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Research article

Disjoint union of fuzzy soft topological spaces

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Abstract: In this work, sums of fuzzy soft topological spaces are defined with free union of a pairwise disjoint non-empty family of fuzzy soft topological spaces. Firstly, we give general information of fuzzy soft topological spaces. Then, we define free union of fuzzy soft topological spaces and disjoint union topology of fuzzy soft topological spaces. We call the free union of a pairwise disjoint non-empty family of fuzzy soft topological spaces the sum of fuzzy soft topological spaces. We show what are the interchangeable properties between fuzzy soft topological spaces and the sum of fuzzy soft topological spaces. For example, there are fuzzy soft interior, fuzzy soft closure, fuzzy soft limit points. Also, we provide some properties showing the relationships between fuzzy soft topological spaces and their sums. Some of these are fuzzy soft base, fuzzy soft sequences, fuzzy soft connected-disconnected, fuzzy soft compact spaces. Also, part of the research for this article is work on fuzzy soft convergence on fuzzy soft topological sum. With this paper, some results, theorems and definitions for fuzzy soft topological sum have been acquired with the help of results, theorems and definitions given in previous studies about fuzzy soft topological spaces.

Keywords: sum of topological spaces; soft sets; fuzzy sets; fuzzy soft sets; sum of soft topological spaces
Mathematics Subject Classification: 54A40, 54A05

1. Introduction

The existence of some problems that cannot be expressed in classical logic and, at the same time, scientific advances have led mathematicians to use new mathematical models and tools. Many theories have been presented in the field of mathematics to find solutions to such problems. For this reason, mathematicians constantly felt the need to come up with new theories. One of these theories is the concept of the fuzzy set introduced by Zadeh [1] in 1965. Later, in 1999, soft set theory was modeled by Molodtsov [2]. Soft set and fuzzy set theories are widely used in expert systems, decision-making, modeling, social sciences, medical diagnostics, etc. In addition to being used in many different areas, it has application areas in the determination of COVID-19 patients, which all of humanity has been following closely recently. There are also many studies in the field of mathematics on soft set and fuzzy set theories, which attract the attention of many researchers.

Based on fuzzy set theory, a definition of fuzzy topology was given by Chang [3] in 1968, and then a more natural definition of fuzzy topology was given by Lowen [4] in 1976, unlike Chang's definition. Since Chang applied fuzzy set theory into topology, many topological notions were introduced in a fuzzy setting. One of the current studies on fuzzy sets is an article [5]. In this article, a generalized framework for orthopairs (pairs of disjoint sets) of fuzzy sets called "(m, n)-Fuzzy sets" is presented. For (m, n)-Fuzzy sets, some operations are given and characterized. Then, the aggregation operators method, which has a wide working area in the computing, is extended with the help of (m, n)- Fuzzy sets.

Molodtsov [2] introduced the concept of a soft set in 1999, and started to develop the basics of the corresponding theory as a new approach for modeling uncertainties. In [6] and [7] Maji et al. have presented their work, respectively, which includes the application of soft set theory to decision making problems and the fundamental notions between two soft sets: soft intersection, soft union and soft equality relations. Later, in [8] Ali et al. have defined new types of soft intersections and union between two soft sets by changing several results of the work in [7]. Also, in [9] Shabir and Naz introduced the notions of soft topological spaces using soft sets that are defined on an initial universe set with a fixed parameter set. Then, many authors explored soft topological concepts similar to researchers in classical topology after the inception of soft topology. Some of them are as follows: [10–13]. The soft mapping concept was first introduced in [14] by Kharal and Ahmad, and its basic properties were proven. Next, [15] introduced the notion of soft homeomorphism maps. Also, in [16], the properties of soft mapping spaces have been explored and the relationships between some soft mapping spaces have been obtained. A new soft separation axiom is defined in [17]. The aim of [17], which describes and gives some properties of some soft topological operators, was to characterize a few soft separation axioms.

In 2001, Maji et al. [18] combined fuzzy set and soft set theories and gave the definition of a fuzzy soft set. In the literature review, it has been seen that a significant part of the recent studies are on fuzzy soft topological spaces. To continue the investigation on fuzzy soft sets, Ahmad and Kharal [19] presented some more properties of fuzzy soft sets and introduced the notion of a mapping on fuzzy soft sets. In 2011, Tanay et al. [20] gave the topological structure of fuzzy soft sets. Then in 2012, Varol and Aygün [21] gave the fuzzy soft topology, and Şimşekler and Yüksel [22] gave the fuzzy Soft Topological Spaces in 2013. Also in [23], the basic set of arithmetic operations for (a, b)-Fuzzy soft sets is introduced and the basic properties are revealed. Then, addition operators for (a, b)-Fuzzy soft sets were introduced, and their relations were discussed. One of the current study by Alcantud [24] on fuzzy soft sets, links soft topology with fuzzy soft topology. This paper explains both soft topology and fuzzy soft topology by analyzing the relations between them.

In this project, first, as preliminaries, we give some basic definitions and results in fuzzy soft set theory. After giving these preliminaries, we give definition of fuzzy soft topology. Although the sum of topological spaces has been studied by Atay and Tutalar [25] in 2015, and sum of soft topological spaces has been studied by Al-shami et al [26] in 2020, no study has been made for the sum of fuzzy soft topological spaces. With this work, our aim is to introduce and examine the notion of sum of fuzzy soft topological spaces using pairwise disjoint fuzzy soft topological spaces. We will then explore the properties that this sum provides. Our results include unchangeable features between fuzzy soft topological spaces and their sums. Many properties such as fuzzy soft compact spaces, fuzzy soft connected-disconnected and fuzzy soft discrete-indiscrete topology are investigated with the help of explanatory examples. Additionally, we got some results related to some important generalized fuzzy soft open sets by using interchangeability of fuzzy soft base, fuzzy soft interior and fuzzy soft closure operators between fuzzy soft topological spaces and their sum . Finally, we examine in which case a fuzzy soft topological space symbolizes the sum of some fuzzy soft topological spaces.

