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Research article

Weibull distribution under indeterminacy with applications

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Abstract: The Weibull distribution has always been important in numerous areas because of its vast variety of applications. In this paper, basic properties of the neutrosophic Weibull distribution are derived. The effect of indeterminacy is studied on parameter estimation. The application of the neutrosophic Weibull distribution will be discussed with the help of two real-life datasets. From the analysis, it can be seen that the neutrosophic Weibull model is adequate, reasonable, and effective to apply in an uncertain environment.

Keywords: Weibull distribution; indeterminacy; wind speed; neutrosophy; estimation
Mathematics Subject Classification: 62A86

1. Introduction

Lifetime probability models are extensively used to analyze datasets in different fields, such as biological systems, renewable energies, health sciences, actuarial and risk measures, economics, and engineering. Various important distributions have been derived over the last century to serve as distributions in applied sciences. Among these models, the so-called Weibull distribution is famous in terms of its usefulness. In terms of reliability, Weibull distribution is most likely the most commonly used. The Weibull distribution is commonly used to characterize the lifetime distributions of systems that fail due to the failure of the "weakest link" [1]. The Weibull distribution is incredibly flexible and may be used with a very small number of samples. The Weibull module is particularly important, as it may provide insight into the process (or the physics of the failure). The Weibull is the best choice and, as a result, the best practice when there are fewer than 20 samples. For example, when the shape parameter is one, it reduces to an exponential, and the two resultant distributions are the Rayleigh distribution [2].

Wind energy is one of the world's fastest-growing renewable green energy sources, particularly in the United States, Europe, Canada, India, and Africa. Wind energy has become a significant element of the global power industry because it provides several benefits, including pure green energy and low rates, as well as the absence of pollutants that create acid rain or greenhouse gases. Wind power forecasting is critical for the efficient and cost-effective integration of wind energy into the electricity supply system. Wind power generated by a wind farm is highly dependent on the stochastic nature of wind speed, and unanticipated changes in wind power production raise the electrical system's operational costs [3,4].

Uncertainty in wind energy production estimations may be significantly decreased by using a probability model that appropriately depicts the wind speed distribution. The Weibull distribution has been utilized in previous studies to analyze wind speed data, and the model has also been used to explore wind energy potential [5]. The Weibull distribution does not always provide results so the authors generalized the Weibull distribution or introduced some more distribution for this modeling of wind speed data. For example, upper-truncated Weibull distribution [6], inverted Kumaraswamy distribution [5], Marshall-Olkin Power Lomax distribution [7], Marshall-Olkin inverse Lindley distribution [8], Alpha logarithmic transformed Log-normal distribution [9], and new Alpha Power Lindley distribution [10].

Classical statistical distributions can be used to model data with hundred percent exact and determinate observations. The classical probability models are not suitable candidates when uncertainty is found in observations or parameters of distributions. So, for these types of datasets, fuzzy-logic-based distributions can be applied. For reliability analysis, fuzzy Rayleigh distribution was introduced and studied [11]. For biomass pyrolysis, the Fuzzy Rayleigh distribution was employed [12]. The estimate of a fuzzy Rayleigh distribution was explored by Van Hecke [13]. Further, Pak et al. [14] estimate the parameter of fuzzy Rayleigh distribution parameter for fuzzy lifetime data. Shafiq et al. [15] provided comprehensive work on distribution reliability concerns utilizing the fuzzy method. Chaturvedi et al. [16] utilized a fuzzy method to analyze hybrid censored data.

Fuzzy logic which is the special case of neutrosophic logic gives information only about the measures of truth and falseness. The neutrosophic logic which is the generalization of fuzzy-based logic and interval-based logic gives information about the measure of indeterminacy additionally. The interval-based statistics did use crisp numbers and captured the data within the intervals. The neutrosophic logic used the set analysis, where any type of set can be used to capture the data inside the intervals. Recent studies showed the efficiency of neutrosophic logic over fuzzy-based logic. Zeema and Christopher [17] worked on the optimization of neutrosophic numbers and discussed the behavior in prediction problems. Sumathi and Sweety [18] introduced different methods to analyze trapezoidal neutrosophic data. Maiti et al. [19] introduced the programming for multilevel objectives under the neutrosophic environment. Abdel-Basset et al. [20,21] discussed the applications of neutrosophic logic in response systems and renewable energy, respectively.

