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Research article

Stability analysis and design of cooperative control for linear delta operator system

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Abstract: This paper investigates the cooperative state feedback control problem for delta operator-based large-scale systems with independent subsystems. First, the state feedback controller is introduced to interconnect the adjacent subsystems into a closed-loop system. Second, the Lyapunov function in delta domain is constructed, and the linear matrix inequality method is used to design the cooperative state feedback stability controller for the whole large-scale interconnected system. Third, a performance index is introduced for the design of the optimal cooperative state feedback controller. Finally, stability of the closed-loop system is proved on the basis of stability theory, and simulation examples are given for showing the effectiveness of the design method.

Keywords: delta operator; interconnected systems; cooperative control; stability analysis
Mathematics Subject Classification: 93A15

1. Introduction

There exist complex large-scale interconnected systems in many control applications, such as networked control systems, power systems, economic systems and so on, they are usually connected by many subsystems and their model structures are complex and robust. Since the 21st century, the problem of large-scale interconnected system has attracted more and more attention from scholars [1–12], including adaptive identification [1], state estimation [2], asynchronous control [3], fault detection [4], longitudinal and laternal control [5], decentralized control [6–12], adaptive tracking control [13–16], and so on. Among them, how to improve the stability of large-scale complex systems has become an increasingly important and concerned topic in the research of dynamical systems.

In traditional stability analysis and control synthesis of large-scale interconnected systems, it needs very conservative requirement that each subsystems are stable. And some examples are given to verify that the subsystems are unstable to make the large-scale system reach a stable equilibrium state via interconnection control actions [17, 18]. In [17], the appropriate interconnection and cooperative feedback controllers are used to make the unstable subsystems into a stable large-scale system. In [18], the algorithms for the interconnected stability and cooperative stabilization of two unstable subsystems are given by using the technique of the bilinear matrix inequality. Up to now, research on interconnected and cooperative control has made many achievements [19–25]. A perturbed cooperative control method is proposed for interconnected systems exchanging data via the communication network in [21]. In [22,23], the design of event-triggered cooperative stability controller of multi-agent systems with interconnected dynamics are presented. In [24], a novel decentralized adaptive control algorithm considering discontinuity caused by state-triggering is shown for uncertain interconnected systems. And a class of cooperative control approach for the constrained interconnected nonlinear systems is proposed in [25].

In recent years, the advantages of the delta operator system in fast sampling make it widely used in industrial automation, power network, computer communication and other fields [26–31]. And some good sliding mode control (SMC) methods by utilizing its robust property have been proposed for delta operator system [32–34]. In [32], two kinds of dynamical SMC design methods are explored, and sufficient conditions for the asymptotic stability of delta operator systems are obtained. In [33], a SMC law for the adaptive control of Markovian jump systems under delta operator framework is proposed. A novel quantized SMC method in delta operator system is developed for dealing with the quantized state feedback problem in [34].

However, there are few researches on the cooperative control problem of delta operator system. In [35], for a delta operator based large-scale system composed of linear subsystems, a stable cooperative state feedback controller is given by constructing the Lyapunov function in the delta domain and using the linear matrix inequality (LMI) technique. However, the design method here can only be used in the case of two indpendent subsystems, which limits the application of the method in actual engineering. Motivated by above mentioned results, this paper mainly investigates the cooperative control design for the large-scale interconnected systems with N independent subsystems in delta operator framework.

The main contribution of this paper is summarized twofold. Sufficient conditions for the asymptotic stability of large-scale interconnected system via cooperative state feedback are derived in the delta operator framework. And the optimal cooperative control with performance index problem is further investigated. Both theoretical analysis and examples show that the designed methods have better performance.

2. Problem statement and preliminaries

The following description is given for N independent subsystems under the delta operator framework:

$$\delta(x_i(t_k)) = A_i x_i(t_k) + B_i u_i(t_k), i = 1, 2, \cdots, N \tag{2.1}$$

where $x_i(t_k) \in \mathrm{R}^{n_i}$, $u_i(t_k) \in \mathrm{R}^{m_i}$ represent the state variables and the control input variables of the subsystem i. The constant value matrices $A_i \in \mathrm{R}^{n_i \times n_i}$ and $B_i \in \mathrm{R}^{n_i \times m_i}$ are known to represent the system matrix and control input matrix of the subsystem i, respectively.

Definition 1. (Yang et al. [26]) Let $t_k = k\mathrm{T}$, delta operator $\delta(\cdot)$ is defined as follows:

$$\delta(v(t_k)) = \begin{cases} \frac{dv(t)}{dt}, & \mathrm{T} = 0 \\ \frac{v(t_k+\mathrm{T})-v(t_k)}{\mathrm{T}}, & \mathrm{T} \neq 0 \end{cases}$$

where $v(t_k)$ is the variable, T represents a high-speed sampling period.

Lemma 1. (Yang et al. [26], Zheng et al. [36]) For any function $x(t)$ and $y(t)$,

$$\delta(x(t)y(t)) = \delta(x(t))y(t) + x(t)\delta(y(t)) + \mathrm{T}\delta(x(t))\delta(y(t)).$$

The N independent subsystems (2.1) are interconnected, and the state feedback controllers are designed as shown in Eq (2.2):

$$\begin{cases} u_i(t_k) = K_i x_{i+1}(t_k),\ i = 1, 2, \cdots, N-1 \\ u_N(t_k) = K_N x_1(t_k) \end{cases} \tag{2.2}$$

where $K_i \in \mathrm{R}^{m_i \times n_{i+1}} (i = 1, 2, \cdots, N-1)$ and $K_N \in \mathrm{R}^{m_N \times n_1}$ are the state feedback gain matrices with appropriate dimensions.

The schematic of the interconnected system with cooperative state feedback controller (2.2) is shown in Figure 1.

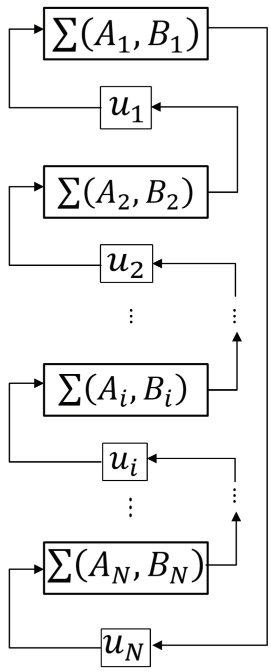

Figure 1. The schematic of the interconnected system with N independent subsystems.

One can see from the schematic of the system that there are information feedback from one subsystem to another, and thus it forms a whole large-scale interconnected system. In the existing literature, most of them consider that two subsystems cooperate as one interconnected system, such as [17–20]. Here, the cooperative control of large-scale system with N subsystems is considered.

By the combination of the subsystems (2.1) and the state feedback controllers (2.2), one can get the large-scale interconnected closed-loop system as follows:

$$\delta(x(t_k)) = Ax(t_k) \tag{2.3}$$

where

$$A = \begin{bmatrix} A_1 & B_1K_1 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ B_NK_N & 0 & \cdots & A_N \end{bmatrix}, \tag{2.4}$$

and

$$x(t_k) = [\ x_1^T(t_k), \quad x_2^T(t_k), \quad \cdots, \quad x_N^T(t_k)\ ]^T.$$

Definition 2. (Duan et al. [17]) The system (2.1) is said to be cooperatively stable if there exist cooperative controllers (2.2) such that the closed-loop system (2.3) is asymptotically stable. To optimize the performance of the designed controller, a performance index J is introduced:

$$\begin{aligned} J =& \mathrm{T}\sum_{k=0}^{\infty}[x^T(t_k)Qx(t_k) + u_1^T(t_k)C_1u_1(t_k) \\ &+ u_2^T(t_k)C_2u_2(t_k) + \cdots + u_N^T(t_k)C_Nu_N(t_k)] \\ =& \mathrm{T}\sum_{k=0}^{\infty}x^T(t_k)(Q + K^TCK)x(t_k) \end{aligned} \tag{2.5}$$

where $Q \in \mathrm{R}^{(n_1+n_2+\cdots+n_N)\times(n_1+n_2+\cdots+n_N)}$ represents the positive definite matrix to be designed, C_i $(i = 1, 2, \cdots, N)$ represents a positive definite known constant matrix, and $C = blockdiag\,[C_N, C_1, \cdots, C_{N-1}]$, $K = blockdiag\,[K_N, K_1, \cdots, K_{N-1}]$.