2. Preliminaries

In this section we will give the primary results and definitions which will be used during this paper.

2.1. Fuzzy soft sets

During this work, $U \neq \emptyset$ will be an initial universal set, $I = [0,1] \subset \mathbb{R}$ and $E \neq \emptyset$ is the set of all possible parameters of U.

Definition 1. *[1] Let $\mu_A(x) : U \to I$ be a mapping. Then, $A = \{(x, \mu_A(x)) : x \in U\}$ is defined as a fuzzy set in U, and $\mu_A(x)$ is defined as a degree of membership of $x \in A$. The family of all fuzzy sets in U is indicated by I^U.*

Definition 2. *[2] Let $A \subset E$. A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \to 2^U$.*

Definition 3. *[16] Let $A \subset E$. f_A is defined to be a fuzzy soft set on (U, E) if $f : A \to I^U$ is a mapping defined by $f(e) = \mu_f^e$ such that*

$$f(e) = \begin{cases} \mu_f^e = \overline{0}, & e \in E - A \\ \mu_f^e \neq \overline{0}, & e \in A \end{cases}$$

where $\overline{0}(u) = 0$ for each $u \in U$.

Definition 4. *[16] The complement of a fuzzy soft set f_A is a fuzzy soft set on (U, E), which is denoted by f_A^c, and $f^c : A \to I^U$ is defined as follows:*

$$f^c = \begin{cases} \mu_{f^c}^e = 1 - \mu_f^e, & e \in A \\ \mu_{f^c}^e = \overline{1}, & e \in E - A \end{cases}$$

where $\overline{1}(u) = 1$ for each $u \in U$.

Definition 5. *[16] The fuzzy soft set f_A is called the null fuzzy soft set if $f_A(e) = 0$ for each $e \in E$ and denoted by $\tilde{0}_E$.*

Definition 6. *[16] The fuzzy soft set f_A is called the universal fuzzy soft set if $f_A(e) = 1$ for each $e \in E$ and denoted by $\tilde{1}_E$.*

Clearly, $(\tilde{1}_E)^c = \tilde{0}_E$, and $(\tilde{0}_E)^c = \tilde{1}_E$.

From now on, we will use $\mathcal{F}(U, E)$ instead of the family of all fuzzy soft sets over U.

Definition 7. *[16] Let f_A, g_B be two fuzzy soft sets on $\mathcal{F}(U, E)$ and $A \subseteq B \subset X$. Then, f_A is called a fuzzy soft subset of g_B, denoted by $f_A \sqsubseteq g_B$, if $f_A(e) \leq g_B(e)$ for every $e \in E$. If g_B is a fuzzy soft subset of f_A, then f_A is called a fuzzy soft superset of g_B and denoted by $f_A \sqsupseteq g_B$.*

Definition 8. *[16] Let $f_A, g_B \in \mathcal{F}(U, E)$. If $f_A \sqsubseteq g_B$ and $g_B \sqsubseteq f_A$, then f_A and g_B are said to be equal and denoted by $f_A = g_B$.*

Definition 9. *[16] Let f_A, g_B and h_C be fuzzy soft sets. For $f_A \sqcup g_B = h_C$, we say that h_C is the union of f_A and g_B, whose membership function $\mu^e_{h_C}(x) = max\left\{\mu^e_{f_A}(x), \mu^e_{g_B}(x)\right\}$ for every $x \in U$.*

Definition 10. *[16] Let f_A, g_B and h_C be fuzzy soft sets. For $f_A \sqcap g_B = h_C$, we say that h_C is the intersection of f_A and g_B, whose membership function $\mu^e_{h_C}(x) = min\left\{\mu^e_{f_A}(x), \mu^e_{g_B}(x)\right\}$ for every $x \in U$.*

Theorem 1. *[17] Let f_A, g_B be two fuzzy soft sets on (U, E). Then, the following holds:*

(1) $f^c_A \sqcap g^c_B = (f_A \sqcup g_B)^c$;
(2) $f^c_A \sqcup g^c_B = (f_A \sqcap g_B)^c$.

Theorem 2. *[17] Let I be an index set and $(f_A)_i$ be a family of fuzzy soft sets on (U, E). Then, the following holds:*

(1) $\sqcap_i((f_A)^c_i) = (\sqcup_i((f_A)_i))^c$;
(2) $\sqcup_i((f_A)^c_i) = (\sqcap_i((f_A)_i))^c$.

Theorem 3. *[19] Let I be an index set and $f_A, g_B, h_C, (f_A)_i, (g_B)_i \in \mathcal{F}(U, E), \forall i \in I$; then, the following holds:*

(1) $f_A \sqcap f_A = f_A, f_A \sqcup f_A = f_A$.
(2) $f_A \sqcap g_B = g_B \sqcap f_A, f_A \sqcup g_B = g_B \sqcup f_A$.
(3) $f_A \sqcap (g_B \sqcap h_C) = (f_A \sqcap g_B) \sqcap h_C, f_A \sqcup (g_B \sqcup h_C) = (f_A \sqcup g_B) \sqcup h_C$.
(4) $f_A \sqcap (\bigsqcup_{i\in I}(g_B)_i) = \bigsqcup_{i\in I}(f_A \sqcap (g_B)_i), f_A \sqcup (\sqcap_{i\in I}(g_B)_i) = \sqcap_{i\in I}(f_A \sqcup (g_B)_i)$.
(5) *If* $f_A \sqsubseteq g_B$, *then* $(g_B)^c \sqsubseteq (f_A)^c$.
(6) $f_A \sqcap g_B \sqsubseteq f_A, g_B$ *and* $f_A, g_B \sqsubseteq f_A \sqcup g_B$.