Neutrosophic statistics which utilizes the idea of neutrosophic logic is found to be more efficient than classical statistics [22]. Neutrosophic statistics deal with the data having imprecise, interval, and uncertain observations. Neutrosophic statistics reduce to classical statistics when no indeterminacy is found in the data or the parameters of statistical distribution. Various applications of neutrosophic logic can be read in [23–25]. The idea of neutrosophic statistics was given by Smarandache [26].

In recent years, there has been a significant increase in the development of neutrosophic distributions. They work on the modeling of a variety of phenomena involving uncertain observations.

Several authors introduced the most useful neutrosophic probability distributions to analyze these types of data sets. Alhabib et al. [27] proposed neutrosophic Uniform, neutrosophic exponential, and neutrosophic Poisson distributions. Alhasan and Smarandache [28] introduced some neutrosophic distributions such as the neutrosophic Weibull distribution, neutrosophic Rayleigh distribution, neutrosophic three-parameter Weibull distribution, neutrosophic five-parameter Weibull distribution, neutrosophic beta Weibull distribution, and neutrosophic inverse Weibull distribution. Patro and Smarandache [29] proposed neutrosophic normal and binomial distributions. Aslam [30] proposed neutrosophic Raleigh distribution and used it for modeling wind speed data. Sherwani et al. [31] proposed neutrosophic Beta distribution, and Ahsan-ul-Haq [32] proposed neutrosophic Kumaraswamy distribution. Aslam [33] introduced the probability density for the Weibull distribution under indeterminacy and used it to design the sampling plan for testing average wind speed and recently in 2023 Ahsan-ul-Haq and Zafar [34] proposed neutrosophic discrete Ramos-Louzada distribution.

A rich literature on various statistical distributions is available that can be applied to model various types of data. The existing classical distributions can be applied only when all observations in the data are uncertain. But, in practice, data is not always precise, certain, and exact and the existing distributions can be applied for modeling this type of data. Aslam [33] introduced the Weibull distribution under indeterminacy and applied it to wind testing. By exploring the literature and according to the best of our knowledge, there is no work on the properties of Weibull distribution under indeterminacy. To fill this gap, in this paper, we will introduce various properties of the Weibull distribution under indeterminacy. We will derive some statistical properties of the neutrosophic Weibull distribution. The parameters are estimated using the maximum likelihood estimation approach and the performance of these derived estimators is assessed via a simulation study. The application of the neutrosophic Weibull distribution will be given using wind speed data. It is expected that the neutrosophic Weibull model will be quite effective to model the wind speed data than the existing Weibull distribution under classical statistics.

The remainder of the article is as follows. The neutrosophic Weibull is recalled in Section 2. Some statistical properties are derived in Section 3. The parameter estimation and simulation are performed in Section 4. Section 5 is based on the application of the neutrosophic Weibull distribution. We conclude our study in the last section.

2. Neutrosophic Weibull distribution

Let $x_N = x_L + x_L I_N$ is a nonnegative neutrosophic random variable and $\alpha_N = \alpha_L + \alpha_U I_N, \beta_N = \beta_L + \beta_U I_N$ are neutrosophic scale and shape parameters, where $I_N \epsilon [I_L, I_U]$ is the measure of indeterminacy and the first part of neutrosophic forms presents the determinate part and the second part presents the indeterminate part? The probability density function (pdf) of the neutrosophic Weibull distribution [33] is given by

$$f(x_N) = \left\{\left(\frac{\beta_L}{\alpha_L}\right)\left(\frac{x_L}{\alpha_L}\right)^{\beta_L - 1} e^{-\left(\frac{x_L}{\alpha_L}\right)^{\beta_L}}\right\} + \left\{\left(\frac{\beta_U}{\alpha_U}\right)\left(\frac{x_U}{\alpha_U}\right)^{\beta_U - 1} e^{-\left(\frac{x_U}{\alpha_U}\right)^{\beta_U}}\right\} I_N;\ I_N \epsilon [I_L, I_U]. \quad (1)$$

