}{\langle[0.1,0.3],[0.4,0.7],[0.3,0.7]\rangle},\frac{l_3}{\langle[0.1,0.2],[0.5,0.8],[0.4,0.9]\rangle}\right\}\right),\\
\left((o_3,\mathcal{P}_3,1),\left\{\frac{l_1}{\langle[0.3,0.5],[0.1,0.2],[0.4,0.7]\rangle},\frac{l_2}{\langle[0.3,0.4],[0.1,0.7],[0.2,0.7]\rangle},\frac{l_3}{\langle[0.6,0.9],[0.2,0.9],[0.3,0.6]\rangle}\right\}\right),\\
\left((o_4,\mathcal{P}_1,1),\left\{\frac{l_1}{\langle[0.2,0.3],[0.2,0.3],[0.1,0.9]\rangle},\frac{l_2}{\langle[0.2,0.4],[0.2,0.6],[0.1,0.6]\rangle},\frac{l_3}{\langle[0.1,0.3],[0.2,0.8],[0.7,0.8]\rangle}\right\}\right),\\
\left((o_4,\mathcal{P}_2,1),\left\{\frac{l_1}{\langle[0.1,0.7],[0.3,0.4],[0.2,0.7]\rangle},\frac{l_2}{\langle[0.3,0.7],[0.3,0.6],[0.2,0.7]\rangle},\frac{l_3}{\langle[0.4,0.5],[0.5,0.8],[0.6,0.7]\rangle}\right\}\right),\\
\left((o_4,\mathcal{P}_3,1),\left\{\frac{l_1}{\langle[0.6,0.8],[0.4,0.7],[0.3,0.5]\rangle},\frac{l_2}{\langle[0.1,0.8],[0.4,0.5],[0.3,0.6]\rangle},\frac{l_3}{\langle[0.1,0.3],[0.2,0.7],[0.1,0.2]\rangle}\right\}\right),\\
\left((o_5,\mathcal{P}_1,1),\left\{\frac{l_1}{\langle[0.2,0.5],[0.2,0.3],[0.1,0.5]\rangle},\frac{l_2}{\langle[0.1,0.5],[0.2,0.5],[0.1,0.6]\rangle},\frac{l_3}{\langle[0.2,0.8],[0.4,0.7],[0.3,0.8]\rangle}\right\}\right),\\
\left((o_5,\mathcal{P}_2,1),\left\{\frac{l_1}{\langle[0.5,0.9],[0.4,0.8],[0.4,0.5]\rangle},\frac{l_2}{\langle[0.2,0.9],[0.7,0.8],[0.4,0.8]\rangle},\frac{l_3}{\langle[0.6,0.9],[0.4,0.7],[0.5.0.8]\rangle}\right\}\right),\\
\left((o_5,\mathcal{P}_3,1),\left\{\frac{l_1}{\langle[0.1,0.3],[0.1,0.5],[0.3,0.5]\rangle},\frac{l_2}{\langle[0.2,0.3],[0.4,0.5],[0.3,0.2]\rangle},\frac{l_3}{\langle[0.1,0.6],[0.1,0.2],[0.3,0.6]\rangle}\right\}\right),
\end{array}\right\},$$

$(\bigvee,\mathbf{M})_0 =$

$$\left\{\begin{array}{l}
\left((o_1,\mathcal{P}_1,0),\left\{\frac{l_1}{\langle[0.3,0.4],[0.1,0.2],[0.4,0.5]\rangle},\frac{l_2}{\langle[0.2,0.4],[0.1,0.5],[0.4,0.7]\rangle},\frac{l_3}{\langle[0.2,0.5],[0.2,0.3],[0.3,0.6]\rangle}\right\}\right),\\
\left((o_1,\mathcal{P}_2,0),\left\{\frac{l_1}{\langle[0.4,0.5],[0.1,0.8],[0.4,0.9]\rangle},\frac{l_2}{\langle[0.1,0.5],[0.1,0.9],[0.2,0.9]\rangle},\frac{l_3}{\langle[0.2,0.8],[0.2,0.4],[0.3,0.9]\rangle}\right\}\right),\\
\left((o_1,\mathcal{P}_3,0),\left\{\frac{l_1}{\langle[0.1,0.3],[0.1,0.7],[0.4,0.5]\rangle},\frac{l_2}{\langle[0.1,0.5],[0.1,0.6],[0.2,0.5]\rangle},\frac{l_3}{\langle[0.1,0.5],[0.2,0.7],[0.1,0.6]\rangle}\right\}\right),\\
\left((o_2,\mathcal{P}_1,0),\left\{\frac{l_1}{\langle[0.4,0.7],[0.1,0.8],[0.3,0.8]\rangle},\frac{l_2}{\langle[0.4,0.6],[0.1,0.7],[0.4,0.8]\rangle},\frac{l_3}{\langle[0.2,0.5],[0.2,0.3],[0.3,0.6]\rangle}\right\}\right),\\
\left((o_2,\mathcal{P}_2,0),\left\{\frac{l_1}{\langle[0.4,0.8],[0.1,0.8],[0.4,0.7]\rangle},\frac{l_2}{\langle[0.2,0.8],[0.5,0.8],[0.4,0.6]\rangle},\frac{l_3}{\langle[0.3,0.7],[0.2,0.7],[0.1,0.9]\rangle}\right\}\right),\\
\left((o_2,\mathcal{P}_3,0),\left\{\frac{l_1}{\langle[0.4,0.7],[0.1,0.2],[0.4,0.4]\rangle},\frac{l_2}{\langle[0.1,0.7],[0.1,0.6],[0.4,0.5]\rangle},\frac{l_3}{\langle[0.2,0.3],[0.2,0.8],[0.3,0.7]\rangle}\right\}\right),\\
\left((o_3,\mathcal{P}_1,0),\left\{\frac{l_1}{\langle[0.4,0.3],[0.1,0.6],[0.4,0.8]\rangle},\frac{l_2}{\langle[0.4,0.3],[0.3,0.6],[0.5,0.8]\rangle},\frac{l_3}{\langle[0.2,0.5],[0.2,0.3],[0.3,0.6]\rangle}\right\}\right),\\
\left((o_3,\mathcal{P}_2,0),\left\{\frac{l_1}{\langle[0.4,0.6],[0.1,0.7],[0.4,0.5]\rangle},\frac{l_2}{\langle[0.4,0.3],[0.1,0.6],[0.4,0.8]\rangle},\frac{l_3}{\langle[0.2,0.8],[0.2,0.9],[0.3,0.8]\rangle}\right\}\right),\\
\left((o_3,\mathcal{P}_3,0),\left\{\frac{l_1}{\langle[0.2,0.3],[0.1,0.6],[0.3,0.5]\rangle},\frac{l_2}{\langle[0.1,0.3],[0.2,0.6],[0.4,0.8]\rangle},\frac{l_3}{\langle[0.1,0.5],[0.2,0.3],[0.3,0.7]\rangle}\right\}\right),\\
\left((o_4,\mathcal{P}_1,0),\left\{\frac{l_1}{\langle[0.4,0.5],[0.1,0.2],[0.4,0.6]\rangle},\frac{l_2}{\langle[0.4,0.3],[0.1,0.6],[0.2,0.8]\rangle},\frac{l_3}{\langle[0.2,0.7],[0.2,0.8],[0.2,0.6]\rangle}\right\}\right),\\
\left((o_4,\mathcal{P}_2,0),\left\{\frac{l_1}{\langle[0.1,0.4],[0.1,0.6],[0.4,0.5]\rangle},\frac{l_2}{\langle[0.1,0.5],[0.1,0.4],[0.2,0.7]\rangle},\frac{l_3}{\langle[0.1,0.5],[0.2,0.4],[0.3,0.6]\rangle}\right\}\right),\\
\left((o_4,\mathcal{P}_3,0),\left\{\frac{l_1}{\langle[0.2,0.3],[0.1,0.2],[0.2,0.5]\rangle},\frac{l_2}{\langle[0.4,0.3],[0.2,0.5],[0.4,0.6]\rangle},\frac{l_3}{\langle[0.2,0.6],[0.1,0.3],[0.1,0.6]\rangle}\right\}\right),\\
\left((o_5,\mathcal{P}_1,0),\left\{\frac{l_1}{\langle[0.1,0.5],[0.1,0.4],[0.4,0.5]\rangle},\frac{l_2}{\langle[0.3,0.6],[0.1,0.7],[0.2,0.8]\rangle},\frac{l_3}{\langle[0.1,0.4],[0.2,0.3],[0.3,0.7]\rangle}\right\}\right),\\
\left((o_5,\mathcal{P}_2,0),\left\{\frac{l_1}{\langle[0.1,0.3],[0.1,0.2],[0.3,0.5]\rangle},\frac{l_2}{\langle[0.4,0.3],[0.3,0.6],[0.5,0.8]\rangle},\frac{l_3}{\langle[0.2,0.5],[0.1,0.3],[0.2,0.6]\rangle}\right\}\right),\\
\left((o_5,\mathcal{P}_3,0),\left\{\frac{l_1}{\langle[0.5,0.3],[0.1,0.3],[0.4,0.6]\rangle},\frac{l_2}{\langle[0.1,0.3],[0.5,0.6],[0.4,0.7]\rangle},\frac{l_3}{\langle[0.2,0.4],[0.2,0.3],[0.3,0.6]\rangle}\right\}\right),
\end{array}\right\},$$