2.2. Fuzzy soft topological spaces

Definition 11. *[18] A fuzzy soft topological space is a pair $(X, \mathcal{T})$ where X is a nonempty set and $\mathcal{T}$ a family of fuzzy soft sets over X satisfying the following properties:*

(1) $\tilde{0}_E, \tilde{1}_E \in \mathcal{T}$,
(2) *If* $f_A, g_B \in \mathcal{T}$, *then* $f_A \sqcap g_B \in \mathcal{T}$,
(3) *If* $(f_A)_i \in \mathcal{T}, \forall i \in I$, *then* $\bigsqcup_{i\in I}(f_A)_i \in \mathcal{T}$.

$\mathcal{T}$ is called a topology of fuzzy soft sets on X. Every member of $\mathcal{T}$ is called fuzzy soft open. If $(g_B)^c \in \mathcal{T}$, then g_B is called fuzzy soft closed in $(X, \mathcal{T})$.

Definition 12. *[20] Let f_A be a fuzzy soft set on (U, E) and $p_{x,i}(x \in U, i \in (0, 1])$ be a fuzzy soft point in I^U. If $i \leq \mu^e_{f_A}(x)$, $\forall e \in A$, then $p_{x,i}$ belongs to f_A, and it is denoted by $p_{x,i} \tilde{\in} f_A$.*

Definition 13. *[20] Let $p_{x,i}$ be a fuzzy soft point in I^U. Then, $f_{p_{x,i}}$ is a fuzzy soft set on (U, E) where $p_{x,i}(e) = \mu^e_{p_{x,i}}$, $\mu^e_{p_{x,i}}(u) = i$, if $u = x$ and $\mu^e_{p_{x,i}}(u) = 0$, if $u \neq x$, $\forall e \in E$ and $\forall u \in U$.*

Definition 14. *[20] Let $(X,\mathcal{T})$ be a fuzzy soft topological space over X, $f_A \sqsubseteq X$ and $p_{x,i}$ be a fuzzy soft point in I^U. f_A is called a fuzzy soft neighborhood of $p_{x,i}$ if there exists a fuzzy soft open set g_A such that $p_{x,i}\tilde{\in}g_A \sqsubseteq f_A$.*

Definition 15. *[19] Let $(X,\mathcal{T})$ be a fuzzy soft topological space over X and $f_A \sqsubseteq X$. The union of all fuzzy soft open subsets of f_A is called the fuzzy soft interior of f_A and is denoted by $(f_A)^o$.*

Definition 16. *[19] Let $(X,\mathcal{T})$ be a fuzzy soft topological space over X and $f_A \sqsubseteq X$. The intersection of all fuzzy soft closed sets containing f_A is called fuzzy soft closure of f_A and is denoted by $\bar{f}_A$.*

Definition 17. *[19] Let $(X,\mathcal{T})$ be a fuzzy soft topological space and $\mathcal{B} \sqsubseteq \mathcal{T}$. If every member of $\mathcal{T}$ can be expressed as a union of members of $\mathcal{B}$, then $\mathcal{B}$ is called a base for $\mathcal{T}$.*

Definition 18. *[19] Let $(X,\mathcal{T})$ be a fuzzy soft topological space and $f_A \neq \emptyset, f_A \sqsubseteq X$. The family $\mathcal{T}_(f_A) = \{f_A \sqcap g_A : g_A \in \mathcal{T}\}$ is called a fuzzy soft relative topology on f_A, and $(f_A, \mathcal{T}_(f_A))$ is called a fuzzy soft subspace of $(X,\mathcal{T})$.*

A family $\{f_A^i : i \in I\}$ of fuzzy soft sets in $(X,\mathcal{T})$ is said be a fuzzy soft cover of $(X,\mathcal{T})$ if $\sqcup_{i\in I} f_A^i = X$. A fuzzy soft cover $\{f_A^i : i \in I\}$ is said to be locally finite if, for each fuzzy soft point, $p_{x,\lambda}$ has a fuzzy soft neighborhood intersecting only finitely many f_A^i.

Definition 19. *A fuzzy soft $(X,\mathcal{T})$ is said to be*

(1) [27] Fuzzy soft compact if every fuzzy soft open cover of X has a finite subcover.
(2) [28] Fuzzy soft connected if it cannot be expressed as a union of two disjoint fuzzy soft open sets.

Definition 20. *[29] A fuzzy soft mapping $\xi : (X,\mathcal{T}) \to (Y,\mathcal{T}')$ is said to be:*

(1) Fuzzy soft continuous if the inverse image of each fuzzy soft open set is fuzzy soft open.
(2) Fuzzy soft open if the image of each fuzzy soft open set is fuzzy soft open.
(3) Fuzzy soft closed if the image of each fuzzy soft closed set is fuzzy soft closed.
(4) Fuzzy soft homeomorphism if it is bijective, fuzzy soft continuous and fuzzy soft open.

3. Disjoint union of fuzzy soft topological spaces

In this section, open sets, interior, closure, base and neighborhoods will be defined on the sums of fuzzy topological spaces after defining the free union of fuzzy soft topological spaces. Some of the results obtained for these topological sums will be mentioned.