Suppose that $x_L = x_U = x_N$, the pdf can be written as

$$f(x_N) = \left\{\left(\frac{\beta_N}{\alpha_N}\right)\left(\frac{x_N}{\alpha_N}\right)^{\beta_N-1} e^{-\left(\frac{x_L}{\alpha_U}\right)^{\beta_N}}\right\}(1+I_N). \tag{2}$$

Note that when $I_L = 0$, the neutrosophic quantities reduce to classical statistics.
The pdf curves of the NW distribution are presented in Figure 1.

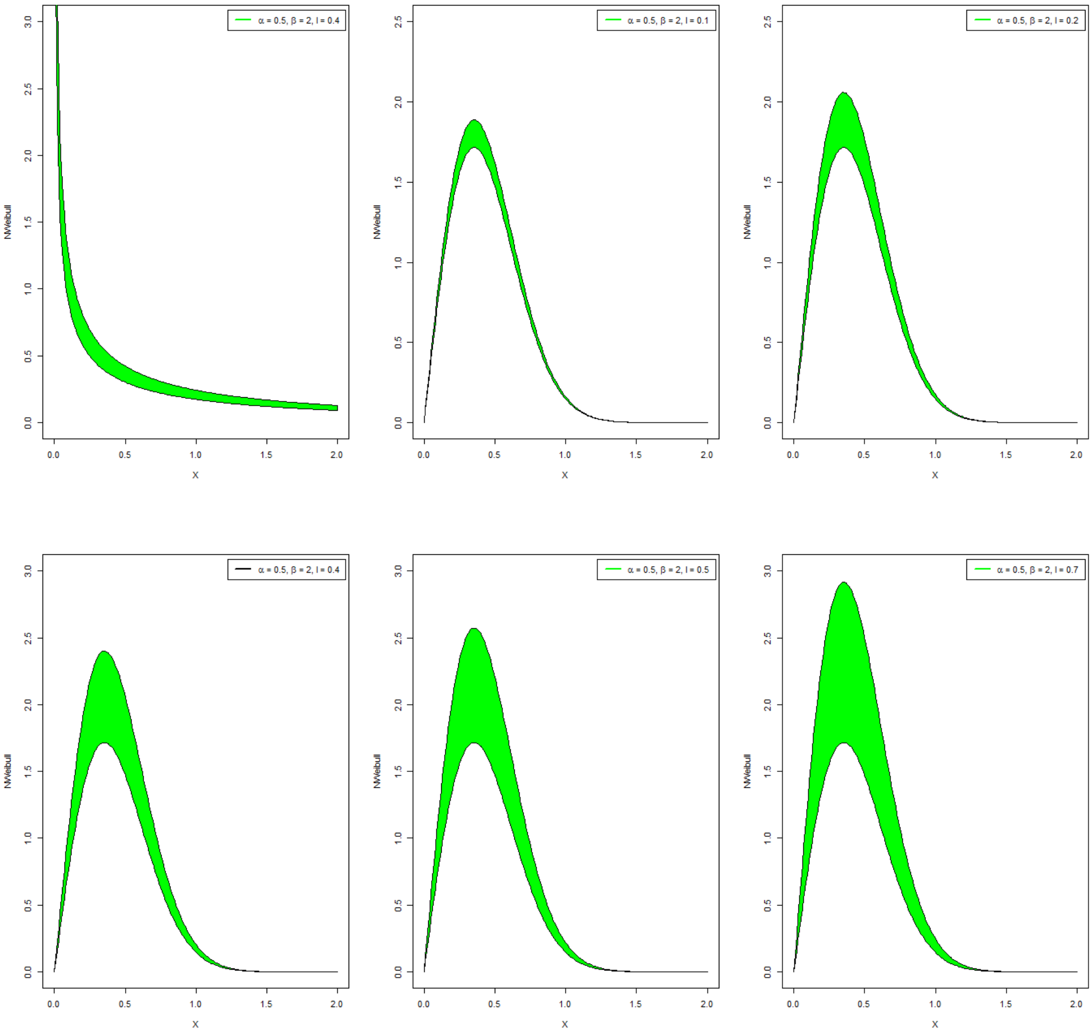

Figure 1. The pdf curves for NNW distribution.

