Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 8(11): 27626–27642.
DOI: 10.3934/math.20231413
Received: 02 May 2023
Revised: 25 July 2023
Accepted: 15 August 2023
Published: 07 October 2023

Research article

Mathematical modeling for the development of traffic based on the theory of system dynamics

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Abstract: This paper is concerned with mathematical modeling for the development of Shandong traffic. The system dynamics model of the development of traffic in Shandong is established. In terms of this model, it is shown that highway operation as well as rail transit promotes the development of traffic, while traffic accidents inhibit traffic development. Moreover, the maximum error between the output data and the statistics bureau, based on which some forecasts for the development of traffic in the future are given, is obtained, some suggestions and optimization schemes for traffic development are given. Finally, a neural network model of the development of Shandong traffic is also derived.

Keywords: traffic development; system dynamics; neural network model
Mathematics Subject Classification: 34A37

1. Introduction

Recently, the traffic situation has been improved, which is reflected by the amount of passenger on a highway of 15 billion people in 2018, and the total length of rail lines in China has reached 5761.4 kilometers, therefore, the study of the traffic system by different theories is crucial. For example, based on mathematical modeling, reference [11] shows that the emergency vehicle management solution can reduce travel times of EVs without causing any performance degradation of normal vehicles. Zhu [30] proposed a bayesian updating approach based on the dirichlet model to describe the traffic system performance. Via machine learning, Saleem [21] proposed a fusion-based intelligent traffic congestion control system to alleviate traffic congestion in smart cities.

The traffic system is a complex dynamic system whose influencing factors are complex and diverse, thus, the mathematical modeling of traffic system is difficult. In order to understand the dynamic behavior of traffic systems comprehensively, the system dynamic (SD) method is one of the most powerful tools [10]. The theory of system dynamics founded by Forrester [7] can be used to simplify the multi-variable system, and it is widely used to deal with social and economic problems. As for the application of the theory of system dynamics, one can refer to [4, 18, 25, 26], to name but a few. The basic concepts of system dynamics are as follows:

(1) Level variable, represented by a rectangle in VENSIM software, are affected by rate variables, which can be expressed by integrating the rate variables.
(2) Rate variables are time functions, and they determine the level variable.
(3) For ease of communication and clarity, it is often to define auxiliary variables, which are neither stock nor flow.
(4) The constant variable is a constant, and it can be characterized by table functions.
(5) Table functions conveniently represent nonlinear relationships.
(6) Flow graph is a characteristic diagram in system dynamics. It simplifies the equations of models. In the sequel, the flow graph can be transformed into VENSIM equations.

The modeling steps for system dynamics in VENSIM software are shown by Figure 1.

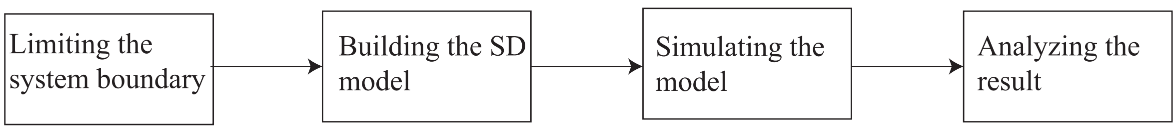


Figure 1. Modeling steps of system dynamics.

Recently, research based on system dynamic theory mainly focuses on urban traffic, low-carbon traffic as well as traffic policies. According to the restriction policy, Wen [27] calculated that the implementation of the tail licensing restriction policy could effectively reduce carbon dioxide emissions by 3.86%. Moreover, they found that the vehicle license limit policy could effectively curb the growth of car ownership, and alleviate traffic congestion. Jia [14] established an SD model of traffic congestion charge and subsidy. Furthermore, it can be found that zero-subsidy and low charge reduce carbon dioxide emissions. He [10] concluded that the railway occupation rate was directly proportional to the railway length. Additionally, in order to assess the relation between numbers of available public traffic and traffic congestion in Jakarta, Sardjono [24] developed an SD model for traffic conditions, which can reduce traffic congestion. Based on road accident statistics, Victor [16] proposed the methodology of evaluated long-term trends in the dynamics of traffic safety improvement. Rajput [20] present a system dynamics simulation model to reduce traffic congestion by implementing an Intelligent Transportation System in metropolitan cities of India. Ye [29] concluded that highway investment had a certain pulling effect on economic growth based on a concrete analysis on the relationship between highway construction and economic growth.