which are $\mathcal{J}$vnse-sets.

▷ **Construction of agree- and disagree-$\mathcal{J}$vnse-sets**

$(\partial_1,\mathbf{M}_1) =$

$$\left\{\begin{array}{l}
\left((o_1,\mathcal{P}_1,1),\left\{\frac{l_1}{\langle[0.4,0.5],[0.1,0.2],[0.7,0.8]\rangle},\frac{l_2}{\langle[0.4,0.6],[0.1,0.3],[0.7,0.9]\rangle},\frac{l_3}{\langle[0.3,0.4],[0.3,0.5],[0.5,0.6]\rangle}\right\}\right),\\
\left((o_2,\mathcal{P}_2,1),\left\{\frac{l_1}{\langle[0.5,0.6],[0.3,0.7],[0.7,0.8]\rangle},\frac{l_2}{\langle[0.1,0.6],[0.3,0.6],[0.5,0.8]\rangle},\frac{l_3}{\langle[0.6,0.7],[0.5,0.8],[0.1,0.1]\rangle}\right\}\right),\\
\left((o_3,\mathcal{P}_3,1),\left\{\frac{l_1}{\langle[0.3,0.5],[0.1,0.2],[0.4,0.7]\rangle},\frac{l_2}{\langle[0.3,0.4],[0.1,0.7],[0.2,0.7]\rangle},\frac{l_3}{\langle[0.6,0.9],[0.2,0.9],[0.3,0.6]\rangle}\right\}\right),
\end{array}\right\},$$

which is an agree-$\mathcal{J}$vnse-set.

$(\partial_0,\mathbf{M}_0) =$

$$\left\{\begin{array}{l}
\left((o_1,\mathcal{P}_1,0),\left\{\frac{l_1}{\langle[0.3,0.4],[0.1,0.2],[0.4,0.5]\rangle},\frac{l_2}{\langle[0.2,0.4],[0.1,0.5],[0.4,0.7]\rangle},\frac{l_3}{\langle[0.2,0.5],[0.2,0.3],[0.3,0.6]\rangle}\right\}\right),\\
\left((o_2,\mathcal{P}_2,0),\left\{\frac{l_1}{\langle[0.4,0.8],[0.1,0.8],[0.4,0.7]\rangle},\frac{l_2}{\langle[0.2,0.8],[0.5,0.8],[0.4,0.6]\rangle},\frac{l_3}{\langle[0.3,0.7],[0.2,0.7],[0.1,0.9]\rangle}\right\}\right),\\
\left((o_3,\mathcal{P}_3,0),\left\{\frac{l_1}{\langle[0.2,0.3],[0.1,0.6],[0.3,0.5]\rangle},\frac{l_2}{\langle[0.1,0.3],[0.2,0.6],[0.4,0.8]\rangle},\frac{l_3}{\langle[0.1,0.5],[0.2,0.3],[0.3,0.7]\rangle}\right\}\right),
\end{array}\right\},$$

which is a disagree-$\mathcal{J}$vnse-set.

▷ **Interval truth membership function components**

$$(\aleph_1,\pounds_1) = \left\{\ \left((o_1,\mathcal{P}_1,1),\left\{\begin{array}{l}\langle[0.4,0.5],[0.4,0.6],[0.3,0.4]\rangle,\langle[0.1,0.2],[0.1,0.3],[0.3,0.5]\rangle,\\ \langle[0.7,0.8],[0.7,0.9],[0.5,0.6]\rangle\end{array}\right\}\right)\ \right\},$$

$$(\aleph_2, \pounds_2) = \left\{ \left((o_2, \mathcal{P}_2, 1), \begin{Bmatrix} \langle [0.5, 0.6], [0.1, 0.6], [0.6, 0.7] \rangle, \langle [0.3, 0.7], [0.3, 0.6], [0.1, 0.1] \rangle, \\ \langle [0.7, 0.8], [0.5, 0.8], [0.1, 0.1] \rangle \end{Bmatrix} \right) \right\},$$

$$(\aleph_3, \pounds_3) = \left\{ \left((o_3, \mathcal{P}_3, 1), \begin{Bmatrix} \langle [0.3, 0.5], [0.3, 0.4], [0.6, 0.9] \rangle, \langle [0.1, 0.2], [0.1, 0.7], [0.2, 0.9] \rangle, \\ \langle [0.4, 0.7], [0.2, 0.9], [0.3, 0.6] \rangle \end{Bmatrix} \right) \right\},$$

$$(\aleph_1, \perp_1) = \left\{ \left((o_1, \mathcal{P}_1, 0), \begin{Bmatrix} \langle [0.3, 0.4], [0.2, 0.4], [0.2, 0.5] \rangle, \langle [0.1, 0.2], [0.1, 0.5], [0.2, 0.3] \rangle, \\ \langle [0.4, 0.5], [0.4, 0.7], [0.3, 0.6] \rangle \end{Bmatrix} \right) \right\},$$

$$(\aleph_2, \perp_2) = \left\{ \left((o_2, \mathcal{P}_2, 0), \begin{Bmatrix} \langle [0.4, 0.8], [0.2, 0.8], [0.3, 0.7] \rangle, \langle [0.1, 0.8], [0.5, 0.8], [0.2, 0.7] \rangle, \\ \langle [0.4, 0.7], [0.4, 0.6], [0.1, 0.9] \rangle \end{Bmatrix} \right) \right\},$$

$$(\aleph_3, \perp_3) = \left\{ \left((o_3, \mathcal{P}_3, 0), \begin{Bmatrix} \langle [0.2, 0.3], [0.1, 0.3], [0.1, 0.5] \rangle, \langle [0.1, 0.6], [0.2, 0.6], [0.2, 0.3] \rangle, \\ \langle [0.3, 0.5], [0.4, 0.8], [0.3, 0.7] \rangle \end{Bmatrix} \right) \right\}.$$