Remark 1. Performance index (2.5) is commonly used in control system analysis and design. For example, for discrete-time systems with state and input quantizations, the robust guaranteed performance control design is investigated in [37]. In [38], robust stability analysis of guaranteed performance control of impulsive switched systems is well studied. In addition, cooperative control of interconnected systems with performance requirements is presented in [17, 18, 20, 35].

Definition 3. (Duan et al. [17]) The systems (2.1) are said to be cooperatively stable and satisfy the performance J if there exist cooperative controllers (2.2) such that the closed-loop system (2.3) is asymptotically stable and satisfy the performance index (2.5).

It can be seen from the above statement that the design of cooperative feedback control stability method is to solve the asymptotic stability problem of interconnected systems (2.3). The design of cooperative feedback optimal control method is to solve the asymptotic stability of driving the state to zero problem of interconnected systems (2.3) that satisfy the performance index (2.5).

3. Main results

Theorem 1. For the interconnected closed-loop system (2.3), if there exist symmetric positive definite matrices X_{ii}, W_{ii}, X_{ij}, $W_{ij}, i \neq j$, and general matrices Y_{ij}, $i, j = 1, 2, \cdots, N$ satisfy the following conditions:

$$X = \begin{bmatrix} X_{11} & \dots & X_{1N} \\ \vdots & \ddots & \vdots \\ * & \dots & X_{NN} \end{bmatrix} > 0$$

$$W = \begin{bmatrix} W_{11} & \dots & W_{1N} \\ \vdots & \ddots & \vdots \\ * & \dots & W_{NN} \end{bmatrix} > 0$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ * & \Sigma_{22} & 0 \\ * & * & \Sigma_{33} \end{bmatrix} < 0 \tag{3.1}$$

where

$$\Sigma_{11} = \begin{bmatrix} -2W_{11} & \dots & -2W_{1N} \\ \vdots & \ddots & \vdots \\ * & \dots & -2W_{NN} \end{bmatrix}$$

$$\Sigma_{12} = \begin{bmatrix} A_1X_{11} + B_1Y_{21} & \cdots & A_1X_{1N} + B_1Y_{2N} \\ \vdots & \ddots & \vdots \\ A_NX_{1N}^T + B_NY_{11} & \cdots & A_NX_{NN} + B_NY_{1N} \end{bmatrix}$$

$$\Sigma_{13} = \begin{bmatrix} \mathrm{T}W_{11} & \dots & \mathrm{T}W_{1N} \\ \vdots & \ddots & \vdots \\ \mathrm{T}W_{N1} & \dots & \mathrm{T}W_{NN} \end{bmatrix}$$

$$\Sigma_{22} = \Sigma_{12} + \Sigma_{12}^T$$

$$\Sigma_{33} = \begin{bmatrix} -\mathrm{T}X_{11} & \dots & -\mathrm{T}X_{1N} \\ \vdots & \ddots & \vdots \\ * & \dots & -\mathrm{T}X_{NN} \end{bmatrix}$$

Then the linear system (2.1) is said to be cooperatively stable, and the gains of the cooperative controllers can be obtained by

$$\begin{cases} K_i = Y_{(i+1)(i+1)}X_{(i+1)(i+1)}^{-1}, \quad i = 1, 2, ..., N-1 \\ K_N = Y_{11}X_{11}^{-1} \end{cases} \tag{3.2}$$

Proof. Taking

$$V(x(t_k)) = x^T(t_k)Px(t_k) \tag{3.3}$$

According to Lemma 1 and the system (2.3), one can see that

$$\delta V(x(t_k)) = x^T(t_k)A^T Px(t_k) + x^T(t_k)PAx(t_k) + \mathrm{T}\delta^T(x(t_k))P\delta(x(t_k)) \tag{3.4}$$

Noting system Eq (2.3), we can get that $A(x(t_k)) - \delta(x(t_k)) = 0$. And for any positive definite matrix $\hat{W} > 0$, we have

$$0 = \delta^T(x(t_k))\hat{W}(A(x(t_k)) - \delta(x(t_k))) \tag{3.5}$$

Substituting (3.5) into (3.4), we can get

$$\delta V(x(t_k)) = \xi^T(t_k)\Sigma_1\xi(t_k) \tag{3.6}$$

where $\Sigma_1 = \begin{bmatrix} \mathrm{T}P - 2\hat{W} & \hat{W}A \\ * & A^TP + PA \end{bmatrix}$ and $\xi(t_k) = \begin{bmatrix} \delta^T(x(t_k)) & x^T(t_k) \end{bmatrix}^T$. Then the inequality $\delta V(x(t_k)) < 0$ holds if and only if

$$\Sigma_1 < 0 \tag{3.7}$$

By multiplying positive definite matrix *blockdiag* $\begin{bmatrix} \hat{W}^{-1} & P^{-1} \end{bmatrix}$ on both sides of (3.7), we get

$$\begin{bmatrix} \hat{W}^{-1}\mathrm{T}P\hat{W}^{-1} - 2\hat{W}^{-1} & AP^{-1} \\ * & P^{-1}A^T + AP^{-1} \end{bmatrix} < 0 \tag{3.8}$$

Applying Schur Complement Lemma [39], one can get that

$$\begin{bmatrix} -2\hat{W}^{-1} & AP^{-1} & \mathrm{T}\hat{W}^{-1} \\ * & P^{-1}A^T + AP^{-1} & 0 \\ * & * & -\mathrm{T}P^{-1} \end{bmatrix} < 0$$

Let $P^{-1} = X$ and $W^{-1} = \hat{W}$, we have

$$\Sigma = \begin{bmatrix} -2W & AX & \mathrm{T}W \\ * & \mathrm{He}(AX) & 0 \\ * & * & -\mathrm{T}X \end{bmatrix} < 0 \tag{3.9}$$

where the notation He(X) represents the sum of X and its transpose. Combining (3.2) and (3.9), one can see that

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ * & \Sigma_{22} & 0 \\ * & * & \Sigma_{33} \end{bmatrix} < 0$$

So $\Sigma_1 < 0$.

It follows from Definition 1 that

$$\begin{aligned} \delta V(x(t_k)) &= \frac{V(x(t_k + \mathrm{T})) - V(x(t_k))}{\mathrm{T}} \\ &= \frac{1}{\mathrm{T}}\left[x^T(t_k + \mathrm{T})Px(t_k + \mathrm{T}) - x^T(t_k)Px(t_k)\right] \\ &< 0 \end{aligned}$$

Then

$$x^T(t_k + \mathrm{T})Px(t_k + \mathrm{T}) - x^T(t_k)Px(t_k) < 0 \tag{3.10}$$

if $x(t_k) \neq 0$. Since P is positive definite, then $x(t_k) \to 0$ when $k \to \infty$.

Therefore, the closed-loop system (2.3) is asymptotically stable and the system (2.1) is cooperatively stable if the LMI $\Sigma < 0$ holds. This completes Theorem 1. □

Theorem 2. For the large-scale interconnected closed-loop system (2.3), if there exist symmetric positive definite matrices X_{ii}, W_{ii}, Z_{ii}, and matrices $X_{ij}(i \neq j)$, W_{ij}, $Z_{ij}(i \neq j)$, $i, j = 1,2,\cdots,N$ such that

$$X = \begin{bmatrix} X_{11} & \dots & X_{1N} \\ \vdots & \ddots & \vdots \\ * & \dots & X_{NN} \end{bmatrix} > 0$$

$$W = \begin{bmatrix} W_{11} & \dots & W_{1N} \\ \vdots & \ddots & \vdots \\ * & \dots & W_{NN} \end{bmatrix} > 0$$

$$Z = \begin{bmatrix} Z_{11} & \dots & Z_{1N} \\ \vdots & \ddots & \vdots \\ * & \dots & Z_{NN} \end{bmatrix} > 0$$

