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Research article

A fuzzy based solution to multiple objective LPP

Junaid Basha M and Nandhini S*

Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu 632014, India

* **Correspondence:** Email: nandhini.s@vit.ac.in.

Abstract: This study presents a Fuzzy Multiple Objective Linear Programming Problem (FMOLPP) method to solve the Linear Programming Problem (LPP). Initially Multiple Objective Linear Programming Problem (MOLPP) is solved using Chandra Sen's approach along with various types of mean approaches. Furthermore, FMOLPP is solved using Chandra Sen's approach and various categories of fuzzy mean techniques. The simplex form is used to solve the LPP, where the three-tuple symmetric triangular fuzzy number with the constraints of the fuzzy objective function is considered. We have presented a comparative study of optimum values of MOLPP with optimum values of FMOLPP, to show the significance of our proposed method.

Keywords: MOLPP; mean techniques; fuzzy MOLPP; symmetric triangular fuzzy number; fuzzy mean methods; Deffuzification Mean of Maxima (MOM)

1. Introduction

Mathematicians use linear programming to simulate and resolve a wide range of challenging real-world issues involving the solution of an objective function which is subject to some restrictions. A primary objective function might be to maximize or minimize. In the present situation, many researchers' focus has shifted to the MOLPP, which incorporates several inconsistent objective functions. In this paper, we present a novel framework to solve MOLPP using multiple fuzzy average approaches, including arithmetic, quadratic, geometric, harmonic, and heronian mean. In addition, to present a comparison of the results, a state-of-the-art algorithm has been proposed with novel mean approaches.

2. Materials and methods

In a 2014 study [1], fuzzy concepts of a lexicographic sequence relationship are used to compare two imperious triangular fuzzy numbers in sequence to check the optimality of the objective function. The Fully Fuzzy Multiple Objective Linear Programming Problem (FFMOLPP) is replaced by a crisp Multi-Level; Multiple Objective Linear Programming Problem (MMOLPP) based on the sequence relationship which can be concluded step by step using the min operator. In this work the FFMOLPP with triangular fuzzy numbers for all parameters and variables is considered. The values of the objective function are also triangular fuzzy numbers. The min operator is used to solve multi-level multi-objective linear programming problems, and the fuzzy optimal solution of the Multiple Objective Fully Fuzzy Linear Programming Problem is obtained. This methodology suggests three approaches for solving the auxiliary multi-objective programming: an optimistic approach, a pessimistic approach, and a linear sum approach based on the membership function. To demonstrate how this strategy is put into practice, a transportation issue and an investment case are given. According to the comparison study, conceivable linear programming with triangular fuzzy numbers cannot be generalized, as efficiently as the fuzzy linear programming with TrFNs created in this work [2]. The 2015 concept [3], developed a new fuzzy mathematical programming technique for solving heterogeneous multi-attribute decision-making problems based on the Linear Programming Technique for Multidimensional Analysis of Preference and hesitancy degrees on pair-wise comparisons of alternatives as interval-valued intuitionistic fuzzy (IVIF) sets (IVIDSs). IVIFSs of ordered pairs of alternatives express the preference relations between the decision maker's provided options. By comparing alternatives using IVIF truth degrees, consistency and inconsistency indices are defined as IVIFSs. In this research [4], a separate interval-valued intuitionistic fuzzy (IVIF) mathematical programming approach for hybrid multi-criteria group decision-making (DM) using alternative evaluations and hesitation degrees is developed. The complete relative intimacy points of the alternatives to such a fuzzy, positive optimal solution are then utilized to complete the instance-based system of the alternatives provided by each decision-making process.To address these issues, an intuitionistic fuzzy-linear goal approach is suggested. Priority weights are first derived by IFPRs using an intuitionistic fuzzy linear goal exemplary. This intuitionistic fuzzy linear goal example is solved using three different strategies: optimistic, pessimistic, and varied strategies. These strategies are dependent on the formulation of non-membership functions [5]. The 2016 study [6] propose a certain class of hybrid multi-attribute decision-making (MADM) issues with partial attribute substance data presents a tentative fuzzy programming approach based on linear programming (LINMAP). By using pair-wise comparisons employing hesitant fuzzy accuracy values, the DM in this approach assigns desires to variants .