▹ **Interval truth membership function components**

$$(\aleph_1, \pounds_1) = \left\{ \left((o_1, \mathcal{P}_1, 1), \begin{Bmatrix} \langle [0.4, 0.5], [0.4, 0.6], [0.3, 0.4] \rangle, \langle [0.1, 0.2], [0.1, 0.3], [0.3, 0.5] \rangle, \\ \langle [0.7, 0.8], [0.7, 0.9], [0.5, 0.6] \rangle \end{Bmatrix} \right) \right\},$$

$$(\aleph_1, \perp_1) = \left\{ \left((o_1, \mathcal{P}_1, 0), \begin{Bmatrix} \langle [0.3, 0.4], [0.2, 0.4], [0.2, 0.5] \rangle, \langle [0.1, 0.2], [0.1, 0.5], [0.2, 0.3] \rangle, \\ \langle [0.4, 0.5], [0.4, 0.7], [0.3, 0.6] \rangle \end{Bmatrix} \right) \right\}.$$

The operation AND between $(\aleph_1, \pounds_1) \wedge (\aleph_1, \perp_1)$ can be applied as

$$\left\{ \begin{array}{ll} \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{l_1}{[0.3,0.4]}, \frac{l_2}{[0.2,0.4]}, \frac{l_3}{[0.2,0.4]} \right\} \right) & \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{l_1}{[0.1,0.2]}, \frac{l_2}{[0.1,0.5]}, \frac{l_3}{[0.2,0.3]} \right\} \right) \\ \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{l_1}{[0.4,0.5]}, \frac{l_2}{[0.4,0.6]}, \frac{l_3}{[0.3,0.4]} \right\} \right) & \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{l_1}{[0.1,0.2]}, \frac{l_2}{[0.1,0.3]}, \frac{l_3}{[0.2,0.5]} \right\} \right) \\ \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{l_1}{[0.1,0.2]}, \frac{l_2}{[0.1,0.3]}, \frac{l_3}{[0.2,0.3]} \right\} \right) & \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{l_1}{[0.1,0.2]}, \frac{l_2}{[0.1,0.3]}, \frac{l_3}{[0.3,0.5]} \right\} \right) \\ \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{l_1}{[0.3,0.4]}, \frac{l_2}{[0.2,0.4]}, \frac{l_3}{[0.2,0.5]} \right\} \right) & \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{l_1}{[0.1,0.2]}, \frac{l_2}{[0.1,0.5]}, \frac{l_3}{[0.2,0.3]} \right\} \right) \\ \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{l_1}{[0.4,0.5]}, \frac{l_2}{[0.4,0.7]}, \frac{l_3}{[0.3,0.6]} \right\} \right) & \end{array} \right\}.$$

▹ **Interval truth membership function components**

The interval truth membership function components of $(\aleph_1, \pounds_1) \wedge (\aleph_1, \perp_1)$ are shown in Table 2.

Table 2. Induced interval truth membership function components.

L	l_1	l_2	l_3
$(o_1, p_1, 1)(o_1, p_1, 0)$	[0.3 0.4]	[0.2 0.4]	[0.2 0.4]
$(o_1, p_1, 1)(o_1, p_1, 0)$	[0.1 0.2]	[0.1 0.5]	[0.2 0.3]
$(o_1, p_1, 1)(o_1, p_1, 0)$	[0.4 0.5]	[0.4 0.6]	[0.3 0.4]
$(o_2, p_1, 1)(o_1, p_1, 0)$	[0.1 0.2]	[0.1 0.3]	[0.2 0.5]
$(o_2, p_1, 1)(o_1, p_1, 0)$	[0.1 0.2]	[0.1 0.3]	[0.2 0.3]
$(o_2, p_1, 1)(o_1, p_1, 0)$	[0.1 0.2]	[0.1 0.3]	[0.3 0.5]
$(o_3, p_1, 1)(o_1, p_1, 0)$	[0.3 0.4]	[0.2 0.4]	[0.2 0.5]
$(o_3, p_1, 1)(o_1, p_1, 0)$	[0.1 0.2]	[0.1 0.5]	[0.2 0.3]
$(o_3, p_1, 1)(o_1, p_1, 0)$	[0.4 0.5]	[0.4 0.7]	[0.3 0.6]

▹ **Induced interval truth membership function components**

The induced interval truth membership function components of $(\aleph_1, \pounds_1) \wedge (\aleph_1, \perp_1)$ are shown in Table 3.

Table 3. Induced interval truth membership function components.

L	$\mathfrak{l}_1$	$\mathfrak{l}_2$	$\mathfrak{l}_3$
$(o_1, p_1, 1)(o_1, p_1, 0)$	**0.35**	0.30	0.30
$(o_1, p_1, 1)(o_1, p_1, 0)$	0.15	**0.75**	0.25
$(o_1, p_1, 1)(o_1, p_1, 0)$	0.45	**0.50**	0.35
$(o_2, p_1, 1)(o_1, p_1, 0)$	0.15	0.20	**0.35**
$(o_2, p_1, 1)(o_1, p_1, 0)$	0.15	0.20	**0.25**
$(o_2, p_1, 1)(o_1, p_1, 0)$	0.15	0.20	**0.40**
$(o_3, p_1, 1)(o_1, p_1, 0)$	**0.35**	0.30	0.25
$(o_3, p_1, 1)(o_1, p_1, 0)$	0.15	**0.30**	0.25
$(o_3, p_1, 1)(o_1, p_1, 0)$	0.45	**0.55**	0.45

▹ **Scores of interval truth membership function components**

We calculate the scores of the interval truth membership function components by adding the highest bold values in the column of Table 3: $\tilde{S}(\mathfrak{l}_1) = 0.35+0.35 = 0.7$, $\tilde{S}(\mathfrak{l}_2) = 0.75+0.5+0.3+0.55 = 2.1$, $\tilde{S}(\mathfrak{l}_3) = 0.35 + 0.25 + 0.4 = 0.10$.

Now, we repeat the process for the interval indeterminacy membership function components.

