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Research article

Intuitionistic fuzzy credibility Dombi aggregation operators and their application of railway train selection in Pakistan

Muhammad Qiyas[1], **Neelam Khan**[1], **Muhammad Naeem** [2,*] **and Saleem Abdullah**[1]

[1] Department of Mathematics, Abdul Wali Khan University Mardan, Mardan, KP, Pakistan
[2] Department of Mathematics Deanship of Applied Sciences Umm Al-Qura University, Makkah, Saudi Arabia

* **Correspondence:** Email: mfaridoon@uqu.edu.sa.

Abstract: The degree of credibility of the fuzzy assessment value demonstrates its significance and necessity in the fuzzy decision making problem. The fuzzy assessment values should be closely related to their credibility measures in order to increase the credibility levels and degrees of fuzzy assessment values. This will increase the abundance and the credibility of the assessment information. As a new extension of the intuitionistic fuzzy concept, this study suggests the idea of an intuitionistic fuzzy credibility number (IFCN). So, based on Dombi norms, we proposed some new operational laws for intuitionistic fuzzy credibility numbers. Different intuitionistic fuzzy credibility aggregation operators are defined using Dombi t-norm and t-conorm operations. i.e., intuitionistic fuzzy credibility Dombi weighted averaging (IFCDWA), intuitionistic fuzzy credibility Dombi ordered weighted averaging (IFCDOWA), intuitionistic fuzzy credibility Dombi hybrid weighted averaging (IFCDHWA) operators. Next, we defined multiple criteria group decisions (MCGDM) approach. To ensure that their results are reliable and applicable, we also gave an example of railway train selection and discussed comparative result analysis.

Keywords: credibility number; intuitionistic fuzzy credibility number; Dombi operation; aggregation operators
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1. Introduction

The best tools for accurate reasoning, computing, and modeling are crisp, deterministic, and precise in nature, where crisp is defined as a dichotomous, or dual logic statement, statement. Common dual logic contains two true or false statements, in which the central term is disregarded [32]. Multi-attribute group decision-making (MAGDM) system is one of the most important factors in the current research situation [8]. Using MAGDM, experts evaluate and select alternatives based on priorities. Taking a chance on making that different decision must be taken into account [11]. The choice of robots for a manufacturing facility is a MAGDM problem that requires non-programmed decision-making and a lengthy period of contracting with the business. Many decision analysts, including those from the fields of research, economics, development, and engineering, make up this type of decision-making group. Actually, it may not be likely for a single decision maker to pay attention. The prominence of the expert in the decision making (DM) process has the potential to change the outcome. The development of a multi-functional team on purchasing the most efficient firms was impacted by the selection and valuation of robots. The representation of the attribute value, which results from clear numbers, is a crucial topic in DM. When the decision-maker has a choice, it can be challenging to demonstrate an attribute using a crisp set. Fuzzy set theory can be useful in many contexts, including engineering, management, and social sciences, to identify DM problems such as ambiguity and data imprecision. The application of fuzzy set theory to DM problems is extremely important. The fuzzy set theory is crucial for solving problems involving decision-making. In 1965, Zadeh [31] found a solution to this problem when he defined the fuzzy set. It is observed that the notion of a negative membership degree occasionally manifests, which is a crucial fact in organizing the entire suggested outcome and problem design. Various fuzzy approaches [2, 14, 21, 22, 27] have been proposed and used for decision-making and evaluation problems in the fuzzy environment in more recent times.

The IFS is presented in order of pairs which are considered by the membership of positive and negative degree following the condition, that the addition of both the function is to be less than or equal to one [17]. Additionally, a decision matrix with ambiguous assessments and a need for human judgment is necessary. Decision-makers may provide fuzzy evaluation values with some degree of credibility in relation to dissimilar attributes. Experts are more knowledgeable about some characteristics, but not all of the requirements. When supposition occurs, three types of decision responses take place: no, yes, and refusal. The most precise response in all of these cases is "refusal", which might not be possible using the usual fuzzy sets [31] and [1]. In [20], entropy for the intuitionistic fuzzy sets is defined. To solve the DM problems based on IFS, Hung and Chen [9] also created a TOPSIS method with entropy weight. Fuzzy multiple attribute GDM problems were studied by Xu [23], where the attribute values are represented in IFNs and the attribute weights are provided by the DMs in accordance with one or more of the various preference structures. The intuitionistic fuzzy set has recently been widely used to solve decision-making problems [3, 4, 6, 15]. IFSs have been discovered to be a very helpful tool for dealing with vagueness. Dombi's t-norm and t-conorm, two new operations with the operation of the parameters prioritized for variability, were introduced in [5]. Liu [16] applied in Dombi operations to IFSs and introduced the MAGDM problem using the Dombi Bonferroni mean operator in the context of IF information to take advantage of Dombi operations. Huang et al. [12] design alternative assessment and selection: A novel Z-cloud rough number-based BWM-MABAC model. Xiao et al. [24] defined a q-rung orthopair fuzzy decision-making model with new score function and best-worst method for manufacturer selection. Huang et al. [13] proposed failure mode and effect analysis Using T-Spherical fuzzy maximizing deviation and combined comparison solution methods. Xiao [26] discussed an integrated risk assessment method using Z-fuzzy clouds and generalized TODIM. Huang [10] developed an assessment and prioritization method of key engineering characteristics for complex products based on cloud rough numbers.

However, due to the haziness and uncertainty of human cognition and judgments for challenging

MADM problems, existing fuzzy MADM methods only indicates fuzzy assessment values and lack the degrees of credibility regarding the fuzzy assessment values in the appropriate assessment of alternatives over attributes. In uncertain and ambiguous contexts, human subjective assessments and judgments are typically at the heart of MADM difficulties. Therefore, decision-makers or experts may provide fuzzy evaluation values using varying degrees of credibility with respect to certain attributes when they may be more familiar with some traits but not so much with others. For instance, in some article review systems, each expert or reviewer is obliged to provide his or her overall evaluation of a paper and the appropriate credibility degree or level, ranging from 1 to 10. Assume that due to any inadequacy, ambiguity, or uncertainty in the expert's knowledge and/or experience, the expert specifies 6 (equal to the fuzzy evaluation value of 0.6) and 9 (corresponding to the credibility degree/level of 0.9). In order to increase the credibility of his or her overall assessment of the text, it is clear that the fuzzy assessment value of 0.6 is closely related to the credibility degree of 0.9 in the pair of fuzzy values (0.6, 0.9). The classical fuzzy and intuitionistic fuzzy concept in [17] cannot convey the information of the pair of fuzzy and intuitionistic fuzzy values since it simply indicates a fuzzy degree without taking into account its credibility degree (0.6, 0.9). Then, they suggest in-depth familiarity with the geology, which is frequently ambiguous, complex, and uncertain in terms of the structural and material characteristics of mineral deposits. In the meantime, designers and decision-makers could lack some expertise and understanding when evaluating slope design schemes based on certain indices or features. Because human judgments might not be totally accurate and trustworthy in ambiguous and unpredictable contexts, they not only provide fuzzy evaluation values, but also indicate their credibility degrees to maintain credibility levels/degrees of the fuzzy evaluation values. In order to increase the quantity and credibility of the assessment information, the intuitionistic fuzzy evaluation values in uncertain and ambiguous situations should be correlated to their credibility levels/degrees. As a new extension of the intuitionistic fuzzy concept, this study suggests the idea of a intuitionistic fuzzy credibility number (IFCN), where a intuitionistic fuzzy value and a credibility degree are both expressed by a pair of intuitionistic fuzzy values.

Ye et al. [30] discussed these issues in detail in the new concept known as the fuzzy credibility set (FCS). Qiyas et al. [18] studied decision support system using fuzzy credibility Dombi aggregation operators and modified TOPSIS method. Yahya et al. [29] developed an analysis of S-box using image encryption application and complex fuzzy credibility Frank AOs. Yahya et al. [28] discussed analysis of medical diagnosis based on fuzzy credibility Dombi Bonferroni mean operator. Qiyas et al. [19] defined extended GRA method for MCGDM problem based on fuzzy credibility geometric aggregation operator.

The novelty and new contributions in this paper are summarized below:

(1). The proposed new concept of IFCN based on the hybrid information on the intuitionistic fuzzy values and degree of credibility can make the information expression more credible and more reasonable.

(2). The operations and score function of IFCNs and the intuitionistic fuzzy credibility averaging and geometric operators of IFCNs can provide useful mathematical tools for the modeling of MAGDM problems with IFCN information.

(3). To determine the verity of the proposed approach with the help of an example by using the defined operators to determine the consistency and validity of the defined approach.

(4). The proposed MAGDM method not only solves the MAGDM problem with IFCNs, but also makes the decision process more credible and more effective.

(5). To demonstrate the benefits of the suggested method, the mathematical manifestations of the determined operators are discussed.

(6). To determine the comparative analysis of the proposed operators by using some existing operators.

The paper is designed as follows: Preliminaries contained some basic definition of FS, IFS, FCN and some new operational laws of IFCNs. In Section 3, we defined intuitionistic fuzzy credibility Dombi weighted averaging (IFCDWA), intuitionistic fuzzy credibility Dombi ordered weighted averaging (IFCDOWA), intuitionistic fuzzy credibility Dombi hybrid averaging (IFCDHWA) operators. In Section 4, we proposed intuitionistic fuzzy credibility Dombi weighted geometric (IFCDWG), intuitionistic fuzzy credibility Dombi ordered weighted geometric (IFCDOWG), intuitionistic fuzzy credibility Dombi hybrid geometric (IFCDHWG) operators. In Section 5, we utilized defined operators and proposed MAGDM approach. In Section 6, effects of parameters the decision-making results are analyzed. Finally, in Section 7, we have placed a conclusion.

2. Preliminaries

The basic concepts about the FS, IFS and FCNs are presented in this section, which will be useful in certain studies.

Definition 2.1. [31] Suppose $\mathbb{N}$ be a universal set. A fuzzy set $\mathfrak{R}$ of $\mathbb{N}$ is given by:

$$\mathfrak{R} = \{(\langle \check{r}, \mu_{\mathfrak{R}}(\check{r}))\rangle \,|\check{r} \in \mathbb{N}\}, \tag{2.1}$$

where $\mu_{\mathfrak{R}}(\check{r}) : \mathbb{N} \to [0, 1]$ be the membership function of $\mathfrak{R}$.

Definition 2.2. [1] Let $\mathbb{N}$ be a universal set. Then, an IFS $\mathfrak{R}$ on set $\mathbb{N}$ is given as:

$$\mathfrak{R} = \{\langle(\check{r}, \mu_{\mathfrak{R}}(\check{r}), \upsilon_{\mathfrak{R}}(\check{r})\rangle \,|\check{r} \in \mathbb{N}\}, \tag{2.2}$$

in which $\mu_{\mathfrak{R}}(\check{r}) : \mathbb{N} \to [0,1]$ and $\upsilon_{\mathfrak{R}}(\check{r}) : \mathbb{N} \to [0,1]$ show the MG and NMG of an alternative $\check{r} \in \mathbb{N}$ with the condition $0 \leq \mu^2_{\mathfrak{R}} + \upsilon^2_{\mathfrak{R}}(\check{r}) \leq 1$. Furthermore, the hesitation index is given by $\pi_{\mathfrak{R}}(\check{r}) = 1 - \mu_{\mathfrak{R}}(\check{r}) - \upsilon_{\mathfrak{R}}(\check{r})$.