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & 0 & 0 \\ * & \Pi_{22} & 0 & \Pi_{24} & \Pi_{25} \\ * & * & \Pi_{33} & 0 & 0 \\ * & * & * & \Pi_{44} & 0 \\ * & * & * & * & \Pi_{55} \end{bmatrix} < 0 \tag{3.11}$$

where

$$\Pi_{11} = \begin{bmatrix} -2W_{11} & \dots & -2W_{1N} \\ \vdots & \ddots & \vdots \\ * & \dots & -2W_{NN} \end{bmatrix},$$

$$\Pi_{13} = \begin{bmatrix} \mathrm{T}W_{11} & \dots & \mathrm{T}W_{1N} \\ \vdots & \ddots & \vdots \\ \mathrm{T}W_{1N}^{\mathrm{T}} & \dots & \mathrm{T}W_{NN} \end{bmatrix},$$

$$\Pi_{12} = \begin{bmatrix} A_1X_{11} + B_1Y_{21} & \cdots & A_1X_{1N} + B_1Y_{2N} \\ \vdots & \ddots & \vdots \\ A_NX_{1N}^T + B_NY_{11} & \cdots & A_NX_{NN} + B_NY_{1N} \end{bmatrix},$$

$$\Pi_{22} = \Pi_{12} + \Pi_{12}^T,$$

$$\Pi_{33} = \begin{bmatrix} -\mathrm{T}X_{11} & \dots & -\mathrm{T}X_{1N} \\ \vdots & \ddots & \vdots \\ * & \dots & -\mathrm{T}X_{NN} \end{bmatrix},$$

$$\Pi_{24} = \begin{bmatrix} X_{11}^T & \dots & X_{1N}^T \\ \vdots & \ddots & \vdots \\ X_{N1}^T & \dots & X_{NN}^T \end{bmatrix},$$

$$\Pi_{25} = \begin{bmatrix} Y_{11}^T & \cdots & Y_{N1}^T \\ \vdots & \ddots & \vdots \\ Y_{1N}^T & \cdots & Y_{NN}^T \end{bmatrix},$$

$$\Pi_{44} = \begin{bmatrix} -Z_{11} & \dots & -Z_{1N} \\ \vdots & \ddots & \vdots \\ * & \dots & -Z_{NN} \end{bmatrix},$$

$$\Pi_{55} = \begin{bmatrix} -C_N^{-1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ * & \cdots & -C_{N-1}^{-1} \end{bmatrix}.$$

Then the linear system (2.1) is said to be cooperatively stable, and the cooperative state feedback controllers are taken as

$$\begin{cases} u_i(t_k) = Y_{(i+1)(i+1)} X_{(i+1)(i+1)}^{-1} x_{i+1}(t_k), i = 1, \cdots, N-1 \\ u_N(t_k) = Y_{11} X_{11}^{-1} x_1(t_k) \end{cases} \tag{3.12}$$

and the performance index (2.5) satisfies

$$J \leq x^T(t_0) P x(t_0) \tag{3.13}$$

Proof. We also take

$$V(x(t_k)) = x^T(t_k) P x(t_k)$$

And suppose that the matrix inequality

$$\delta V(x(t_k)) + x^T(t_k)(Q + K^T C K) x(t_k) < 0 \tag{3.14}$$

holds.

Substituting (3.14) into (3.5), one can obtain that

$$\begin{aligned} &\delta V(x(t_k)) + x^T(t_k)(Q + K^T C K) x(t_k) \\ =& x^T(t_k)(\mathrm{He}(A^T P) + Q + K^T C K) x(t_k) \\ &+ \delta^T(x(t_k))(\mathrm{T}P - 2\hat{W})\delta(x(t_k)) + \delta^T(x(t_k))\hat{W} A x(t_k) + x^T(t_k) A^T \hat{W} \delta(x(t_k)) \\ =& \xi^T(t_k) \Pi_1 \xi(t_k) < 0 \end{aligned} \tag{3.15}$$

where $\mathrm{He}(A^T P) = A^T P + PA$, $\xi(t_k) = \begin{bmatrix} \delta^T(x(t_k)) & x^T(t_k) \end{bmatrix}^T$ and $\Pi_1 = \begin{bmatrix} \mathrm{T}P - 2\hat{W} & \hat{W} A \\ * & A^T P + PA + Q + K^T C K \end{bmatrix}$.

Then the inequality (3.15) holds if and only if

$$\Pi_1 < 0 \tag{3.16}$$

Multiplying the positive definite matrix *blockdiag* $\begin{bmatrix} \hat{W}^{-1} & P^{-1} \end{bmatrix}$ on both sides of (3.16), we get

$$\begin{bmatrix} \hat{W}^{-1} \mathrm{T} P \hat{W}^{-1} - 2\hat{W}^{-1} & A P^{-1} \\ * & \Omega \end{bmatrix} < 0 \tag{3.17}$$

where $\Omega = \mathrm{He}(AP^{-1}) + P^{-1} Q P^{-1} + P^{-1} K^T C K P^{-1}$.

Applying Schur Complement Lemma, and let $\hat{W}^{-1} = W$, $P^{-1} = X$, we have

$$\Pi = \begin{bmatrix} -2W & AX & \mathrm{T}W & 0 & 0 \\ * & \mathrm{He}\,(AX) & 0 & X^T & Y^T \\ * & * & -\mathrm{T}X & 0 & 0 \\ * & * & * & -Q^{-1} & 0 \\ * & * & * & * & -C^{-1} \end{bmatrix} < 0 \tag{3.18}$$

It follows from (3.2) and (3.9) that

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & 0 & 0 \\ * & \Pi_{22} & 0 & \Pi_{24} & \Pi_{25} \\ * & * & \Pi_{33} & 0 & 0 \\ * & * & * & \Pi_{44} & 0 \\ * & * & * & * & \Pi_{55} \end{bmatrix} < 0 \tag{3.19}$$

Then the inequality $\Pi < 0$ holds if and only if the inequality $\Pi_1 < 0$ holds.

According to Definition 1 and formula (3.14), one can obtain that

$$\mathrm{T} \sum_{k=0}^{\infty} x^T(t_k) \left(Q + K^T C K \right) x(t_k) \leq V\left(x(t_0)\right)$$

Further

$$J \leq x^T(t_0) P x(t_0)$$

This completes Theorem 2. □

The above two theorems design methods of the cooperative state feedback control of large-scale interconnected system (2.1), where Theorem 1 gives the feasible solution of the cooperative controller design, and Theorem 2 presents the cooperative controller design with performance index (2.5). The upper bound condition of the optimization performance index J for large-scale interconnected close-loop system (2.1) is given in Theorem 3.

Theorem 3. For delta operator based linear system (2.1), if the following LMI-based optimization problems

$$\min \quad \gamma > 0$$

$$s.t.\ (\mathrm{i}) X = \begin{bmatrix} X_{11} & \dots & X_{1N} \\ \vdots & \ddots & \vdots \\ * & \dots & X_{NN} \end{bmatrix} > 0$$

$$(\mathrm{ii})\, W = \begin{bmatrix} W_{11} & \dots & W_{1N} \\ \vdots & \ddots & \vdots \\ * & \dots & W_{NN} \end{bmatrix} > 0$$

$$(\mathrm{iii})\, Z = \begin{bmatrix} Z_{11} & \dots & Z_{1N} \\ \vdots & \ddots & \vdots \\ * & \dots & Z_{NN} \end{bmatrix} > 0$$

$$\text{(iv)}\ \Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & 0 & 0 \\ * & \Pi_{22} & 0 & \Pi_{24} & \Pi_{25} \\ * & * & \Pi_{33} & 0 & 0 \\ * & * & * & \Pi_{44} & 0 \\ * & * & * & * & \Pi_{55} \end{bmatrix} < 0$$

$$\text{(v)} \begin{bmatrix} -\gamma & x_1^T(0) & \cdots & x_N^T(0) \\ * & -X_{11} & \cdots & -X_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & -X_{NN} \end{bmatrix} < 0$$

can be solvable, then the controller defined as (3.12) is the cooperative state feedback guaranteed cost controller with the performance $J \le x^T(t_0)Px(t_0) < \gamma$.

