Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 8(9): 21069–21088.
DOI:10.3934/math.20231073
Received: 03 May 2023
Revised: 13 June 2023
Accepted: 24 June 2023
Published: 03 July 2023

Research article

Applications of picture fuzzy filters: performance evaluation of an employee using clustering algorithm

K. Tamilselvan[1], **V. Visalakshi**[1] **and Prasanalakshmi Balaji**[2,*]

[1] Department of Mathematics, Faculty of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur, Tamil Nadu, India

[2] Department of Computer Science, King Khalid University, Abha-62529, Saudi Arabia

* **Correspondence:** Email: prengaraj@kku.edu.sa.

Abstract: This article defines the concepts of picture fuzzy filter, picture fuzzy grill, picture fuzzy section, picture fuzzy base, picture fuzzy subbase, picture fuzzy ultrafilter, as well as their fundamental features. Characteristics of the aforementioned concepts are addressed, and equivalence between the picture fuzzy filter and picture fuzzy grills is established. Real-world examples are offered to demonstrate the advantages of picture fuzzy filters in the classification of sets using a clustering technique. Illustration is provided to show the advantages of picture fuzzy sets and the results are compared with intuitionistic fuzzy sets. Clustering technique is applied to the picture fuzzy filter collection reduces the computational process which helps the decision makers to classify the sets with fewer iterations.

Keywords: picture fuzzy filter; picture fuzzy grill; picture fuzzy section; picture fuzzy ultrafilter; picture fuzzy normal family; clustering analysis for picture fuzzy sets
Mathematics Subject Classification: 03E72, 54A40

1. Introduction

Zadeh [32] inculcated the theory of vagueness and uncertainty into a new class of fuzzy sets. Contributions to the theory of ambiguousness play a significant part in solving many predicament problems involved with impreciseness. Applications of fuzzy sets are extended and disseminated to various fields such as information [22], control [23], robotics [14–16], etc.

Chang [8] has made a promising contribution to applying fuzzy sets in topological structures. Atanassov [6, 7] generalized the fuzzy set intuitionistically, and the intuitionistic fuzzy set theory emerged. Coker [9] developed the theory on intuitionistic fuzzy set. The fuzzy set theory provides the degree of membership, while the intuitionistic fuzzy set also aggregates the degree of non-membership.

B. Cong and V. Kerinovich [11] defined the concept of picture fuzzy set, which was deduced from the fuzzy set and intuitionistic fuzzy set. Abdul Razaaq et al. [19] defined the rank of picture fuzzy topological space and properties related to continuous functions. Tareq M. Al-Shami et al. [1] introduced SR fuzzy set and its relationship with generalizations of fuzzy sets, weighted aggregated operators to facilitate the multiattribute decision makers. An effective approach in decision making problems using aggregation operations for (m,n) fuzzy sets are established by Tareq M. Al-Shami et al. [2]. Multi criteria decision making problems under (a,b) fuzzy soft set and (2,1) fuzzy sets are obtained by aggregated operators defined by Tareq M. Al-Shami et al. [3, 4]. A new fuzzy ordered weighted averaging (OWA) operator is proposed by Juan-juan Peng et al. [17] to solve the aggregation problem associated with many fuzzy numbers. Moreover, various operators are defined with their desirable properties. Chao Tian et al. [26] developed the weighted picture fuzzy power Choquet ordered geometric (WPFPCOG) operator and a weighted picture fuzzy power Shapley Choquet ordered geometric (WPFPSCOG) operator based on fuzzy measure to deal with multi criteria decision making problems. Sustainability evaluation index system for water environment treatment public-private-partnership (WET-PPP) projects is constructed by Chao Tian et al. [27] to improve the accuracy of decision-making problems and applied effectively to evaluation problems.

Picture fuzzy set has adequate applications in various situations involving many human perspectives in addition to yes, no, refusal, etc. Picture fuzzy set incorporates the degree of neutrality, membership, and non-membership. Manufacturing the components in a fabrication industry by an employee emulates the picture fuzzy set, where the completion of the product by the employee is the degree of membership, the incomplete products contribute the degree of non-membership, and the damaged product is the degree of neutrality.

Statistical data analysis can be effectively implemented by clustering analysis techniques, which are extensively applied in several domains, such as pattern recognition, microbiology analysis, data mining, information retrieval, etc. In an empirical world, the data considered for clustering may be linguistic and uncertain. Abundant clustering algorithms corresponding to various fuzzy environments have been proposed, e.g., intuitionistic clustering algorithm [29, 30] concerning the correlation coefficient formulas for IFSs, classification of picture fuzzy sets using correlation coefficients [21] and Sanchez et al. [20] created a new method, Fuzzy Granular Gravitational Clustering Algorithm (FGGCA) and also compared FGGCA with other clustering techniques. The correlation coefficient analyzes the association and interdependencies between variables. The correlation coefficient is observed under probability distribution in classical statistics, whereas many real situations are subjective. Correlation coefficients between intuitionistic fuzzy sets are applied to linguistic variables, which overcome the limitations obtained in fuzzy correlation measures. The correlation coefficient between two picture fuzzy sets is the one in which the membership values have different and unique consequences, which helps the decision makers to classify their attributes more effectively. Picture fuzzy clustering [25] is one of the computational intelligence methods used in pattern recognition. The enhancement of the traditional and intuitionistic fuzzy sets is picture fuzzy sets. In computational intelligence, a picture fuzzy set provides a better clustering quality than other admissible clustering algorithms involved with different fuzzy sets.

Picture fuzzy set has a robust application in medical diagnosis [12]. In the medical diagnosis of a specific disease, some symptoms do not directly affect the particular disorder, and those symptoms have neutral membership. In this way, the picture fuzzy set constitutes a good effect on the medical diagnosis. The topological structures: filters, grills, clusters, etc., have many applications in the field of pattern analysis in the context of camouflaged objects [18], their applications of obtaining a C structure compactification [28], intuitionistic fuzzy C-ends [31] and Q neighbourhoods, infra fuzzy topological spaces, infra fuzzy homeomorphism, infra fuzzy isomorphisms [5] triggered us to define the picture fuzzy topological structures like filters, grills, ultrafilter. Picture fuzzy filters have a wide variety of applications in the field of Science and Technology, including pattern recognition, image analysis, digital image processing, and forgery detection. Picture fuzzy filters, picture fuzzy grills, and picture fuzzy ultrafilters may contribute to better analysis of various pattern recognition in the context of camouflaged objects.

Many clustering algorithms are available to classify the data set among picture fuzzy sets, which reflects the significance of the degree of positive, negative and neutral membership. In this paper, the clustering algorithm defined using the correlation coefficient between picture fuzzy sets belonging to the picture fuzzy filter collection enhances the data set's classification method. A straightforward approach based on the picture fuzzy filter is applied to the clustering algorithm, which classifies the data set more effectively in the picture fuzzy topological space domain than the other existing classifications. Classification of picture fuzzy clusters among the picture fuzzy filter collection of any cardinality can be obtained at the fourth stage of the iteration process of the equivalent coefficient matrix involved in the clustering algorithm employed in the paper. An illustration is provided in this paper to experience the ease of classification using a picture fuzzy filter collection. It is compared with some intuitionistic fuzzy set collection and intuitionistic fuzzy filter collection.

The paper is structured as follows: Section 2 deals with the fundamentals of Picture fuzzy sets and the corresponding topological structures. Section 3 explores the fundamental properties of various structures like picture fuzzy filter, grill and ultrafilter. Section 4 deals with an illustration of the application of the clustering algorithm of picture fuzzy sets. For a practical example, the cotton industry is considered. The primary four processes involved in producing yarn are assumed as the attributes. An employee whose performance is based on completion, damage and incomplete of the product plays the role of picture fuzzy sets. A clustering algorithm for picture fuzzy sets is applied to the filter collection to classify the picture fuzzy sets in a filter collection, which helps the industry analyze the employee's performance. The abbreviations and acronyms used in the paper are listed in Table 1.

Table 1. List of abbreviations and acronyms used in the paper.

Abbreviations	Definitions
PFS	Picture Fuzzy Set
PFS(X)	Collection of all Picture Fuzzy Sets on X
PFTS	Picture Fuzzy Topological Space
IFS	Intuitionistic Fuzzy Set
IFTS	Intuitionistic Fuzzy Topological Space
PFOS	Picture Fuzzy Open Set
PFCS	Picture Fuzzy Closed Set
PFNF	Picture Fuzzy Normal Family
int(D)	Interior of D
cl(D)	Closure of D
$E_p(D)$	Informational energy of D
$C_{p_2}(D,E)$	Correlation between D and E
$K_{p_3}(D,E)$	Correlation Coefficient between D and E
$PFsec(\mathfrak{C})$	Picture Fuzzy Section of ($\mathfrak{C}$)
M_C	Correlation Matrix
C	Association Matrix

2. Preliminaries

2.1. Picture fuzzy sets and associated topological space

Definition 2.1. *[10] A picture fuzzy set(PFS) D on X is of the form of* $D=\{(x, \gamma_D(x), \nu_D(x), \eta_D(x))|x \in X\}$*. In this form* $\gamma_D(x)$, $\nu_D(x)$, $\eta_D(x)$ *denote the degree of positive membership, the degree of negative membership, the degree of neutral membership of x in D respectively which satisfying,* $\forall x \in X, \gamma_D(x) + \nu_D(x) + \eta_D(x) \leq 1$*. The degree of refusal membership of x in D is given by* $\rho_D(x) = (1 - (\gamma_D(x) + \nu_D(x) + \eta_D(x)))$*. Such collection of sets is represented as* $PFS(X)$*.*

Definition 2.2. *[10] Let D and E any two PFS s, then*

(i) $D \subseteq E$ *iff* ($y \in X$, $\gamma_D(y) \leq \gamma_E(y)$ *and* $\nu_D(y) \geq \nu_E(y)$ *and* $\eta_D(y) \leq \eta_E(y)$);
(ii) $D = E$ *iff* ($D \subseteq E$ *and* $E \subseteq D$);
(iii) $D \cup E=\{(x, \vee(\gamma_D(x), \gamma_E(x)), \wedge(\nu_D(x), \nu_E(x)), \wedge(\eta_D(x), \eta_E(x)))|x \in X\}$;
(iv) $D \cap E=\{(x, \wedge(\gamma_D(x), \gamma_E(x)), \vee(\nu_D(x), \nu_E(x)), \wedge(\eta_D(x), \eta_E(x)))|x \in X\}$;
(v) $CO(D) = \overline{D} = \{(\nu_D(x), \gamma_D(x), \eta_D(x))|x \in X\}$.