Definition 2.3. [30] Let $\mathbb{N}$ be a universal set. Then, a FCNs on $\mathbb{N}$ is defined as:

$$\mathfrak{I} = \{(\check{r}, \mu_{\mathfrak{I}}(\check{r}), \phi_{\mathfrak{I}}(\check{r})|\check{r} \in \mathbb{N}\}, \tag{2.3}$$

for all $\mu_{\mathfrak{I}} : \mathbb{N} \to [0,1], \phi_{\mathfrak{I}} : \mathbb{N} \to [0,1]$, are the membership degree of the element $\check{r}$ to $\mathbb{N}$ and the degree of credibility related to $\mu_{\hat{i}}(\check{r})$ respectively. Then, the pair $(\mu_{\mathfrak{I}}(\check{r}), \phi_{\mathfrak{I}}(\check{r}))$ is the FCN, such that $\mu_{\mathfrak{I}}(\check{r}) \in [0,1], \phi_{\mathfrak{I}}(\check{r}) \in [0,1]$.

Definition 2.4. [30] Let $\mathfrak{I}_1 = (\mu_1, \phi_1)$ and $\mathfrak{I}_2 = (\mu_2, \phi_2)$ are two fuzzy credibility numbers. Then, the basic relations are the following:

(1). $\mathfrak{I}_1^c = (\langle 1 - \mu_{\mathfrak{I}_1}(\check{r})\rangle, \langle 1 - \phi_{\mathfrak{I}_1}(\check{r})\rangle);$

(2). $\mathfrak{I}_1 \cap \mathfrak{I}_2 = (\langle \mu_{\mathfrak{I}_1}(\check{r}) \wedge \mu_{\mathfrak{I}_2}(\check{r})\rangle, \langle \phi_{\mathfrak{I}_1}(\check{r}) \wedge \phi_{\mathfrak{I}_2}(\check{r})\rangle);$

(3). $\mathfrak{I}_1 \cup \mathfrak{I}_2 = (\langle \mu_{\mathfrak{I}_1}(\check{r}) \vee \mu_{\mathfrak{I}_2}(\check{r})\rangle, \langle \phi_{\mathfrak{I}_1}(\check{r}) \vee \phi_{\mathfrak{I}_2}(\check{r})\rangle);$

(4). $\mathfrak{I}_1 \oplus \mathfrak{I}_2 = (\langle \mu_{\mathfrak{I}_1}(\check{r}) + \mu_{\mathfrak{I}_2}(\check{r}) - \mu_{\mathfrak{I}_1}(\check{r})\mu_{\mathfrak{I}_2}(\check{r})\rangle, \langle \phi_{\mathfrak{I}_1}(\check{r}) + \phi_{\mathfrak{I}_2}(\check{r}) - \phi_{\mathfrak{I}_1}(\check{r})\phi_{\mathfrak{I}_2}(\check{r})\rangle);$

(5). $\mathfrak{I}_1 \otimes \mathfrak{I}_2 = (\langle \mu_{\mathfrak{I}_1}(\check{r})\mu_{\mathfrak{I}_2}(\check{r})\rangle, \langle \phi_{\mathfrak{I}_1}(\check{r})\phi_{\mathfrak{I}_2}(\check{r})\rangle);$

(6). $\psi\mathfrak{I}_1 = (\langle 1-(1-\mu_1(\check{r})^{\psi}\rangle, \langle 1-(1-\phi_1(\check{r})^{\psi}\rangle);$

(7). $\mathfrak{I}_1^{\psi} = (\mu_1(\check{r})^{\psi}, \phi_1(\check{r})^{\psi}).$

Definition 2.5. [5] Let x and y are two numbers from real number $\mathbb{R}$, i.e., $x, y \in \mathbb{R}$. Then, Dombi's t-norm and t-conorm are defined with the assistance of an expression such that,

$$\begin{aligned} Dom(x,y) &= \frac{1}{1+\left\{\left(\frac{1-x}{x}\right)^{\varpi}+\left(\frac{1-y}{y}\right)^{\varpi}\right\}^{1/\varpi}}, \\ Dom^{\mathbb{C}}(x,y) &= 1-\frac{1}{1+\left\{\left(\frac{x}{1-x}\right)^{\varpi}+\left(\frac{y}{1-y}\right)^{\varpi}\right\}^{1/\varpi}}, \end{aligned} \tag{2.4}$$

where, $\varpi \geq 1$ and $(x, y) \in [0,1] * [0,1]$.

2.1. Intuitionistic fuzzy credibility number

Definition 2.6. Let $\mathbb{N}$ be a universal set. Then, the IFCNs on $\mathbb{N}$ is defined as:

$$\mathfrak{I} = \{(\check{r}, (\langle \mu_{\mathfrak{I}}(\check{r}), \phi_{\mathfrak{I}}(\check{r})\rangle, \langle \upsilon_{\mathfrak{I}}(\check{r}), \varphi_{\mathfrak{I}}(\check{r})\rangle) \,|\check{r} \in \mathbb{N}\}, \tag{2.5}$$

where the function $\mu_{\mathfrak{I}}(\check{r}), \phi_{\mathfrak{I}}(\check{r}), \upsilon_{\mathfrak{I}}(\check{r}), \varphi_{\mathfrak{I}}(\check{r}) : \mathbb{N} \to [0,1]$, are the MD and NMD of the element $\check{r}$ to $\mathbb{N}$ and the degree of credibility related to $\mu_{\mathfrak{I}}(\check{r}), \upsilon_{\mathfrak{I}}(\check{r})$ respectively. Then, the pair $(\langle \mu_{\mathfrak{I}}(\check{r}), \phi_{\mathfrak{I}}(\check{r})\rangle, \langle \upsilon_{\mathfrak{I}}(\check{r}), \varphi_{\mathfrak{I}}(\check{r})\rangle)$ is the IFCN, such that $\mu_{\mathfrak{I}}(\check{r}), \upsilon_{\mathfrak{I}}(\check{r}), \phi_{\mathfrak{I}}(\check{r}), \varphi_{\mathfrak{I}}(\check{r}) \in [0,1]$, and satisfies the condition $0 \leq \mu_{\mathfrak{I}}(\check{r}) + \phi_{\mathfrak{I}}(\check{r}) \leq 1, 0 \leq \upsilon_{\mathfrak{I}}(\check{r}) + \varphi_{\mathfrak{I}}(\check{r}) \leq 1$.

Definition 2.7. [30] Let $\mathfrak{I}_1 = (\langle \mu_{\mathfrak{I}_1}(\check{r}), \phi_{\mathfrak{I}_1}(\check{r})\rangle, \langle \upsilon_{\mathfrak{I}_1}(\check{r}), \varphi_{\mathfrak{I}_1}(\check{r})\rangle)$ and $\mathfrak{I}_2 = (\langle \mu_{\mathfrak{I}_2}(\check{r}), \phi_{\mathfrak{I}_2}(\check{r})\rangle, \langle \upsilon_{\mathfrak{I}_2}(\check{r}), \varphi_{\mathfrak{I}_2}(\check{r})\rangle)$ are two fuzzy credibility numbers. Then, their relation are defined as follows:

(1). $\mathfrak{I}_1^c = (\langle (1-\mu_{\mathfrak{I}_1}(\check{r})), (1-\phi_{\mathfrak{I}_1}(\check{r}))\rangle, \langle 1-\upsilon_{\mathfrak{I}_1}(\check{r}), (1-\varphi_{\mathfrak{I}_1}(\check{r}))\rangle);$

(2). $\mathfrak{I}_1 \cap \mathfrak{I}_2 = (\langle (\mu_{\mathfrak{I}_1}(\check{r}) \wedge \mu_{\mathfrak{I}_2}(\check{r})), (\phi_{\mathfrak{I}_1}(\check{r}) \wedge \phi_{\mathfrak{I}_2}(\check{r}))\rangle, \langle (\upsilon_{\mathfrak{I}_1}(\check{r}) \wedge \upsilon_{\mathfrak{I}_2}(\check{r})), (\varphi_{\mathfrak{I}_1}(\check{r}) \wedge \varphi_{\mathfrak{I}_2}(\check{r}))\rangle);$

(3). $\mathfrak{I}_1 \cup \mathfrak{I}_2 = (\langle (\mu_{\mathfrak{I}_1}(\check{r}) \vee \mu_{\mathfrak{I}_2}(\check{r})), (\phi_{\mathfrak{I}_1}(\check{r}) \vee \phi_{\mathfrak{I}_2}(\check{r}))\rangle, \langle (\upsilon_{\mathfrak{I}_1}(\check{r}) \vee \upsilon_{\mathfrak{I}_2}(\check{r})), (\varphi_{\mathfrak{I}_1}(\check{r}) \vee \varphi_{\mathfrak{I}_2}(\check{r}))\rangle);$

(4). $\mathfrak{I}_1 \oplus \mathfrak{I}_2 = \begin{pmatrix} \langle (\mu_{\mathfrak{I}_1}(\check{r}) + \mu_{\mathfrak{I}_2}(\check{r}) - \mu_{\mathfrak{I}_1}(\check{r})\mu_{\mathfrak{I}_2}(\check{r})), (\phi_{\mathfrak{I}_1}(\check{r}) + \phi_{\mathfrak{I}_2}(\check{r}) - \phi_{\mathfrak{I}_1}(\check{r})\phi_{\mathfrak{I}_2}(\check{r}))\rangle, \\ \langle (\upsilon_{\mathfrak{I}_1}(\check{r})\upsilon_{\mathfrak{I}_2}(\check{r})), (\varphi_{\mathfrak{I}_1}(\check{r})\varphi_{\mathfrak{I}_2}(\check{r}))\rangle \end{pmatrix};$

(5). $\mathfrak{I}_1 \otimes \mathfrak{I}_2 = \begin{pmatrix} \langle (\mu_{\mathfrak{I}_1}(\check{r})\mu_{\mathfrak{I}_2}(\check{r})), (\phi_{\mathfrak{I}_1}(\check{r})\phi_{\mathfrak{I}_2}(\check{r}))\rangle \\ \langle (\upsilon_{\mathfrak{I}_1}(\check{r}) + \upsilon_{\mathfrak{I}_2}(\check{r}) - \upsilon_{\mathfrak{I}_1}(\check{r})\upsilon_{\mathfrak{I}_2}(\check{r})), (\varphi_{\mathfrak{I}_1}(\check{r}) + \varphi_{\mathfrak{I}_2}(\check{r}) - \varphi_{\mathfrak{I}_1}(\check{r})\varphi_{\mathfrak{I}_2}(\check{r}))\rangle, \end{pmatrix};$

(6). $\psi\mathfrak{I}_1 = (\langle (1-(1-\mu_1(\check{r})^{\psi}), (1-(1-\phi_1(\check{r})^{\psi})\rangle, \langle \upsilon_1(\check{r})^{\psi}, \varphi_1(\check{r})^{\psi}\rangle);$

(7). $\mathfrak{I}_1^{\psi} = (\mu_1(\check{r})^{\psi}, \phi_1(\check{r})^{\psi}, \langle (1-(1-\upsilon_1(\check{r})^{\psi}), (1-(1-\varphi_1(\check{r})^{\psi})\rangle).$

Definition 2.8. Let $\mathfrak{I}_{\hat{\imath}} = (\langle \mu_{\mathfrak{I}_{\hat{\imath}}}(\check{r}), \phi_{\mathfrak{I}_{\hat{\imath}}}(\check{r})\rangle, \langle \upsilon_{\mathfrak{I}_{\hat{\imath}}}(\check{r}), \varphi_{\mathfrak{I}_{\hat{\imath}}}(\check{r})\rangle)$ be a IFCNs. Then, the score function $S(\mathfrak{I}_{\hat{\imath}})$ are described:

$$S(\mathfrak{I}_{\hat{\imath}}) = [\mu_{\mathfrak{I}_{\hat{\imath}}}(\check{r}) - \phi_{\mathfrak{I}_{\hat{\imath}}}(\check{r}) - \upsilon_{\mathfrak{I}_{\hat{\imath}}}(\check{r}) - \varphi_{\mathfrak{I}_{\hat{\imath}}}(\check{r})]/4, \text{ where } \mathfrak{I}_{\hat{\imath}} \in [-1,1]. \tag{2.6}$$