The focus of 2017 work [7] is on the FFMOLPP, in which all measures and decision variables are LR flat fuzzy numbers, the further generalized performance of fuzzy numbers, and all constraints are fuzzy inequalities. A new strategy for solving the FFMOLPP has been given in this study, which first turns the fuzzy problem into a Multiple Objective Interval Linear Programming Problem (MOILPP) by utilizing inner interval approximations of fuzzy numbers to avoid traps of important information. The interval programming problem is then turned into an LPP employing the center, width, and scalarization techniques. Thus, proving that the Fuzzy Pareto optimal solution of the FFMOLPP is the optimal solution of the LPP. In this approach [8], various kinds of assessment data on criteria are expressed, respectively, by intuitionistic fuzzy valuation, hesitant fuzzy sets, trapezoidal fuzzy values, intermissions, and actual values. An established multi-objective assignment model is modified into a single-objective goal that is exemplary in structure to obtain the overall ranking sequence of alternatives. In order to solve hybrid MCGDM, a novel fuzzy mathematical goal approach is suggested. In this research [9], proposes a new interval-valued fuzzy preference relations (IVFPR) programming approach for solving group decision-making (GDM) problems. An Inter-valued Fuzzy programming problem is constructed to determine the priority weights of alternatives in the context of additive and consistent IVFPR. The Intuitionistic Fuzzy (IF) limitations, in this case, are seen as additive and consistent conditions. The Pythagorean fuzzy (PF) mathematical goal strategy is advanced in this study [10] to address MADM issues in PF settings. The primary job can be broken down into four aspects. The ability of the suggested strategy is then confirmed by analyzing a case study of a green quantity sequence. The suggested approach in this research is not only computationally efficient, but it can also appropriately explain the acceptable quality of the fuzzy restrictions. In this study [11], fuzzy knapsack issue and the investment problem, as well as the validity and perfection of the recommended strategy, are demonstrated. Finally, a decision-support system is created for the suggested methodology. This 2019 study [12] proposes the usage of duality to simplify secularizing multiple objective functions. Both multi-objective optimization strategies are identical in that they achieve all four objectives at the same time. As a result, multiple objective optimization outcomes can be handled by creating a multiple objective functions after all the objective functions have been transformed to a specific maximizing or minimizing mode. It has been demonstrated that duality facilitates the observation of multiple objective functions. The current means for solving MOO problems are evaluated in this work. In order to pick an engineering project portfolio, a new two-stage strategy for fuzzy multi-objective linear program is developed in this study. All of the technological coefficients, resources, and goal coefficients in the fuzzy multi-objective linear program are trapezoidal fuzzy numbers (TrFNs). The interval expectation of TrFNs is used to introduce an order connection for them. The suggested approach is not only rigorous theoretically, but it can also take into account the decision maker's level of acceptance of the possibility that the fuzzy restrictions may be exceeded [13].

This 2020 study proposes improved averaging techniques based on the geometric mean and harmonic mean. These techniques were tested using appropriate examples and found to be superior to existing MOO averaging techniques. The findings reveal that improved averaging techniques outperform existing averaging strategies in tackling MOO problems [14]. This paper's main concern is group decision-making issues with interval-valued Atanassov intuitionistic fuzzy preference relations (IV-AIFPRs). To measure an AIFPR's incremental accuracy degree, a new structure has been devised. Then, an IV-AIFPR is divided into two AIFPRs to define an incremental stability definition and an appropriate additive continuity definition, respectively. To confirm the competence of the suggested strategy, a structural design illustration is tested [15].This article [16], attempts to address three significant and challenging problems: How to build a 0–1 mixed integer linear program-based emergency distribution center (EDC) location, model for large-scale, emergencies how to develop a workable method for sharpening the trapezoid ally fuzzy EDC location model, how to present evidence for the constructed model's reliability, adaptability, and superiority. To support the adaptability and advantages of the suggested method, sensitivity and comparison studies are also included.

An innovative method for analyzing or testing defuzzification methods is presented with varying asymmetries, amounts of overlapping, α-cut values, and types of membership functions being produced and tried. The final result of a fuzzy system is determined by the defuzzification procedure. The defuzzification value is chosen from the core by maxima-based defuzzification procedures. Because Best of Maxima (BM) prefers one of the maxima first, last, or center as a conclusion, because Proportionate Maxima (PM) prefers a conclusion based on the difference between the left and right areas in balance with the center area, PM can address asymmetries in regions more precisely while maintaining continuity. From extensive testing and analysis, the proposed approaches are more productive than previous maxima approaches and are comparable to the Center of Gravity (COG) method [17]. This 2022 study [18] demonstrates the application of the Fuzzy Harmonic mean approach to conclude Fully Fuzzy Multilevel; Multiple Objective Linear Programming (FFMMLP) issues using the Fuzzy Harmonic mean approach. Initially, using the crisp linear approach, the FFMMLP issue is transformed into three crisp MOLPP at each level. The Fuzzy Harmonic Mean approach is then employed to combine the multiple objectives of each crisp problem into a single objective. Secondly, the harmonic mean for each stage is used to generate the final single-objective problem. Finally, the problem is solved by obtaining a fuzzy compromise result for the FFMMLP overall. An extensive study has been done on LPP and Chandra Sen's mean techniques, including the Harmonic method, Average method, Defuzzification method, FFMMLP, Multiple Objective Interval Linear Programming (MOILP), and Priority of MOLPP. Their characteristics and drawbacks are also discussed. It is evident from analyzing these MOLPP and Chandra Sen's mean models that the MOLPP and Chandra Sen's Fuzzy mean models will need to be strengthened and improved to deliver the optimal outcome. In order to address the need of the hour, we have presented an FMOLPP which gives optimum results when compared with the previous models.