Definition 2.3. *[10] Some Special PFS s are as follows:*

(i) *A constant picture fuzzy set is the PFS* $(\widehat{\vartheta, \varepsilon, \varrho}) = \{(y, \vartheta, \varepsilon, \varrho)|y \in X\}$;
(ii) *Picture fuzzy universe set is* 1_X *defined as* $1_X = (\widehat{1, 0, 0}) = \{(y, 1, 0, 0)|y \in X\}$;
(iii) *Picture fuzzy empty set is* ϕ *defined as* $\phi = 0_X = (\widehat{0, 0, 1}) = \{(y, 0, 0, 1)|y \in X\}$.

Definition 2.4. *[24] A picture fuzzy topology on X is a collection* σ *of PFS satisfying*

(1) $(\widehat{\vartheta, \varepsilon, \varrho}) \in \sigma, (\widehat{\vartheta, \varepsilon, \varrho}) \in PFS(X)$;
(2) $G \cap H \in \sigma$ *for any* $G, H \in \sigma$;
(3) $\cup_{i \in I} H_i$ *for* $\{H_i | i \in I\} \subseteq \sigma$.

Then (X, σ) *is said to be a picture fuzzy topological space (PFTS) and the member of* σ *is picture fuzzy open set (PFOS) in X. The picture fuzzy closed set (PFCS) is the complement of it.* σ^c *denote the collection of all PFCS s.*

Definition 2.5. *[24] For a picture fuzzy topological space* (X, σ), $int(D)$ *and* $cl(D)$ *denotes the interior and closure operator of a picture fuzzy set D in* (X, σ) *and is defined as follows:*

$$int(D) = \cup\{H | H \text{ is a } PFOS,\ H \subseteq D\},$$
$$cl(D) = \cap\{K | K \text{ is a } PFOS,\ D \subseteq K\}.$$

Definition 2.6. *[10] The image of* $D \in PFS(X)$ *under the function f from X into Y is defined as follows:*

$$f(D)(b) = \begin{cases} \left(\bigvee_{a \in f^{-1}(b)} \gamma_D(a), \bigwedge_{a \in f^{-1}(b)} \nu_D(a), \bigwedge_{a \in f^{-1}(b)} \eta_D(a) \right), & \text{if } f^{-1}(b) \neq \phi, \\ (0, 0, 0), & \text{if } f^{-1}(b) = \phi. \end{cases}$$

The pre image of $E \in PFS(Y)$ *under f is* $f^{-1}(E)(a)= (\gamma_E(f(a)), \nu_E(f(a)), \eta_E(f(a)))$.

2.2. Linear relationship between two IFSs

Let $X = \{x_1, x_2, \cdots, x_n\}$ be a discrete universe of discourse, D and E be a two IFSs on X denoted as $D = \{(x_i, \gamma_D(x_i), \nu_D(x_i))|x_i \in X, i = 1, 2, \cdots, n\}$ and $E = \{(x_i, \gamma_E(x_i), \nu_E(x_i))|x_i \in X, i = 1, 2, \cdots, n\}$ respectively.

Definition 2.7. *[13] For IFS D=$\{(x_i, \gamma_D(x_i), \nu_D(x_i))|x_i \in X, i = 1, 2, \cdots, n\}$, the informational energy of the set D is defined as*

$$E_{IFS}(D) = \sum_{i=1}^{n}(\gamma_D^2(x_i) + \nu_D^2(x_i)). \tag{2.1}$$

Definition 2.8. *[13] For $D, E \in IFS\,s$, the correlation $C_{p_2}(D, E)$ is defined by*

$$C_{IFS_1}(D, E) = \sum_{i=1}^{n}(\gamma_D(x_i)\gamma_E(x_i) + \nu_D(x_i)\nu_E(x_i)). \tag{2.2}$$

Definition 2.9. *[13] The correlation coefficient between any two intuitionistic fuzzy sets D and E is given by,*

$$\begin{aligned} K_{IFS_1}(D, E) &= \frac{C_{IFS_1}(D, E)}{(E_{IFS}(D))^{\frac{1}{2}}(E_{IFS}(E))^{\frac{1}{2}}} \\ &= \sum_{i=1}^{n}(\gamma_D(x_i)\gamma_E(x_i) + \nu_D(x_i)\nu_E(x_i)\Big/\Big\{\sum_{i=1}^{n}u_i(\gamma_D^2(x_i) + \nu_D^2(x_i))\Big\}^{\frac{1}{2}} \cdot \Big\{\sum_{i=1}^{n}(\gamma_E^2(x_i) + \nu_E^2(x_i))\Big\}^{\frac{1}{2}}. \end{aligned} \tag{2.3}$$

Proposition 2.1. *[13] The correlation coefficient between two IFSs D and E defined in Eq (2.3), satisfies:*

(1) $K_{IFS_1}(D, E) = K_{IFS_1}(E, D)$*;*
(2) $0 \leq K_{IFS_1}(D, E) \geq 1$*;*
(3) $K_{IFS_1}(D, E) = 1$ *iff* $D = E$*.*

Definition 2.10. *[13] Let $D_j(j = 1, 2, \cdots, m)$ be m IFSs, and $C = (K_{ij})_{m\times m}$ be a correlation matrix, where $K_{ij} = K(D_i, D_j)$ denotes the correlation coefficient of two IFSs D_i and D_j and satisfies:*

(1) $0 \leq K_{ij} \leq 1$*;*
(2) $K_{ii} = 1$*;*
(3) $K_{ij} = K_{ji}$*.*

Definition 2.11. *[30] The correlation matrix of m IFSs is given by $M_C = (K_{ij})_{m\times m}$, the composition matrix of a correlation matrix is $M_C^2 = M_C \circ M_C = (\overline{K}_{ij})_{m\times m}$, where*

$$\overline{K}_{ij} = max_n\Big\{min\{K_{in}, K_{nj}\}\Big\}. \tag{2.4}$$

Definition 2.12. *[30] Let $M_C = (K_{ij})_{m\times m}$ be a correlation matrix, if $M_C^2 \subseteq M_C$, i.e.,*

$$max_n\Big\{min\{K_{in}, K_{nj}\}\Big\} \leq K_{ij}\ i, j = 1, 2, \cdots, m. \tag{2.5}$$

Then M_C is called an equivalent correlation matrix.

Definition 2.13. *[30] Let $M_C = (K_{ij})_{m\times m}$ be an equivalent correlation matrix. Then we call $(M_C)_\lambda = ({}_\lambda K_{ij})_{m\times m}$ the λ-cutting matrix of M_C, where*

$$
{}_\lambda K_{ij} = \begin{cases} 0, & if\ K_{ij} < \lambda, \\ 1, & if\ K_{ij} \geq \lambda, \end{cases} \tag{2.6}
$$

and λ is the confidence level with $\lambda \in [0, 1]$.

2.3. Linear relationship between two PFSs

Let $X = \{x_1, x_2, \cdots, x_n\}$ be a discrete universe of discourse, D and E be a two PFSs on X denoted as D=$\{(x_i, \gamma_D(x_i), \nu_D(x_i),\ \eta_D(x_i))|x_i \in X, i = 1, 2, \cdots, n\}$ and E=$\{(x_i, \gamma_E(x_i), \nu_E(x_i),\ \eta_E(x_i))|x_i \in X, i = 1, 2, \cdots, n\}$ respectively. Let $u = (u_1, u_2, \cdots, u_n)^T$ be the weight vector of $x_i (i = 1, 2, \cdots, n)$ with $u_i \geq 0$ and $\sum_{i=1}^{n} u_i = 1$.

Definition 2.14. *[21] For PFS D=$\{(x_i, \gamma_D(x_i), \nu_D(x_i), \eta_D(x_i))|x_i \in X, i = 1, 2, \cdots, n\}$, the informational energy of the set D is defined as*

$$
E_p(D) = \sum_{i=1}^{n} u_i(\gamma_D^2(x_i) + \nu_D^2(x_i) + \eta_D^2(x_i) + \rho_D^2(x_i)). \tag{2.7}
$$

Definition 2.15. *[21] For $D, E \in PFS\,s$, the correlation $C_{p_2}(D, E)$ is defined by*

$$
C_{p_2}(D, E) = \sum_{i=1}^{n} u_i(\gamma_D(x_i)\gamma_E(x_i) + \nu_D(x_i)\nu_E(x_i) + \eta_D(x_i)\eta_E(x_i) + \rho_D(x_i)\rho_E(x_i)). \tag{2.8}
$$

Definition 2.16. *[21] The correlation coefficient between any two picture fuzzy sets D and E is given by,*

$$
\begin{aligned}
K_{p_3}(D, E) =& \frac{C_{p_2}(D, E)}{(E_p(D))^{\frac{1}{2}}(E_p(E))^{\frac{1}{2}}} \\
=& \sum_{i=1}^{n} u_i(\gamma_D(x_i)\gamma_E(x_i) + \nu_D(x_i)\nu_E(x_i) + \eta_D(x_i)\eta_E(x_i) \\
& + \rho_D(x_i)\rho_E(x_i)) \Big/ \Big\{ \sum_{i=1}^{n} u_i(\gamma_D^2(x_i) + \nu_D^2(x_i) + \eta_D^2(x_i) \\
& + \rho_D^2(x_i)) \Big\}^{\frac{1}{2}} \cdot \Big\{ \sum_{i=1}^{n} u_i(\gamma_E^2(x_i) + \nu_E^2(x_i) + \eta_E^2(x_i) + \rho_E^2(x_i)) \Big\}^{\frac{1}{2}}.
\end{aligned} \tag{2.9}
$$

$K_{p_3}(D, E)$ in Eq (2.9) depends on the following factors:

(1) The amount of information expressed by the degree of positive membership, the degree of neutral membership, the degree of negative membership.
(2) The reliability of the information expressed by refusal membership.

Proposition 2.2. *[21] Let $u = (u_1, u_2, \cdots, u_n)^T$ be the weight vector of $x_i (i = 1, 2, \cdots, n)$ with $u_i \geq 0$ and $\sum_{i=1}^{n} u_i = 1$ then correlation coefficient between two PFSs D and E defined in Eq (2.9), satisfies:*

(1) $K_{p_3}(D,E) = K_{p_3}(E,D)$;
(2) $0 \le K_{p_3}(D,E) \ge 1$;
(3) $K_{p_3}(D,E) = 1$ *iff* $D = E$.

