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Research article

Stabilization of discrete-time positive switched T-S fuzzy systems subject to actuator saturation

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Abstract: The stabilization of discrete-time positive switched Takagi-Sugeno (T-S) fuzzy systems with actuator saturation is investigated in this paper. It is assumed that the switched subsystems are partially stabilizable. Based on the convex hull technique (CHT) and parallel distribution compensation (PDC) algorithm, a saturated fuzzy controller and slow-fast combined mode-dependent average dwell time (MDADT) switching signal are co-designed and sufficient conditions for the positivity and stability of closed-loop positive switched T-S fuzzy systems (PSTSFSs) are developed, which can be reduced to the ones under the case where all switched subsystems are stabilizable. Moreover, the largest attraction domain estimation (ADE) is given for PSTSFSs by formulating an optimization problem. Finally, the designed control scheme is applied to two illustrative examples to verify its availability and superiority.

Keywords: stabilization; actuator saturation; positive systems; switched systems; T-S fuzzy systems

1. Introduction

Positive switched systems consist of a series of subsystems and a switching signal that determines the switching behavior of the subsystems [1]. In recent years, positive switched systems have received extensive attention [2–5] due to their wide application in various fields, including formation flying [6], communication [7], water pollution control [8].

The nonlinear characteristic is ubiquitous in practical positive switched systems. T-S fuzzy model, as a powerful tool to deal with nonlinearity [9, 10], is used to approximate positive switched nonlinear systems. Such systems are called PSTSFSs in this paper.

The existing research results on PSTSFSs mainly focus on the stabilization problem under the case that switched subsystems are all stabilizable [11–13]. However, switched subsystems may be unstabilizable in practice [14–16]. Recently, the stabilization has been investigated for discrete-time PSTSFSs with partially stabilizable switched subsystems in [17].

However, little attention is paid to the problem of actuator saturation in [17]. In fact, actuator saturation is an unavoidable phenomenon in the practical system due to the inherent constraints on physical actuators. Actuator saturation often leads to system performance degradation or even instability. Hence, the actuator saturation has attracted extensive attention in the past few years [18–23]. For continuous-time PSTSFSs with partially stabilizable switched subsystems, the actuator saturation was discussed and sufficient conditions were proposed to ensure the positivity and stability of closed-loop systems in [24]. Nevertheless, the control scheme proposed in [24] is invalid for discrete-time saturated PSTSFSs. In fact, the controller of practical systems implements in a digital manner. Thus, it is an important and meaningful issue that the stabilization of discrete-time saturated PSTSFSs with actuator saturation. Moreover, Fornasini Ettore also pointed out that the stability condition in the discrete-time case is much more difficult than the stability condition in the continuous-time case in [25]. Hence, it is natural to ask the following two questions: how to design the controller to ensure the positivity and stability of closed-loop PSTSFSs subject to actuator saturation in discrete-time domain? Whether the largest ADE of the system can be estimated in discrete-time domain? These two issues motivate us to carry out this work.

The stabilization of discrete-time PSTSFSs with actuator saturation is investigated in this paper. The main innovations include three aspects:

(1) The saturated state feedback fuzzy controller as well as the slow-fast combined MDADT switching signal are co-designed to ensure the positivity and stability of the closed-loop PSTSFSs.
(2) The sufficient conditions for the positivity and stability of discrete-time saturated PSTSFSs with partially stabilizable subsystems are proposed, which can be reduced to the ones applied to the case that switched subsystems are all stabilizable.
(3) The largest ADE for discrete-time saturated PSTSFSs is given by formulating an optimization problem.

The remainder of this paper is arranged as follows. The problem formulation and preliminaries of discrete-time saturated PSTSFSs are presented in Section 2. A saturated fuzzy controller is designed and sufficient conditions for the positivity and stability of discrete-time saturated PSTSFSs are developed in Section 3. The largest ADE of saturated PSTSFSs is given in Section 4. Section 5 provides two simulation examples to verify the availability and superiority of the proposed control scheme. Section 6 summarizes the work of this paper and points out the research in the future.

Notation: I stands for n-order identity matrix. A^T represents the transpose of matrix A. $A \geq 0 (A \prec 0)$ denotes all elements of matrix A are nonnegative (negative). $\|\cdot\|$ represents Euclidean norm. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ represent the set of all n-dimensional vectors and the set of all $m \times n$ matrices over the real number field, respectively. $\mathbb{R}^n_+$ and $\mathbb{R}^{m\times n}_+$ represent the set of all n-dimensional vectors and the set of all $m \times n$ matrices over the positive real number field, respectively. $L(H)$ stands for the set $\{x \in \mathbb{R}^n | |h_i x| \leq 1, H \in \mathbb{R}^{m\times n}\}$, where h_i is the ith row of matrix H. For any constant $c > 0$, $\varepsilon(\xi_p, c)$ represents the set $\left\{x(k) \in \mathbb{R}^n | \, x^T(k)\xi_p \leq c\right\}$.

2. Problem formulation and preliminaries

Consider the following discrete-time PSTSFS with actuator saturation, where each switched subsystem is described by T-S fuzzy model:
R_p^i: IF $z_{p1}(k)$ is M_{p1}^i and $z_{p2}(k)$ is M_{p2}^i and $\cdots$ and $z_{pl}(k)$ is M_{pl}^i, THEN

$$x(k+1) = A_{pi}x(k) + B_{pi}\mathrm{sat}(u(k)), \tag{2.1}$$

where $R_p^i, i \in \mathcal{R}, p \in \mathcal{N}$ is the i-th fuzzy rule of the p-th positive switched subsystems. $\mathcal{R} \triangleq \{1, 2, \cdots, r\}$ is a set of fuzzy rule numbers with total number r. $\mathcal{N} \triangleq \{1, 2, \cdots, N\}$ is a set of positive switched subsystem numbers with total number N, and $\mathcal{N} = \mathcal{N}_c \cup \mathcal{N}_{uc}$, where $\mathcal{N}_c = \{1, 2, \ldots, n\}$ and $\mathcal{N}_{uc} = \{n+1, n+2, \ldots, N\}$ represent the set of stabilizable subsystems and the set of unstabilizable subsystems, respectively. $z_p(k) = [z_{p1}(k)\ z_{p2}(k)\ \cdots\ z_{pn}(k)]^T$ is a vector composed of premise variables $z_{pi}(k)$. $M_{pj}^i, i \in \mathcal{R}, p \in \mathcal{N}, j \in \mathcal{L}$ are fuzzy sets, where $\mathcal{L} \triangleq \{1, 2, \cdots, l\}$ is a set of fuzzy set numbers with total number l. $x(k) \in \mathbb{R}_+^n$ and $u(k) \in \mathbb{R}^m$ are the system state and control input. A_{pi} and B_{pi} are constant matrices of the pth subsystems. Define the saturation function $\mathrm{sat}(u(k)) = [\mathrm{sat}(u_1(k)), \mathrm{sat}(u_2(k)), \cdots, \mathrm{sat}(u_m(k))]^T$, where $\mathrm{sat}(u_j(k)) = \mathrm{sign}(u_j(k))\min\left\{\left|u_j(k)\right|, 1\right\}$, $j = 1, 2, \cdots, m$.

Applying the fuzzy blending method, the final discrete-time PSTSFS is

$$x(k+1) = \sum_{i=1}^{r} \theta_{pi}(z_p(k))[A_{pi}x(k) + B_{pi}\mathrm{sat}(u(k))], \tag{2.2}$$

where the normalized membership functions $\theta_{pi}(z_p(k))$ satisfy

$$\theta_{pi}(z_p(k)) = \frac{\prod_{j=1}^{l} M_{pj}^i(z_{pj}(k))}{\sum_{i=1}^{r}\prod_{j=1}^{l} M_{pj}^i(z_{pj}(k))} \geq 0, \sum_{i=1}^{r} \theta_{pi}(z_p(k)) = 1. \tag{2.3}$$

Some definitions and lemmas related to this paper will be introduced in the follows.