In China, the traffic system mainly includes taxi, road vehicles, private cars and rail transit [8]. The interactive communication between them forms a complex dynamic system of traffic. To the knowledge of the author, there still lacks proper mathematical models to study traffic systems, including the six subsystem by the SD method. Based on the actual situation of the traffic system in Shandong Province, this paper proposes an SD model to describe the relationship among the six subsystem which contains the highway vehicle subsystem, the private car subsystem, the highway mileage subsystem, the rail transit subsystem and the traffic accident subsystem. Based on the SD model of the traffic system, some forecasts for the development of traffic in the future are given and some suggestions and optimization schemes for traffic development are given. This model is helpful for improving the convenience of traffic service and to promote stable traffic.

2. Main results

2.1. SD model

In highway vehicle subsystems, the number of operating vehicles is affected by highway mileage and highway density [6]. Generally speaking, highway has positive effects on the number of vehicles. In addition, the scrap of private cars can force people to take public traffic, which requires the increasing of investment in public traffic [3]. Thus, the above factors are defined as the influencing factors of the number of operating vehicles.

In taxi subsystems, taxi growth and taxi scrap are considered as two main factors which lead to the overall change in the number of taxis. As for the private car subsystems, it is used in traffic frequently, but it brings a lot of exhaust pollution [13]. In highway mileage subsystem, with the increasing of all kinds of vehicles, highway construction has been accelerated. On the contrary, old roads reduce highway mileage [6]. Highway mileage is an important embodiment of traffic development. In order to indicate the situation of highway mileage, we represent it by old roads and the number of vehicles.

In rail transit subsystems, with the increase of damage to the track, the length of rail transit has decreased. Track is influenced by the use of various vehicles and track investment [9, 28]. Then, the number of private cars and track damage are chosen as the influencing factors of rail transit length.

In traffic accident subsystems, the national economy can be affected by traffic accidents, and a suitable traffic condition can reduce the number of occurrences of traffic accidents effectively. Public traffic can affect the number of traffic accidents directly, which changes the choice of travel modes. Hence, we can get the influencing factors for the number of operating vehicles.

In terms of the fact indicated above, the following parameters are selected as Level variables: traffic accidents, number of road vehicles, highway mileage, number of taxis, number of private cars owned and length of rail transit line. For the sake of convenience, in the following context, all the elements are summarized in Table 1. In addition, the scenario diagram of the SD model is displayed in Figure 2.

According to the scenario diagram of traffic shown in Figure 2, each subsystem is associated with one state variable, such as T_1, T_9, T_{15}, T_{20}, T_{26}, T_{33} and so on. All the subsystems and their influence parameters can be found in Table 1.

In order to evaluate the situation of traffic, an SD model of the traffic system in Shandong Province was established by VENSIM software, the feedback process of flow diagram is marked by a blue arrow in Figure 3. Furthermore, it can be obtained that the influence between the parameters is multiplex and the influences between the six subsystems are mutual.

Table 1. All parameters in the SD model.

Subsystem	Parameter	Initial value	Description
Traffic accident	T		Traffic development index
	T_1	14.56 thousand times	Traffic accident
	T_2		Growth of traffic accident
	T_3		Mortality rate of traffic accidents
	T_4		Number of deaths in traffic accident
	T_5		Reduction of traffic accident
	T_6		Input of relevant personnel
	T_7		Relevant policy
	T_8		Education influence coefficient
Highway vehicle	T_9	913.8 thousand units	Number of vehicles in operation
	T_{10}		Road density
	T_{11}		Travel volume of highway operation
	T_{12}		Growth of road vehicles
	T_{13}		Car scrapping in highway operation
	T_{14}		Average car scrap rate
Highway mileage	T_{15}	229.9 thousand km	Highway mileage
	T_{16}		Growth of highway mileage
	T_{17}		Growth rate of highway mileage
	T_{18}		Reduction of highway mileage
	T_{19}		Reduction rate of highway mileage
Taxis	T_{20}	57.7 thousand units	Number of taxi
	T_{21}		Taxi trips
	T_{22}		Growth of taxi
	T_{23}		Growth rate of taxi
	T_{24}		Taxi scrap
	T_{25}		Taxi scrap rate
Private car	T_{26}	5.77 million units	Number of private car
	T_{27}		Private car trip
	T_{28}		Damage of private car
	T_{29}		Growth of private car
	T_{30}		GDP
	T_{31}		Growth rate of private cars
	T_{32}		Private car scrapping
Rail transit	T_{33}	38 million km	Length of rail transit
	T_{34}		Line down
	T_{35}		Travel volume of rail transit
	T_{36}		Growth of rail transit line length
	T_{37}		Reduction of rail transit line
	T_{38}		Track damage
	T_{39}		Growth rate of track
	T_{40}		Investment