Definition 2.17. *[21] Let $D_j(j = 1, 2, \cdots, m)$ be m PFSs, and $C = (K_{ij})_{m\times m}$ be a correlation matrix, where $K_{ij} = K(D_i, D_j)$ denotes the correlation coefficient of two PFSs D_i and D_j and satisfies:*

(1) $0 \le K_{ij} \le 1$;
(2) $K_{ii} = 1$;
(3) $K_{ij} = K_{ji}$.

Definition 2.18. *[30] The correlation matrix of m PFSs is given by $M_C = (K_{ij})_{m\times m}$, the composition matrix of a correlation matrix is $M_C^2 = M_C \circ M_C = (\overline{K}_{ij})_{m\times m}$, where*

$$\overline{K}_{ij} = max_n\Big\{min\{K_{in}, K_{nj}\}\Big\}. \tag{2.10}$$

Definition 2.19. *[30] Let $M_C = (K_{ij})_{m\times m}$ be a correlation matrix, if $M_C^2 \subseteq M_C$, i.e.,*

$$max_n\Big\{min\{K_{in}, K_{nj}\}\Big\} \le K_{ij}\ i, j = 1, 2, \cdots, m. \tag{2.11}$$

Then M_C is called an equivalent correlation matrix.

Definition 2.20. *[30] Let $M_C = (K_{ij})_{m\times m}$ be an equivalent correlation matrix. Then we call $(M_C)_\lambda = ({}_\lambda K_{ij})_{m\times m}$ the λ-cutting matrix of M_C, where*

$${}_\lambda K_{ij} = \begin{cases} 0, & if\ K_{ij} < \lambda, \\ 1, & if\ K_{ij} \ge \lambda, \end{cases} \tag{2.12}$$

and λ is the confidence level with $\lambda \in [0, 1]$.

3. Picture fuzzy filter

Throughout the paper picture fuzzy topology is observed in the sense of Chang [8].

Definition 3.1. *Let (X, σ) be a $PFTS$. Picture fuzzy filter $\mathfrak{E} \subset \sigma^c$ on X is a collection of $PFS(X)$ satisfying*

(1) $\mathfrak{E}$ *is nonempty and* $0_X \notin \mathfrak{E}$;
(2) *If* $D_1, D_2 \in \mathfrak{E}$ *then* $D_1 \cap D_2 \in \mathfrak{E}$;
(3) *If* $D \subseteq E$ *where* $D \in \mathfrak{E}$ *and* $E \in \sigma^c$ *then* $E \in \mathfrak{E}$.

Definition 3.2. *If (X, σ) is a $PFTS$ and $Y \subseteq X$, the collection $\sigma_Y = \{D \cap 1_Y : D \in \sigma\}$ is a picture fuzzy topology on Y. (Y, σ_Y) is called a picture fuzzy subspace on X.*

Example 3.1. *Let $X = \{k,\ l,\ m\}$ and $\sigma = \{1_X,\ 0_X,\ H,\ K,\ E = H \cup K,\ D = H \cap K\}$, the membership values of H, K, E and D are provided in Table 2.*

Table 2. Membership values.

		H	K	E	D	1_X	0_X
	γ	0.3	0.3	0.3	0.3	1	0
k	η	0.2	0.2	0.2	0.2	0	0
	ν	0.4	0.4	0.4	0.4	0	1
	γ	0.2	0.7	0.2	0.7	1	0
l	η	0.1	0.0	0.0	0.0	0	0
	ν	0.3	0.2	0.3	0.2	0	1
	γ	0.4	0.3	0.3	0.4	1	0
m	η	0.2	0.1	0.1	0.1	0	0
	ν	0.3	0.4	0.4	0.3	0	1

Thus $(X,\ \sigma)$ is a picture fuzzy topological space. Let $Y = \{k,\ l\}$. $\sigma_Y = \{D \cap 1_Y | D \in \sigma\}$ is a picture fuzzy topology on Y.

Definition 3.3. *Let (X, σ) be a PFTS. Picture fuzzy grill $\mathscr{G} \subset \sigma^c$ on X is a collection of $PFS(X)$ satisfying*

(1) $\mathscr{G}$ is non empty and $0_X \notin \mathscr{G}$;
(2) If $D \in \mathscr{G}$ and $D \subseteq E$ then $E \in \mathscr{G}$;
(3) If $D \cup E \in \mathscr{G}$ then $D \in \mathscr{G}$ or $E \in \mathscr{G}$.

Definition 3.4. *Picture fuzzy section of $\mathfrak{E}$ is $PFsec(\mathfrak{E}) = \{E \in \sigma^c : D \cap E \neq 0_X, D \in \mathfrak{E}\}$.*

Definition 3.5. *$\mathscr{B} \subset \mathfrak{E}$ is a picture fuzzy base for $\mathfrak{E}$ if for every $D \in \mathfrak{E}\ \exists\ E \in \mathscr{B}$ with $E \subseteq D$.*

Definition 3.6. *$\mathscr{H} \subset \sigma^c$ is a picture fuzzy subbase for $\mathfrak{E}$ if $\{\cap_{i=1}^n D_i | D_i \in \mathscr{H}\}$ is a picture fuzzy base for $\mathfrak{E}$.*

Proposition 3.1. *Let (X, σ) be a PFTS and $\mathscr{B} \subset \sigma^c$. Then (i) and (ii) are equivalent.*

(i) There is only one picture fuzzy filter $\mathfrak{E}$ having $\mathscr{B}$ as a picture fuzzy base;
(ii) (a) $\mathscr{B}$ is non empty and $0_X \notin \mathscr{B}$;
(b) If $E_1, E_2 \in \mathscr{B}$ there is $E_3 \in \mathscr{B}$ with $E_3 \subseteq E_1 \cap E_2$.

Proof.

(i)⇒(ii). Assume that there is only one picture fuzzy filter $\mathfrak{E}$ having $\mathscr{B}$ as a picture fuzzy base. Since $\mathscr{B}$ is a picture fuzzy base for $\mathfrak{E}$ and $0_X \notin \mathfrak{E}$, $0_X \notin \mathscr{B}$ and also $\mathscr{B}$ is non empty. Also if $E_1, E_2 \in \mathscr{B}$ such that $E_1 \subseteq D_1$ and $E_2 \subseteq D_2$, for $D_1, D_2 \in \mathfrak{E}$. Since $D_1, D_2 \in \mathfrak{E}$, $D_1 \cap D_2 = D_3 \in \mathfrak{E}$, $E_1 \cap E_2 \subseteq D_1 \cap D_2 = D_3$. Thus $\exists\ E_3 \in \mathscr{B} \ni E_3 = E_1 \cap E_2 \subseteq D_3$.

(ii)⇒ (i). Suppose that $\mathscr{B}$ is a picture fuzzy base for two different picture fuzzy filters $\mathfrak{E}_1$ and $\mathfrak{E}_2$. For each $D_1 \in \mathfrak{E}_1\ \exists\ E_1 \in \mathscr{B} \ni E_1 \subseteq D_1$. Similarly for each $D_2 \in \mathfrak{E}_2\ \exists\ E_2 \in \mathscr{B} \ni E_2 \subseteq A_2$. Also if $D_1 \cap D_2 = 0_X$ then $0_X \in \mathscr{B}$ which is impossible. Hence $D_1 \cap D_2 \neq 0_X$. Therefore $D_1 \cap D_2$ lies in both $\mathfrak{E}_1, \mathfrak{E}_2$ and thus the picture fuzzy filters are same. □

Proposition 3.2. *If there is a picture fuzzy base $\mathscr{B}$ satisfies (a) and (b) of Proposition 3.1, then*

$$\mathfrak{E} = \{D \in \sigma^c | \exists E \in \mathscr{B} \text{ with } E \subseteq D\}$$

is a picture fuzzy filter generated by $\mathscr{B}$.

Proof. Since $\mathscr{B}$ is nonempty implies $\mathfrak{E}$ is nonempty. If $D \in \mathfrak{E}$ then $D \supseteq E$ for some $E \in \mathscr{B}$. By assumption $E \neq 0_X$ thus $D \neq 0_X$. If $D_1, D_2 \in \mathfrak{E}$ there exists $E_1, E_2 \in \mathscr{B}$ with $E_i \subseteq D_i$ for $i = 1, 2$. Then $D_1 \cap D_2 \supseteq E_1 \cap E_2 \supseteq E_3$ for some $E_3 \in \mathscr{B}$. Thus $D_1 \cap D_2 \in \mathfrak{E}$. This proves that $\mathfrak{E}$ is a picture fuzzy filter on X. □

Proposition 3.3. *Let $\mathfrak{E}$ be a picture fuzzy filter and $D \in \sigma^c$. $\mathfrak{E} \cup \{D\}$ lies in some picture fuzzy filter iff for each $E \in \mathfrak{E}$, $E \cap D \neq 0_X$.*

Definition 3.7. *A picture fuzzy ultrafilter $\mathfrak{E}$ is a maximal picture fuzzy filter among the set of all picture fuzzy filters $\{\mathfrak{E}_j\}_{j \in J}$.*

Proposition 3.4. *Every picture fuzzy filter $\mathfrak{E}$ extends to picture fuzzy ultrafilter $\mathscr{V}$.*

Proposition 3.5. *For any picture fuzzy filter $\mathfrak{E}$ on X, we have the equivalence.*

(i) $\mathfrak{E}$ is a picture fuzzy ultrafilter;
(ii) If $D \in \sigma^c$ and $\forall\ E \in \mathfrak{E}$ with $D \cap E \neq 0_X$, then $D \in \mathfrak{E}$;
(iii) If D is $PFCS$ and D is not in $\mathfrak{E}$, then there is $E \in \mathfrak{E} \ni D \cap E = 0_X$.

Proof.

(i)⇒(ii). Suppose D be a $PFCS$ and $D \cap E \neq 0_X$, for all $E \in \mathfrak{E}$. From Proposition 3.3, $\mathfrak{E} \cup \{D\}$ lies in some picture fuzzy filter $\mathfrak{E}^*$. By (i), $\mathfrak{E} = \mathfrak{E}^*$.