Proof. Let $J \le x^T(t_0)Px(t_0) < \gamma$. The matrix $-X < 0$ can be known from condition (i). Using Schur complement Lemma 2 for the condition (v), we can get

$$-\gamma - x^T(t_0)(-X)^{-1}x(t_0) < 0$$

Since $P^{-1} = X$, we have

$$-\gamma + x^T(t_0)Px(t_0) < 0$$

Then

$$J \le x^T(t_0)Px(t_0) < \gamma$$

This completes Theorem 3. □

4. Numerical simulation

To illustrate the effectiveness and superiority of the design method in this paper, the comparions and discussions with the existing method and open loop system are presented in this section.

Example 1. Three independent subsystems described by delta operator framework are considered. Choosing the sampling period $\mathrm{T} = 0.05$, the initial state value $x_1(0) = x_2(0) = x_3(0) = \begin{bmatrix} 0.5 & -0.5 \end{bmatrix}^T$. Other parameters are presented as follows:

$$A_1 = \begin{bmatrix} -0.0055 & 0.005 \\ 0.0025 & -0.005 \end{bmatrix}, A_2 = \begin{bmatrix} -0.05 & 0 \\ -0.05 & -0.05 \end{bmatrix}, A_3 = \begin{bmatrix} -0.005 & 0.015 \\ 0 & -0.005 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.01 & 0.05 \\ 0.01 & 0.07 \end{bmatrix}, B_2 = \begin{bmatrix} 0.01 & 0.05 \\ 0.03 & 0.01 \end{bmatrix}, B_3 = \begin{bmatrix} 0.01 & 0.07 \\ 0.07 & 0.01 \end{bmatrix}.$$

Applying the existing cooperative control design method in [35], one can find that both the feasible solution algorithm in Theorem 1 and the optimal solution algorithm in Theorem 2 there have no solution.

Applying the proposed cooperative control design method in this paper, one can get a set of feasible solutions K_1, K_2, K_3 according to Theorem 1.

$$K_1 = \begin{bmatrix} 16.3636 & -2.1197 \\ -2.1591 & 1.5437 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 3.2301 & -0.5071 \\ 0.2166 & 2.2275 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 1.0064 & 0.4912 \\ 0.2741 & 1.2461 \end{bmatrix}.$$

Further, one can get a set of optimal solutions K_1, K_2, K_3 via the design method in Theorem 3.

$$K_1 = \begin{bmatrix} -0.0678 & -1.3225 \\ 3.5019 & 3.3100 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.8520 & -0.4758 \\ 0.0734 & 0.8885 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 0.5000 & -0.0906 \\ 0.0906 & 0.5000 \end{bmatrix}.$$

At the same time, the optimized performance index $J = 5.0068$ is obtained.

The simulation results are presented in Figures 2-10 (Performed on Matlab R2021b under LENOVO Yoga Pro14c, intel i7, Windows 10).

The open loop state response curves of the three subsystems are shown in Figures 2-4. It can be easily observed that none of the three subsystems is stable with good convergence performance.

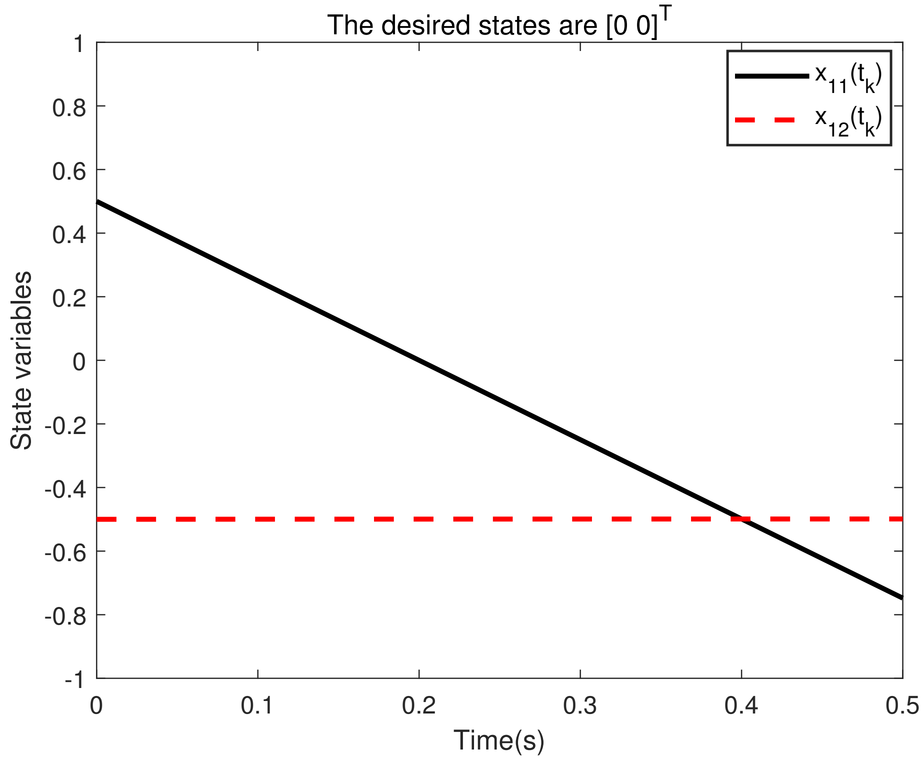

Figure 2. State response curves of subsystem I under open loop mode.

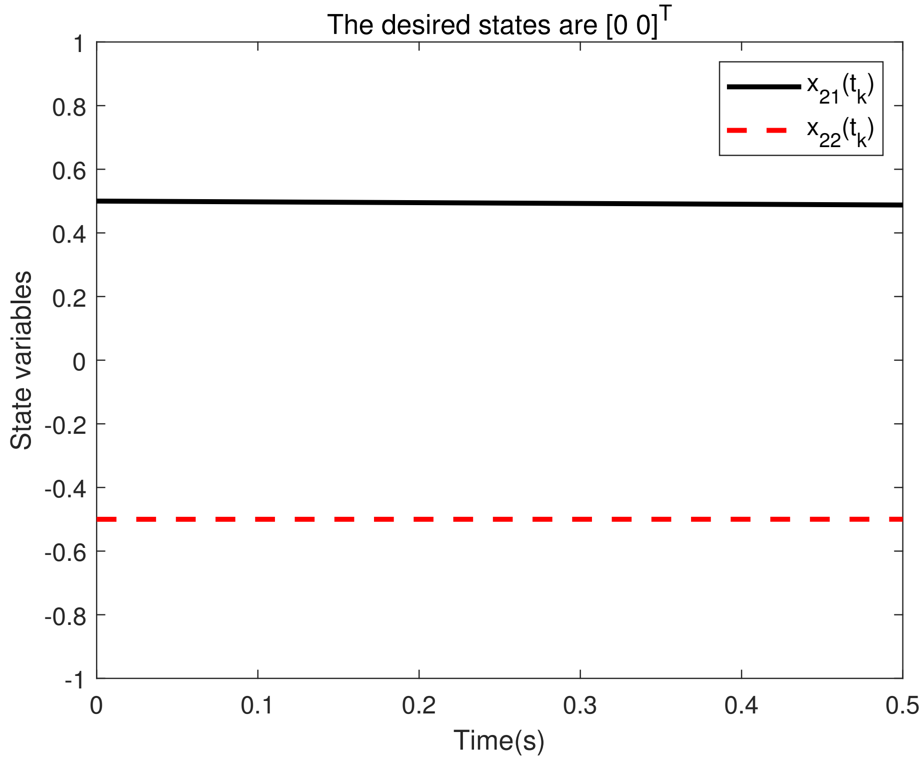

Figure 3. State response curves of subsystem II under open loop mode.