Definition 1 ([26]). *The system is called stabilizable if one can find an unconstrained control input $u(k)$ such that for any finite time $[k_0, k_f]$, the system can always arrive at any terminal state $x(k_f)$.*

Definition 2 ([27]). *System* (2.1) *is controlled positive if under a certain control input $u(k)$, the system state $x(k)$ is non-negative for any non-negative initial state $x(k_0)$.*

Lemma 1 ([27]). *For any $0 < \theta_{pi}(z_p(k)) < 1$, the positivity of system* (2.2) *with* $\mathrm{sat}(u(k)) = 0$ *is equivalent to $A_{pi} \geq 0$, $p \in \mathcal{N}$, $i \in \mathcal{R}$.*

Definition 3 ([28]). *System* (2.2) *is exponentially stable if for any switching signal $\sigma(k)$, we can find two constants $\alpha > 0$ and $0 < \beta < 1$ such that $\|x(k)\| \leq \alpha\beta^{(k-k_0)}\|x(k_0)\|$ holds for any $k \geq k_0$.*

Definition 4 ([29]). *For given switching signal $\sigma(k) = p$, if one can find two positive constants κ_{ap} and N_{0p} such that*

$$N_{\sigma p}(k_2, k_1) \leq N_{0p} + \frac{T_p(k_2, k_1)}{\kappa_{ap}},$$

holds for any $k_2 \geq k_1 \geq 0$*, then* κ_{ap} *is called a MDADT, where* $N_{\sigma p}(k_2, k_1)$ *and* $T_p(k_2, k_1)$ *denote the total switching numbers and the total time of the pth subsystem on* $[k_1, k_2)$*, respectively.*

Definition 5 ([30]). *For given switching signal* $\sigma(k) = p$*, if one can find two positive constants* κ_{ap} *and* N_{0p} *such that*

$$N_{\sigma q}(k_2, k_1) \geq N_{0q} + \frac{T_q(k_2, k_1)}{\kappa_{aq}},$$

holds for any $k_2 \geq k_1 \geq 0$*, then* κ_{ap} *is called a fast MDADT, where* $N_{\sigma p}(k_2, k_1)$ *and* $T_p(k_2, k_1)$ *are the same as Definition 4.*

Lemma 2 ([31]). *Given matrices* $K, H \in \mathbb{R}^{m\times n}$*,*

$$\mathrm{sat}(Kx(k)) \in co\{E_s Kx(k) + E_s^- Hx(k)\}, s \in Q \overset{\Delta}{=} \{1, 2, \cdots, 2^m\}, \tag{2.4}$$

holds for any $x(k) \in L(H)$*, where* $E_s \in \mathbb{R}^{m\times m}$ *are diagonal matrices whose diagonal elements are 0 or 1, and* $E_s^- = I - E_s$*. Since* $s \in Q$*, the numbers of matrices* E_s *and* E_s^- *are both* 2^m*, thus,* $\mathrm{sat}(Kx(k))$ *can be further expressed as follows:*

$$\mathrm{sat}(Kx(k)) = \sum_{s=1}^{2^m} \eta_s(k)(E_s K + E_s^- H)x(k), s \in Q, \tag{2.5}$$

where $\eta_s(k)$ *are nonnegative scalar functions with* $0 \leq \eta_s(k) \leq 1$ *and* $\sum_{s=1}^{2^m} \eta_s(k) = 1$.

3. Saturated controller design and stabilization

In this section, our goal is to ensure system (3.4) is controlled positive and exponentially stable by designing a saturated fuzzy controller. And the sufficient conditions for the positivity and stability of closed-loop system with partially stabilizable subsystems are given in Theorem 1. These conditions can be reduced to the ones for the positivity and stability of closed-loop system with all stabilizable subsystems, which is given in Corollary 1.

3.1. Saturated controller design

If $\sigma(k) = q \in \mathcal{N}_{uc}$, it is impossible to find a suitable controller to stabilize the subsystem. Therefore, we can ignore the design of the controller, that is to say, $\mathrm{sat}(u(k)) = 0$. If $\sigma(k) = p \in \mathcal{N}_c$, a suitable controller can be found to stabilize the subsystem, which means $\mathrm{sat}(u(k)) \neq 0$. For any $x(k) \in L(H)$, by Lemma 2, a PDC-based fuzzy controller is designed as follows:
Rule R_p^i: IF $z_{p1}(k)$ is M_{p1}^i and $\cdots$ and $z_{pn}(k)$ is M_{pn}^i, THEN

$$\mathrm{sat}(u(k)) = \mathrm{sat}(K_{pj}x(k)) = \sum_{s=1}^{2^m} \eta_s(k)(E_{sj}K_{pj} + E_{sj}^- H_{pj})x(k). \tag{3.1}$$

The final fuzzy controller is

$$\mathrm{sat}(u(k)) = \sum_{j=1}^{r} \theta_{pj}(z_p(k)) \sum_{s=1}^{2^m} \eta_s(k)(E_{sj}K_{pj} + E_{sj}^- H_{pj})x(k), \tag{3.2}$$

with K_{pj}, H_{pj} are matrices to be determined.

(3.2) together with (2.2) implies that

$$x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{2^m} \theta_{pi}(z_p(k))\theta_{pj}(z_p(k))\eta_s(k)\bar{A}_{pij}x(k), \tag{3.3}$$

where $\bar{A}_{pij} = A_{pi} + B_{pi}(E_{sj}K_{pj} + E_{sj}^{-}H_{pj})$.

The final closed-loop PSTSFS with partially stabilizable subsystems in discrete-time domain is

$$x(k+1) = \begin{cases} \sum\limits_{i=1}^{r} \theta_{qi}(z_q(k))A_{qi}x(k), q \in \mathcal{N}_{uc}, \\ \sum\limits_{i=1}^{r}\sum\limits_{j=1}^{r}\sum\limits_{s=1}^{2^m} \theta_{pi}(z_p(k))\theta_{pj}(z_p(k))\eta_s(k)\bar{A}_{pij}x(k), p \in \mathcal{N}_c. \end{cases} \tag{3.4}$$

3.2. Stabilization

Theorem 1. *For $p \in \mathcal{N}_c$, $q \in \mathcal{N}_{uc}$, given constants $\mu_p > 1$, $0 < \lambda_p < 1$, $0 < \mu_q < 1$, $\lambda_q > 1$. If we can find a group of vectors $c_p \in \mathbb{R}_+^m$, $\xi_p = [\xi_{p1}, \xi_{p2}, \cdots, \xi_{pn}] \in \mathbb{R}_+^n$, $\xi_q = [\xi_{q1}, \xi_{q2}, \cdots, \xi_{qn}] \in \mathbb{R}_+^n$, $g_{pij} \in \mathbb{R}^n$, $w_{pij} \in \mathbb{R}^n$, $z_{pij} \in \mathbb{R}$, such that*

$$c_p^T E_{sj}^T B_{pi}^T \xi_p A_{pi} + B_{pi}E_{sj}c_p g_{pij}^T + B_{pi}E_{sj}^{-}c_p w_{pij}^T \geq 0,$$
$$p \in \mathcal{N}_c, s \in Q, i, j \in \mathcal{R}, \tag{3.5}$$
$$A_{qi} \geq 0, q \in \mathcal{N}_{uc}, i \in \mathcal{R}, \tag{3.6}$$
$$A_{pi}^T \xi_p + g_{pij} + w_{pij}z_{pij} - \lambda_p \xi_p \prec 0, p \in \mathcal{N}_c, i, j \in \mathcal{R}, \tag{3.7}$$
$$A_{qi}^T \xi_q - \lambda_q \xi_q \prec 0, q \in \mathcal{N}_{uc}, i \in \mathcal{R}, \tag{3.8}$$
$$\xi_p - \mu_p \xi_r \prec 0, \quad p, r \in \mathcal{N}_c, p \neq r, \tag{3.9}$$
$$\xi_p - \mu_p \xi_q \prec 0, \quad p \in \mathcal{N}_c, q \in \mathcal{N}_{uc}, \tag{3.10}$$
$$\xi_q - \mu_q \xi_p \prec 0, \quad p \in \mathcal{N}_c, q \in \mathcal{N}_{uc}, \tag{3.11}$$
$$\varepsilon(\xi_p, 1) \subset L(H_{pj}), p \in \mathcal{N}_c, j \in \mathcal{R}, \tag{3.12}$$

then system (3.4) *is controlled positive and exponentially stable under the saturated fuzzy controller* (3.2) *and the slow-fast combined MDADT with*

$$\kappa_{ap} > -\frac{\ln \mu_p}{\ln \lambda_p}, \, p \in \mathcal{N}_c, \tag{3.13}$$

$$\kappa_q^* \leq \kappa_{aq} < -\frac{\ln \mu_q}{\ln \lambda_q}, \, q \in \mathcal{N}_{uc}, \tag{3.14}$$

where

$$H_{pj} = \frac{c_p w_{pij}^T}{c_p^T E_{sj}^T B_{pi}^T \xi_p}, p \in \mathcal{N}_c, \tag{3.15}$$

and the controller gains are

$$K_{pj} = \frac{c_p g_{pij}^T}{c_p^T E_{sj}^T B_{pi}^T \xi_p}, p \in \mathcal{N}_c. \tag{3.16}$$

Proof. It is assumed that the switching instants on $[0, K)$ are $k_i, i \in \{1, 2, \ldots, N_\sigma\}$ and when $k \in [k_i, k_{i+1})$, the $\sigma(k)$th subsystem is activated.