▹ **Interval indeterminacy membership function components**

$$(\aleph_2, \pounds_2) = \left\{ \left((o_1, \mathcal{P}_1, 1), \begin{Bmatrix} \langle [0.5, 0.6], [0.1, 0.6], [0.6, 0.7] \rangle, \langle [0.3, 0.7], [0.3, 0.6], [0.1, 0.1] \rangle, \\ \langle [0.7, 0.8], [0.5, 0.8], [0.1, 0.1] \rangle \end{Bmatrix} \right) \right\},$$

$$(\aleph_2, \perp_2) = \left\{ \left((o_1, \mathcal{P}_1, 0), \begin{Bmatrix} \langle [0.4, 0.8], [0.2, 0.8], [0.3, 0.7] \rangle, \langle [0.1, 0.8], [0.5, 0.8], [0.2, 0.7] \rangle, \\ \langle [0.4, 0.7], [0.4, 0.6], [0.1, 0.9] \rangle \end{Bmatrix} \right) \right\}.$$

The operation AND between $(\aleph_2, \pounds_2) \wedge (\aleph_2, \perp_2)$ can be applied as

$$\left\{ \begin{matrix} \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{\mathfrak{l}_1}{[0.5,0.8]}, \frac{\mathfrak{l}_2}{[0.2,0.8]}, \frac{\mathfrak{l}_3}{[0.6,0.7]} \right\} \right) \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{\mathfrak{l}_1}{[0.5,0.8]}, \frac{\mathfrak{l}_2}{[0.5,0.8]}, \frac{\mathfrak{l}_3}{[0.6,0.7]} \right\} \right) \\ \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{\mathfrak{l}_1}{[0.5,0.7]}, \frac{\mathfrak{l}_2}{[0.4,0.6]}, \frac{\mathfrak{l}_3}{[0.6,0.9]} \right\} \right) \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{\mathfrak{l}_1}{[0.4,0.8]}, \frac{\mathfrak{l}_2}{[0.3,0.8]}, \frac{\mathfrak{l}_3}{[0.3,0.7]} \right\} \right) \\ \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{\mathfrak{l}_1}{[0.3,0.8]}, \frac{\mathfrak{l}_2}{[0.5,0.8]}, \frac{\mathfrak{l}_3}{[0.2,0.7]} \right\} \right) \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{\mathfrak{l}_1}{[0.4,0.7]}, \frac{\mathfrak{l}_2}{[0.4,0.6]}, \frac{\mathfrak{l}_3}{[0.1,0.9]} \right\} \right) \\ \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{\mathfrak{l}_1}{[0.7,0.8]}, \frac{\mathfrak{l}_2}{[0.5,0.8]}, \frac{\mathfrak{l}_3}{[0.3,0.7]} \right\} \right) \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{\mathfrak{l}_1}{[0.7,0.8]}, \frac{\mathfrak{l}_2}{[0.5,0.8]}, \frac{\mathfrak{l}_3}{[0.2,0.7]} \right\} \right) \\ \left(((o_1, \mathcal{P}_1, 1), (o_1, \mathcal{P}_1, 0)), \left\{ \frac{\mathfrak{l}_1}{[0.7,0.8]}, \frac{\mathfrak{l}_2}{[0.5,0.8]}, \frac{\mathfrak{l}_3}{[0.1,0.9]} \right\} \right) \end{matrix} \right\}.$$

▹ **Interval indeterminacy membership function components**

The interval indeterminacy membership function components of $(\aleph_2, \pounds_2) \wedge (\aleph_2, \perp_2)$ are shown in Table 4.

Table 4. Interval indeterminacy membership function components.

L	l_1	l_2	l_3
$(o_1, p_1, 1)(o_1, p_1, 0)$	[0.5 0.8]	[0.2 0.8]	[0.6 0.8]
$(o_1, p_1, 1)(o_1, p_1, 0)$	[0.5 0.8]	[0.5 0.9]	[0.6 0.7]
$(o_1, p_1, 1)(o_1, p_1, 0)$	[0.5 0.7]	[0.4 0.6]	[0.6 0.9]
$(o_2, p_1, 1)(o_1, p_1, 0)$	[0.4 0.8]	[0.3 0.8]	[0.3 0.7]
$(o_2, p_1, 1)(o_1, p_1, 0)$	[0.3 0.8]	[0.5 0.8]	[0.2 0.7]
$(o_2, p_1, 1)(o_1, p_1, 0)$	[0.4 0.7]	[0.4 0.6]	[0.1 0.9]
$(o_3, p_1, 1)(o_1, p_1, 0)$	[0.7 0.8]	[0.5 0.8]	[0.3 0.7]
$(o_3, p_1, 1)(o_1, p_1, 0)$	[0.7 0.8]	[0.5 0.8]	[0.2 0.7]
$(o_3, p_1, 1)(o_1, p_1, 0)$	[0.7 0.8]	[0.5 0.8]	[0.1 0.9]

▷ **Induced interval indeterminacy membership function components**

The induced interval indeterminacy membership function components of $(\aleph_2, £_2) \wedge (\aleph_2, \perp_2)$ are shown in Table 5.

Table 5. Induced interval indeterminacy membership function components.

L	l_1	l_2	l_3
$(o_1, p_1, 1)(o_1, p_1, 0)$	0.65	0.50	**0.70**
$(o_1, p_1, 1)(o_1, p_1, 0)$	0.65	**0.70**	0.65
$(o_1, p_1, 1)(o_1, p_1, 0)$	0.60	0.50	**0.75**
$(o_2, p_1, 1)(o_1, p_1, 0)$	**0.60**	0.55	0.50
$(o_2, p_1, 1)(o_1, p_1, 0)$	0.55	**0.65**	0.45
$(o_2, p_1, 1)(o_1, p_1, 0)$	**0.65**	0.50	0.50
$(o_3, p_1, 1)(o_1, p_1, 0)$	**0.75**	0.65	0.50
$(o_3, p_1, 1)(o_1, p_1, 0)$	**0.75**	0.65	0.45
$(o_3, p_1, 1)(o_1, p_1, 0)$	**0.75**	0.65	0.50

▷ **Scores of interval indeterminacy membership function components**.

We calculate the scores of the interval indeterminacy membership function components by adding the highest bold values in the column of Table 5: $\dot{S}(l_1) = 0.6 + 0.65 + 0.75 + 0.75 + 0.75 = 3.5$ $\dot{S}(l_2) = 0.7 + 0.65 = 0.72$, $\dot{S}(l_3) = 0.7 + 0.65 = 0.72$.

Again, we repeat the process for the interval falsity membership function components.