(ii)⇒ (iii). Let D is $PFCS$ and is not in $\mathfrak{E}$. By (ii), for some $E \in \mathfrak{E}$ we have $D \cap E \neq 0_X$.

(iii)⇒ (i). Let $\mathscr{M}$ be a picture fuzzy filter with $\mathfrak{E} \subset \mathscr{M}$ and $\mathfrak{E} \neq \mathscr{M}$. Let $D \in \mathscr{M} \ni D \notin \mathfrak{E}$. By (iii), $\exists$ $E \in \mathfrak{E}$ with $D \cap E = 0_X$. Since $E, D \in \mathscr{M}$, $E \cap D \in \mathscr{M}$ implies $0_X \in \mathscr{M}$, contradicts our assumption. Hence $\mathfrak{E} = \mathscr{M}$ which is a picture fuzzy ultrafilter. □

Proposition 3.6. *If $\mathscr{V}_1$, $\mathscr{V}_2$ the two different picture fuzzy ultrafilters on X, then $(\cap_i D_i) \cap (\cap_j D_j) = 0_X$ for all $D_i \in \mathscr{V}_1$ and $D_j \in \mathscr{V}_2$.*

Proof. Suppose $(\cap_i D_i) \cap (\cap_j D_j) \neq 0_X$, for all $D_i \in \mathscr{V}_1$ and $D_j \in \mathscr{V}_2$. Then there exists an $x \in X$ for which, $\wedge_i(\gamma_{D_i}(x)) \neq 0$, $\vee_i(\nu_{D_i}(x)) \neq 1$, $\wedge_i(\eta_{D_i}(x)) \neq 0$. Also $\wedge_j(\gamma_{D_j}(x)) \neq 0$, $\vee_j(\nu_{D_j}(x)) \neq 1$, $\wedge_j(\eta_{D_j}(x)) \neq 0$, $\Rightarrow \wedge(\gamma_{D_i}(x), \gamma_{D_j}(x)) > 0$, $\vee(\nu_{D_i}(x), \nu_{D_j}(x)) < 1$, $\wedge(\eta_{D_i}(x), \eta_{D_j}(x)) > 0$, for all i, j. Which implies $D_i \cap D_j \neq 0_X$. By Proposition 3.5, $D_i \in \mathscr{V}_2$ and $D_j \in \mathscr{V}_1$ for all i, j. Then it leads to contradiction. □

Proposition 3.7. *Every picture fuzzy ultrafilter is a picture fuzzy grill.*

Proof. Let D, E be $PFCS$ with $D \cup E$ lies in picture fuzzy ultrafilter $\mathscr{V}$. Suppose D, E is not in $\mathscr{V}$. Then there exists $D_1, E_1 \in \mathscr{V}$ with $D \cap D_1 = 0_X$ and $E \cap E_1 = 0_X$. Since $\mathscr{V}$ is a picture fuzzy ultrafilter, $(D \cup E) \cap D_1 \cap E_1 \in \mathscr{V}$. Now, $[(D \cup E) \cap D_1] \cap E_1 = [(D \cap D_1) \cup (E \cap E_1)] \cap E_1 = [0_X \cup (E \cap D_1)] \cap E_1$ $= [0_X \cap E_1] \cup [(E \cap D_1) \cap E_1] = 0_X \cup [E \cap E_1 \cap D_1] = 0_X \cup 0_X = 0_X$. This leads a contradiction. Hence $\mathscr{V}$ is a picture fuzzy grill on X. □

Proposition 3.8. *If $\mathscr{P}(\mathfrak{E})$ is the collection of picture fuzzy grills containing $\mathfrak{E}$, then we have $\mathfrak{E} = \cap_{\mathscr{G} \in \mathscr{P}(\mathfrak{E})} \mathscr{G}$.*

Proof. If A be a $PFCS$ and is not in $\mathfrak{E}$. Now, $\mathscr{L}$ denotes the inductive set consisting of all picture fuzzy filters $\mathscr{G}$ containing $\mathfrak{E}$ and $A \notin \mathscr{G}$. $\mathscr{L}$ posses the maximal filter $\mathscr{V}$. We claim $\mathscr{V}$ is a picture fuzzy grill. Let $B_1, B_2 \in \sigma^c$ with $B_1 \cup B_2 \in \mathscr{V}$ such that $B_1, B_2 \notin \mathscr{V}$. Consider the family $\mathscr{J} = \{F \in \sigma^c | F \cup B_2 \in \mathscr{V}\}$. Since $B_1 \in \sigma^c$ and $B_1 \cup B_2 \in \mathscr{V}$, implies that $B_1 \in \mathscr{J}$. This implies $\mathscr{J}$ is non empty. Suppose if 0_X is in $\mathscr{J}$, $B_2 \in \mathscr{V}$. Contradicts our assumption. Hence $0_X \notin \mathscr{J}$. If $F_1, F_2 \in \mathscr{J}$. By definition of $\mathscr{J}$, $F_1 \cup B_2 \in \mathscr{V}$ and $F_2 \cup B_2 \in \mathscr{V}$. Since $\mathscr{V}$ is a picture fuzzy filter. $[F_1 \cup B_2] \cap [F_2 \cup B_2] \in \mathscr{V} \Rightarrow (F_1 \cap F_2) \cup B_2 \in \mathscr{V} \Rightarrow F_1 \cap F_2 \in \mathscr{J}$. If $F \in \mathscr{J}$ and $B \in \sigma^c$ such that $F \subseteq B$, $F \cup B_2 \in \mathscr{V}$, $\mathscr{V}$ is ultrafilter. $B_1 \cup B_2 \in \mathscr{V}$, implies that $B \in \mathscr{J}$. Thus $\mathscr{J}$ is a picture fuzzy filter. Since $B_1 \cup B_2 \in \mathscr{V}$, $(B_1 \cup B_2) \cup B_2 \in \mathscr{V}$, implies that $B_1 \cup B_2 \in \mathscr{J}$. Thus $\mathscr{V} \subset \mathscr{J}$. Since $B_1 \in \mathscr{J}$ and $B_1 \notin \mathscr{V}$. Thus $\mathscr{V} \neq \mathscr{J}$. Let $\mathscr{K} = \{C \in \sigma^c | A \cup C \in \mathscr{V}\}$. Suppose $0_X \in \mathscr{K}$, then by definition of $\mathscr{K}$, $A \in \mathscr{V}$. Contradicts our assumption $\mathscr{V} \in \mathscr{L}$ and $A \notin \mathscr{V}$. Hence $0_X \notin \mathscr{K}$. Since $1_x \in \mathscr{V}$, implies that $1_X \in \mathscr{K}$. $\mathscr{K}$ satisfies the first condition of picture fuzzy filter. If $A^*,\ A^{**} \in \mathscr{K}$. By definition of $\mathscr{K}$, the picture fuzzy sets $A^* \cup A$ and $A^{**} \cup A$ are in $\mathscr{V}$. Since $\mathscr{V}$ is a picture fuzzy filter, $A^* \cup A^{**} \cup A \in \mathscr{V}$. Therefore $A^* \cup A^{**} \in \mathscr{K}$. If $A \in \mathscr{K}$ and $A^* \in \sigma^c$ such that $A^* \supseteq A$ then $A^{**} \in \mathscr{K}$. Hence $\mathscr{K}$ is a picture fuzzy filter.

Now, $\mathfrak{E} \subset \mathscr{K}$. $A \notin \mathscr{K}$ for $A \notin \mathscr{V}$. Hence $\mathscr{K}$ also lies in $\mathscr{L}$ and $\mathscr{V} \subset \mathscr{K}$. $\mathscr{V}$=$\mathscr{K}$ since $\mathscr{V}$ is maximal. If $A \in \mathscr{J}$, then $A \cup B_2 \in \mathscr{V}$, implies that $B_2 \in \mathscr{K} = \mathscr{V}$. Contradicts our assumption $B_2 \notin \mathscr{V}$. Thus $A \notin \mathscr{J}$. $\mathscr{V} = \mathscr{J}$, since $\mathscr{J} \in \mathscr{L}$, $\mathscr{V} \subset \mathscr{J}$. However, $\mathscr{V} \neq \mathscr{J}$. So it is absurd to assume $B_1, B_2 \notin \mathscr{V}$. Thus $B_1, B_2 \in \mathscr{V}$. Therefore $\mathscr{V}$ is a picture fuzzy grill and $A \notin \mathscr{V}$. Hence $\cap_{\mathscr{G} \in \mathscr{P}(\mathfrak{E})} \mathscr{G} \subset \mathfrak{E}$. □

Definition 3.8. *A picture fuzzy filter* $\mathfrak{E}_{x_{(\vartheta,\varepsilon,\varrho)}}$ *generated by picture fuzzy point* $x_{(\vartheta,\varepsilon,\varrho)}$*, if the non empty collection* $\mathfrak{E}_{x_{(\vartheta,\varepsilon,\varrho)}} = \{E \in \sigma^c | x_{(\vartheta,\varepsilon,\varrho)} \subseteq E\}$ *is a picture fuzzy grill on* X*.*

Definition 3.9. *Picture fuzzy normal family*($PFNF$) *is a collection of* $PFCS$ *if given* $D_1, D_2 \in \sigma^c$ *such that* $D_1 \cap D_2 = 0_X$ *there exist* $E_1, E_2 \in \sigma^c$ *with* $E_1 \cup E_2 = 1_X$*,* $D_1 \cap E_1 = 0_X$ *and* $D_2 \cap E_2 = 0_X$*.*

Proposition 3.9. *Let* (X, σ) *be any* $PFTS$ *and* σ^c *be a* $PFNF$*. Every picture fuzzy grill* $\mathscr{G}$ *on* X *lies exactly in one picture fuzzy ultrafilter.*