Definition 21. *Let $(X,\mathcal{T})$ be a fuzzy soft topological space and $\{(X_\lambda,\mathcal{T}_\lambda)\}_{\lambda\in\Lambda}$ is a subspace of fuzzy soft topological space $(X,\mathcal{T})$ such that $X = \sqcup_{\lambda\in\Lambda} X_\lambda$. If $T \sqsubseteq X$ is fuzzy soft open (or fuzzy soft closed), then $T \sqcap X_\lambda$ is fuzzy soft open (or fuzzy soft closed) in X_λ, for all $\lambda \in \Lambda$. Hence, if $T \in \mathcal{T}(T^c \in \mathcal{T}) \iff \forall\lambda \in \Lambda, T \sqcap X_\lambda \in \mathcal{T}_\lambda((T \sqcap X_\lambda)^c \in \mathcal{T}_\lambda)$ is satisfied, then we say that X is free union of $(X_\lambda,\mathcal{T}_\lambda)$. If $A \sqsubseteq X$ is fuzzy soft open or fuzzy soft closed, then the subspace $(A,\mathcal{T}_A)$ is free union of $(A \sqcap X_\lambda, \mathcal{T}_{A\sqcap X_\lambda})$ subspaces.*

Theorem 4. *Let $(X,\mathcal{T})$ be a fuzzy soft topological space, $(X_\lambda,\mathcal{T}_\lambda)$ be a subspace of fuzzy soft topological spaces $(X,\mathcal{T})$ and $X = \sqcup_{\lambda\in\Lambda} X_\lambda$. If X_λ is fuzzy soft open subset of fuzzy soft set X for all $\lambda \in \Lambda$ (or $\{(X_\lambda,\mathcal{T}_\lambda)\}_{\lambda\in\Lambda}$ is a local finite, and X_λ is fuzzy soft closed subset of fuzzy soft set X for all $\lambda \in \Lambda$), then X is free union of $\{(X_\lambda,\mathcal{T}_\lambda)\}_{\lambda\in\Lambda}$.*

Proof. Let X_λ be a fuzzy soft open subset of fuzzy soft set X for all $\lambda \in \Lambda$. Since $\{T \sqcap X_\lambda\}_{\lambda\in\Lambda}$, $T \sqsubseteq X$, is family of fuzzy soft open sets in X, then $\sqcup_{\lambda\in\Lambda}(T \sqcap X_\lambda) = T \sqcap \sqcup_{\lambda\in\Lambda} X_\lambda = T \sqcap X = T$ is fuzzy soft open in X. It is clear that if $T \in \mathcal{T}$ then $T \sqcap X_\lambda$ is fuzzy soft open in X_λ for all $\lambda \in \Lambda$. Thus, X is free union of $\{(X_\lambda, \mathcal{T}_\lambda)\}_{\lambda\in\Lambda}$.

Let X_λ be a fuzzy soft closed subset of fuzzy soft set X for all $\lambda \in \Lambda$ and $\{(X_\lambda, \mathcal{T}_\lambda)\}_{\lambda\in\Lambda}$ be a local finite. Let $F \sqcap X_\lambda$ be a fuzzy soft closed set of X_λ for $F \sqsubseteq X$ and for all $\lambda \in \Lambda$. Then, $F \sqcap X_\lambda$ is a fuzzy soft closed set of X. Since $\{(X_\lambda, \mathcal{T}_\lambda)\}_{\lambda\in\Lambda}$ is a local finite, $\sqcup_{\lambda\in\Lambda}(F \sqcap X_\lambda) = F \sqcap (\sqcup_{\lambda\in\Lambda} X_\lambda) = F \sqcap X = F$. We know that finite union of fuzzy soft closed sets is fuzzy soft closed. Thus, F is fuzzy soft closed in X. □

Remark 1. *The space $(X_\lambda, \mathcal{T}_\lambda)$ does not have to be a subspace of X. So, we will give the definition of disjoint union fuzzy soft topology and topological summed of fuzzy soft topologies and investigate some of results about topological sums of fuzzy soft topologies in this paper. Let $(X_\lambda, \mathcal{T}_\lambda)$ be a disjoint non-empty collection of fuzzy soft topological spaces $(X_\lambda, \mathcal{T}_\lambda)$ indexed by a set Λ. The disjoint union $X = \sqcup_{\lambda\in\Lambda} X_\lambda$ is a fuzzy soft topological space with the following fuzzy soft topology; $\mathcal{T} = \{T \in X : T \cap X_\lambda \in \mathcal{T}_\lambda \quad \text{for each} \quad \lambda \in \Lambda\}$. $\mathcal{T}$ is a disjoint union fuzzy soft topology, and X is a fuzzy soft topological sum of a disjoint non-empty collection of fuzzy soft topological spaces X_λ. Hence, $(X, \mathcal{T})$ is a free union of $(X_\lambda, \mathcal{T}_\lambda)$. Now, we will give free union of a disjoint non-empty collection of fuzzy soft topological spaces. Definitions, theorems and some results for fuzzy soft topological sums have been obtained by using the known definitions and theorems for the fuzzy soft topological spaces.*

Proposition 1. *Let $(X_\lambda, \mathcal{T}_\lambda)$ be a disjoint non-empty collection of fuzzy soft topological spaces indexed by a set Λ and $X = \sqcup_{\lambda\in\Lambda} X_\lambda$. Then, $\mathcal{T} = \{T \in X : T \sqcap X_\lambda \in \mathcal{T}_\lambda \quad \text{for each} \quad \lambda \in \Lambda\}$ defines a fuzzy soft topology on X.*

Proof. (1) For each $\lambda \in \Lambda$, $\tilde{0}_E \sqcap X_\lambda = \tilde{0}_E \in \mathcal{T}_\lambda \Longrightarrow \tilde{0}_E \in \mathcal{T}_\lambda$, $\tilde{1}_E \sqcap X_\lambda = X_\lambda \in \mathcal{T}_\lambda \Longrightarrow \tilde{1}_E \in \mathcal{T}_\lambda$.