▷ **Interval falsity membership function components**

$$(\aleph_3, £_3) = \left\{ \left((o_3, \mathcal{P}_3, 1), \begin{Bmatrix} \langle [0.3, 0.5], [0.3, 0.4], [0.6, 0.9] \rangle, \langle [0.1, 0.2], [0.1, 0.7], [0.2, 0.9] \rangle, \\ \langle [0.4, 0.7], [0.2, 0.9], [0.3, 0.6] \rangle \end{Bmatrix} \right) \right\},$$

$$(\aleph_3, \perp_3) = \left\{ \left((o_3, \mathcal{P}_3, 0), \begin{Bmatrix} \langle [0.2, 0.3], [0.1, 0.3], [0.1, 0.5] \rangle, \langle [0.1, 0.6], [0.2, 0.6], [0.2, 0.3] \rangle, \\ \langle [0.3, 0.5], [0.4, 0.8], [0.3, 0.7] \rangle \end{Bmatrix} \right) \right\}.$$

The operation AND between $(\aleph_3, £_3) \wedge (\aleph_3, \perp_3)$ can be applied as

$$\left\{\begin{array}{ll} \left(((o_3,\mathcal{P}_3,1),(o_3,\mathcal{P}_3,0)),\left\{\frac{l_1}{[0.3,0.5]},\frac{l_2}{[0.3,0.4]},\frac{l_3}{[0.6,0.9]}\right\}\right) & \left(((o_3,\mathcal{P}_3,1),(o_3,\mathcal{P}_3,0)),\left\{\frac{l_1}{[0.3,0.6]},\frac{l_2}{[0.3,0.6]},\frac{l_3}{[0.6,0.9]}\right\}\right) \\ \left(((o_3,\mathcal{P}_3,1),(o_3,\mathcal{P}_3,0)),\left\{\frac{l_1}{[0.3,0.5]},\frac{l_2}{[0.4,0.8]},\frac{l_3}{[0.6,0.9]}\right\}\right) & \left(((o_3,\mathcal{P}_3,1),(o_3,\mathcal{P}_3,0)),\left\{\frac{l_1}{[0.2,0.3]},\frac{l_2}{[0.1,0.7]},\frac{l_3}{[0.2,0.9]}\right\}\right) \\ \left(((o_3,\mathcal{P}_3,1),(o_3,\mathcal{P}_3,0)),\left\{\frac{l_1}{[0.3,0.5]},\frac{l_2}{[0.4,0.8]},\frac{l_3}{[0.3,0.9]}\right\}\right) & \left(((o_3,\mathcal{P}_3,1),(o_3,\mathcal{P}_3,0)),\left\{\frac{l_1}{[0.3,0.5]},\frac{l_2}{[0.4,0.8]},\frac{l_3}{[0.2,0.9]}\right\}\right) \\ \left(((o_3,\mathcal{P}_3,1),(o_3,\mathcal{P}_3,0)),\left\{\frac{l_1}{[0.4,0.7]},\frac{l_2}{[0.2,0.9]},\frac{l_3}{[0.3,0.5]}\right\}\right) & \left(((o_3,\mathcal{P}_3,1),(o_3,\mathcal{P}_3,0)),\left\{\frac{l_1}{[0.4,0.7]},\frac{l_2}{[0.4,0.9]},\frac{l_3}{[0.4,0.7]}\right\}\right) \\ \left(((o_3,\mathcal{P}_3,1),(o_3,\mathcal{P}_3,0)),\left\{\frac{l_1}{[0.4,0.7]},\frac{l_2}{[0.4,0.9]},\frac{l_3}{[0.3,0.7]}\right\}\right) & \end{array}\right\}.$$

▷ **Interval falsity membership function components**

The interval falsity membership function components of $(\aleph_3, £_3) \wedge (\aleph_3, \perp_3)$ are shown in Table 6.

Table 6. Interval indeterminacy membership function components.

L	l_1	l_2	l_3
$(o_1, p_1, 1)(o_1, p_1, 0)$	[0.3 0.5]	[0.3 0.4]	[0.6 0.9]
$(o_1, p_1, 1)(o_1, p_1, 0)$	[0.3 0.6]	[0.3 0.6]	[0.6 0.9]
$(o_1, p_1, 1)(o_1, p_1, 0)$	[0.3 0.5]	[0.4 0.8]	[0.6 0.9]
$(o_2, p_1, 1)(o_1, p_1, 0)$	[0.2 0.3]	[0.1 0.7]	[0.2 0.9]
$(o_2, p_1, 1)(o_1, p_1, 0)$	[0.3 0.5]	[0.4 0.8]	[0.3 0.9]
$(o_2, p_1, 1)(o_1, p_1, 0)$	[0.3 0.5]	[0.4 0.8]	[0.2 0.9]
$(o_3, p_1, 1)(o_1, p_1, 0)$	[0.4 0.7]	[0.2 0.9]	[0.3 0.5]
$(o_3, p_1, 1)(o_1, p_1, 0)$	[0.4 0.7]	[0.4 0.9]	[0.4 0.7]
$(o_3, p_1, 1)(o_1, p_1, 0)$	[0.4 0.7]	[0.4 0.9]	[0.3 0.7]

▷ **Induced interval falsity membership function components**

The induced interval falsity membership function components of $(\aleph_3, £_3) \wedge (\aleph_3, \perp_3)$ are shown in Table 7, and they are obtained by taking the averaging value of each interval.

Table 7. Induced interval indeterminacy membership function components.

L	l_1	l_2	l_3
$(o_1, \mathbb{P}_1, 1)(o_1, \mathbb{P}_1, 0)$	0.40	0.35	**0.75**
$(o_1, \mathbb{P}_1, 1)(o_1, \mathbb{P}_1, 0)$	0.45	0.45	**0.75**
$(o_1, \mathbb{P}_1, 1)(o_1, \mathbb{P}_1, 0)$	0.40	0.60	**0.75**
$(o_1, \mathbb{P}_1, 1)(o_1, \mathbb{P}_1, 0)$	0.25	0.40	**0.65**
$(o_1, \mathbb{P}_1, 1)(o_1, \mathbb{P}_1, 0)$	0.40	**0.60**	0.50
$(o_1, \mathbb{P}_1, 1)(o_1, \mathbb{P}_1, 0)$	0.40	**0.60**	0.55
$(o_1, \mathbb{P}_1, 1)(o_1, \mathbb{P}_1, 0)$	**0.55**	0.45	0.40
$(o_1, \mathbb{P}_1, 1)(o_1, \mathbb{P}_1, 0)$	**0.65**	0.55	0.45
$(o_1, \mathbb{P}_1, 1)(o_1, \mathbb{P}_1, 0)$	**0.56**	0.55	0.50

▷ **Scores of interval falsity membership function components**

We calculate the scores of the interval falsity membership function components by adding the highest

bold values in the column of Table 7:
$\ddot{S}(l_1) = 0.55+0.65+0.56 = 1.76$, $\ddot{S}(l_2) = 0.6+0.6 = 1.2$, $\ddot{S}(l_3) = 0.75+0.75+0.75+0.65 = 2.90$.

▷ **Decision**

The final decision can be made by evaluating the final scores in the following way:
$S(l_i) =| \tilde{S}(l_i) + \dot{S}(l_i) + \ddot{S}(l_i)/3 |$, and it is shown in Table 8.

Table 8. Final decision.

$\tilde{S}(l_1)$	$\dot{S}(l_2)$	$\ddot{S}(l_3)$
$SV = 0.7$	$SV = 3.5$	$SV = 1.76$
$SV = 2.1$	$SV = 1.35$	$SV = 1.2$
$SV = 1.0$	$SV = 1.45$	$SV = 2.90$
$3.8/3 = 1.0267$	$6.30/3 = 2.100$	$5.86/3 = 1.9533$

Because l_2 is satisfactory, the alternative l_2 is adopted. Figure 5 shows the graphical representation of ranking of alternatives.