The remaining research is organized as follows: 2. Preliminaries, 3. Solving MOLPP by Chandra Sen's approach, 4. An average method algorithm for multiple objective LPP, 5. Numerical example 6. A proposed algorithm for solving FMOLPP using new average methods, 7. Defuzzification, 8. Numerical example and 9. Conclusions.

3. Preliminaries

3.1. Fuzzy set

Let S be a non-empty set. A fuzzy set T in S is identified by its membership function $\mu_{\tilde{T}}(z)$: $S \rightarrow [0,1]$ and $\mu_{\tilde{T}}(z)$ is described as the degree to which an element is such a member z in fuzzy set T for each $z \in S$. Then a fuzzy set T in S is a collection of ordered pairs [19].

$$\tilde{T} = \{(z, \mu_{\tilde{T}}(z)/z \in S\}.$$

3.2. Fuzzy number

A fuzzy number is preceding to be a non-negative if and only if $\mu_{\tilde{T}}(z) = 0, \forall z = 0$.

A fuzzy set $\tilde{T}$ on R must possess at least one of the three properties.

(1) $\tilde{T}$ must be a normal fuzzy set;

(2) $\tilde{T}$ must be a convex fuzzy set; and

(3) It should be piecewise continuous.

3.3. Triangular fuzzy number

A triangular fuzzy number is represented as $\tilde{T} = (t_1, t_2, t_3)$. Which satisfies the following conditions:

(1) t_1 to t_2 is an increasing function;

(2) t_2 to t_3 is a decreasing function; and

(3) $t_1 \leq t_2 \leq t_3$.

$$\mu_{\tilde{T}}(z) = \begin{cases} 0, & \text{for} \quad z < t_1 \\ \frac{z-t_1}{t_2-t_1}, & \text{for} \quad t_1 \leq z \leq t_2 \\ \frac{t_3-z}{t_3-t_2}, & \text{for} \quad t_2 \leq z \leq t_3 \\ 0, & \text{for} \quad z > t_3. \end{cases}$$

The membership function $\mu_{\tilde{T}}(z)$ satisfies the following characteristics :

$$z_2 > z_1 \Rightarrow \mu_{\tilde{T}}(z_2) > \mu_{\tilde{T}}(z_1) \; \forall z_1, z_2 \in [t_1, t_2].$$

$$z_2 > z_1 \Rightarrow \mu_{\tilde{T}}(z_2) < \mu_{\tilde{T}}(z_1) \; \forall z_1, z_2 \in [t_2, t_3].$$

If $t_2 - t_1 = t_3 - t_2$, then it is a symmetrical Triangular Fuzzy number otherwise, is said to be a asymmetrical Triangular Fuzzy Number [20].

3.4. Operation of triangular fuzzy number

Let $\tilde{T} = (t_1, t_2, t_3)$ and $\tilde{S} = (s_1, s_2, s_3)$. Then

(1) Addition :
$\tilde{T} + \tilde{S} = (t_1 + s_1, t_2 + s_2, t_3 + s_3)$.

(2) Subtraction :
$\tilde{T} - \tilde{S} = (t_1 - s_3, t_2 - s_2, t_3 - s_1)$.

(3) Multiplication:
$\tilde{T} \times \tilde{S} = (\min(t_1 s_1, t_1 s_3, t_3 s_1, t_3 s_3), t_2 s_2, \; \max(t_1 s_1, t_1 s_3, t_3 s_1, t_3 s_3))$.

(4) Division :
$\tilde{T}/\tilde{S} = (\min(t_1/s_1, t_1/s_3, t_3/s_1, t_3/s_3), t_2/s_2, \; \max(t_1/s_1, t_1/s_3, t_3/s_1, t_3/s_3))$.