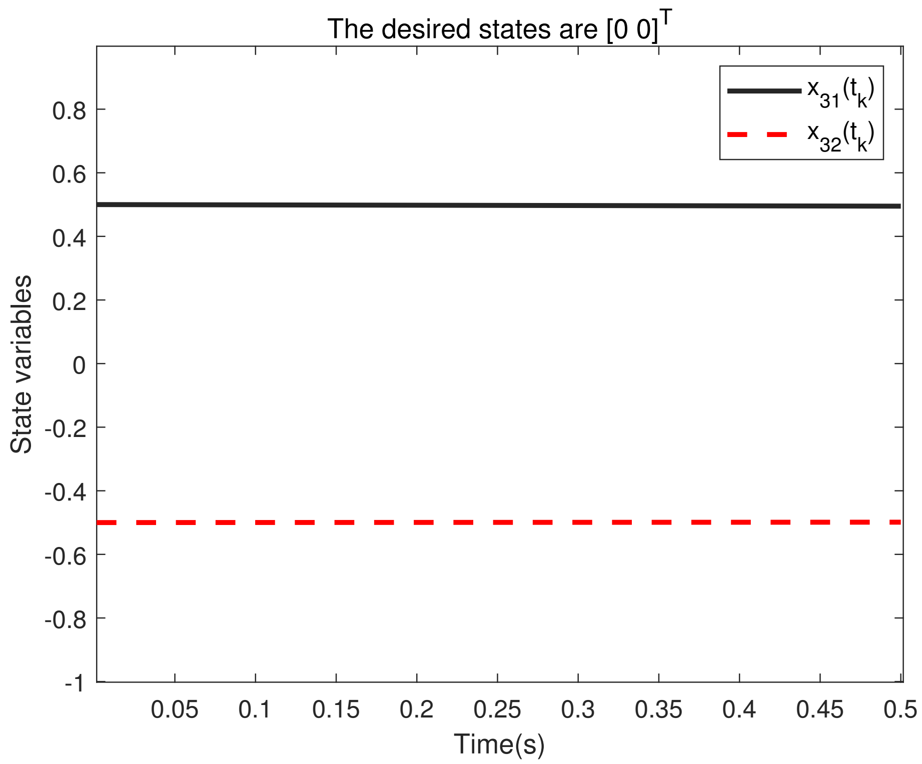

Figure 4. State response curves of subsystem III under open loop mode.

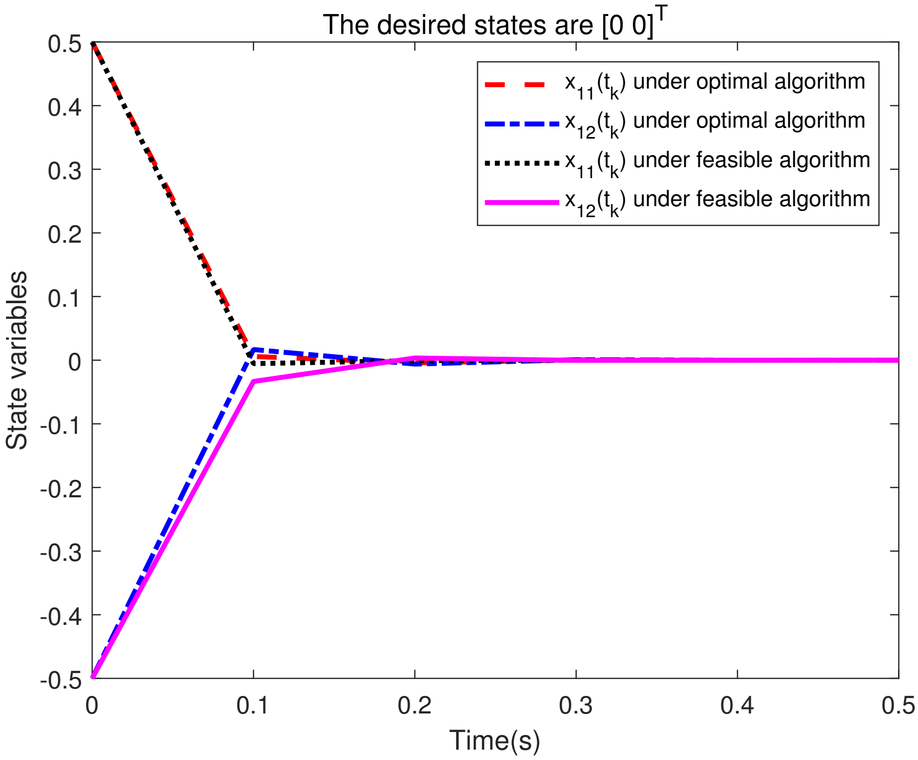

Figure 5. State response curves of subsystem I.

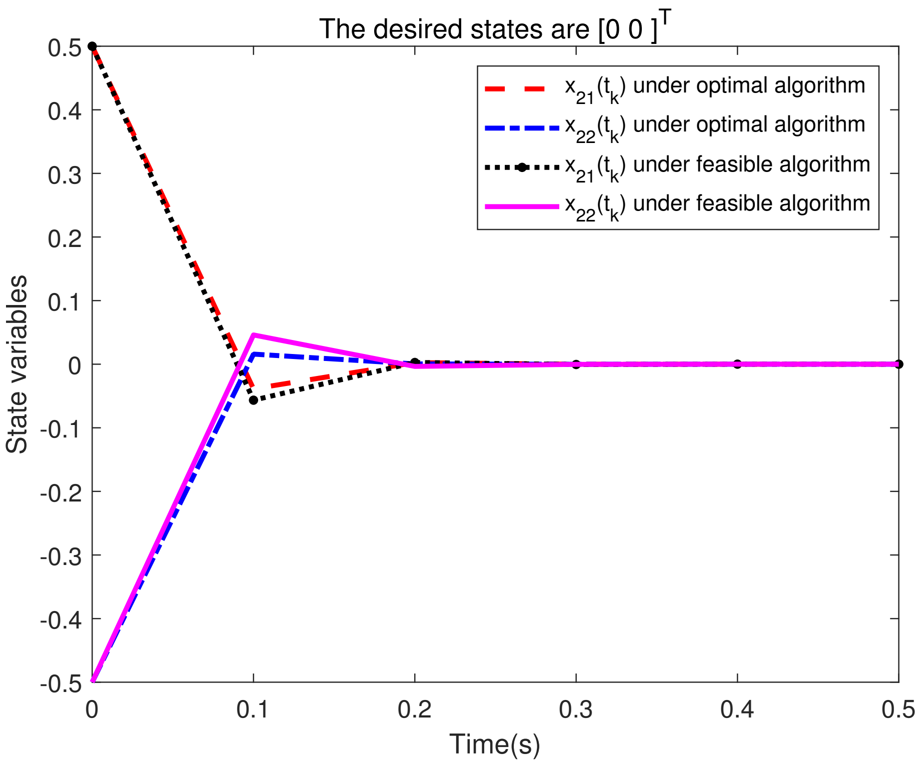

Figure 6. State response curves of subsystem II.

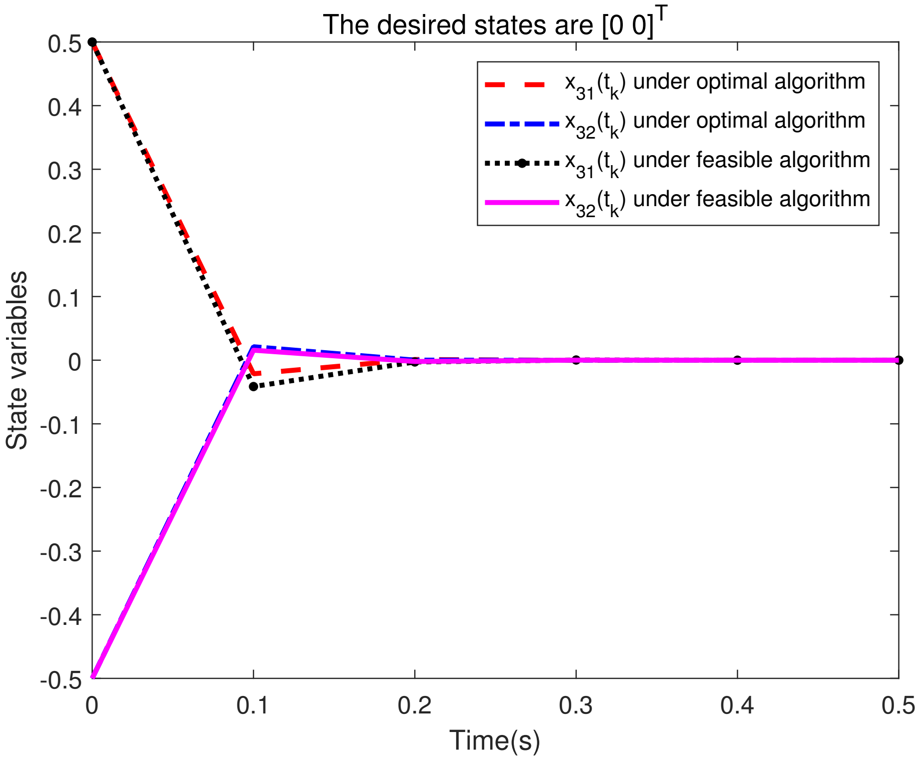

Figure 7. State response curves of subsystem III.

