AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 8(3): 6176–6190.
DOI: 10.3934/math.2023312
Received: 11 October 2022
Revised: 05 December 2022
Accepted: 07 December 2022
Published: 03 January 2023

Research article

The stability of anti-periodic solutions for fractional-order inertial BAM neural networks with time-delays

Yuehong Zhang, Zhiying Li*, **Wangdong Jiang and Wei Liu**

Shaoxing University Yuanpei College, Shaoxing 312000, China

* **Correspondence:** Email: 1401509674@qq.com.

Abstract: The dynamic signal transmission process can be regarded as an anti-periodic process, and fractional-order inertial neural networks are widely used in signal processing and other fields, so anti-periodicity is also regarded as an important dynamic feature of inertial neural networks. This paper mainly studies the existence and Mittag-Leffler stability of anti-periodic solutions for a class of fractional-order inertial BAM neural networks with time-delays. By introducing variable substitution, the model with two different fractional-order derivatives is transformed into a model with only one fractional-order derivative of the same order. Using the properties of fractional-order calculus, the relationship between the fractional-order integral of the state function with and without time-delays is given. Firstly, the sufficient conditions for the boundedness and the Mittag-Leffler stability of the solutions for the system are derived. Secondly, by constructing the sequence solution of the function for the system and applying Ascoli-Arzela theorem, the sufficient conditions for the existence and Mittag-Leffler stability of the anti-periodic solution are given. Finally, the correctness of the conclusion is verified by a numerical example.

Keywords: fractional-order; inertial BAM neural networks; Ascoli-Arzela theorem; anti-periodic solutions; Mittag-Leffler stability
Mathematics Subject Classification: 92B20, 34K20

1. Introduction

As a special case of periodic solutions, the existence and stability of anti-periodic solutions are extremely important in the dynamic behavior of nonlinear differential equations. Their phenomena are widely found in various physical phenomena [1–5]. In recent years, the scholars have obtained many results on the existence and stability. For example, [6–8] studied the stability of inertial BAM neural networks and Cohen-Grossberg BAM neural networks. [9, 10] studied the global exponential stability of anti-periodic solutions for BAM neural networks. The global exponential stability of the anti-periodic solution of Cohen-Grossberg neural networks is determined by [11]. The existence and stability of anti-periodic solutions of BAM type Cohen-Grossberg neural networks are studied by [12]. The existence and stability of anti-periodic solutions of inertial neural networks are studied by [13]. Almost anti-periodic solution of inertial neural networks with leakage and time-varying delays on timescales is studied in [14]. The paper [15] discussed the anti-periodic dynamics on high-order inertial Hopfield neural networks with time-varying delays. The paper [16] studied the anti-periodicity on high-order inertial Hopfield neural networks with mixed delays.

From [9–16], it can be seen that the models are all models with integer derivative. In recent years, the study of periodic solutions of fractional-order neural networks has been attracted, some important results have been obtained, such as: asymptotic periodic solutions, s-asymptotic periodic solutions, almost periodic solutions of fractional-order neural networks in [17–21].

Fractional-order calculus is a generalization of integer-order. Fractional-order differential equations are considered as a powerful tool for modeling practical problems in biology, chemistry, physics, medicine, economics and other sciences.

With the further study of the periodicity of nonlinear differential equations, many scholars have paid attention to the anti-periodic problem, which is another characteristic of periodic changes, such as [22–24]. The phenomena are widely seen in biology, economics, medicine, physics and many other disciplines. Because the fractional-order inertial neural networks models are the nonlinear differential equations, which are widely used in signal processing, fluid mechanics, biology and other fields. According to the data we have consulted, there have been many achievements in the research on the global asymptotic stability, Mittag-Leffler stability, single stability and multi stability of periodic solutions of fractional-order inertial neural networks, such as [17–21]. However, the research results on the anti-periodic dynamic behavior of fractional-order inertial neural networks have not been seen yet. In this paper, we study the stability of the anti-periodic solutions of a class of fractional-order inertial BAM neural networks. It is a new topic worthy of study, which will provide a new criterion for theoretical analysis in the exploration of dynamic behavior and practical application. The following are the main innovations of this article:

1) The fractional-order inertial BAM neural networks model with two different fractional-order derivatives is transformed into a model with only one fractional-order derivative with the same order through appropriate variable replacement;

2) When the time variable changes in the finite interval with less than or equal to time-delays and in the infinite interval with more than or equal to time-delays, the relationship between the fractional-order integral of the state function with and without time-delay is derived;

3) By constructing the function sequence solutions of the system, we can derive that it converges uniformly to a continuous function by applying Ascoli-Arzela theorem, then the sufficient conditions for judging the existence and Mittag-Leffler stability of the anti-periodic solutions for the system are given;

4) The results given provides a new theoretical reference for further research on the theory and practical application of fractional-order inertial BAM neural networks with time-delays.

We Consider a class of fractional-order inertial BAM neural networks with time-delays:

$$\begin{cases} D_t^{2\alpha} x_i(t) = -c_i D_t^{\alpha} x_i(t) - \alpha_i x_i(t) + \sum\limits_{j=1}^{n} a_{ij} f_j(y_j(t)) + \sum\limits_{j=1}^{n} b_{ij} f_j(y_j(t-\tau_{ij})) + I_i(t), \\ D_t^{2\alpha} y_j(t) = -d_j D_t^{\alpha} y_j(t) - \beta_j y_j(t) + \sum\limits_{i=1}^{n} g_{ji} f_i(x_i(t)) + \sum\limits_{i=1}^{n} h_{ji} f_i(x_i(t-\sigma_{ji})) + J_j(t), \end{cases} \tag{1.1}$$

where $t \geq 0$, $i, j = 1, 2, \cdots, n$. D_t^{α} is the Riemann-Liouville fractional-order derivative of order α, $0 < \alpha < 1$, $x_i(t)$ and $y_j(t)$ represent the state of ith and jth neurons of t. $c_i > 0$, $d_j > 0$, $\alpha_i > 0$, $\beta_j > 0$, a_{ij}, b_{ij}, g_{ji}, h_{ji} respectively represent the connection weights between neurons, $f_i(\cdot)$ represents the excitation function of the ith neuron, $I_i(t)$ and $J_j(t)$ represent the external input of the ith and jth neurons at time t, τ_{ij} and σ_{ji} represent the signal transmission delay of the ith neuron and the jth neuron at the time t and satisfies $0 < \tau_{ij} \leq \tau$, $0 < \sigma_{ji} \leq \sigma$.

The initial conditions of (1.1) are given as follows:

$$\begin{cases} x_i(s) = \varphi_{1i}(s), \ D_t^{\alpha} x_i(s) = \varphi_{2i}(s), \ -\delta \leq s \leq 0, \\ y_j(s) = \psi_{1j}(s), \ D_t^{\alpha} y_j(s) = \psi_{2j}(s), \ -\delta \leq s \leq 0, \end{cases} \tag{1.2}$$

where $i, j = 1, 2, \cdots, n$, and $\varphi_{1i}(s)$, $\varphi_{2i}(s)$, $\psi_{1j}(s)$, $\psi_{2j}(s)$ are continuous and bounded in $[-\delta, 0]$, $\delta = \max\{\sigma, \tau\}$.