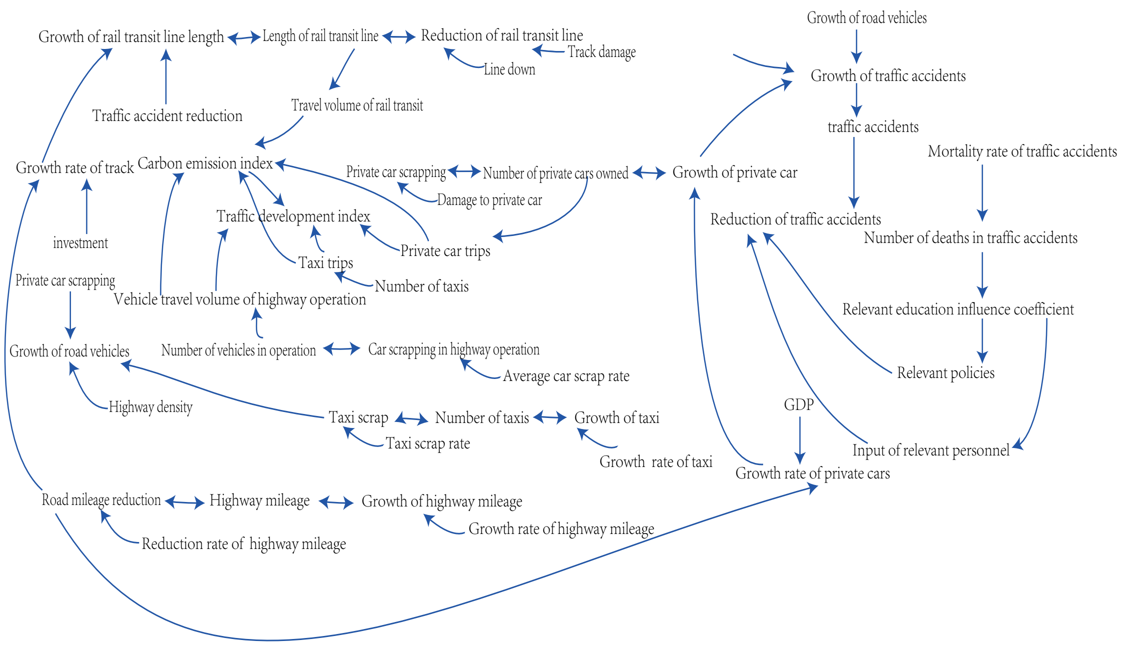


Figure 2. Scenario diagram of SD model.