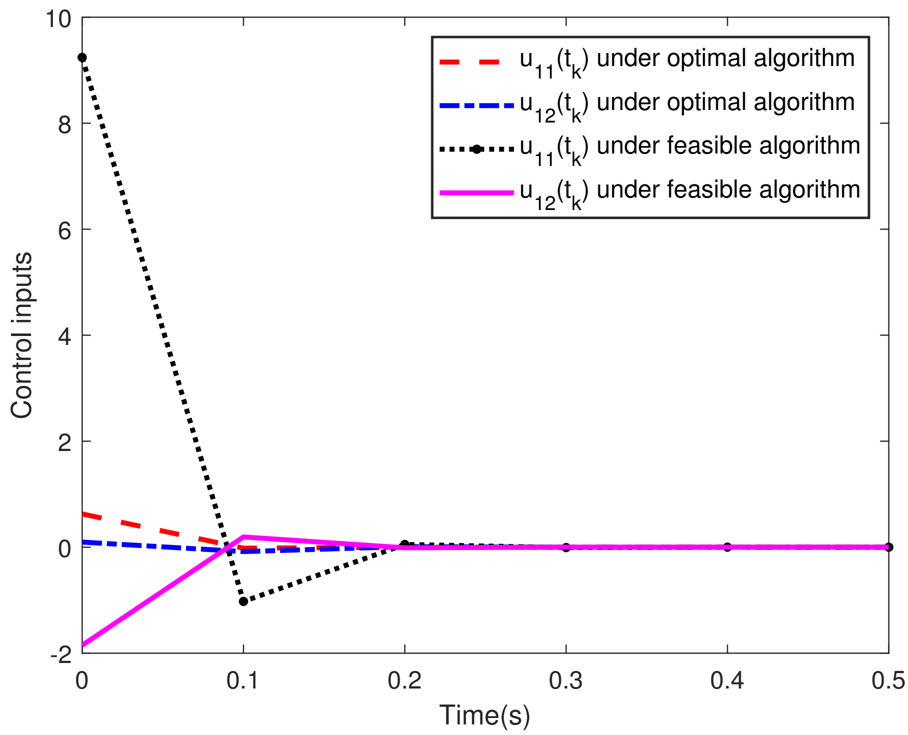

Figure 8. Control input response curves of subsystem I.

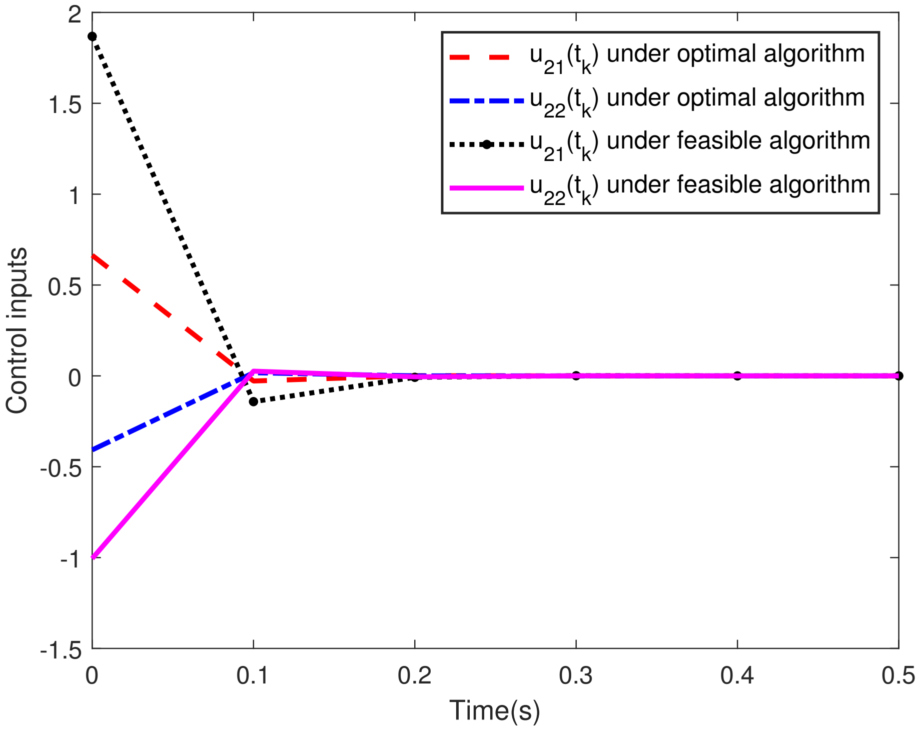

Figure 9. Control input response curves of subsystem II.

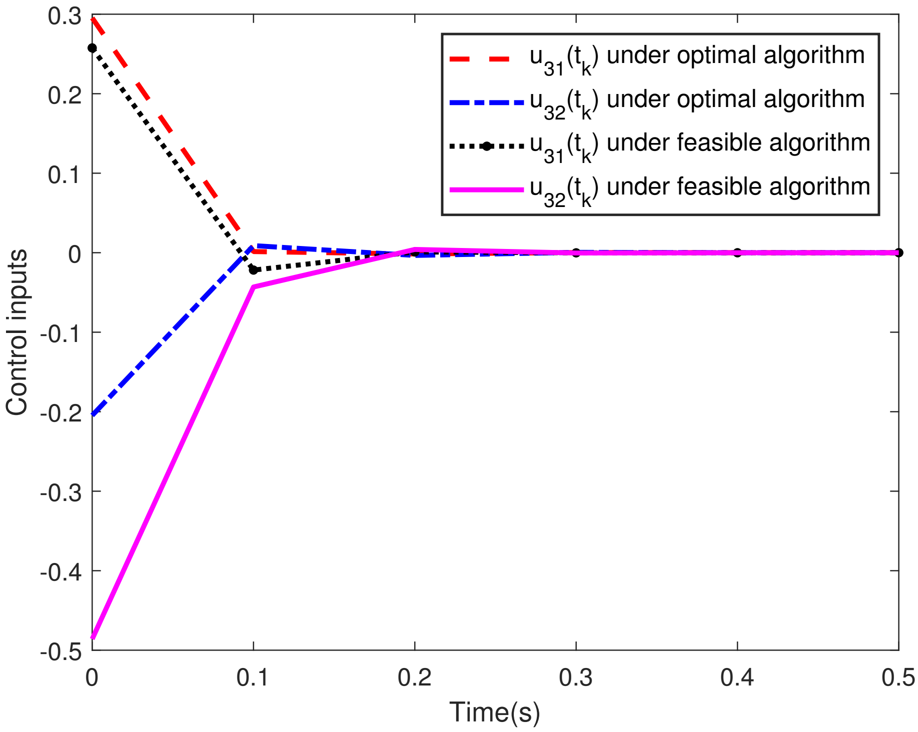

Figure 10. Control input response curves of subsystem III.

Figures 5-10 show the state response and control input simulation results of the three subsystems under the proposed cooperative control algorithms. It can be seen from the state response curves 5-7 that both the proposed feasible solution and optimal solution methods can guarantee the interconnected closed-loop systems are stable. Among them, all the three subsystems can converge to the desired origin in 0.2 seconds using the proposed feasible solution method. While applying the proposed optimal solution method, subsystem 1 can converge to zero in 0.1 seconds and subsystem 2 and subsystem 3 can converge to zero in less than 0.2 seconds, respectively. Furthermore, combined with the control input response curves 8-10, it can be seen under the proposed optimal solution algorithm that the large-scale interconnected systems can achieve faster convergence speed with lower control input cost, i.e., lower control input amplitude.