4. Solving MOLPP by Chandra Sen's approach

Consider our MOLPP : The primary goal of this research is to solve MOLPP.
MOLPP has the following mathematical form.

$$\text{Max } p_1 = K_1^r + g_1.$$
$$\text{Max } p_2 = K_2^r + g_2.$$

....

....

$$\text{Max } p_g = K_g^u + g_g.$$
$$\text{Min } p_{g+1} = K_{g+1}^r + g_{g+1}.$$
$$\text{Min } p_{g+2} = K_{g+2}^r + g_{g+2}.$$

....

$$\text{Min } p_h = K_h^u + g_h.$$

Subject to

$$T\tilde{u} = \tilde{v}.$$
$$\tilde{u} \geq 0.$$

Where $\tilde{v}$ is a m - dimensional vector of constants, $\tilde{u}$ is a n - dimensional vector of decision variables, moreover, T is a $m \times n$ constants matrix. Both types of objective functions are required.

In Sen's approach, the Multiple Objective functions are solved individually using the Simplex method [21].

$$\begin{aligned} \text{Max } p_1 &= \chi_1. \\ \text{Max } p_2 &= \chi_2. \\ \ldots\ldots\ldots\ldots &\ldots \ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots &\ldots \ldots\ldots\ldots \\ \text{Max } p_g &= \chi_g. \\ \text{Max } p_{g+1} &= \chi_{g+1}. \\ \text{Max } p_{g+2} &= \chi_{g+2}. \\ \ldots\ldots\ldots\ldots &\ldots \ldots\ldots\ldots \\ \text{Max } p_h &= \chi_h. \end{aligned}$$

Where $\chi_1, \chi_2, \ldots\ldots, \chi_h$ are the optimal valuations of the objective function.

By summing (for maximization) and deducting (for minimization) the results of dividing every p_k by χ, these valuations are employed to structure a single objective function. Mathematically,

$$\text{Max } p = \sum_{k=1}^{g} \frac{p_k}{|\chi_k|} - \sum_{k=g+1}^{h} \frac{p_k}{|\chi_k|}, \tag{4.1}$$

where, $|\chi_k| \neq 0$.

Subject to the constraints, equations remain the same.

Then the linear programming problem with a single objective is optimized.

5. Average method algorithm for multiple objective LPP

The steps involved in the algorithm are

(1) Apply the simplex approach to determine the optimum valuations for every objective function in Eq (4.1).

(2) Tab the feasibility of step one; if the solution is feasible, proceed to step three; else, apply the dual simplex method.

(3) Let Max $p_k = \chi_k, k = 1, 2, ...g$ and Min $p_k = \chi_k, k = g + 1, g + 2, ...h$.

(4) Calculate B_1 & B_2 where $B_1 = \max(|\chi_k|), k = 1, 2, ..., g$ and $B_2 = \min(|\chi_k|), k = g+1, g+2, ..., h$.

(5) Calculate the values of Arithmetic Mean method (AM_{avg}), Quadratic Mean method (QM_{avg}), Geometric Mean method (GM_{avg}), Harmonic Mean method (HM_{avg}) and Heronian Mean method (HeM_{avg}).

(6) Using the same constraints as before, optimize the combined objective function [21]:

$$\text{Max } p = \left(\sum_{k=1}^{g} p_k - \sum_{k=g+1}^{h} p_k\right) / AM_{avg}. \tag{5.1}$$

$$\text{Max } p = \left(\sum_{k=1}^{g} p_k - \sum_{k=g+1}^{h} p_k\right) / QM_{avg}. \tag{5.2}$$

$$\text{Max } p = \left(\sum_{k=1}^{g} p_k - \sum_{k=g+1}^{h} p_k\right) / GM_{avg}. \tag{5.3}$$

$$\text{Max } p = \left(\sum_{k=1}^{g} p_k - \sum_{k=g+1}^{h} p_k\right) / HM_{avg}. \tag{5.4}$$

$$\text{Max } p = \left(\sum_{k=1}^{g} p_k - \sum_{k=g+1}^{h} p_k\right) / HeM_{avg}. \tag{5.5}$$

5.1. Arithmetic Mean method

$$\text{Max } p = \left(\sum_{k=1}^{g} p_k - \sum_{k=g+1}^{h} p_k\right) / AM_{avg}, \tag{5.6}$$

where, $AM_{avg} = \frac{B_1+B_2}{2}$.

5.2. Quadratic Mean method

$$\text{Max } p = \left(\sum_{k=1}^{g} p_k - \sum_{k=g+1}^{h} p_k\right) / QM_{avg}, \tag{5.7}$$

where, $QM_{avg} = \sqrt{\frac{1}{2}(B_1^2 + B_2^2)}$.