(2) For $f_A, g_B \in \mathcal{T}$, we have $(f_A \sqcap g_B) \sqcap X_\lambda = (f_A \sqcap X_\lambda) \sqcap (g_B \sqcap X_\lambda)$ for each $\lambda \in \Lambda$. Since $f_A, g_B \in \mathcal{T}$, for every $\lambda \in \Lambda$ we have $f_A \sqcap X_\lambda \in \mathcal{T}_\lambda, g_B \sqcap X_\lambda \in \mathcal{T}_\lambda$, and we know $(X_\lambda, \mathcal{T}_\lambda)$ is topological space, so $(f_A \sqcap X_\lambda) \sqcap (g_B \sqcap X_\lambda) \in \mathcal{T}_\lambda$. Then, $(f_A \sqcap X_\lambda) \sqcap (g_B \sqcap X_\lambda) = (f_A \sqcap g_B) \sqcap X_\lambda \in \mathcal{T}_\lambda$, and hence $(f_A \sqcap g_B) \in \mathcal{T}$.

(3) For every subfamily $\{f_A^1, f_A^2, ...\} \sqsubseteq \mathcal{T}$ and for each $\lambda \in \Lambda$ we have, $(\sqcup_{i\in I} f_A^i) \sqcap X_\lambda = \sqcup_{i\in I}(f_A^i \sqcap X_\lambda) \in \mathcal{T}_\lambda$, and hence $(\sqcup_{i\in I} f_A^i) \in \mathcal{T}$.

□

Definition 22. *The disjoint union $X = \sqcup_{\lambda\in\Lambda} X_\lambda$ is a fuzzy soft topological space with the fuzzy soft topology given in the above proposition 1. X is a fuzzy soft topological sum of a disjoint non-empty collection of fuzzy soft topological spaces X_λ. Hence, $(X, \mathcal{T})$ is a free union of $(X_\lambda, \mathcal{T}_\lambda)$ and is denoted by $(\uplus_{\lambda\in\lambda} X_\lambda, \mathcal{T})$.*

In this paper we understand $(X_\lambda, \mathcal{T}_\lambda)$, $\lambda \in \Lambda$, is the disjoint non-empty collection of fuzzy soft topological spaces indexed by a set Λ. $(X, \mathcal{T})$ is the fuzzy soft topological sum of collection $(X_\lambda, \mathcal{T}_\lambda)$, $\lambda \in \Lambda$ and is denoted by $(\uplus_{\lambda\in\lambda} X_\lambda, \mathcal{T})$. Now, we will give free union of a disjoint non-empty collection of fuzzy soft topological spaces. Definitions, theorems and some results for fuzzy soft topological sums have been obtained by using the known definitions and theorems for the fuzzy soft topological spaces.

Proposition 2. *Fuzzy soft topology $\mathcal{T}_\lambda$ is subfamily of fuzzy soft topology $\mathcal{T}$, for each $\lambda \in \Lambda$.*

Proof. Let $f_A \in \mathcal{T}_\lambda$. Then, we have $f_A \sqcap X_\lambda = f_A \in \mathcal{T}_\lambda$. Also, for $\lambda \neq \lambda'$, $f_A \sqcap X_{\lambda'} = \tilde{0}_E \in \mathcal{T}_\lambda$. Hence, we obtain $f_A \sqcap X_\lambda \in \mathcal{T}_\lambda$, for each $\lambda \in \Lambda$, and so $f_A \in \mathcal{T}$. □

Example 1. *Let $(X_\lambda, \mathcal{T}_\lambda)_{\lambda\in\Lambda}$ be a disjoint non-empty collection of fuzzy soft topological spaces. It is clear that $\sqcup_{i\in I} f_A^i \in \mathcal{T}$ for each $\lambda \in \Lambda$, $f_A^i \in \mathcal{T}_\lambda$. Also, by Proposition 2 we obtain $\mathcal{T} = \{\sqcup_{i\in I} f_A^i : \exists\lambda \in \Lambda, f_A^i \in \mathcal{T}_\lambda\}$. Suppose now that $\mathcal{T}_\lambda$ is indiscrete fuzzy soft topology. For every $\lambda \in \Lambda$, we have $X_\lambda \in \mathcal{T}$. Hence, $\mathcal{T}$ cannot be indiscrete fuzzy soft topology except for $\Lambda = 1$.*

Corollary 1. *Fuzzy soft topological space $(X, \mathcal{T})$ is fuzzy soft indiscrete spaces if and only if cardinality of Λ is equal to 1.*

Proposition 3. *Fuzzy soft topological space $(X, \mathcal{T})$ is fuzzy soft discrete space if and only if for each $\lambda \in \Lambda$, $\mathcal{T}_\lambda$ is fuzzy soft discrete.*

Proof. Because of $X = \sqcup_{\lambda\in\Lambda} X_\lambda$, X_λ is a fuzzy soft subset of X. However, for every $\lambda \in \Lambda$ and for $\mathcal{T}' = f_A \sqcap X_\lambda : f_A \in \mathcal{T}$, we must show that $\mathcal{T}_\lambda = \mathcal{T}'$.

On the other hand $f_A \in \mathcal{T}' \implies \exists g_A \in \mathcal{T} : f_A = g_A \sqcap X_\lambda \implies g_A \sqcap X_\lambda \in \mathcal{T}_\lambda \implies f_A \in \mathcal{T}_\lambda$ $h_A \in \mathcal{T}_\lambda \implies h_A \in \mathcal{T} \implies h_A = h_A \sqcap X_\lambda, h_A \in \mathcal{T} \implies h_A \in \mathcal{T}'$. □

Theorem 5. *Let $p_{x,i}$ be a fuzzy soft point and f_A, $f_{p_{x,i}}$ be fuzzy soft sets. If $f_A \sqcap X_j = f_{p_{x,i}}$ and $f_{p_{x,i}} \notin \mathcal{T}_j$, for fuzzy soft point $p_{x,i}$ and $j \in \Lambda$, f_A cannot be the neighborhood of $p_{x,i}$.*

