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Research article

Combinative distance-based assessment method for decision-making with 2-tuple linguistic q-rung picture fuzzy sets

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Abstract: Multi-criteria group decision-making (MCGDM) approaches have a substantial effect on decision-making in a range of critical sectors, including science, business, and real-life research. These strategies also efficiently assist researchers in resolving challenges that may arise throughout their study activity. The current work's major purpose is to research and develop the combinative distance-based assessment (CODAS) approach by employing 2-tuple linguistic q-rung picture fuzzy sets (2TLq-RPFSs) as a background. The CODAS technique computes the distances from the negative ideal solutions and ranks the alternatives in increasing order. To compute the normal weights of attributes, the entropy weighting information process is used. Furthermore, two aggregation operators, namely the 2-tuple linguistic q-rung picture fuzzy Einstein weighted average and the 2-tuple linguistic q-rung picture fuzzy Einstein order weighted average, are introduced. Our inspiration for employing the notion of 2TLq-RPFSs is the ability of q-RPFSs to support a wide range of information and the significant qualities of 2-tuple linguistic term sets to handle qualitative data. Congested transportation networks may be made more efficient by leveraging digital transformation. Real-time traffic management is one solution to the problem of road congestion. As a result of connected autonomous vehicle (CAV) advances, the benefits of real-time traffic management systems have grown dramatically. CAVs can help manage traffic by acting as enforcers. To complement the extended approach, the proposed technique is used to select the best alternative for a real-time traffic management system. The performance of the suggested technique is validated using scenario analysis. The results show that the suggested strategy is efficient and relevant to real-world situations.

Keywords: Einstein operators; CODAS method; autonomous vehicles; decision-making

1. Introduction

The benefits and consequences of the digital transformation of vehicle technology for sustainability are becoming obvious, but the literature on the issue is still scarce. Increased traffic safety and efficiency are among the benefits of autonomous vehicles (AVs). Infrastructures must be upgraded as AV technology progresses to truly enjoy these benefits [1]. Technologies such as vehicle-to-infrastructure (V2I), vehicle-to-vehicle (V2V), and 5G-enabled AVs can communicate and receive data from other cars and infrastructure. AVs with such technology have been called "connected autonomous vehicles" (CAVs). According to Molnar et al. [2], the application of V2X (vehicle-to-everything) technology and CAVs for traffic management problems has the ability to minimize growing congestion and offer energy efficiency. Traditional control methods, like traffic lights, can occasionally cause bottlenecks, according to Azadi [3]. Moreover, deploying CAVs in traffic flow decreases expenses by eliminating the requirement for sensors because the data transmission characteristic of CAVs assume sensor duties [4]. As a consequence, the expense of placing sensors on infrastructure is avoided. Fuzzy systems have been effectively implemented into a variety of issues by the researchers [5, 6]. In order to better represent the viewpoints of experts when presented with confusing and imprecise information, Zadeh [7] provided the framework for an astonishing theory called fuzzy set (FS) theory (specified by a membership function μ). Eventually, researchers created certain FS extensions that are quite useful for dealing with multi-criteria group decision-making (MCGDM) challenges. For example, Atanassov [8] offered the concept of intuitionistic FS (IFS), Yager [9, 10] introduced the idea of Pythagorean FS (PyFS), Senapati and Yager [11] created the notion of Fermatean FS (FFS), and Yager [12] proposed q-rung orthopair FS (q-ROFS) by introducing a negative membership grade (ν) having the conditions, $\mu + \nu \leq 1$, $\mu^2 + \nu^2 \leq 1$, $\mu^3 + \nu^3 \leq 1$, and $\mu^q + \nu^q \leq 1$ $(q \geq 1)$, respectively. Cuong and Kreinovich [13] created Picture FS (PFS) to collect more accurate assessment data by taking into consideration a neutral membership grade (γ). With the condition $\mu + \nu + \gamma \leq 1$, a PFS can accept three membership degrees: positive membership (PM), neutral membership (NM), and negative membership (N_eM). Gündoğdu and Kahraman [14], and Mahmood et al. [15] proposed the idea of spherical FS (SFS), where $(\mu^2 + \nu^2 + \gamma^2 \leq 1)$ is the condition. Li et al. [16] introduced the q-rung PFS (q-RPFS), which employs Yager's q-ROFS $(\mu^q + \nu^q + \gamma^q \leq 1, q \geq 1)$. Gurmani et al. [17] presented some Dombi operations on linguistic T-spherical fuzzy numbers along with an application. Gurmani et al. [18] extended various decision-making methodologies and implemented them on different problems.

Real-time traffic control systems are often employed to significantly reduce congestion. There are various studies in the literature on the efficiency of traffic management systems. Variable speed limit (VSL) is one of the most extensively used real-time traffic control technique. Research on the enhancements given by VSL found that sometimes decreasing speed limits during problematic traffic situations reduced incident risk by 5%-17% ([19]). In a separate work, Hegyi et al. [20] offer a model predictive control (MPC) to optimally coordinate VSL for highway traffic, and advantages connected to the use of this technique are examined. The results reveal less congestion, more outflow, and shorter travel times. Another real-time traffic control strategy is ramp metering. The findings of simulations in several studies on the traffic efficiency benefits of ramp metering management systems reveal that ramp metering improves traffic flow by improving travel speeds, lowering travel times, and increasing volumes. Various research presents conclusions relating to safety improvements in ramp metering [21]. Dynamic route guidance is one form of real-time traffic management. The benefits of dynamic route guiding are studied in separate research, in addition to efficiency gains, pollution reduction, and fuel economy [22]. With the newest advancements in autonomous and communication technology over the last decade, the focus of traffic management has changed towards integrating linked autonomous cars into traditional traffic control approaches. The use of CAVs in traffic management approaches appears to be improving control system efficiency. Many recent studies in the literature corroborate this efficiency boost. An evaluation of bottleneck discharge rates and system delay reductions is undertaken in research concerning the integration of CAVs with VSL [23]. Wang et al. [24] examined the merging CAVs and dynamic route guiding systems in a separate research. A simulated analysis of the subject reveals that the capacity of CAVs to acquire and send data permits the discovery of shorter routes in terms of journey durations. In a separate research, the impact of deploying CAVs on lane control signals (LCS) traffic management approaches is explored [25–27]. A simulation research is carried out in an environment that has been calibrated using real-world traffic data. The results reveal that the CAV-enabled LCS management technique beats real-world LCS applications by an average of 12.8% in throughput. Apart from combining traditional traffic management approaches and CAVs, another study topic is the use of CAVs alone to enhance traffic conditions. While there is less research on this issue in the literature, current studies demonstrate that this sort of traffic management is also beneficial ([28]). Gokasar et al. [30] used rough fuzzy numbers to expand the MACBETH (Measuring Attractiveness by a Categorical Based Evaluation Method). In this study, the benefits of RTTM strategies with and without CAVs are prioritized by comparing these approaches using the criteria. The most advantageous option is picked based on the comparative findings.

1.1. Study motivations and contributions

In this paper, we use Einstein aggregation operators (AOs) to build the 2TLq-RPF-CODAS approach. The motivations for using the 2TLq-RPFSs, extending CODAS method and Einstein AOs, and implementing the proposed methodology to an application are stated as follows:

- In DM circumstances, professionals frequently utilize language words such as “good”, “bad”, “great”, and “poor” to appraise the options. Zadeh [31] proposed the concept of linguistic variables as a technique to more effectively transmit human viewpoints in DM. He established the notion of linguistic variables and stressed that linguistic terms may effectively describe linguistic information. Experts can communicate their views more precisely thanks to linguistic variables. Herrera and Martínez [32, 33] developed the 2-tuple linguistic term set (2TLTS) to better transmit judgement and evaluation information in MCGDM settings. 2TLTS is a promising approach for more efficiently conveying evaluation information in DM situations. Because of these traits, the researchers concentrated their efforts on them. Many studies have focused on the 2TLRM, and it has emerged as an important technique for treating linguistic DM difficulties. Researchers have used this idea, including Zhang [34] and Garg [35]. Lin et al. [36] introduced the idea of linguistic q-ROFS (Lq-ROFS). More information can be found by reading some related researches ([37]). Many AOs have been developed by researchers and are presently in use in linguistic contexts. Akram et al. [38] recently proposed the notion of 2TLq-RPFSs and applied it to solve an MCGDM problem. The aforementioned research inspired us to present our work in the context of the 2TLq-RPFSs.

- TOPSIS [39], MABAC [40], VIKOR [41], and other decision-making approaches were invented and expanded by researchers. Lihong et al. [42] presented an improved version of VIKOR method for the selection of thermal power enterprise's coal suppliers. The fundamental goal of these methodologies is to solve DM difficulties. These strategies were initially created using crisp data. First, Bellman and Zadeh [43] developed a DM technique for dealing with ambiguity in real-world issues in a fuzzy environment. The combinative distance-based assessment (CODAS) approach was pioneered by Ghorabaee et al. [44]. Moreover, Lei et al. [45] offered a combined assessment of objective criteria weights method. Simic et al. [46] created a picture fuzzy expansion of the CODAS algorithm. Wang [47] provided an interval-valued IF extension of the CODAS approach and evaluated a college's teaching quality. He et al. [48] proposed a 2TLPyF-COADS approach to tackle a financial management performance evaluation challenge. Naz et al. [49] demonstrated an issue related to arc welding robot selection by providing a unique extension of the CODAS technique. Vinodh et al. [50] integrated the DEMATEL (Decision-Making Trial and Evaluation Laboratory) method with the CODAS method and addressed a case study. An interval-valued IF CODAS method was presented by Deveci et al. [51], in which an example for the selection of renewable energy alternatives in Turkey was discussed. Aydogmus et al. [52] presented an enterprise resource planning (ERP) problem in picture fuzzy environment. Researchers proposed various extensions of the CODAS method [53, 54] and implemented them to tackle various real-world problems. Akram et al. [55–57] discussed the CODAS method and MABAC method in a 2-tuple linguistic setting. We choose the CODAS technique because of its simplicity and dependability in dealing with actual MCGDM difficulties. Moreover, an extension of the CODAS approach in a 2TLq-RPF context is still lacking. To fill the research gap, this work introduces a novel CODAS technique based on Einstein AOs for prioritizing connected autonomous cars in real-time traffic management (RTTM) systems.
- To get at the final findings, it is critical to synthesize data received from multiple perspectives during the decision-making process. The researchers used AOs to aggregate information during the MCGDM process. Peng and Yang [58] showed several characteristics of interval-valued PyF AOs. Yager [59] created ordered weighted average AOs for multi-criteria DM. Gassert [60] presented the operators on FSs. The AOs on IFSs were proposed by Xu [61]. t-norms and t-conorms on IFSs were created by Deschrijver et al. [62]. Garg [63] created generalized IF AOs for decision-making utilizing Einstein t-norm and t-conorm. Wang and Liu [64] presented several Einstein AOs against the backdrop of IFS. Researchers and intellectuals created and used many AOs and their expansions [65–67]. Some picture fuzzy AOs for DM were proposed by Wei [68]. Akram et al. [69] used Lq-ROFSs as a background for their work on Einstein AOs. Readers are directed to [70–73] for more study on Einstein AOs. According to earlier research, there is a requirement to establish Einstein AOs against the backdrop of 2TLq-RPFSs. As a result, we want to build two AOs: 2-tuple linguistic q-rung picture fuzzy Einstein weighted average (2TLq-RPFEWA) and 2-tuple linguistic q-rung picture fuzzy Einstein ordered weighted average (2TLq-RPFEOWA).
- The implementation of RTTM systems has a wide range of applications, ranging from traffic safety and efficiency to economic, employment, and energy-related concerns. Conventional traffic management approaches rely heavily on information systems, such as electronic traffic

boards, to instruct vehicles on how to adjust their speeds and change lanes. Researchers focus has shifted to the benefits of autonomous vehicle traffic management and incorporating autonomous cars into traffic management approaches due to advancements in autonomous vehicles and communication technology. Although it is an important and intriguing subject of research, there are numerous studies on the efficiency of traffic management systems in the literature. As a result of these investigations, we choose the example of a RTTM system with three options: 1) RTTM; 2) RTTM combined with linked autonomous cars; and 3) RTTM employing connected autonomous vehicles.

Table 1. Acronyms and their descriptions.

Acronym	Description	Acronym	Description
FS	Fuzzy set	2TLq-RPFS	2-Tuple linguistic q-RPFS
IFS	Intuitionistic FS	2TLq-RPFEWA	2TLq-RPF Einstein weighted average
PyFS	Pythagorean FS	2TLq-RPFEOWA	2TLq-RPF Einstein ordered weighted average
q-ROFS	q-Rung orthopair FS	SWARA	Stepwise weight assessment ratio analysis
FFS	Fermatean FS	IDOCRIW	Integrated determination of objective criteria weights
PFS	Picture FS	MCGDM	Multi-criteria group decision-making
q-RPFS	q-Rung PFS	PROMETHEE	Preference ranking organization method
SFS	Spherical FS		for enrichment evaluation
2TLTS	2-Tuple linguistic term set	ELECTRE	Elimination and choice expressing reality
AOs	Aggregation operators	RTTM	Real time traffic management
Lq-ROFS	Linguistic q-ROFS	WASPAS	Weighted aggregated sum product assessment
2TLPyS	2-Tuple linguistic PyFS	CODAS	Combinative distance-based assessment
2TLFFS	2-Tuple linguistic FFS	MACBETH	Measuring Attractiveness by a Categorical
PM	Positive membership		Based Evaluation Method
NM	Neutral membership	MULTIMOORA	Multi-objective optimization ratio analysis
N_eM	Negative membership		plus full multiplicative form
CAVs	Connected autonomous vehicles	TOPSIS	Technique for order preference by
VSL	Variable speed limit		similarity to an ideal solution
AHP	Analytical hierarchy process	VIKOR	VlseKriterijumska Optimizacija I Kompromisno Resenje
DEMATEL	Decision-making trial and	MABAC	Multi-attributive border approximation
	evaluation laboratory		area comparison method

The following are the contributions of this research:

- Our goal in this paper is to build some Einstein AOs against the backdrop of 2TLq-RPFSs. We achieve our aim by creating two Einstein AOs, notably the 2TLq-RPFEWA operator and the 2TLq-RPFEOWA operator, and studying their features.
- Our next goal is to develop a more comprehensive CODAS approach to address MCGDM problems. We create the CODAS technique in a 2TLq-RPF environment and use the AOs indicated above for aggregation. Based on the generalized properties of 2TLq-RPFS, we develop the 2TLq-RPF-CODAS approach. To calculate the weights of the criterion, we used the entropy measure. The suggested CODAS technique extension may be used to solve difficult MCGDM issues.
- The provided technique is applied to the problem of selecting the optimal alternative for a real-time traffic control system, demonstrating its applicability. Apart from that, the impact of parameters on the final findings is examined.
- To illustrate the validity of the established method, we conduct a comparison analysis of the

provided methodology with other recent techniques.

This article is organized into seven sections. Section 2 goes over some key principles related to the 2TLq-RPFSs. Section 3 introduces the 2TLq-RPFEWA and 2TLq-RPFEOWA AOs. Section 4 describes the 2TLq-RPF-CODAS method technique. Section 5 applies the current technique to a problem. Section 6 compares the provided technique with two previously published research. Section 7 concludes with closing comments. The acronyms used in this paper are described in Table 1.

2. Preliminaries

This section goes through a few fundamental 2TLq-RPFSs concepts to help readers comprehend the recommended method. First, we review the definition of 2TLq-RPFS and its related introductory concepts.

The notion of 2TLq-RPFSs proposed by Akram et al. [38] is described in the following definition.

Definition 2.1. [38] A 2TLq-RPFS A ($q \geq 1$) is defined as:

$$A = \{(g, ((\wp_\alpha(g), E(g)), (\wp_\beta(g), F(g)), (\wp_\gamma(g), G(g)))) : g \in K\}, \tag{2.1}$$

where $(\wp_\alpha(g), E(g))$, $(\wp_\beta(g), F(g))$ and $(\wp_\gamma(g), G(g))$ are the degrees of PM, NM and N_eM, respectively. A 2TLq-RPFS should satisfy the following conditions:

$$\wp_\alpha(g), \wp_\beta(g), \wp_\gamma(g) \in A, \quad E(g), F(g), G(g) \in [-0.5, 0.5),$$

$$0 \leq \Lambda^{-1}(\wp_\alpha(g), E(g)) \leq \Theta + 1,\ 0 \leq \Lambda^{-1}(\wp_\beta(g), F(g)) \leq \Theta + 1,\ 0 \leq \Lambda^{-1}(\wp_\gamma(g), G(g)) \leq \Theta + 1,$$
$$0 \leq (\Lambda^{-1}(\wp_\alpha(g), E(g)))^q + (\Lambda^{-1}(\wp_\beta(g), F(g)))^q + (\Lambda^{-1}(\wp_\gamma(g), G(g)))^q \leq (\Theta + 1)^q.$$

For simplicity, we call $A = ((\wp_\alpha, E), (\wp_\beta, F), (\wp_\gamma, G))$, a 2TL$q$-RPF number (2TL$q$-RPFN) with $0 \leq \Lambda^{-1}(\wp_\alpha, E) \leq \Theta + 1, 0 \leq \Lambda^{-1}(\wp_\beta, F) \leq \Theta + 1, 0 \leq \Lambda^{-1}(\wp_\gamma, G) \leq \Theta + 1$, $0 \leq (\Lambda^{-1}(\wp_\alpha, E))^q + (\Lambda^{-1}(\wp_\beta, F))^q + (\Lambda^{-1}(\wp_\gamma, G))^q \leq (\Theta + 1)^q$.

Definition 2.2. [38] Let a, b, c denote the values obtained after performing an operation on the LTs' indices in a LTS S with $a, b, c \in [0, \Theta]$. In addition, suppose that

$$\alpha = round(a), \quad \beta = round(b), \quad \gamma = round(c), \quad \text{and}$$

$$E = a - \alpha, \quad F = b - \beta, \quad G = c - \gamma,$$

where, $E, F, G \in [-0.5, 0.5)$, then E, F, G are the values of symbolic translation.

The following definition is used to transform a numerical value into a 2-tuple.

Definition 2.3. [38] Let $S = \{\wp_k : k = 0, 1, 2, \ldots, \Theta\}$ represents a LTS and $a, b, c \in [0, \Theta]$. Then the function $\Lambda : [0, \Theta] \to S \times [-0.5, 0.5)$ to acquire the 2TL information (equivalent to a, b, c) can be defined as:

$$\Lambda(a) = \begin{cases} \wp_\alpha, & \alpha = round(a) \\ E = a - \alpha, & E \in [-0.5, 0.5), \end{cases}$$

$$\Lambda(b)=\begin{cases}\wp_\beta, & \beta=round(b)\\ F=b-\beta, & F\in[-0.5,0.5),\end{cases}$$

$$\Lambda(c)=\begin{cases}\wp_\gamma, & \gamma=round(c)\\ G=c-\gamma, & G\in[-0.5,0.5),\end{cases}$$

The following definition converts a 2-tuple into its equivalent numerical value.