The corresponding cumulative distribution function (cdf) is

$$F(x_N) = 1 - \left\{e^{-\left(\frac{x_N}{\alpha_N}\right)^{\beta_N}}(1+I_N)\right\} + I_N;\ I_N \epsilon [I_L, I_U]. \tag{3}$$

The survival and hazard rate functions of NW distribution are

$$\lambda(x_N) = \left\{e^{-\left(\frac{x_N}{\alpha_N}\right)^{\beta_N}}(1+I_N)\right\} + I_N \tag{4}$$

and

$$h(x_N) = \frac{\left\{\left(\frac{\beta_N}{\alpha_N}\right)\left(\frac{x_N}{\alpha_N}\right)^{\beta_N-1} e^{-\left(\frac{x_N}{\alpha_N}\right)^{\beta_N}}\right\}+\left\{\left(\frac{\beta_N}{\alpha_N}\right)\left(\frac{x_N}{\alpha_N}\right)^{\beta_N-1} e^{-\left(\frac{x_N}{\alpha_N}\right)^{\beta_N}}\right\}I_N}{\left\{e^{-\left(\frac{x_N}{\alpha_N}\right)^{\beta_N}}(1+I_N)\right\}+I_N}. \tag{5}$$

3. Statistical properties

Some mathematical properties of NW distribution derived, such as moments, incomplete moments, and quantile function.

Theorem 1. Let X be a random variable that follows NW distribution, then the neutrosophic ordinary moment given by

$$E(X^r) = \alpha_N^r \Gamma\left(1+\frac{r}{\beta_N}\right)(1+I_N). \tag{6}$$

Proof. The ordinary moments can be obtained as

$$E(x^r) = \int_0^\infty x^r f(x)dx$$

$$E(X^r) = \int_0^\infty x^r \left[\left\{\left(\frac{\beta_N}{\alpha_N}\right)\left(\frac{x_N}{\alpha_N}\right)^{\beta_N-1} e^{-\left(\frac{x_N}{\alpha_N}\right)^{\beta_N}}\right\}+\left\{\left(\frac{\beta_N}{\alpha_N}\right)\left(\frac{x_N}{\alpha_N}\right)^{\beta_N-1} e^{-\left(\frac{x_N}{\alpha_N}\right)^{\beta_N}}\right\}I_N\right]dx.$$

Using transformation $y = \left(\frac{x_N}{\alpha_N}\right)^{\beta_N}$, we get

$$E(X^r) = \alpha_N^r \int_0^\infty y^{\frac{r}{\beta_N}} e^{-y} dy + \alpha_N^r I_N \int_0^\infty y^{\frac{r}{\beta_N}} e^{-y} dy.$$

After some algebraic simplification, we get the following expression.

$$E(X^r) = \alpha_N^r \Gamma\left(1+\frac{r}{\beta_N}\right)(1+I_N). \tag{7}$$

The first four moments about the origin are

$$E(X) = \alpha_N^1\, \Gamma\left(1+\frac{1}{\beta_N}\right)(1+I_N)$$

$$E(X^2) = \alpha_N^2\, \Gamma\left(1+\frac{2}{\beta_N}\right)(1+I_N)$$

$$E(X^3) = \alpha_N^3 \Gamma\left(1+\frac{3}{\beta_N}\right)(1+I_N)$$

$$E(X^4) = \alpha_N^4 \Gamma\left(1+\frac{4}{\beta_N}\right)(1+I_N).$$

The neutrosophic mean, neutrosophic variance, and neutrosophic dispersion index of the distribution are given by