Case 1: $\sigma(k) = p \in \mathcal{N}_c$.

Since $c_p \in \mathbb{R}^m_+, B_{pi} \in \mathbb{R}^{n\times m}_+, \xi_p \in \mathbb{R}^n_+$, one can find $E_{sj} \in \mathbb{R}^{m\times m}_+$ such that $c_p^T E_{sj}^T B_{pi}^T \xi_p \in \mathbb{R}_+$. It is obtained from (3.5) that

$$A_{pi} + B_{pi}(E_{sj}\frac{c_p g_{pij}^T}{c_p^T E_{sj}^T B_{pi}^T \xi_p} + E_{sj}^- \frac{c_p w_{pij}^T}{c_p^T E_{sj}^T B_{pi}^T \xi_p}) \geq 0. \tag{3.17}$$

Combining (3.15) and (3.16) yields that

$$A_{pi} + B_{pi}(E_{sj}K_{pj} + E_{sj}^- H_{pj}) \geq 0. \tag{3.18}$$

Thus, the positivity of system (3.4) is proved by Lemma 1.

Case 2: $\sigma(k) = q \in \mathcal{N}_{uc}$.

From (3.6), we can also get the positivity of system (3.4).

In what follows, we will prove that system (3.4) is exponentially stable.

Case 1: $\sigma(k) = p \in \mathcal{N}_c$.

Constructing the following multiple linear co-positive Lyapunov function candidate:

$$V_p(x(k)) = x^T(k)\xi_p. \tag{3.19}$$

Let $d_1 = \min_{(p,n)\in\mathcal{N}\times\mathcal{N}_*}\{\xi_{pn}\}$, $d_2 = \max_{(p,n)\in\mathcal{N}\times\mathcal{N}_*}\{\xi_{pn}\}$, where $\mathcal{N}_*$ belongs to the set $\{1, 2, \cdots, n\}$.

From (3.19), one can get that

$$d_1 \|x(k)\| \leq V_p(x(k)) \leq d_2 \|x(k)\|. \tag{3.20}$$

From (3.4) and (3.20), one has

$$\begin{aligned} V_p(x(k+1)) - \lambda_p V_p(x(k)) =& x^T(k+1)\xi_p - \lambda_p x^T(k)\xi_p \\ =& \sum_{i=1}^r \mu_{pi}(z_p(k)) \sum_{j=1}^r \mu_{pj}(z_p(k)) \sum_{s=1}^{2^m} \eta_s(t) x^T(k)\{[A_{pi}^T \\ &+ (K_{pj}^T E_{sj}^T + H_{pj}^T (E_{sj}^-)^T) B_{pi}^T]\xi_p - \lambda_p \xi_p\} \\ =& \sum_{i=1}^r \mu_{pi}(z_p(k)) \sum_{j=1}^r \mu_{pj}(z_p(k)) \sum_{s=1}^{2^m} \eta_s(t) x^T(k)\{[A_{pi}^T \xi_p \\ &+ K_{pj}^T E_{sj}^T B_{pi}^T \xi_p + H_{pj}^T (E_{sj}^-)^T B_{pi}^T \xi_p] - \lambda_p \xi_p\} \\ =& \sum_{i=1}^r \mu_{pi}(z_p(k)) \sum_{j=1}^r \mu_{pj}(z_p(k)) \sum_{s=1}^{2^m} \eta_s(t) x^T(k)(A_{pi}^T \xi_p \\ &+ g_{pij} + w_{pij} z_{pij} - \lambda_p \xi_p). \end{aligned} \tag{3.21}$$

It follows from (3.7) and (3.21) that

$$V_p(x(k+1)) - \lambda_p V_p(x(k)) < 0. \tag{3.22}$$

Case 2: $\sigma(k) = q \in \mathcal{N}_{uc}$.
From (3.8), one can also get

$$V_q(x(k+1)) - \lambda_q V_q(x(k)) < 0. \tag{3.23}$$

Combining (3.22) and (3.23) gives that

$$V_{\sigma(k)}(x(k+1)) - \lambda_{\sigma(k)} V_{\sigma(k)}(x(k)) < 0, \forall k \in [k_i, k_{i+1}). \tag{3.24}$$

When $k = k_i$, the subsystem is assumed that switched from the $\sigma(k_i - 1)$th to the $\sigma(k_i)$th.
Case 1: $\sigma(k_i - 1) = q \in \mathcal{N}_{uc}$, $\sigma(k_i) = p \in \mathcal{N}_c$.
It is obtained from (3.10) that

$$\begin{aligned} V_{\sigma(k_i)}(x(k_i)) - \mu_{\sigma(k_i)} V_{\sigma(k_i-1)}(x(k_i - 1)) =& V_p(x(k_i)) - \mu_p V_q(x(k_i - 1)) \\ =& x^T(k_i)\xi_p - \mu_p x^T(k_i)\xi_q \\ =& x^T(k_i)(\xi_p - \mu_p \xi_q) \\ <& 0. \end{aligned} \tag{3.25}$$

Case 2: $\sigma(k_i - 1) = p' \in \mathcal{N}_c$, $\sigma(k_i) = p \in \mathcal{N}_c$.
We can also derive from (3.9) that (3.25) holds.
Case 3: $\sigma(k_i - 1) = p \in \mathcal{N}_c$, $\sigma(k_i) = q \in \mathcal{N}_{uc}$.
We can also get from (3.11) that (3.25) holds.
From (3.24) and (3.25), it follows that

$$\begin{aligned} V_{\sigma(k)}(x(k)) \le& \lambda_{\sigma(k_i)}^{k-k_i} \mu_{\sigma(k_i)} V_{\sigma(k_{i-1})}(x(k_i)) \\ &\vdots \\ \le& \lambda_{\sigma(k_i)}^{k-k_i} \prod_{i=0}^{k_{N_\sigma}} \lambda_{\sigma(k_i)}^{k_{i+1}-k_i} \prod_{j=1}^{i} \mu_{\sigma(k_j)} V_{\sigma(k_0)}(x(k_0)) \\ \le& \prod_{p=1}^{m} \lambda_p^{T_p(k_0,k)} \mu_p^{N_{\sigma p}(k_0,k)} \prod_{q=m+1}^{M} \lambda_q^{T_q(k_0,k)} \mu_q^{N_{\sigma q}(k_0,k)} V_{\sigma(k_0)}(x(k_0)). \end{aligned} \tag{3.26}$$

Due to $\mu_p > 1$, $0 < \lambda_p < 1$ and $0 < \mu_q < 1$,$\lambda_q > 0$, according to Definitions 4 and 5, one can get

$$\lambda_p^{T_p(k_0,k)} \mu_p^{N_{\sigma p}(k_0,k)} \le (\lambda_p \mu_p^{1/\tau_{ap}})^{T_p(k_0,k)} \mu_p^{N_{0p}}. \tag{3.27}$$

and

$$\lambda_p^{T_q(k_0,k)} \mu_q^{N_{\sigma q}(k_0,k)} \le (\lambda_q \mu_q^{1/\tau_{aq}})^{T_q(k_0,k)} \mu_q^{N_{0q}}. \tag{3.28}$$

From (3.26)–(3.28), we have

$$V_{\sigma(k)}(x(k)) \le \prod_{p=1}^{n} \lambda_p^{T_p(k_0,k)} \mu_p^{N_{\sigma p}(k_0,k)} \prod_{q=n+1}^{N} \lambda_q^{T_q(k_0,k)} \mu_q^{N_{\sigma q}(k_0,k)} V_{\sigma(k_0)}(x(k_0))$$

$$
\begin{aligned}
&\leq \prod_{p=1}^{n} (\lambda_p \mu_p^{1/\tau_{ap}})^{T_p(k_0,k)} \mu_p^{N_{0p}} \prod_{q=n+1}^{N} (\lambda_q \mu_q^{1/\tau_{aq}})^{T_q(k_0,k)} \times \mu_q^{N_{0q}} V_{\sigma(k_0)}(x(k_0)) \\
&\leq \exp\left\{\sum_{p=1}^{n} T_p(k_0,k)\left(\frac{1}{\kappa_{ap}} \ln \mu_p + \ln \lambda_p\right)\right\} \times \exp\left\{\sum_{q=n+1}^{N} T_q(k_0,k)\left(\frac{1}{\kappa_{aq}} \ln \mu_q + \ln \lambda_q\right)\right\} \\
&\quad \times \prod_{p=1}^{n} \mu_p^{N_{0p}} \prod_{q=n+1}^{N} \mu_q^{N_{0q}} V_{\sigma(k_0)}(x(k_0)) \\
&\leq \gamma \zeta^{k-k_0} V_{\sigma(k_0)}(x(k_0)),
\end{aligned} \tag{3.29}
$$

where

$$
\gamma = \prod_{p=1}^{n} \mu_p^{N_{0p}} \prod_{q=n+1}^{N} \mu_q^{N_{0p}}, \tag{3.30}
$$

$$
\zeta = \exp\left\{\sum_{p=1}^{n}\left(\frac{1}{\kappa_{ap}} \ln \mu_p + \ln \lambda_p\right) + \sum_{q=n+1}^{N}\left(\frac{1}{\kappa_{aq}} \ln \mu_q + \ln \lambda_q\right)\right\}. \tag{3.31}
$$