Definition 2.4. [38] Let $S=\{\wp_k : k=0,1,2,\ldots,\Theta\}$ represents a LTS and consider a 2TLq-RPFN

$$A=((\wp_\alpha,E),(\wp_\beta,F),(\wp_\gamma,G)),$$

then, each 2TLq-RPFN is restored to its equivalent numerical value $a,b,c\in[0,\Theta]$, using the function $\Lambda^{-1}: S\times[-0.5,0.5)\to[0,\Theta]$, where

$$\Lambda^{-1}(\wp_\alpha,E)=\alpha+E=a,$$
$$\Lambda^{-1}(\wp_\beta,F)=\beta+F=b,$$
$$\Lambda^{-1}(\wp_\gamma,G)=\gamma+G=c.$$

Some basic operational laws between two 2TLq-RPFNs are presented in the following definition.

Definition 2.5. [38] Let $A=((\wp_\alpha,E),(\wp_\beta,F),(\wp_\gamma,G))$, $A_1=((\wp_{\alpha_1},E_1),(\wp_{\beta_1},F_1),(\wp_{\gamma_1},G_1))$ and $A_2=((\wp_{\alpha_2},E_2),(\wp_{\beta_2},F_2),(\wp_{\gamma_2},G_2))$ are three 2TLq-RPFNs with $q\geq 1$ and $\rho>0$. Then

$$1)\ A_1\bigoplus A_2=\left(\begin{array}{c}\Lambda\Big(\Theta\big(1-\big(1-\big(\frac{\Lambda^{-1}(\wp_{\alpha_1},E_1)}{\Theta}\big)^q\big)\big(1-\big(\frac{\Lambda^{-1}(\wp_{\alpha_2},E_2)}{\Theta}\big)^q\big)\big)^{\frac{1}{q}}\Big),\\ \Lambda\Big(\Theta\big(\frac{\Lambda^{-1}(\wp_{\beta_1},F_1)}{\Theta}\big)\big(\frac{\Lambda^{-1}(\wp_{\beta_2},F_2)}{\Theta}\big)\Big),\\ \Lambda\Big(\Theta\big(\frac{\Lambda^{-1}(\wp_{\gamma_1},G_1)}{\Theta}\big)\big(\frac{\Lambda^{-1}(\wp_{\gamma_2},G_2)}{\Theta}\big)\Big)\end{array}\right).$$

$$2)\ A_1\bigotimes A_2=\left(\begin{array}{c}\Lambda\Big(\Theta\big(\frac{\Lambda^{-1}(\wp_{\alpha_1},E_1)}{\Theta}\big)\big(\frac{\Lambda^{-1}(\wp_{\alpha_2},E_2)}{\Theta}\big)\Big),\\ \Lambda\Big(\Theta\big(1-\big(1-\big(\frac{\Lambda^{-1}(\wp_{\beta_1},F_1)}{\Theta}\big)^q\big)\big(1-\big(\frac{\Lambda^{-1}(\wp_{\beta_2},F_2)}{\Theta}\big)^q\big)\big)^{\frac{1}{q}}\Big),\\ \Lambda\Big(\Theta\big(1-\big(1-\big(\frac{\Lambda^{-1}(\wp_{\gamma_1},G_1)}{\Theta}\big)^q\big)\big(1-\big(\frac{\Lambda^{-1}(\wp_{\gamma_2},G_2)}{\Theta}\big)^q\big)\big)^{\frac{1}{q}}\Big)\end{array}\right).$$

$$3)\ \rho A=\left(\Lambda\Big(\Theta\big(1-\big(1-\big(\frac{\Lambda^{-1}(\wp_\alpha,E)}{\Theta}\big)^q\big)^\rho\big)^{\frac{1}{q}}\Big),\Lambda\Big(\Theta\big(\frac{\Lambda^{-1}(\wp_\beta,F)}{\Theta}\big)^\rho\Big),\Lambda\Big(\Theta\big(\frac{\Lambda^{-1}(\wp_\gamma,G)}{\Theta}\big)^\rho\Big)\right).$$

$$4)\ A^\rho=\left(\Lambda\Big(\Theta\big(\frac{\Lambda^{-1}(\wp_\alpha,E)}{\Theta}\big)^\rho\Big),\Lambda\Big(\Theta\big(1-\big(1-\big(\frac{\Lambda^{-1}(\wp_\beta,F)}{\Theta}\big)^q\big)^\rho\big)^{\frac{1}{q}}\Big),\Lambda\Big(\Theta\big(1-\big(1-\big(\frac{\Lambda^{-1}(\wp_\gamma,G)}{\Theta}\big)^q\big)^\rho\big)^{\frac{1}{q}}\Big)\right).$$

3. 2TLq-RPF Einstein aggregation operators

In this section, we present the Einstein aggregation operators for 2TLq-RPFNs. First, we present the basic Einstein operational laws between two 2TLq-RPFNs in the following definition.

Definition 3.1. Consider three 2TLq-RPFNs $L = ((\wp_\alpha, E), (\wp_\beta, F), (\wp_\gamma, G))$, $L_1 = ((\wp_{\alpha_1}, E_1), (\wp_{\beta_1}, F_1), (\wp_{\gamma_1}, G_1))$ and $L_2 = ((\wp_{\alpha_2}, E_2), (\wp_{\beta_2}, F_2), (\wp_{\gamma_2}, G_2))$ with $q \geq 1$ and $\rho > 0$. Then the 2TLq-RPF Einstein operations between L_1 and L_2 are:

$$1)\ L_1 \bigoplus L_2 = \left(\begin{array}{c} \Lambda\Bigg(\Theta\Bigg(\sqrt[q]{\dfrac{\big(\frac{\Lambda^{-1}(\wp_{\alpha_1}, E_1)}{\Theta}\big)^q + \big(\frac{\Lambda^{-1}(\wp_{\alpha_2}, E_2)}{\Theta}\big)^q}{1 + \big(\frac{\Lambda^{-1}(\wp_{\alpha_1}, E_1)}{\Theta}\big)^q \big(\frac{\Lambda^{-1}(\wp_{\alpha_2}, E_2)}{\Theta}\big)^q}}\Bigg)\Bigg), \\ \Lambda\Bigg(\Theta\Bigg(\dfrac{\big(\frac{\Lambda^{-1}(\wp_{\beta_1}, F_1)}{\Theta}\big)\big(\frac{\Lambda^{-1}(\wp_{\beta_2}, F_2)}{\Theta}\big)}{\sqrt[q]{1 + \Big(1 - \big(\frac{\Lambda^{-1}(\wp_{\beta_1}, F_1)}{\Theta}\big)^q\Big)\Big(1 - \big(\frac{\Lambda^{-1}(\wp_{\beta_2}, F_2)}{\Theta}\big)^q\Big)}}\Bigg)\Bigg), \\ \Lambda\Bigg(\Theta\Bigg(\dfrac{\big(\frac{\Lambda^{-1}(\wp_{\gamma_1}, G_1)}{\Theta}\big)\big(\frac{\Lambda^{-1}(\wp_{\gamma_2}, G_2)}{\Theta}\big)}{\sqrt[q]{1 + \Big(1 - \big(\frac{\Lambda^{-1}(\wp_{\gamma_1}, G_1)}{\Theta}\big)^q\Big)\Big(1 - \big(\frac{\Lambda^{-1}(\wp_{\gamma_2}, G_2)}{\Theta}\big)^q\Big)}}\Bigg)\Bigg) \end{array} \right).$$

$$2)\ L_1 \bigotimes L_2 = \left(\begin{array}{c} \Lambda\Bigg(\Theta\Bigg(\dfrac{\big(\frac{\Lambda^{-1}(\wp_{\alpha_1}, E_1)}{\Theta}\big)\big(\frac{\Lambda^{-1}(\wp_{\alpha_2}, E_2)}{\Theta}\big)}{\sqrt[q]{1 + \Big(1 - \big(\frac{\Lambda^{-1}(\wp_{\alpha_1}, E_1)}{\Theta}\big)^q\Big)\Big(1 - \big(\frac{\Lambda^{-1}(\wp_{\alpha_2}, E_2)}{\Theta}\big)^q\Big)}}\Bigg)\Bigg), \\ \Lambda\Bigg(\Theta\Bigg(\sqrt[q]{\dfrac{\big(\frac{\Lambda^{-1}(\wp_{\beta_1}, F_1)}{\Theta}\big)^q + \big(\frac{\Lambda^{-1}(\wp_{\beta_2}, F_2)}{\Theta}\big)^q}{1 + \big(\frac{\Lambda^{-1}(\wp_{\beta_1}, F_1)}{\Theta}\big)^q \big(\frac{\Lambda^{-1}(\wp_{\beta_2}, F_2)}{\Theta}\big)^q}}\Bigg)\Bigg), \\ \Lambda\Bigg(\Theta\Bigg(\sqrt[q]{\dfrac{\big(\frac{\Lambda^{-1}(\wp_{\gamma_1}, G_1)}{\Theta}\big)^q + \big(\frac{\Lambda^{-1}(\wp_{\gamma_2}, G_2)}{\Theta}\big)^q}{1 + \big(\frac{\Lambda^{-1}(\wp_{\gamma_1}, G_1)}{\Theta}\big)^q \big(\frac{\Lambda^{-1}(\wp_{\gamma_2}, G_2)}{\Theta}\big)^q}}\Bigg)\Bigg), \end{array} \right).$$

$$3)\ \rho L=\begin{pmatrix}\Lambda\left(\Theta\left(\sqrt[q]{\frac{\left(1+\frac{\Lambda^{-1}(\wp_{\alpha},E)}{\Theta}\right)^{\rho}-\left(1-\frac{\Lambda^{-1}(\wp_{\alpha},E)}{\Theta}\right)^{\rho}}{\left(1+\frac{\Lambda^{-1}(\wp_{\alpha},E)}{\Theta}\right)^{\rho}+\left(1-\frac{\Lambda^{-1}(\wp_{\alpha},E)}{\Theta}\right)^{\rho}}}\right)\right),\\ \Lambda\left(\Theta\left(\frac{\sqrt[q]{2}\left(\frac{\Lambda^{-1}(\wp_{\beta},F)}{\Theta}\right)^{\rho}}{\sqrt[q]{\left(2-\left(\frac{\Lambda^{-1}(\wp_{\beta},F)}{\Theta}\right)^{q}\right)^{\rho}+\left(\left(\frac{\Lambda^{-1}(\wp_{\beta},F)}{\Theta}\right)^{q}\right)^{\rho}}}\right)\right),\\ \Lambda\left(\Theta\left(\frac{\sqrt[q]{2}\left(\frac{\Lambda^{-1}(\wp_{\gamma},G)}{\Theta}\right)^{\rho}}{\sqrt[q]{\left(2-\left(\frac{\Lambda^{-1}(\wp_{\gamma},G)}{\Theta}\right)^{q}\right)^{\rho}+\left(\left(\frac{\Lambda^{-1}(\wp_{\gamma},G)}{\Theta}\right)^{q}\right)^{\rho}}}\right)\right)\end{pmatrix}.$$

$$4)\ L^{\rho}=\begin{pmatrix}\Lambda\left(\Theta\left(\frac{\sqrt[q]{2}\left(\frac{\Lambda^{-1}(\wp_{\alpha},E)}{\Theta}\right)^{\rho}}{\sqrt[q]{\left(2-\left(\frac{\Lambda^{-1}(\wp_{\alpha},E)}{\Theta}\right)^{q}\right)^{\rho}+\left(\left(\frac{\Lambda^{-1}(\wp_{\alpha},E)}{\Theta}\right)^{q}\right)^{\rho}}}\right)\right),\\ \Lambda\left(\Theta\left(\sqrt[q]{\frac{\left(1+\frac{\Lambda^{-1}(\wp_{\beta},F)}{\Theta}\right)^{\rho}-\left(1-\frac{\Lambda^{-1}(\wp_{\beta},F)}{\Theta}\right)^{\rho}}{\left(1+\frac{\Lambda^{-1}(\wp_{\beta},F)}{\Theta}\right)^{\rho}+\left(1-\frac{\Lambda^{-1}(\wp_{\beta},F)}{\Theta}\right)^{\rho}}}\right)\right),\\ \Lambda\left(\Theta\left(\sqrt[q]{\frac{\left(1+\frac{\Lambda^{-1}(\wp_{\gamma},G)}{\Theta}\right)^{\rho}-\left(1-\frac{\Lambda^{-1}(\wp_{\gamma},G)}{\Theta}\right)^{\rho}}{\left(1+\frac{\Lambda^{-1}(\wp_{\gamma},G)}{\Theta}\right)^{\rho}+\left(1-\frac{\Lambda^{-1}(\wp_{\gamma},G)}{\Theta}\right)^{\rho}}}\right)\right)\end{pmatrix}.$$

Now we present the 2TLq-RPFEWA operator and its related fundamental properties:

Definition 3.2. The 2TLq-RPFEWA operator is a mapping $H^n \to H$ such that: for each collection of 2TLq-RPFNs $M_k = ((\wp_{\alpha_k}, E_k), (\wp_{\beta_k}, F_k), (\wp_{\gamma_k}, G_k))$ $(k = 1, 2, \ldots, n)$,

$$2TLq-RPFEWA(M_1, M_2, \ldots, M_n) = \bigoplus_{k=1}^{n} \mathcal{W}_k M_k, \tag{3.1}$$

where $\mathcal{W} = (\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_n)^T$ is the weight vector of M_k $(k = 1, 2, \ldots, n)$ with $\mathcal{W}_k \in [0, 1]$ and $\sum_{k=1}^{n} \mathcal{W}_k = 1$.

Theorem 3.1. *Consider a collection of 2TLq-RPFNs* $M_k = ((\wp_{\alpha_k}, E_k), (\wp_{\beta_k}, F_k), (\wp_{\gamma_k}, G_k))$ $(k = 1, 2, \ldots, n)$ *having weight vector* $\mathcal{W} = (\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_n)^T$ *with* $\mathcal{W}_k \in [0, 1]$ *and* $\sum_{k=1}^{n} \mathcal{W}_k = 1$.

Then

$$2TLq-RPFEWA(M_1,M_2,\ldots,M_n)=$$

$$\left(\begin{array}{c}\Lambda\left(\Theta\left(\sqrt[q]{\frac{\prod_{k=1}^{n}\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}-\prod_{k=1}^{n}\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}}{\prod_{k=1}^{n}\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}+\prod_{k=1}^{n}\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}}}\right)\right),\\ \Lambda\left(\Theta\left(\frac{\sqrt[q]{2}\prod_{k=1}^{n}\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^{\mathcal{W}_k}}{\sqrt[q]{\prod_{k=1}^{n}\left(2-\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^q\right)^{\mathcal{W}_k}+\prod_{k=1}^{n}\left(\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^q\right)^{\mathcal{W}_k}}}\right)\right),\\ \Lambda\left(\Theta\left(\frac{\sqrt[q]{2}\prod_{k=1}^{n}\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^{\mathcal{W}_k}}{\sqrt[q]{\prod_{k=1}^{n}\left(2-\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^q\right)^{\mathcal{W}_k}+\prod_{k=1}^{n}\left(\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^q\right)^{\mathcal{W}_k}}}\right)\right)\end{array}\right). \quad (3.2)$$

Proof. We use induction method and Definition 3.1 to prove this theorem. For $n=2$, we have

$$\mathcal{W}_1M_1\bigoplus\mathcal{W}_2M_2=$$

$$\left(\begin{array}{c}\Lambda\left(\Theta\left(\sqrt[q]{\frac{\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_1},E_1)}{\Theta}\right)^{\mathcal{W}_1}-\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_1},E_1)}{\Theta}\right)^{\mathcal{W}_1}}{\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_1},E_1)}{\Theta}\right)^{\mathcal{W}_1}+\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_1},E_1)}{\Theta}\right)^{\mathcal{W}_1}}}\right)\right),\\ \Lambda\left(\Theta\left(\frac{\sqrt[q]{2}\left(\frac{\Lambda^{-1}(\wp_{\beta_1},F_1)}{\Theta}\right)^{\mathcal{W}_1}}{\sqrt[q]{\left(2-\left(\frac{\Lambda^{-1}(\wp_{\beta_1},F_1)}{\Theta}\right)^q\right)^{\mathcal{W}_1}+\left(\left(\frac{\Lambda^{-1}(\wp_{\beta_1},F_1)}{\Theta}\right)^q\right)^{\mathcal{W}_1}}}\right)\right),\\ \Lambda\left(\Theta\left(\frac{\sqrt[q]{2}\left(\frac{\Lambda^{-1}(\wp_{\gamma_1},G_1)}{\Theta}\right)^{\mathcal{W}_1}}{\sqrt[q]{\left(2-\left(\frac{\Lambda^{-1}(\wp_{\gamma_1},G_1)}{\Theta}\right)^q\right)^{\mathcal{W}_1}+\left(\left(\frac{\Lambda^{-1}(\wp_{\gamma_1},G_1)}{\Theta}\right)^q\right)^{\mathcal{W}_1}}}\right)\right)\end{array}\right)\bigoplus$$

$$\left(\begin{array}{l}\Lambda\Bigg(\Theta\Bigg(\sqrt[q]{\dfrac{\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_2},E_2)}{\Theta}\right)^{w_2}-\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_2},E_2)}{\Theta}\right)^{w_2}}{\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_2},E_2)}{\Theta}\right)^{w_2}+\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_2},E_2)}{\Theta}\right)^{w_2}}}\Bigg)\Bigg),\\ \Lambda\Bigg(\Theta\Bigg(\dfrac{\sqrt[q]{2}\left(\frac{\Lambda^{-1}(\wp_{\beta_2},F_2)}{\Theta}\right)^{w_2}}{\sqrt[q]{\left(2-\left(\frac{\Lambda^{-1}(\wp_{\beta_2},F_2)}{\Theta}\right)^q\right)^{w_2}+\left(\left(\frac{\Lambda^{-1}(\wp_{\beta_2},F_2)}{\Theta}\right)^q\right)^{w_2}}}\Bigg)\Bigg),\\ \Lambda\Bigg(\Theta\Bigg(\dfrac{\sqrt[q]{2}\left(\frac{\Lambda^{-1}(\wp_{\gamma_2},G_2)}{\Theta}\right)^{w_2}}{\sqrt[q]{\left(2-\left(\frac{\Lambda^{-1}(\wp_{\gamma_2},G_2)}{\Theta}\right)^q\right)^{w_2}+\left(\left(\frac{\Lambda^{-1}(\wp_{\gamma_2},G_2)}{\Theta}\right)^q\right)^{w_2}}}\Bigg)\Bigg)\end{array}\right).$$

$$=\left(\begin{array}{l}\Lambda\Bigg(\Theta\Bigg(\sqrt[q]{\dfrac{\prod_{k=1}^{2}\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{w_k}-\prod_{k=1}^{2}\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{w_k}}{\prod_{k=1}^{2}\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{w_k}+\prod_{k=1}^{2}\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{w_k}}}\Bigg)\Bigg),\\ \Lambda\Bigg(\Theta\Bigg(\dfrac{\sqrt[q]{2}\prod_{k=1}^{2}\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^{w_k}}{\sqrt[q]{\prod_{k=1}^{2}\left(2-\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^q\right)^{w_k}+\prod_{k=1}^{2}\left(\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^q\right)^{w_k}}}\Bigg)\Bigg),\\ \Lambda\Bigg(\Theta\Bigg(\dfrac{\sqrt[q]{2}\prod_{k=1}^{2}\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^{w_k}}{\sqrt[q]{\prod_{k=1}^{2}\left(2-\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^q\right)^{w_k}+\prod_{k=1}^{2}\left(\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^q\right)^{w_k}}}\Bigg)\Bigg)\end{array}\right).$$