$$Mean_N = E(X) = \alpha_N \Gamma\left(1+\frac{1}{\beta_N}\right)(1+I_N).$$

$$\sigma_N^2 = \alpha_N^2\,\Gamma\left(1+\frac{2}{\beta_N}\right)(1+I_N) - \left[\alpha_N\Gamma\left(1+\frac{1}{\beta_N}\right)(1+I_N)\right]^2 \tag{8}$$

$$DI_N = \frac{\alpha_N^2\Gamma\left(1+\frac{2}{\beta_N}\right)(1+I_N) - \left[\alpha_N\Gamma\left(1+\frac{1}{\beta_N}\right)(1+I_N)\right]^2}{\alpha_N\Gamma\left(1+\frac{1}{\beta_N}\right)(1+I_N)}.$$

Theorem 2. The quantile function of neutrosophic Weibull distribution is

$$x_p = \alpha_N\left\{-\log\left[1-\frac{p}{1+I_N}\right]\right\}^{\frac{1}{\beta_N}} \quad for \quad 0<p<1. \tag{9}$$

Proof. the quantile function can be obtained as

$$F(X_N) = p$$

$$e^{-\left(\frac{x_N}{\alpha_N}\right)^{\beta_N}} = 1-\frac{p}{(1+I_N)}$$

$$\left(\frac{x_N}{\alpha_N}\right)^{\beta_N} = -\log\left[1-\frac{p}{(1+I_N)}\right]$$

$$x_N = \alpha_N\left\{-\log\left[1-\frac{p}{(1+I_N)}\right]\right\}^{\frac{1}{\beta_N}}.$$

The first, second, and third quartiles can be obtained by taking $p = \frac{1}{4}, \frac{1}{2}, \& \ \frac{3}{4}$ as:

$$Q_{1N} = \alpha_N\left\{-\log\left[1-\frac{\frac{1}{4}}{(1+I_N)}\right]\right\}^{\frac{1}{\beta_N}},$$

$$Q_{2N} = \alpha_N\left\{-\log\left[1-\frac{\frac{1}{2}}{(1+I_N)}\right]\right\}^{\frac{1}{\beta_N}},$$

and

$$Q_{3N} = \alpha_N\left\{-\log\left[1-\frac{\frac{3}{4}}{(1+I_N)}\right]\right\}^{\frac{1}{\beta_N}}.$$

The Eq (9) is also called a random number generator when p follows a uniform distribution with ranges 0 and 1.

Theorem 3. The neutrosophic mean time between failures of NW distribution given by

Proof. The MTBF is defined as

$$MTBF_N = \int_0^\infty R(x)dx$$

$$= (1+I_N)\int_0^\infty e^{-\left(\frac{x_N}{\alpha_N}\right)^{\beta_N}} dx.$$

Using substitution $y = \left(\frac{x_N}{\alpha_N}\right)^{\beta_N}$

$$= (1+I_N)\frac{\alpha_N}{\beta_N}\int_0^\infty y^{\frac{1}{\beta_N}-1} e^{-y} dy$$

$$= \frac{\alpha_N}{\beta_N}\Gamma\left(\frac{1}{\beta_N}\right)(1+I_N).$$

Theorem 4. The neutrosophic value at risk (VaR) of the distribution is given by

$$VaR_{Np} = \alpha_N\left\{-\log\left[1-\frac{p}{(1+I_N)}\right]\right\}^{\frac{1}{\beta_N}}.$$

Researchers use the VaR as a common measure of financial market risk.