(3.20) together with (3.29) gives that

$$
\begin{aligned}
\|x(k)\| &\leq \frac{1}{d_1} V_{\sigma(k)}(x(k)) \\
&\leq \frac{1}{d_1} \gamma \zeta^{(k-k_0)} V_{\sigma(k_0)}(k_0) \\
&\leq \frac{d_2}{d_1} \gamma \zeta^{(k-k_0)} \|x(k_0)\| \\
&\leq \eta \zeta^{(k-k_0)} \|x(k_0)\|,
\end{aligned}
$$

where $\eta = \frac{d_2}{d_1}\gamma$.

From $\mu_p > 1$, $p \in \mathcal{N}_c$, $0 < \mu_p < 1$, $p \in \mathcal{N}_{uc}$ and $d_1 > 0$, $d_2 > 0$, it follows that $\eta > 0$. From (3.13) and (3.14), one can obtain that $0 < \zeta < 1$. By Definition 3, system (3.4) is exponentially stable.

Then, it will be proved that an ADE of system (3.4) is the set $\bigcap_{p=1}^{N} \varepsilon(\xi_p, \gamma^{-1})$. For any initial state $x(k_0) \in \bigcap_{p=1}^{N} \varepsilon(\xi_p, \gamma^{-1}) \subset \varepsilon(\xi_p, \gamma^{-1})$, one has $x(k_0) \in \varepsilon(\xi_p, \gamma^{-1})$, which implies that $x^T(k_0)\xi_p \leq \gamma^{-1}$. It is known from (3.30) that $\gamma > 0$. We can get that $\gamma V_{\sigma(t_0)}(x(t_0)) \leq 1$ through multiplying both sides of $x^T(k_0)\xi_p \leq \gamma^{-1}$ by γ. It yields from $0 < \zeta < 1$ that $0 < \zeta^{k-k_0} < 1$. Thereby, it is obtained that $V_{\sigma(t)}(x(k)) \leq \gamma \zeta^{k-k_0} V_{\sigma(t_0)}(x(k_0)) \leq 1$, that is to say, $x(k) \in \varepsilon(\xi_p, 1) \subset L(H_{pj})$. Thus, the set $\bigcap_{p=1}^{N} \varepsilon(\xi_p, \gamma^{-1})$ is an ADE of system (3.4). □

Remark 1. *Note that the designed switching signal is invalid for the switching between unstabilizable subsystems. Suppose that the qth subsystems is switched to the rth one, and then is switched to the qth one again, where $p, q, r \in \mathcal{N}_{uc}$, one has $0 < \mu_r < 1, 0 < \mu_q < 1$. But $\xi_r - \mu_r \xi_q \prec 0$ and $\xi_q - \mu_q \xi_r \prec 0$ imply that $\mu_r \mu_q > 1$. This is in contradiction with $0 < \mu_r < 1, 0 < \mu_q < 1$.*

Remark 2. *In [32], the stabilization for positive switched linear systems with partially stabilizable subsystems and actuator saturation was investigated, which is not applicable to nonlinear systems. Different from [32], this paper studies the actuator saturation control of discrete-time PSTSFSs, the obtained result is applicable to nonlinear systems. In addition, the solution to controller gains K_{pj} requires a given matrix H_{pj} in advance in [32], while this paper can solve K_{pj} and H_{pj} at the same time, which removes the requirement for given H_{pj}.*

When $N = n$, sufficient conditions in Theorem 1 will reduce to the conditions where subsystems are all stabilizable.

Corollary 1. *For any $p, q \in \mathcal{N}_c$, $p \neq q$ and $i, j \in \mathcal{R}$, given a group of constants $0 < \lambda_p < 1$, $\mu_p > 1$, and vectors $c_p \in \mathbb{R}^m_+$. If there exist a set of vectors $\xi_p \in \mathbb{R}^n_+$, $g_{pij} \in \mathbb{R}^n$, $w_{pij} \in \mathbb{R}^n$, $z_{pij} \in \mathbb{R}$ such that*

$$c_p^T E_{sj}^T B_{pi}^T \xi_p A_{pi} + B_{pi} E_{sj} c_p g_{pij}^T + B_{pi} E_{sj}^- c_p w_{pij}^T \geq 0, \tag{3.32}$$

$$A_{pi}^T \xi_p + g_{pij} + w_{pij} z_{pij} - \lambda_p \xi_p \prec 0, \tag{3.33}$$

$$\xi_p - \mu_p \xi_q \prec 0, \tag{3.34}$$

and $\varepsilon(\xi_p, 1) \subset L(H_{pj})$, then system (3.4) is controlled positive and exponentially stable under the saturated fuzzy controller (3.2) as well as the switching signal satisfying (3.13), where H_{pj}, K_{pj} are the same as (3.15) and (3.16).

Note that it is impossible to solve H_{pj} and K_{pj} from the conditions of Theorem 1 due to the existence of (3.12). The conditions for directly solving H_{pj} and K_{pj} will be given in Theorem 2.

Theorem 2. *For $p \in \mathcal{N}_c$ and $q \in \mathcal{N}_{uc}$, given a group of constants $\mu_p > 1$, $0 < \lambda_p < 1$, $0 < \mu_q < 1$, $\lambda_q > 1$, and vectors $c_p \in \mathbb{R}^m_+$. If one can find a family of vectors $\xi_p \in \mathbb{R}^n_+$, $\xi_q \in \mathbb{R}^n_+$, $g_{pij} \in \mathbb{R}^n$, $w_{pij} \in \mathbb{R}^n$, $z_{pij} \in \mathbb{R}$ such that*

$$c_p^T E_{sj}^T B_{pi}^T \xi_p A_{pi} + B_{pi} E_{sj} c_p g_{pij}^T + B_{pi} E_{sj}^- c_p w_{pij}^T \geq 0, \tag{3.35}$$

$$p \in \mathcal{N}_c, s \in Q, i, j \in \mathcal{R}, \tag{3.36}$$

$$A_{qi} \geq 0, q \in \mathcal{N}_{uc}, i \in \mathcal{R}, \tag{3.37}$$

$$A_{pi}^T \xi_p + g_{pij} + w_{pij} z_{pij} - \lambda_p \xi_p \prec 0, p \in \mathcal{N}_c, i, j \in \mathcal{R}, \tag{3.38}$$

$$A_{qi}^T \xi_q - \lambda_q \xi_q \prec 0, q \in \mathcal{N}_{uc}, i \in \mathcal{R}, \tag{3.39}$$

$$\xi_p - \mu_p \xi_r \prec 0, \quad p, r \in \mathcal{N}_c, p \neq r, \tag{3.40}$$

$$\xi_p - \mu_p \xi_q \prec 0, \quad p \in \mathcal{N}_c, q \in \mathcal{N}_{uc}, \tag{3.41}$$

$$\xi_q - \mu_q \xi_p \prec 0, \quad p \in \mathcal{N}_c, q \in \mathcal{N}_{uc}, \tag{3.42}$$

$$\xi_p - \left| h_{pji}^T \right| \geq 0, \quad p \in \mathcal{N}_c, j \in \mathcal{R}, \tag{3.43}$$

then system (3.4) is controlled positive and exponentially stable under the saturated fuzzy controller (3.2) as well as the slow-fast combined MDADT satisfying (3.13) and (3.14).

Proof. To prove Theorem 2, we only need to prove that (3.12) $\Leftrightarrow$ (3.43).

1) (3.12) $\Rightarrow$ (3.43).