Equation 3.2 holds for $n = 2$. Assume that Eq (3.2) holds for $n = m$.

$$2TLq-RPFEWA(M_1,M_2,\ldots,M_m)=$$

$$\left(\begin{array}{c}\Lambda\left(\Theta\left(\sqrt[q]{\frac{\prod_{k=1}^{m}\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}-\prod_{k=1}^{m}\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}}{\prod_{k=1}^{m}\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}+\prod_{k=1}^{m}\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}}}\right)\right),\\ \Lambda\left(\Theta\left(\frac{\sqrt[q]{2}\prod_{k=1}^{m}\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^{\mathcal{W}_k}}{\sqrt[q]{\prod_{k=1}^{m}\left(2-\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^q\right)^{\mathcal{W}_k}+\prod_{k=1}^{m}\left(\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^q\right)^{\mathcal{W}_k}}}\right)\right),\\ \Lambda\left(\Theta\left(\frac{\sqrt[q]{2}\prod_{k=1}^{m}\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^{\mathcal{W}_k}}{\sqrt[q]{\prod_{k=1}^{m}\left(2-\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^q\right)^{\mathcal{W}_k}+\prod_{k=1}^{m}\left(\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^q\right)^{\mathcal{W}_k}}}\right)\right)\end{array}\right).$$

For $n = m + 1$ by the induction hypothesis, we have

$$2TLq-RPFEWA(M_1,M_2,\ldots,M_m,M_{m+1}) = 2TLq-RPFEWA(M_1,M_2,\ldots,M_m)\bigoplus\mathcal{W}_{m+1}M_{m+1}$$

$$=\left(\begin{array}{c}\Lambda\left(\Theta\left(\sqrt[q]{\frac{\prod_{k=1}^{m}\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}-\prod_{k=1}^{m}\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}}{\prod_{k=1}^{m}\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}+\prod_{k=1}^{m}\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}}}\right)\right),\\ \Lambda\left(\Theta\left(\frac{\sqrt[q]{2}\prod_{k=1}^{m}\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^{\mathcal{W}_k}}{\sqrt[q]{\prod_{k=1}^{m}\left(2-\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^q\right)^{\mathcal{W}_k}+\prod_{k=1}^{m}\left(\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^q\right)^{\mathcal{W}_k}}}\right)\right),\\ \Lambda\left(\Theta\left(\frac{\sqrt[q]{2}\prod_{k=1}^{m}\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^{\mathcal{W}_k}}{\sqrt[q]{\prod_{k=1}^{m}\left(2-\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^q\right)^{\mathcal{W}_k}+\prod_{k=1}^{m}\left(\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^q\right)^{\mathcal{W}_k}}}\right)\right)\end{array}\right)\bigoplus$$

$$\left(\begin{array}{c}\Lambda\Bigg(\Theta\Bigg(\sqrt[q]{\frac{\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_{m+1}},E_{m+1})}{\Theta}\right)^{\mathcal{W}_{m+1}}-\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_{m+1}},E_{m+1})}{\Theta}\right)^{\mathcal{W}_{m+1}}}{\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_{m+1}},E_{m+1})}{\Theta}\right)^{\mathcal{W}_{m+1}}+\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_{m+1}},E_{m+1})}{\Theta}\right)^{\mathcal{W}_{m+1}}}}\Bigg)\Bigg),\\ \Lambda\Bigg(\Theta\Bigg(\frac{\sqrt[q]{2}\left(\frac{\Lambda^{-1}(\wp_{\beta_{m+1}},F_{m+1})}{\Theta}\right)^{\mathcal{W}_{m+1}}}{\sqrt[q]{\left(2-\left(\frac{\Lambda^{-1}(\wp_{\beta_{m+1}},F_{m+1})}{\Theta}\right)^{q}\right)^{\mathcal{W}_{m+1}}+\left(\left(\frac{\Lambda^{-1}(\wp_{\beta_{m+1}},F_{m+1})}{\Theta}\right)^{q}\right)^{\mathcal{W}_{m+1}}}}\Bigg)\Bigg),\\ \Lambda\Bigg(\Theta\Bigg(\frac{\sqrt[q]{2}\left(\frac{\Lambda^{-1}(\wp_{\gamma_{m+1}},G_{m+1})}{\Theta}\right)^{\mathcal{W}_{m+1}}}{\sqrt[q]{\left(2-\left(\frac{\Lambda^{-1}(\wp_{\gamma_{m+1}},G_{m+1})}{\Theta}\right)^{q}\right)^{\mathcal{W}_{m+1}}+\left(\left(\frac{\Lambda^{-1}(\wp_{\gamma_{m+1}},G_{m+1})}{\Theta}\right)^{q}\right)^{\mathcal{W}_{m+1}}}}\Bigg)\Bigg)\end{array}\right).$$

$$=\left(\begin{array}{c}\Lambda\Bigg(\Theta\Bigg(\sqrt[q]{\frac{\prod_{k=1}^{m+1}\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}-\prod_{k=1}^{m+1}\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}}{\prod_{k=1}^{m+1}\left(1+\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}+\prod_{k=1}^{m+1}\left(1-\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\right)^{\mathcal{W}_k}}}\Bigg)\Bigg),\\ \Lambda\Bigg(\Theta\Bigg(\frac{\sqrt[q]{2}\prod_{k=1}^{m+1}\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^{\mathcal{W}_k}}{\sqrt[q]{\prod_{k=1}^{m+1}\left(2-\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^{q}\right)^{\mathcal{W}_k}+\prod_{k=1}^{m+1}\left(\left(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\right)^{q}\right)^{\mathcal{W}_k}}}\Bigg)\Bigg),\\ \Lambda\Bigg(\Theta\Bigg(\frac{\sqrt[q]{2}\prod_{k=1}^{m+1}\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^{\mathcal{W}_k}}{\sqrt[q]{\prod_{k=1}^{m+1}\left(2-\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^{q}\right)^{\mathcal{W}_k}+\prod_{k=1}^{m+1}\left(\left(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\right)^{q}\right)^{\mathcal{W}_k}}}\Bigg)\Bigg)\end{array}\right).$$

Hence, Eq (3.2) holds for all positive integers $n \geq 1$. □

Proposition 3.1. *Consider two collections of 2TLq-RPFNs* $M_{a_k} = ((\wp_{\alpha_{a_k}}, E_{a_k}), (\wp_{\beta_{a_k}}, F_{a_k}), (\wp_{\gamma_{a_k}}, G_{a_k}))$ $(k = 1, 2, \ldots, n)$ *and* $M_{b_k} = ((\wp_{\alpha_{b_k}}, E_{b_k}), (\wp_{\beta_{b_k}}, F_{b_k}), (\wp_{\gamma_{b_k}}, G_{b_k}))$ $(k = 1, 2, \ldots, n)$. *Then the 2TLq-RPFEWA operator has the following properties:*

1) *(Idempotency) If* $M_k = ((\wp_{\alpha_k}, E_k), (\wp_{\beta_k}, F_k), (\wp_{\gamma_k}, G_k)) = M$ *for all* $(k = 1, 2, \ldots, n)$, *then*

$$2TLq - RPFEWA(M_1, M_2, \ldots, M_n) = M. \tag{3.3}$$

Proof. Suppose $M_k = ((\wp_{\alpha_k}, E_k), (\wp_{\beta_k}, F_k), (\wp_{\gamma_k}, G_k))$ is a collection of 2TLq-RPFNs such that $M_k = M$ for all $(k = 1, 2, \ldots, n)$, $\mathcal{W}_k \in [0, 1]$ and $\sum_{k=1}^{n} \mathcal{W}_k = 1$. From Eq 3.2, we get

$$2TLq - RPFEWA(M_1, M_2, \ldots, M_n) =$$

$$\begin{pmatrix} \Lambda\Bigg(\Theta\Bigg(\sqrt[q]{\frac{\prod_{k=1}^{n}\Big(1+\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\Big)^{w_k}-\prod_{k=1}^{n}\Big(1-\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\Big)^{w_k}}{\prod_{k=1}^{n}\Big(1+\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\Big)^{w_k}+\prod_{k=1}^{n}\Big(1-\frac{\Lambda^{-1}(\wp_{\alpha_k},E_k)}{\Theta}\Big)^{w_k}}}\Bigg)\Bigg), \\ \Lambda\Bigg(\Theta\Bigg(\frac{\sqrt[q]{2}\prod_{k=1}^{n}\Big(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\Big)^{w_k}}{\sqrt[q]{\prod_{k=1}^{n}\Big(2-\Big(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\Big)^{q}\Big)^{w_k}+\prod_{k=1}^{n}\Big(\Big(\frac{\Lambda^{-1}(\wp_{\beta_k},F_k)}{\Theta}\Big)^{q}\Big)^{w_k}}}\Bigg)\Bigg), \\ \Lambda\Bigg(\Theta\Bigg(\frac{\sqrt[q]{2}\prod_{k=1}^{n}\Big(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\Big)^{w_k}}{\sqrt[q]{\prod_{k=1}^{n}\Big(2-\Big(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\Big)^{q}\Big)^{w_k}+\prod_{k=1}^{n}\Big(\Big(\frac{\Lambda^{-1}(\wp_{\gamma_k},G_k)}{\Theta}\Big)^{q}\Big)^{w_k}}}\Bigg)\Bigg) \end{pmatrix}$$

$$= \begin{pmatrix} \Lambda\Bigg(\Theta\Bigg(\sqrt[q]{\frac{\Big(1+\frac{\Lambda^{-1}(\wp_{\alpha},E)}{\Theta}\Big)-\Big(1-\frac{\Lambda^{-1}(\wp_{\alpha},E)}{\Theta}\Big)}{\Big(1+\frac{\Lambda^{-1}(\wp_{\alpha},E)}{\Theta}\Big)+\Big(1-\frac{\Lambda^{-1}(\wp_{\alpha},E)}{\Theta}\Big)}}\Bigg)\Bigg), \\ \Lambda\Bigg(\Theta\Bigg(\frac{\sqrt[q]{2}\Big(\frac{\Lambda^{-1}(\wp_{\beta},F)}{\Theta}\Big)}{\sqrt[q]{\Big(2-\Big(\frac{\Lambda^{-1}(\wp_{\beta},F)}{\Theta}\Big)^{q}\Big)+\Big(\Big(\frac{\Lambda^{-1}(\wp_{\beta},F)}{\Theta}\Big)^{q}\Big)}}\Bigg)\Bigg), \\ \Lambda\Bigg(\Theta\Bigg(\frac{\sqrt[q]{2}\Big(\frac{\Lambda^{-1}(\wp_{\gamma},G)}{\Theta}\Big)}{\sqrt[q]{\Big(2-\Big(\frac{\Lambda^{-1}(\wp_{\gamma},G)}{\Theta}\Big)^{q}\Big)+\Big(\Big(\frac{\Lambda^{-1}(\wp_{\gamma},G_k)}{\Theta}\Big)^{q}\Big)}}\Bigg)\Bigg) \end{pmatrix}$$

$$= \Bigg(\Lambda\Big(\Theta(\frac{\Lambda^{-1}(\wp_{\alpha},E)}{\Theta})\Big), \Lambda\Big(\Theta(\frac{\Lambda^{-1}(\wp_{\beta},F)}{\Theta})\Big), \Lambda\Big(\Theta(\frac{\Lambda^{-1}(\wp_{\gamma},G)}{\Theta})\Big) \Bigg)$$

$$= \Big((\wp_{\alpha},E),(\wp_{\beta},F),(\wp_{\gamma},G) \Big)$$

$$= M.$$

□

2) *(Monotonicity) If* $M_{a_k} \le M_{b_k}$, *for all* $(k = 1, 2, \ldots, n)$, *then*

$$2TLq-RPFEWA(M_{a_1}, M_{a_2}, \ldots, M_{a_n}) \le 2TLq-RPFEWA(M_{b_1}, M_{b_2}, \ldots, M_{b_n}). \tag{3.4}$$

3) *(Boundedness) Consider a collection of 2TLq-RPFNs:*

$$M_{a_k} = ((\wp_{\alpha_{a_k}}, E_{a_k}), (\wp_{\beta_{a_k}}, F_{a_k}), (\wp_{\gamma_{a_k}}, G_{a_k})) \ \ (k = 1, 2, \ldots, n)$$

with

$$M^{+}_{a_k} = (\max_{a_k}(\wp_{\alpha_{a_k}}, E_{a_k}), \min_{a_k}(\wp_{\beta_{a_k}}, F_{a_k}), \min_{a_k}(\wp_{\gamma_{a_k}}, G_{a_k}))$$

and

$$M^{-}_{a_k} = (\min_{a_k}(\wp_{\alpha_{a_k}}, E_{a_k}), \max_{a_k}(\wp_{\beta_{a_k}}, F_{a_k}), \max_{a_k}(\wp_{\gamma_{a_k}}, G_{a_k})),$$

then

$$M^{-} \leq 2TLq - RPFEWA(M_1, M_2, \ldots, M_n) \leq M^{+}. \tag{3.5}$$

Now we extend the Einstein order weighted average operator to 2TLq-RPFEOWA operator. Further, we state the elementary properties of 2TLq-RPFEOWA operator.

Definition 3.3. The 2TLq-RPFEOWA operator is a mapping $H^n \rightarrow H$ such that: for each collection of 2TLq-RPFNs $M_k = ((\wp_{\alpha_k}, E_k), (\wp_{\beta_k}, F_k), (\wp_{\gamma_k}, G_k))$ $(k = 1, 2, \ldots, n)$,

$$2TLq - RPFEOWA(M_1, M_2, \ldots, M_n) = \bigoplus_{k=1}^{n} \mathcal{W}_k M_{\mu(k)}, \tag{3.6}$$

where $\mu(k)$ is such that $M_{\mu(k-1)} \geq M_{\mu(k)}$ for all k, $\mathcal{W} = (\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_n)^T$ is the weight vector of $M_k (k = 1, 2, \ldots, n)$ with $\mathcal{W}_k \in [0, 1]$ and $\sum_{k=1}^{n} \mathcal{W}_k = 1$.

Theorem 3.2. *Consider a collection of 2TLq-RPFNs* $M_k = ((\wp_{\alpha_k}, E_k), (\wp_{\beta_k}, F_k), (\wp_{\gamma_k}, G_k))$ $(k = 1, 2, \ldots, n)$ *having weight vector* $\mathcal{W} = (\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_n)^T$ *with* $\mathcal{W}_k \in [0, 1]$ *and* $\sum_{k=1}^{n} \mathcal{W}_k = 1$. *Then*

$$2TLq - RPFEOWA(M_1, M_2, \ldots, M_n) =$$

$$\left(\begin{array}{l} \Lambda\Bigg(\Theta\Bigg(\sqrt[q]{\dfrac{\prod_{k=1}^{n}\Big(1 + \dfrac{\Lambda^{-1}(\wp_{\alpha_{\mu(k)}}, E_{\mu(k)})}{\Theta}\Big)^{\mathcal{W}_k} - \prod_{k=1}^{n}\Big(1 - \dfrac{\Lambda^{-1}(\wp_{\alpha_{\mu(k)}}, E_{\mu(k)})}{\Theta}\Big)^{\mathcal{W}_k}}{\prod_{k=1}^{n}\Big(1 + \dfrac{\Lambda^{-1}(\wp_{\alpha_{\mu(k)}}, E_{\mu(k)})}{\Theta}\Big)^{\mathcal{W}_k} + \prod_{k=1}^{n}\Big(1 - \dfrac{\Lambda^{-1}(\wp_{\alpha_{\mu(k)}}, E_{\mu(k)})}{\Theta}\Big)^{\mathcal{W}_k}}}\Bigg)\Bigg), \\ \Lambda\Bigg(\Theta\Bigg(\dfrac{\sqrt[q]{2}\prod_{k=1}^{n}\Big(\dfrac{\Lambda^{-1}(\wp_{\beta_{\mu(k)}}, F_{\mu(k)})}{\Theta}\Big)^{\mathcal{W}_k}}{\sqrt[q]{\prod_{k=1}^{n}\Big(2 - \Big(\dfrac{\Lambda^{-1}(\wp_{\beta_{\mu(k)}}, F_{\mu(k)})}{\Theta}\Big)^{q}\Big)^{\mathcal{W}_k} + \prod_{k=1}^{n}\Big(\Big(\dfrac{\Lambda^{-1}(\wp_{\beta_{\mu(k)}}, F_{\mu(k)})}{\Theta}\Big)^{q}\Big)^{\mathcal{W}_k}}}\Bigg)\Bigg), \\ \Lambda\Bigg(\Theta\Bigg(\dfrac{\sqrt[q]{2}\prod_{k=1}^{n}\Big(\dfrac{\Lambda^{-1}(\wp_{\gamma_{\mu(k)}}, G_{\mu(k)})}{\Theta}\Big)^{\mathcal{W}_k}}{\sqrt[q]{\prod_{k=1}^{n}\Big(2 - \Big(\dfrac{\Lambda^{-1}(\wp_{\gamma_{\mu(k)}}, G_{\mu(k)})}{\Theta}\Big)^{q}\Big)^{\mathcal{W}_k} + \prod_{k=1}^{n}\Big(\Big(\dfrac{\Lambda^{-1}(\wp_{\gamma_{\mu(k)}}, G_{\mu(k)})}{\Theta}\Big)^{q}\Big)^{\mathcal{W}_k}}}\Bigg)\Bigg) \end{array} \right). \tag{3.7}$$