5.3. Geometric Mean method

$$\text{Max } p = \left(\sum_{k=1}^{g} p_k - \sum_{k=g+1}^{h} p_k\right) / GM_{avg}, \tag{5.8}$$

where, $GM_{avg} = \sqrt{B_1 \times B_2}$.

5.4. Harmonic Mean method

$$\text{Max } p = \left(\sum_{k=1}^{g} p_k - \sum_{k=g+1}^{h} p_k\right) / HM_{avg}, \tag{5.9}$$

where, $HM_{avg} = \frac{2}{\frac{1}{B_1} + \frac{1}{B_2}}$.

5.5. Heronian Mean method

$$\text{Max } p = \left(\sum_{k=1}^{g} p_k - \sum_{k=g+1}^{h} p_k\right) / HeM_{avg}, \tag{5.10}$$

where, $HeM_{avg} = \frac{1}{3}(B_1 + \sqrt{B_1 B_2} + B_2)$.

6. Numerical example

Consider the MOLPP:

$$\text{Max } p_1 = 2u_1 + u_2$$

$$\text{Max } p_2 = u_1$$

$$\text{Min } p_3 = -3u_1 - 2u_2$$

$$\text{Min } p_4 = -u_1 - u_2.$$

Subject to

$$8u_1 + 6u_2 \leq 45$$

$$2u_1 + u_2 \leq 6$$

$$u_1 \geq 3$$

$$u_1 + 2u_2 \leq 8$$

$$u_1, u_2 \geq 0.$$

The procedure to find the optimum valuation of the individual objective function using the simplex method.

First objective function:

$$\text{Max } p_1 = 2u_1 + u_2.$$

Subject to

$$8u_1 + 6u_2 + u_3 = 45$$
$$2u_1 + u_2 + u_4 = 6$$
$$-u_1 + u_5 = -3$$
$$u_1 + 2u_2 + u_6 = 8$$
$$u_1, u_2, u_3, u_4, u_5, u_6 \geq 0.$$

The optimized value is Max $p_1 = 11.25$.
Second objective function:

$$\text{Max } p_2 = u_1.$$

Subject to

$$8u_1 + 6u_2 + u_3 = 45$$
$$2u_1 + u_2 + u_4 = 6$$
$$-u_1 + u_5 = -3$$
$$u_1 + 2u_2 + u_6 = 8$$
$$u_1, u_2, u_3, u_4, u_5, u_6 \geq 0.$$

The optimized value is Max $p_2 = 5.625$.
Third objective function:

$$\text{Min } p_3 = -3u_1 - 2u_2.$$

Subject to

$$8u_1 + 6u_2 + u_3 = 45$$
$$2u_1 + u_2 + u_4 = 6$$
$$-u_1 + u_5 = -3$$
$$u_1 + 2u_2 + u_6 = 8$$
$$u_1, u_2, u_3, u_4, u_5, u_6 \geq 0.$$

The optimized value is Min $p_3 = -16.875$.
Fourth objective function:

$$\text{Min } p_4 = -u_1 - u_2.$$

Subject to

$$8u_1 + 6u_2 + u_3 = 45$$
$$2u_1 + u_2 + u_4 = 6$$
$$-u_1 + u_5 = -3$$
$$u_1 + 2u_2 + u_6 = 8$$
$$u_1, u_2, u_3, u_4, u_5, u_6 \geq 0.$$

The optimized value is Min $p_4 = -6.1$.

Table 1. Initial table.

i	χ_k	$\lvert\chi_k\rvert$	Values of B_1 and B_2
1	11.25	11.25	$B_1 = 11.25$
2	5.625	5.625	
3	-16.875	16.875	$B_2 = 6.1$
4	-6.1	6.1	

Now we only have to prove Eq (5.9):

Let $B_1 = \max\ (|\chi_k|), k = 1, 2, ..., g$, and

$B_2 = \min\ (|\chi_k|), k = g + 1, g + 2, ..., h.$

Harmonic Mean [21]:

$HM_{avg} = \left(\frac{2}{\frac{1}{11.25} + \frac{1}{6.1}}\right)$

$HM_{avg} = 7.905.$

Now for New Harmonic Mean Methods [21].

Max $p = \{(p_1 + p_2) - (p_3 + p_4)\}/HM_{avg}.$

$= \{(2u_1 + u_2 + u_1) - (-3u_1 - 2u_2 - u_1 - u_2)\}.$

Max $p = (7u_1 + 4u_2)/7.905.$

$$\text{Max } p = 0.886u_1 + 0.506u_2.$$

Subject to

$$8u_1 + 6u_2 + u_3 = 45$$

$$2u_1 + u_2 + u_4 = 6$$

$$-u_1 + u_5 = -3$$

$$u_1 + 2u_2 + u_6 = 8$$

$$u_1, u_2, u_3, u_4, u_5, u_6 \geq 0.$$

The optimized value of MOLPP is Max $p = 4.984$.