Definition 2.9. Let we have two IFCNs $\mathfrak{I}_1 = (\langle \mu_{\mathfrak{I}_1}(\check{r}), \phi_{\mathfrak{I}_1}(\check{r})\rangle, \langle \upsilon_{\mathfrak{I}_1}(\check{r}), \varphi_{\mathfrak{I}_1}(\check{r})\rangle)$ and $\mathfrak{I}_2 = (\langle \mu_{\mathfrak{I}_2}(\check{r}), \phi_{\mathfrak{I}_2}(\check{r})\rangle, \langle \upsilon_{\mathfrak{I}_2}(\check{r}), \varphi_{\mathfrak{I}_2}(\check{r})\rangle)$, $\varpi \geq 1, \psi > 0$. Then, Dombi's t-norm and t-conorm operation of IFCNs are defined as follows:

(1). $\mathfrak{I}_1 \oplus \mathfrak{I}_2 = \left\{ \begin{array}{c} \left(1-\frac{1}{1+\left\{\left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}+\left(\frac{\mu_2}{1-\mu_2}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\left(\frac{\phi_1}{1-\phi_1}\right)^{\varpi}+\left(\frac{\phi_2}{1-\phi_2}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\left(\frac{1-\upsilon_1}{\upsilon_1}\right)^{\varpi}+\left(\frac{1-\upsilon_2}{\upsilon_2}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\left(\frac{1-\varphi_1}{\varphi_1}\right)^{\varpi}+\left(\frac{1-\varphi_2}{\varphi_2}\right)^{\varpi}\right\}^{1/\varpi}}\right) \end{array} \right\};$

$$(2).\ \Im_1 \otimes \Im_2 = \left\{ \begin{array}{c} \left(\frac{1}{1+\left\{\left(\frac{1-\mu_1}{\mu_1}\right)^{\varpi}+\left(\frac{1-\mu_2}{\mu_2}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\left(\frac{1-\phi_1}{\phi_1}\right)^{\varpi}+\left(\frac{1-\phi_2}{\phi_2}\right)^{\varpi}\right\}^{1/\varpi}} \right), \\ \left(1-\frac{1}{1+\left\{\left(\frac{\upsilon_1}{1-\upsilon_1}\right)^{\varpi}+\left(\frac{\upsilon_2}{1-\upsilon_2}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\left(\frac{\varphi_1}{1-\varphi_1}\right)^{\varpi}+\left(\frac{\varphi_2}{1-\varphi_2}\right)^{\varpi}\right\}^{1/\varpi}} \right) \end{array} \right\};$$

$$(3).\ \psi.\Im_1 = \left\{ \begin{array}{c} \left(1-\frac{1}{1+\left\{\psi\left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\psi\left(\frac{\phi_1}{1-\phi_1}\right)^{\varpi}\right\}^{1/\varpi}} \right), \\ \left(\frac{1}{1+\left\{\psi\left(\frac{1-\upsilon_1}{\upsilon_1}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\psi\left(\frac{1-\varphi_1}{\varphi_1}\right)^{\varpi}\right\}^{1/\varpi}} \right) \end{array} \right\};$$

$$(4).\ \Im_1^{\psi} = \left\{ \begin{array}{c} \left(\frac{1}{1+\left\{\psi\left(\frac{1-\mu_1}{\mu_1}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\psi\left(\frac{1-\phi_1}{\phi_1}\right)^{\varpi}\right\}^{1/\varpi}} \right), \\ \left(1-\frac{1}{1+\left\{\psi\left(\frac{\upsilon_1}{1-\upsilon_1}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\psi\left(\frac{\varphi_1}{1-\varphi_1}\right)^{\varpi}\right\}^{1/\varpi}} \right) \end{array} \right\}.$$

Theorem 2.1. Let $\Im_1 = (\langle \mu_{\Im_1}(\check{r}), \phi_{\Im_1}(\check{r})\rangle, \langle \upsilon_{\Im_1}(\check{r}), \varphi_{\Im_1}(\check{r})\rangle)$ and $\Im_2 = (\langle \mu_{\Im_2}(\check{r}), \phi_{\Im_2}(\check{r})\rangle, \langle \upsilon_{\Im_2}(\check{r}), \varphi_{\Im_2}(\check{r})\rangle)$, be two IFCNs. Then, we have the following equations:

(1). $\Im_1 \oplus \Im_2 = \Im_2 \oplus \Im_1$;

(2). $\Im_1 \otimes \Im_2 = \Im_2 \otimes \Im_1$;

(3). $\psi(\Im_1 \oplus \Im_2) = \psi\Im_1 \oplus \psi\Im_2, \psi \succ 0$;

(4). $(\Im_1 \otimes \Im_2)^{\psi} = \Im_1^{\psi} \otimes \Im_2^{\psi}$;

(5). $\psi_1\Im \oplus \psi_2\Im = (\psi_1 \oplus \psi_2)\Im$;

(6). $\Im^{\psi_1} \otimes \Im^{\psi_2} = \Im^{(\psi_1 \otimes \psi_2)}$.

Proof. Proof is obvious.

3. Intuitionistic fuzzy credibility Dombi averaging operators

In this section, using the above operational laws, we proposed IFC Dombi averaging operators.

3.1. Intuitionistic fuzzy credibility Dombi weighted averaging operator

Definition 3.1. Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r})\rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r})\rangle)\,(\hat{\imath} = 1, ..., n)$ be IFCNs. Then, intuitionistic fuzzy credibility Dombi weighted average (IFCDWA) operator is a mapping $\Im^n \to \Im$, such that:

$$IFCDWA_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \bigoplus_{\hat{\imath}=1}^{n} (\varrho_{\hat{\imath}}\Im_{\hat{\imath}}), \tag{3.1}$$

where $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)^T$ are the weight vector of $\Im_{\hat{\imath}}(\hat{\imath} = 1, ..., n)$ with $\varrho_{\hat{\imath}} \succ 0$ and $\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} = 1$.

Theorem 3.1. Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r})\rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r})\rangle)\,(\hat{\imath} = 1, ..., n)$ be the set of IFCNs. Then, the aggregated value by using the intuitionistic fuzzy credibility Dombi weighted averaging (IFCDWA) operator is also a IFCNs, defined as:

$$IFCDWA_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \bigoplus_{\hat{\imath}=1}^{n}(\varrho_{\hat{\imath}}\Im_{\hat{\imath}})$$
$$= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}}\left(\frac{\mu_{\hat{\imath}}}{1-\mu_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}}\left(\frac{\phi_{\hat{\imath}}}{1-\phi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}}\left(\frac{1-\upsilon_{\hat{\imath}}}{\upsilon_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}}\left(\frac{1-\varphi_{\hat{\imath}}}{\varphi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right) \end{array} \right\}, \tag{3.2}$$

where $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)^T$ be the weight vector of $\Im_{\hat{\imath}}(\hat{\imath} = 1, ..., n)$ with $\varrho_{\hat{\imath}} > 0$ and $\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}} = 1$.

Proof. This theorem is proved by using mathematical induction principle:

Let $n = 2$, bases on the operations of IFCNs, we get result in left hand side

$$\begin{aligned} IFCDWA_{\varrho}(\Im_1, \Im_2) &= \Im_1 \oplus \Im_2 \\ &= \left(\left\langle\mu_{\Im_1}(\check{r}), \phi_{\Im_1}(\check{r})\right\rangle, \left\langle\upsilon_{\Im_1}(\check{r}), \varphi_{\Im_1}(\check{r})\right\rangle\right) \oplus \left(\left\langle\mu_{\Im_2}(\check{r}), \phi_{\Im_2}(\check{r})\right\rangle, \left\langle\upsilon_{\Im_2}(\check{r}), \varphi_{\Im_2}(\check{r})\right\rangle\right). \end{aligned}$$

For right hand side we get

$$\left\{ \begin{array}{c} \left(1 - \frac{1}{1+\left\{\left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}+\left(\frac{\mu_2}{1-\mu_2}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\left(\frac{\phi_1}{1-\phi_1}\right)^{\varpi}+\left(\frac{\phi_2}{1-\phi_2}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\left(\frac{1-\upsilon_1}{\upsilon_1}\right)^{\varpi}+\left(\frac{1-\upsilon_2}{\upsilon_2}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\left(\frac{1-\varphi_1}{\varphi_1}\right)^{\varpi}+\left(\frac{1-\varphi_2}{\varphi_2}\right)^{\varpi}\right\}^{1/\varpi}}\right) \end{array} \right\}$$
$$= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{2}\varrho_{\hat{\imath}}\left(\frac{\mu_{\hat{\imath}}}{1-\mu_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{2}\varrho_{\hat{\imath}}\left(\frac{\phi_{\hat{\imath}}}{1-\phi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{2}\varrho_{\hat{\imath}}\left(\frac{1-\upsilon_{\hat{\imath}}}{\upsilon_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{2}\varrho_{\hat{\imath}}\left(\frac{1-\varphi_{\hat{\imath}}}{\varphi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right) \end{array} \right\}$$

show that it is true for $n = 2$.

Now, for $n = k$, we have

$$IFCDWA_{\varrho}(\Im_1, \Im_2, ..., \Im_k) = \bigoplus_{\hat{\imath}=1}^{n}(\varrho_{\hat{\imath}}\Im_{\hat{\imath}})$$
$$= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k}\varrho_{\hat{\imath}}\left(\frac{\mu_{\hat{\imath}}}{1-\mu_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k}\varrho_{\hat{\imath}}\left(\frac{\phi_{\hat{\imath}}}{1-\phi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k}\varrho_{\hat{\imath}}\left(\frac{1-\upsilon_{\hat{\imath}}}{\upsilon_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k}\varrho_{\hat{\imath}}\left(\frac{1-\varphi_{\hat{\imath}}}{\varphi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right) \end{array} \right\}.$$

For $n = k + 1$, then we have,

$$IFCDWA_{\varrho}(\Im_1, \Im_2, ..., \Im_k, \Im_{k+1}) = \bigoplus_{\hat{\imath}=1}^{n}(\varrho_{\hat{\imath}}\Im_{\hat{\imath}}) \oplus (\varrho_{k+1}\Im_{k+1})$$

$$= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k} \varrho_{\hat{\imath}}\left(\frac{\mu_{\hat{\imath}}}{1-\mu_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k} \varrho_{\hat{\imath}}\left(\frac{\phi_{\hat{\imath}}}{1-\phi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k} \varrho_{\hat{\imath}}\left(\frac{1-\upsilon_{\hat{\imath}}}{\upsilon_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k} \varrho_{\hat{\imath}}\left(\frac{1-\varphi_{\hat{\imath}}}{\varphi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right) \end{array} \right\}$$

$$\oplus \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\left\{\varrho_{\hat{\imath}}\left(\frac{\mu_{k+1}}{1-\mu_{k+1}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\varrho_{\hat{\imath}}\left(\frac{\phi_{k+1}}{1-\phi_{k+1}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\varrho_{\hat{\imath}}\left(\frac{1-\upsilon_{k+1}}{\upsilon_{k+1}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\varrho_{\hat{\imath}}\left(\frac{1-\varphi_{k+1}}{\varphi_{k+1}}\right)^{\varpi}\right\}^{1/\varpi}}\right) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k+1} \varrho_{\hat{\imath}}\left(\frac{\mu_{\hat{\imath}}}{1-\mu_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k+1} \varrho_{\hat{\imath}}\left(\frac{\phi_{\hat{\imath}}}{1-\phi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k+1} \varrho_{\hat{\imath}}\left(\frac{1-\upsilon_{\hat{\imath}}}{\upsilon_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k+1} \varrho_{\hat{\imath}}\left(\frac{1-\varphi_{\hat{\imath}}}{\varphi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right) \end{array} \right\}.$$

Thus, the result is true for $n = k + 1$. As a result of the above proof, it is clear that it is true for any value of n.