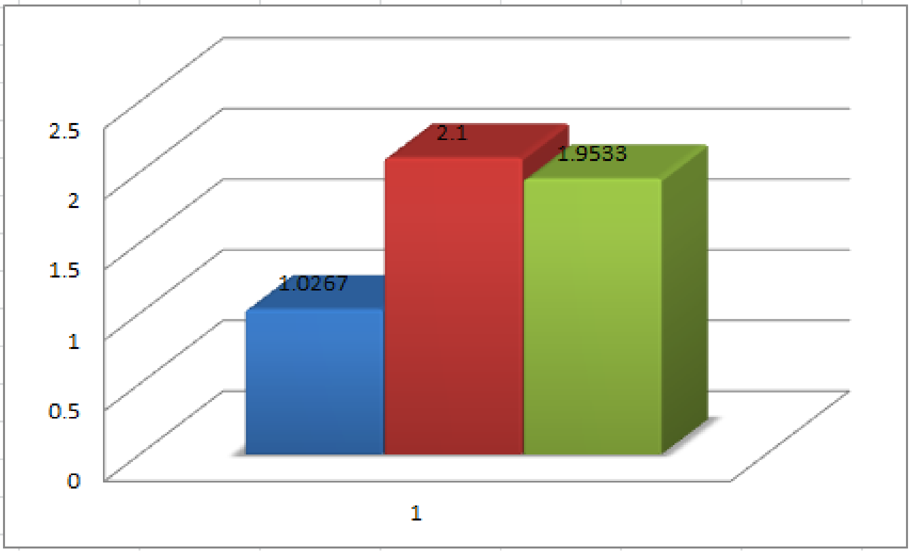

Figure 5. Graphical representation of ranking.

5. Discussion, sensitivity and comparison analysis

In this study, and for the readers, the proposed approach has been evaluated in terms of computational straightforwardness and logical reflection. It is assumed that the presented approach realizes the adopted decisive factors better than pre-developed approaches. Therefore, it is confirmed that this approach is superior in terms of both measures.

1. Companies have been turning out to be more prescriptive about their procurement approaches. Procurement has become a significant aspect of production groups, and it shapes their effectiveness, endurance and growth. This study highlights that SS decisions have become mandatory for success in the current competitive environment.

2. The major issue raised in the literature review regards the expert-identified factors of the buyer-seller relationship. Although the views of experts reflect their partiality with respect to each decisive factor, the computational process may stand out as influenced. Therefore, it would be imperative for researchers to scrutinize whether such influences are shown in their own studies by examining whether their computational results match those obtained by other researchers. However, strict observation is warranted for this purpose as well due to the limited number of sources available for each study conducted by researchers during this vast era.
3. The majority of strategies for MADM search use knowledge-based strategies, which have also been applied in the SS process. In this regard, we require a series of questionnaires to be defined, but it is not clear how much more it will help to improve the results. In other words, the knowledge base for MADM is built upon established knowledge about their existence, characteristics and functionalities. It provides a clear idea about how far such information can be used in decision-making by specifying the types of decision problems that may arise during MADM decision-making. The latter is significant at the moment when the knowledge base becomes inadequate under some circumstances, as it may hinder the overall goal of any method or technique of decision-making, which includes the SS process.
4. The above discussion proved that the researchers Xiao et al. [39], Zhao et al. [41] and Rouyendegh et al. [42] have dedicated a significant amount of research work to the topic. The contributions of these researchers are seen as most suitable for the SS process. But, in these structures, soft expert and interval-valued settings have been found to be missing. The suggested structure is highly effective in controlling all of the above features of the project effectively, collectively and efficiently.
5. With the proposed approach and its computational results, it is shown that the proposed approach is thoroughly distinct from existing approaches, so it may not be comparable with existing models. Table 9 summarizes the structural comparison among different approaches. The evaluating factors in this decision are as follows: (a) multi-decisive opinions (IES); (b) single-argument approximate function (ETEN); (c) fuzzy membership function (FMF); (d) intuitive fuzzy non-membership function (IFNMF); (e) neutrosophic values (NV) and (f) interval-valued conditions (IVC). Different structures of SS, which have been discussed in the literature review section, are compared with certain characteristics. The purpose of the first feature is to check the multi-decision opinions of different experts in a single model. The second property is applied to the assessment of a soft-set environment. The third one is applied to test whether or not the impreciseness of experts' opinions was tackled in an appropriate way. The fourth one is applied to see the non-membership functions in different structures, and the next one represents indeterminacy. The last characteristic is for structural interval-valued settings. A comparison analysis is shown in Tables 10, 11 and 12. The symbols ¥ and ℵ are written here for yes and no, respectively.
6. Similar to Example 4.1, the notion of arithmetic mean is applied to determine the score values of alternatives based on parametric values, and, as a result, results are seen. The calculated scores, however, might change if other methods are used to identify score values that unquestionably change the results. The following cases of geometric and harmonic means can be taken into consideration in the following regard.

Table 9. Comparison analysis.

Models/Features	IES	ETEN	FMF	IFNMF	NV	IVC
Hoseini et al. [30]	ℵ	ℵ	¥	ℵ	ℵ	ℵ
Aretoulis et al. [36]	ℵ	ℵ	¥	¥	ℵ	ℵ
Safa et al. [37]	ℵ	ℵ	¥	¥	¥	ℵ
Xiao et al. [39]	ℵ	¥	ℵ	ℵ	ℵ	ℵ
Zhao et al. [41]	ℵ	¥	¥	¥	ℵ	ℵ
Rouyendegh et al. [42]	ℵ	¥	¥	¥	¥	ℵ
Zhang et al. [43]	ℵ	¥	¥	¥	¥	ℵ
Proposed Model	¥	¥	¥	¥	¥	¥

Case 1. The geometric means of the items in their corresponding rows of the choice phase are used to calculate the score values for the alternatives.

Table 10. Scores of alternatives calculated by using the geometric mean.

Alternatives	Scores
$\mathfrak{l}_1$	1.1370
$\mathfrak{l}_2$	1.8992
$\mathfrak{l}_3$	1.8296

Case 2. The harmonic means of the items in their corresponding rows of the choice phase are used to calculate the score values for the alternatives.

Table 11. Scores of alternatives calculated by using the harmonic mean.

Alternatives	Scores
$\mathfrak{l}_1$	1.0328
$\mathfrak{l}_2$	1.7481
$\mathfrak{l}_3$	1.7179

Table 12. Computational results-based comparison.

Alternatives	$\mathfrak{l}_1$	$\mathfrak{l}_2$	$\mathfrak{l}_3$	Ranking
A.M	1.3667	2.100	1.9533	$\mathfrak{l}_2 > \mathfrak{l}_3 > \mathfrak{l}_1$
G.M	1.1370	1.8992	1.8296	$\mathfrak{l}_2 > \mathfrak{l}_3 > \mathfrak{l}_1$
H.M	1.0328	1.7481	1.7179	$\mathfrak{l}_2 > \mathfrak{l}_3 > \mathfrak{l}_1$

Figure 6 shows the graphical representation of ranking of alternatives.