Through the above mean approaches we have proved the harmonic mean approach alone and the remaining mean approaches are calculated in the following table.

7. A proposed algorithm for solving FMOLPP using new average methods

The steps involved in the algorithm are:

(1) Here, we define constraints of the objective functions of $\tilde{u_1}, \tilde{u_2}, \tilde{u_3}, ..., \tilde{u_n}$ and $\tilde{s_k}$ (Procedure $z_j - c_j$ is denoted as $\tilde{s}_k$ in the ordinary simplex method) as a fuzzy number in the form of a symmetric triangular fuzzy number.

(2) Apply the simplex approach to obtain the optimum valuation of the individual fuzzy objective functions.

(3) Tab the feasibility of step one; if the solution is feasible, proceed to step three; else, apply the fuzzy dual simplex method [22].

(4) Let Max $\tilde{p_k} = \tilde{\chi_k}, k = 1, 2, ...g$ & Min $\tilde{p_k} = \tilde{\chi}_k,\ j = g + 1, g + 2, ...h$.

(5) Calculate $\tilde{B_1}$ & $\tilde{B_2}$ where $\tilde{B_1}$ = max $(|\tilde{\chi_k}|), k = 1, 2, ..., g$ and $\tilde{B_2}$ = min $(|\tilde{\chi_k}|), k = g+1, g+2, ...h$.

(6) Calculate the values of the Fuzzy Arithmetic Mean method(FAM_{avg}), Fuzzy Quadratic Mean method (FQM_{avg}), Fuzzy Geometric Mean method (FGM_{avg}), Fuzzy Harmonic Mean method (FHM_{avg}) and Fuzzy Heronian Mean method ($FHeM_{avg}$).

(7) Maximize or Minimize the associated fuzzy Objective function using similar constraints as follows:

$$\text{Max } \tilde{p} = \left(\sum_{k=1}^{g} \tilde{p_k} - \sum_{k=g+1}^{h} \tilde{p_k} \right) / FAM_{avg}. \tag{7.1}$$

$$\text{Max } \tilde{p} = \left(\sum_{k=1}^{g} \tilde{p_k} - \sum_{j=g+1}^{h} \tilde{p_k} \right) / FQM_{avg}. \tag{7.2}$$

$$\text{Max } \tilde{p} = \left(\sum_{k=1}^{g} \tilde{p_k} - \sum_{k=g+1}^{h} \tilde{p_k} \right) / FGM_{avg}. \tag{7.3}$$

$$\text{Max } \tilde{p} = \left(\sum_{j=1}^{g} \tilde{p_k} - \sum_{k=g+1}^{h} \tilde{p_k} \right) / FHM_{avg}. \tag{7.4}$$

$$\text{Max } \tilde{p} = \left(\sum_{k=1}^{g} \tilde{p_k} - \sum_{k=g+1}^{h} \tilde{p_k} \right) / FHeM_{avg}. \tag{7.5}$$

7.1. Fuzzy Quadratic Mean technique

$$\text{Max } \tilde{p} = \left(\sum_{k=1}^{g} \tilde{p_k} - \sum_{k=g+1}^{h} \tilde{p_k} \right) / FQM_{avg}, \tag{7.6}$$

where, $FQM_{avg} = \sqrt{\frac{1}{2}(\tilde{B}_1^2 + \tilde{B}_2^2)}$.

7.2. Fuzzy Geometric Mean technique

$$\text{Max } \tilde{p} = \left(\sum_{k=1}^{g} \tilde{p}_k - \sum_{k=g+1}^{h} \tilde{p}_k \right) / FGM_{avg}, \tag{7.7}$$

where, $FGM_{avg} = \sqrt{\tilde{B}_1 \times \tilde{B}_2}$.

7.3. Fuzzy Harmonic Mean technique

$$\text{Max } \tilde{p} = \left(\sum_{k=1}^{g} \tilde{p}_k - \sum_{k=g+1}^{h} \tilde{p}_k \right) / FHM_{avg}, \tag{7.8}$$

where, $FHM_{avg} = \frac{2}{\frac{1}{\tilde{B}_1} + \frac{1}{\tilde{B}_2}}$.