Example 2. Another three independent subsystems described by delta operator framework are considered here. Choosing the sampling period $\mathrm{T} = 0.001$, the initial state values $x_1(0) = \begin{bmatrix} 0.2 & -0.1 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}^T$, $x_3(0) = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix}^T$. Other parameters are presented as follows:

$$A_1 = \begin{bmatrix} -0.0165 & 0.015 \\ 0.0075 & -0.015 \end{bmatrix}, A_2 = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}, A_3 = \begin{bmatrix} -0.005 & 0.015 \\ 0 & -0.005 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.01 & 0.03 \\ 0.01 & 0.05 \end{bmatrix}, B_2 = \begin{bmatrix} 0.02 & 0.05 \\ 0.03 & 0.06 \end{bmatrix}, B_3 = \begin{bmatrix} 0.02 & 0.07 \\ 0.04 & 0.01 \end{bmatrix}.$$

One can also find that both the feasible solution algorithm and the optimal solution algorithm in [35] have no solution. Now applying the proposed cooperative control design method in this paper, one can get a set of feasible solutions K_1, K_2, K_3 according to Theorem 1.

$$K_1 = \begin{bmatrix} 20.1585 & -3.6572 \\ -3.2223 & 3.2373 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 24.4035 & -7.2248 \\ -9.7245 & 5.4192 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 1.6566 & 0.1375 \\ 0.1138 & 1.6238 \end{bmatrix}.$$

Further, one can get a set of optimal solutions K_1, K_2, K_3 via the design method in Theorem 3.

$$K_1 = \begin{bmatrix} 2.5833 & 0.2654 \\ 0.4578 & 1.6416 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 1.7718 & 0.5237 \\ -0.5631 & 1.3065 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 0.6799 & -0.2418 \\ 0.2028 & 0.5021 \end{bmatrix}.$$

At the same time, the optimized performance index $J = 5$ is obtained.

The simulation results are presented in Figures 11-16. Figures 11-16 show the state response and control input simulation results of the three subsystems under the proposed cooperative control algorithms. Similar to that in example 1, it also shows the effectiveness and advantage of the proposed method in this paper.

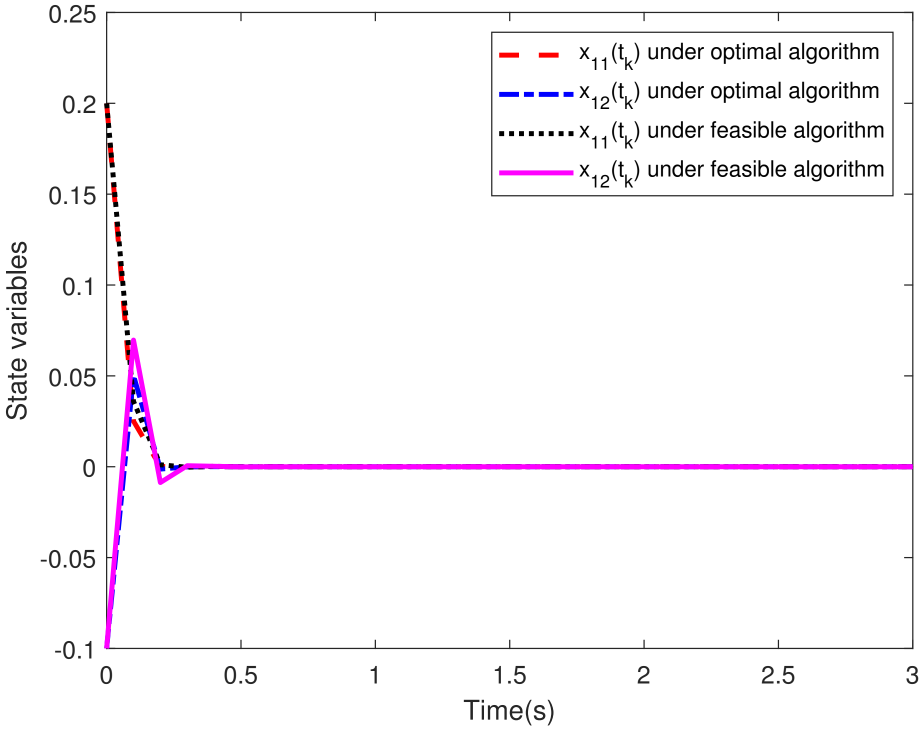

Figure 11. State response curves of subsystem I.

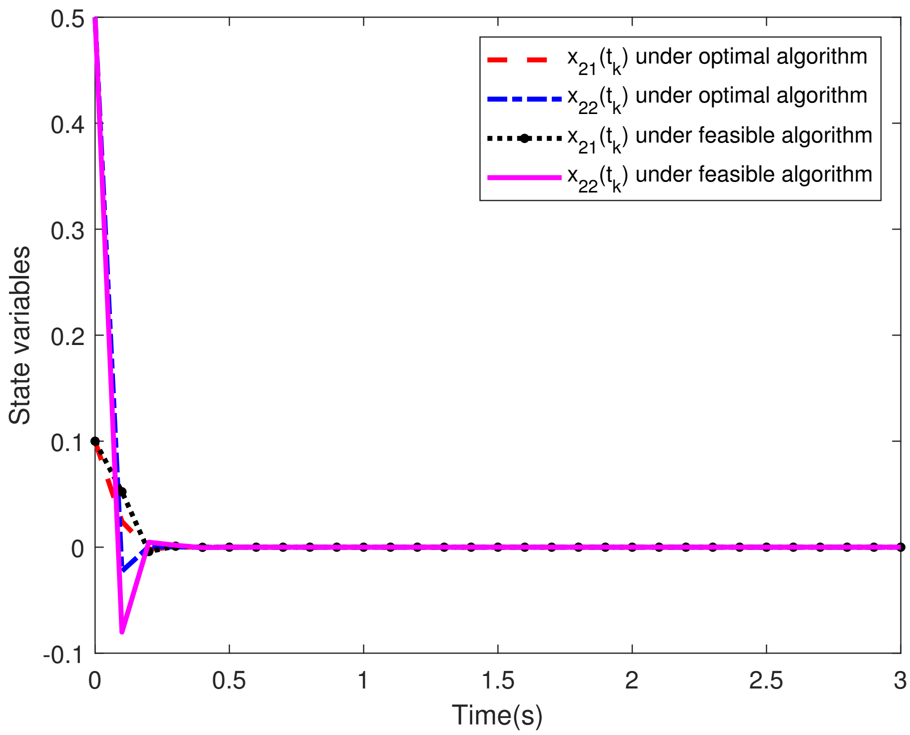

Figure 12. State response curves of subsystem II.

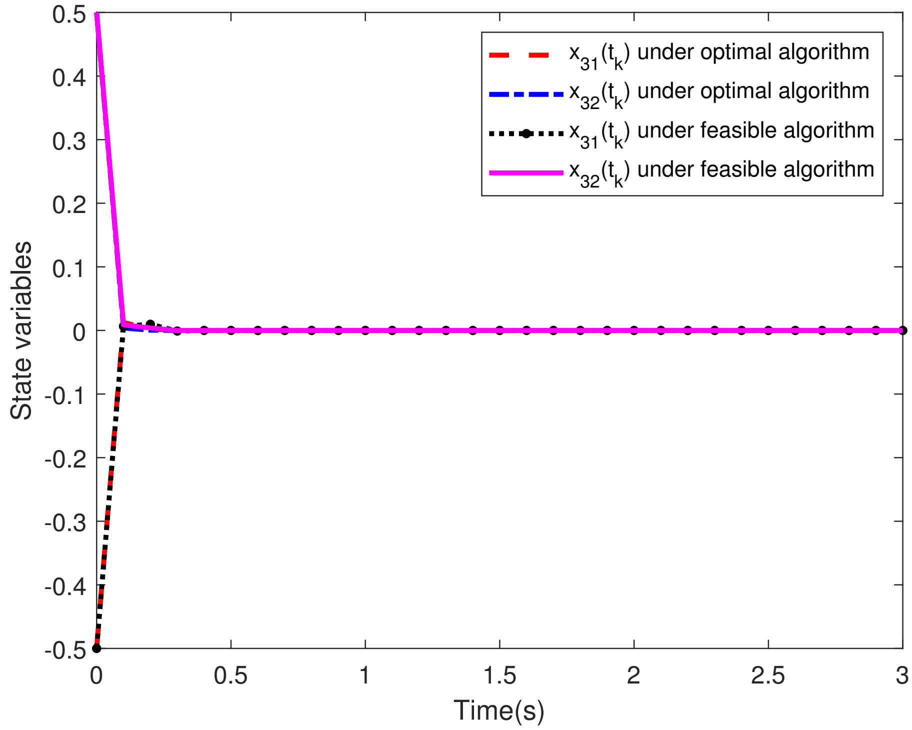

Figure 13. State response curves of subsystem III.