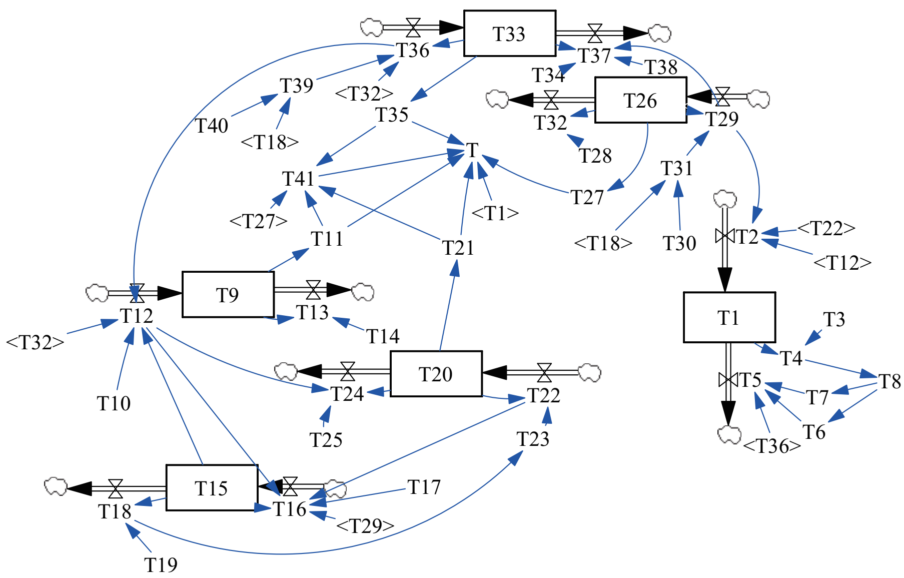


Figure 3. Flow diagram of the SD model.

2.2. Mathematical model

In light of the flow diagram (see Figure 3), the SD model can be established by translating the SD flow diagram into VENSIM equations, and thus the level variable can be expressed as the following integral equation [25]

$$Level(t) = \int_{t_0}^{t} Rate_{in}(s) - Rate_{out}(s)ds + Level(t_0), \tag{2.1}$$

in which, $Level(t)$ represents level variables, $Rate_{in}$ and $Rate_{in}$ signify inflow rate variables and outflow rate variables respectively. Thus, we have

$$\begin{cases} T_1(t) = \int_{t_0}^{t} T_2(s) - T_5(s)ds, \\ T_9(t) = \int_{t_0}^{t} T_{12}(s) - T_{13}(s)ds, \\ T_{15}(t) = \int_{t_0}^{t} T_{16}(s) - T_{18}(s)ds + 25.96, \\ T_{20}(t) = \int_{t_0}^{t} T_{22}(s) - T_{24}(s)ds + 60.119, \\ T_{26}(t) = \int_{t_0}^{t} T_{29}(s) - T_{32}(s)ds, \\ T_{33}(t) = \int_{t_0}^{t} T_{36}(s) - T_{37}(s)ds + 0.5, \end{cases} \tag{2.2}$$

where s is time at any time between the initial time t_o to the current time t, the unit of time assumed here is a year. On the other hand, we find

$$\frac{dLevel(t)}{dt} = Rate_{in}(t) - Rate_{out}(t),$$

hence

$$\begin{cases} \frac{dT_1(t)}{dx} = T_2(t) - T_5(t), \\ \frac{dT_9(t)}{dx} = T_{12}(t) - T_{13}(t), \\ \frac{dT_{15}(t)}{dx} = T_{16}(t) - T_{18}(t), \\ \frac{dT_{20}(t)}{dx} = T_{22}(t) - T_{24}(t), \\ \frac{dT_{26}(t)}{dx} = T_{29}(t) - T_{32}(t), \\ \frac{dT_{33}(t)}{dx} = T_{36}(t) - T_{37}(t). \end{cases} \tag{2.3}$$

Following $Increment = Growth\ rate \times Original\ data$, we have

$$\begin{cases} T_4(t) = T_1(t) \times T_3, \\ T_{13}(t) = T_9(t) \times T_{14}, \\ T_{18}(t) = T_{15}(t) \times T_{19}, \\ T_{22}(t) = T_{20}(t) \times T_{23}(t), \\ T_{29}(t) = T_{26}(t) \times T_{31}(t). \end{cases} \tag{2.4}$$

In traffic accident subsystems, the mortality rate is affected by travel modes, highway vehicles and taxis, and these are often selected as the analysis objects [15, 23], rail transit can decrease the accident rate [17]. In light of the data on the traffic accidents, we can find that the increase in the number of car leads to an increase in the number of traffic accidents, and the implementation of rail transit as well as positive policy can reduce the number of accidents. In addition, education on the safety awareness for people can also reduce the number of accidents. With respect to the highway subsystem, the number of cars is proportional to the number of roads and the miles of highway [12]. Investment in traffic can increase the mileage of highway [29]. In taxi subsystems, the expenditure on taking a taxi is higher than use of public traffic, but it is necessary to increase taxis reasonably, and the use of taxis is accompanied by the scrapping of taxis [19]. As for the private car subsystem, private cars are contradictory to public traffic, and they are positively related to highway mileage [3, 6]. In rail transit subsystem, the increase of the length of rail transit results in increasing of the investment of rail [9]. On the other hand, there exists the conflict between rail transit and private car [28]. The state of GDP is an important index of national development and it can reflect the government investment on traffic infrastructure construction. As indicated above, we obtain the following expressions on the rest of variables,