Proof. Assume that $\mathscr{V}_1$ and $\mathscr{V}_2$ are the picture fuzzy ultrafilters having $\mathscr{G} \subset \mathscr{V}_1$, $\mathscr{G} \subset \mathscr{V}_2$, $\mathscr{V}_1 \neq \mathscr{V}_2$. Then $\exists\ D_1 \in \mathscr{V}_1$ and $D_2 \in \mathscr{V}_2$ with $D_1 \cap D_2 = 0_X$. Since σ^c is a $PFNF$, there exist $E_1, E_2 \in \sigma^c$ with $E_1 \cup E_2 = 1_X$, $D_1 \cap E_1 = 0_X$ and $D_2 \cap E_2 = 0_X$. Since $E_1 \cup E_2 = 1_X$ and $\mathscr{G}$ is a picture fuzzy grill, $E_1 \in \mathscr{G}$ or $E_2 \in \mathscr{G}$. Suppose if $E_1 \in \mathscr{G}$, then $E_1 \in \mathscr{V}_1$ and $E_1 \in \mathscr{V}_2$. Thus $D_1 \cap E_1 = 0_X$ with $D_1, E_1 \in \mathscr{V}_1$, contradicts our assumption. Similarly, $E_2 \in \mathscr{G}$, then $E_2 \in \mathscr{V}_1$ and $E_2 \in \mathscr{V}_2$. Thus $D_2 \cap E_2 = 0_X$ with $D_2, E_2 \in \mathscr{V}_2$, contradicts our assumption. Hence $\mathscr{V}_1 = \mathscr{V}_2$. □

Proposition 3.10. *If* σ^c *be a* $PFNF$ *and for every picture fuzzy point* $x_{(\vartheta,\varepsilon,\varrho)}$ *there exists a unique picture fuzzy ultrafilter* $\mathscr{V}_{x_{(\vartheta,\varepsilon,\varrho)}}$ *which contains* $\mathfrak{E}_{x_{(\vartheta,\varepsilon,\varrho)}}$*.*

Proof. Proof follows from Definition 3.8 and Proposition 3.9. □

Proposition 3.11. *For any two picture fuzzy points* $x_{(\vartheta,\varepsilon,\varrho)}$*,* $y_{(\gamma,\delta,\varphi)}$ *with* $x = y$*, we have* $\mathscr{V}_{x_{(\vartheta,\varepsilon,\varrho)}} = \mathscr{V}_{y_{(\gamma,\delta,\varphi)}}$*.*

Proof. Proof is obtained from Proposition 3.6. □

Proposition 3.12. *Let* (X, σ) *is a picture fuzzy topological space. Then*

(a) $\mathfrak{E}$ is a picture fuzzy filter on X iff $PFsec(\mathfrak{E})$ is picture fuzzy grill on X.
(b) $\mathscr{G}$ is a picture fuzzy grill on X iff $PFsec(\mathscr{G})$ is picture fuzzy filter on X.

Proof.

(a) Let $\mathfrak{E}$ be a picture fuzzy filter on X. First two conditions of $PFsec(\mathfrak{E})$ is true by the nature of $\mathfrak{E}$. Let $D \cup E \in PFsec(\mathfrak{E})$, then for all $C \in \mathfrak{E}$, $C \cap (D \cup E) \in \mathfrak{E}$. By definition of $PFsec(\mathfrak{E})$, $D, E \in PFsec(\mathfrak{E})$. Hence $PFsec(\mathfrak{E})$ is a picture fuzzy grill on X. Conversely, $PFsec(\mathfrak{E})$ satisfies first and third condition of picture fuzzy filter. If $D_1, D_2 \in PFsec(\mathfrak{E})$, $D_1 \cap D_2 \in PFsec(\mathfrak{E})$. Hence $PFsec(\mathfrak{E})$ is a picture fuzzy filter on X.

(b) Let $\mathfrak{E}$ be a picture fuzzy grill on X. First and second conditions of picture fuzzy filter is true for $PFsec(\mathfrak{E})$. For the second condition let $D_1, D_2 \in PFsec(\mathfrak{E})$, $(D_1 \cap C) \neq 0_X$ and $(D_2 \cap C) \neq 0_X$, $\forall C \in \mathfrak{E}$). Therefore $D_1 \cap D_2 \in PFsec(\mathfrak{E})$. Hence $PFsec(\mathfrak{E})$ is a picture fuzzy filter on X. Conversely, $PFsec(\mathfrak{E})$ is a picture fuzzy filter on X. First and second conditions of picture fuzzy grill is obvious. For the third condition $D \cup E \in PFsec(\mathfrak{E})$, $C \cap (D \cup E) \neq 0_X$. Hence both D, E is in $PFsec(\mathfrak{E})$. Hence $PFsec(\mathfrak{E})$ is a picture fuzzy grill on X. □

By the above Proposition, it is easy to analyze that there is a one to one correspondence between the set of all picture fuzzy filters and the set of all picture fuzzy grills.

4. Implementation of clustering algorithm on PFSs

P. Singh [21] proposed the clustering algorithm for picture fuzzy set. The proposed algorithm is applied to the picture fuzzy filter collection and the classification of picture fuzzy sets is obtained.

Proposition 4.1. *[30] The composition matrix M_C^2 is also a correlation matrix.*

Proposition 4.2. *[30] Let M_C be a correlation matrix. Then for any non-negative integers p_1 and p_2, the composition matrix. $M_C^{p_1 p_2}$ derived from $M_C^{p_1 p_2} = M_C^{p_1} \circ M_C^{p_2}$ is still a correlation matrix.*

Proposition 4.3. *[30] Let $M_C = (K_{ij})_{m \times m}$ be a correlation matrix. Then after the finite times of compositions:*
$M_C \to M_C^2 \to M_C^4 \to \cdots \to M_C^{2^k} \to \cdots$, there must exist a positive integer k such that $M_C^{2^k} = M_C^{2^{k+1}}$ and $M_C^{2^k}$ is also an equivalent correlation matrix.

4.1. Algorithm for PFSs [21]

Step 1. Let $\{D_1, D_2, \cdots, D_m\}$ be a set of PFSs in $X = \{x_1, x_2, \cdots, x_n\}$. Using the formula, correlation coefficient of picture fuzzy set can be calculated and the correlation matrix $M_C = (K_{ij})_{m \times m}$, where $K_{ij} = K(D_i, D_j)$ can be constructed.
Step 2. Check whether $M_C^2 \subseteq M_C$, where $M_C^2 = M_C \circ M_C = (\overline{K}_{ij})_{m \times m} = max_n\Big\{min\{K_{in}, K_{nj}\}\Big\} \leq K_{ij}$ $i, j = 1, 2, \cdots, m$. If it does not hold, construct the equivalent correlation matrix $M_C^{2^k}$: $M_C \to M_C^2 \to M_C^4 \to \cdots \to M_C^{2^k} \to \cdots$, until $M_C^{2^k} = M_C^{2^{k+1}}$.
Step 3. For confidence level λ, construct a λ-cutting matrix $(M_C)_\lambda = ({}_\lambda K_{ij})_{m \times m}$ through Definition 2.20 in order to classify the PFSs $P_j (j = 1, 2, \cdots, m)$. If all element of the ith column in $(M_C)_\lambda$ are the same as the corresponding elements of the jth column in $(M_C)_\lambda$, then the PFSs D_i and D_j are of the same type. The classification of picture fuzzy sets can be done by the above principle.

4.2. Illustration: performance measure of an employee in a cotton industry

Illustration 1. For a practical example, an employee from a subunit of the cotton industry is considered. As the production of yarn depends on four crucial processes blowing, carding, drawing, and roving can be performed by the workers. After completing the above-said stages, the product yarn can be obtained through machines automatically. While doing the first four processes, there are positive, negative, and flaws in an employee's performance. We attempted to define a picture fuzzy topological space on the collection of picture fuzzy sets obtained from employee performance. Later picture fuzzy filter collection is obtained and applied with the clustering algorithm leads to a classification of the employee based on their performance. Each employee is associated with four different attributes denoted by X = {k, l, m, n}, k: Blowing; l: Carding; m: Drawing; n: Roving with weight vector u = (0.4, 0.2, 0.3, 0.1). Based on the expert's information, the evaluation of each employee is expressed in the form of PFSs. Table 3 represents the degree of positive, negative, and neutral membership of each attribute of X given by the experts.

Let $\sigma = \{1_X, 0_X, E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9\}$ be the picture fuzzy topology on X where

$E_1 = \{(k, 0.7, 0.1, 0.2), (l, 0.9, 0.0, 0.1), (m, 0.8, 0.1, 0.0), (n, 0.7, 0.1, 0.2)\}$,

$E_2 = \{(k, 0.8, 0.1, 0.1), (l, 0.8, 0.1, 0.1), (m, 0.1, 0.1, 0.7), (n, 0.6, 0.2, 0.1)\}$,

$E_3 = \{(k, 0.4, 0.2, 0.2), (l, 0.7, 0.1, 0.2), (m, 0.6, 0.1, 0.3), (n, 0.6, 0.3, 0.1)\}$,

$E_4 = E_1 \cup E_2, E_5 = E_1 \cup E_3, E_6 = E_2 \cup E_3, E_7 = E_1 \cap E_2, E_8 = E_1 \cap E_3, E_9 = E_2 \cap E_3$

and (X, σ) be the PFTS. Then $\sigma^c = \{\overline{1_X}, \overline{0_X}, \overline{E_1}, \overline{E_2}, \overline{E_3}, \overline{E_4}, \overline{E_5}, \overline{E_6}, \overline{E_7}, \overline{E_8}, \overline{E_9}\}$. Consider the picture fuzzy filter $\mathcal{F}_1 = \{\overline{0_x}, \overline{E_2}, \overline{E_3}, \overline{E_6}, \overline{E_9}\}$.

Table 3. Employees experts membership values.

		$\overline{E_1}$	$\overline{E_2}$	$\overline{E_3}$	$\overline{E_4}$	$\overline{E_5}$	$\overline{E_6}$	$\overline{E_7}$	$\overline{E_8}$	$\overline{E_9}$	$\overline{0_X}$	$\overline{1_X}$
	γ	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.2	0.2	1	0
k	η	0.1	0.1	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0	0
	ν	0.7	0.8	0.4	0.8	0.7	0.8	0.7	0.4	0.4	0	1
	γ	0.1	0.1	0.2	0.1	0.1	0.1	0.1	0.2	0.2	1	0
l	η	0.0	0.1	0.1	0.0	0.0	0.1	0.0	0.0	0.1	0	0
	ν	0.9	0.8	0.7	0.9	0.9	0.8	0.8	0.7	0.7	0	1
	γ	0.0	0.7	0.3	0.0	0.0	0.3	0.7	0.3	0.3	1	0
m	η	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0	0
	ν	0.8	0.1	0.6	0.8	0.8	0.6	0.1	0.6	0.1	0	1
	γ	0.2	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.1	1	0
n	η	0.1	0.2	0.3	0.1	0.1	0.2	0.1	0.1	0.2	0	0
	ν	0.7	0.6	0.6	0.7	0.7	0.6	0.6	0.6	0.6	0	1

Applying clustering algorithm to picture fuzzy sets from $\mathcal{F}_1$.