2. Preliminaries

Definition 2.1. [25] Let $q > 0$ be any positive real number, the fractional-order integral (Riemann-Liourille integral) of $f(t)$ with q order is defined as

$$D_t^{-q} f(t) = \frac{1}{\Gamma(q)} \int_0^t (t-r)^{q-1} f(r) \mathrm{d}r,$$

where $\Gamma(\cdot)$ is a Γ function, that is $\Gamma(s) = \int_0^{+\infty} e^{-x} x^{s-1} \mathrm{d}x$, $s > 0$.

Definition 2.2. [25] Let $q > 0$ be any positive real number, $n-1 \leq q < n$, Riemann-Liourille fractional-order derivative is defined as

$$D_t^{q} f(t) = \frac{1}{\Gamma(n-q)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \int_0^t \frac{f(s)}{(t-s)^{q-n+1}} \mathrm{d}s.$$

Definition 2.3. [25] The Mittag-Leffler function with a parameter q is defined as

$$E_q(z) = \sum_{k=0}^{+\infty} \frac{z^k}{\Gamma(kq+1)},$$

where the real part $Re(q) > 0$ of the complex number q, z is a complex number, and $\Gamma(\cdot)$ is a Γ function.

Definition 2.4. Let $X^{\mathrm{T}}(t)$, $\overline{X}^{\mathrm{T}}(t)$ be two solutions of system (1.1) with initial conditions

$$\begin{cases} x_i(s) = \varphi_{1i}(s), \ D_t^{\alpha} x_i(s) = \varphi_{2i}(s), \ -\sigma \leq s \leq 0, \\ y_j(s) = \psi_{1j}(s), \ D_t^{\alpha} y_j(s) = \psi_{2j}(s), \ -\tau \leq s \leq 0, \end{cases}$$

and

$$\begin{cases} \overline{x}_i(s) = \overline{\varphi}_{1i}(s), \ D_t^{\alpha} \overline{x}_i(s) = \overline{\varphi}_{2i}(s), \ -\sigma \leq s \leq 0, \\ \overline{y}_j(s) = \overline{\psi}_{1j}(s), \ D_t^{\alpha} \overline{y}_j(s) = \overline{\psi}_{2j}(s), \ -\tau \leq s \leq 0. \end{cases}$$

If there are two normal numbers $\rho_1 > 0$ and $\rho_2 > 0$, which satisfy

$$\|X(t) - \overline{X}(t)\| \leq [M(\varphi - \overline{\varphi})E_q(-\rho_1 t^q)]^{\rho_2},\ t \geq 0,$$

then, the solution of system (1.1) is said to be globally Mittag-Leffler stable, where

$$X(t) = (x_1(t), x_2(t), \cdots, x_n(t), y_1(t), y_2(t), \cdots, y_n(t))^{\mathrm{T}},$$

$$\overline{X}(t) = (\overline{x}_1(t), \overline{x}_2(t), \cdots, \overline{x}_n(t), \overline{y}_1(t), \overline{y}_2(t), \cdots, \overline{y}_n(t))^{\mathrm{T}},$$

$$\varphi(t) = (\varphi_{11}(t), \varphi_{12}(t), \cdots, \varphi_{1n}(t), \psi_{11}(t), \psi_{12}(t), \cdots, \psi_{1n}(t))^{\mathrm{T}},$$

$$\overline{\varphi}(t) = (\overline{\varphi}_{11}(t), \overline{\varphi}_{12}(t), \cdots, \overline{\varphi}_{1n}(t), \overline{\psi}_{11}(t), \overline{\psi}_{12}(t), \cdots, \overline{\psi}_{1n}(t))^{\mathrm{T}},$$

$$M(\varphi - \overline{\varphi}) \geq 0, M(0) = 0.$$

$E_q(\cdot)$ is a Mittag-Leffler function with one parameter.

Definition 2.5. If $u(t) \in C(R)$, $u(t+\omega) = -u(t)$ for $t \in R$, then, $u(t)$ is an anti-periodic function, where ω is a positive number.

Lemma 2.1. [26] If $x(t) \in R$ is continuous and differentiable in $[0, \delta]$ ($\delta > 0$), and $0 < q < 1$, $n - 1 < p < n$, $n \in Z^+$, then,

(1) $D_t^p D_t^q x(t) = D_t^{p+q} x(t)$;

(2) $D_t^{-q} D_t^q x(t) = x(t)$.

Lemma 2.2. [27] If $r(t)$ is derivable and $r'(t)$ is continuous, then,

$$\frac{1}{2} D_t^q r^2(t) \leq r(t) D_t^q r(t),\ 0 < q \leq 1.$$

Lemma 2.3. [26] Let $u(t)$ be a continuous function which is defined in $[0, +\infty)$, if there exist constants $d_1 > 0$ and $d_2 > 0$, such that $u(t) \leq -d_1 D_t^{-q} u(t) + d_2,\ t \geq 0$, then,

$$u(t) \leq d_2 E_q(-d_1 t^q),$$

where $0 < q < 1$ and $E_q(\cdot)$ is the Mittag-Leffler function with one parameter.

The discussion is based on the assumptions below for $i, j = 1, 2, ..., n$.

H$_1$. The function $f_j(\cdot)$ is continuous and bounded which satisfies the Lipschitz condition, that is, there exist constants $l_j > 0$, $\bar{f}_j > 0$, satisfing

$$|f_j(u) - f_j(v)| \leq l_j |u - v|,\ |f_j(u)| \leq \bar{f}_j,\ u, v \in R,$$

and

$$f_j(u) = -f_j(-u),\ u \in R.$$

H$_2$. $I_i(t)$ and $J_j(t)$ are bounded, that is, there exist constants $\bar{I}_i > 0$ and $\bar{J}_j > 0$, satisfing

$$|I_i(t)| \leq \bar{I}_i,\ |J_j(t)| \leq \bar{J}_j.$$

H$_3$. There exists a constant $\omega > 0$, satisfing $I_i(t+\omega) = -I_i(t),\ J_j(t+\omega) = -J_j(t)$.

For system (1.1), we introduce variable replacement:

$$\begin{cases} u_i(t) = D_t^\alpha x_i(t) + \gamma_i x_i(t), \\ v_j(t) = D_t^\alpha y_j(t) + \gamma_j y_j(t), \end{cases} \gamma_i > 0,\ \gamma_j > 0,\ i, j = 1, 2, \cdots, n.$$

From Lemma 2.1, system (1.1) can be transformed into

$$\begin{cases} D_t^\alpha x_i(t) = -\gamma_i x_i(t) + u_i(t), \\ D_t^\alpha u_i(t) = -(\alpha_i + \gamma_i^2 - \gamma_i c_i) x_i(t) - (c_i - \gamma_i) u_i(t) + \sum\limits_{j=1}^n a_{ij} f_j(y_j(t)) + \sum\limits_{j=1}^n b_{ij} f_j(y_j(t - \tau_{ij})) + I_i(t), \\ D_t^\alpha y_j(t) = -\gamma_j y_j(t) + v_j(t), \\ D_t^\alpha v_j(t) = -(\beta_j + \gamma_j^2 - \gamma_j d_j) y_j(t) - (d_j - \gamma_j) v_j(t) + \sum\limits_{i=1}^n g_{ji} f_i(x_i(t)) + \sum\limits_{i=1}^n h_{ji} f_i(x_i(t - \sigma_{ji})) + J_j(t). \end{cases} \tag{2.1}$$

3. Results

Theorem 3.1. For the system (1.1), assume that $\mathbf{H_1}$ and $\mathbf{H_2}$ are true, if

$$\eta_1 = \min_{1 \le i \le n} \{\gamma_i - |\alpha_i + \gamma_i^2 - \gamma_i c_i|, c_i - \gamma_i - 1\} > 0,\ \eta_2 = \min_{1 \le j \le n} \{\gamma_j - |\beta_j + \gamma_j^2 - \gamma_j d_j|, d_j - \gamma_j - 1\} > 0,$$

then, the solutions $x_i(t), y_j(t)$ of (1.1) and $D_t^\alpha x_i(t), D_t^\alpha y_j(t)$ $(i, j = 1, 2, \cdots, n)$ are bounded in $[0, T]$ $(T < +\infty)$.