Suppose (3.43) does not hold. It means that

$$\xi_p - \left| h_{pji}^T \right| \prec 0. \tag{3.44}$$

Due to the positivity of $x(k)$, multiplying both sides of (3.44) by $x^T(k)$, we can obtain

$$x^T(k)\xi_p - x^T(k)\left|h^T_{pji}\right| < 0. \tag{3.45}$$

For any $x(k) \in L(H_{pj})$, one has $\left|h_{pji}x(k)\right| \leq 1$, namely, $\left|x^T(k)h^T_{pji}\right| \leq 1$. Since $x(k)$ is positive, we can get from (3.45) that

$$x^T(k)\xi_p < x^T(k)\left|h^T_{pji}\right| = \left|x^T(k)h^T_{pji}\right| \leq 1. \tag{3.46}$$

It is obtained from (3.46) that $x(k) \in \varepsilon(\xi_p, 1)$. Thus, $L(H_{pj}) \subset \varepsilon(v_p, 1)$, which is in contradiction with (3.12), Therefore, (3.12) $\Rightarrow$ (3.43).

2) (3.43) $\Rightarrow$ (3.12).

For any $x(k) \in \varepsilon(\xi_p, 1)$, one has

$$x^T(k)\xi_p \leq 1. \tag{3.47}$$

Multiplying both sides of (3.43) by $x^T(k)$ yields that

$$x^T(k)\xi_p - x^T(k)\left|h^T_{pji}\right| \geq 0. \tag{3.48}$$

Combining (3.47) and (3.48) gives that

$$1 \geq x^T(k)\xi_p \geq x^T(k)\left|h^T_{pji}\right| = \left|x^T(k)h^T_{pji}\right| = \left|h_{pji}x(k)\right|. \tag{3.49}$$

Thus, $x(k) \in L(H_{pj})$, which implies that $\varepsilon(\xi_p, 1) \subset L(H_{pj})$. Therefore, (3.43) $\Rightarrow$ (3.12). The proof is completed. □

4. Attraction domain estimation

In this section, we expect to seek a larger attraction domain to reduce the conservatism of the ADE in Theorem 1. The largest ADE is proposed in Theorem 3.

Theorem 3. *For $p \in \mathcal{N}_c$ and $q \in \mathcal{N}_{uc}$, given constants $\mu_p > 1$, $0 < \lambda_p < 1$, $0 < \mu_q < 1$, $\lambda_q > 1$, $\gamma > 0$ and vectors $\xi^*, c_p \in \mathbb{R}^m_+$. If there exist a family of vectors $\xi_p, \xi_q \in \mathbb{R}^n_+$, $g_{pij} \in \mathbb{R}^n$, $w_{pij} \in \mathbb{R}^n$, $z_{pij} \in \mathbb{R}$ such that the following optimization problem*

$$\begin{aligned} &\sup \ \ \rho \\ &\quad \text{s.t.} \ \ (a)\ \xi^* \geq \gamma\rho\xi_p, \\ &\qquad\quad (b) \text{ Inequalities (3.35) to (3.43)}, \end{aligned} \tag{4.1}$$

is solvable, then $\varepsilon(\xi_p, \gamma^{-1})$ is the largest ADE for system (3.4).

Proof. Given a shape reference set $X_\xi \subset \mathbb{R}^n$, which is a bounded convex set. For the set $S \subset \mathbb{R}^n$ and the parameter $\rho > 0$, define $\rho_\xi(S)$ as follows:

$$\rho_\xi(S) := \sup\{\rho > 0 : \rho X_\xi \subset S\}. \tag{4.2}$$

It is obvious that $\rho_\xi(S) \geq 1$, which implies $X_\xi \subset S$. When $\rho_\xi(S) < 1$, the definition of X_ξ is given as follows:

$$X_\xi = \{x(k) \in R^n | \, x^T(k)\xi^* \leq 1, \xi^* > 0\}. \tag{4.3}$$

In order to obtain the largest ADE, the maximum of ρ is expected to be found such that $\rho X_\xi \subset \varepsilon(\xi_p, \gamma^{-1})$. Thus, we can transform the ADE problem into the following optimization problem:

$$\begin{aligned} \sup \quad & \rho \\ \text{s.t.} \quad & (a') \; \rho X_\xi \subset \varepsilon(\xi_p, \gamma^{-1}), \\ & (b) \text{ Inequalities (3.35) to (3.43) hold.} \end{aligned}$$

However, the above optimization problem is unsolvable due to the existence of (a'). In what follows, it is proved that $(a') \Leftrightarrow (a)$.

Necessity($(a') \Leftarrow (a)$). Suppose $\xi^* \prec \gamma\rho\xi_p$, then

$$(\frac{1}{\rho}x(k))^T\xi^* < x^T(k)\gamma\xi_p. \tag{4.4}$$

For any $x(k) \in \varepsilon(\xi_p, \gamma^{-1})$, one can obtain that

$$x^T(k)\gamma\xi_p \leq 1. \tag{4.5}$$

Combining (4.4) and (4.5) gives that

$$(\frac{1}{\rho}x(k))^T\xi^* \leq 1, \tag{4.6}$$

which means that $\frac{1}{\rho}x(k) \in X_\xi$, namely, $x(k) \in \rho X_\xi$. It follows that $\varepsilon(\xi_p, \gamma^{-1}) \subset \rho X_\xi$. Since it is unconformity to (a), the necessity is proved.

Sufficiency ($(a) \Rightarrow (a')$). Multiplying both sides of (a) by $x^T(k)$, one has

$$(\frac{1}{\rho}x(k))^T\xi^* \geq x^T(k)\gamma\xi_p. \tag{4.7}$$

For any $x(k) \in \rho X_\xi$, it can be found that $\frac{1}{\rho}x(k) \in X_\xi$. From (4.3), one can get

$$(\frac{1}{\rho}x(k))^T\xi^* \leq 1. \tag{4.8}$$

Combining (4.7) and (4.8) yields that

$$x^T(k)\gamma\xi_p \leq 1. \tag{4.9}$$

It follows from (4.9) that $x(k) \in \varepsilon(\xi_p, \gamma^{-1})$, which means that (a) holds. The sufficiency is proved. Hence, (a') are equivalent to (a). The proof is completed. □

Remark 3. *By substituting* (3.32)–(3.34) *for* (3.35)–(3.42) *in the condition (b) of Theorem 3, it is obtained that the largest ADE of discrete-time PSTSFSs with actuator saturation when switched subsystems are all stabilizable.*

5. Illustrative examples

Two illustrative examples will be provided to verify the availability and advantages of the proposed control scheme.

Example 1. The positive switched nonlinear system with partially stabilizable subsystems

Considering the following positive switched nonlinear system with actuator saturation in discrete-time domain:

$$\Xi_1:\begin{cases} x_1(k+1)=1.5x_1(k)+0.3x_2(k)+0.3\sin^2(x_1(k))x_1(k)\\ \qquad\qquad\quad -0.1\sin^2(x_1(k))x_2(k),\\ x_2(k+1)=0.1x_1(k)+0.5x_2(k)+0.3\sin^2(x_1(k))x_1(k)\\ \qquad\qquad\quad +0.3\sin^2(x_1(k))x_2(k),\end{cases}$$

$$\Xi_2:\begin{cases} x_1(k+1)=0.8x_1(k)+0.5x_2(k)-0.1\sin^2(x_2(k))x_1(k)\\ \qquad\qquad\quad +0.4\sin^2(x_2(k))x_2(k)+\mathrm{sat}(u(k)),\\ x_2(k+1)=0.7x_1(k)+0.6x_2(k)-0.1\sin^2(x_2(k))x_1(k)\\ \qquad\qquad\quad -0.2\sin^2(x_2(k))x_2(k)+\mathrm{sat}(u(k)),\end{cases}$$

$$\Xi_3:\begin{cases} x_1(k+1)=1.6x_1(k)+0.3x_2(k)-0.4\sin^2(x_2(k))x_1(k)\\ \qquad\qquad\quad -0.2\sin^2(x_2(k))x_2(k),\\ x_2(k+1)=0.2x_1(k)+0.5x_2(k)+0.4\sin^2(x_2(k))x_1(k)\\ \qquad\qquad\quad -0.1\sin^2(x_2(k))x_2(k).\end{cases}$$

Define $z_1(k)=\sin^2(x_1(k))$, $z_2(k)=\sin^2(x_2(k))$, $z_3(k)=\sin^2(x_2(k))$, by sector nonlinearity [33], we construct the following T–S fuzzy model:

Rule R_p^i: IF $z_{p1}(k)$ is M_{p1}^i, THEN

$$x(k+1)=A_{pi}x(k)+B_{pi}\mathrm{sat}(u(k)),\ p\in\{1,2,3\},\ i\in\{1,2\},$$

where

$$A_{11}=\begin{bmatrix}1.5 & 0.3\\ 0.1 & 0.5\end{bmatrix},A_{12}=\begin{bmatrix}1.8 & 0.2\\ 0.4 & 0.8\end{bmatrix},$$
$$A_{21}=\begin{bmatrix}0.4 & 0.3\\ 0.2 & 0.1\end{bmatrix},A_{22}=\begin{bmatrix}0.2 & 0.1\\ 0.5 & 0.7\end{bmatrix},$$
$$A_{31}=\begin{bmatrix}1.6 & 0.3\\ 0.2 & 0.5\end{bmatrix},A_{32}=\begin{bmatrix}1.2 & 0.1\\ 0.6 & 0.4\end{bmatrix},$$
$$B_{11}=\begin{bmatrix}0\\ 0\end{bmatrix},B_{12}=\begin{bmatrix}0\\ 0\end{bmatrix},B_{21}=\begin{bmatrix}1\\ 1\end{bmatrix},$$
$$B_{22}=\begin{bmatrix}1\\ 1\end{bmatrix},B_{31}=\begin{bmatrix}0\\ 0\end{bmatrix},B_{32}=\begin{bmatrix}0\\ 0\end{bmatrix}.$$

The final discrete-time PSTSFS with actuator saturation is:

$$x(k+1)=\sum_{i=1}^{2}\theta_{pi}(z(k))[A_{pi}x(k)+B_{pi}\mathrm{sat}(u(k))],\ p\in\{1,2,3\},$$

with the following normalized membership functions:

$$\theta_{11}(z_1(k)) = \sin^2(x_1(k)), \theta_{12}(z_1(k)) = 1 - \sin^2(x_1(k)),$$
$$\theta_{21}(z_2(k)) = \sin^2(x_2(k)), \theta_{22}(z_2(k)) = 1 - \sin^2(x_2(k)),$$
$$\theta_{31}(z_3(k)) = \sin^2(x_2(k)), \theta_{32}(z_3(k)) = 1 - \sin^2(x_2(k)).$$

Let $A_p = \sum_{i=1}^{2} \theta_{pi}(z_p(k))A_{pi}, p \in \{1,2,3\}$, $B_p = \sum_{i=1}^{2} \theta_{pi}(z_p(k))B_{pi}$, $p \in \{1,2,3\}$, $K_p = \sum_{j=1}^{2} \theta_{pj}(z_p(k))K_{pj}, p \in \{2\}$.

For $\text{sat}(u(k)) = 0$, by Lemma 1, $A_{p1} \geq 0$, $A_{p2} \geq 0$, $A_{p3} \geq 0$ imply the positivity of subsystems Ξ_1, Ξ_2 and Ξ_3. By calculating the eigenvalues of matrices A_{p1}, A_{p2}, A_{p3}, we can get that subsystems Ξ_1, Ξ_2 and Ξ_3 are all unstable.

For $\text{sat}(u(k)) \neq 0$, given $\lambda_1 = 2$, $\lambda_2 = 0.8$, $\lambda_3 = 1.8$, $\mu_1 = 0.45$, $\mu_2 = 3$, $\mu_3 = 0.5$, $c_1 = c_2 = c_3 = 1$, $D_{11} = D_{12} = 1$, $D_{11}^- = D_{12}^- = 0$, $D_{21} = D_{22} = 0$, $D_{21}^- = D_{22}^- = 1$, $\xi^* = [3 \ 6]$. By Theorem 3, the designed switching signal satisfies $\kappa_{a1} < -\frac{\ln \mu_1}{\lambda_1} = 1.1520$, $\tau_{a2} > -\frac{\ln \mu_2}{\lambda_2} = 4.9233$, $\tau_{a3} < -\frac{\ln \mu_3}{\lambda_3} = 1.1792$, the feasible solutions can be obtained and the saturated controller gains are $K_{21} = [-0.4111 - 0.3618]$, $K_{22} = [-0.4111 - 0.3618]$.

We can check that $A_2 + B_2K_2 \geq 0$. By Lemma 1, one can get the positivity of closed-loop subsystem Ξ_2. Given the initial condition $x(k_0) = [1,1]^T$, Figure 1 depicts the state trajectories of closed-loop subsystem Ξ_2. From Figure 1, we can observe that the closed-loop subsystem Ξ_2 is stable.

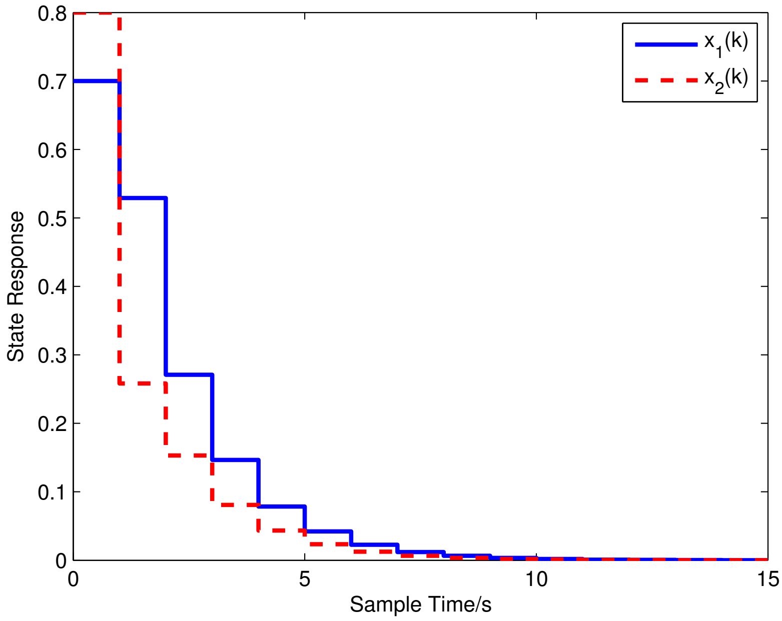

Figure 1. State trajectories of closed-loop subsystem Ξ_2.

According to (3.15) and (3.16), we design the switching signal as follows: $\kappa_{a1} = 1.1 < 1.1520$, $\kappa_{a2} = 5 > 4.9233$ and $\kappa_{a3} = 1.1 < 1.1792$. Hence, it is possible to generate a switching sequence $(1,2,3,2,1,3,2,1,2,3,2,1,2)$. Given $x(k_0) = [0.7, 0.8]^T$, the state trajectories of the system are depicted in Figure 2, which shows that the closed-loop system satisfies the positivity and stability. Figures 3 and 4 depict the saturated control input and the largest attraction domain, respectively.

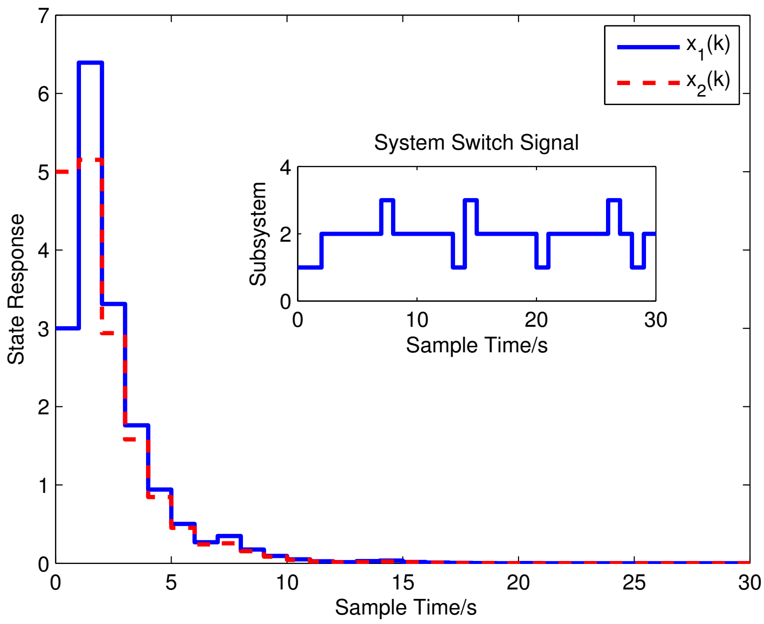

Figure 2. State trajectories of the closed-loop system and switching signal.