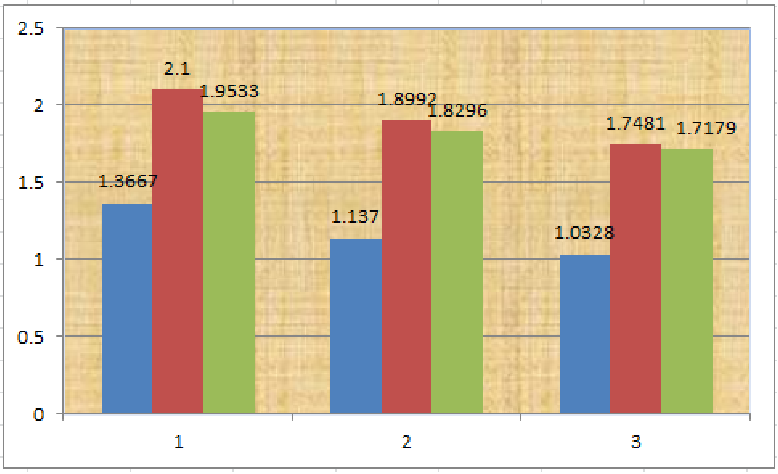

Figure 6. Graphical representation of ranking.

6. Flexibility of proposed structure

Here, a meaningful discussion about the flexibility of this structure, i.e., the $\mathfrak{I}$vnse-set, is made.

1. The $\mathfrak{I}$vnse-set becomes an $\mathfrak{I}$vns-set if the expert set is excluded.
2. It becomes an $\mathfrak{I}$vifse-set if indeterminacy is ended.
3. It becomes an $\mathfrak{I}$vfse-set if falsity and indeterminacy are removed.
4. It becomes an $\mathfrak{I}$vfs-set if falsity, indeterminacy and the expert set are removed.
5. It becomes an $\mathfrak{I}$vf-set if falsity, indeterminacy and the expert set are excluded.
6. It changes into an $\mathfrak{N}$se-set if the interval-valued condition is removed.
7. It changes into an $\mathfrak{N}$s-set if the interval-valued condition and the expert set are removed.
8. It becomes an $\mathfrak{N}$-set if the interval-valued condition, expert set and soft set are omitted.
9. It takes the form of an intuitionistic fuzzy set when the indeterminacy, the interval-valued condition, the soft set and the expert set are fuzzy are excluded.
10. It takes the form of the fuzzy set when the indeterminacy, falsity, the interval-valued condition, the soft set and the expert set are omitted.

6.1. Advantages of the proposed structure

The $\mathfrak{I}$vnse-set has the following advantages.

1. The suggested work is very useful and has great importance in dealing with real-life decision-making issues.
2. The current model focuses on the primary investigation of characteristics under the conditions of multi-decisive opinions. It pursues the best decision-making path, is delicate and is additionally stable.
3. The suggested model has all of the features and characteristics of existing models, like the $\mathfrak{I}$vf-set, $\mathfrak{I}$vif-set, $\mathfrak{I}$vn-set, $\mathfrak{I}$vfse-set and $\mathfrak{I}$vifse-set.

The benefits of $\mathfrak{I}$vnse-sets are shown in Table 13. In this table, some prominent characteristics of existing models are compared with those of $\mathfrak{I}$vnse-set. These characteristics include truth membership

(TM), false membership (FM), indeterminate membership (IM), interval-valued condition (IV), single-argument approximate function (SAAF) and multi-decisive opinion (MDO). Yes and no are respectively be denoted by $\uparrow$ and $\downarrow$ in the following Table 13.

Table 13. Comparison with some structures.

Contributors	Structures	TM	FM	IM	SAAF	MDO	IV
Gorzalczany [18]	$\mathfrak{I}$vf-set	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Atanassov et al. [19]	$\mathfrak{I}$vif-set	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Wang et al. [20]	$\mathfrak{I}$vn-set	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$
Yang et al. [21]	$\mathfrak{I}$vfs-set	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Jiang et al. [22]	$\mathfrak{I}$vifs-set	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Deli [23]	$\mathfrak{I}$vns-set	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$
Proposed model	$\mathfrak{I}$vnse-set	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$

From the Table 13, it is obvious that the one we suggest is more versatile than the models mentioned before.

6.2. *Objective comparison analysis*

The $\mathfrak{I}$vnse-set is a recently developed decision-making technique that blends soft expert sets and interval-valued neutrosophic sets. It offers a flexible framework to handle ambiguity and insufficient information in problem-solving situations. The particular context and demands of the current DMP must be taken into account in order to fairly compare the $\mathfrak{I}$vnse-set with alternative approaches. However, the following broad comparisons between the $\mathfrak{I}$vnse-set and other widely used decision-making techniques are provided:

▷**$\mathfrak{I}$vnse-set vs. uzzy sets:**

Both $\mathfrak{I}$vnse-sets and fuzzy sets are methods that can handle uncertainty and imprecision in decision-making. However, $\mathfrak{I}$vnse-sets can better deal with conflicting and indeterminate information since it uses a three-component representation (truth, indeterminacy and falsity) rather than the component representation (truth) used in fuzzy sets.

▷**$\mathfrak{I}$vnse-set vs. intuitionistic fuzzy sets:**

Intuitionistic fuzzy sets extend the notion of fuzzy sets by introducing the concept of falsity in addition to the degrees of membership and non-membership. The $\mathfrak{I}$vnse-set also deals with falsity but uses the neutrosophic component to represent it, which provides a more precise representation of uncertainty.

▷**$\mathfrak{I}$vnse-set vs. MCDM**:

MCDM methods involve a comparison of alternatives based on multiple criteria. The $\mathfrak{I}$vnse-set can be used as a tool within MCDM to handle uncertain and incomplete information in the criteria and preference evaluations.

▷**$\mathfrak{I}$vnse-set vs. AHP**:

Pairwise comparisons and hierarchical structures are used in the common decision-making process called the AHP. In pairwise comparisons and criteria evaluations, the $\mathfrak{I}$vnse-set can be used as a tool within the AHP to handle uncertain and partial information. The AHP derives weights for criteria and alternatives through pairwise comparisons, which may result in inconsistent judgements.

Contrarily, the $\mathfrak{I}$vnse-set communicates uncertainty through interval-valued neutrosophic sets, allowing for a more adaptable and accurate representation of insufficient and contradictory information. The AHP heavily relies on expert knowledge and demands that the experts make reliable assessments in pairwise comparisons. The $\mathfrak{I}$vnse-set similarly relies on expert knowledge, but it allows for language expressions and soft constraints, making it more appropriate in circumstances when expert knowledge is ambiguous or lacking. In conclusion, $\mathfrak{I}$vnse-sets constitute versatile decision-making technique that can deal with different kinds of uncertainty and insufficient knowledge. Compared to other techniques, its distinctive three-component model offers a more accurate representation of uncertainty. However, the particular context and demands of the DMP will determine how well the $\mathfrak{I}$vnse-set fits the bill.

▷ **$\mathfrak{I}$vnse-set vs. COPRAS**:

While COPRAS employs a linguistic term set to convey uncertain information, the $\mathfrak{I}$vnse-set uses interval-valued neutrosophic sets. As opposed to language concepts, which might not be able to express the subtleties of uncertainty as effectively, interval-valued neutrosophic sets provide a more accurate and adaptable representation of partial and inconsistent information. Both the $\mathfrak{I}$vnse-set and COPRAS rely on expert knowledge; however, the $\mathfrak{I}$vnse-set permits the expression of ambiguous information and soft constraints, while COPRAS calls for precise language judgments from experts. While $\mathfrak{I}$vnse-sets enable a more flexible and accurate representation of decision-making criteria, which can be interval-valued, neutrosophic or soft, COPRAS employs a weighted aggregation approach to identify the optimal alternative.