Proof. This proof is similar to the proof of Theorem 3.1. □

Proposition 3.2. *Consider two collections of* $2TLq$*-RPFNs* $M_{a_k} = ((\wp_{\alpha_{a_k}}, E_{a_k}), (\wp_{\beta_{a_k}}, F_{a_k}), (\wp_{\gamma_{a_k}}, G_{a_k}))$ $(k = 1, 2, \ldots, n)$ *and* $M_{b_k} = ((\wp_{\alpha_{b_k}}, E_{b_k}), (\wp_{\beta_{b_k}}, F_{b_k}), (\wp_{\gamma_{b_k}}, G_{b_k}))$ $(k = 1, 2, \ldots, n)$*. Then the* $2TLq$*-RPFEOWA operator has the following properties:*

1) *(Idempotency) If* $M_k = ((\wp_{\alpha_k}, E_k), (\wp_{\beta_k}, F_k), (\wp_{\gamma_k}, G_k)) = M$ *for all* $(k = 1, 2, \ldots, n)$*, then*

$$2TLq - RPFEOWA(M_1, M_2, \ldots, M_n) = M. \tag{3.8}$$

2) *(Monotonicity) If* $M_{a_k} \leq M_{b_k}$*, for all* $(k = 1, 2, \ldots, n)$*, then*

$$2TLq - RPFEOWA(M_{a_1}, M_{a_2}, \ldots, M_{a_n}) \leq 2TLq - RPFEOWA(M_{b_1}, M_{b_2}, \ldots, M_{b_n}). \tag{3.9}$$

3) *(Boundedness) Consider a collection of* $2TLq$*-RPFNs:*

$$M_{a_k} = ((\wp_{\alpha_{a_k}}, E_{a_k}), (\wp_{\beta_{a_k}}, F_{a_k}), (\wp_{\gamma_{a_k}}, G_{a_k})) \ \ (k = 1, 2, \ldots, n)$$

with

$$M^+_{a_k} = (\max_{a_k}(\wp_{\alpha_{a_k}}, E_{a_k}), \min_{a_k}(\wp_{\beta_{a_k}}, F_{a_k}), \min_{a_k}(\wp_{\gamma_{a_k}}, G_{a_k}))$$

and

$$M^-_{a_k} = (\min_{a_k}(\wp_{\alpha_{a_k}}, E_{a_k}), \max_{a_k}(\wp_{\beta_{a_k}}, F_{a_k}), \max_{a_k}(\wp_{\gamma_{a_k}}, G_{a_k})),$$

then

$$M^- \leq 2TLq - RPFEOWA(M_1, M_2, \ldots, M_n) \leq M^+. \tag{3.10}$$

4. 2TLq-RPF-CODAS method

This section will develop the CODAS approach for the 2TLq-RPF environment in order to address MCGDM challenges. Our objective is to find the best alternative by employing the terminology of CODAS method in a 2TLq-RPF background. The extended Einstein aggregation operators are utilized to fuse the judgements. Furthermore, the entropy measure is used to generate the weights for the criterion. The 2TLq-RPF-CODAS technique is capable of addressing the challenging MCGDM problems.

The selection of decision data, choice of practicable actions, performance assessments, and environment are the most frequent deciding elements. Each MCGDM challenge includes a panel of experts, a collection of realistic options, and a set of criteria. To choose the best option from a list of m possible options or alternatives $\mathbb{A} = \{\mathbb{A}_1, \mathbb{A}_2, \mathbb{A}_3, \ldots, \mathbb{A}_m\}$ that are to be reviewed based on n criteria $\mathbb{C} = \{\mathbb{C}_1, \mathbb{C}_2, \mathbb{C}_3, \ldots, \mathbb{C}_n\}$, a group of l decision-making professionals $\mathbf{E} = \{\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3, \ldots, \mathbf{E}_l\}$, for the purpose of judging the various options are invited. The steps of the proposed method are also shown in Figure 1. The recommended strategy's steps fall under the following steps:

Step 1: *Formation of linguistic terms.* The right linguistic expressions must be used to explain how alternate possibilities behave during the DM process. Decision experts frequently employ linguistic terms to evaluate the options or alternatives presented in MCGDM issues. The experts in this methodology first identify the linguistic expressions and their associated 2TLq-RPFNs for the

MCGDM procedure. The seven point linguistic scale used in this study is presented in Table 2.

Step 2: *Construction of individual matrices by the experts.* During this stage, each expert creates an assessment matrix and delivers evaluation data in the form of 2TLq-RPFNs. Let $\mathbb{J} = (\mathbb{A}^k_{ij})_{m\times n}$ be the evaluation matrices presented by the experts'. The judgement matrix of the kth expert can be represented as follows:

$$\mathbb{J} = (\mathbb{A}^k_{ij})_{m\times n}\begin{bmatrix} a^k_{11} & a^k_{12} & \dots & a^k_{1n} \\ a^k_{21} & a^k_{22} & \dots & a^k_{2n} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ a^k_{m1} & a^k_{m2} & \dots & a^k_{mn} \end{bmatrix}.$$

Each term of the evaluation matrix $\mathbb{J} = (\mathbb{A}^k_{ij})_{m\times n}$ $(k = 1, 2, \dots, l)$ represents a 2TLq-RPFN as $a^k_{ij} = ((\wp^k_{\alpha_{ij}}, E^k_{ij}), (\wp^k_{\beta_{ij}}, F^k_{ij}), (\wp^k_{\gamma_{ij}}, G^k_{ij}))$, where $(\wp^k_{\alpha_{ij}}, E^k_{ij})$, $(\wp^k_{\beta_{ij}}, F^k_{ij})$ and $(\wp^k_{\gamma_{ij}}, G^k_{ij})$ are the degrees of membership, abstinence and non-membership, respectively.

Step 3: *Normalization of individual matrices.* Each individual matrix is normalized on the basis of following rule:

$$a_{ij} = \begin{cases} ((\wp_{\alpha_{ij}}, E_{ij}), (\wp_{\beta_{ij}}, F_{ij}), (\wp_{\gamma_{ij}}, G_{ij})), & \text{for benefit type criteria;} \\ ((\wp_{\gamma_{ij}}, G_{ij}), (\wp_{\beta_{ij}}, F_{ij}), (\wp_{\alpha_{ij}}, E_{ij})), & \text{for cost type criteria.} \end{cases} \tag{4.1}$$

Step 4: *Computation of combined aggregated matrix.* To create an aggregated evaluation matrix $\mathbb{G} = (\mathbb{A}_{ij})_{m\times n}$, the individual evaluation matrices of the experts must be added together. The 2TLq-RPFEWA operator, which is provided in (4.2), is utilized for this purpose.

$$c_{ij} = 2TLq - RPFEWA(a^1_{ij}, a^2_{ij}, \dots, a^l_{ij}) =$$

$$\left(\begin{array}{c}\Lambda\Bigg(\Theta\Bigg(\sqrt[q]{\dfrac{\prod_{k=1}^{l}\Big(1+\dfrac{\Lambda^{-1}(\wp_{\alpha_{ij}^{k}},E_{ij}^{k})}{\Theta}\Big)-\prod_{k=1}^{l}\Big(1-\dfrac{\Lambda^{-1}(\wp_{\alpha_{ij}^{k}},E_{ij}^{k})}{\Theta}\Big)}{\prod_{k=1}^{l}\Big(1+\dfrac{\Lambda^{-1}(\wp_{\alpha_{ij}^{k}},E_{ij}^{k})}{\Theta}\Big)+\prod_{k=1}^{l}\Big(1-\dfrac{\Lambda^{-1}(\wp_{\alpha_{ij}^{k}},E_{ij}^{k})}{\Theta}\Big)}}\Bigg)\Bigg),\\ \Lambda\Bigg(\Theta\Bigg(\dfrac{\sqrt[q]{2}\prod_{k=1}^{l}\Big(\dfrac{\Lambda^{-1}(\wp_{\beta_{ij}^{k}},F_{ij}^{k})}{\Theta}\Big)}{\sqrt[q]{\prod_{k=1}^{l}\Big(2-\big(\dfrac{\Lambda^{-1}(\wp_{\beta_{ij}^{k}},F_{ij}^{k})}{\Theta}\big)^{q}\Big)+\prod_{k=1}^{l}\Big(\big(\dfrac{\Lambda^{-1}(\wp_{\beta_{ij}^{k}},F_{ij}^{k})}{\Theta}\big)^{q}\Big)}}\Bigg)\Bigg),\\ \Lambda\Bigg(\Theta\Bigg(\dfrac{\sqrt[q]{2}\prod_{k=1}^{l}\Big(\dfrac{\Lambda^{-1}(\wp_{\gamma_{ij}^{k}},G_{ij}^{k})}{\Theta}\Big)}{\sqrt[q]{\prod_{k=1}^{l}\Big(2-\big(\dfrac{\Lambda^{-1}(\wp_{\gamma_{ij}^{k}},G_{ij}^{k})}{\Theta}\big)^{q}\Big)+\prod_{k=1}^{l}\Big(\big(\dfrac{\Lambda^{-1}(\wp_{\gamma_{ij}^{k}},G_{ij}^{k})}{\Theta}\big)^{q}\Big)}}\Bigg)\Bigg)\end{array}\right). \quad (4.2)$$

Step 5: *Calculation of criteria weights.* By using the entropy measure, the criterion weights are determined. The following are the key steps:

Step 1∗*: Find scores of aggregated matrix.* The combined aggregated matrix $\mathbb{G}$ scores that were obtained in **Step 4** should be calculated. Equation 4.3 provides the scoring formula.

$$\Im(a_{ij}) = \Lambda\Bigg(\Bigg(\Theta + \Big(\frac{\Lambda^{-1}(\wp_{\alpha}, E)}{\Theta}\Big)^{q} - \Big(\frac{\Lambda^{-1}(\wp_{\beta}, F)}{\Theta}\Big)^{q} - \Big(\frac{\Lambda^{-1}(\wp_{\gamma}, G)}{\Theta}\Big)^{q}\Bigg)\Bigg). \quad (4.3)$$

Step 2∗*: Calculate projection values.* In this step, the projection values $\mathcal{P}_{ij}$ of each criteria are computed based on Eq 4.4:

$$\mathcal{P}_{ij} = \frac{\Im(a_{ij})}{\sum_{i=1}^{m}\Im(a_{ij})}. \quad (4.4)$$

Step 3∗*: Compute entropy values.* The projection values are used in the following way to calculate the entropy value $\mathcal{E}_j$ for each attribute:

$$\mathcal{E}_j = \frac{-1}{\log(m)}\sum_{i=1}^{m}\mathcal{P}_{ij}\log(\mathcal{P}_{ij}). \quad (4.5)$$

Step 4∗*: Find divergence values.* Now, using the entropy values, the following formula is used to get the degree of divergence $\daleth_j$ for each attribute as the inherent contrast intensity of the attribute:

$$\daleth_j = 1 - \mathcal{E}_j. \quad (4.6)$$

Step 5∗*: Determine weights.* The criteria weights are computed based on Eq 4.7:

$$\mathcal{W}_j = \frac{\daleth_j}{\sum_{j=1}^{n}\daleth_j}, \quad (4.7)$$

where $\sum_{j=1}^{n}\mathcal{W}_j = 1$.

Step 6: *Computation of weighted aggregated matrix.* Weighted aggregated matrix is established by using the criteria weights $\mathcal{W}_j$ and the matrix $\mathbb{G}$ by applying Eq 4.8.

$$b_{ij} = \mathcal{W}_j\mathbb{G} = \left(\Lambda\Big(\Theta\big(1-\big(1-\big(\frac{\Lambda^{-1}(\wp_\alpha, E)}{\Theta}\big)^q\big)^{\mathcal{W}_j}\big)^{\frac{1}{q}}\Big), \Lambda\Big(\Theta\big(\frac{\Lambda^{-1}(\wp_\beta, F)}{\Theta}\big)^{\mathcal{W}_j}\Big), \Lambda\Big(\Theta\big(\frac{\Lambda^{-1}(\wp_\gamma, G)}{\Theta}\big)^{\mathcal{W}_j}\Big)\right). \tag{4.8}$$

Step 7: *Calculation of scores and identification of NIS.* The scores of weighted aggregated matrix are calculated and the values of negative ideal solution (NIS) corresponding to each criteria are determined based on score values. The NIS is identified on the following fact:

$$NIS = [NIS_j]_{1\times n}; \tag{4.9}$$

$$NIS_j = \min_i \mathfrak{S}(b_{ij}). \tag{4.10}$$

Step 8: *Determination of Euclidean distance and Hamming distance.* In this step, the Hamming distances $\mathcal{HD}_i$ and Euclidean distances $\mathcal{ED}_i$ between the values of the weighted aggregated matrix and the NIS are computed based on the following Equations:

$$\mathcal{HD}_i = \sum_{j=1}^{n}\mathcal{HD}(b_{ij}, NIS_j), \tag{4.11}$$

$$\mathcal{ED}_i = \sum_{j=1}^{n}\mathcal{ED}(b_{ij}, NIS_j). \tag{4.12}$$

Let $A_1 = ((\wp_{\alpha_1}, E_1), (\wp_{\beta_1}, F_1), (\wp_{\gamma_1}, G_1))$ and $A_2 = ((\wp_{\alpha_2}, E_2), (\wp_{\beta_2}, F_2), (\wp_{\gamma_2}, G_2))$ be two 2TLq-RPFNs. Then the values of $\mathcal{HD}$ and $\mathcal{ED}$ between two 2TLq-RPFNs A_1 and A_2 can be computed as:

$$\mathcal{HD}(A_1, A_2) = \Lambda\Big(\frac{1}{2\Theta}\big(\big|\Lambda^{-1}(\wp_{\alpha_1}, E_1) - \Lambda^{-1}(\wp_{\alpha_2}, E_2)\big| + \big|\Lambda^{-1}(\wp_{\beta_1}, F_1) - \Lambda^{-1}(\wp_{\beta_2}, F_2)\big| + \big|\Lambda^{-1}(s_{\gamma_1}, G_1) - \Lambda^{-1}(\wp_{\gamma_2}, G_2)\big|\big)\Big), \tag{4.13}$$

$$\mathcal{ED}(A_1, A_2) = \Lambda\Big(\frac{\Theta}{2}\big(\big|\big(\frac{\Lambda^{-1}(\wp_{\alpha_1}, E_1) - \Lambda^{-1}(\wp_{\alpha_2}, E_2)}{\Theta}\big)^q\big| + \big|\big(\frac{\Lambda^{-1}(\wp_{\beta_1}, F_1) - \Lambda^{-1}(\wp_{\beta_2}, F_2)}{\Theta}\big)^q\big| + \big|\big(\frac{\Lambda^{-1}(\wp_{\gamma_1}, G_1) - \Lambda^{-1}(\wp_{\gamma_2}, G_2)}{\Theta}\big)^q\big|\big)\Big). \tag{4.14}$$

Step 9: *Finding of $\mathcal{RA}$ matrix.* In this step, the relative assessment $\mathcal{RA}$ matrix is constructed:

$$\mathcal{RA} = [c_{ik}]_{m\times n}; \tag{4.15}$$

$$c_{ik} = (\mathcal{ED}_i - \mathcal{ED}_k) + (g(\mathcal{ED}_i - \mathcal{ED}_k) \times (\mathcal{HD}_i - \mathcal{HD}_k)); \quad k = 1, 2, 3, \ldots, m. \tag{4.16}$$

Here g represents a function which could be defined as:

$$g(\xi) = \begin{cases} 1, & |\xi| \geq \tau; \\ 0, & |\xi| < \tau, \end{cases} \tag{4.17}$$

where $\tau \in [0.01, 0.05]$ which is given by the experts. In current study, $\tau = 0.02$.
Step 10: *Final rank of alternatives.* The average solution is computed based on Eq 4.18:

$$AS_i = \sum_{k=1}^{m} c_{ik}. \tag{4.18}$$

Finally, the alternatives are arranged based on the AS_i values. The alternative having highest AS_i value will be regarded the best one.

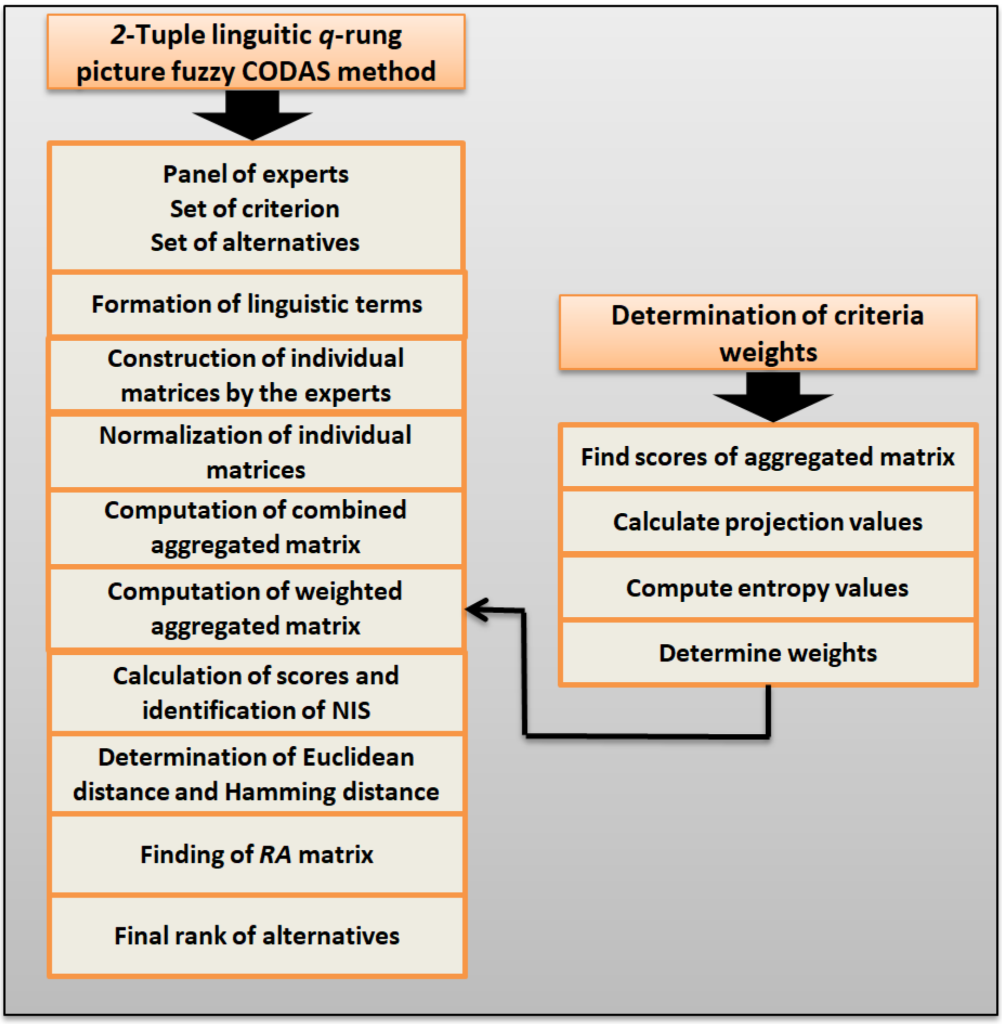

Figure 1. Steps of the proposed methodology.