Theorem 3.2. (Idempotency). Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r})\rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r})\rangle)$ be a set of IFCNs are all identical where $(\hat{\imath} = 1, ..., n)$ such as $\Im_{\hat{\imath}} = \Im$, $\forall \hat{\imath}$. Then,

$$IFCDWA_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \Im. \tag{3.3}$$

Proof. Since, $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r})\rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r})\rangle) = \Im$, where $(\hat{\imath} = 1, ..., n)$. Then, we have,

$$IFCDWA_{\varrho}(\Im_1, \Im_2, ..., \Im_{\hat{\imath}}) = \bigoplus_{\hat{\imath}=1}^{n}(\varrho_{\hat{\imath}}\Im_{\hat{\imath}})$$

$$= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\mu_{\hat{\imath}}}{1-\mu_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\phi_{\hat{\imath}}}{1-\phi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\upsilon_{\hat{\imath}}}{\upsilon_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\varphi_{\hat{\imath}}}{\varphi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\left\{\left(\frac{\mu}{1-\mu}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\left(\frac{\phi}{1-\phi}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\left(\frac{1-\upsilon}{\upsilon}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\left(\frac{1-\varphi}{\varphi}\right)^{\varpi}\right\}^{1/\varpi}}\right) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\frac{\mu}{1-\mu}}, 1 - \frac{1}{1+\frac{\phi}{1-\phi}}\right), \\ \left(\frac{1}{1+\frac{1-\upsilon}{\upsilon}}, \frac{1}{1+\frac{1-\varphi}{\varphi}}\right) \end{array} \right\}$$

$$= (\langle \mu_{\Im}(\check{r}), \phi_{\Im}(\check{r})\rangle, \langle \upsilon_{\Im}(\check{r}), \varphi_{\Im}(\check{r})\rangle)$$

$$= \Im,$$

thus, $IFCDWA_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \Im$, holds.

Theorem 3.3. (Boundedness). Suppose $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r})\rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r})\rangle)\, (\hat{\imath} = 1, ..., n)$ be a set of IFCNs and $\Im^- = \min(\Im_1, \Im_2, ..., \Im_n)$ and $\Im^+ = \max(\Im_1, \Im_2, ..., \Im_n)$. Then,

$$\Im^- \leq IFCDWA_{\varrho}(\Im_1, \Im_2, ..., \Im_{\hat{\imath}}) \leq \Im^+. \tag{3.4}$$

Proof. Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r})\rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r})\rangle)\, (\hat{\imath} = 1, ..., n)$ be a number of IFCNs. Let $\Im^- = \min(\Im_1, \Im_2, ..., \Im_{\hat{\imath}}) = \left(\left\langle \mu^-_{\Im_{\hat{\imath}}}(\check{r}), \phi^-_{\Im_{\hat{\imath}}}(\check{r})\right\rangle, \left\langle \upsilon^-_{\Im_{\hat{\imath}}}(\check{r}), \varphi^-_{\Im_{\hat{\imath}}}(\check{r})\right\rangle\right)$ and $\Im^+ = \max(\Im_1, \Im_2, ..., \Im_n) = \left(\left\langle \mu^+_{\Im_{\hat{\imath}}}(\check{r}), \phi^+_{\Im_{\hat{\imath}}}(\check{r})\right\rangle, \left\langle \upsilon^+_{\Im_{\hat{\imath}}}(\check{r}), \varphi^+_{\Im_{\hat{\imath}}}(\check{r})\right\rangle\right)$. We have, $\mu^-_{\Im_{\hat{\imath}}}(\check{r}) = \min_{\hat{\imath}}(\mu_{\Im_{\hat{\imath}}}(\check{r})), \phi^-_{\Im_{\hat{\imath}}}(\check{r}) = \min_{\hat{\imath}}(\phi_{\Im_{\hat{\imath}}}(\check{r})), \upsilon^-_{\Im_{\hat{\imath}}}(\check{r}) = \max_{\hat{\imath}}(\upsilon_{\Im_{\hat{\imath}}}(\check{r})), \varphi^-_{\Im_{\hat{\imath}}}(\check{r}) = \max_{\hat{\imath}}(\varphi_{\Im_{\hat{\imath}}}(\check{r}))$ and $\mu^+_{\Im_{\hat{\imath}}}(\check{r}) = \max_{\hat{\imath}}(\mu_{\Im_{\hat{\imath}}}(\check{r})), \phi^+ = \max_{\hat{\imath}}(\phi_{\Im_{\hat{\imath}}}(\check{r})), \upsilon^+_{\Im_{\hat{\imath}}}(\check{r}) = \min_{\hat{\imath}}(\upsilon_{\Im_{\hat{\imath}}}(\check{r})), \varphi^+_{\Im_{\hat{\imath}}}(\check{r}) = \min_{\hat{\imath}}(\varphi_{\Im_{\hat{\imath}}}(\check{r}))$. Hence, we have the subsequent inequalities,

$$\begin{aligned}
1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\mu^-_{\Im_{\hat{\imath}}}(\check{r})}{1-\mu^-_{\Im_{\hat{\imath}}}(\check{r})}\right)^{\varpi}\right\}^{1/\varpi}} &\leq 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\mu_{\Im_{\hat{\imath}}}(\check{r})}{1-\mu_{\Im_{\hat{\imath}}}(\check{r})}\right)^{\varpi}\right\}^{1/\varpi}} \leq 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\mu^+_{\Im_{\hat{\imath}}}(\check{r})}{1-\mu^+_{\Im_{\hat{\imath}}}(\check{r})}\right)^{\varpi}\right\}^{1/\varpi}}, \\
1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\phi^-_{\Im_{\hat{\imath}}}(\check{r})}{1-\phi^-_{\Im_{\hat{\imath}}}(\check{r})}\right)^{\varpi}\right\}^{1/\varpi}} &\leq 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\phi_{\Im_{\hat{\imath}}}(\check{r})}{1-\phi_{\Im_{\hat{\imath}}}(\check{r})}\right)^{\varpi}\right\}^{1/\varpi}} \leq 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\phi^+_{\Im_{\hat{\imath}}}(\check{r})}{1-\phi^+_{\Im_{\hat{\imath}}}(\check{r})}\right)^{\varpi}\right\}^{1/\varpi}}, \\
\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\upsilon^-_{\Im_{\hat{\imath}}}(\check{r})}{\upsilon^-_{\Im_{\hat{\imath}}}(\check{r})}\right)^{\varpi}\right\}^{1/\varpi}} &\leq \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\upsilon_{\Im_{\hat{\imath}}}(\check{r})}{\upsilon_{\Im_{\hat{\imath}}}(\check{r})}\right)^{\varpi}\right\}^{1/\varpi}} \leq \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\upsilon^+_{\Im_{\hat{\imath}}}(\check{r})}{\upsilon^+_{\Im_{\hat{\imath}}}(\check{r})}\right)^{\varpi}\right\}^{1/\varpi}}, \\
\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\varphi^-_{\Im_{\hat{\imath}}}(\check{r})}{\varphi^-_{\Im_{\hat{\imath}}}(\check{r})}\right)^{\varpi}\right\}^{1/\varpi}} &\leq \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\varphi_{\Im_{\hat{\imath}}}(\check{r})}{\varphi_{\Im_{\hat{\imath}}}(\check{r})}\right)^{\varpi}\right\}^{1/\varpi}} \leq \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\varphi^+_{\Im_{\hat{\imath}}}(\check{r})}{\varphi^+_{\Im_{\hat{\imath}}}(\check{r})}\right)^{\varpi}\right\}^{1/\varpi}}.
\end{aligned}$$

Therefore,

$$\Im^- \leq IFCDWA_{\varrho}(\Im_1, \Im_2, ..., \Im_n) \leq \Im^+.$$

Theorem 3.4. (Monotonicity). Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r})\rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r})\rangle)$ and $\Im^*_{\hat{\imath}} = \left(\left\langle \mu^*_{\Im_{\hat{\imath}}}(\check{r}), \phi^*_{\Im_{\hat{\imath}}}(\check{r})\right\rangle, \left\langle \upsilon^*_{\Im_{\hat{\imath}}}(\check{r}), \varphi^*_{\Im_{\hat{\imath}}}(\check{r})\right\rangle\right)(\hat{\imath} = 1, ..., n)$ be a number of IFCNs, if $\Im_{\hat{\imath}} \leq \Im'_{\hat{\imath}}$, for all $\hat{\imath}$. Then,

$$IFCDWA_{\varrho}(\Im_1, \Im_2, ..., \Im_n) \leq IFCDWA_{\varrho}(\Im'_1, \Im'_2, ..., \Im'_n). \tag{3.5}$$

Proof. Since, it's given $\mu_{\Im_{\hat{\imath}}}(\check{r}) \leq \mu^*_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r}) \leq \phi^*_{\Im_{\hat{\imath}}}(\check{r})$, $\upsilon_{\Im_{\hat{\imath}}}(\check{r}) \leq \upsilon^*_{\Im_{\hat{\imath}}}(\check{r})$ and $\varphi_{\Im_{\hat{\imath}}}(\check{r}) \leq \varphi^*_{\Im_{\hat{\imath}}}(\check{r})$ for all $\hat{\imath}$. Then,

$$\begin{aligned}
1 - \mu^*_{\Im_{\hat{\imath}}}(\check{r}) &\leq 1 - \mu_{\Im_{\hat{\imath}}}(\check{r}) \\
\prod_{\hat{\imath}=1}^{n} (1 - \mu_{\Im_{\hat{\imath}}}(\check{r}))^{\varrho_{\hat{\imath}}} &\leq 1 - \prod_{\hat{\imath}=1}^{n} \left(1 - \mu^*_{\Im_{\hat{\imath}}}(\check{r})\right)^{\varrho_{\hat{\imath}}} \\
&\Longrightarrow \left(1 - \prod_{\hat{\imath}=1}^{n} (1 - \mu_{\Im_{\hat{\imath}}}(\check{r}))^{\varrho_{\hat{\imath}}}\right) \leq \left(1 - \prod_{\hat{\imath}=1}^{n} \left(1 - \mu^*_{\Im_{\hat{\imath}}}(\check{r})\right)^{\varrho_{\hat{\imath}}}\right),
\end{aligned}$$

and

$$\begin{aligned}
1 - \phi^*_{\Im_{\hat{\imath}}}(\check{r}) &\leq 1 - \phi_{\Im_{\hat{\imath}}}(\check{r}) \\
\prod_{\hat{\imath}=1}^{n} (1 - \phi_{\Im_{\hat{\imath}}}(\check{r}))^{\varrho_{\hat{\imath}}} &\leq 1 - \prod_{\hat{\imath}=1}^{n} \left(1 - \phi^*_{\Im_{\hat{\imath}}}(\check{r})\right)^{\varrho_{\hat{\imath}}}
\end{aligned}$$

$$\Longrightarrow \left(1-\prod_{\hat{\imath}=1}^{n}\left(1-\phi_{\mathfrak{I}_{\hat{\imath}}}(\check{r})\right)^{\varrho_{\hat{\imath}}}\right) \leq \left(1-\prod_{\hat{\imath}=1}^{n}\left(1-\phi^{*}_{\mathfrak{I}_{\hat{\imath}}}(\check{r})\right)^{\varrho_{\hat{\imath}}}\right).$$

Similarly, we can show that $\upsilon_{\mathfrak{I}_{\hat{\imath}}}(\check{r}) \leq \upsilon^{*}_{\mathfrak{I}_{\hat{\imath}}}(\check{r})$ and $\varphi_{\mathfrak{I}_{\hat{\imath}}}(\check{r}) \leq \varphi^{*}_{\mathfrak{I}_{\hat{\imath}}}(\check{r})$. Thus, we obtain

$$\left\{\begin{array}{c}\left(1-\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}}\left(\frac{\mu_{\hat{\imath}}}{1-\mu_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}}\left(\frac{\phi_{\hat{\imath}}}{1-\phi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}}\left(\frac{1-\upsilon_{\hat{\imath}}}{\upsilon_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}}\left(\frac{1-\varphi_{\hat{\imath}}}{\varphi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right)\end{array}\right\}$$

$$\leq \left\{\begin{array}{c}\left(1-\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}}\left(\frac{\mu^{*}_{\hat{\imath}}}{1-\mu^{*}_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k+1}\varrho_{\hat{\imath}}\left(\frac{\phi^{*}_{\hat{\imath}}}{1-\phi^{*}_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k+1}\varrho_{\hat{\imath}}\left(\frac{1-\upsilon^{*}_{\hat{\imath}}}{\upsilon^{*}_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{k+1}\varrho_{\hat{\imath}}\left(\frac{1-\varphi^{*}_{\hat{\imath}}}{\varphi^{*}_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}\right)\end{array}\right\}.$$

Hence, from the above equation we prove that, $IFCDWA\left(\mathfrak{I}_1, ..., \mathfrak{I}_n\right) \leq IFCDWA\left(\mathfrak{I}^{*}_1, ..., \mathfrak{I}^{*}_n\right)$.