Proof. According to the first two formulas in (2.1), we can get

$$\begin{cases} D_t^\alpha |x_i(t)| \le -\gamma_i |x_i(t)| + |u_i(t)|, \\ D_t^\alpha |u_i(t)| \le |\alpha_i + \gamma_i^2 - \gamma_i c_i| |x_i(t)| - (c_i - \gamma_i) |u_i(t)| + \sum\limits_{j=1}^n (|a_{ij}| + |b_{ij}|) \bar{f}_j + \bar{I}_i. \end{cases} \tag{3.1}$$

According to Lemma 2.1, we can get from (3.1) that

$$\begin{aligned} |x_i(t)| + |u_i(t)| &\le -\gamma_i D_t^{-\alpha} |x_i(t)| + D_t^{-\alpha} |u_i(t)| + |\alpha_i + \gamma_i^2 - \gamma_i c_i| D_t^{-\alpha} |x_i(t)| \\ &\quad -(c_i - \gamma_i) D_t^{-\alpha} |u_i(t)| + D_t^{-\alpha} [\sum_{j=1}^n (|a_{ij}| + |b_{ij}|) \bar{f}_j + \bar{I}_i] \\ &\le -(\gamma_i - |\alpha_i + \gamma_i^2 - \gamma_i c_i|) D_t^{-\alpha} |x_i(t)| - (c_i - \gamma_i - 1) D_t^{-\alpha} |u_i(t)| \\ &\quad + [\sum_{j=1}^n (|a_{ij}| + |b_{ij}|) \bar{f}_j + \bar{I}_i] \frac{T^\alpha}{\Gamma(\alpha + 1)} \\ &\le -\eta_1 D_t^{-\alpha} [|x_i(t)| + |u_i(t)|] + m_1, \end{aligned} \tag{3.2}$$

where $m_1 = [\sum\limits_{j=1}^n (|a_{ij}| + |b_{ij}|) \bar{f}_j + \bar{I}_i] \frac{T^\alpha}{\Gamma(\alpha+1)}$. According to Lemma 2.3 and (3.2), we have

$$|x_i(t)| + |u_i(t)| \le m_1 E_\alpha(-\eta_1 t^\alpha),\ t > 0,$$

so $x_i(t)$ and $u_i(t)$ are bounded. Due to $D_t^\alpha x_i(t) = -\gamma_i x_i(t) + u_i(t)$, when $x_i(t)$ and $u_i(t)$ are bounded, and

$$D_t^\alpha |x_i(t)| \le -\gamma_i |x_i(t)| + |u_i(t)| \le (\gamma_i + 1) m_1 E_\alpha(-\eta_1 t^\alpha),$$

so $D_t^\alpha x_i(t)$ are bounded.

Similarly, it can be deduced from the last two formulas of (2.1) that

$$|y_j(t)| + |v_j(t)| \le m_2 E_\alpha(-\eta_2 t^\alpha),\ D_t^\alpha |y_j(t)| \le (\gamma_j + 1) m_2 E_\alpha(-\eta_2 t^\alpha),$$

where $m_2 = [\sum_{i=1}^{n}(|g_{ji}| + |h_{ji}|)\bar{f}_i + \bar{J}_j]\frac{T^\alpha}{\Gamma(\alpha+1)}$. So $y_j(t)$ and $D_t^\alpha y_j(t)$ are bounded. □

Theorem 3.2. Assume that the condition $\mathbf{H_1}$ is true, if $\eta = \min\{\min_{1\le i\le n}\{2\gamma_i - |1-\alpha_i-\gamma_i^2+\gamma_i c_i| - \sum_{j=1}^{n}(|g_{ji}|+|h_{ji}|)l_i,\ 2c_i - 2\gamma_i - |1-\alpha_i-\gamma_i^2+\gamma_i c_i| - \sum_{j=1}^{n}(|a_{ij}| + |b_{ij}|)l_j\},\ \min_{1\le j\le n}\{2\gamma_j - |1-\beta_j-\gamma_j^2+\gamma_j d_j| - \sum_{i=1}^{n}(|a_{ij}|+|b_{ij}|)l_j,\ 2d_j - 2\gamma_j - |1-\beta_j-\gamma_j^2+\gamma_j d_j| - \sum_{i=1}^{n}(|g_{ji}|+|h_{ji}|)l_i\}\} > 0$, then the solutions of system (1.1) are globally Mittag-Leffler stable.

Proof. Let $X(t) = (x_1(t), x_2(t), \cdots, x_n(t), y_1(t), y_2(t), \cdots, y_n(t))^{\mathrm{T}}$ and $\overline{X}(t) = (\overline{x}_1(t), \overline{x}_2(t), \cdots, \overline{x}_n(t), \overline{y}_1(t), \overline{y}_2(t), \cdots, \overline{y}_n(t))^{\mathrm{T}}$ be two solutions of system (1.1) with the initial value $x_i(s) = \varphi_{1i}(s),\ D_t^\alpha x_i(s) = \varphi_{2i}(s),\ y_i(s) = \psi_{1i}(s),\ D_t^\alpha x_i(s) = \psi_{2i}(s)$ and $\overline{x}_i(s) = \overline{\varphi}_{1i}(s),\ D_t^\alpha \overline{x}_i(s) = \overline{\varphi}_{2i}(s),\ \overline{y}_i(s) = \overline{\psi}_{1i}(s),\ D_t^\alpha \overline{y}_i(s) = \overline{\psi}_{2i}(s)$. Let $z_i(t) = x_i(t)-\overline{x}_i(t),\ w_i(t) = y_i(t)-\overline{y}_i(t),\ p_i(t) = u_i(t)-\overline{u}_i(t),\ q_i(t) = v_i(t)-\overline{v}_i(t),\ i = 1, 2, \cdots, n$. According to (2.1), we get

$$\begin{cases} D_t^\alpha z_i(t) = -\gamma_i z_i(t) + p_i(t), \\ D_t^\alpha p_i(t) = -(\alpha_i + {\gamma_i}^2 - \gamma_i c_i) z_i(t) - (c_i - \gamma_i) p_i(t) \\ \qquad + \sum_{j=1}^{n} a_{ij}(f_j(y_j(t)) - f_j(\overline{y}_j(t))) + \sum_{j=1}^{n} b_{ij}(f_j(y_j(t-\tau_{ij})) - f_j(\overline{y}_j(t-\tau_{ij}))), \\ D_t^\alpha w_j(t) = -\gamma_j w_j(t) + q_j(t), \\ D_t^\alpha q_j(t) = -(\beta_j + {\gamma_j}^2 - \gamma_j d_j) w_j(t) - (d_j - \gamma_j) q_j(t) \\ \qquad + \sum_{i=1}^{n} g_{ji}(f_i(x_i(t)) - f_i(\overline{x}_i(t))) + \sum_{i=1}^{n} h_{ji}(f_i(x_i(t-\sigma_{ji})) - f_i(\overline{x}_i(t-\sigma_{ji}))). \end{cases} \tag{3.3}$$