5. Application

This section presents a numerical example that is related to the prioritization of autonomous vehicles in RTTM. In the first subsection, we provided the problem description along with the definitions of alternatives and criteria. In the second subsection, we presented the necessary numerical steps along with the calculations. At the end, we discussed the impact of parameter q on ranking results.

5.1. Problem description

Although improving traffic conditions and reducing congestion through RTTM is a very effective method, it has several drawbacks, such as the inability to place sensors at all strategically important locations for data collection. The tendency is to look for methods to either employ CAVs to control traffic in real-time or combine them with RTTM systems as connectivity technologies and CAVs advance. For real-time management systems, this study defines three basic options. The experts use the CODAS approach to prioritization of the system's economics, public support, energy consumption, traffic safety and order, and traffic management elements. RTTM techniques can reduce congestion, but they can also lengthen travel times and cause delays in traffic. The use of traffic management systems is crucial for ensuring safer traffic flow, in addition to improving traffic efficiency and drivers' quality of life. Numerous real-time traffic control techniques have been used for years with great success in the real world. However, because studies pertaining to this integration reveal incredibly positive and effective outcomes, the usage of CAVs and/or AVs in RTTM approaches has grown to be an attractive issue for researchers. The option of deploying CAVs and/or AVs just for in-the-moment traffic management exists as well. For the prioritizing of CAVs in a RTTM system, we took into consideration an issue that has been adopted from [30]. The ambiguous information is handled by the 2TLq-RPFNs. A panel of four experts $\{E_1, E_2, E_3, E_4\}$ evaluates the three alternatives $\{\mathbb{A}_1, \mathbb{A}_2, \mathbb{A}_3\}$ on the basis of five major criteria $\{EA, PF, EC, TS, TM\}$ each having different sub criteria. Suppose, $S = \{\wp_0, \wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_8\}$ be the set of nine linguistic terms. At first, the experts' panel form the linguistic terms (see Table 2) to judge the alternatives in the form of 2TLq-RPFNs. After that, the experts' are requested to make evaluation to judge the most optimal alternative in the form of 2TLq-RPFNs. In this paper, the initial data presented in Table 2 and Table 3 has been taken from the paper [30], i.e, we have considered the linguistic terms and assigned them their corresponding 2TLq-RPFNs. The considered alternatives are briefly described as follows:

- $\mathbb{A}_1$: **RTTM.** Real-time traffic management's main objective is to enhance traffic flow while reducing congestion and increasing safety. However, there are restrictions, such as driver compliance rates with the system's regulations and the inability to collect data for effective management from crucial areas. But with connecting technologies, this procedure may be carried out more precisely and successfully. A couple of the often employed traffic control strategies include ramp metering and variable speed limits (VSL).
- $\mathbb{A}_2$: **RTTM integrated with connected autonomous vehicles.** The performance of real-time traffic control systems can be considerably enhanced by the addition of networked autonomous vehicles. Findings show that connection and collaboration will be essential elements of future transportation systems, in addition to automation, which can remove driver mistakes. The benefits of RTTM systems might greatly increase, leading to significantly improved traffic conditions if linked autonomous cars outperform human-driven vehicles in terms of performance and adherence to the laws of the road. Examples of conventional RTTM techniques include variable speed limits (VSL) and lane control signals (LCS). These techniques have been shown to work; however, adding CAVs to existing traffic management techniques or developing a brand-new technique that just employs CAVs may be more efficient.
- $\mathbb{A}_3$: **RTTM using connected autonomous vehicles.** According to research published in the literature, linked autonomous vehicles can act as traffic enforcers by using V2I connections to

connect to traffic control centers and receive instructions and information ([29]). RTTM restrictions, such as not being able to deploy sensors everywhere, can also be removed by using linked autonomous cars.

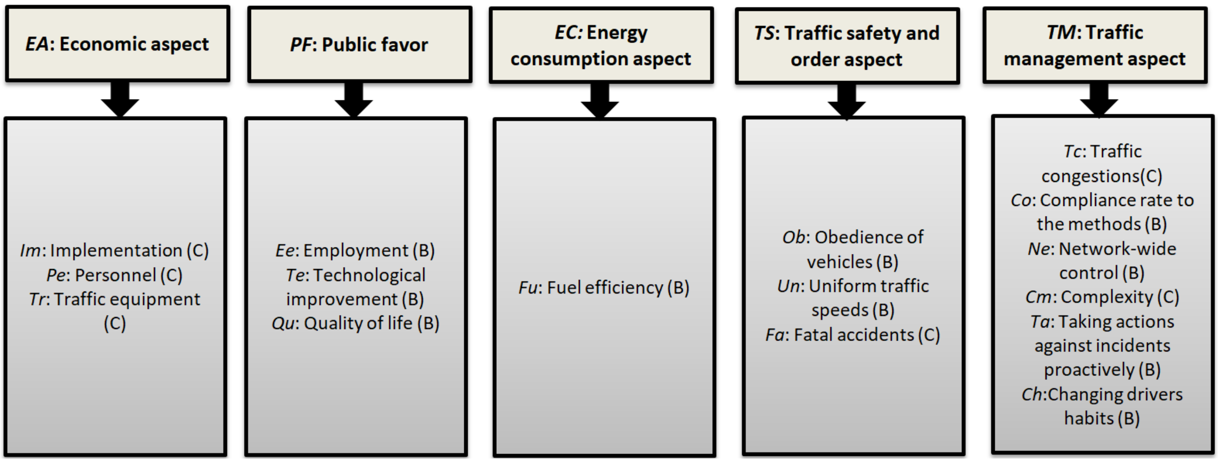

Figure 2. Criteria and their corresponding sub criteria.

A model is constructed by asking experts questions and utilizing their answers as inputs to determine the importance of various factors such as the economy, public opinion, energy consumption, traffic safety, order, and traffic management. Each criteria, their corresponding sub-criterion along with their types are shown in Figure 2. In Figure 2, C stands for cost-type criteria, and B stands for benefit-type criteria. The definition of each criteria is given as follows:

1) EA : **Economic aspect.** This component primarily addresses the RTTM alternative's financial characteristics.

- Im : *Implementation.* A specific budget must be set out for the implementation of RTTM. As a result, the method becomes more appealing by lowering the necessary budget amount.
- Pe : *Personnel.* The operation and maintenance of a few RTTM techniques need the employment of human resources. Personnel must assess the traffic situation in order to show the relevant signs on electronic traffic boards when using RTTM techniques, such as Lane Control Signals (LCS). This approach is more appealing due to the cost reduction.
- Tr : *Traffic equipment.* This feature shows the overall cost of the machinery needed to put the management system in place, such as the sensors and traffic lights. That method will become more appealing due to lower equipment prices.

2) PF : **Public favor.** It focuses on how the RTTM option is perceived and supported by the general population.

- Em : *Employment.* Qualified professionals and human resources are needed for creating, running, and maintaining RTTM systems. That system gains allure as more work possibilities are produced by it.
- Te : *Technological improvement.* Technology advancements can lead to a better optimal system with fewer operating and maintenance costs and more advantages. Therefore, using

technology-based approaches for traffic management will make that process more appealing than constructing new facilities or acquiring extra equipment.

- Qu : *Quality of life.* The RTTM system will help the city's residents enjoy a better quality of life as traffic conditions improve.

3) EC : **Energy consumption aspect.** This component focuses on the energy savings that are offered for the RTTM solution under consideration.

- Fu : *Fuel efficiency.* Road traffic may be made more uniform with RTTM, which enables cars to travel at steady speeds and with high levels of fuel economy. When real-time management offers greater levels of fuel economy, it becomes more appealing.

TS : **Traffic safety and order aspect.** This element primarily addresses the advantages of RTTM and the significance of abiding by the system's regulations in terms of traffic safety.

- Ob : *Obedience of the vehicles.* Traffic conditions and safety get better as a result of drivers following the management's guidelines more frequently. The system's regulations are perfectly followed by autonomous cars. This has a significant benefit for traffic safety since it prevents drivers from making the mistake of breaking the regulations.
- Un : *Uniform traffic speeds.* Significant speed variations lead to unstable traffic situations and more overtaking movements. Accident risk declines when traffic speeds are more consistently maintained. Autonomous cars are anticipated to have significantly superior driving skills than human drivers and to adhere to prescribed speed restrictions much more successfully.
- Fa : *Fatal accidents.* In the United States (US), one of the leading causes of mortality is traffic accidents. RTTM technologies can improve traffic safety while providing a major advantage in areas like health.

4) TM : **Traffic management aspect.** The advantages of traffic management, such as decreased congestion and enhanced traffic stability and system features, are the main focus of this component.

- Tc : *Traffic congestion.* All traffic management strategies aim to increase traffic while preventing the development of additional road infrastructure. The public will profit from the implementation of these systems.
- Co : *Compliance rate to the methods.* The potential benefits of traffic management are anticipated to rise as more drivers adhere to the regulations set out by the traffic management systems.
- Ne : *Network-wide control.* The efficacy of the traffic management system is anticipated to improve if additional data from the whole road network can be collected.
- Cm : *Complexity.* Effective calibration is necessary for some traffic control systems, such as ramp metering, to work satisfactorily. The level of motorist adherence is a key factor in how well other traffic management methods operate. The complexity of the traffic management systems is increased by these elements, and as the complexity levels rise, the system loses appeal. The strict adherence to the law that autonomous cars are projected to exhibit will also reduce the complexity of traffic control systems.

- Ta : *Taking actions against incidents pro actively.* By establishing traffic management systems, it is crucial to lower the chance of an incident. Proactive systems allow for the prediction of hazardous traffic flow conditions and the implementation of safety measures. CAVs may be employed as dynamic sensors in the future. This constant data flow from the deployment of these cars enables event detection algorithms to anticipate traffic accidents and congestion before they happen.
- Ch : *Changing drivers habits.* The elimination of driver mistakes via strict adherence to traffic laws and the decrease in traffic accidents brought on by these errors are two of the anticipated advantages of autonomous cars. Drivers are also obliged to adhere to the laws in order to adapt the changing traffic circumstances brought about by the increased market penetration of autonomous cars in traffic.

5.2. Procedural steps

Assume that $\Theta = 8$ and $q = 2$. The stages involved in carrying out the suggested approach are listed below:

Step 1. Table 2 lists the seven-point linguistic term scale together with the 2TLq-RPFNs that the experts assigned to each term.
Step 2. The four experts' evaluations of the alternatives corresponding to criteria they supplied in the form of 2TLq-RPFNs are collected in Table 3.
Step 3. Table 4 contains the alternatives' normalized assessment values based on the 4.1.
Step 4. To create an aggregated matrix, the individual ratings need to be integrated. Equation 4.2 is used to do this. The values of the options that correspond to each criterion are shown in Table 5 as an aggregate.
Step 5. At this stage, the entropy measure is used to determine the criterion weights.
Step 1.* Using Eq 4.3 and the results shown in Table 6, the scores of the combined aggregated matrix are first calculated.
Step 2.* Now, using Eq 4.4 and the values listed in Table 7, the projection values are determined.
Step 3.* The entropy values by employing Eq 4.5 are summarized in Table 7.
Step 4.* In Table 7, the divergence values of the criteria based on Eq 4.6 are also shown.
Step 5.* The criteria weights are also determined in Table 7 using Eq 4.7.
Step 6. Table 8 contains the terms of the weighted aggregated matrix based on Eq 4.8.
Step 7. To identify the NIS, first the scores of Table 8 are calculated. Further, the values of NIS are identified based on 4.9 and 4.10, given in Table 9.
Step 8. The values of $\mathcal{HD}_i$ (4.11) and $\mathcal{ED}_i$ (4.12) are computed as given below:

$$\mathcal{ED}_1 = 0.039050, \quad \mathcal{ED}_2 = 1.655277, \quad \mathcal{ED}_3 = 2.780990.$$
$$\mathcal{HD}_1 = 0.049313, \quad \mathcal{HD}_2 = 2.512104, \quad \mathcal{HD}_3 = 4.415909.$$

Step 9. The values of $\mathcal{RA}$ matrix with their sum are given in Table 10.
Step 10. The best option is shown in Table 10 as $\mathbb{A}_3$.

Table 2. LTs and their corresponding 2TLq-RPFNs.

2TLq-RPFNs	LTs
$((\wp_1,0),(\wp_4,0),(\wp_7,0))$	Very poor (VP)
$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	Poor (P)
$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	Medium poor (MP)
$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	Medium (M)
$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	Medium good (MG)
$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	Good (G)
$((\wp_7,0),(\wp_4,0),(\wp_1,0))$	Very good (VG)

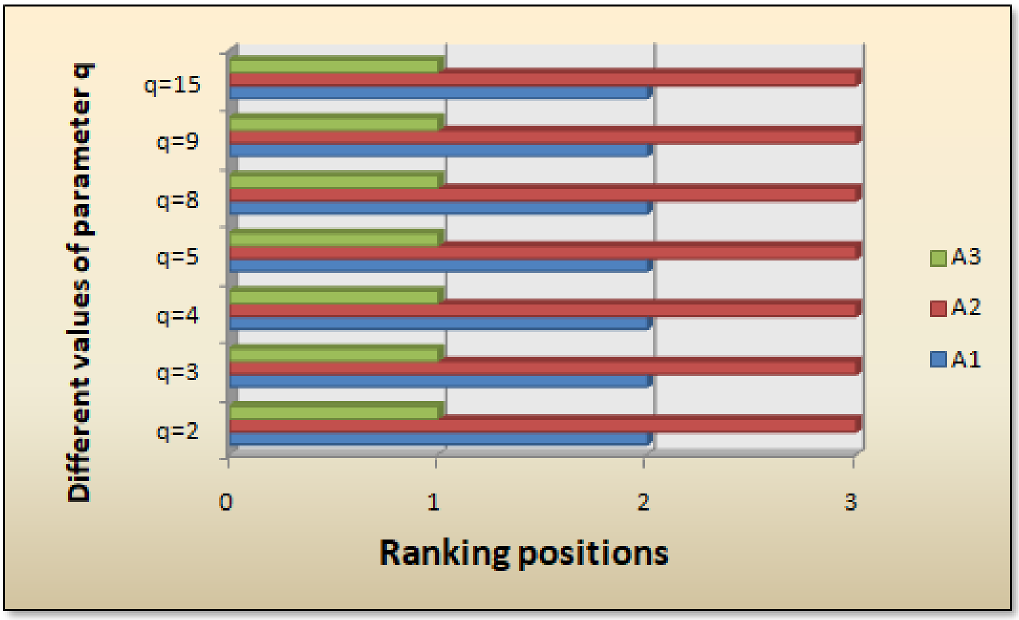

Figure 3. Ranking positions by using distinct values of q.

5.3. Sensitivity analysis

By adjusting seven distinct parameter values, we can solve this numerical case, i.e., $q = 2$, $q = 3$, $q = 4$, $q = 5$, $q = 8$, $q = 9$ and $q = 15$, in order to see the effects of modifying parameter q values on the outcomes of decision-making. With the change in value of q, the ranking order of the alternatives can be effected. Therefore, we show a sensitivity analysis using various parameter values in proposed strategy. The values of the ranking orders of the three options are shown in Table 11. Table 11 demonstrates that, given a range of feasible choices, alternative $\mathbb{A}_3$ is always the best choice. Also, alternative $\mathbb{A}_1$ is in the second slot, whereas $\mathbb{A}_2$ is the worst option. The value of the parameter q has no bearing on the veracity of the findings. As it retains accuracy beyond the bounds of the current sets, the adjustable q's major goal is to broaden the range of applications for the recommended approach. As a result, it is concluded that the suggested methodology has good stability with all parameters. Hence the stability and reliability of designed 2TLq-RPF-CODAS method for MCGDM problems is verified through evaluation of sensitive analysis for the prioritization of CAVs in a RTTM system. Figure 3 also displays the ranking places obtained by using various q values.

6. Comparative study and discussion

This section offers a few studies that compare the suggested methodology with the previous approaches in an effort to demonstrate the usefulness, underline the significance, and emphasize the advantages of the suggested procedures over the existing. In order to address the numerical problem, we emphasized the key components of each of the previous strategies. We use the 2TLPyF-CODAS [48] and 2TLFF-CODAS [55] methods.

6.1. Comparison with 2TLPyF-CODAS method

Using the 2TLPyF-CODAS approach, we solved the numerical example in this subsection. The neutral membership grade has not been taken into account. First, Table 4 compiles the 2TLPyFNs representing the normalized expert evaluations (ignore neutral part). The combined decision matrix is shown in Table 12. The score degrees of Table 12 are shown in Table 13. The authors explicitly gave the weights to the criterion in the 2TLPyF-CODAS technique. In computing the criterion weights, we use the entropy measure. After calculating the projection values, entropy values and divergence values the criteria weights are presented as follows:

$$\mathcal{W}_1 = 0.0010, \quad \mathcal{W}_2 = 0.0100, \quad \mathcal{W}_3 = 0.0030, \quad \mathcal{W}_4 = 0.0400,$$
$$\mathcal{W}_5 = 0.0650, \quad \mathcal{W}_6 = 0.0340, \quad \mathcal{W}_7 = 0.0640, \quad \mathcal{W}_8 = 0.0850,$$
$$\mathcal{W}_9 = 0.0930, \quad \mathcal{W}_{10} = 0.0010, \quad \mathcal{W}_{11} = 0.0020, \quad \mathcal{W}_{12} = 0.0980,$$
$$\mathcal{W}_{13} = 0.1420, \quad \mathcal{W}_{14} = 0.0100, \quad \mathcal{W}_{15} = 0.1270, \quad \mathcal{W}_{16} = 0.1910.$$

The weighted aggregated matrix's values are shown in Table 14. Score values of Table 14 and the values of NIS are demonstrated in Table 15. The values of $\mathcal{ED}_i$ and $\mathcal{HD}_i$ are given as follows:

$$\mathcal{ED}_1 = 0.1297, \quad \mathcal{ED}_2 = 0.8914, \quad \mathcal{ED}_3 = 1.7877.$$
$$\mathcal{HD}_1 = 0.1703, \quad \mathcal{HD}_2 = 1.1769, \quad \mathcal{HD}_3 = 2.4228.$$

The $\mathcal{RA}$ matrix is shown in Table 16 together with the related options sum and rankings.