Step 1. The correlation coefficient of PFSs $\overline{E}_j (j = 2, 3, 6, 9)$ can be computed by using Eq (2.9) and the correlation matrix M_C is constructed:

$$M_C = \begin{pmatrix} 1 & 0.7940 & 0.8924 & 0.9064 & 0.7395 \\ 0.7940 & 1 & 0.8364 & 0.8364 & 0.8495 \\ 0.8924 & 0.8364 & 1 & 0.7752 & 0.9486 \\ 0.9064 & 0.8364 & 0.7752 & 1 & 0.5975 \\ 0.7395 & 0.8495 & 0.9486 & 0.5975 & 1 \end{pmatrix}.$$

Step 2. Construct equivalent correlation matrices:

$$M_C^2 = \begin{pmatrix} 1 & 0.8364 & 0.8924 & 0.9064 & 0.8924 \\ 0.8364 & 1 & 0.8495 & 0.8364 & 0.8495 \\ 0.8924 & 0.8495 & 1 & 0.8924 & 0.9486 \\ 0.9064 & 0.8364 & 0.8924 & 1 & 0.8364 \\ 0.8924 & 0.8495 & 0.9486 & 0.8364 & 1 \end{pmatrix},$$

$$M_C^4 = \begin{pmatrix} 1 & 0.8495 & 0.8924 & 0.9064 & 0.8924 \\ 0.8495 & 1 & 0.8495 & 0.8495 & 0.8495 \\ 0.8924 & 0.8495 & 1 & 0.8924 & 0.9486 \\ 0.9064 & 0.8495 & 0.8924 & 1 & 0.8924 \\ 0.8924 & 0.8495 & 0.9486 & 0.8924 & 1 \end{pmatrix},$$

$$M_C^8 = \begin{pmatrix} 1 & 0.8495 & 0.8924 & 0.9064 & 0.8924 \\ 0.8495 & 1 & 0.8495 & 0.8495 & 0.8495 \\ 0.8924 & 0.8495 & 1 & 0.8924 & 0.9486 \\ 0.9064 & 0.8495 & 0.8924 & 1 & 0.8924 \\ 0.8924 & 0.8495 & 0.9486 & 0.8924 & 1 \end{pmatrix}.$$

Therefore $M_C^8 = M_C^4$. Hence M_C^4 is equivalent matrix.

Step 3. λ-cutting matrix $M_{C_\lambda} = (\lambda K_{ij})_{m\times m}$ is computed using the Eq (2.12), based on which, we get all possible classification of the employee $\overline{E}_j (j = 2, 3, 6, 9)$: classification shown in Table 6.

Thus the above illustration leads to classifying an employee from the picture fuzzy filter collection obtained in the third iteration. The number of iterations is more for some collection of picture fuzzy sets. Thus if the collection is a picture fuzzy filter, the classification is obtained at the earliest.

4.3. Illustration: classification of intuitionistic fuzzy sets from intuitionistic fuzzy filter

Illustration 2. Let $\sigma = \{1_X, 0_X, A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}\}$ be the IFT and (X, σ) be the IFTs. The membership values of intuitionistic fuzzy sets belongs to σ^c is given in Table 4. Consider the intuitionistic fuzzy filter $\mathcal{F}_2 = \{\overline{0_x}, \overline{A_1}, \overline{A_6}, \overline{A_8}, \overline{A_{10}}\}$.

Table 4. The data of σ^c.

		$\overline{A_1}$	$\overline{A_2}$	$\overline{A_3}$	$\overline{A_4}$	$\overline{A_5}$	$\overline{A_6}$	$\overline{A_7}$	$\overline{A_8}$	$\overline{A_9}$	$\overline{A_{10}}$	$\overline{0_x}$	$\overline{1_x}$
x_1	γ_{A_i}	0.40	0.30	0.20	0.40	0.10	0.30	0.40	0.10	0.40	0.10	1.00	0.00
	ν_{A_i}	0.30	0.40	0.40	0.30	0.80	0.40	0.60	0.90	0.40	0.90	0.00	1.00
x_2	γ_{A_i}	0.70	0.10	0.10	0.00	0.20	0.50	0.20	0.20	0.00	0.00	1.00	0.00
	ν_{A_i}	0.20	0.50	0.60	0.90	0.70	0.30	0.40	0.70	1.00	0.80	0.00	1.00
x_3	γ_{A_i}	0.50	0.20	0.10	0.10	0.00	0.60	0.20	0.10	0.10	0.30	1.00	0.00
	ν_{A_i}	0.40	0.60	0.80	0.80	0.70	0.20	0.70	0.70	0.90	0.60	0.00	1.00
x_4	γ_{A_i}	0.80	0.70	0.60	0.10	0.10	0.20	0.60	0.50	0.20	0.20	1.00	0.00
	ν_{A_i}	0.10	0.20	0.20	0.70	0.40	0.50	0.30	0.40	0.60	0.50	0.00	1.00
x_5	γ_{A_i}	0.50	0.30	0.70	0.80	0.20	0.40	0.70	0.50	0.70	0.10	1.00	0.00
	ν_{A_i}	0.40	0.60	0.30	0.10	0.80	0.50	0.30	0.40	0.20	0.80	0.00	1.00
x_6	γ_{A_i}	0.70	0.20	0.20	0.80	0.60	0.60	0.10	0.00	0.80	0.40	1.00	0.00
	ν_{A_i}	0.20	0.70	0.50	0.20	0.40	0.30	0.60	0.80	0.10	0.60	0.00	1.00

Applying clustering algorithm to intuitionistic fuzzy sets from $\mathcal{F}_2$.

Step 1. The correlation coefficient of IFSs $\overline{A}_j(j = 1, 6, 8, 10)$ can be computed by using Eq (2.3) and the correlation matrix M_C is constructed:

$$M_C = \begin{pmatrix} 1 & 0.4306 & 0.9436 & 0.5204 & 0.1428 \\ 0.4306 & 1 & 0.3345 & 0.8750 & 0.5285 \\ 0.9436 & 0.3345 & 1 & 0.4511 & 0.0571 \\ 0.5204 & 0.8750 & 0.4511 & 1 & 0.3857 \\ 0.1428 & 0.5285 & 0.0571 & 0.3857 & 1 \end{pmatrix}.$$

Step 2. Construct equivalent correlation matrices:

$$M_C^2 = \begin{pmatrix} 1 & 0.5204 & 0.9436 & 0.8750 & 0.4306 \\ 0.5204 & 1 & 0.4511 & 0.8750 & 0.5285 \\ 0.9436 & 0.4511 & 1 & 0.5204 & 0.3857 \\ 0.8750 & 0.8750 & 0.5204 & 1 & 0.5285 \\ 0.4306 & 0.5285 & 0.3857 & 0.5285 & 1 \end{pmatrix},$$

$$M_C^4 = \begin{pmatrix} 1 & 0.8750 & 0.9436 & 0.8750 & 0.5285 \\ 0.8750 & 1 & 0.5204 & 0.8750 & 0.5204 \\ 0.9436 & 0.5204 & 1 & 0.8750 & 0.5204 \\ 0.8750 & 0.8750 & 0.8750 & 1 & 0.5285 \\ 0.5285 & 0.5204 & 0.5204 & 0.5285 & 1 \end{pmatrix},$$

$$M_C^8 = \begin{pmatrix} 1 & 0.8750 & 0.9436 & 0.8750 & 0.5285 \\ 0.8750 & 1 & 0.5204 & 0.8750 & 0.5204 \\ 0.9436 & 0.5204 & 1 & 0.8750 & 0.5204 \\ 0.8750 & 0.8750 & 0.8750 & 1 & 0.5285 \\ 0.5285 & 0.5204 & 0.5204 & 0.5285 & 1 \end{pmatrix}.$$

Therefore $M_C^8 = M_C^4$. Hence M_C^4 is equivalent matrix.
Step 3. λ-cutting matrix $M_{C_\lambda} = ({}_\lambda K_{ij})_{m\times m}$ is computed using the Eq (2.6) , based on which, we get all possible classification of the $A_j(j = 1, 6, 8, 10)$: classification is shown in Table 6.

4.4. Illustration: classification of car data set

Illustration 3.

Now we utilize the algorithm-IFSC to cluster the ten new cars $A_i(i = 1, 1, \cdots, 10)$ whose positive and negative membership values are provided in Table 5, which involves the following steps:

Table 5. The car data set.

		A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
x_1	γ_{A_i}	0.30	0.40	0.40	0.30	0.80	0.40	0.60	0.90	0.40	0.90
	ν_{A_i}	0.40	0.30	0.20	0.40	0.10	0.30	0.40	0.10	0.40	0.10
x_2	γ_{A_i}	0.20	0.50	0.60	0.90	0.70	0.30	0.40	0.70	1.00	0.80
	ν_{A_i}	0.70	0.10	0.10	0.00	0.20	0.50	0.20	0.20	0.00	0.00
x_3	γ_{A_i}	0.40	0.60	0.80	0.80	0.70	0.20	0.70	0.70	0.90	0.60
	ν_{A_i}	0.50	0.20	0.10	0.10	0.00	0.60	0.20	0.10	0.10	0.30
x_4	γ_{A_i}	0.80	0.20	0.20	0.70	0.40	0.50	0.30	0.40	0.60	0.50
	ν_{A_i}	0.10	0.70	0.60	0.10	0.10	0.20	0.60	0.50	0.20	0.20
x_5	γ_{A_i}	0.40	0.30	0.30	0.10	0.80	0.50	0.30	0.40	0.20	0.80
	ν_{A_i}	0.50	0.60	0.70	0.80	0.20	0.40	0.70	0.50	0.70	0.10
x_6	γ_{A_i}	0.20	0.70	0.50	0.20	0.40	0.30	0.60	0.80	0.10	0.60
	ν_{A_i}	0.70	0.20	0.20	0.80	0.60	0.60	0.10	0.00	0.80	0.40