According to Lemma 2.2 and (3.3), we have

$$\begin{cases} D_t^\alpha z_i^2(t) \le -2\gamma_i z_i^2(t) + 2 z_i(t) p_i(t), \\ D_t^\alpha p_i^2(t) \le -2(\alpha_i + {\gamma_i}^2 - \gamma_i c_i) z_i(t) p_i(t) - 2(c_i - \gamma_i) p_i^2(t) \\ \qquad +2|p_i(t)|[\sum_{j=1}^{n} |a_{ij}| l_j |w_j(t)| + \sum_{j=1}^{n} |b_{ij}| l_j |w_j(t-\tau_{ij})|], \\ D_t^\alpha w_j^2(t) \le -2\gamma_j w_j^2(t) + 2 w_j(t) q_j(t), \\ D_t^\alpha q_j^2(t) \le -2(\beta_j + {\gamma_j}^2 - \gamma_j d_j) q_j(t) w_j(t) - 2(d_j - \gamma_j) q_j^2(t) \\ \qquad +2|q_j(t)|[\sum_{i=1}^{n} |g_{ji}| l_i |z_i(t)| + \sum_{i=1}^{n} |h_{ji}| l_i |z_i(t-\sigma_{ji})|]. \end{cases} \tag{3.4}$$

From (3.4), we have

$$\begin{aligned} & D_t^\alpha \sum_{i=1}^{n}[z_i^2(t) + p_i^2(t)] + D_t^\alpha \sum_{j=1}^{n}[w_j^2(t) + q_j^2(t)] \\ \le\ & \sum_{i=1}^{n}\{-2\gamma_i z_i^2(t) + 2(1-\alpha_i-\gamma_i^2+\gamma_i c_i) z_i(t) p_i(t) - 2(c_i-\gamma_i) p_i^2(t) + \sum_{j=1}^{n} |a_{ij}| l_j (p_i^2(t) + w_j^2(t)) \end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}|b_{ij}|l_j(p_i^2(t)+w_j^2(t-\tau_{ij}))\}+\sum_{j=1}^{n}\{-2\gamma_j w_j^2(t)+2(1-\beta_j-\gamma_j^2+\gamma_j d_j)w_j(t)q_j(t)\\
&-2(d_j-\gamma_j)q_j^2(t)+\sum_{i=1}^{n}|g_{ji}|l_i(z_i^2(t)+q_j^2(t))+\sum_{i=1}^{n}|h_{ji}|l_i(q_j^2(t)+z_i^2(t-\sigma_{ji}))\}\\
=\ &\sum_{i=1}^{n}\{-[2\gamma_i-|1-\alpha_i-\gamma_i^2+\gamma_i c_i|-\sum_{j=1}^{n}|g_{ji}|l_i]z_i^2(t)-[2c_i-2\gamma_i-|1-\alpha_i-\gamma_i^2+\gamma_i c_i|\\
&-\sum_{j=1}^{n}(|a_{ij}|+|b_{ij}|)l_j]p_i^2(t)\}+\sum_{j=1}^{n}\{-[2\gamma_j-|1-\beta_j-\gamma_j^2+\gamma_j d_j|-\sum_{i=1}^{n}|a_{ij}|l_j]w_j^2(t)\\
&-[2d_j-2\gamma_j-|1-\beta_j-\gamma_j^2+\gamma_j d_j|-\sum_{i=1}^{n}(|g_{ji}|+|h_{ji}|)l_i]q_j^2(t)\}\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}|b_{ij}|l_j w_j^2(t-\tau_{ij})+\sum_{j=1}^{n}\sum_{i=1}^{n}|h_{ji}|l_i z_i^2(t-\sigma_{ji}).
\end{aligned}
\tag{3.5}
$$

From (3.5), we can obtain

$$
\begin{aligned}
&\sum_{i=1}^{n}[z_i^2(t)+p_i^2(t)]+\sum_{j=1}^{n}[w_j^2(t)+q_j^2(t)]\\
\le\ &\sum_{i=1}^{n}\{-[2\gamma_i-|1-\alpha_i-\gamma_i^2+\gamma_i c_i|-\sum_{j=1}^{n}|g_{ji}|l_i]D_t^{-\alpha}z_i^2(t)-[2c_i-2\gamma_i-|1-\alpha_i-\gamma_i^2+\gamma_i c_i|\\
&-\sum_{j=1}^{n}(|a_{ij}|+|b_{ij}|)l_j]D_t^{-\alpha}p_i^2(t)\}+\sum_{j=1}^{n}\{-[2\gamma_j-|1-\beta_j-\gamma_j^2+\gamma_j d_j|-\sum_{i=1}^{n}|a_{ij}|l_j]D_t^{-\alpha}w_j^2(t)\\
&-[2d_j-2\gamma_j-|1-\beta_j-\gamma_j^2+\gamma_j d_j|-\sum_{i=1}^{n}(|g_{ji}|+|h_{ji}|)l_i]D_t^{-\alpha}q_j^2(t)\}\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}|b_{ij}|l_j D_t^{-\alpha}w_j^2(t-\tau_{ij})+\sum_{j=1}^{n}\sum_{i=1}^{n}|h_{ji}|l_i D_t^{-\alpha}z_i^2(t-\sigma_{ji}).
\end{aligned}
\tag{3.6}
$$

When $t\in[0,\tau_{ij}]$, we have $t-\tau_{ij}\le 0$, then,

$$
\begin{aligned}
D_t^{-\alpha}w_j^2(t-\tau_{ij}) &= \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}w_j^2(s-\tau_{ij})\mathrm{d}s\\
&= \frac{1}{\Gamma(\alpha)}\int_{-\tau_{ij}}^{t-\tau_{ij}}(t-u-\tau_{ij})^{\alpha-1}w_j^2(u)\mathrm{d}u\\
&\le \frac{w_j^*}{\Gamma(\alpha)}\int_{-\tau_{ij}}^{t-\tau_{ij}}(t-u-\tau_{ij})^{\alpha-1}\mathrm{d}u\\
&= \frac{w_j^* t^\alpha}{\Gamma(\alpha+1)}\le\frac{w_j^*\tau^\alpha}{\Gamma(\alpha+1)},
\end{aligned}
\tag{3.7}
$$

where $w_j^*=\sup_{-\tau\le s\le 0}\{|\psi_{1j}(s)-\overline{\psi}_{1j}(s)|\}$, $j=1,2,\cdots,n$.