7.4. Fuzzy Heronian Mean technique

$$\text{Max } \tilde{p} = \left(\sum_{k=1}^{g} \tilde{p}_k - \sum_{k=g+1}^{h} \tilde{p}_k \right) / FHeM_{avg}, \tag{7.9}$$

where, $FHeM_{avg} = \frac{1}{3}(\tilde{B}_1 + \sqrt{\tilde{B}_1 \tilde{B}_2} + \tilde{B}_2)$.

8. Defuzzification approach

We are using the Mean of Maxima approach in which the valuation is taken as the elements along with the FMOLP values. If the maximum values are not unique, the average of the values is taken as follows [23, 24].

$$F = \frac{\sum_{\tilde{u}_k \in D} \tilde{u}_k}{|D|} \quad \text{Where, } |D| \text{ is the number of values.} \tag{8.1}$$

$$F = \frac{\tilde{u}_1 + \tilde{u}_2 + \tilde{u}_3}{|D|} \quad \text{Where, } |D| \text{ is the number of values.} \tag{8.2}$$

In our proposed method, we have taken the three-tuple triangular fuzzy number of mean techniques and then using the Mean of Maxima method, the problem is converted to Multiple Objective LPP using the Eq (8.2).

9. Numerical example

Consider the FMOLPP :

$$\text{Max } \tilde{p}_1 = 2\tilde{u}_1 + \tilde{u}_2$$

$$\text{Max } \tilde{p}_2 = \tilde{u}_1$$

$$\text{Min } \tilde{p}_3 = -3\tilde{u}_1 - 2\tilde{u}_2$$

$$\text{Min } \tilde{p}_4 = -\tilde{u}_1 - \tilde{u}_2.$$

Subject to

$$8\tilde{u}_1 + 6\tilde{u}_2 \leq (44, 45, 46)$$

$$2\tilde{u}_1 + \tilde{u}_2 \leq (5, 6, 7)$$

$$\tilde{u}_1 \geq (1, 3, 5)$$

$$\tilde{u}_1 + 2\tilde{u}_2 \leq (7, 8, 9)$$

$$\tilde{u}_1, \tilde{u}_2 \geq 0.$$

The procedure to find the optimum value of individual Fuzzy objective functions using the simplex method.

First fuzzy objective function:

$$\text{Max } \tilde{p}_1 = 2\tilde{u}_1 + \tilde{u}_2.$$

Subject to

$$8\tilde{u}_1 + 6\tilde{u}_2 + \tilde{u}_3 = (44, 45, 46)$$

$$2\tilde{u}_1 + \tilde{u}_2 + \tilde{u}_4 = (5, 6, 7)$$

$$-\tilde{u}_1 + \tilde{u}_5 = (-5, -3, -1)$$

$$\tilde{u}_1 + 2\tilde{u}_2 + \tilde{u}_6 = (7, 8, 9)$$

$$\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{u}_4, \tilde{u}_5, \tilde{u}_6 \geq 0.$$

The optimized value is Max $\tilde{p}_1 = (3, 11.25, 19.5)$.

Second fuzzy objective function:

$$\text{Max } \tilde{p}_2 = \tilde{u}_1.$$

Subject to

$$8\tilde{u}_1 + 6\tilde{u}_2 + \tilde{u}_3 = (44, 45, 46)$$

$$2\tilde{u}_1 + \tilde{u}_2 + \tilde{u}_4 = (5, 6, 7)$$

$$-\tilde{u}_1 + \tilde{u}_5 = (-5, -3, -1)$$

$$\tilde{u}_1 + 2\tilde{u}_2 + \tilde{u}_6 = (7, 8, 9)$$

$$\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{u}_4, \tilde{u}_5, \tilde{u}_6 \geq 0.$$

The optimized value is Max $\tilde{p}_2 = (1.5, 5.625, 9.75)$.

Third fuzzy objective function:

$$\text{Min } \tilde{p}_3 = -3\tilde{u}_1 - 2\tilde{u}_2.$$

Subject to

$$8\tilde{u}_1 + 6\tilde{u}_2 + \tilde{u}_3 = (44, 45, 46)$$

$$2\tilde{u}_1 + \tilde{u}_2 + \tilde{u}_4 = (5, 6, 7)$$

$$-\tilde{u}_1 + \tilde{u}_5 = (-5, -3, -1)$$

$$\tilde{u}_1 + 2\tilde{u}_2 + \tilde{u}_6 = (7, 8, 9)$$

$$\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{u}_4, \tilde{u}_5, \tilde{u}_6 \geq 0.$$

The optimized value is Min $\tilde{p}_3 = (-29.25, -16.875, -4.5)$.
Fourth fuzzy objective function:

$$\text{Min } \tilde{p}_4 = -\tilde{u}_1 - \tilde{u}_2.$$

Subject to

$$8\tilde{u}_1 + 6\tilde{u}_2 + \tilde{u}_3 = (44, 45, 46)$$

$$2\tilde{u}_1 + \tilde{u}_2 + \tilde{u}_4 = (5, 6, 7)$$

$$-\tilde{u}_1 + \tilde{u}_5 = (-5, -3, -1)$$

$$\tilde{u}_1 + 2\tilde{u}_2 + \tilde{u}_6 = (7, 8, 9)$$

$$\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{u}_4, \tilde{u}_5, \tilde{u}_6 \geq 0.$$

The optimized value is Min $\tilde{p}_4 = (-11.25, -6.1, -0.95)$.