Proof. Since $(X_\lambda, \mathcal{T}_\lambda)_{\lambda\in\Lambda}$ is the disjoint collection of fuzzy soft topological spaces, X_j is the only subset such that $p_{x,i} \in X_j$. On the contrary, f_A is the neighborhood of $p_{x,i}$ such that $f_A \sqcap X_j = f_{p_{x,i}}$ and $f_{p_{x,i}} \notin \mathcal{T}_j$. In this case, there is $g_A \in \mathcal{T}$ such that $p_{x,i} \in g_A \sqsubseteq f_A$. So $g_A \sqcap X_\lambda \in \mathcal{T}_\lambda$, for each $\lambda \in \Lambda$, i.e., $g_A \sqcap X_j = f_{p_{x,i}} \in \mathcal{T}_j$, for $j \in \Lambda$, contradiction. □

Proposition 4. *A fuzzy soft subset f_A of $(\uplus X_i, \mathcal{T})$ is fuzzy soft closed if and only if $f_A \sqcap X_i$ is a soft closed subset of $(X_i, \mathcal{T}_i)$ for every $i \in I$.*

Proof. $(f_A)^c \in \mathcal{T} \iff \forall i \in I, (f_A)^c \sqcap X_i \in \mathcal{T}_i \iff f_A \sqcap X_i$ is fuzzy soft closed set of $(X_i, \mathcal{T}_i)$. □

Corollary 2. *All fuzzy soft sets X_λ are fuzzy soft clopen in $(\uplus_{\lambda\in\Lambda} X_\lambda, \mathcal{T})$.*

Corollary 3. *Every sum of fuzzy soft topological spaces is fuzzy soft disconnected.*

Proposition 5. *If $\{(X_\lambda, \mathcal{T}_\lambda)\}_{\lambda\in\Lambda}$ is the disjoint collection of fuzzy soft topological spaces, and Y_λ is a subspace of X_λ for every $\lambda \in \Lambda$, then the fuzzy soft topology of the sum of subspaces $\{(X_\lambda, \mathcal{T}_{X_\lambda})\}_{\lambda\in\Lambda}$ and the fuzzy soft topological subspace on $\sqcup_{\lambda\in\Lambda} X_\lambda$ of the sum fuzzy soft topology $(\uplus_{\lambda\in\Lambda} X_\lambda, \mathcal{T})$ coincide.*

Proof. Straightforward. □

Theorem 6. *For fuzzy soft point $x \in X_\lambda \sqsubseteq X$ and $x \in A \sqsubseteq X$, A is fuzzy soft neighborhood of fuzzy soft point $x \in X$ such that $A \neq X_\lambda$ iff $A \sqcap X_\lambda$ is fuzzy soft neighborhood of $x \in X_\lambda$.*

Proof. Necessity: Let A be a fuzzy soft neighborhood of x in X. So, there is $U \in \mathcal{T}$ such that $x \in U \sqsubseteq A$. So that $A \sqcap X_\lambda$ is the fuzzy soft neighborhood of x in X_λ because of $x \in U \sqcap X_\lambda \sqsubseteq A \sqcap X_\lambda$ and $U \sqcap X_\lambda \in \mathcal{T}_\lambda$. *Sufficiency*: If $A \sqcap X_\lambda$ is the fuzzy soft neighborhood of x in X_λ, there is $U \in \mathcal{T}_\lambda$ such that $x \in U \sqsubseteq A \sqcap X_\lambda$. $U \in \mathcal{T}$, because of $\mathcal{T}_\lambda \sqsubseteq \mathcal{T}$. Also, we know that $U \sqsubseteq A \sqcap X_\lambda \sqsubseteq A$ and so A is neighborhood of x in X. □

Theorem 7. *Let $f_A \sqsubseteq X$. For $\bar{f_A}$ is fuzzy soft closure of f_A, $\bar{f_A} = \sqcup_{\lambda\in\Lambda} f_A \bar{\sqcap} X_\lambda$.*

Proof. Let $x \in \bar{f_A}$. Then, $x \in g_A$ such that $(g_A)^c \in \mathcal{T}$ for each $f_A \sqsubseteq g_A$. Hence, $x \in g_A \sqcap X_\lambda$ such that $(g_A \sqcap X_\lambda)^c \in \mathcal{T}_\lambda$, for at least one $\lambda \in \Lambda$. So, $x \in \sqcup_{\lambda\in\Lambda} f_A \bar{\sqcap} X_\lambda$.

Let $x \in \sqcup_{\lambda\in\Lambda} f_A \bar{\sqcap} X_\lambda$. Then $x \in f_A \bar{\sqcap} X_\lambda$, for at least one $\lambda \in \Lambda$. So $x \in \bar{f_A}$ because of $f_A \sqcap X_\lambda$. □

Theorem 8. *Let $f_A \sqsubseteq X$. For $(f_A)^o$ is fuzzy soft interior of f_A, $(f_A)^o = \sqcup_{\lambda\in\Lambda}(f_A \sqcap X_\lambda)^o$.*

Proof. Let $x \in (f_A)^o$. Then, $x \in U$ such that $U \in \mathcal{T}$ for at least one $U \sqsubseteq f_A$. Therefore, $x \in U \sqcap X_\lambda$ for at least one $\lambda \in \Lambda$ and $U \sqsubseteq f_A$. So, $x \in U \sqcap X_\lambda \sqsubseteq f_A \sqcap X_\lambda$ such that $x \in U \sqcap X_\lambda$. Thus, $x \in \sqcup_{\lambda\in\Lambda}(f_A \sqcap X_\lambda)^o$, so $(f_A)^o \sqsubseteq \sqcup_{\lambda\in\Lambda}(f_A \sqcap X_\lambda)^o$.