When $t \in [\tau_{ij}, +\infty)$, we have $t - \tau_{ij} \geq 0$, then

$$\begin{aligned} D_t^{-\alpha} w_j^2(t-\tau_{ij}) &= \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} w_j^2(s-\tau_{ij}) \mathrm{d}s \\ &= \frac{1}{\Gamma(\alpha)} \int_{-\tau_{ij}}^{t-\tau_{ij}} (t-u-\tau_{ij})^{\alpha-1} w_j^2(u) \mathrm{d}u \\ &= \frac{1}{\Gamma(\alpha)} \int_{-\tau_{ij}}^{0} (t-u-\tau_{ij})^{\alpha-1} w_j^2(u) \mathrm{d}u + \frac{1}{\Gamma(\alpha)} \int_{0}^{t-\tau_{ij}} (t-u-\tau_{ij})^{\alpha-1} w_j^2(u) \mathrm{d}u \\ &\leq \frac{w_j^* \tau^\alpha}{\Gamma(\alpha+1)} + \frac{1}{\Gamma(\alpha)} \int_0^t (t-u)^{\alpha-1} w_j^2(u) \mathrm{d}u \\ &= \frac{w_j^* \tau^\alpha}{\Gamma(\alpha+1)} + D_t^{-\alpha} w_j^2(t). \end{aligned} \tag{3.8}$$

From (3.7) and (3.8), we get

$$D_t^{-\alpha} w_j^2(t-\tau_{ij}) \leq \frac{w_j^* \tau^\alpha}{\Gamma(\alpha+1)} + D_t^{-\alpha} w_j^2(t), \tag{3.9}$$

where $w_j^* = \sup\limits_{-\tau \leq s \leq 0} \{|\psi_{1j}(s) - \overline{\psi}_{1j}(s)|\},\ j = 1, 2, \cdots, n$. The same can be deduced

$$D_t^{-\alpha} z_i^2(t-\sigma_{ji}) \leq \frac{z_i^* \sigma^\alpha}{\Gamma(\alpha+1)} + D_t^{-\alpha} z_i^2(t), \tag{3.10}$$

where $z_i^* = \sup\limits_{-\sigma \leq s \leq 0} \{(\varphi_{1i}(s) - \overline{\varphi}_{1i}(s))^2\},\ i = 1, 2, \cdots, n$.

We substitute (3.9) and (3.10) into (3.6) to get

$$\begin{aligned} &\sum_{i=1}^n [z_i^2(t) + p_i^2(t)] + \sum_{j=1}^n [w_j^2(t) + q_j^2(t)] \\ \leq\ & \sum_{i=1}^n \{-[2\gamma_i - |1-\alpha_i - \gamma_i^2 + \gamma_i c_i| - \sum_{j=1}^n (|g_{ji} + |h_{ji}|) l_i] D_t^{-\alpha} z_i^2(t) \\ & -[2c_i - 2\gamma_i - |1-\alpha_i - \gamma_i^2 + \gamma_i c_i| - \sum_{j=1}^n (|a_{ij}| + |b_{ij}|) l_j] D_t^{-\alpha} p_i^2(t)\} \\ & + \sum_{j=1}^n \{-[2\gamma_j - |1-\beta_j - \gamma_j^2 + \gamma_j d_j| - \sum_{i=1}^n (|a_{ij}| + |b_{ij}|) l_j] D_t^{-\alpha} w_j^2(t) \\ & -[2d_j - 2\gamma_j - |1-\beta_j - \gamma_j^2 + \gamma_j d_j| - \sum_{i=1}^n (|g_{ji}| + |h_{ji}|) l_i] D_t^{-\alpha} q_j^2(t)\} \\ & + \frac{1}{\Gamma(\alpha+1)} \sum_{i=1}^n \sum_{j=1}^n [|b_{ij}| l_j w_j^* \tau^\alpha + |h_{ji}| l_i z_i^* \sigma^\alpha] \\ \leq\ & -\eta D_t^{-\alpha} [\sum_{i=1}^n (z_i^2(t) + p_i^2(t)) + \sum_{j=1}^n (w_j^2(t) + q_j^2(t))] + M(\varphi - \overline{\varphi}), \end{aligned}$$

where

$$M(\varphi-\overline{\varphi})=\frac{1}{\Gamma(\alpha+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}[|b_{ij}|l_jw_j^*\tau^\alpha+|h_{ji}|l_iz_i^*\sigma^\alpha],$$

$$\varphi=(\varphi_{11}(t),\varphi_{12}(t),\cdots,\varphi_{1n}(t),\psi_{11}(t),\psi_{12}(t),\cdots,\psi_{1n}(t))^{\mathrm{T}},$$

$$\overline{\varphi}=(\overline{\varphi}_{11}(t),\overline{\varphi}_{12}(t),\cdots,\overline{\varphi}_{1n}(t),\overline{\psi}_{11}(t),\overline{\psi}_{12}(t),\cdots,\overline{\psi}_{1n}(t))^{\mathrm{T}}.$$

According to Lemma 2.3, we have

$$\sum_{i=1}^{n}[z_i^2(t)+p_i^2(t)]+\sum_{j=1}^{n}[w_j^2(t)+q_j^2(t)]\le M(\varphi-\overline{\varphi})E_\alpha(-\eta t^\alpha).$$

Then,

$$\sum_{i=1}^{n}z_i^2(t)+\sum_{i=1}^{n}w_i^2(t)\le M(\varphi-\overline{\varphi})E_\alpha(-\eta t^\alpha).$$

Using the inequality $\sum\limits_{i=1}^{n}|a_i|\le(n\sum\limits_{i=1}^{n}a_i^2)^{\frac{1}{2}}$, we have

$$\|X(t)-\overline{X}(t)\|=\sum_{i=1}^{n}[|z_i(t)|+|w_i(t)|]\le 2[nM(\varphi-\overline{\varphi})E_\alpha(-\eta t^\alpha)]^{\frac{1}{2}}. \tag{3.11}$$

Obviously, $M(0)=0$, $M(\varphi-\overline{\varphi})\ge 0$. According to Definition 2.4, the solutions of the system (1.1) are globally Mittag-Leffler stable. □

Theorem 3.3. Suppose that $\mathbf{H_1}$–$\mathbf{H_3}$ hold, if the conditions of Theorems 3.1 and 3.2 are satisfied, then the system (1.1) has an anti-periodic solution which is globally Mittag-Leffler stable.

Proof. Let $X(t)=(x_1(t),x_2(t),\cdots,x_n(t),y_1(t),y_2(t),\cdots,y_n(t))^{\mathrm{T}}$ be any solution of system (1.1). For any natural number k and $\omega>0$ which is given in $\mathbf{H_3}$, we can get from (2.1) that

$$\begin{cases} D_t^\alpha[(-1)^{k+1}x_i(t+(k+1)\omega)]=-\gamma_i(-1)^{k+1}x_i(t+(k+1)\omega)+(-1)^{k+1}u_i(t+(k+1)\omega),\\ D_t^\alpha[(-1)^{k+1}u_i(t+(k+1)\omega)]=-(\alpha_i+{\gamma_i}^2-\gamma_ic_i)(-1)^{k+1}x_i(t+(k+1)\omega)\\ \qquad -(c_i-\gamma_i)(-1)^{k+1}u_i(t+(k+1)\omega)+\sum\limits_{j=1}^{n}a_{ij}f_j((-1)^{k+1}y_j(t+(k+1)\omega))\\ \qquad +\sum\limits_{j=1}^{n}b_{ij}(f_j((-1)^{k+1}y_j(t+(k+1)\omega-\tau_{ij}))+I_i(t),\\ D_t^\alpha[(-1)^{k+1}y_j(t+(k+1)\omega)]=-\gamma_j(-1)^{k+1}y_j(t+(k+1)\omega)+(-1)^{k+1}v_j(t+(k+1)\omega),\\ D_t^\alpha[(-1)^{k+1}v_j(t+(k+1)\omega)]=-(\beta_j+{\gamma_j}^2-\gamma_jd_j)(-1)^{k+1}y_j(t+(k+1)\omega)\\ \qquad -(d_j-\gamma_j)(-1)^{k+1}v_j(t+(k+1)\omega)+\sum\limits_{i=1}^{n}g_{ji}f_i((-1)^{k+1}x_i(t+(k+1)\omega))\\ \qquad +\sum\limits_{i=1}^{n}h_{ji}(f_i((-1)^{k+1}x_i(t+(k+1)\omega-\sigma_{ji}))+J_j(t). \end{cases} \tag{3.12}$$