Step 1. Utilize to calculate the association coefficients of $A_i(i = 1, 1, \cdots, 10)$, and then construct an association matrix:

$$C = \begin{pmatrix} 1.000 & 0.667 & 0.645 & 0.709 & 0.633 & 0.919 & 0.696 & 0.609 & 0.666 & 0.611 \\ 0.667 & 1.000 & 0.909 & 0.661 & 0.666 & 0.665 & 0.913 & 0.820 & 0.665 & 0.640 \\ 0.645 & 0.909 & 1.000 & 0.768 & 0.740 & 0.576 & 0.937 & 0.862 & 0.771 & 0.670 \\ 0.709 & 0.661 & 0.768 & 1.000 & 0.755 & 0.610 & 0.717 & 0.728 & 0.968 & 0.711 \\ 0.633 & 0.666 & 0.740 & 0.755 & 1.000 & 0.623 & 0.713 & 0.476 & 0.764 & 0.861 \\ 0.919 & 0.665 & 0.576 & 0.610 & 0.623 & 1.000 & 0.634 & 0.579 & 0.566 & 0.622 \\ 0.696 & 0.913 & 0.937 & 0.717 & 0.713 & 0.634 & 1.000 & 0.889 & 0.722 & 0.692 \\ 0.609 & 0.820 & 0.862 & 0.728 & 0.476 & 0.579 & 0.889 & 1.000 & 0.740 & 0.811 \\ 0.666 & 0.665 & 0.771 & 0.968 & 0.764 & 0.566 & 0.722 & 0.740 & 1.000 & 0.732 \\ 0.611 & 0.640 & 0.670 & 0.711 & 0.861 & 0.622 & 0.692 & 0.811 & 0.732 & 1.000 \end{pmatrix}.$$

Step 2. Similarly, C^2, C^4 has be computed and C^8, C^{16} are as follows:

$$C^8 = \begin{pmatrix} 1.000 & 0.709 & 0.709 & 0.709 & 0.709 & 0.919 & 0.709 & 0.709 & 0.709 & 0.709 \\ 0.709 & 1.000 & 0.913 & 0.771 & 0.811 & 0.709 & 0.913 & 0.889 & 0.771 & 0.811 \\ 0.709 & 0.913 & 1.000 & 0.771 & 0.811 & 0.709 & 0.937 & 0.889 & 0.771 & 0.811 \\ 0.709 & 0.771 & 0.771 & 1.000 & 0.771 & 0.709 & 0.771 & 0.771 & 0.968 & 0.771 \\ 0.709 & 0.811 & 0.811 & 0.768 & 1.000 & 0.709 & 0.811 & 0.811 & 0.771 & 0.861 \\ 0.919 & 0.709 & 0.709 & 0.709 & 0.709 & 1.000 & 0.709 & 0.709 & 0.709 & 0.709 \\ 0.709 & 0.913 & 0.937 & 0.771 & 0.811 & 0.709 & 1.000 & 0.889 & 0.771 & 0.811 \\ 0.709 & 0.889 & 0.889 & 0.771 & 0.811 & 0.709 & 0.889 & 1.000 & 0.771 & 0.811 \\ 0.709 & 0.771 & 0.771 & 0.968 & 0.771 & 0.709 & 0.771 & 0.771 & 1.000 & 0.771 \\ 0.709 & 0.811 & 0.811 & 0.771 & 0.861 & 0.709 & 0.811 & 0.811 & 0.771 & 1.000 \end{pmatrix},$$

$$C^{16} = \begin{pmatrix} 1.000 & 0.709 & 0.709 & 0.709 & 0.709 & 0.919 & 0.709 & 0.709 & 0.709 & 0.709 \\ 0.709 & 1.000 & 0.913 & 0.771 & 0.811 & 0.709 & 0.913 & 0.889 & 0.771 & 0.811 \\ 0.709 & 0.913 & 1.000 & 0.771 & 0.811 & 0.709 & 0.937 & 0.889 & 0.771 & 0.811 \\ 0.709 & 0.771 & 0.771 & 1.000 & 0.771 & 0.709 & 0.771 & 0.771 & 0.968 & 0.771 \\ 0.709 & 0.811 & 0.811 & 0.768 & 1.000 & 0.709 & 0.811 & 0.811 & 0.771 & 0.861 \\ 0.919 & 0.709 & 0.709 & 0.709 & 0.709 & 1.000 & 0.709 & 0.709 & 0.709 & 0.709 \\ 0.709 & 0.913 & 0.937 & 0.771 & 0.811 & 0.709 & 1.000 & 0.889 & 0.771 & 0.811 \\ 0.709 & 0.889 & 0.889 & 0.771 & 0.811 & 0.709 & 0.889 & 1.000 & 0.771 & 0.811 \\ 0.709 & 0.771 & 0.771 & 0.968 & 0.771 & 0.709 & 0.771 & 0.771 & 1.000 & 0.771 \\ 0.709 & 0.811 & 0.811 & 0.771 & 0.861 & 0.709 & 0.811 & 0.811 & 0.771 & 1.000 \end{pmatrix},$$

hence, $C^8 = C^{16}$, i.e., C^8 is an equivalent association matrix.

Step 3. Since the confidence level λ has a close relationship with the element of the equivalent association matrix C^8, in the following, we give a detailed sensitivity analysis with respect to the confidence level λ and we get all possible classifications of the 10 new cars $A_i(i = 1, 2, ..., 10)$: classification is shown in Table 6.

Classification of the above Illustrations are provided in Table 6.

Table 6. Classification of intuitionistic and picture fuzzy sets using clustering algorithm.

	Class	Confidence level	Clustering result
	5	$0.9486 \leq \lambda \leq 1$	$\{\overline{E}_2\}, \{\overline{E}_3\}, \{\overline{E}_6\}, \{\overline{E}_9\}, \{\overline{0_X}\}$
	4	$0.9064 \leq \lambda \leq 0.9486$	$\{\overline{E}_2\}, \{\overline{E}_3\}\{\overline{E}_6, \overline{0_X}\}, \{\overline{E}_9\}$
PFSf	3	$0.8924 \leq \lambda \leq 0.9064$	$\{\overline{E}_2, \overline{E}_9\}, \{\overline{E}_3\}, \{\overline{0_X}, \overline{E}_6\}$
	2	$0.8495 \leq \lambda \leq 0.8924$	$\{\overline{E}_3\}, \{\overline{E}_2, \overline{E}_6, \overline{E}_9, \overline{0_X}\}$
	1	$0 \leq \lambda \leq 0.8495$	$\{\overline{E}_2, \overline{E}_3, \overline{E}_6, \overline{E}_9, \overline{0_X}\}$
	5	$0.9436 \leq \lambda \leq 1$	$\{\overline{A}_6\}, \{\overline{A}_8\}, \{\overline{A}_1\}, \{\overline{A}_{10}\}, \{\overline{0_X}\}$
	4	$0.8750 \leq \lambda \leq 0.9436$	$\{\overline{A}_6, \overline{A}_8\}, \{\overline{A}_1\}, \{\overline{A}_{10}\}, \{\overline{0_X}\}$
IFSf	4	$0.5285 \leq \lambda \leq 0.8750$	$\{\overline{A}_6, \overline{A}_8\}, \{\overline{A}_1\}, \{\overline{A}_{10}\}, \{\overline{0_X}\}$
	4	$0.5204 \leq \lambda \leq 0.5285$	$\{\overline{A}_6, \overline{A}_8\}, \{\overline{A}_1\}, \{\overline{A}_{10}\}, \{\overline{0_X}\}$
	1	$0 \leq \lambda \leq 0.5204$	$\{\overline{A}_1, \overline{A}_6, \overline{A}_8, \overline{A}_{10}, \overline{0_X}\}$
	10	$0.968 \leq \lambda \leq 1$	$\{A_3\}, \{A_7\}, \{A_5\}, \{A_{10}\}, \{A_6\}, \{A_1\}\ \{A_2\}, \{A_8\}\ \{A_4\}, \{A_9\}$
	9	$0.937 \leq \lambda \leq 0.968$	$\{A_3\}, \{A_7\}, \{A_5\}, \{A_{10}\}, \{A_6\}, \{A_1\}\ \{A_2\}, \{A_8\}\ \{A_4, A_9\}$
	8	$0.919 \leq \lambda \leq 0.937$	$\{A_3, A_7\}, \{A_5\}, \{A_{10}\}, \{A_6\}, \{A_1\}\ \{A_2\}, \{A_8\}\ \{A_4, A_9\}$
	7	$0.913 \leq \lambda \leq 0.919$	$\{A_3, A_7\}, \{A_5\}, \{A_{10}\}, \{A_6, A_1\}\ \{A_2\}, \{A_8\}\ \{A_4, A_9\}$
IFS	6	$0.889 \leq \lambda \leq 0.913$	$\{A_3, A_2, A_7\}, \{A_5\}, \{A_{10}\}, \{A_6, A_1\}, \{A_8\}\ \{A_4, A_9\}$
	7	$0.861 \leq \lambda \leq 0.889$	$\{A_3, A_7\}, \{A_5\}, \{A_{10}\}, \{A_6, A_1\}\ \{A_2\}, \{A_8\}\ \{A_4, A_9\}$
	6	$0.811 \leq \lambda \leq 0.861$	$\{A_3, A_7\}, \{A_5, A_{10}\}, \{A_6, A_1\}\ \{A_2\}, \{A_8\}\ \{A_4, A_9\}$
	5	$0.771 \leq \lambda \leq 0.811$	$\{A_3, A_5, A_7, A_{10}\}, \{A_6, A_1\}, \{A_2\}, \{A_8\}\ \{A_4, A_9\}$
	2	$0.709 \leq \lambda \leq 0.771$	$\{A_2, A_3, A_4, A_5, A_7, A_8, A_9, A_{10}\}, \{A_6, A_1\}$
	1	$0 \leq \lambda \leq 0.709$	$\{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}\}$

4.5. *Comparative analysis*

The clustering algorithm for picture fuzzy sets applied to picture fuzzy filter collection to classify the picture fuzzy sets can accommodate situations in which the inputs are picture fuzzy in nature. As the picture fuzzy set is the generalization of the fuzzy set and intuitionistic fuzzy set and hence the proposed method can be widely used. In Illustration 3, the classification of the intuitionistic fuzzy set is obtained by the C^{16} associative matrix, whereas the classification of the intuitionistic fuzzy set belonging to the intuitionistic fuzzy filter by the clustering technique is obtained at the fourth stage. In Illustration 1, the classification of picture fuzzy sets belonging to picture fuzzy filter collection is obtained at the fourth stage, and the result is more generalized than the intuitionistic fuzzy set.