Let $x \in \sqcup_{\lambda\in\Lambda}(f_A \sqcap X_\lambda)^o$. Then, $x \in (f_A \sqcap X_\lambda)^o$, for at least one $\lambda \in \Lambda$. So, $x \in (f_A)^o$ because of $f_A \sqcap X_\lambda \sqsubseteq f_A$. So, $\sqcup_{\lambda\in\Lambda}(f_A \sqcap X_\lambda)^o \sqsubseteq (f_A)^o$. □

Theorem 9. *The family $\mathcal{B} = \{f_A \sqsubseteq X : \exists\lambda \in \Lambda, f_A \in \mathcal{T}_\lambda\}$ is the fuzzy soft base of $(\uplus_{\lambda\in\Lambda} X_\lambda, \mathcal{T})$ topological space.*

Proof. We know that for every $\lambda \in \Lambda, X_\lambda \in \mathcal{T}_\lambda$ and $\sqcup_{\lambda\in\Lambda} X_\lambda = X$. On the other hand, let $f_A, g_B \in \mathcal{B}$. So, $\exists\lambda_i, \lambda_j \in \Lambda, f_A \in \mathcal{T}_{\lambda_i}, g_B \in \mathcal{T}_{\lambda_j}$. For $i \neq j$, the condition is obvious. Let $i = j$ and $f_A \sqcap g_B \neq \tilde{0}$. Then, $f_A \sqcap g_B \in \mathcal{T}_{\lambda_i}$, and so $f_A \sqcap g_B \in \mathcal{B}$. □

Theorem 10. *Let the family $\mathcal{B}_\lambda, \lambda \in \Lambda$, be a fuzzy soft base of $X\lambda$ fuzzy soft topological spaces. Then, $\mathfrak{B} = \{f_A : \exists\lambda \in \Lambda, f_A \in \mathcal{B}_\lambda\}$ is the fuzzy soft base of $(\uplus_{\lambda\in\Lambda} X_\lambda, \mathcal{T})$ topological space.*

Proof. Since $\mathcal{B}_\lambda$ is the fuzzy soft base of X_λ fuzzy soft topological spaces, it is obvious that $\sqcup_{\mathcal{T}\in\mathcal{B}_\lambda} \mathcal{T} = X$.

It is obvious that each $\mathcal{B}_\lambda$ is discrete, $\forall\lambda \in \Lambda$. On the other hand, let $f_A \in, g_B \in \mathcal{B}_i$ and $f_A \sqcap g_B \neq \tilde{0}$ for $i \in \lambda$. As $\mathcal{B}_i$ is a fuzzy soft base of X_i, there exist $h_C \in \mathcal{B}_i$ such that $x \in h_C \sqsubseteq f_A \sqcap g_B, \forall x \in f_A \sqcap g_B$. □

Definition 23. *Let $(x_n)_{n\in\mathbb{N}}$ be fuzzy soft sequence of $X = (\uplus_{\lambda\in\Lambda} X_\lambda, \mathcal{T})$ and $b \in X$. For every $T^(k) \in \mathcal{T}_\lambda$ containing b, $(x_n) \to b \iff \exists\lambda \in \Lambda, \exists N_k \in \mathbb{N} : n \geq N_k \implies x_n \in T^(k)$. We say that (x_n) converges to b, or b is limit point of (x_n).*

Theorem 11. *The convergent sequence of $(X_\lambda, \mathcal{T}_\lambda)$ converges in $(\uplus_{\lambda\in\Lambda} X_\lambda, \mathcal{T})$ also.*

Proof. Let (x_n) be such a fuzzy soft sequence of X_λ that converges the point $b \in X_\lambda$ in X_λ. x_n also is a fuzzy soft sequence of $\uplus_{\lambda\in\Lambda} X_\lambda$ because of $X_\lambda \sqsubseteq \uplus_{\lambda\in\Lambda} X_\lambda$. From the definition of convergence we have $\forall (f_A)^\lambda \in \mathcal{T}_\lambda(b \in (f_A)^\lambda), \exists (n_0)^\lambda \in \mathbb{N} : n \geq (n_0)^\lambda \implies x_n \in (f_A)^\lambda$. On the other hand, $\forall f_A \in \mathcal{T}(b \in f_A)$ we know that $f_A = (f_A)^\lambda$ or $f_A \sqsupseteq (f_A)^\lambda$. So, for $n_0 = (n_0)^\lambda$, if $n \geq n_0, x_n \in f_A$. Then, (x_n) converges in $\uplus_{\lambda\in\Lambda} X_\lambda$. We note that the sequence of $\uplus_{\lambda\in\Lambda} X_\lambda$ need not be converging in X_λ. □

The following example shows that the converse of the previous theorem is not true.

Example 2. *Let $A = \{e_1, e_2, e_3\}$, $X_1 = \{1, 2, 3\}, X_2 = \{a, b\}, X_3 = \{x, y, z\}$ and $(X_1, \mathcal{T}_1), (X_2, \mathcal{T}_2), (X_3, \mathcal{T}_3)$ be fuzzy soft discrete topological spaces. Then, $(X, \mathcal{T})$ is a fuzzy soft discrete topological space for $X = \uplus_{\lambda\in\Lambda} X_\lambda, \Lambda = \{1, 2, 3\}$. $(x_n) = (1, a, x, y, y, y, ...)$ is a fuzzy soft sequence of X and converges to $y \in X$. However, (x_n) is a fuzzy soft sequence of neither X_1 nor X_2, X_3. So, convergence of the sequence (x_n) cannot be mentioned in any of the spaces X_1, X_2, X_3.*

Let us define the set $Z_{f_A} = \{n \in \mathbb{N} : x_n \notin f_A\}$, for (x_n) is a sequence of $\uplus_{\lambda\in\Lambda} X_\lambda$ and $f_A \in \mathcal{T}$. Now, we will talk about a different approach for convergence with $\max Z_{f_A}$. Also, we will take $\max Z_{f_A} = 1$ while $Z_{f_A} = \tilde{0}$.