Theorem 5. The neutrosophic Tail value at risk (TVaR) of the distribution is given by

$$TVaR_{Np} = \alpha_N\Gamma\left(1+\frac{1}{\beta_N},\left(\frac{VaR_{Np}}{\alpha_N}\right)^{\beta_N}\right)(1+I_N).$$

Proof. The $TVaR_{Np}$ is obtained as

$$TVaR_{Np} = \frac{1}{1-p}\int_{VaR_{Np}}^\infty xf(x)\,dx$$

$$TVaR_{Np} = \frac{1}{1-p}\int_{VaR_{Np}}^\infty x\left[\left(\frac{\beta_N}{\alpha_N}\right)\left(\frac{x_N}{\alpha_N}\right)^{\beta_N-1} e^{-\left(\frac{x_N}{\alpha_N}\right)^{\beta_N}} + \left(\frac{\beta_N}{\alpha_N}\right)\left(\frac{x_N}{\alpha_N}\right)^{\beta_N-1} e^{-\left(\frac{x_N}{\alpha_N}\right)^{\beta_N}} I_N\right]dx.$$

Using substitution $y = \left(\frac{x_N}{\alpha_N}\right)^{\beta_N}$

$$TVaR_{Np} = \frac{1}{1-p}\left[\int_{\left(\frac{VaR_{Np}}{\alpha_N}\right)^{\beta_N}}^\infty \alpha_N y^{\frac{1}{\beta_N}} e^{-y}\,dy + I_N\int_{\left(\frac{VaR_{Np}}{\alpha_N}\right)^{\beta_N}}^\infty \alpha_N y^{\frac{1}{\beta_N}} e^{-y}\,dy\right]$$

$$TVaR_{Np} = \frac{1}{1-p}\alpha_N\Gamma\left(1+\frac{1}{\beta_N},\left(\frac{VaR_{Np}}{\alpha_N}\right)^{\beta_N}\right)(1+I_N).$$

4. Parameters estimation and simulation

The parameters are estimated using the famous maximum likelihood approach. The likelihood function is

$$\prod_{i=1}^n f(x_i) = \prod_{i=1}^n\left[\left\{\left(\frac{\beta_N}{\alpha_N}\right)\left(\frac{x_{i_N}}{\alpha_N}\right)^{\beta_N-1} e^{-\left(\frac{x_N}{\alpha_N}\right)^{\beta_N}}\right\} + \left\{\left(\frac{\beta_N}{\alpha_N}\right)\left(\frac{x_{i_N}}{\alpha_N}\right)^{\beta_N-1} e^{-\left(\frac{x_{i_N}}{\alpha_N}\right)^{\beta_N}}\right\}I_N\right].$$

The log-likelihood function is written as

$$L(\alpha_N, \beta_N, I_N) = \sum_{i=1}^{n} \log \left[\left\{ \left(\frac{\beta_N}{\alpha_N}\right) \left(\frac{x_{i_N}}{\alpha_N}\right)^{\beta_N - 1} e^{-\left(\frac{x_N}{\alpha_N}\right)^{\beta_N}} \right\} (1 + I_N) \right]. \tag{10}$$

Now by differentiating the above log-likelihood equation w.r.t. α can be written

$$\frac{\partial l}{\partial \alpha_N} = \sum_{i=1}^{n} \frac{\beta_N\left(\left(\frac{x_{i_N}}{\alpha_N}\right)^{\beta_N} - 1\right)}{\alpha_N}. \tag{11}$$

$$\frac{\partial l}{\partial \beta_N} = \sum_{i=1}^{n} \frac{1 - \beta_N\left(\left(\frac{x_{i_N}}{\alpha_N}\right)^{\beta_N} - 1\right) \log\left(\frac{x_{i_N}}{\alpha_N}\right)}{\beta_N}. \tag{12}$$

The derived ML estimates are asymptotically normally distributed with a joint bivariate normal distribution given by

$$\left(\hat{\alpha}_N, \hat{\beta}_N\right) \sim N_2[(\alpha_N, \beta_N), I^{-1}(\alpha_N, \beta_N)] \;\; for\; n \to \infty,$$

where $I^{-1}(\alpha_N, \beta_N)$ are the diagonal elements of the Fisher information matrix (FIM), which is defined as