3.2. Intuitionistic fuzzy credibility Dombi ordered weighted averaging operator

Definition 3.2. Let $\mathfrak{I}_{\hat{\imath}} = \left(\left\langle\mu_{\mathfrak{I}_{\hat{\imath}}}(\check{r}), \phi_{\mathfrak{I}_{\hat{\imath}}}(\check{r})\right\rangle, \left\langle\upsilon_{\mathfrak{I}_{\hat{\imath}}}(\check{r}), \varphi_{\mathfrak{I}_{\hat{\imath}}}(\check{r})\right\rangle\right)(\hat{\imath} = 1, ..., n)$ be a set of IFCNs. Then, the intuitionistic fuzzy credibility Dombi ordered weighted averaging (IFCDOWA) operator of dimension n is a function $IFCDOWA : \mathfrak{I}^n \rightarrow \mathfrak{I}$, such as

$$IFCDOWA_{\varrho}(\mathfrak{I}_1, \mathfrak{I}_2, ..., \mathfrak{I}_n) = \bigoplus_{\hat{\imath}=1}^{n}(\varrho_{\hat{\imath}}\mathfrak{I}_{\tau(\hat{\imath})}), \tag{3.6}$$

where $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)^T$ be the corresponding weight vector of $\mathfrak{I}_{\hat{\imath}}(\hat{\imath} = 1, ..., n)$ with $\varrho_{\hat{\imath}} > 0$ and $\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}} = 1$, and the permutation $(\tau(1), ..., \tau(n))$ of $(\hat{\imath} = 1, ..., n)$, for which $\mathfrak{I}_{\tau(\hat{\imath}-1)} \geq \mathfrak{I}_{\tau(\hat{\imath})} \forall \hat{\imath} = 1, ..., n$.

Theorem 3.5. Let $\mathfrak{I}_{\hat{\imath}} = \left(\left\langle\mu_{\mathfrak{I}_{\hat{\imath}}}(\check{r}), \phi_{\mathfrak{I}_{\hat{\imath}}}(\check{r})\right\rangle, \left\langle\upsilon_{\mathfrak{I}_{\hat{\imath}}}(\check{r}), \varphi_{\mathfrak{I}_{\hat{\imath}}}(\check{r})\right\rangle\right)(\hat{\imath} = 1, ..., n)$ be a set of IFCNs. Then, intuitionistic fuzzy credibility Dombi ordered weighted averaging (IFCDOWA) operator of dimension n and mapping $IFCDOWA : \mathfrak{I}^n \rightarrow \mathfrak{I}$, such that

$$\begin{aligned} &IFCDOWA_{\varrho}(\mathfrak{I}_1, \mathfrak{I}_2, ..., \mathfrak{I}_n) = \bigoplus_{\hat{\imath}=1}^{n}(\varrho_{\hat{\imath}}\mathfrak{I}_{\tau(\hat{\imath})}) \\ &= \left\{\begin{array}{c}\left(1-\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}}\left(\frac{\mu_{\tau(\hat{\imath})}}{1-\mu_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}}\left(\frac{\phi_{\tau(\hat{\imath})}}{1-\phi_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}}\left(\frac{1-\upsilon_{\tau(\hat{\imath})}}{\upsilon_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}}\left(\frac{1-\varphi_{\tau(\hat{\imath})}}{\varphi_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}\right)\end{array}\right\}, \end{aligned} \tag{3.7}$$

with the corresponding weight vector $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)^T$ of $\mathfrak{I}_{\hat{\imath}}(\hat{\imath} = 1, ..., n)$ with $\varrho_{\hat{\imath}} > 0$ and $\sum_{\hat{\imath}=1}^{n}\varrho_{\hat{\imath}} = 1$, and the permutation $(\tau(1), ..., \tau(n))$ of $(\hat{\imath} = 1, ..., n)$, for which $\mathfrak{I}_{\tau(\hat{\imath}-1)} \geq \mathfrak{I}_{\tau(\hat{\imath})}(\hat{\imath} = 1, ..., n)$.

Theorem 3.6. (Idempotency). Let $\mathfrak{I}_{\hat{\imath}} = \left(\left\langle\mu_{\mathfrak{I}_{\hat{\imath}}}(\check{r}), \phi_{\mathfrak{I}_{\hat{\imath}}}(\check{r})\right\rangle, \left\langle\upsilon_{\mathfrak{I}_{\hat{\imath}}}(\check{r}), \varphi_{\mathfrak{I}_{\hat{\imath}}}(\check{r})\right\rangle\right)(\hat{\imath} = 1, ..., n)$ are identical, i.e., $\mathfrak{I}_{\hat{\imath}} = \mathfrak{I}$, for all n. Then,

$$IFCDOWA_{\varrho}(\mathfrak{I}_1, \mathfrak{I}_2, ..., \mathfrak{I}_n) = \mathfrak{I}. \tag{3.8}$$

Theorem 3.7. (Boundedness). Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r})\rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r})\rangle)\,(\hat{\imath} = 1, ..., n)$ be a number of IFCNs and $\Im^- = \min(\Im_1, \Im_2, ..., \Im_n)$ and $\Im^+ = \max(\Im_1, \Im_2, ..., \Im_n)$. Then,

$$\Im^- \leq IFCDOWA_{\varrho}(\Im_1, \Im_2, ..., \Im_n) \leq \Im^+. \tag{3.9}$$

Theorem 3.8. (Monotonicity). Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r})\rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r})\rangle)\,(\hat{\imath} = 1, ..., n)$ be a number of IFCNs, if $\Im_{\hat{\imath}} \leq \Im'_{\hat{\imath}}$ for all $\hat{\imath}$. Then,

$$IFCDOWA_{\varrho}(\Im_1, \Im_2, ..., \Im_n) \leq IFCDOWA_{\varrho}(\Im'_1, \Im'_2, ..., \Im'_n). \tag{3.10}$$

3.3. Intuitionistic fuzzy credibility Dombi hybrid weighted averaging operator

Definition 3.3. Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r})\rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r})\rangle)\,(\hat{\imath} = 1, ..., n)$ be a set of IFCNs. Then, the intuitionistic fuzzy credibility Dombi hybrid weighted averaging (IFCDHWA) operator of dimension n and function $IFCDHWA : \Im^n \rightarrow \Im$ with correlated weight vector $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)|\varrho_{\hat{\imath}} > 0$ and $\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} = 1$. Therefore, IFCDHWA operator can be evaluated as:

$$\begin{aligned} IFCDHWA_{\varrho}(\Im_1, \Im_2, ..., \Im_n) &= \bigoplus_{\hat{\imath}=1}^{n} (\varrho_{\hat{\imath}} \Im^*_{\tau(\hat{\imath})}) \\ &= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{\mu^*{\tau(\hat{\imath})}}{1-\mu_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{\phi^*_{\tau(\hat{\imath})}}{1-\phi_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}} \right), \\ \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{1-\upsilon^*_{\tau(\hat{\imath})}}{\upsilon^*_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{1-\varphi^*_{\tau(\hat{\imath})}}{\varphi^*_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}} \right) \end{array} \right\}, \end{aligned} \tag{3.11}$$

where $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)^T$ be the corresponding weight vector of $\Im_{\hat{\imath}}(\hat{\imath} = 1, ..., n)|\varrho_{\hat{\imath}} > 0$ and $\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} = 1$, and $\Im_{\tau(\hat{\imath})}$ is the $\hat{\imath}^{th}$ biggest weighted intuitionistic fuzzy credibility values $\Im^*_{\hat{\imath}}(\Im^*_{\hat{\imath}} = nw_{\hat{\imath}}\Im_{\hat{\imath}}, \hat{\imath} = 1, ..., n)$ and $w = (w_1, w_2, ..., w_n)^T$ be the weight vector of $\Im_{\hat{\imath}}$ with $w_{\hat{\imath}} > 0$ and $\sum_{\hat{\imath}=1}^{n} w_{\hat{\imath}} = 1$, where the balancing coefficient is n.

4. Intuitionistic fuzzy credibility Dombi geometric operators

In this section, using the above operational laws, we proposed IFC Dombi geometric operators.

4.1. Intuitionistic fuzzy credibility Dombi weighted geometric operator

Definition 4.1. Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r})\rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r})\rangle)\,(\hat{\imath} = 1, ..., n)$ be a IFCNs. Then, the intuitionistic fuzzy credibility Dombi weighted geometric (IFCDWG) operator is a mapping $\Im^n \rightarrow \Im$, such as:

$$IFCDWG_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \bigotimes_{\hat{\imath}=1}^{n} (\Im_{\hat{\imath}})^{\varrho_{\hat{\imath}}}, \tag{4.1}$$

where $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)^T$ be the weight vector of $\Im_{\hat{\imath}}(\hat{\imath} = 1, ..., n)$ with $\varrho_{\hat{\imath}} > 0$ and $\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} = 1$.

Theorem 4.1. Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r})\rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r})\rangle)\,(\hat{\imath} = 1, ..., n)$ be the set of IFCNs. Then, the aggregated value by using the intuitionistic fuzzy credibility Dombi weighted geometric (IFCDWG) operator is also a IFCNs, defined as:

$$IFCDWG_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \bigotimes_{\hat{\imath}=1}^{n} (\Im_{\hat{\imath}})^{\varrho_{\hat{\imath}}}$$
$$= \left\{ \begin{array}{c} \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\mu_{\hat{\imath}}}{\mu_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\phi_{\hat{\imath}}}{\phi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}} \right), \\ \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\upsilon_{\hat{\imath}}}{1-\upsilon_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\varphi_{\hat{\imath}}}{1-\varphi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}} \right), \end{array} \right\} \tag{4.2}$$

where $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)^T$ be the weight vector of $\varrho_{\hat{\imath}}(\hat{\imath} = 1, ..., n)$ with $\varrho_{\hat{\imath}} \succ 0$ and $\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} = 1$.
Proof. Proof is same as the proof of the Theorem (3.1) .