$$\begin{cases} T_2(t) = 0.316T_1(t) + 0.126T_{12}(t) + 0.512T_{22}(t), \\ T_5(t) = T_6(t) + 0.223T_7(t) + 0.125T_{36}(t), \\ T_7(t) = 0.656T_8(t), \\ T_8(t) = 5.89T_4(t), \\ T_{12}(t) = 0.316T_{10} + 0.233T_{15}(t) + 0.14T_{36}(t), \\ T_{16}(t) = T_{40} \times T_{15}(t), \\ T_{23}(t) = 0.205T_{25} - 0.065, \\ T_{24}(t) = T_{25} \times T_{20}(t) + 0.19T_{12}, \\ T_{31}(t) = 0.344T_{18}(t) + 0.756T_{28}, \\ T_{32}(t) = 0.625T_{22}(t) + 0.365T_{28}, \\ T_{36}(t) = 0.872T_{39} \times T_{33} + 0.128T_{32}, \\ T_{37}(t) = 0.233T_{29}(t) + 0.569T_{38}, \\ T_{39}(t) = 0.456T_{18}(t) + 0.544T_{40}. \end{cases} \tag{2.5}$$

2.3. Optimization of SD model

This subsection is devoted to optimize the SD model (see Figures 4 and 5). By comparing the output data with real data, four factors were selected in the model to judge the relative error of an SD model. The data of the National Bureau of Statistics and output date are expressed by R and C respectively, O denotes the output data of optimized model, and the relative error of the optimized model is signified by Oe. The following expressions to calculate the Oe of four curves is defined as