6.2. Comparison with 2TLFF-CODAS method

Applying the 2TLFF-CODAS method, we solved the numerical example in this subsection. We did not take into account the neutral membership grade. First, the normalized assessments of the experts are compiled in Table 4 (ignore neutral part). Additionally, Table 17 presents the aggregated decision matrix. Table 18 shows the scores of Table 17. The weights for the criteria were directly assigned by the authors in the 2TLFF-CODAS methodology. While computing the weights for the criteria, we use the entropy measure. The criteria weights are given below:

$$\mathcal{W}_1 = 0.0028, \quad \mathcal{W}_2 = 0.0188, \quad \mathcal{W}_3 = 0.0050, \quad \mathcal{W}_4 = 0.0547,$$
$$\mathcal{W}_5 = 0.0796, \quad \mathcal{W}_6 = 0.0509, \quad \mathcal{W}_7 = 0.0807, \quad \mathcal{W}_8 = 0.1028,$$
$$\mathcal{W}_9 = 0.1017, \quad \mathcal{W}_{10} = 0.0022, \quad \mathcal{W}_{11} = 0.0033, \quad \mathcal{W}_{12} = 0.0973,$$
$$\mathcal{W}_{13} = 0.1227, \quad \mathcal{W}_{14} = 0.0149, \quad \mathcal{W}_{15} = 0.1233, \quad \mathcal{W}_{16} = 0.1393.$$

The values of the weighted aggregated matrix are shown in Table 19. Table 19's score values and the NIS values are shown in Table 20. The values of $\mathcal{ED}_i$ and $\mathcal{HD}_i$ are given as follows:

$$\mathcal{ED}_1 = 0.1143, \quad \mathcal{ED}_2 = 2.267, \quad \mathcal{ED}_3 = 3.964.$$
$$\mathcal{HD}_1 = 0.1021, \quad \mathcal{HD}_2 = 2.1401, \quad \mathcal{HD}_3 = 3.9146.$$

Finally, the $\mathcal{RA}$ matrix with their corresponding alternatives sum and ranks is presented in Table 21.

Table 3. Assessments of experts in the form of 2TLq-RPFNs.

	Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
$\mathbf{E}_1$	Im	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$
	Pe	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$
	Tr	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$
	Em	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Te	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Qu	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$
	Fu	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Ob	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Un	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Fa	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$
	Tc	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$
	Co	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Ne	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Cm	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$
	Ta	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Ch	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
$\mathbf{E}_2$	Im	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$
	Pe	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$
	Tr	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$
	Em	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Te	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Qu	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Fu	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Ob	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$
	Un	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$
	Fa	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$
	Tc	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$
	Co	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Ne	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Cm	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$
	Ta	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$
	Ch	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$
$\mathbf{E}_3$	Im	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$
	Pe	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$
	Tr	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Em	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$
	Te	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Qu	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Fu	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$
	Ob	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Un	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$
	Fa	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$
	Tc	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_5,0),(\varphi_3,0),(\varphi_2,0))$
	Co	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$
	Ne	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Cm	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$
	Ta	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Ch	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_3,0))$
$\mathbf{E}_4$	Im	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$
	Pe	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$
	Tr	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$
	Em	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Te	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Qu	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Fu	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$
	Ob	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$
	Un	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Fa	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$
	Tc	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$
	Co	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Ne	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$	$((\varphi_4,0),(\varphi_1,0),(\varphi_4,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Cm	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_2,0),(\varphi_3,0),(\varphi_6,0))$
	Ta	$((\varphi_3,0),(\varphi_2,0),(\varphi_5,0))$	$((\varphi_6,0),(\varphi_3,0),(\varphi_2,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$
	Ch	$((\varphi_1,0),(\varphi_4,0),(\varphi_7,0))$	$((\varphi_5,0),(\varphi_2,0),(\varphi_3,0))$	$((\varphi_7,0),(\varphi_4,0),(\varphi_1,0))$

Table 4. Normalized assessments of experts in the form of 2TLq-RPFNs.

	Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
$\mathbf{E}_1$	Im	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$
	Pe	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$
	Tr	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Em	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Te	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Qu	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Fu	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Ob	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Un	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Fa	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Tc	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Co	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Ne	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Cm	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_1,0),(\wp_4,0),(\wp_7,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$
	Ta	$((\wp_1,0),(\wp_4,0),(\wp_7,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Ch	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
$\mathbf{E}_2$	Im	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$
	Pe	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Tr	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Em	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Te	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Qu	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Fu	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Ob	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Un	$((\wp_1,0),(\wp_4,0),(\wp_7,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Fa	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Tc	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Co	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Ne	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Cm	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_1,0),(\wp_4,0),(\wp_7,0))$
	Ta	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Ch	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
$\mathbf{E}_3$	Im	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$
	Pe	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Tr	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_1,0),(\wp_4,0),(\wp_7,0))$
	Em	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Te	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Qu	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Fu	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Ob	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Un	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Fa	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Tc	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	$((\wp_2,0),(\wp_3,0),(\wp_5,0))$
	Co	$((\wp_1,0),(\wp_4,0),(\wp_7,0))$	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Ne	$((\wp_1,0),(\wp_4,0),(\wp_7,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Cm	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$
	Ta	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Ch	$((\wp_1,0),(\wp_4,0),(\wp_7,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
$\mathbf{E}_4$	Im	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Pe	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$
	Tr	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$
	Em	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Te	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Qu	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Fu	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Ob	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$
	Un	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Fa	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$
	Tc	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$
	Co	$((\wp_1,0),(\wp_4,0),(\wp_7,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Ne	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$	$((\wp_4,0),(\wp_1,0),(\wp_4,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Cm	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	$((\wp_2,0),(\wp_3,0),(\wp_6,0))$
	Ta	$((\wp_3,0),(\wp_2,0),(\wp_5,0))$	$((\wp_6,0),(\wp_3,0),(\wp_2,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$
	Ch	$((\wp_1,0),(\wp_4,0),(\wp_7,0))$	$((\wp_5,0),(\wp_2,0),(\wp_3,0))$	$((\wp_7,0),(\wp_4,0),(\wp_1,0))$

Table 5. Aggregated decision matrix in the form of 2TLq-RPFNs.

Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
Im	$((\wp_8, -0.036400), (\wp_0, 0.058085), (\wp_0, 0.008906))$	$((\wp_8, -0.283031), (\wp_0, 0.006047), (\wp_0, 0.070656))$	$((\wp_8, -0.097574), (\wp_0, 0.037935), (\wp_0, 0.024430))$
Pe	$((\wp_8, -0.076589), (\wp_0, 0.042263), (\wp_0, 0.018026))$	$((\wp_7, 0.275450), (\wp_0, 0.001420), (\wp_0, 0.167980))$	$((\wp_8, -0.278678), (\wp_0, 0.013637), (\wp_0, 0.067958))$
Tr	$((\wp_8, -0.056894), (\wp_0, 0.037935), (\wp_0, 0.013637))$	$((\wp_8, -0.345896), (\wp_0, 0.004400), (\wp_0, 0.079829))$	$((\wp_8, -0.214266), (\wp_0, 0.079829), (\wp_0, 0.069960))$
Em	$((\wp_7, -0.499937), (\wp_0, 0.013637), (\wp_0, 0.459790))$	$((\wp_8, -0.345896), (\wp_0, 0.004400), (\wp_0, 0.079829))$	$((\wp_8, -0.005238), (\wp_0, 0.167980), (\wp_0, 0.001420))$
Te	$((\wp_6, 0.150333), (\wp_0, 0.027602), (\wp_1, -0.401024))$	$((\wp_8, -0.077800), (\wp_0, 0.018742), (\wp_0, 0.018742))$	$((\wp_8, -0.002484), (\wp_0, 0.230844), (\wp_0, 0.000701))$
Qu	$((\wp_7, -0.250790), (\wp_0, 0.008906), (\wp_0, 0.362234))$	$((\wp_8, -0.163239), (\wp_0, 0.013637), (\wp_0, 0.037935))$	$((\wp_8, -0.005238), (\wp_0, 0.167980), (\wp_0, 0.001420))$
Fu	$((\wp_6, -0.001802), (\wp_0, 0.006737), (\wp_0, 0.631457))$	$((\wp_7, 0.464836), (\wp_0, 0.002874), (\wp_0, 0.122230))$	$((\wp_8, -0.011044), (\wp_0, 0.122230), (\wp_0, 0.002874))$
Ob	$((\wp_6, -0.132958), (\wp_0, 0.008906), (\wp_1, 0.351810))$	$((\wp_8, -0.077800), (\wp_0, 0.018742), (\wp_0, 0.018742))$	$((\wp_8, -0.011044), (\wp_0, 0.122230), (\wp_0, 0.002874))$
Un	$((\wp_6, -0.322617), (\wp_0, 0.012240), (\wp_0, 0.815291))$	$((\wp_8, -0.163239), (\wp_0, 0.013637), (\wp_0, 0.037935))$	$((\wp_8, -0.011044), (\wp_0, 0.122230), (\wp_0, 0.002874))$
Fa	$((\wp_8, -0.023647), (\wp_0, 0.039441), (\wp_0, 0.006047))$	$((\wp_8, -0.204460), (\wp_0, 0.027602), (\wp_0, 0.049447))$	$((\wp_8, -0.104685), (\wp_0, 0.020881), (\wp_0, 0.024775))$
Tc	$((\wp_8, -0.114154), (\wp_0, 0.058085), (\wp_0, 0.031019))$	$((\wp_8, -0.317763), (\wp_0, 0.018026), (\wp_0, 0.075711))$	$((\wp_8, -0.155899), (\wp_0, 0.028698), (\wp_0, 0.042631))$
Co	$((\wp_6, -0.374006), (\wp_0, 0.012726), (\wp_1, -0.001432))$	$((\wp_8, -0.152188), (\wp_0, 0.024775), (\wp_0, 0.037407))$	$((\wp_8, -0.005238), (\wp_0, 0.167980), (\wp_0, 0.001420))$
Ne	$((\wp_5, -0.089963), (\wp_0, 0.028698), (\wp_1, 0.308967))$	$((\wp_7, 0.272545), (\wp_0, 0.001420), (\wp_0, 0.167980))$	$((\wp_8, -0.002484), (\wp_0, 0.230844), (\wp_0, 0.000701))$
Cm	$((\wp_7, 0.269041), (\wp_0, 0.042263), (\wp_0, 0.218628))$	$((\wp_8, -0.165807), (\wp_0, 0.006047), (\wp_0, 0.039441))$	$((\wp_8, -0.049799), (\wp_0, 0.028698), (\wp_0, 0.012240))$
Ta	$((\wp_5, 0.320921), (\wp_0, 0.018742), (\wp_1, 0.033545))$	$((\wp_8, -0.056894), (\wp_0, 0.037935), (\wp_0, 0.013637))$	$((\wp_8, -0.005238), (\wp_0, 0.167980), (\wp_0, 0.001420))$
Ch	$((\wp_5, -0.364778), (\wp_0, 0.039441), (\wp_2, -0.358421))$	$((\wp_8, -0.121428), (\wp_0, 0.012240), (\wp_0, 0.028698))$	$((\wp_8, -0.011044), (\wp_0, 0.122230), (\wp_0, 0.002874))$

Table 6. Scores of Table 5.

Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
Im	$(\wp_9, -0.009133)$	$(\wp_9, -0.069585)$	$(\wp_9, -0.024277)$
Pe	$(\wp_9, -0.019089)$	$(\wp_9, -0.173376)$	$(\wp_9, -0.068531)$
Tr	$(\wp_9, -0.014198)$	$(\wp_9, -0.084704)$	$(\wp_9, -0.053025)$
Em	$(\wp_9, -0.343137)$	$(\wp_9, -0.084704)$	$(\wp_9, -0.001750)$
Te	$(\wp_9, -0.414577)$	$(\wp_9, -0.019366)$	$(\wp_9, -0.001454)$
Qu	$(\wp_9, -0.299614)$	$(\wp_9, -0.040419)$	$(\wp_9, -0.001750)$
Fu	$(\wp_9, -0.444069)$	$(\wp_9, -0.129550)$	$(\wp_9, -0.002993)$
Ob	$(\wp_9, -0.468719)$	$(\wp_9, -0.019366)$	$(\wp_9, -0.002993)$
Un	$(\wp_8, 0.493247)$	$(\wp_9, -0.040419)$	$(\wp_9, -0.002993)$
Fa	$(\wp_9, -0.005928)$	$(\wp_9, -0.050512)$	$(\wp_9, -0.026016)$
Tc	$(\wp_9, -0.028403)$	$(\wp_9, -0.077958)$	$(\wp_9, -0.038636)$
Co	$(\wp_8, 0.478977)$	$(\wp_9, -0.037717)$	$(\wp_9, -0.001750)$
Ne	$(\wp_8, 0.349910)$	$(\wp_9, -0.173376)$	$(\wp_9, -0.001454)$
Cm	$(\wp_9, -0.175166)$	$(\wp_9, -0.041047)$	$(\wp_9, -0.012426)$
Ta	$(\wp_8, 0.425682)$	$(\wp_9, -0.014198)$	$(\wp_9, -0.001750)$
Ch	$(\wp_8, 0.293577)$	$(\wp_9, -0.030142)$	$(\wp_9, -0.002993)$

Table 7. Projection values, entropy values, divergence values and weights.

Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$	Entropy values	Divergence values	Weights
Im	0.334270	0.332023	0.333707	0.999996	0.000004	0.001278
Pe	0.335873	0.330103	0.334024	0.999976	0.000024	0.007670
Tr	0.334691	0.332065	0.333245	0.999995	0.000005	0.001598
Em	0.325808	0.335535	0.338657	0.999877	0.000123	0.039310
Te	0.323190	0.338068	0.338742	0.999788	0.000212	0.067753
Qu	0.326368	0.336091	0.337541	0.999899	0.000101	0.032279
Fu	0.323801	0.335704	0.340494	0.999797	0.000203	0.064877
Ob	0.321827	0.338778	0.339395	0.999727	0.000273	0.087248
Un	0.321108	0.338739	0.340154	0.999691	0.000309	0.098754
Fa	0.334134	0.332478	0.333388	0.999998	0.000002	0.000639
Tc	0.334075	0.33223	0.333694	0.999997	0.000003	0.000959
Co	0.320693	0.338973	0.340333	0.999669	0.000331	0.105785
Ne	0.319002	0.337215	0.343783	0.999547	0.000453	0.144775
Cm	0.329637	0.334647	0.335716	0.999971	0.000029	0.009268
Ta	0.319037	0.340246	0.340717	0.999578	0.000422	0.134867
Ch	0.31582	0.341573	0.342607	0.999365	0.000635	0.202940

Table 8. Weighted aggregated decision matrix in the form of 2TLq-RPFNs.

Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
Im	$((\wp_1, -0.380796), (\wp_8, -0.050198), (\wp_8, -0.069227))$	$((\wp_0, 0.466599), (\wp_8, -0.07315), (\wp_8, -0.048207))$	$((\wp_1, -0.449084), (\wp_8, -0.054525), (\wp_8, -0.058993))$
Pe	$((\wp_1, 0.383780), (\wp_8, -0.315345), (\wp_8, -0.365404))$	$((\wp_1, -0.074989), (\wp_7, 0.487233), (\wp_8, -0.233578))$	$((\wp_1, 0.141438), (\wp_8, -0.381726), (\wp_8, -0.287298))$
Tr	$((\wp_1, -0.341338), (\wp_8, -0.068120), (\wp_8, -0.081077))$	$((\wp_1, -0.497914), (\wp_8, -0.095378), (\wp_8, -0.058684))$	$((\wp_1, -0.452272), (\wp_8, -0.058684), (\wp_8, -0.060358))$
Em	$((\wp_2, -0.369489), (\wp_6, 0.226806), (\wp_7, 0.150308))$	$((\wp_2, 0.433401), (\wp_6, -0.044014), (\wp_7, -0.325274))$	$((\wp_4, -0.165955), (\wp_7, -0.127189), (\wp_6, -0.303003))$
Te	$((\wp_2, -0.060398), (\wp_5, 0.448438), (\wp_7, -0.288467))$	$((\wp_4, -0.125701), (\wp_5, 0.307390), (\wp_5, 0.307390))$	$((\wp_5, 0.019373), (\wp_6, 0.291669), (\wp_4, 0.248062))$
Qu	$((\wp_2, -0.432923), (\wp_6, 0.423281), (\wp_7, 0.239420))$	$((\wp_3, -0.490477), (\wp_7, -0.487770), (\wp_7, -0.269118))$	$((\wp_4, -0.486557), (\wp_7, 0.062057), (\wp_6, 0.053658))$
Fu	$((\wp_2, -0.172703), (\wp_5, 0.053795), (\wp_7, -0.215043))$	$((\wp_3, -0.179762), (\wp_5, -0.217946), (\wp_6, 0.099287))$	$((\wp_5, -0.490689), (\wp_6, 0.099287), (\wp_5, -0.217946))$
Ob	$((\wp_2, 0.041578), (\wp_4, 0.419886), (\wp_6, 0.424933))$	$((\wp_4, 0.316997), (\wp_5, -0.283674), (\wp_5, -0.283674))$	$((\wp_5, 0.072184), (\wp_6, -0.445368), (\wp_4, 0.004564))$
Un	$((\wp_2, 0.068176), (\wp_4, 0.217616), (\wp_6, 0.384810))$	$((\wp_4, 0.169251), (\wp_4, 0.262872), (\wp_5, -0.283924))$	$((\wp_5, 0.313761), (\wp_5, 0.293725), (\wp_4, -0.344712))$
Fa	$((\wp_0, 0.457762), (\wp_8, -0.027111), (\wp_8, -0.036659))$	$((\wp_0, 0.349315), (\wp_8, -0.028929), (\wp_8, -0.025959))$	$((\wp_0, 0.386112), (\wp_8, -0.030350), (\wp_8, -0.029479))$
Tc	$((\wp_0, 0.467279), (\wp_8, -0.037698), (\wp_8, -0.042486))$	$((\wp_0, 0.395587), (\wp_8, -0.046627), (\wp_8, -0.035674))$	$((\wp_0, 0.446593), (\wp_8, -0.043008), (\wp_8, -0.040059))$
Co	$((\wp_2, 0.111096), (\wp_4, 0.046331), (\wp_6, 0.419346))$	$((\wp_4, 0.330831), (\wp_4, 0.341775), (\wp_5, -0.4648))$	$((\wp_6, -0.317582), (\wp_5, 0.31619), (\wp_3, 0.208569))$
Ne	$((\wp_2, 0.057549), (\wp_4, -0.459370), (\wp_6, 0.155644))$	$((\wp_4, -0.210731), (\wp_2, 0.291225), (\wp_5, -0.427204))$	$((\wp_6, 0.482821), (\wp_5, -0.21183), (\wp_2, 0.068635))$
Cm	$((\wp_1, 0.013693), (\wp_8, -0.379463), (\wp_8, -0.262502))$	$((\wp_1, 0.366219), (\wp_7, 0.484443), (\wp_8, -0.384343))$	$((\wp_2, -0.402751), (\wp_8, -0.406753), (\wp_8, -0.466484))$
Ta	$((\wp_2, 0.201813), (\wp_4, -0.465298), (\wp_6, 0.070515))$	$((\wp_5, 0.287030), (\wp_4, -0.112662), (\wp_3, 0.386321))$	$((\wp_6, 0.152814), (\wp_5, -0.248773), (\wp_2, 0.495920))$
Ch	$((\wp_2, 0.257889), (\wp_3, -0.278060), (\wp_6, -0.199010))$	$((\wp_6, -0.293962), (\wp_2, 0.146601), (\wp_3, -0.448161))$	$((\wp_7, -0.318362), (\wp_3, 0.424276), (\wp_2, -0.400287))$

Table 9. Scores of Table 8 and NIS.

Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$	NIS
Im	$(\wp_6, 0.035733)$	$(\wp_6, 0.033621)$	$(\wp_6, 0.033021)$	$((\wp_1, -0.449084), (\wp_8, -0.054525), (\wp_8, -0.058993))$
Pe	$(\wp_6, 0.196467)$	$(\wp_6, 0.194995)$	$(\wp_6, 0.184047)$	$((\wp_1, 0.141438), (\wp_8, -0.381726), (\wp_8, -0.287298))$
Tr	$(\wp_6, 0.043903)$	$(\wp_6, 0.042258)$	$(\wp_6, 0.034337)$	$((\wp_1, -0.452272), (\wp_8, -0.058684), (\wp_8, -0.060358))$
Em	$(\wp_7, -0.363148)$	$(\wp_7, -0.15788)$	$(\wp_7, -0.015491)$	$((\wp_2, -0.369489), (\wp_6, 0.226806), (\wp_7, 0.150308))$
Te	$(\wp_7, -0.108876)$	$(\wp_7, 0.354272)$	$(\wp_7, 0.493171)$	$((\wp_2, -0.060398), (\wp_5, 0.448438), (\wp_7, -0.288467))$
Qu	$(\wp_7, -0.425188)$	$(\wp_7, -0.272128)$	$(\wp_7, -0.158987)$	$((\wp_2, -0.432923), (\wp_6, 0.423281), (\wp_7, 0.23942))$
Fu	$(\wp_7, -0.06621)$	$(\wp_7, 0.185694)$	$(\wp_7, 0.379133)$	$((\wp_2, -0.172703), (\wp_5, 0.053795), (\wp_7, -0.215043))$
Ob	$(\wp_7, 0.114889)$	$(\wp_8, -0.403922)$	$(\wp_8, -0.330678)$	$((\wp_2, 0.041578), (\wp_4, 0.419886), (\wp_6, 0.424933))$
Un	$(\wp_7, 0.151926)$	$(\wp_8, -0.359856)$	$(\wp_8, -0.205447)$	$((\wp_2, 0.068176), (\wp_4, 0.217616), (\wp_6, 0.384810))$
Fa	$(\wp_6, 0.019184)$	$(\wp_6, 0.015605)$	$(\wp_6, 0.017259)$	$((\wp_0, 0.349315), (\wp_8, -0.028929), (\wp_8, -0.025959))$
Tc	$(\wp_6, 0.023407)$	$(\wp_6, 0.022967)$	$(\wp_6, 0.023847)$	$((\wp_0, 0.395587), (\wp_8, -0.046627), (\wp_8, -0.035674))$
Co	$(\wp_7, 0.169936)$	$(\wp_8, -0.322859)$	$(\wp_8, -0.097921)$	$((\wp_2, 0.111096), (\wp_4, 0.046331), (\wp_6, 0.419346))$
Ne	$(\wp_7, 0.278211)$	$(\wp_8, -0.184400)$	$(\wp_8, 0.231580)$	$((\wp_2, 0.057549), (\wp_4, -0.459370), (\wp_6, 0.155644))$
Cm	$(\wp_6, 0.173221)$	$(\wp_6, 0.247679)$	$(\wp_6, 0.252187)$	$((\wp_1, 0.013693), (\wp_8, -0.379463), (\wp_8, -0.262502))$
Ta	$(\wp_7, 0.304730)$	$(\wp_8, 0.021471)$	$(\wp_8, 0.141459)$	$((\wp_2, 0.201813), (\wp_4, -0.465298), (\wp_6, 0.070515))$
Ch	$(\wp_7, 0.438088)$	$(\wp_8, 0.334986)$	$(\wp_8, 0.474368)$	$((\wp_2, 0.257889), (\wp_3, -0.278060), (\wp_6, -0.199010))$

Table 10. $\mathcal{RA}$ matrix with sum and ranks.

$\mathcal{RA}$	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$	Sum	Rank
$\mathbb{A}_1$	0	2.364202	9.231004	11.595206	2
$\mathbb{A}_2$	5.596656	0	1.017425	6.614081	3
$\mathbb{A}_3$	14.714884	3.268851	0	17.983735	1

Table 11. Results using different values of q.

Alternatives	$q = 2$	$q = 3$	$q = 4$	$q = 5$	$q = 8$	$q = 9$	$q = 15$
$\mathbb{A}_1$	2	2	2	2	2	2	2
$\mathbb{A}_2$	3	3	3	3	3	3	3
$\mathbb{A}_3$	1	1	1	1	1	1	1

Table 12. Aggregated decision matrix in the form of 2TLPyFNs.

Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
Im	$((\wp_8, -0.1101), (\wp_0, 0.0234))$	$((\wp_8, -0.3281), (\wp_0, 0.1563))$	$((\wp_8, -0.2178), (\wp_0, 0.0586))$
Pe	$((\wp_8, -0.2068), (\wp_0, 0.0469))$	$((\wp_7, -0.1046), (\wp_0, 0.3750))$	$((\wp_7, 0.4903), (\wp_0, 0.1563))$
Tr	$((\wp_8, -0.1538), (\wp_0, 0.0352))$	$((\wp_7, 0.3758), (\wp_0, 0.1875))$	$((\wp_8, -0.3548), (\wp_0, 0.1367))$
Em	$((\wp_6, 0.1881), (\wp_1, -0.1562))$	$((\wp_7, 0.3758), (\wp_0, 0.1875))$	$((\wp_8, -0.0226), (\wp_0, 0.0039))$
Te	$((\wp_6, -0.1227), (\wp_1, 0.0547))$	$((\wp_8, -0.1897), (\wp_0, 0.0469))$	$((\wp_8, -0.0121), (\wp_0, 0.0020))$
Qu	$((\wp_6, 0.3588), (\wp_1, -0.2969))$	$((\wp_8, -0.3579), (\wp_0, 0.0938))$	$((\wp_8, -0.0226), (\wp_0, 0.0039))$
Fu	$((\wp_6, -0.3115), (\wp_1, 0.1250))$	$((\wp_7, 0.1156), (\wp_0, 0.2813))$	$((\wp_8, -0.0422), (\wp_0, 0.0078))$
Ob	$((\wp_6, -0.4533), (\wp_1, 0.1719))$	$((\wp_8, -0.1897), (\wp_0, 0.0469))$	$((\wp_8, -0.0422), (\wp_0, 0.0078))$
Un	$((\wp_5, 0.3949), (\wp_1, 0.3672))$	$((\wp_8, -0.3579), (\wp_0, 0.0938))$	$((\wp_8, -0.0422), (\wp_0, 0.0078))$
Fa	$((\wp_8, -0.0724), (\wp_0, 0.0156))$	$((\wp_8, -0.4115), (\wp_0, 0.1172))$	$((\wp_8, -0.2553), (\wp_0, 0.0625))$
Tc	$((\wp_8, -0.2379), (\wp_0, 0.0703))$	$((\wp_7, 0.4206), (\wp_0, 0.1758))$	$((\wp_8, -0.2938), (\wp_0, 0.0938))$
Co	$((\wp_5, 0.3959), (\wp_2, -0.4687))$	$((\wp_8, -0.3050), (\wp_0, 0.0879))$	$((\wp_8, -0.0226), (\wp_0, 0.0039))$
Ne	$((\wp_5, -0.2596), (\wp_2, -0.0312))$	$((\wp_7, -0.1046), (\wp_0, 0.3750))$	$((\wp_8, -0.0121), (\wp_0, 0.0020))$
Cm	$((\wp_7, 0.0003), (\wp_0, 0.4219))$	$((\wp_8, -0.3281), (\wp_0, 0.0938))$	$((\wp_8, -0.1357), (\wp_0, 0.0313))$
Ta	$((\wp_5, 0.0924), (\wp_2, -0.3594))$	$((\wp_8, -0.1538), (\wp_0, 0.0352))$	$((\wp_8, -0.0226), (\wp_0, 0.0039))$
Ch	$((\wp_5, -0.4839), (\wp_2, 0.2969))$	$((\wp_8, -0.2655), (\wp_0, 0.0703))$	$((\wp_8, -0.0422), (\wp_0, 0.0117))$

Table 13. Scores of Table 12.

Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
Im	$(\wp_8, -0.1094)$	$(\wp_8, -0.3229)$	$(\wp_8, -0.2150)$
Pe	$(\wp_8, -0.2043)$	$(\wp_7, -0.0371)$	$(\wp_8, -0.4950)$
Tr	$(\wp_8, -0.1524)$	$(\wp_7, 0.3980)$	$(\wp_8, -0.3481)$
Em	$(\wp_6, 0.3488)$	$(\wp_7, 0.3980)$	$(\wp_8, -0.0226)$
Te	$(\wp_6, 0.0894)$	$(\wp_8, -0.1876)$	$(\wp_8, -0.0121)$
Qu	$(\wp_6, 0.4962)$	$(\wp_8, -0.3504)$	$(\wp_8, -0.0226)$
Fu	$(\wp_6, -0.0567)$	$(\wp_7, 0.1595)$	$(\wp_8, -0.0421)$
Ob	$(\wp_6, -0.1630)$	$(\wp_8, -0.1876)$	$(\wp_8, -0.0421)$
Un	$(\wp_6, -0.2978)$	$(\wp_8, -0.3504)$	$(\wp_8, -0.0421)$
Fa	$(\wp_8, -0.0721)$	$(\wp_8, -0.4018)$	$(\wp_8, -0.2515)$
Tc	$(\wp_8, -0.2347)$	$(\wp_7, 0.4396)$	$(\wp_8, -0.2890)$
Co	$(\wp_6, -0.3268)$	$(\wp_8, -0.2997)$	$(\wp_8, -0.0226)$
Ne	$(\wp_5, 0.1622)$	$(\wp_7, -0.0371)$	$(\wp_8, -0.0121)$
Cm	$(\wp_7, 0.0516)$	$(\wp_8, -0.3219)$	$(\wp_8, -0.1346)$
Ta	$(\wp_5, 0.4526)$	$(\wp_8, -0.1524)$	$(\wp_8, -0.0226)$
Ch	$(\wp_5, -0.0550)$	$(\wp_8, -0.2614)$	$(\wp_8, -0.0421)$

Table 14. Weighted aggregated decision matrix in the form of 2TLPyFNs.

Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
Im	$((\wp_8,-0.0144),(\wp_8,-0.0465))$	$((\wp_8,-0.0101),(\wp_8,-0.0314))$	$((\wp_8,-0.0117),(\wp_8,-0.0392))$
Pe	$((\wp_8,-0.1181),(\wp_8,-0.4007))$	$((\wp_8,-0.0541),(\wp_8,-0.2411))$	$((\wp_8,-0.0833),(\wp_8,-0.3087))$
Tr	$((\wp_8,-0.0391),(\wp_8,-0.1292))$	$((\wp_8,-0.0227),(\wp_8,-0.0896))$	$((\wp_8,-0.0293),(\wp_8,-0.0971))$
Em	$((\wp_8,-0.1446),(\wp_7,0.3117))$	$((\wp_8,-0.2979),(\wp_7,-0.1153))$	$((\wp_7,0.2130),(\wp_6,-0.1033))$
Te	$((\wp_8,-0.1992),(\wp_7,0.0128))$	$((\wp_7,0.2426),(\wp_6,-0.2718))$	$((\wp_7,-0.3747),(\wp_5,-0.3339))$
Qu	$((\wp_8,-0.1347),(\wp_7,0.3652))$	$((\wp_8,-0.3246),(\wp_7,-0.1224))$	$((\wp_7,0.3260),(\wp_6,0.1728))$
Fu	$((\wp_8,-0.1783),(\wp_7,0.0561))$	$((\wp_8,-0.3910),(\wp_6,0.4571))$	$((\wp_7,-0.0849),(\wp_5,0.1332))$
Ob	$((\wp_8,-0.2197),(\wp_7,-0.2051))$	$((\wp_7,0.0243),(\wp_5,0.1686))$	$((\wp_7,-0.4078),(\wp_4,0.4377))$
Un	$((\wp_8,-0.2225),(\wp_7,-0.2121))$	$((\wp_7,0.1431),(\wp_5,0.2908))$	$((\wp_6,0.4732),(\wp_4,0.1982))$
Fa	$((\wp_8,-0.0160),(\wp_8,-0.0498))$	$((\wp_8,-0.0092),(\wp_8,-0.0337))$	$((\wp_8,-0.0111),(\wp_8,-0.0387))$
Tc	$((\wp_8,-0.0227),(\wp_8,-0.0754))$	$((\wp_8,-0.0157),(\wp_8,-0.0609))$	$((\wp_8,-0.0210),(\wp_8,-0.0708))$
Co	$((\wp_8,-0.2344),(\wp_7,-0.1966))$	$((\wp_7,0.0455),(\wp_5,0.1416))$	$((\wp_6,0.2074),(\wp_4,-0.2111))$
Ne	$((\wp_8,-0.2419),(\wp_7,-0.4442))$	$((\wp_7,0.2645),(\wp_5,0.1804))$	$((\wp_5,0.2991),(\wp_2,0.4638))$
Cm	$((\wp_8,-0.0578),(\wp_8,-0.2320))$	$((\wp_8,-0.1002),(\wp_8,-0.3479))$	$((\wp_8,-0.1345),(\wp_8,-0.4314))$
Ta	$((\wp_8,-0.2596),(\wp_7,-0.4581))$	$((\wp_7,-0.4992),(\wp_4,0.0161))$	$((\wp_6,-0.2415),(\wp_3,0.0371))$
Ch	$((\wp_8,-0.2879),(\wp_6,0.3034))$	$((\wp_6,0.1645),(\wp_3,0.2387))$	$((\wp_5,0.1785),(\wp_2,0.2994))$

Table 15. Scores of Table 14 and NIS.

Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$	NIS
Im	$(\wp_4,0.0320)$	$(\wp_4,0.0212)$	$(\wp_4,0.0274)$	$((\wp_8,-0.0101),(\wp_8,-0.0314))$
Pe	$(\wp_4,0.2734)$	$(\wp_4,0.1835)$	$(\wp_4,0.2199)$	$((\wp_8,-0.0541),(\wp_8,-0.2411))$
Tr	$(\wp_4,0.0892)$	$(\wp_4,0.0664)$	$(\wp_4,0.0673)$	$((\wp_8,-0.0227),(\wp_8,-0.0896))$
Em	$(\wp_5,-0.4846)$	$(\wp_5,-0.2548)$	$(\wp_5,0.0785)$	$((\wp_8,-0.1446),(\wp_7,0.3117))$
Te	$(\wp_5,-0.2704)$	$(\wp_5,0.2277)$	$(\wp_5,0.3826)$	$((\wp_8,-0.1992),(\wp_7,0.0128))$
Qu	$(\wp_4,0.4760)$	$(\wp_5,-0.2744)$	$(\wp_5,-0.0271)$	$((\wp_8,-0.1347),(\wp_7,0.3652))$
Fu	$(\wp_5,-0.2881)$	$(\wp_5,0.0127)$	$(\wp_5,0.3418)$	$((\wp_8,-0.1783),(\wp_7,0.0561))$
Ob	$(\wp_5,-0.1023)$	$(\wp_5,0.4141)$	$(\wp_5,0.4852)$	$((\wp_8,-0.2197),(\wp_7,-0.2051))$
Un	$(\wp_5,-0.0991)$	$(\wp_5,0.4395)$	$(\wp_6,-0.4827)$	$((\wp_8,-0.2225),(\wp_7,-0.2121))$
Fa	$(\wp_4,0.0337)$	$(\wp_4,0.0244)$	$(\wp_4,0.0275)$	$((\wp_8,-0.0092),(\wp_8,-0.0337))$
Tc	$(\wp_4,0.0524)$	$(\wp_4,0.0450)$	$(\wp_4,0.0495)$	$((\wp_8,-0.0227),(\wp_8,-0.0754))$
Co	$(\wp_5,-0.1239)$	$(\wp_5,0.4502)$	$(\wp_6,-0.4890)$	$((\wp_7,0.0455),(\wp_5,0.1416))$
Ne	$(\wp_5,0.0756)$	$(\wp_6,-0.3790)$	$(\wp_5,0.3756)$	$((\wp_8,-0.2419),(\wp_7,-0.4442))$
Cm	$(\wp_4,0.1710)$	$(\wp_4,0.2408)$	$(\wp_4,0.2864)$	$((\wp_8,-0.0578),(\wp_8,-0.2320))$
Ta	$(\wp_5,0.0698)$	$(\wp_6,-0.3668)$	$(\wp_5,0.4960)$	$((\wp_8,-0.2596),(\wp_7,-0.4581))$
Ch	$(\wp_5,0.2340)$	$(\wp_6,-0.2805)$	$(\wp_5,0.3456)$	$((\wp_8,-0.2879),(\wp_6,0.3034))$

Table 16. $\mathcal{RA}$ matrix with sum and ranks.

$\mathcal{RA}$	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$	Sum	Rank
$\mathbb{A}_1$	0	0.0050	2.0766	2.0816	2
$\mathbb{A}_2$	1.5284	0	0.2204	1.7488	3
$\mathbb{A}_3$	5.3926	2.0130	0	7.4056	1

Table 17. Aggregated decision matrix in the form of 2TLFFNs.

Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
Im	$((\wp_8, -0.1052), (\wp_0, 0.0119))$	$((\wp_7, 0.4442), (\wp_0, 0.0852))$	$((\wp_8, -0.2311), (\wp_0, 0.0310))$
Pe	$((\wp_8, -0.2150), (\wp_0, 0.0238))$	$((\wp_7, -0.4241), (\wp_0, 0.2018))$	$((\wp_7, 0.4036), (\wp_0, 0.0838))$
Tr	$((\wp_8, -0.1567), (\wp_0, 0.0179))$	$((\wp_7, 0.2360), (\wp_0, 0.0990))$	$((\wp_8, -0.3757), (\wp_0, 0.0820))$
Em	$((\wp_6, -0.1795), (\wp_1, -0.4907))$	$((\wp_7, 0.2360), (\wp_0, 0.0990))$	$((\wp_8, -0.0167), (\wp_0, 0.0020))$
Te	$((\wp_5, 0.4799), (\wp_1, -0.3493))$	$((\wp_8, -0.2001), (\wp_0, 0.0242))$	$((\wp_8, -0.0081), (\wp_0, 0.0010))$
Qu	$((\wp_6, -0.0264), (\wp_0, 0.4096))$	$((\wp_8, -0.4068), (\wp_0, 0.0486))$	$((\wp_8, -0.0167), (\wp_0, 0.0020))$
Fu	$((\wp_5, 0.2041), (\wp_1, -0.3124))$	$((\wp_7, -0.1240), (\wp_0, 0.1494))$	$((\wp_8, -0.0344), (\wp_0, 0.0039))$
Ob	$((\wp_5, -0.0150), (\wp_1, -0.2935))$	$((\wp_8, -0.2001), (\wp_0, 0.0242))$	$((\wp_8, -0.0344), (\wp_0, 0.0039))$
Un	$((\wp_5, -0.1053), (\wp_1, -0.1274))$	$((\wp_8, -0.4068), (\wp_0, 0.0486))$	$((\wp_8, -0.0344), (\wp_0, 0.0039))$
Fa	$((\wp_8, -0.0656), (\wp_0, 0.0080))$	$((\wp_8, -0.4690), (\wp_0, 0.0621))$	$((\wp_8, -0.2743), (\wp_0, 0.0322))$
Tc	$((\wp_8, -0.2486), (\wp_0, 0.0385))$	$((\wp_7, 0.3072), (\wp_0, 0.0937))$	$((\wp_8, -0.3171), (\wp_0, 0.0520))$
Co	$((\wp_5, 0.0385), (\wp_1, 0.0487))$	$((\wp_8, -0.3431), (\wp_0, 0.0467))$	$((\wp_8, -0.0167), (\wp_0, 0.0020))$
Ne	$((\wp_4, 0.3207), (\wp_1, 0.3483))$	$((\wp_7, -0.4241), (\wp_0, 0.2018))$	$((\wp_8, -0.0081), (\wp_0, 0.0010))$
Cm	$((\wp_7, -0.1663), (\wp_0, 0.2499))$	$((\wp_8, -0.3789), (\wp_0, 0.0494))$	$((\wp_8, -0.1344), (\wp_0, 0.0161))$
Ta	$((\wp_5, -0.3724), (\wp_1, 0.0848))$	$((\wp_8, -0.1567), (\wp_0, 0.0179))$	$((\wp_8, -0.0167), (\wp_0, 0.0020))$
Ch	$((\wp_4, 0.1949), (\wp_2, -0.3358))$	$((\wp_8, -0.2973), (\wp_0, 0.0366))$	$((\wp_8, -0.0344), (\wp_0, 0.0039))$

Table 18. Scores of Table 17.

Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
Im	$(\wp_8, -0.1557)$	$(\wp_7, 0.2229)$	$(\wp_8, -0.3367)$
Pe	$(\wp_8, -0.3139)$	$(\wp_6, 0.2215)$	$(\wp_7, 0.1704)$
Tr	$(\wp_8, -0.2305)$	$(\wp_7, -0.0400)$	$(\wp_7, 0.4625)$
Em	$(\wp_6, -0.4605)$	$(\wp_7, -0.0400)$	$(\wp_8, -0.0250)$
Te	$(\wp_5, 0.2835)$	$(\wp_8, -0.2927)$	$(\wp_8, -0.0121)$
Qu	$(\wp_6, -0.3352)$	$(\wp_7, 0.4203)$	$(\wp_8, -0.0250)$
Fu	$(\wp_5, 0.0986)$	$(\wp_7, -0.4602)$	$(\wp_8, -0.0514)$
Ob	$(\wp_5, -0.0350)$	$(\wp_8, -0.2927)$	$(\wp_8, -0.0514)$
Un	$(\wp_5, -0.0890)$	$(\wp_7, 0.4203)$	$(\wp_8, -0.0514)$
Fa	$(\wp_8, -0.0976)$	$(\wp_7, 0.3369)$	$(\wp_8, -0.3975)$
Tc	$(\wp_8, -0.3614)$	$(\wp_7, 0.0482)$	$(\wp_8, -0.4570)$
Co	$(\wp_5, -0.0097)$	$(\wp_8, -0.4929)$	$(\wp_8, -0.0250)$
Ne	$(\wp_5, -0.3890)$	$(\wp_6, 0.2215)$	$(\wp_8, -0.0121)$
Cm	$(\wp_6, 0.4931)$	$(\wp_7, 0.4581)$	$(\wp_8, -0.1982)$
Ta	$(\wp_5, -0.2358)$	$(\wp_8, -0.2305)$	$(\wp_8, -0.0250)$
Ch	$(\wp_5, -0.4593)$	$(\wp_8, -0.4296)$	$(\wp_8, -0.0514)$

Table 19. Weighted aggregated decision matrix in the form of 2TLFFNs.

Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
Im	$((\wp_2,-0.3330),(\wp_8,-0.1445))$	$((\wp_1,0.3283),(\wp_8,-0.1011))$	$((\wp_2,-0.4766),(\wp_8,-0.1234))$
Pe	$((\wp_3,-0.1187),(\wp_7,0.1712))$	$((\wp_2,-0.0217),(\wp_7,0.4653))$	$((\wp_2,0.4619),(\wp_7,0.3429))$
Tr	$((\wp_2,-0.0642),(\wp_8,-0.2404))$	$((\wp_2,-0.4909),(\wp_8,-0.1738))$	$((\wp_2,-0.2773),(\wp_8,-0.1811))$
Em	$((\wp_2,0.3776),(\wp_7,-0.1188))$	$((\wp_3,0.3132),(\wp_6,0.2915))$	$((\wp_5,-0.0118),(\wp_5,0.0823))$
Te	$((\wp_2,0.4966),(\wp_7,-0.4484))$	$((\wp_5,-0.4178),(\wp_5,0.0415))$	$((\wp_6,-0.2585),(\wp_4,-0.0880))$
Qu	$((\wp_2,0.4010),(\wp_7,-0.1231))$	$((\wp_4,-0.3672),(\wp_6,0.1698))$	$((\wp_5,-0.1151),(\wp_5,0.2450))$
Fu	$((\wp_2,0.3592),(\wp_7,-0.4373))$	$((\wp_3,0.4196),(\wp_6,-0.1980))$	$((\wp_5,0.3335),(\wp_4,0.3232))$
Ob	$((\wp_2,0.4314),(\wp_6,0.2336))$	$((\wp_5,-0.0582),(\wp_4,0.4066))$	$((\wp_6,-0.3043),(\wp_4,-0.3473))$
Un	$((\wp_2,0.3732),(\wp_6,0.3860))$	$((\wp_5,-0.4969),(\wp_5,-0.2392))$	$((\wp_6,-0.3205),(\wp_4,-0.3165))$
Fa	$((\wp_2,-0.391),(\wp_8,-0.1207))$	$((\wp_1,0.2642),(\wp_8,-0.0851))$	$((\wp_1,0.3740),(\wp_8,-0.0965))$
Tc	$((\wp_2,-0.4066),(\wp_8,-0.1397))$	$((\wp_1,0.3426),(\wp_8,-0.1165))$	$((\wp_2,-0.4600),(\wp_8,-0.1319))$
Co	$((\wp_2,0.4171),(\wp_7,-0.4351))$	$((\wp_5,-0.4472),(\wp_6,-0.1500))$	$((\wp_6,-0.1565),(\wp_4,-0.4305))$
Ne	$((\wp_2,0.2006),(\wp_6,0.4299))$	$((\wp_4,-0.3539),(\wp_5,0.0932))$	$((\wp_6,0.3876),(\wp_3,-0.3443))$
Cm	$((\wp_2,-0.0518),(\wp_8,-0.4027))$	$((\wp_2,0.4676),(\wp_7,0.4160))$	$((\wp_3,-0.1804),(\wp_7,0.2932))$
Ta	$((\wp_2,0.3753),(\wp_6,0.2531))$	$((\wp_5,0.3354),(\wp_4,-0.2302))$	$((\wp_6,0.1985),(\wp_3,-0.1229))$
Ch	$((\wp_2,0.2229),(\wp_6,0.4284))$	$((\wp_5,0.1524),(\wp_4,-0.2227))$	$((\wp_6,0.1523),(\wp_3,-0.2348))$

Table 20. Scores of Table 19 and NIS.

Criteria	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$	NIS
Im	$(\wp_0,0.2490)$	$(\wp_0,0.1681)$	$(\wp_0,0.2099)$	$((\wp_1,0.3283),(\wp_8,-0.1011))$
Pe	$(\wp_1,0.3057)$	$(\wp_1,-0.1899)$	$(\wp_1,0.0235)$	$((\wp_2,-0.0217),(\wp_7,0.4653))$
Tr	$(\wp_0,0.4065)$	$(\wp_0,0.2819)$	$(\wp_0,0.3055)$	$((\wp_2,-0.4909),(\wp_8,-0.1738))$
Em	$(\wp_2,-0.4406)$	$(\wp_2,0.3385)$	$(\wp_4,-0.0559)$	$((\wp_2,0.3776),(\wp_7,-0.1188))$
Te	$(\wp_2,-0.0754)$	$(\wp_4,-0.2494)$	$(\wp_5,0.0109)$	$((\wp_2,0.4966),(\wp_7,-0.4484))$
Qu	$(\wp_2,-0.4327)$	$(\wp_3,-0.4603)$	$(\wp_4,-0.2166)$	$((\wp_2,0.4010),(\wp_7,-0.1231))$
Fu	$(\wp_2,-0.1056)$	$(\wp_3,-0.2135)$	$(\wp_5,-0.4460)$	$((\wp_2,0.3592),(\wp_7,-0.4373))$
Ob	$(\wp_2,0.2199)$	$(\wp_4,0.2744)$	$(\wp_5,0.0628)$	$((\wp_2,0.4314),(\wp_6,0.2336))$
Un	$(\wp_2,0.0698)$	$(\wp_4,-0.1296)$	$(\wp_5,0.0408)$	$((\wp_2,0.3732),(\wp_6,0.3860))$
Fa	$(\wp_0,0.2109)$	$(\wp_0,0.1421)$	$(\wp_0,0.1633)$	$((\wp_1,0.2642),(\wp_8,-0.0851))$
Tc	$(\wp_0,0.2375)$	$(\wp_0,0.1911)$	$(\wp_0,0.2231)$	$((\wp_1,0.3426),(\wp_8,-0.1165))$
Co	$(\wp_2,-0.1001)$	$(\wp_4,-0.1540)$	$(\wp_5,0.2036)$	$((\wp_2,0.4171),(\wp_7,-0.4351))$
Ne	$(\wp_2,0.0064)$	$(\wp_3,0.3465)$	$(\wp_6,-0.1102)$	$((\wp_2,0.2006),(\wp_6,0.4299))$
Cm	$(\wp_1,-0.3681)$	$(\wp_1,-0.0690)$	$(\wp_1,0.1444)$	$((\wp_2,-0.0518),(\wp_8,-0.4027))$
Ta	$(\wp_2,2.1945)$	$(\wp_5,-0.2320)$	$(\wp_6,-0.3255)$	$((\wp_2,0.3753),(\wp_6,0.2531))$
Ch	$(\wp_2,0.0104)$	$(\wp_5,-0.3524)$	$(\wp_6,-0.3459)$	$((\wp_2,0.2229),(\wp_6,0.4284))$

Table 21. $\mathcal{RA}$ matrix with sum and ranks.

$\mathcal{RA}$	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$	Sum	Rank
$\mathbb{A}_1$	0	2.2345	10.8273	13.0618	2
$\mathbb{A}_2$	6.5399	0	-4.7083	1.8315	3
$\mathbb{A}_3$	18.5267	4.7083	0	23.2350	1

6.3. Discussion

The following facts have been observed when addressing the problem using the suggested and existing approaches:

- Using the 2TLPyF-CODAS technique [48], the results are shown in Table 16. These findings, which we obtained are identical to our suggested approach. The best option using the suggested and existing methodologies is the same, which is $\mathbb{A}_3$, demonstrating the validity and correctness of the technique. The 2TLPyF-CODAS technique, however, simply considers the positive and negative membership degrees as the two membership degrees. On the other hand, our current strategy, which consists of three membership levels, can accommodate a wide range of content. Moreover, the authors explicitly give weights to the criterion, whereas we compute the weights of the criteria using the entropy measure.
- The outcomes of using the 2TLFF-CODAS approach are shown in Table 21. These calculations we made using the previous method produced results that are consistent. The best option by utilizing the previous and suggested methodologies is the same, which is $\mathbb{A}_3$ and proves the validity and correctness of the approach offered. The 2TLFF-CODAS technique, on the other hand, consists of just two membership degrees: the positive membership degree and the negative membership degree, and since there isn't a neutral portion, it might not produce correct findings. On the other hand, our presented strategy, which consists of three membership levels and can handle a wide range of obscure information, is built on a solid foundation. Furthermore, we use the entropy measure to determine the criteria weights, compared to the 2TLFF-CODAS method that explicitly assign weights to the criteria.
- Table 22 displays the final rankings of the three options along with the best alternative using the suggested and existing methodologies. Figure 4 also shows the graphic representation of the ranking positions of all options.

The following salient characteristics of our created technique serve to distinguish the given approach from the approaches already in use:

1) As 2TLPyFS and 2TLFFS are special instances of 2TLq-RPFS (owing to the lack of neutral membership grade), for $q = 2$ and $q = 3$, respectively, the range of the 2TLq-RPFS is wider than the 2TLPyFS and 2TLFFS. As a result, the suggested strategy is generalized. We suggest the new AOs to fuse the provided information using 2TLq-RPFSs. These novel operators, namely the 2TLq-RPFEWA and 2TLq-RPFEOWA operators, limit the effects of unclear data throughout the information integrating process and establish the link between aggregated arguments.

2) We present a novel aggregation concept for Einstein AOs in the 2TLq-RPF setting. This concept has a wider range (i.e., includes degrees of membership, neutral membership, and non-membership to explain the uncertainty and vagueness) and is capable of illuminating the maximum discrepancies of the data set.

3) The most often used way for resolving MCGDM issues is the CODAS approach. The application of the CODAS approach with 2TLq-RPFSs is also lacking.

4) In conclusion, the approach we provide is general and suited to solving 2TLq-RPFS MCGDM issues. While contrasting the suggested technique with the 2TLPyF-CODAS and 2TLFF-CODAS methodologies, we came to the same ranking conclusions.

Table 22. Comparison with existing methods.

Method	Ranking order	Best alternative
2TLq-RPF-CODAS (proposed)	$\mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_2$	$\mathbb{A}_3$
2TLPyF-CODAS [48]	$\mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_2$	$\mathbb{A}_3$
2TLFF-CODAS [55]	$\mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_2$	$\mathbb{A}_3$

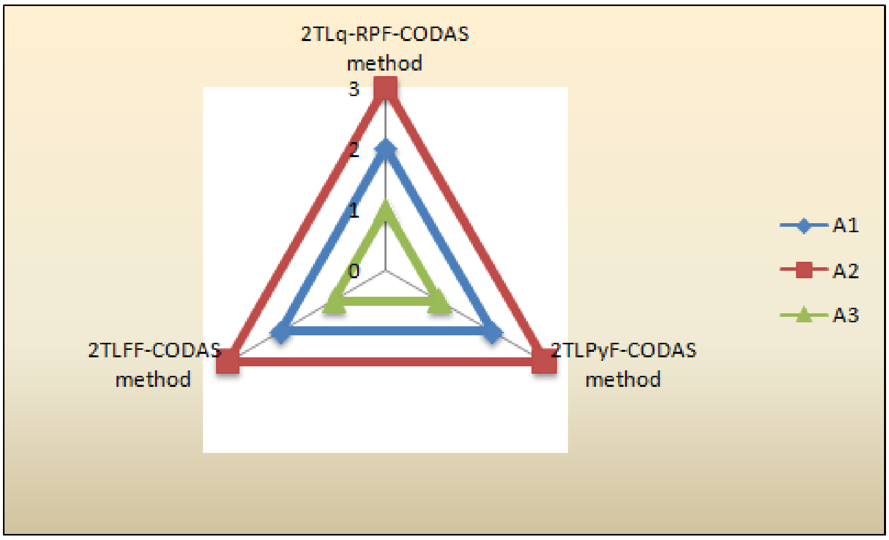

Figure 4. Comparison using proposed and existing approaches.

7. Conclusions

In order to illustrate linguistic assessment values using LTs, it has been acknowledged that the 2TLTSs are adequate structures. The 2TLq-RPFS is a more extensive and universal structure since there is a wide range of permissible triplets. Furthermore, the 2TLq-RPFSs provide remarkable qualities to express the experts' preference ratings utilizing LTs and within the constraints of q-RPFSs. The 2TLq-RPF context has been introduced with an improved and efficient CODAS approach. We use the entropy measure approach to compute the weights for the criteria. Moreover, under the umbrella of 2TLq-RPFSs, two Einstein AOs, 2TLq-RPFEWA and 2TLq-RPFEOWA, have been investigated. The recommended method has been used to select the best alternative for RTTM system in order to show its value and effectiveness. In addition to this, parameter analysis is done to verify the stability of the suggested technique. The importance and reliability of the developed approach have been assessed and demonstrated by comparison with other procedures. As a result, we draw the conclusion that the technique is generic, simple to comprehend, practical to use, and capable of capturing the linguistic MCGDM difficulties. The major contributions of the proposed study are outlined in the following essential points: (1) suggested a novel approach to MCGDM issues in a linguistic context; (2) presented two AOs for 2TLq-RPFSs; (3) provided a more reliable and effective CODAS approach for the 2TLq-RPF scenario; (4) used a case study to implement the suggested MCGDM methodology. Although the structure of 2TLq-RPFS has a broader structure and is capable to accommodate a wide range of information by increasing the powers of parameter q. But it can only handle fuzzy information, and we have to consider the same powers of three triplets. In addition, the proposed strategies may not provide stable results for a large number of alternatives and criteria. Furthermore, the proposed methodology is able to capture only one-dimensional data. Our future research directions are stated as follows:

- We aim to integrate the CODAS approach with other methodologies to compute criteria weights, such as AHP, SWARA and IDOCRIW.
- In the future, we will extend more MCGDM approaches for 2TLq-RPFSs using the CODAS methodology to address more real-world MCGDM problems in other disciplines. The application of the current work can be looked into other pertinent subject topics.
- Furthermore, we are planning to extend more MCGDM methods, such as PROMETHEE, ELECTRE, and WASPAS for 2TLq-RPFSs.

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Conflict of interest

The authors declare no conflict of interest.

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