It can be seen that for any natural number k,

$$(-1)^{k+1}x_i(t+(k+1)\omega),(-1)^{k+1}u_i(t+(k+1)\omega),(-1)^{k+1}y_j(t+(k+1)\omega),(-1)^{k+1}v_j(t+(k+1)\omega)$$

are the solutions of (2.1), then,

$$(-1)^k x_i(t+k\omega), (-1)^k u_i(t+k\omega), (-1)^k y_j(t+k\omega), (-1)^{kk} v_j(t+k\omega)\ (i,j=1,2,\cdots,n)$$

are also the solutions of (2.1). This means $(-1)^k x_i(t+k\omega), (-1)^k y_j(t+k\omega)$ are the solution of system (1.1). From Theorem 3.1, the solutions of system (1.1) $x_i(t),\ y_j(t)$ and $D_t^\alpha x_i(t),\ D_t^\alpha y_j(t)$ are bounded, due to

$$u_i(t) = D_t^\alpha x_i(t) + \gamma_i x_i(t),\ v_j(t) = D_t^\alpha y_j(t) + \gamma_j y_j(t),$$

we get $(-1)^k x_i(t+k\omega), (-1)^k u_i(t+k\omega), (-1)^k y_j(t+k\omega), (-1)^k v_j(t+k\omega)$ are also bounded, $i,j=1,2,\cdots,n$.

On the other hand, due to

$$ {}_a^{RL}D_t^\alpha g(t) = {}_a^C D_t^\alpha g(t) + \sum_{k=1}^{n-1} \frac{(t-a)^{k-\alpha}}{\Gamma(k-\alpha+1)} f^k(a),$$

where ${}_a^{RL}D_t^\alpha g(t)$ is Riemann-Liourille fractional-order derivative, ${}_a^C D_t^\alpha g(t)$ is Caputo fractional-order derivative, that is

$${}_a^C D_t^\alpha g(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau,\ n-1<\alpha<n.$$

So that we can get $x_i(t),\ y_j(t),\ u_i(t),\ v_j(t)\ (i,j=1,2,\cdots,n)$ are differentiable, then we obtain the function sequence $(-1)^k x_i(t+k\omega), (-1)^k u_i(t+k\omega), (-1)^k y_j(t+k\omega), (-1)^k v_j(t+k\omega)$ is equicontinuity and uniform boundedness. Using the Ascoli-Arzela theorem, we can select a subsequence $\{k\omega\}_{k\in N}$, such that $\{(-1)^k x_i(t+k\omega)\}_{k\in N}, \{(-1)^k y_j(t+k\omega)\}_{k\in N}, \{(-1)^k u_i(t+k\omega)\}_{k\in N}, \{(-1)^k v_j(t+k\omega)\}_{k\in N}$ $(N = 1,2,\cdots,n,\cdots)$ uniformly converge to continuous functions $x_i^*(t),\ y_j^*(t),\ u_i^*(t),\ v_j^*(t)$ on any compact set in $[0,+\infty)$, that is,

$$\lim_{k\to+\infty} (-1)^k x_i(t+k\omega) = x_i^*(t),\ \lim_{k\to+\infty} (-1)^k y_j(t+k\omega) = y_j^*(t),$$

$$\lim_{k\to+\infty} (-1)^k u_i(t+k\omega) = u_i^*(t),\ \lim_{k\to+\infty} (-1)^k v_j(t+k\omega) = v_j^*(t).$$

Let

$$X^*(t) = (x_1^*(t), x_2^*(t), \cdots, x_n^*(t), y_1^*(t), y_2^*(t), \cdots, y_n^*(t))^{\mathrm{T}},$$
$$U^*(t) = (u_1^*(t), u_2^*(t), \cdots, u_n^*(t), v_1^*(t), v_2^*(t), \cdots, v_n^*(t))^{\mathrm{T}}.$$

We can prove that $X^*(t)$ and $U^*(t)$ are anti-periodic functions. In fact,

$$X^*(t) = \lim_{k\to+\infty} (-1)^k X(t+\omega+k\omega) = -\lim_{k\to+\infty} (-1)^{k+1} X(t+(k+1)\omega) = -X^*(t),$$

$$U^*(t) = \lim_{k\to+\infty} (-1)^k U(t+\omega+k\omega) = -\lim_{k\to+\infty} (-1)^{k+1} U(t+(k+1)\omega) = -U^*(t),$$

from this, $X^*(t)$ and $U^*(t)$ are the ant-periodic functions. Because

$$\lim_{k\to+\infty} (-1)^{k+1} x_i(t+(k+1)\omega) = x_i^*(t),\ \lim_{k\to+\infty} (-1)^{k+1} y_j(t+(k+1)\omega) = y_j^*(t),$$

$$\lim_{k\to+\infty} (-1)^{k+1} u_i(t+(k+1)\omega) = u_i^*(t),\ \lim_{k\to+\infty} (-1)^{k+1} v_j(t+(k+1)\omega) = v_j^*(t).$$

When $f_i(\cdot)$ is continuous, it can be inferred from (3.12) that

$$\begin{cases} D_t^\alpha x_i^*(t) = -\gamma_i x_i^*(t) + u_i^*(t), \\ D_t^\alpha u_i^*(t) = -(\alpha_i + \gamma_i^2 - \gamma_i c_i)x_i^*(t) - (c_i - \gamma_i)u_i^*(t) + \sum\limits_{j=1}^n a_{ij} f_j(y_j^*(t)) + \sum\limits_{j=1}^n b_{ij} f_j(y_j^*(t-\tau_{ij})) + I_i(t), \\ D_t^\alpha y_j^*(t) = -\gamma_j y_j^*(t) + v_j^*(t), \\ D_t^\alpha v_j^*(t) = -(\beta_j + \gamma_j^2 - \gamma_j d_j)y_j^*(t) - (d_j - \gamma_j)v_j^*(t) + \sum\limits_{i=1}^n g_{ji} f_i(x_i^*(t)) + \sum\limits_{i=1}^n h_{ji} f_i(x_i^*(t-\sigma_{ji})) + J_j(t). \end{cases} \tag{3.13}$$

That is, $X^*(t)$ and $U^*(t)$ are the anti-periodic solutions of (2.1). This means $X^*(t)$ is the anti-periodic solution of system (1.1). According to (3.11), we have

$$\|X(t) - X^*(t)\| \le 2[nM(\varphi - \varphi^*)E_\alpha(-\eta t^\alpha)]^{\frac{1}{2}},$$

where $X^*(t)$ is with the initial value $\varphi^* = (\varphi_{11}^*(s), \varphi_{12}^*(s), \cdots, \varphi_{1n}^*(s), \psi_{11}^*(s), \psi_{12}^*(s), \cdots, \psi_{1n}^*(s))^{\mathrm{T}}$, $-\delta \le s \le 0$, $\delta = \max\{\tau, \sigma\}$, $M(\varphi - \varphi^*) \ge 0$, $M(0) = 0$. Therefore, the system (1.1) has an anti-periodic solution which is Mittag-Leffler stable. □