$$\text{FIM} = \text{I} = \begin{bmatrix} I_{\alpha_N \alpha_N} & I_{\alpha_N \beta_N} \\ I_{\alpha_N \beta_N} & I_{\beta_N \beta_N} \end{bmatrix} = -\text{E} \begin{bmatrix} \frac{\partial^2 l}{\partial^2 \alpha_N} & \frac{\partial^2 l}{\partial \alpha_N \partial \beta_N} \\ \frac{\partial^2 l}{\partial \alpha_N \partial \beta_N} & \frac{\partial^2 l}{\partial^2 \beta_N} \end{bmatrix}.$$

$$\frac{\partial^2 l}{\partial^2 \alpha_N} = -\frac{\beta_N\left(-1 + \left(\frac{x_{i_N}}{\alpha_N}\right)^{\beta_N} + \left(\frac{x_{i_N}}{\alpha_N}\right)^{\beta_N} \beta_N\right)}{{\alpha_N}^2},$$

$$\frac{\partial^2 l}{\partial \alpha_N \partial \beta_N} = \frac{-1 + \left(\frac{x_{i_N}}{\alpha_N}\right)^{\beta_N} + \left(\frac{x_{i_N}}{\alpha_N}\right)^{\beta_N} \beta_N \log\left(\frac{x_{i_N}}{\alpha_N}\right)}{\alpha_N},$$

$$\frac{\partial^2 l}{\partial^2 \beta_N} = -\frac{1}{\beta^2{}_N} - \left(\frac{x_{i_N}}{\alpha_N}\right)^{\beta_N} \log\left(\frac{x_{i_N}}{\alpha_N}\right)^2.$$

If the information matrix is replaced with the actual information matrix, the asymptotic behavior remains true. So, the approximate $100(1-\theta)\%$ two-sided confidence interval for the parameters α and β are given by

$$\hat{\alpha}_N = \pm Z_{\theta/2}\sqrt{I^{-1}_{\alpha_N \alpha_N}} \quad \text{and} \quad \hat{\beta}_N = \pm Z_{\theta/2}\sqrt{I^{-1}_{\beta_N \beta_N}}$$

where Z_θ is the θ-th percentile of standard normal distribution.

Next, we present a simulation study to check the performance of estimates. To run simulations, we generate N=10000 random samples of sizes $n = 30{,}50{,}80, \text{and}\; 100$ from NNW distribution. The average estimates (AEs) and mean square errors (MSEs) are reported in Table 1. The numerical results are derived using R (version 4.2.2, 2022) software.

Table 1. Simulation results for the NWD.

Sample	Actual Values			AE		MSE	
n	α	β	I_N	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
30	0.5	0.5	0.0	0.5257	0.5224	0.0391	0.0066
50				0.5109	0.5113	0.0224	0.0032
80				0.5094	0.5084	0.0147	0.0021
100				0.5054	0.5078	0.0106	0.0017
30	0.5	0.5	0.1	0.3231	0.5858	0.0347	0.0095
50				0.3220	0.5893	0.0381	0.0128
80				0.3196	0.5864	0.0364	0.0101
100				0.3220	0.5849	0.0349	0.0094
30	1.0	1.0	0.1	0.7959	1.1914	0.0580	0.0689
50				0.7952	1.1816	0.0527	0.0517
80				0.8005	1.1766	0.0465	0.0435
100				0.7957	1.1670	0.0471	0.0376
30	1.5	1.5	0.5	0.9339	2.1178	0.3283	0.5364
50				0.9268	2.0417	0.3329	0.3639
80				0.9271	2.0406	0.3310	0.3403
100				0.9303	2.0399	0.3269	0.3285
300				0.9275	2.0109	0.3285	0.2730

The simulation results show that the average bias decreases with an increase in sample size. It means the agreement between practice and theory improves as the sample size increases. The MSEs of the estimators also decreases with an increase in sample size. It is evident that the derived estimators are consistent, and the maximum likelihood estimator of the parameters performs well, yielding exact and accurate results.

5. Applications

In this section, the distribution is applied to real-life data sets. For this purpose, wind speed data of Bahawalpur station is considered. Some summary measures of this data set are given in Table 2.

Table 2. Some descriptive measures for the first dataset.