▷ **$\mathfrak{I}$vnse-set vs. MARCOS-G**:

While MARCOS-G handles uncertain information using the rank-order centroid similarity method, the $\mathfrak{I}$vnse-set depicts uncertainty using interval-valued neutrosophic sets. In contrast to MARCOS-G, which employs rank-order centroid similarity to describe uncertainty, interval-valued neutrosophic sets allow for a more accurate and adaptable representation of imperfect and inconsistent information. For decision-makers who are unfamiliar with interval-valued neutrosophic sets, MARCOS-G may be simpler to use and comprehend. However, the $\mathfrak{I}$vnse-set offers a more transparent representation of the issue and can aid decision-makers in better understanding the sources and nature of uncertainty in the decision-making process.

▷ **$\mathfrak{I}$vnse-set vs. FBWM-FIS**:

The advantages of $\mathfrak{I}$vnse-set over FBWM-FIS are as follows:

More flexible representation of uncertainty: In contrast to the fuzzy sets used in FBWM-FIS, the $\mathfrak{I}$vnse-set enables experts to express their opinions as intervals, which offers a more flexible and accurate representation of their uncertainty.

Easy to use: The $\mathfrak{I}$vnse-set, unlike FBWM-FIS, does not necessitate in-depth familiarity with fuzzy logic or mathematical programming, making it simple to utilize in decision-making issues.

Better accuracy: When the expert opinions are very ambiguous or contradictory, the $\mathfrak{I}$vnse-set can yield results that are more precise than FBWM-FIS.

Robustness: When dealing with ambiguous or partial information, the $\mathfrak{I}$vnse-set is more resilient than FBWM-FIS since it can manage missing or unknown values better.

Transparency: The $\mathfrak{I}$vnse-set gives decision-makers a transparent and comprehensible approach to represent expert opinions, making it simpler for them to comprehend and convey the analysis' findings.

▷ **$\mathfrak{I}$vnse-set vs. Model based on fuzzy PCA**:

The advantages of $\mathfrak{I}$vnse-set over fuzzy PCA are as follows:

Handling of uncertainty: The $\mathfrak{J}$vnse-set is better equipped to deal with ambiguity and haziness in professional judgments. Uncertainty in the data is not taken into consideration by fuzzy PCA.
Flexible representation: Expert opinions can be represented as intervals in a more flexible manner thanks to the $\mathfrak{J}$vnse-set. A data-driven method that depends on the input data is fuzzy PCA.
Better accuracy: When dealing with ambiguous and hazy expert opinions, the $\mathfrak{J}$vnse-set can yield more precise findings than fuzzy PCA.
Transparency: The $\mathfrak{J}$vnse-set gives decision-makers a transparent and comprehensible approach to represent expert opinions, making it simpler for them to comprehend and convey the analysis' findings.

6.3. Advantages of the proposed method

The $\mathfrak{J}$vnse-set has several advantages over existing fuzzy and soft set-like structures in the SS process. Some of these advantages are as follows:

▷ **More flexible representation of information:**

The $\mathfrak{J}$vnse-set allows for the representation of information in a more flexible way, which can better reflect the expert's opinions about the supplier's performance on various criteria. This is because the $\mathfrak{J}$vnse-set allows for the incorporation of linguistic terms and soft information, which may not be easily represented using traditional fuzzy and soft set-like structures.

▷**Ability to handle incomplete and indeterminate information:**

$\mathfrak{J}$vnse-sets can handle incomplete and indeterminate information more efficiently than traditional fuzzy and soft set-like structures. This is because $\mathfrak{J}$vnse-sets can represent uncertain information in terms of intervals and the degree of indeterminacy using neutrosophic sets.

▷ **Incorporation of expert opinions:**

The $\mathfrak{J}$vnse-set allows for the incorporation of multiple experts' opinions in the decision-making process. This can help to reduce bias and increase the accuracy of the decision.

▷ **More comprehensive evaluation of suppliers:**

The $\mathfrak{J}$vnse-set can provide a more comprehensive evaluation of suppliers by considering both quantitative and qualitative criteria. This can lead to a more informed decision that takes into account a broader range of factors.

▷ **Better handling of imprecise and vague data:**

The $\mathfrak{J}$vnse-set can handle imprecise and vague data more efficiently than traditional fuzzy and soft set-like structures. This is because the $\mathfrak{J}$vnse-set allows for the representation of imprecise and vague data in terms of linguistic terms, which can provide a more accurate reflection of the experts' opinions.
In summary, the advantages of using the $\mathfrak{J}$vnse-set in the SS process over existing fuzzy and soft set-like structures are its more flexible representation of information, ability to handle incomplete and indeterminate information, incorporation of expert opinions, more comprehensive evaluation of suppliers and better handling of imprecise and vague data.

7. Conclusions

The concepts of the $\mathfrak{J}$vnse-set are discussed in this article, along with its fundamental operations, such as union, intersection, AND and OR operations. All of these elements help to reinforce the study's methodology. An effort has been made to comprehend the nature of attributes and how they can be used effectively in supply chain management under the conditions of multi-decisive opinions. The discussion of a decision-making algorithm that uses the $\mathfrak{J}$vnse-set and its fundamental operations is based on a real-world application for SS in a building project. A structural comparison of different approaches is presented. In particular, a preliminary comparison of the proposed approach with some existing approaches is made. The advantages are judged through these comparisons. This study has been developed on the basis of $\mathfrak{J}$vnse-set, and the flexibility of the proposed structure is much better as compared to the existing structures involving SS processes. The main reason behind this flexibility is the multi-decisive opinions within a single model. But, still, there are certain situations where this structure is not useful: (a) when attributes exist in the form of sub-attributes, and these sub-attributes have disjoint set values (hypersoft set environment), (b) when certain types of data exist with a periodic nature (complex form), (c) when data and information have a rough nature, (d) when attributes exist in the form of sub-attributes and these sub-attributes have overlapping set values (multi-soft set environment). We used an empirical example to assess how well the suggested approach performed while choosing suppliers for the construction project. To determine the best supplier to provide all different forms of construction supplies for their building projects when given three primary choices, multi-decisive opinions were used. As a result, the second option was given priority over the others. Additionally, the performance and validity of the introduced methodology were proven by comparisons between the proposed model and other approaches, like COPRAS, AHP, MCDM and MARCOS-G. Additionally, the final ranking results were analyzed after the sensitivity analysis was used to calculate the score values for alternatives based on parametric values. By adding more qualities, the proposed study can be easily expanded as a case study for various uncertain environments, such as fuzzy-set like models with neutrosophic settings, in order to avoid computational complexity while maintaining the simplicity and generality of the framework it provides for the SS process. It can be expanded to include structures that resemble hypersoft structures under the condition of MADM, where attributes are further separated into disjoint attribute-valued sets. A hybridized study of structures like the interval-valued neutrosophic hypersoft set and its uses in SS projects for construction may also be included in future work for the single-argument approximate function for soft expert set environments.

Use of AI tools declaration

The authors declare that they have not used artificial intelligence tools in the creation of this article.

Conflict of interest

The authors declare no conflict of interest.

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