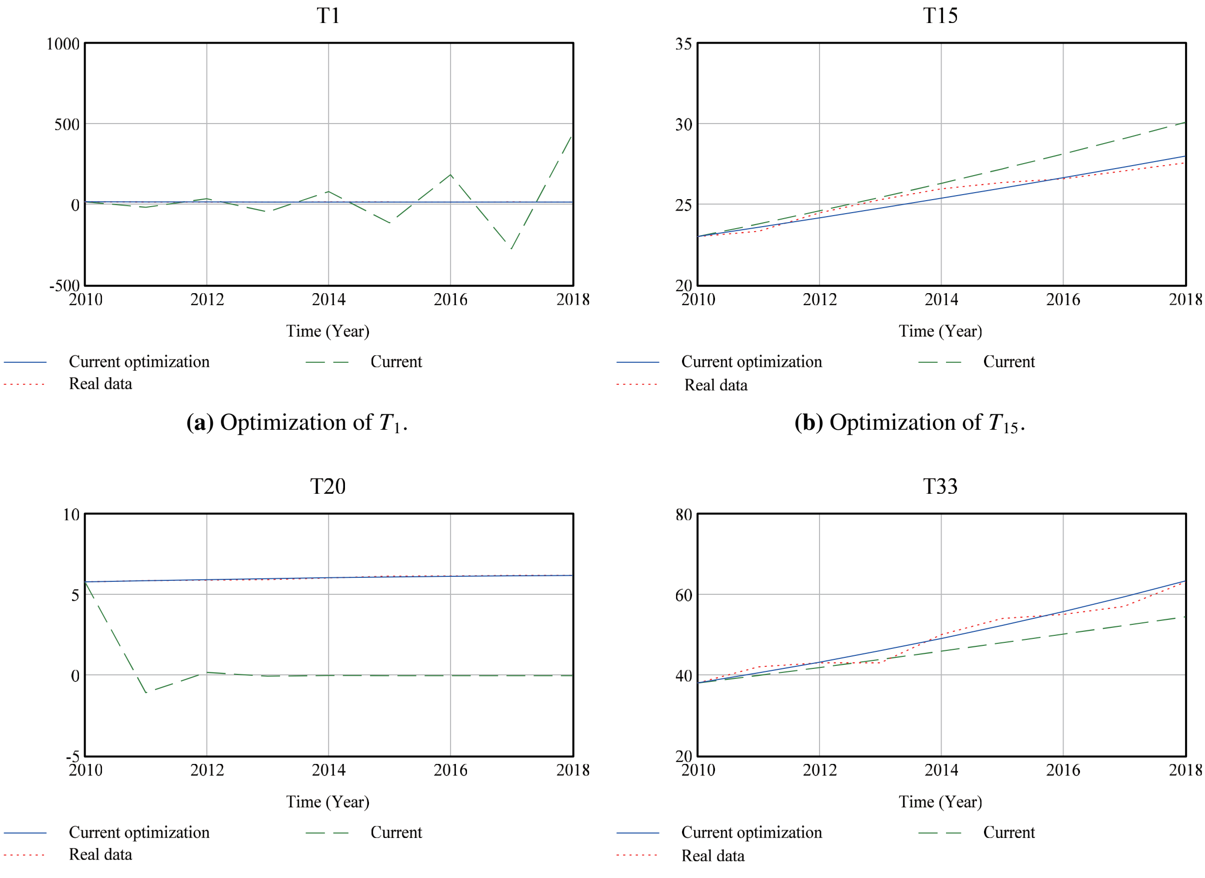
$$Oe = \frac{|R - O|}{|R|}. \tag{2.6}$$

$T_3, T_{10}, T_{14}, T_{17}, T_{19}, T_{25}, T_{28}, T_{30}, T_{34}, T_{38}$ and T_{40} are selected as the optimized parameters to examine. There are four optimization models related to the eleven objectives, see Figure 5. It is shown that the optimized curves O 1:4 are close to the real curves R 1:4. It can be seen that the O curves are closer to the R curves.

Table 2 shows that the relative error of the output data is less than the limit of error of 10% in system dynamics, which means that the precision of the model is enough.

Maximum payoff found at:
T3 = 0.0966794
T10 = 80
T14 = 0.291712
T17 = 1.74955
T19 = -0.010023
T25 = -0.135105
T28 = 0.0669325
T30 = 2.49007
T34 = 0.977206
T38 = 0.233964
T4O = 41.878

Figure 4. Optimization results of the SD model.



(a) Optimization of T_1.

(b) Optimization of T_{15}.

(c) Optimization of T_{20}.

(d) Optimization of T_{33}.

Figure 5. Optimization of SD model.

Table 2. The relative error of SD model.

Year	2013	2014	2015	2016	2017	2018
C T_1	-48.05	77.94	-115.93	182.81	-277.03	431.36
C T_{15}	25.42	26.29	27.18	28.11	29.07	30.06
C T_{20}	-0.08	-0.04	-0.04	-0.04	-0.04	-0.05
C T_{33}	43.82	45.88	47.98	50.10	52.25	54.39
R T_1	12.88	13.57	13.38	13.16	13.40	13.23
R T_{15}	25.28	25.95	26.34	26.57	27.06	27.56
R T_{20}	5.91	6.01	6.12	6.13	6.17	6.17
R T_{33}	43	50	54	55	57	63
O T_1	13.39	13.21	13.12	13.10	13.15	13.27
O T_{15}	24.75	25.36	25.99	26.64	27.30	27.98
O T_{20}	5.96	6.024	6.071	6.11	6.14	6.16
O T_{33}	45.98	48.88	51.98	55.24	58.66	62.23
Oe T_1	3.97%	2.62%	1.89%	0.46%	1.87%	0.36%
Oe T_{15}	2.09%	2.25%	1.3%	0.28%	0.91%	1.55%
Oe T_{20}	1.04%	0.02%	0.84%	0.32%	0.39%	0.07%
Oe T_{33}	6.83%	2.23%	3.73%	0.45%	2.92%	1.22%

Note: The date of this table comes from China's National Bureau of Statistics.

2.4. Parameter analysis

Based on the SD model, ten debugging models are established, in which T is taken as the objective function, and T_3, T_{10}, T_{30} and T_{40} are taken as the design variables. By using these debugging models, the optimal design of traffic can be obtained to provide a better traffic control strategy. Based on the results of debugging, the effects of different debugging models on the traffic are evaluated.