4.6. *Advantages of proposed method*

The correlation coefficient for the intuitionistic fuzzy set has some limitations and cannot reflect the complete information about the nature of the fuzzy set. Picture fuzzy set is an extension of the intuitionistic fuzzy set, which reflects the information about positive, negative, and neutral membership and also the degree of refusal membership. The correlation coefficient between picture fuzzy sets proposed by P. Singh [21] is applied to the picture fuzzy filter collection to effectively classify picture fuzzy sets from the picture fuzzy topological space. Classification of picture fuzzy set from picture fuzzy filter collection of any cardinality is obtained at the fourth stage of an equivalent correlation coefficient. The classification is compared with other intuitionistic fuzzy set collection to show fewer iterations required to classify the sets.

5. Conclusions

This paper introduces the notion of picture fuzzy filter, picture fuzzy grill, and picture fuzzy ultrafilter. Properties of the picture fuzzy base and subbase are discussed. Interrelations between picture fuzzy filter, picture fuzzy grill and picture fuzzy ultrafilter are established along with their characterization. Picture fuzzy compact space is studied, and its characterization based on picture fuzzy filter, grill, and ultrafilter has been studied. A clustering algorithm for picture sets in a picture fuzzy filter is implemented with an illustration. Picture fuzzy filter collection reduces the number of iterations required to classify the picture fuzzy sets.

The clustering algorithm based on the correlation coefficient between picture fuzzy sets reflects the significance of positive, negative, and neutral membership. Classification of picture fuzzy sets using the clustering algorithm proposed by P. Singh [21] is applied to the collection of filters obtained from the picture fuzzy topological space. Thus the paper shows that the decision-making problem in picture fuzzy topological space can be performed in a better way by using the picture fuzzy filter collection. The computational process for the correlation matrix in this work is obtained using MAPLE. The iteration for the equivalent correlation matrix will end at the fourth stage for any cardinality of picture fuzzy filter collection obtained from the picture fuzzy topological space, and the comparison among picture fuzzy filter and Intuitionistic fuzzy filter collection of different cardinalities have been classified at the fourth stage of the equivalent correlation matrix. In the future, the proposed work can be explored more precisely by defining a new clustering algorithm using picture fuzzy topological distance measure and picture fuzzy filter to analyze the classification in the topological structure and compare the accuracy with the other existing clustering algorithms and also pattern recognition problems.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

The authors extend their appreciation to the Deanship of Scientific Research at King Khalid University for funding this work through Small group Research Project under grant number RGP1/325/44.

The authors would like to express their gratitude to King Khalid University, Saudi Arabia for providing administrative and technical support. Also, the authors would like to thank the referee for their comments and suggestions on the manuscript.

Conflict of interest

The authors declare no conflicts of interest regarding the publication of this article.

References

1. T. Al-shami, H. Ibrahim, A. Azzam, A. El-Maghrabi, SR-fuzzy sets and their weighted aggregated operators in application to decision-making, *J. Funct. Space.*, **2022** (2022), 3653225. http://dx.doi.org/10.1155/2022/3653225
2. T. Al-shami, A. Mhemdi, Generalized frame for orthopair fuzzy sets: (m,n)-fuzzy sets and their applications to multi-criteria decision-making methods, *Information*, **14** (2023), 56. http://dx.doi.org/10.3390/info14010056
3. T. Al-shami, (2,1)-Fuzzy sets: properties, weighted aggregated operators and their applications to multi-criteria decision-making methods, *Complex Intell. Syst.*, **9** (2023), 1687–1705. http://dx.doi.org/10.1007/s40747-022-00878-4
4. T. Al-shami, J. Alcantud, A. Mhemdi, New generalization of fuzzy soft sets: (a,b)-fuzzy soft sets, *AIMS Mathematics*, **8** (2023), 2995–3025. http://dx.doi.org/10.3934/math.2023155
5. Z. Ameen, T. Al-shami, A. Azzam, A. Mhemdi, A novel fuzzy structure: infra-fuzzy topological spaces, *J. Funct. Space.*, **2022** (2022), 9778069. http://dx.doi.org/10.1155/2022/9778069
6. K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Set. Syst.*, **20** (1986), 87–96. http://dx.doi.org/10.1016/S0165-0114(86)80034-3
7. K. Atanassov, G. Gargov, Elements of intuitionistic fuzzy logic-Part I, *Fuzzy Set. Syst.*, **95** (1998), 39–52. http://dx.doi.org/10.1016/S0165-0114(96)00326-0
8. C. Chang, Fuzzy topological spaces, *J. Math. Anal. Appl.*, **24** (1968), 182–190. http://dx.doi.org/10.1016/0022-247X(68)90057-7
9. D. Coker, An introduction to intuitionistic fuzzy topological spaces, *Fuzzy Set. Syst.*, **88** (1997), 81–89. http://dx.doi.org/10.1016/S0165-0114(96)00076-0
10. B. Cuong, Picture fuzzy sets, *Journal of Computer Science and Cybernetics*, **30** (2014), 409–420. http://dx.doi.org/10.15625/1813-9663/30/4/5032
11. B. Cuong, V. Kerinovich, Picture fuzzy sets-a new concept for computational intelligence problems, *Proceedings of Third World Congress on Information and Communication Technologies (WICT 2013)*, 2013, 1–6. http://dx.doi.org/10.1109/WICT.2013.7113099
12. P. Dutta, Medical diagnosis based on distance measures between picture fuzzy sets, *International Journal of Fuzzy System Applications*, **7** (2018), 15–36. http://dx.doi.org/10.4018/IJFSA.2018100102
13. T. Gerstenkorn, J. Mańko, Correlation of intuitionistic fuzzy sets, *Fuzzy Set. Syst.*, **44** (1991), 39–43. http://dx.doi.org/10.1016/0165-0114(91)90031-K
14. G. Klir, B. Yuan, *Fuzzy sets and fuzzy logic: theory and applications*, New Jersey: Prentice-Hall Inc, 1995.
15. O. Montiel, J. Camacho, R. Sepulveda, O. Castillo, Fuzzy system to control the movement of a wheeled mobile robot, In: *Soft computing for intelligent control and mobile robotics*, Berlin: Springer, 2010, 445–463. http://dx.doi.org/10.1007/978-3-642-15534-5_27
16. O. Montiel, R. Sepulveda, O. Castillo, A. Basturk, High performance fuzzy systems for real world problems, *Adv. Fuzzy Syst.*, **2012** (2012), 316187. http://dx.doi.org/10.1155/2012/316187
17. J. Peng, J. Wang, H. Zhang, T. Sun, X. Chen, OWA aggregation over a continuous fuzzy argument with applications in fuzzy multi-criteria decision-making, *J. Intell. Fuzzy Syst.*, **27** (2014), 1407–1417. http://dx.doi.org/10.3233/IFS-131107

18. J. Peters, Associated near sets of distance functions in pattern analysis, In: *Multi-disciplinary trends in artificial intelligence*, Berlin: Springer, 2011, 1–13. http://dx.doi.org/10.1007/978-3-642-25725-4_1
19. A. Razaq, I. Masmali, H. Garg, U. Shuaib, Picture fuzzy topological spaces and associated continuous functions, *AIMS Mathematics*, **7** (2022), 14840–14861. http://dx.doi.org/10.3934/math.2022814
20. M. Sanchez, O. Castillo, J. Castro, P. Melin, Fuzzy granular gravitational clustering algorithm for multivariate data, *Inform. Sci.*, **279** (2014), 498–511. http://dx.doi.org/10.1016/j.ins.2014.04.005
21. P. Singh, Correlation coefficients for picture fuzzy sets, *J. Intell. Fuzzy Syst.*, **28** (2015), 591–604. http://dx.doi.org/10.3233/IFS-141338
22. P. Smets, The degree of belief in a fuzzy event, *Inform. Sci.*, **25** (1981), 1–19. http://dx.doi.org/10.1016/0020-0255(81)90008-6
23. M. Sugeno, An introductory survey of fuzzy control, *Inform. Sci.*, **36** (1985), 59–83. http://dx.doi.org/10.1016/0020-0255(85)90026-X
24. N. Thao, N. Dinh, Rough picture fuzzy set and picture fuzzy topologies, *Journal of Computer Science and Cybernetics*, **31** (2015), 245–253. http://dx.doi.org/10.15625/1813-9663/31/3/5046
25. P. Thong, L. Son, Picture fuzzy clustering: a new computational intelligence method, *Soft Comput.*, **20** (2016), 3549–3562. http://dx.doi.org/10.1007/s00500-015-1712-7
26. C. Tian, J. Peng, S. Zhang, W. Zhang, J. Wang, Weighted picture fuzzy aggregation operators and their applications to multi-criteria decision-making problems, *Comput. Ind. Eng.*, **137** (2019), 106037. http://dx.doi.org/10.1016/j.cie.2019.106037
27. C. Tian, J. Peng, S. Zhang, J. Wang, M. Goh, A sustainability evaluation framework for WET-PPP projects based on a picture fuzzy similarity-based VIKOR method, *J. Clean. Prod.*, **289** (2021), 125130. http://dx.doi.org/10.1016/j.jclepro.2020.125130
28. V. Visalakshi, M. Uma, E. Roja, On soft fuzzy C-structure compactification, *Kochi Journal of Mathematics*, **8** (2013), 119–133.
29. Z. Wang, Z. Xu, S. Liu, J. Tang, A netting clustering analysis method under intuitionistic fuzzy environment, *Appl. Soft Comput.*, **11** (2011), 5558–5564. http://dx.doi.org/10.1016/j.asoc.2011.05.004
30. Z. Xu, J. Chen, J. Wu, Clustering algorithm for intuitionistic fuzzy sets, *Inform. Sci.*, **178** (2008), 3775–3790. http://dx.doi.org/10.1016/j.ins.2008.06.008
31. T. Yogalakshmi, O. Castillo, On intuitionistic fuzzy C-ends, In: *Advances in algebra and analysis*, Cham: Birkhäuser, 2018, 177–184. https://dx.doi.org/10.1007/978-3-030-01120-8_21
32. L. Zadeh, Fuzzy sets, *Information and Control*, **8** (1965), 338–353. http://dx.doi.org/10.1016/S0019-9958(65)90241-X

AIMS Press