Theorem 12. *Let (x_n) be a sequence of $\uplus_{\lambda\in\Lambda} X_\lambda$. (x_n) converges the point $b \in \uplus_{\lambda\in\Lambda} X_\lambda$ iff there exists a $\max Z_{f_A}$ for each $f_A \in \mathcal{T}(b \in f_A)$.*

Proof. Let there be a $\max Z_{f_A}$ for each $f_A \in \mathcal{T}(b \in f_A)$. If the number N_k in Definition 23 is chosen as $N_k = \max\{n_{f_A} : n_{f_A} = \max Z_{f_A}, f_A \in \mathcal{T}, b \in f_A\}$, then (x_n) converges the point $b \in \uplus_{\lambda\in\Lambda} X_\lambda$ in the meanings given in Definition 23.

Let (x_n) be a sequence of $\uplus_{\lambda\in\Lambda} X_\lambda$. (x_n) converges the point $b \in \uplus_{\lambda\in\Lambda} X_\lambda$. Then, we can write from Definition 23, for every $T^(k) \in \mathcal{T}_\lambda$ containing b, $(x_n) \to b \iff \exists\lambda \in \Lambda, \exists N_k \in \mathbb{N} : n \geq N_k \implies x_n \in T^(k)$. So, there exists a $\max Z_{f_A}$ for each $f_A \in \mathcal{T}(b \in f_A)$. □

Theorem 13. *Let (x_n) be a sequence of $\uplus_{\lambda\in\Lambda} X_\lambda$, $\lambda, \mu \in \Lambda$, $b \in X_\lambda$ and $b' \in X_\mu$. If b and b' is limit point for (x_n), then $\lambda = \mu$.*

Proof. On the contrary, let $\lambda \neq \mu$. Then, $X_\lambda \sqcap X_\mu = \tilde{0}$. Since (x_n) converges the point $b \in X_\lambda$, for $n \geq n_0$, there exist $n_0 \in \mathbb{N}$ such that $x_n \in X_\lambda$. On the other hand, since (x_n) converges the point $b' \in X_\mu$, for $n \geq m_0$, there exist $m_0 \in \mathbb{N}$ such that $x_n \in X_\mu$. Let $p_0 = \max\{n_0, m_0\}$. For $n \geq p_0, x_n \in X_\lambda \sqcap X_\mu$, contradiction. □

Proposition 6. *Let $\{(X_i, \mathcal{T}_i) : i \in I, I = \{1, 2, ..., n\}\}$ be a finite family of pairwise disjoint fuzzy soft topological spaces. For every $i \in \{1, 2, ..., n\}$, $(X_i, \mathcal{T}_i)$ is fuzzy soft compact space if and only if the sum of fuzzy soft topological spaces $(\uplus_{i\in I} X_i, \mathcal{T})$ is fuzzy soft compact.*

Proof. Necessity: Let $f_A^j, j \in J$ be a fuzzy soft open cover of $X = \sqcup_{i=1}^n X_i$. So, $X_i = \sqcup_{j\in J}(f_A^j \sqcap X_i), \forall i \leq n$. We know that $(X_i, \mathcal{T}_i)$ is fuzzy soft compact for every $i \leq n$. Hence, there exist finite subsets $S_1, S_2, ..., S_n$ of J such that $X_1 = \sqcup_{j\in S_1}(f_A^j \sqcap X_1), X_2 = \sqcup_{j\in S_2}(f_A^j \sqcap X_2), ..., X_n = \sqcup_{j\in S_n}(f_A^j \sqcap X_n)$. Then, for $S = \sqcup_{i=1}^n S_i$, $X = \sqcup_{j\in S}(f_A^j \sqcap X_i), \forall i \leq n$. $(\uplus_{i\in I} X_i, \mathcal{T})$ is fuzzy soft compact because S is finite.

Sufficiency: $(X_i, \mathcal{T}_i)$ is a fuzzy soft closed subspace of $(\uplus_{i\in I} X_i, \mathcal{T})$ because of Corollary 2. Since a fuzzy soft closed set in a fuzzy soft compact space is also fuzzy soft compact, for every $i \in I$, $(X_i, \mathcal{T}_i)$ is fuzzy soft compact. □

Proposition 7. *Let $\{(X_\lambda, \mathcal{T}_\lambda) :\}_\lambda \in \Lambda$ be a family of nonempty pairwise disjoint fuzzy soft topological spaces. Then, the sum of fuzzy soft topological spaces $(\uplus_{i\in I} X_i, \mathcal{T})$ is disconnected.*

Proof. Straightforward. □

4. Conclusions

Topology is an important and major area of mathematics, and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied and improved the soft set theory, which was initiated by Molodtsov [2] and easily applied to many problems having uncertainties from social life. In this paper, we first gave the definition of the "fuzzy soft topology" and then presented its basic properties with some examples. Then, we defined free union of fuzzy soft topological spaces and disjoint union topology of fuzzy soft topological spaces. We call the free union of a pairwise disjoint non-empty family of fuzzy soft topological spaces the sum of fuzzy soft topological spaces. We show what are the interchangeable properties between fuzzy soft topological spaces and the sum of fuzzy soft topological spaces. Also, we provide some properties which are considered a link between fuzzy soft topological spaces and their sum. With this paper, definitions, theorems and some results for fuzzy soft topological sum have been obtained by using the known definitions and theorems for the fuzzy soft topological spaces.

The following are the articles that will guide us in our paper on the homogeneity of fuzzy soft topological sums and separation axioms in fuzzy soft topological sums, which we will discuss in our future studies. The first one is [30]: This article deals with the soft homogeneity of soft topological space produced by a family of topological spaces and the similarity between some soft topological concepts and general topological concepts. The second one is [17]: The aim of this article, which describes and gives some properties of some soft topological operators, is to characterize a few soft separation axioms. A new soft separation axiom is defined in this article.

Conflict of interest

The author declare that he has no conflict of interest.

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