4. Numerical example

Example 1. Consider the following fractional-order inertial BAM neural networks with time-delays:

$$\begin{cases} D_t^{2\alpha} x_i(t) = -c_i D_t^\alpha x_i(t) - \alpha_i x_i(t) + \sum\limits_{j=1}^2 a_{ij} f_j(y_j(t)) + \sum\limits_{j=1}^2 b_{ij} f_j(y_j(t-\tau_{ij})) + I_i(t), \\ D_t^{2\alpha} y_j(t) = -d_j D_t^\alpha y_j(t) - \beta_j y_j(t) + \sum\limits_{i=1}^2 g_{ji} f_i(x_i(t)) + \sum\limits_{i=1}^2 h_{ji} f_i(x_i(t-\sigma_{ji})) + J_j(t), \end{cases} \tag{4.1}$$

for $t \ge 0$, $i, j = 1, 2$. The parameters settings are as follows: $\alpha = 0.85$, $f_i(x_i) = \frac{1}{50}(|x_i+1| - |x_i-1|)$, $\tau_{ij} = \sigma_{ij} = 1$, $i, j = 1, 2$. $I_1(t) = 0.01\cos(t)$, $I_2(t) = 0.01\cos(t)$, $J_1(t) = 0.03\sin(t)$, $J_2(t) = 0.01\sin(t)$, $\alpha_1 = 2.5$, $\alpha_2 = 2.6$, $\beta_1 = 2.5$, $\beta_2 = 3$, $c_1 = 2.5$, $c_2 = 2.6$, $d_1 = 2.8$, $d_2 = 2.6$, $a_{11} = 0.4$, $a_{12} = 0.5$, $a_{21} = 0.5$, $a_{22} = 0.8$, $b_{11} = -0.1$, $b_{12} = 0.1$, $b_{21} = 0.2$, $b_{22} = 0.2$, $g_{11} = 0.6$, $g_{12} = 0.5$, $g_{21} = 0.5$, $g_{22} = 0.6$, $h_{11} = 0.1$, $h_{12} = 0.5$, $h_{21} = 0.4$, $h_{22} = 0.2$.

We choose $l_i = 1$, $i = 1, 2$, $\omega = \pi$, $\gamma_1 = 1.25$, $\gamma_2 = 1.35$. After calculation, we get

$$\eta_1 = \min_{1\le i\le n}\{\gamma_i - |\alpha_i + \gamma_i^2 - \gamma_i c_i|, c_i - \gamma_i - 1\} = 0.25 > 0,$$

$$\eta_2 = \min_{1\le j\le n}\{\gamma_j - |\beta_j + \gamma_j^2 - \gamma_j d_j|, d_j - \gamma_j - 1\} = 0.0375 > 0,$$

$$\eta = \min\{\min_{1\le i\le n}\{2\gamma_i - |1 - \alpha_i - \gamma_i^2 + \gamma_i c_i| - \sum_{j=1}^n (|g_{ji}| + |h_{ji}|)l_i,\ 2c_i - 2\gamma_i - |1 - \alpha_i - \gamma_i^2 + \gamma_i c_i| - \sum_{j=1}^n (|a_{ij}| + |b_{ij}|)l_j\},$$

$$\min_{1\le j\le n}\{2\gamma_j - |1 - \beta_j - \gamma_j^2 + \gamma_j d_j| - \sum_{i=1}^n (|a_{ij}| + |b_{ij}|)l_j,\ 2d_j - 2\gamma_j - |1 - \beta_j - \gamma_j^2 + \gamma_j d_j| - \sum_{i=1}^n (|g_{ji}| + |h_{ji}|)l_i\}\} = 0.4875 > 0.$$

That is to say, the conditions in Theorem 3.3 are satisfied. It can be seen that system (4.1) has an anti-periodic solution with a period of π and it is Mittag-Leffler stable.

On the other hand, we get the state of the example through numerical simulation in Figure 1. It can be seen from the figure that it is consistent with the theoretical result of Theorem 3.3.

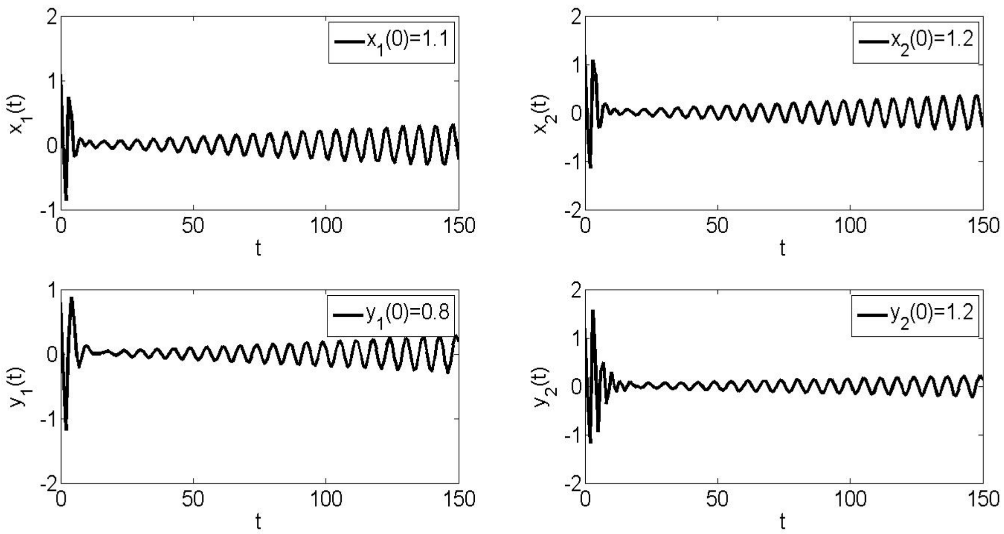

Figure 1. The state variables $x_1(t)$, $x_2(t)$, $y_1(t)$, $y_2(t)$ of example at time t.

5. Conclusions

The fractional-order inertial neural networks considers the damping factor and can simulate more complex networks systems. Therefore, compared with fractional-order neural networks without inertia term, it has more research value in theory and application. The anti-periodic phenomena of nonlinear differential equations appear widely in biology, physics and many other fields. The fractional-order inertial BAM neural networks model is a system of nonlinear differential equations, which has a wide application background. It is important to study the existence and stability of the anti-periodic solution for the system in dynamic behaviors.

In this paper, we study the existence and Mittag-Leffler stability of anti-periodic solutions for fractional-order inertial BAM neural networks with time-delays. It is a new topic. Studying the existence and stability of the anti-periodic solution for the system has certain research value in theoretical exploration. By introducing variable substitution, the BAM neural networks model with two fractional-order derivatives of different orders is transformed into a model with only one fractional-order neural networks of the same order. Using the fractional-order calculus properties, the boundedness and Mittag-Leffler stability of the system solution are given. By constructing the sequence solution of the function for the system and applying the Ascoli-Arzela theorem, the sufficient conditions for the existence and Mittag-Leffler stability of anti-periodic solutions are obtained, and the validity of the conclusions derived in this paper is verified by numerical simulation. We give the sufficient conditions for the existence and stability of anti-periodic solutions for a class of fractional-order inertial BAM neural networks, that is, the results of Theorems 3.1–3.3. From the data we have consulted, there are no results of the study on the stability of anti-periodic solutions for fractional-order inertial BAM neural networks. Therefore, our research topic is new, and the results obtained are also new. The results provide theoretical criteria for further research on innovation and practical application of system dynamic behavior.

Using the research ideas and methods in this paper, we can further study other types of fractional-order inertial neural networks with time-delays, such as the existence and Mittag-Leffler stability of anti-periodic solutions for fractional-order inertial Cohen-Grossberg type BAM neural networks with time-delays.