Min.	Q1	Median	Q3	Mean	var	skewness	Kurtosis	Max.
0.0004	2.4118	3.3181	4.3006	3.5049	2.8053	1.1346	6.0846	20.073

For comparison of the distribution is done with the classical Weibull distribution. The model parameters are estimated using the maximum likelihood method. The best model is selected using log-likelihood, Akaike information criteria (AIC), and Bayesian Information Criteria (BIC).

$$AIC = 2k - 2\ln(\hat{L})$$

$$BIC = k\ln(n) - 2\ln(\hat{L})$$

where $\hat{L}$ is the maximized loglikelihood value, k is the number of parameters, and n is the sample size.

The maximum likelihood estimates and model selection information are listed in Table 3. We plot the fitted density curves over observed wind speed observations.

Table 3. MLEs and goodness-of-it measures for the first dataset.

Model	$\widehat{\alpha}$ (SE)	$\widehat{\beta}$ (SE)	I_N	LogLik	AIC	BIC
WD	3.943394	2.155532	0	-130683.9	261371.8	261390.1
	4.337733	2.371085	0.1	-125756.0	251518.1	251545.5
NWD	4.732073	2.586638	0.2	-124587.8	249181.7	249209.1
	5.126412	2.802192	0.3	-126932.5	253871.0	253898.4

Table 3 lists the MLEs and goodness-of-fit measures from the classical Weibull distribution and neutrosophic Weibull distribution with different indeterminacy parameter values. It is found that the model of the distribution could be chosen as the best distribution as compared to the classical Weibull distribution. It is worth noting that the indeterminacy parameter plays a crucial role in better fitting.

The second dataset is about the case study on the light-emitting diodes (LED) manufacturing process that focuses on the luminous intensities of LED sources. The process distribution has been justified and is fairly close to the Weibull distribution. A sample of size n = 30 is taken from the stable process. The data observations are; 2.163, 5.972, 1.032, 0.628, 2.995, 3.766, 0.974, 4.352, 3.920, 1.375, 0.618, 4.575, 1.027, 6.279, 2.821, 7.125, 5.443, 1.766, 7.155, 0.830, 3.590, 5.965, 3.177, 4.634, 7.261, 2.247, 6.032, 4.065, 5.434, and 1.336. Some summary measures of this data set are given in Table 4.

Table 4. Some descriptive measures for the second dataset.

Min.	Q1	Median	Q3	Mean	var	Skewness	Kurtosis	Max.
0.618	1.473	3.678	5.441	3.619	4.705	0.1522	1.7497	7.261

Table 5 shows the MLEs and goodness-of-fit metrics for the classical Weibull and neutrosophic Weibull distributions with varying indeterminacy parameter values. It is found that the model of the distribution could be chosen as the best distribution as compared to the classical Weibull distribution. It is worth mentioning that the indeterminacy parameter is very important in the improved fitting.

Table 5. MLEs and goodness-of-it measures for the second dataset.

Model	$\widehat{\alpha}$ (SE)	$\widehat{\beta}$ (SE)	I_N	LogLik	AIC	BIC
WD	4.055706	1.717136	0	-63.4225	130.845	133.647
	4.461277	1.888850	0.1	-61.0455	128.091	132.295
NWD	4.866847	2.060563	0.2	-59.8764	125.753	129.956
	5.272418	2.232277	0.3	-59.8570	125.714	129.918

6. Conclusions

Some basic properties of neutrosophic Weibull distribution were studied in this paper. The application of the neutrosophic Weibull distribution was given using the wind speed data. From the analysis, it was concluded that the neutrosophic Weibull distribution is more flexible than the Weibull distribution under classical statistics. The decision-makers can apply the neutrosophic Weibull distribution when uncertainty is presented in observations or parameters. Some more properties of the neutrosophic Weibull distribution can be studied in future research. Other statistical distributions can be derived using the same method as future research. The distribution for multivariate distribution can be extended in future research. The G-families of distributions under neutrosophic statistics can be considered as future research, see [35].

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Conflict of interest

No conflict of interest regarding the paper.

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