Theorem 4.2. (Idempotency). Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r}) \rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r}) \rangle)$ be a set of IFCNs are all identical where $(\hat{\imath} = 1, ..., n)$ such as $\Im_{\hat{\imath}} = \Im$, for all $\hat{\imath}$. Then,

$$IFCDWG_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \Im. \tag{4.3}$$

Theorem 4.3 (Boundedness). Suppose $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r}) \rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r}) \rangle)\,(\hat{\imath} = 1, ..., n)$ be a set of IFCNs and $\Im^- = \min(\Im_1, \Im_2, ..., \Im_n)$ and $\Im^+ = \max(\Im_1, \Im_2, ..., \Im_n)$. Then,

$$\Im^- \leq \hat{\imath}FCDWG_{\varrho}(\Im_1, \Im_2, ..., \Im_{\hat{\imath}}) \leq \Im^+. \tag{4.4}$$

Theorem 4.4. (Monotonicity). Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r}) \rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r}) \rangle)$ and $\Im_{\hat{\imath}}^* = \left(\left\langle \mu_{\Im_{\hat{\imath}}}^*(\check{r}), \phi_{\Im_{\hat{\imath}}}^*(\check{r}) \right\rangle, \left\langle \upsilon_{\Im_{\hat{\imath}}}^*(\check{r}), \varphi_{\Im_{\hat{\imath}}}^*(\check{r}) \right\rangle \right)(\hat{\imath} = 1, ..., n)$ be a number of IFCNs, if $\Im_{\hat{\imath}} \leq \Im_{\hat{\imath}}^{'}$, for all $\hat{\imath}$. Then,

$$IFCDWG_{\varrho}(\Im_1, \Im_2, ..., \Im_n) \leq IFCDWG_{\varrho}(\Im_1^{'}, \Im_2^{'}, ..., \Im_n^{'}). \tag{4.5}$$

4.2. Intuitionistic fuzzy credibility Dombi ordered weighted geometric operator

Definition 4.2. Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r}) \rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r}) \rangle)\,(\hat{\imath} = 1, ..., n)$ be a set of IFCNs. Then, the intuitionistic fuzzy credibility Dombi ordered weighted geometric (IFCDOWG) operator of dimension n is a mapping $IFCDOWG : \Im^n \to \Im$, such as

$$IFCDOWG_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \bigotimes_{\hat{\imath}=1}^{n} (\Im_{\hat{\imath}})^{\varrho_{\hat{\imath}}}, \tag{4.6}$$

where $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)^T$ be the corresponding weight vector of $\Im_{\hat{\imath}}(\hat{\imath} = 1, ..., n)$ with $\varrho_{\hat{\imath}} \succ 0$ and $\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} = 1$, and the permutation $(\tau(1), ..., \tau(n))$ of $(\hat{\imath} = 1, ..., n)$, for which $\Im_{\tau(\hat{\imath}-1)} \geq \Im_{\tau(\hat{\imath})} \forall \hat{\imath} = 1, ..., n$.

Theorem 4.5. Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r}) \rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r}) \rangle)\,(\hat{\imath} = 1, ..., n)$ be a set of IFCNs. Then, intuitionistic fuzzy credibility Dombi ordered weighted geometric (IFCDOWG) operator of dimension n and mapping $IFCDOWG : \Im^n \to \Im$, such that

$$IFCDOWG_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \bigotimes_{\hat{\imath}=1}^{n} (\Im_{\tau(\hat{\imath})})^{\varrho_{\hat{\imath}}}$$
$$= \left\{ \begin{array}{c} \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\mu_{\tau(\hat{\imath})}}{\mu_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\phi_{\tau(\hat{\imath})}}{\phi_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}} \right), \\ \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\upsilon_{\tau(\hat{\imath})}}{1-\upsilon_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\varphi_{\tau(\hat{\imath})}}{1-\varphi_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}} \right) \end{array} \right\}, \tag{4.7}$$

with the corresponding weight vector $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)^T$ of $\Im_{\hat{\imath}}(\hat{\imath} = 1, ..., n)$ with $\varrho_{\hat{\imath}} > 0$ and $\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} = 1$, and the permutation $(\tau(1), ..., \tau(n))$ of $(\hat{\imath} = 1, ..., n)$, for which $\Im_{\tau(\hat{\imath}-1)} \geq \Im_{\tau(\hat{\imath})}(\hat{\imath} = 1, ..., n)$.

Theorem 4.6 (Idempotency). Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r}) \rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r}) \rangle)\,(\hat{\imath} = 1, ..., n)$ are identical, i.e., $\Im_{\hat{\imath}} = \Im$, for all n. Then,

$$IFCDOWG_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \Im. \tag{4.8}$$

Theorem 4.7. (Boundedness). Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r}) \rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r}) \rangle)\,(\hat{\imath} = 1, ..., n)$ be a number of IFCNs and $\Im^- = \min(\Im_1, \Im_2, ..., \Im_n)$ and $\Im^+ = \max(\Im_1, \Im_2, ..., \Im_n)$. Then,

$$\Im^- \leq IFCDOWG_{\varrho}(\Im_1, \Im_2, ..., \Im_n) \leq \Im^+. \tag{4.9}$$

Theorem 4.8. (Monotonicity). Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r}) \rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r}) \rangle)\,(\hat{\imath} = 1, ..., n)$ be a number of IFCNs, if $\Im_{\hat{\imath}} \leq \Im_{\hat{\imath}}^{'}$ for all $\hat{\imath}$. Then,

$$IFCDOWG_{\varrho}(\Im_1, \Im_2, ..., \Im_n) \leq IFCDOWG_{\varrho}(\Im_1^{'}, \Im_2^{'}, ..., \Im_n^{'}). \tag{4.10}$$

4.3. Intuitionistic fuzzy credibility Dombi hybrid weighted geometric operator

Definition 4.3. Let $\Im_{\hat{\imath}} = (\langle \mu_{\Im_{\hat{\imath}}}(\check{r}), \phi_{\Im_{\hat{\imath}}}(\check{r}) \rangle, \langle \upsilon_{\Im_{\hat{\imath}}}(\check{r}), \varphi_{\Im_{\hat{\imath}}}(\check{r}) \rangle)\,(\hat{\imath} = 1, ..., n)$ be a set of IFCNs. Then, the intuitionistic fuzzy credibility Dombi hybrid weighted geometric (IFCDHWG) operator of dimension n and mapping $IFCDHWG : \Im^n \to \Im$ with correlated weight vector $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)|\varrho_{\hat{\imath}} > 0$ and $\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} = 1$. Therefore, IFCDHWG operator can be evaluated as:

$$\begin{aligned} &IFCDHWG_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \bigotimes_{\hat{\imath}=1}^{n} \left(\Im_{\tau(\hat{\imath})}^{*}\right)^{\varrho_{\hat{\imath}}} \\ &= \left\{ \begin{array}{c} \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{1-\mu_{\tau(\hat{\imath})}^{*}}{\mu_{\tau(\hat{\imath})}^{*}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{1-\phi_{\tau(\hat{\imath})}^{*}}{\phi_{\tau(\hat{\imath})}^{*}}\right)^{\varpi}\right\}^{1/\varpi}} \right), \\ \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{\upsilon^{*}\tau(\hat{\imath})}{1-\upsilon_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{\varphi_{\tau(\hat{\imath})}^{*}}{1-\varphi_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}} \right), \end{array} \right\}, \end{aligned} \tag{4.11}$$

with the corresponding weight vector $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)^T$ of $\Im_{\hat{\imath}}(\hat{\imath} = 1, ..., n)|\varrho_{\hat{\imath}} > 0$ and $\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} = 1$, and $\Im_{\tau(\hat{\imath})}$ is the $\hat{\imath}^{th}$ biggest weighted intuitionistic fuzzy credibility values $\Im_{\hat{\imath}}^{*}(\Im_{\hat{\imath}}^{*} = (\Im_{\hat{\imath}})^{nw_{\hat{\imath}}}, \hat{\imath} = 1, ..., n)$ and $w = (w_1, w_2, ..., w_n)^T$ be the weight vector of $\Im_{\hat{\imath}}$ with $w_{\hat{\imath}} > 0$ and $\sum_{\hat{\imath}=1}^{n} w_{\hat{\imath}} = 1$, where the balancing coefficient is n.

5. MAGDM approach for intuitionistic fuzzy credibility numbers

In this section, we defined an approach to solve a MAGDM problem of intuitionistic fuzzy credibility information. The problems of MAGDM can also be addressed in the decision matrix form where the columns and rows represents the attributes and alternatives respectively. That's why, decision-matrix (DM) is represented by $D_{n\times m}$. A set $(\Im_1, \Im_2, ..., \Im_n)$ is considered which represents n alternatives and $(C_1, C_2, ..., C_m)$ represents m criteria/attributes. $\varrho_j \in [0, 1]$ be the known weight vector of m criteria/attributes such that $\sum_{j=1}^{m} C_j = 1$. Suppose $D^{(k)} = \left[\Im_{\hat{\imath}j}^{(k)}\right]_{n\times m} = \left\{\left\langle \mu_{\Im_{\hat{\imath}j}}^{(k)}, \phi_{\Im_{\hat{\imath}j}}^{(k)} \right\rangle, \left\langle \upsilon_{\Im_{\hat{\imath}j}}^{(k)}, \varphi_{\Im_{\hat{\imath}j}}^{(k)} \right\rangle\right\}_{n\times m,}$ $(k = 1, ..., p)$, denotes the intuitionistic fuzzy credibility Dombi decision matrix, where the degree of alternatives is represented by $\Im_{\hat{\imath}j}$ and $C_{\hat{\imath}j}$ be the criteria contemplated by the experts, $E_k(k = 1, ..., p)$.

$$D^{(k)}_{n\times m} = \begin{array}{c} \Re_1 \\ \Re_2 \\ \cdot \\ \cdot \\ \cdot \\ \Re_{\hat{\imath}} \end{array} \begin{pmatrix} C_1 & . \; . \; . & C_m \\ \left(\left\langle \mu^{(k)}_{\Im_{11}}, \phi^{(k)}_{\Im_{11}} \right\rangle, \left\langle \upsilon^{(k)}_{\Im_{11}}, \varphi^{(k)}_{\Im_{11}} \right\rangle\right) & . \; . \; . & \left(\left\langle \mu^{(k)}_{\Im_{1m}}, \phi^{(k)}_{\Im_{1m}} \right\rangle, \left\langle \upsilon^{(k)}_{\Im_{1m}}, \varphi^{(k)}_{\Im_{1m}} \right\rangle\right) \\ \left(\left\langle \mu^{(k)}_{\Im_{21}}, \phi^{(k)}_{\Im_{21}} \right\rangle, \left\langle \upsilon^{(k)}_{\Im_{21}}, \varphi^{(k)}_{\Im_{21}} \right\rangle\right) & . \; . \; . & \left(\left\langle \mu^{(k)}_{\Im_{2m}}, \phi^{(k)}_{\Im_{2m}} \right\rangle, \left\langle \upsilon^{(k)}_{\Im_{2m}}, \varphi^{(k)}_{\Im_{2m}} \right\rangle\right) \\ . & . \; . \; . & . \\ . & . \; . \; . & . \\ \left(\left\langle \mu^{(k)}_{\Im_{n1}}, \phi^{(k)}_{\Im_{n1}} \right\rangle, \left\langle \upsilon^{(k)}_{\Im_{n1}}, \varphi^{(k)}_{\Im_{n1}} \right\rangle\right) & . \; . \; . & \left(\left\langle \mu^{(k)}_{\Im_{nm}}, \phi^{(k)}_{\Im_{nm}} \right\rangle, \left\langle \upsilon^{(k)}_{\Im_{nm}}, \varphi^{(k)}_{\Im_{nm}} \right\rangle\right) \end{pmatrix}.$$

It should be noted that all the data about the weights of DM and criteria are known in the context of DM.