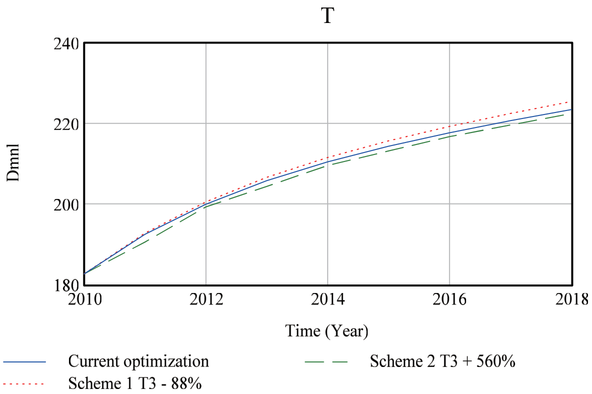
From Figure 6, which shows the current optimization curve of T, we have:

(1) The variation of T associated with the traffic accident subsystem is the maximal one in scheme 1 (see Figure 6a), which means that the government should take measures to reduce the number of traffic accidents.

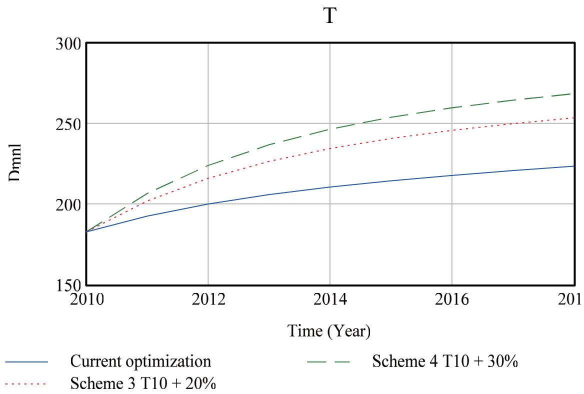
(2) Although there exist relationships between T and all parameters of the global system, the road density is more important than others, therefore, by adjusting road density, we can control the process of development of traffic effectively. As the length of highway can increase T, the government should improve the quality and length of a high utilization factor of highways must be kept.

(3) Figure 6 shows the comparison between current optimization curves and the schemes proposed. By Figure 6c and Table 3, we assert that T_{30} exerts dominant influence on the increasing of T, and the best way to promote development of traffic is to improve the quality and length of highway and rail transit [5].

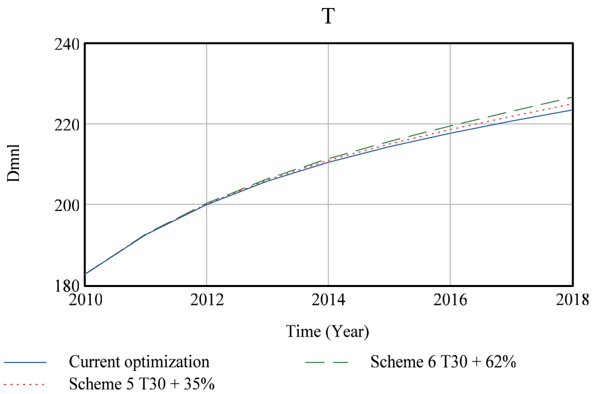
(4) Figure 7 indicates that the relationship between debugging two parameters and debugging one parameter is not linear. Furthermore, the effect of two parameters is better than one parameter.



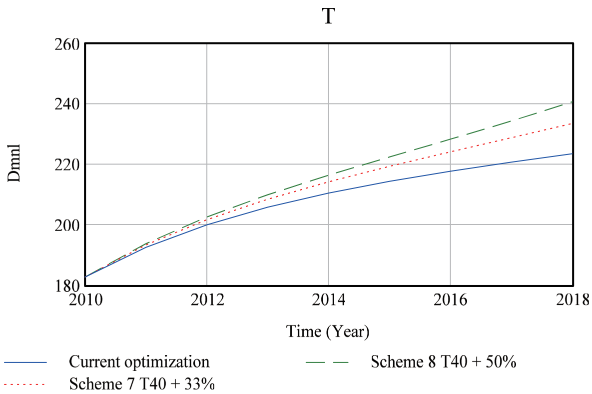
(a) Debugging T_{13}.



(b) Debugging T_{10}.