Table 2. Fuzzy initial table.

j	$\tilde{\chi}_k$	$\lvert\tilde{\chi}_k\rvert$	Values of $\tilde{B}_1$ and $\tilde{B}_2$
1	(3,11.25,19.5)	(3,11.25,19.5)	$\tilde{B}_1 = (3, 11.25, 19.5)$
2	(1.5,5.625,9.75)	(1.5,5.625,9.75)	
3	(-29.25,-16.875,-4.5)	(4.5,16.875,29.25)	$\tilde{B}_2 = (0.95, 6.1, 11.25)$
4	(-11.25,-6.1,-0.95)	(0.95,6.1,11.25)	

Now to prove Eq (7.8):

Let $\tilde{B}_1 = \max\ (|\tilde{\chi}_k|), k = 1, 2, ..., g$, and

$\tilde{B}_2 = \min\ (|\tilde{\chi}_k|), k = g + 1, g + 2, ..., h.$

Fuzzy Harmonic Mean Method:
$FHM_{avg} = \left(\frac{2}{\frac{1}{3}+\frac{1}{5}}, \frac{2}{\frac{1}{8.5}+\frac{1}{6}}, \frac{2}{\frac{1}{14}+\frac{1}{7}}\right)$

$FHM_{avg} = (3.750, 7.034, 9.333)$.

Now for Fuzzy New Harmonic Mean Method.

Max $\tilde{p} = \{(\tilde{p}_1 + \tilde{p}_2) - (\tilde{p}_3 + \tilde{p}_4)\}/FHM_{avg}$.

$= \{(2\tilde{u}_1 + \tilde{u}_2 + \tilde{u}_1) - (-3\tilde{u}_1 - 2\tilde{u}_2 - \tilde{u}_1 - \tilde{u}_2)\}$.

Max $\tilde{p} = (7\tilde{u}_1 + 4\tilde{u}_2)/FHM_{avg}$.

Max $\tilde{p} = (0.490, 0.886, 4.851)\tilde{u_1} + (0.280, 0.506, 2.772)\tilde{u_2}$.

In the following section of the Fuzzy New Harmonic Mean Method, we converted FMOLPP to MOLP using Eq (8.2) Defuzzification Mean of Maxima Method and obtained the following objective function [25].

$$\text{Max } p = 2.076u_1 + 1.186u_2.$$

Subject to constraints :

$$8u_1 + 6u_2 + u_3 = 45$$
$$2u_1 + u_2 + u_4 = 6$$
$$-u_1 + u_5 = -3$$
$$u_1 + 2u_2 + u_6 = 8$$
$$u_1, u_2, u_3, u_4, u_5, u_6 \geq 0.$$

The optimized value is Max $p = 11.678$.

The results of the remaining Defuzzification Mean of Maxima MOLPP mean methods are given in the following table.

Table 3. Comparison table of MOLPP and FMOLPP.

Methods	MOLPP Value of p	FMOLPP of p
Arithmetic mean	4.539	9.011
Quadratic mean	4.354	8.173
Geometric mean	4.753	10.249
Harmonic mean	4.984	11.678
Heronian mean	4.607	9.383

Table 3 presents the comparative study on the optimal values of MOLPP method versus our proposed FMOLPP method and it is evident that our proposed FMOLPP method gives the optimal results than MOLPP method which exhibits the advantage of our proposed method.

10. Conclusions

In this paper, we have solved MOLPP using Chandra Sen's average approach along with the various types of mean methods, and then we solved Fuzzy MOLPP using Chandra Sen's approach in

Fuzzy environs, along with the various types of Fuzzy mean methods. Then the result of Fuzzy MOLPP New mean methods are transformed into a single primary objective function employing the Defuzzification Mean of Maxima method. We compare the optimum values of MOLPP with the optimum values of Fuzzy MOLPP, and we can see that solving the Linear Programming Problem through Fuzzy MOLPP gives optimum values.

Conflict of interest

The Authors declare that there is no conflict of interest.

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