As a result, the following steps represent the MAGDM process

Step 1. Normalized the decision-metrics $D^k_{\hat{\imath}j}$, using the following equation.

$$D^k_{\hat{\imath}j} = \begin{cases} \left(\left\langle \mu_{\Im_{\hat{\imath}j}}, \phi_{\Im_{\hat{\imath}j}} \right\rangle, \left\langle \upsilon_{\Im_{\hat{\imath}j}}, \varphi_{\Im_{\hat{\imath}j}} \right\rangle\right) \text{ for benefit type attribute,} \\ \left(\left\langle \upsilon_{\Im_{\hat{\imath}j}}, \varphi_{\Im_{\hat{\imath}j}} \right\rangle, \left\langle \mu_{\Im_{\hat{\imath}j}}, \phi_{\Im_{\hat{\imath}j}} \right\rangle\right) \text{ for cost type attribute.} \end{cases} \tag{5.1}$$

Step 2a. Using the following equation with the perimeter $\varpi = 1$, to obtain the aggregated values of the experts for alternatives $\Im_{\hat{\imath}}(\hat{\imath} = 1, ..., m)$.

$$IFCDWA_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \bigoplus_{\hat{\imath}=1}^{n} (\varrho_{\hat{\imath}} \Im_{\hat{\imath}})$$
$$= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{\mu_{\hat{\imath}}}{1-\mu_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{\phi_{\hat{\imath}}}{1-\phi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}} \right), \\ \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{1-\upsilon_{\hat{\imath}}}{\upsilon_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{1-\varphi_{\hat{\imath}}}{\varphi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}} \right) \end{array} \right\}. \tag{5.2}$$

Step 2b. Using the following equation with the perimeter $\varpi = 1$, to obtain the aggregated values of the experts for $\Im_{\hat{\imath}}(\hat{\imath} = 1, ..., m)$.

$$IFCDWG_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \bigotimes_{\hat{\imath}=1}^{n} (\Im_{\hat{\imath}})^{\varrho_{\hat{\imath}}}$$
$$= \left\{ \begin{array}{c} \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{1-\mu_{\hat{\imath}}}{\mu_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{1-\phi_{\hat{\imath}}}{\phi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}} \right), \\ \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{\upsilon_{\hat{\imath}}}{1-\upsilon_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}} \left(\frac{\varphi_{\hat{\imath}}}{1-\varphi_{\hat{\imath}}}\right)^{\varpi}\right\}^{1/\varpi}} \right), \end{array} \right\}. \tag{5.3}$$

Step 3a. Using the following equation with the perimeter $\varpi = 1$, to obtain the aggregated IFCN value for alternative $\Im_{\hat{\imath}}(\hat{\imath} = 1, ..., m)$.

$$IFCDOWA_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \bigoplus_{\hat{\imath}=1}^{n}(\varrho_{\hat{\imath}}\Im_{\tau(\hat{\imath})})$$

$$= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\mu_{\tau(\hat{\imath})}}{1-\mu_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\phi_{\tau(\hat{\imath})}}{1-\phi_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\upsilon_{\tau(\hat{\imath})}}{\upsilon_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\varphi_{\tau(\hat{\imath})}}{\varphi_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}\right) \end{array} \right\}. \tag{5.4}$$

Step 3b. Using the following equation with the perimeter $\varpi = 1$, to obtain the aggregated IFCN value for alternative $\Im_{\hat{\imath}}(\hat{\imath} = 1, ..., m)$.

$$IFCDOWG_{\varrho}(\Im_1, \Im_2, ..., \Im_n) = \bigotimes_{\hat{\imath}=1}^{n}(\Im_{\tau(\hat{\imath})})^{\varrho_{\hat{\imath}}}$$

$$= \left\{ \begin{array}{c} \left(\frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\mu_{\tau(\hat{\imath})}}{\mu_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{1-\phi_{\tau(\hat{\imath})}}{\phi_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \\ \left(1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\upsilon_{\tau(\hat{\imath})}}{1-\upsilon_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{\hat{\imath}=1}^{n} \varrho_{\hat{\imath}}\left(\frac{\varphi_{\tau(\hat{\imath})}}{1-\varphi_{\tau(\hat{\imath})}}\right)^{\varpi}\right\}^{1/\varpi}}\right) \end{array} \right\}. \tag{5.5}$$

Step 4. Find the score values $S(\Im_{\hat{\imath}})$ $(\hat{\imath} = 1, ..., m)$ by using Eq. (2.6).

Step 5. Alternatives are ranked according to the score values for $\Im_{\hat{\imath}}(\hat{\imath} = 1, ..., m)$ and the best one is chosen.

Step 6. End.

6. Example

To develop the service quality of domestic railway trains, the Ministry of Railways (MOR) of the government of Pakistan needs to know which railways train is the most excellent in Pakistan. After initial information, four main domestic railway trains which are represented by $\Im_{\hat{\imath}}(\hat{\imath} = 1, 2, 3, 4)$ are reminded on the applicant record. They are: Allama Iqbal Express ($\Im_1$), Badar Express ($\Im_2$), Hazara Express ($\Im_3$), Jinnah Express ($\Im_4$). To select the most excellent alternatives,four main domestic railway trains are evaluated from four major characteristics (attributes): C_1: ticketing and booking service; C_2: better and poor condition; C_3: cabin service; C_4: responsiveness; assume that decision-makers provide the rating values by utilizing IFCNs, and the IFC decision matrix is presented in Tables 1–3.

We used a group of three experts with weight vectors of $\varrho = (0.2, 0.3, 0.5)^T$ to apply and validate our own proposed methods. Using the suggested aggregation operators, we also have a criteria weight vector with the notation $\xi = (0.26, 0.24, 0.28, 0.22)^T$. Additionally, in order to select the best option from all of the available options, we will use the score and accuracy functions, respectively. Additionally, the specifics of the criteria and alternatives were discussed above, and in this case, the only aggregation operators that can produce the best results are those that have been suggested.

Table 1. Intuitionistic fuzzy credibility information given by expert E^1.

	C_1	C_2	C_3	C_4
$\mathfrak{I}_1$	$\begin{pmatrix}\langle 0.8, 0.2\rangle,\\ \langle 0.3, 0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6, 0.3\rangle,\\ \langle 0.4, 0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.5, 0.2\rangle,\\ \langle 0.1, 0.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4, 0.5\rangle,\\ \langle 0.6, 0.3\rangle\end{pmatrix}$
$\mathfrak{I}_2$	$\begin{pmatrix}\langle 0.4, 0.4\rangle,\\ \langle 0.7, 0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.3, 0.6\rangle,\\ \langle 0.5, 0.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.3, 0.4\rangle,\\ \langle 0.6, 0.1\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6, 0.2\rangle,\\ \langle 0.2, 0.7\rangle\end{pmatrix}$
$\mathfrak{I}_3$	$\begin{pmatrix}\langle 0.6, 0.3\rangle,\\ \langle 0.5, 0.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.2, 0.7\rangle,\\ \langle 0.4, 0.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7, 0.2\rangle,\\ \langle 0.4, 0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.3, 0.6\rangle,\\ \langle 0.7, 0.1\rangle\end{pmatrix}$
$\mathfrak{I}_4$	$\begin{pmatrix}\langle 0.3, 0.6\rangle,\\ \langle 0.8, 0.1\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7, 0.2\rangle,\\ \langle 0.3, 0.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4, 0.6\rangle,\\ \langle 0.2, 0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.8, 0.1\rangle,\\ \langle 0.4, 0.3\rangle\end{pmatrix}$

Table 2. Intuitionistic fuzzy credibility information given by expert E^2.

	C_1	C_2	C_3	C_4
$\mathfrak{I}_1$	$\begin{pmatrix}\langle 0.7, 0.2\rangle,\\ \langle 0.4, 0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4, 0.4\rangle,\\ \langle 0.7, 0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.5, 0.2\rangle,\\ \langle 0.1, 0.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4, 0.6\rangle,\\ \langle 0.2, 0.5\rangle\end{pmatrix}$
$\mathfrak{I}_2$	$\begin{pmatrix}\langle 0.6, 0.3\rangle,\\ \langle 0.4, 0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.3, 0.6\rangle,\\ \langle 0.5, 0.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.3, 0.4\rangle,\\ \langle 0.6, 0.1\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.3, 0.3\rangle,\\ \langle 0.5, 0.2\rangle\end{pmatrix}$
$\mathfrak{I}_3$	$\begin{pmatrix}\langle 0.8, 0.1\rangle,\\ \langle 0.4, 0.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7, 0.2\rangle,\\ \langle 0.3, 0.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.5, 0.2\rangle,\\ \langle 0.3, 0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.8, 0.2\rangle,\\ \langle 0.3, 0.5\rangle\end{pmatrix}$
$\mathfrak{I}_4$	$\begin{pmatrix}\langle 0.3, 0.6\rangle,\\ \langle 0.8, 0.1\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7, 0.2\rangle,\\ \langle 0.3, 0.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4, 0.5\rangle,\\ \langle 0.2, 0.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.2, 0.7\rangle,\\ \langle 0.4, 0.3\rangle\end{pmatrix}$

Table 3. Intuitionistic fuzzy credibility information given by expert E^3.

	C_1	C_2	C_3	C_4
$\mathfrak{I}_1$	$\begin{pmatrix}\langle 0.7, 0.2\rangle,\\ \langle 0.3, 0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6, 0.4\rangle,\\ \langle 0.7, 0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.3, 0.6\rangle,\\ \langle 0.1, 0.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4, 0.6\rangle,\\ \langle 0.2, 0.5\rangle\end{pmatrix}$
$\mathfrak{I}_2$	$\begin{pmatrix}\langle 0.6, 0.3\rangle,\\ \langle 0.4, 0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4, 0.5\rangle,\\ \langle 0.2, 0.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.5, 0.2\rangle,\\ \langle 0.3, 0.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6, 0.2\rangle,\\ \langle 0.2, 0.7\rangle\end{pmatrix}$
$\mathfrak{I}_3$	$\begin{pmatrix}\langle 0.5, 0.4\rangle,\\ \langle 0.3, 0.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7, 0.2\rangle,\\ \langle 0.3, 0.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7, 0.1\rangle,\\ \langle 0.3, 0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6, 0.3\rangle,\\ \langle 0.5, 0.4\rangle\end{pmatrix}$
$\mathfrak{I}_4$	$\begin{pmatrix}\langle 0.3, 0.4\rangle,\\ \langle 0.6, 0.1\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.3, 0.6\rangle,\\ \langle 0.7, 0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4, 0.5\rangle,\\ \langle 0.6, 0.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.2, 0.7\rangle,\\ \langle 0.5, 0.3\rangle\end{pmatrix}$

Step 1. The decision matrix does not need to be normalized because all attributes are the benefit attribute.

Step 2a. Using IFCDWA operators with the perimeter $\varpi = 1$, on Tables 1–3, we obtained Table 4.

Table 4. Aggregated value obtained by using IFCDWA operator.

	C_1	C_2	C_3	C_4
$\Im_1$	$\begin{pmatrix} \langle 0.325, 0.413\rangle, \\ \langle 0.431, 0.363\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.479, 0.515\rangle, \\ \langle 0.531, 0.284\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.364, 0.286\rangle, \\ \langle 0.253, 0.480\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.429, 0.325\rangle, \\ \langle 0.201, 0.327\rangle \end{pmatrix}$
$\Im_2$	$\begin{pmatrix} \langle 0.371, 0.351\rangle, \\ \langle 0.481, 0.252\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.410, 0.261\rangle, \\ \langle 0.528, 0.200\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.380, 0.283\rangle, \\ \langle 0.275, 0.417\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.289, 0.420\rangle, \\ \langle 0.327, 0.213\rangle \end{pmatrix}$
$\Im_3$	$\begin{pmatrix} \langle 0.475, 0.297\rangle, \\ \langle 0.321, 0.409\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.217, 0.498\rangle, \\ \langle 0.259, 0.321\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.328, 0.314\rangle, \\ \langle 0.426, 0.219\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.275, 0.308\rangle, \\ \langle 0.421, 0.245\rangle \end{pmatrix}$
$\Im_4$	$\begin{pmatrix} \langle 0.284, 0.364\rangle, \\ \langle 0.482, 0.299\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.268, 0.428\rangle, \\ \langle 0.521, 0.379\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.246, 0.501\rangle, \\ \langle 0.364, 0.301\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.288, 0.329\rangle, \\ \langle 0.418, 0.383\rangle \end{pmatrix}$

Step 2b. Using IFCDWG operators with the perimeter $\varpi = 1$, on Tables 1–3. We obtained Table 5.