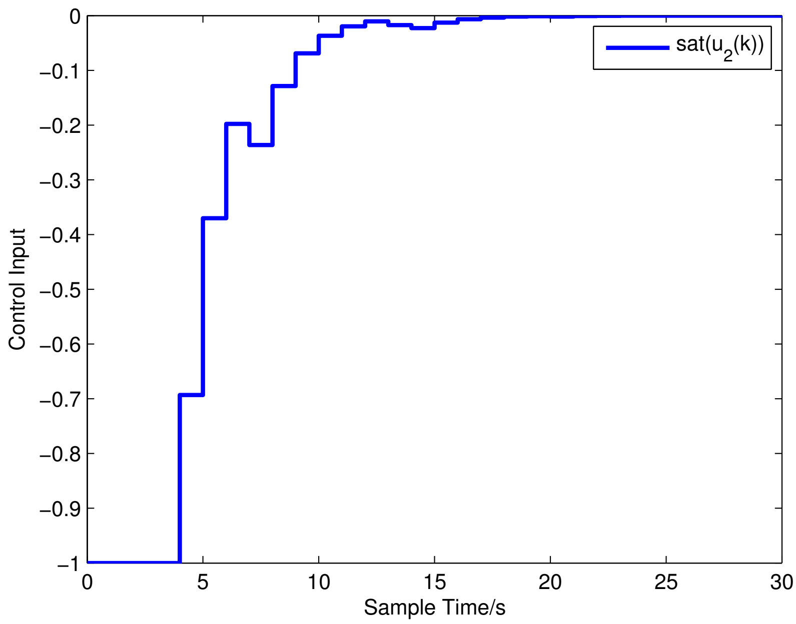

Figure 3. Saturated control input.

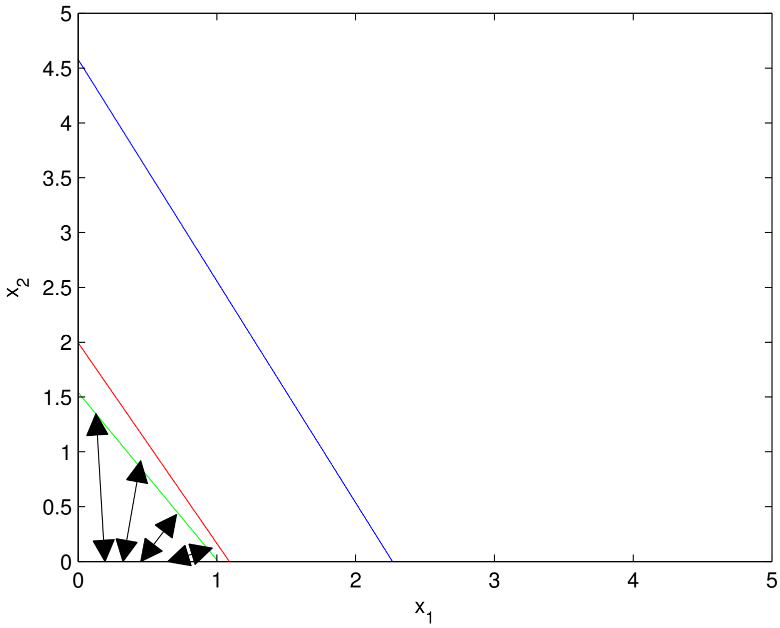

Figure 4. The largest ADE.

Example 2. The positive switched nonlinear system with all stabilizable subsystems

The water-quality model, borrowed from [34,35], can be modeled as the following positive switched nonlinear system:

$$\Xi_1:\begin{cases} x_1(k+1) = 1.2x_1(k) + 0.1x_2(k) - 0.1\sin^2(x_1(k))x_1(k) \\ \qquad\qquad\quad +\mathrm{sat}(u(k)), \\ x_2(k+1) = 0.1x_1(k) + 0.2x_2(k) + 0.3\sin^1(x_2(k))x_1(k) \\ \qquad\qquad\quad +0.1\sin^2(x_1(k))x_2(k) + \mathrm{sat}(u(k)), \end{cases}$$

$$\Xi_2:\begin{cases} x_1(k+1) = 0.8x_1(k) + 0.2x_2(k) + 0.7\sin^2(x_2(k))x_1(k) \\ \qquad\qquad\quad +\mathrm{sat}(u(k)), \\ x_2(k+1) = 0.3x_1(k) + 0.5x_2(k) + 0.3\sin^2(x_2(k))x_1(k) \\ \qquad\qquad\quad -0.4\sin^2(x_2(k))x_2(k) + \mathrm{sat}(u(k)). \end{cases}$$

Similar to Example 1, the final discrete-time PSTSFS with actuator saturation is as follows:

$$x(k+1) = \sum_{i=1}^{2} \theta_{pi}(z(k))[A_{pi}x(k) + B_{pi}\mathrm{sat}(u(k))],\, p \in \{1,2\},$$

where

$$A_{11} = \begin{bmatrix} 1.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, A_{12} = \begin{bmatrix} 1.1 & 0.1 \\ 0.4 & 0.3 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.5 \end{bmatrix}, A_{22} = \begin{bmatrix} 1.5 & 0.2 \\ 0.6 & 0.1 \end{bmatrix},$$

$$B_{11} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_{12} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_{21} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_{22} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$\theta_{11}(z_1(k)) = \sin^2(x_1(k)), \theta_{12}(z_1(k)) = 1 - \sin^2(x_1(k)),$$
$$\theta_{21}(z_2(k)) = \sin^2(x_2(k)), \theta_{22}(z_2(k)) = 1 - \sin^2(x_2(k)).$$

Let $A_p = \sum_{i=1}^{2} \theta_{pi}(z_p(k))A_{pi}$, $B_p = \sum_{i=1}^{2} \theta_{pi}(z_p(k))B_{pi}$, $K_p = \sum_{j=1}^{2} \theta_{pj}(z_p(k))K_{pj}$, where $p \in \{1, 2\}$.

When $\mathrm{sat}(u(k)) = 0$, according to Lemma 1, it follows from $A_{pi} \geq 0, p, j \in \{1, 2\}$ that subsystems Ξ_1 and Ξ_2 are both positive. It is obtained from the eigenvalues of matrices A_{pi}, $p, j \in \{1, 2\}$ that subsystems Ξ_1 and Ξ_2 are both unstable.

When $\mathrm{sat}(u(k)) \neq 0$, both subsystems Ξ_1 and Ξ_2 are checked to be stabilizable. Given $\lambda_1 = 0.5$, $\mu_1 = 3.1$, $\lambda_2 = 0.28$, $\mu_2 = 4$, $c_1 = c_2 = 1$, $D_{11} = D_{12} = 1$, $D^-_{11} = D^-_{12} = 0$, $D_{21} = D_{22} = 0$, $D^-_{21} = D^-_{22} = 1$, $v^* = [3 \ 6]$, according to Corollary 1, the corresponding feasible solutions can be obtained and the switching signals satisfy $\tau_{a1} \geq -\frac{\ln \mu_1}{\lambda_1} = 1.6323$, $\tau_{2a} \geq -\frac{\ln \mu_2}{\lambda_2} = 1.0890$, the saturated controller gains are $K_{11} = [-0.1278 \ 0.0192]$, $K_{12} = [-0.1278 \ 0.0192]$, $K_{21} = [-0.3217 \ -0.2029]$ and $K_{22} = [-0.3217 \ -0.2029]$.

We can check that $A_1 + B_1K_1 \geq 0$ and $A_2 + B_2K_2 \geq 0$, which ensure that the positivity of the saturated closed-loop subsystems Ξ_1 and Ξ_2. And Figures 5 and 6 depict the state trajectories of the system with the initial condition $x(t_0) = [0.7, 0.8]^T$. From Figures 5 and 6, we can observe that closed-loop subsystems Ξ_1 and Ξ_2 are both exponentially stable.

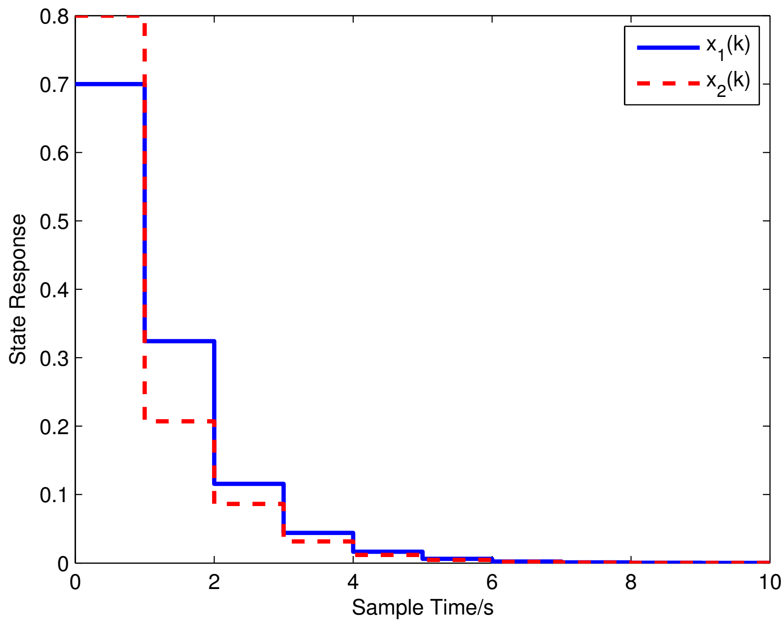

Figure 5. State trajectories of closed-loop subsystem Ξ_1.