Acknowledgments

The work was supported by Science Project of Zhejiang Educational Department (No. Y202145903), the Ministry of Education's Cooperative Education Project (220603284143545), Science Project of Shaoxing University (No. 2020LG1009) and Science Project of Shaoxing University Yuanpei College (KY2021C04).

Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this article.

References

1. A. R. Aftabizadeh, S. Aizicovici, N. H. Pavel, On a class of second-order anti-periodic boundary value problems, *J. Math. Anal. Appl.*, **171** (1992), 301–320. https://doi.org/10.1016/0022-247X(92)90345-E
2. S. Aizicovici, M. Mckibben, S. Reich, Anti-periodic solutions to nonmonotone evolution equations with discontinuous nonlinearities, *Nonlinear Anal.*, **43** (2001), 233–251. https://doi.org/10.1016/S0362-546X(99)00192-3
3. Y. Q. Chen, J. J. Nieto, D. O'Regan, Anti-periodic solutions for fully nonlinear first-order differential equations, *Math. Comput. Model.*, **46** (2007), 1183–1190. https://doi.org/10.1016/j.mcm.2006.12.006
4. H. L. Chen, Antiperiodic wavelets, *J. Comput. Math.*, **14** (1996), 32–39.
5. Y. Q. Li, L. H. Huang, Anti-periodic solutions for a class of Liénard-type systems with continuously distributed delays, *Nonlinear Anal. Real World Appl.*, **10** (2009), 2127–2132. https://doi.org/10.1016/j.nonrwa.2008.03.020
6. Z. Q. Zhang, Z. Y. Quan, Global exponential stability via inequality technique for inertial BAM neural networks with time delays, *Neurocomputing*, **151** (2015), 1316–1326. https://doi.org/10.1016/j.neucom.2014.10.072
7. Z. Q. Zhang, W. B. Liu, D. M. Zhou, Global asymptotic stability to a generalized Cohen-Grossberg BAM neural networks of neutral type delays, *Neural Networks*, **25** (2012), 94–105. https://doi.org/10.1016/j.neunet.2011.07.006
8. Z. Q. Zhang, J. D. Cao, D. M. Zhou, Novel LMI-based condition on global asymptotic stability for a class of Cohen-Grossberg BAM networks with extended activation functions, *IEEE Trans. Neural Net. Learn. Syst.*, **25** (2014), 1161–1172. https://doi.org/10.1109/TNNLS.2013.2289855

9. X. Y. Fu, F. C. Kong, Global exponential stability analysis of anti-periodic solutions of discontinuous bidirectional associative memory (BAM) neural networks with time-varying delays, *Int. J. Nonlinear Sci. Numer. Simul.*, **21** (2020), 807–820. https://doi.org/10.1515/ijnsns-2019-0220

10. N. Radhakrishnan, R. Kodeeswaran, R. Raja, C. Maharajan, A. Stephen, Global exponential stability analysis of anti-periodic of discontinuous BAM neural networks with time-varying delays, *J. Phys. Conf. Ser.*, **1850** (2021), 1–20. https://doi.org/10.1088/1742-6596/1850/1/012098

11. C. F. Xu, F. C. Kong, Global exponential stability of anti-periodic solutions for discontinuous Cohen-Grossberg neural networks with time-varying delays, *J. Exp. Theor. Artif. Intell.*, **33** (2021), 263–281. https://doi.org/10.1080/0952813X.2020.1737244

12. P. Cui, Z. B. Li, Anti-periodic solutions for BAM-type Cohen-Grossberg neural networks with time delays, *J. Nonlinear Sci. Appl.*, **10** (2017), 2171–2180. https://doi.org/10.22436/jnsa.010.04.69

13. Y. Q. Ke, C. F. Miao, Anti-periodic solutions of inertial neural networks with time delays, *Neural Process. Lett.*, **45** (2017), 523–538. https://doi.org/10.1007/s11063-016-9540-z

14. A. Arbi, N. Tahri, C. Jammazi, C. X. Huang, J. D. Cao, Almost anti-periodic solution of inertial neural networks with leakage and time-varying delays on timescales, *Circuits Syst. Signal Process.*, **41** (2022), 1940–1956. https://doi.org/10.1007/s00034-021-01894-4

15. Q. Cao, X. J. Guo, Anti-periodic dynamics on high-order inertial Hopfield neural networks involving time-varying delays, *AIMS Math.*, **5** (2020), 5402–5421. https://doi.org/10.3934/math.2020347

16. L. G. Yao, Q. Cao, Anti-periodicity on high-order inertial Hopfield neural networks involving mixed delays, *J. Inequal. Appl.*, **2020** (2020), 182. https://doi.org/10.1186/s13660-020-02444-3

17. B. S. Chen, J. J. Chen, Global asymptotical ω-periodicity of a fractional-order non-autonomous neural networks, *Neural Networks*, **68** (2015), 78–88. https://doi.org/10.1016/j.neunet.2015.04.006

18. Y. Y. Hou, L. H. Dai, S-asymptotically ω-periodic solutions of fractional-order complex-valued recurrent neural networks with delays, *IEEE Access*, **9** (2021), 37883–37893. https://doi.org/10.1109/ACCESS.2021.3063746

19. A. L. Wu, Z. G. Zeng, Boundedness, Mittag-Leffler stability and asymptotical ω-periodicity of fractional-order fuzzy neural networks, *Neural Networks*, **74** (2016), 73–84. https://doi.org/10.1016/j.neunet.2015.11.003

20. Y. K. Li, M. Huang, B. Li, Besicovitch almost periodic solutions for fractional-order quaternion-valued neural networks with discrete and distributed delays, *Math. Methods Appl. Sci.*, **45** (2022), 4791–4808. https://doi.org/10.1002/mma.8070

21. P. Wan, D. H. Sun, M. Zhao, H. Zhao, Monostability and multistability for almost-periodic solutions of fractional-order neural networks with unsaturating piecewise linear activation functions, *IEEE Trans. Neural Net. Learn. Syst.*, **31** (2020), 5138–5152. https://doi.org/10.1109/TNNLS.2020.2964030

22. C. J. Xu, P. L. Li, On anti-periodic solutions for neutral shunting inhibitory cellular neural networks with time-varying delays and D operator, *Neurocomputing*, **275** (2018), 377–382. https://doi.org/10.1016/j.neucom.2017.08.030

23. C. X. Huang, S. G. Wen, L. H. Huang, Dynamics of anti-periodic solutions on shunting inhibitory cellular neural networks with multi-proportional delays, *Neurocomputing*, **357** (2019), 47–52. https://doi.org/10.1016/j.neucom.2019.05.022

24. Y. K. Li, J. L. Xiang, Existence and global exponential stability of anti-periodic solution for Clifford-valued inertial Cohen-Grossberg neural networks with delays, *Neurocomputing*, **332** (2019), 259–269. https://doi.org/10.1016/j.neucom.2018.12.064

25. I. Podlubny, *Fractional differential equations*, New York: Academic Press, 1998.

26. L. Ke, Mittag-Leffler stability and asymptotic ω-periodicity of fractional-order inertial neural networks with time-delays, *Neurocomputing*, **465** (2021), 53–62. https://doi.org/10.1016/j.neucom.2021.08.121

27. Y. J. Gu, H. Wang, Y. G. Yu, Stability and synchronization for Riemann-Liouville fractional-order time-delayed inertial neural networks, *Neurocomputing*, **340** (2019), 270–280. https://doi.org/10.1016/j.neucom.2019.03.005