Table 5. Aggregated value obtained by using IFCDWG operator.

	C_1	C_2	C_3	C_4
$\Im_1$	$\begin{pmatrix} \langle 0.283, 0.511\rangle, \\ \langle 0.319, 0.432\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.274, 0.265\rangle, \\ \langle 0.426, 0.317\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.487, 0.238\rangle, \\ \langle 0.302, 0.346\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.431, 0.428\rangle, \\ \langle 0.391, 0.317\rangle \end{pmatrix}$
$\Im_2$	$\begin{pmatrix} \langle 0.218, 0.329\rangle, \\ \langle 0.410, 0.267\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.361, 0.470\rangle, \\ \langle 0.327, 0.535\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.501, 0.298\rangle, \\ \langle 0.362, 0.241\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.317, 0.206\rangle, \\ \langle 0.451, 0.241\rangle \end{pmatrix}$
$\Im_3$	$\begin{pmatrix} \langle 0.237, 0.396\rangle, \\ \langle 0.328, 0.408\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.312, 0.298\rangle, \\ \langle 0.279, 0.420\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.286, 0.321\rangle, \\ \langle 0.395, 0.387\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.336, 0.427\rangle, \\ \langle 0.521, 0.251\rangle \end{pmatrix}$
$\Im_4$	$\begin{pmatrix} \langle 0.310, 0.267\rangle, \\ \langle 0.426, 0.383\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.511, 0.327\rangle, \\ \langle 0.261, 0.285\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.398, 0.217\rangle, \\ \langle 0.270, 0.422\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.307, 0.231\rangle, \\ \langle 0.341, 0.523\rangle \end{pmatrix}$

Step 3a. Using IFCDOWA operator with the perimeter $\varpi = 1$, on the Table 4 and 5. We obtained Table 6, the aggregated IFCN value for $\Im_{\hat{\imath}}(\hat{\imath} = 1, ..., m)$.

Table 6. Aggregated value obtained by using IFCDOWA operator.

	C_1	C_2	C_3	C_4
$\Im_1$	$\begin{pmatrix} \langle 0.327, 0.514\rangle, \\ \langle 0.486, 0.528\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.342, 0.400\rangle, \\ \langle 0.472, 0.202\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.468, 0.283\rangle, \\ \langle 0.376, 0.265\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.583, 0.273\rangle, \\ \langle 0.331, 0.382\rangle \end{pmatrix}$
$\Im_2$	$\begin{pmatrix} \langle 0.292, 0.343\rangle, \\ \langle 0.526, 0.512\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.453, 0.501\rangle, \\ \langle 0.273, 0.427\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.531, 0.341\rangle, \\ \langle 0.376, 0.487\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.287, 0.471\rangle, \\ \langle 0.436, 0.320\rangle \end{pmatrix}$
$\Im_3$	$\begin{pmatrix} \langle 0.365, 0.490\rangle, \\ \langle 0.551, 0.601\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.480, 0.328\rangle, \\ \langle 0.331, 0.601\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.318, 0.394\rangle, \\ \langle 0.492, 0.288\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.327, 0.582\rangle, \\ \langle 0.521, 0.274\rangle \end{pmatrix}$
$\Im_4$	$\begin{pmatrix} \langle 0.364, 0.297\rangle, \\ \langle 0.523, 0.364\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.355, 0.403\rangle, \\ \langle 0.472, 0.383\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.522, 0.299\rangle, \\ \langle 0.612, 0.343\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.283, 0.472\rangle, \\ \langle 0.542, 0.376\rangle \end{pmatrix}$

Step 3b. Using IFCDOWG operator with the perimeter $\varpi = 1$, on the Table 4 and 5. We obtained

Table 7, the aggregated IFCN value for $\mathfrak{I}_{\hat{\imath}}(\hat{\imath} = 1, ..., m)$.

Table 7. Aggregated value obtained by using IFCDOWG operator.

	C_1	C_2	C_3	C_4
$\mathfrak{I}_1$	$\begin{pmatrix} \langle 0.427, 0.383\rangle, \\ \langle 0.341, 0.575\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.502, 0.446\rangle, \\ \langle 0.483, 0.377\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.521, 0.382\rangle, \\ \langle 0.378, 0.366\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.428, 0.253\rangle, \\ \langle 0.381, 0.510\rangle \end{pmatrix}$
$\mathfrak{I}_2$	$\begin{pmatrix} \langle 0.298, 0.310\rangle, \\ \langle 0.418, 0.362\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.436, 0.223\rangle, \\ \langle 0.354, 0.384\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.263, 0.343\rangle, \\ \langle 0.490, 0.432\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.354, 0.388\rangle, \\ \langle 0.428, 0.401\rangle \end{pmatrix}$
$\mathfrak{I}_3$	$\begin{pmatrix} \langle 0.373, 0.419\rangle, \\ \langle 0.481, 0.286\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.309, 0.365\rangle, \\ \langle 0.531, 0.452\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.471, 0.411\rangle, \\ \langle 0.495, 0.264\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.427, 0.324\rangle, \\ \langle 0.263, 0.442\rangle \end{pmatrix}$
$\mathfrak{I}_4$	$\begin{pmatrix} \langle 0.526, 0.497\rangle, \\ \langle 0.461, 0.327\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.332, 0.371\rangle, \\ \langle 0.511, 0.423\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.384, 0.532\rangle, \\ \langle 0.345, 0.379\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.426, 0.377\rangle, \\ \langle 0.554, 0.344\rangle \end{pmatrix}$

Step 4. Find the score values $S(\mathfrak{I}_{\hat{\imath}})\,(\hat{\imath} = 1, ..., m)$ by using Eq. (2.6) on Tables 6 and 7.

$$S(\mathfrak{I}_1) = 0.382, S(\mathfrak{I}_2) = 0.472, S(\mathfrak{I}_3) = 0.427, S(\mathfrak{I}_4) = 0.498,$$

and

$$S(\mathfrak{I}_1) = 0.441, S(\mathfrak{I}_2) = 0.517, S(\mathfrak{I}_3) = 0.463, S(\mathfrak{I}_4) = 0.549.$$

Step 5. Thus, the ranking of the alternatives as:

$$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1.$$

6.1. Sensitivity analysis

We must do a sensitivity analysis to evaluate the stability of our suggested method, because the Dombi norms' coefficient (decision method parameter) has a significant impact on the rankings. In order to find different outcomes and to examine the ranking results, we therefore assign different values to the parameter in the proposed aggregation operators. In Table 8, we show the ranking of the alternatives using different values of the parameter ϖ and IFCDOWA operator.

Table 8. Ranking of the alternatives based on the influence of the parameter.

Parameter	Score values				Ranking
	$S(\mathfrak{I}_1)$	$S(\mathfrak{I}_2)$	$S(\mathfrak{I}_3)$	$S(\mathfrak{I}_4)$	
$\varpi = 1$	0.382	0.472	0.427	0.498	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$
$\varpi = 2$	0.361	0.448	0.399	0.460	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$
$\varpi = 3$	0.336	0.435	0.363	0.448	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$
$\varpi = 5$	0.301	0.416	0.332	0.413	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$
$\varpi = 10$	0.269	0.342	0.274	0.364	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$

In Table 9, we show the ranking of the alternatives using different values of the parameter ϖ and IFCDOWG operator.

Table 9. Ranking of the alternatives based on the influence of the parameter.

Parameter	Score values				Ranking
	$S(\mathfrak{I}_1)$	$S(\mathfrak{I}_2)$	$S(\mathfrak{I}_3)$	$S(\mathfrak{I}_4)$	
$\varpi = 1$	0.441	0.517	0.463	0.549	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$
$\varpi = 2$	0.437	0.499	0.448	0.530	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$
$\varpi = 3$	0.416	0.488	0.425	0.517	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$
$\varpi = 5$	0.384	0.454	0.392	0.484	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$
$\varpi = 10$	0.328	0.411	0.356	0.452	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$

The ranking results are the same when different values are assigned to the parameter in the suggested aggregation operators, as can be seen in the above Table 8 and 9.

6.2. Comparison and discussion analysis

To verify the validity of our proposed work, we must compare our proposed method with other existing methods in this section. As a result, there are two ways to compare: one is aggregation operator-wise, and the other is a technique (approach)-wise. There is research being done on aggregation operators using different types of intuitionistic fuzzy data. As a result, we must now compare our findings to intuitionistic fuzzy data.

The data in the current approach takes the form of intuitionistic fuzzy numbers, and a set of aggregation operators is defined based on these numbers' operational laws. A type of different aggregation operators [3, 6, 15], and [16] respectively. Because the data was taken in the form of intuitionistic fuzzy credibility information, our results cannot be compared to this result. Therefore, we disregard the MD and NMD's credibility terms.

Table 10. Different methods and their ranking.

Method	Score values				Ranking
	$S(\mathfrak{I}_1)$	$S(\mathfrak{I}_2)$	$S(\mathfrak{I}_3)$	$S(\mathfrak{I}_4)$	
Boran et al. [3]	0.521	0.562	0.538	0.573	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$
De et al. [6]	0.124	0.147	0.149	0.164	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$
Li [15]	0.241	0.286	0.250	0.308	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$
Liu et al. [16]	0.724	0.803	0.779	0.836	$\mathfrak{I}_4 > \mathfrak{I}_2 > \mathfrak{I}_3 > \mathfrak{I}_1$

Since, there is a significant difference between the ranking results of the proposed method and those of the existing methods, it can be inferred from the ranking results of Table 10 that some changes in the credibility degrees in the decision-making example are sensitive to the ranking order of the four alternatives. The credibility scores thus show their significance and applicability in the MAGDM problem. The usage of credibility degrees can make the decision result more credible and reasonable in the context of IFCNs, which is an advantage of the suggested MAGDM method. The MAGDM problem's evaluation of credibility levels, however, is restricted by the use of a subjective rather than an objective evaluation method. Our research will be improved if we use an objective measurement method or algorithm for the credibility levels.

Due to the significance of credibility degrees in MAGDM problems, all of these authors fail to take credibility measures of fuzzy evaluation values into account when making decisions or evaluating data. As a result, the decision information may be incomplete or miss credibility measure values of intuitionistic fuzzy evaluation values. The intuitionistic fuzzy evaluation information and decision results can then be made more credible and reasonable in the IFC information in the proposed MAGDM method.

7. Conclusions

Aggregation plays an important role in decision making process. In the existing papers, the authors have studied many aggregation operators with the intuitionistic fuzzy information. In recent years, IFS theory has received considerable attention and have been used in MAGDM procedures as an effective means of expressing fuzzy information. But This paper's main contribution is the suggestion of a new MAGDM method in which attribute values are provided as IFCNs. In order to do this, we firstly introduced new operational laws of IFCNs based on Dombi t-norm and Dombi t-conorm. Then, we defined intuitionistic fuzzy credibility Dombi weighted averaging (IFCDWA), intuitionistic fuzzy credibility Dombi ordered weighted averaging (IFCDOWA) and intuitionistic fuzzy credibility hybrid weighted averaging (IFCDHWA), intuitionistic fuzzy credibility Dombi weighted geometric (IFCDWG), intuitionistic fuzzy credibility Dombi ordered weighted geometric (IFCDOWG) and intuitionistic fuzzy credibility hybrid weighted geometric (IFCDHWG) operators and investigated their properties. Further, we used the defined aggregation operators and proposed an approach fro MAGDM problem. Finally, we provided an example, as well as a discussion of the comparative results analysis, to demonstrate that their findings are reliable and viable. We have discussed the sensitivity analysis of the proposed modal using different values of the parameter. To verify and check the correctness of our proposed method we can compare with existing methods that is also be done and give us the best result as like our proposed methods.

In the future, the proposed aggregation operator can be modified by taking the IFCNs, Hamacher norms, Frank norms, Einstein norms, Yager norms and Bonferroni mean operators to define such a new work for solution of decision making problems.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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