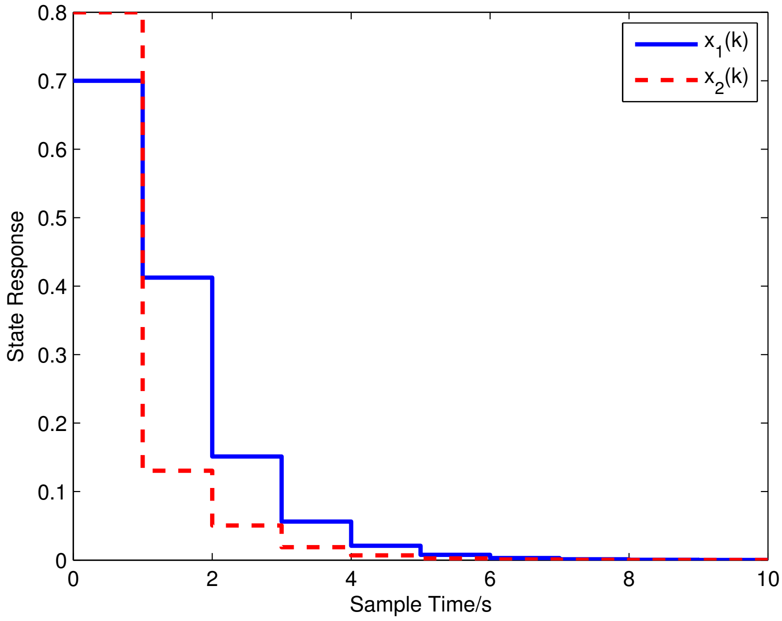

Figure 6. State trajectories of closed-loop subsystem Ξ_2.

By Corollary 1, the switching signals of the system are designed as $\tau_{a1} = 2 > 1.6323$ and $\tau_{a2} = 2 > 1.0890$. Based on the designed swiching signal, we can generate a possible switching sequence. Choose the initial condition $x(t_0) = [3\ 5]^T$, the system state trajectories, together with the switching signal are depicted in Figure 7. From Figure 7, it is observed that the closed-loop system satisfies both positivity and stability. Figure 8 depicts the saturated control input.

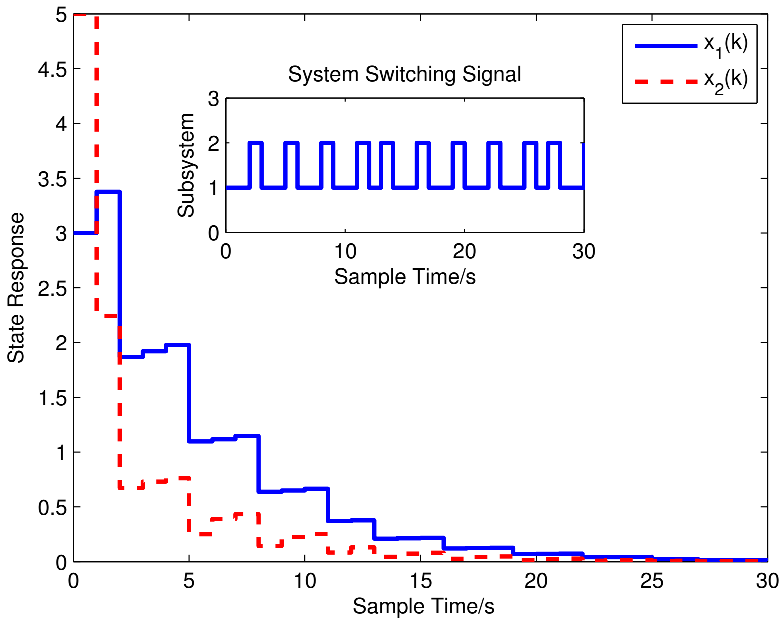

Figure 7. State trajectories of the closed-loop system.

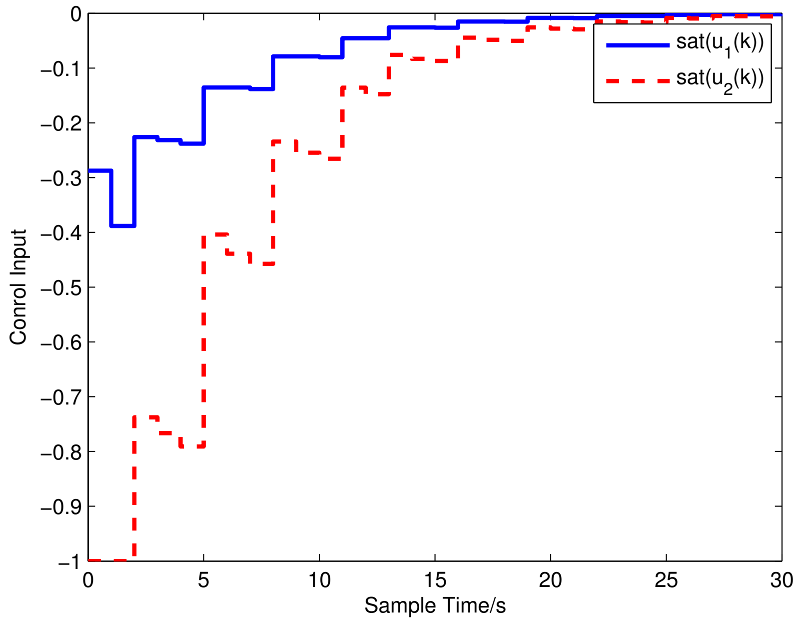

Figure 8. Saturated control input.

Moreover, under the same initial conditions, the comparison results of state trajectories under the controller designed for the system without actuator saturation in [17] and the controller proposed in this paper are shown in Figure 9. From Figure 9, it can be verified that the proposed controller in this paper makes the system converge faster than the one in [17]. Thus, for the PSTSFS in discrete-time domain, the designed control scheme is superior to the one in [17].

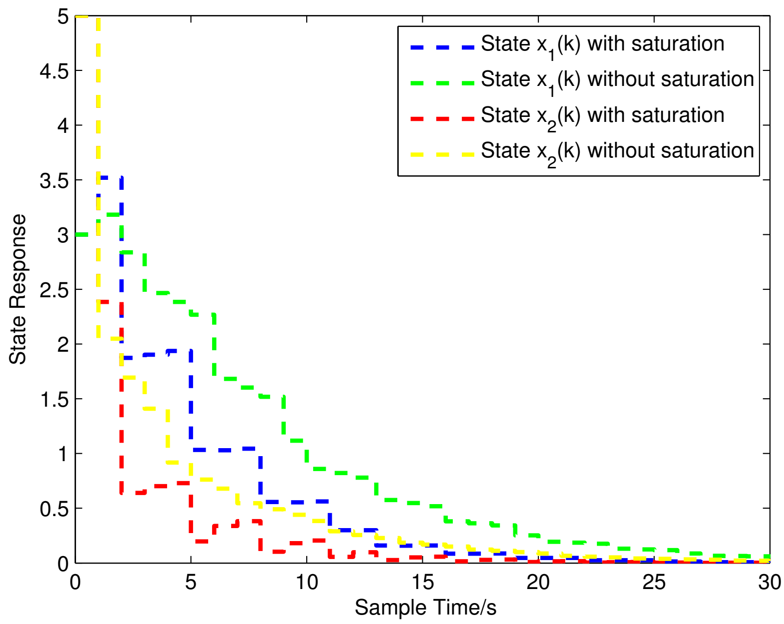

Figure 9. Comparison of state trajectories.

6. Conclusions

The stabilization of discrete-time PSTSFSs with partially stabilizable subsystems has been investigated in the presence of actuator saturation. By the CHT, a PDC-based saturated fuzzy controller is designed and sufficient conditions for the positivity and stability of the closed-loop PSTSFSs are developed by the discrete multiple linear co-positive Lyapunov function and slow-fast combined MDADT approach. The obtained conditions are also applicable to the PSTSFSs with all stabilizable switched subsystems. Furthermore, the largest ADE is presented by an optimization problem. Finally, two illustrative examples are provided to demonstrate the effectiveness and advantages of the proposed control scheme. In practice, due to factors such as measurement costs, the state is often unmeasurable. Thus, the observer-based saturated control of discrete-time PSTSFSs deserves further investigation.

Conflict of interest

The authors declare that there is no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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