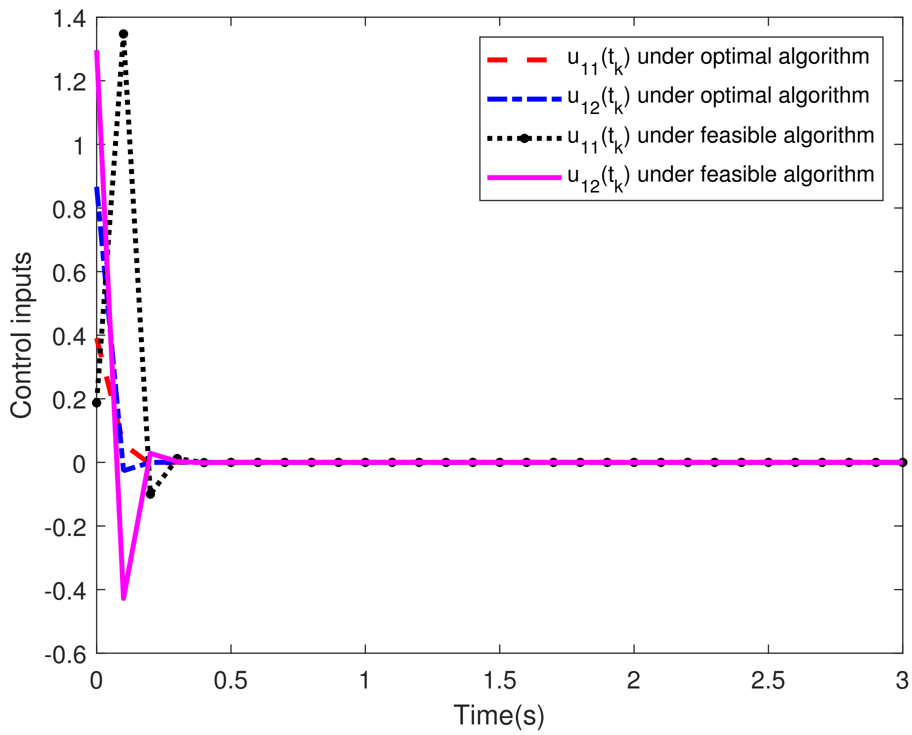

Figure 14. Control input response curves of subsystem I.

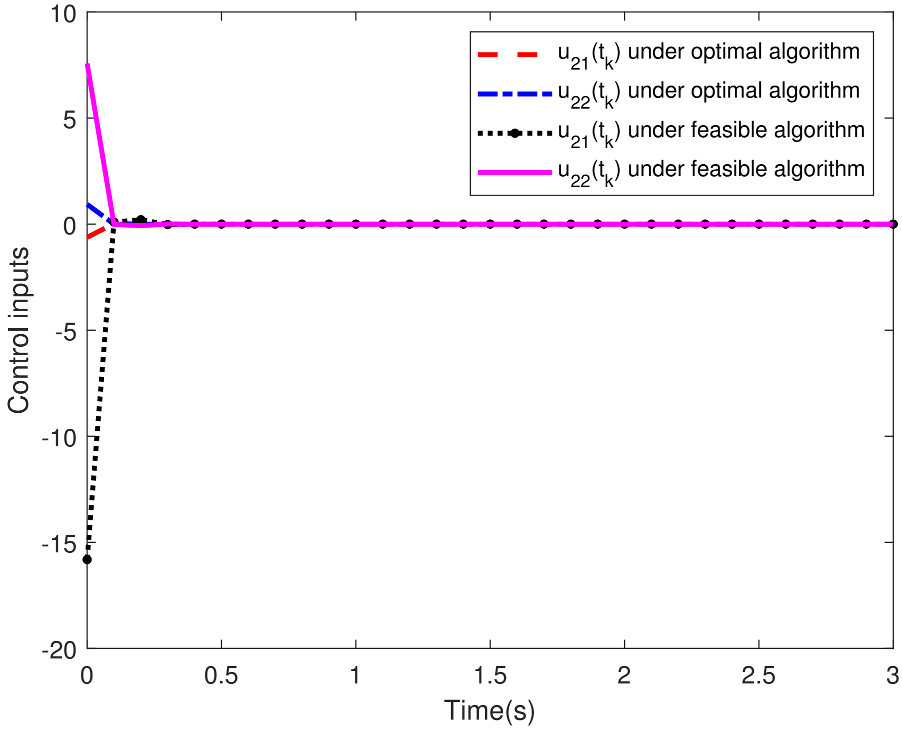

Figure 15. Control input response curves of subsystem II.

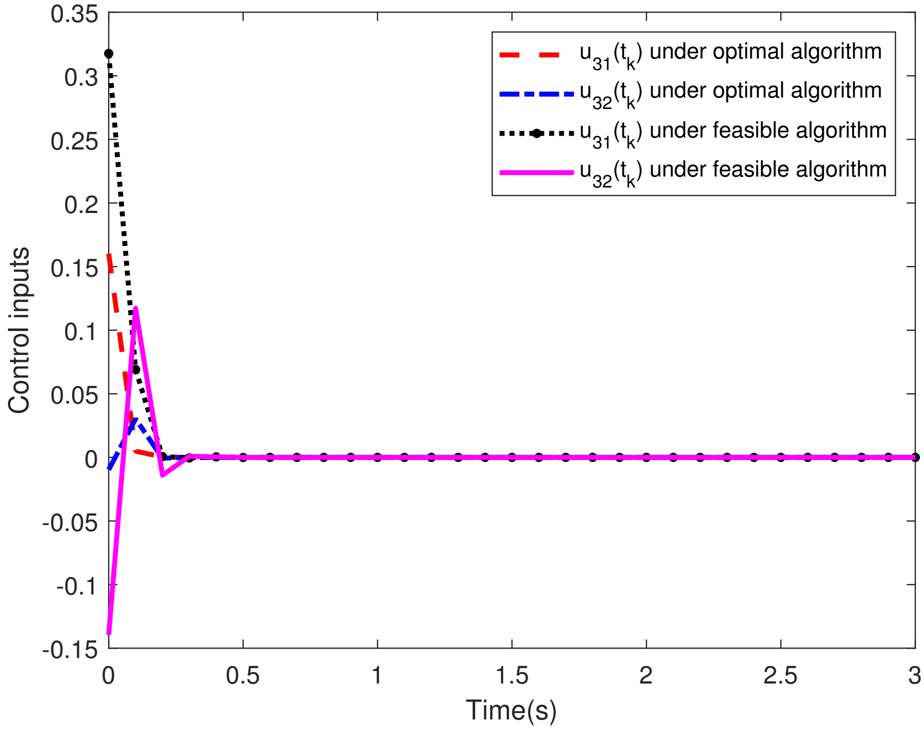

Figure 16. Control input response curves of subsystem III.

5. Conclusions

This paper discusses the cooperative control problem with N independent subsystems via delta operator, and gives the asymptotic stability control method and the optimal control method based on performance index of closed-loop system. Compared with the existing algorithms, this method has broader restrictions on the positive definite matrix in the design of collaborative feedback controller, and extends the system to general N subsystems, so that it can better describe the dynamic system in more actual engineerings. Simulation examples verify the feasibility and superiority of the two control methods designed in this paper. Generally, large-scale interconnected systems will suffer from unpredictable cyber attacks and uncertainties during actual operation process. However, the design of this paper does not consider its impact to system stability and performance. In the future, the design problem of security cooperative control for interconnected uncertain systems subject to cyber attacks and the practical applications of the designed methods will become the focuses of our research.

Acknowledgments

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Conflict of interest

The authors declared that they have no conflicts of interest to this work.

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