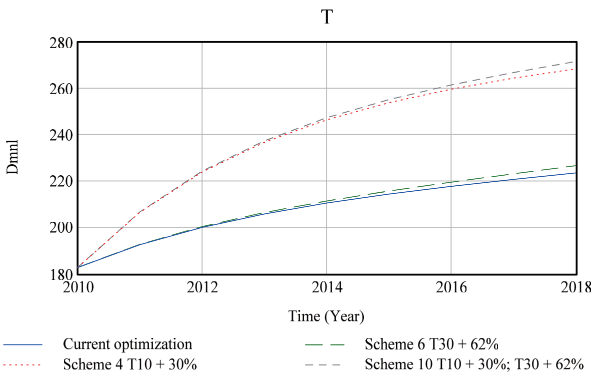


(c) Debugging T_{30}.

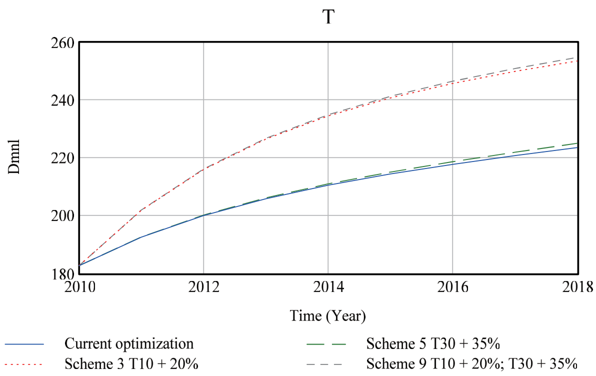


(d) Debugging T_{40}.

Figure 6. Debugging results of one parameter.



(a) Debugging T_{10} and T_{35}.



(b) Debugging T_{10} and T_{35}.

Figure 7. Debugging results of two parameters.

2.5. Data estimation

Table 3 shows that the relative error of $T_1,\ T_9,\ T_{15},\ T_{20}$ and T_{33} are under 6.83% and thus the model has high accuracy. In light of this SD model, Table 4 displays the next ten years developments of traffic in Shandong Province, and it is found that the following trends of traffic in the future.

(1) The number of traffic accidents is increasing every year, which may be related to the growth of private cars and taxis. The growth of private cars and taxis is inevitable. Therefore, the government must take active measures to reduce the number of traffic accidents.
(2) Due to the needs of the citizen and environmental protection, the number of operating vehicles will always keep rising. The increase of operating vehicles means that the number of buses on remote roads and the diversification of routes are improved, which is convenient for citizens.
(3) The larger number of all kinds of vehicles leads to the increase in loading of roads, which will bring more traffic accidents. Obviously, the length of highway mileage should be enlarged.
(4) Although the number of taxis will continue to rise in the next ten years, its growth is slower than other means of traffic. Taxis and buses are convenient for people to take, but the taxi results in more serious pollution than buses. Furthermore, unlike the bus, the taxi is not as safe. Therefore, the number of taxis will decrease in the future.
(5) The number of private cars will grow rapidly in the future. The private cars not only bring convenience to people, but also bring great burden to traffic. The citizens should appropriately reduce the use of private cars and take public traffic.
(6) The rail transit's length will maintain rapid growth in the next decades, which means that government will pay more attention to the development of rail transit, and the rail transit can greatly reduce traffic accidents and environmental pollution.

Table 3. Growth rate of Debugged models.

	Index of 2018	Growth rate
Scheme 1 $T_3 - 88\%$	226.36	0.73%
Scheme 2 $T_3 + 560\%$	223.67	-0.46%
Scheme 3 $T_{10} + 20\%$	254.68	13.34%
Scheme 4 $T_{10} + 30\%$	269.67	20.01%
Scheme 5 $T_{30} + 35\%$	243.78	8.49%
Scheme 6 $T_{30} + 62\%$	229.28	2.03%
Scheme 7 $T_{40} + 33\%$	234.68	4.44%
Scheme 8 $T_{40} + 50\%$	241.82	7.61%
Scheme 9 $T_{10} + 20\%, T_{30} + 35\%$	258.25	14.91%
Scheme 10 $T_{10} + 30\%, T_{30} + 62\%$	276.45	23.02%

Table 4. Data estimation based on SD model.

Year	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028
T_1	13.46	13.74	14.11	14.59	15.18	15.92	16.83	17.93	19.25	20.85
T_9	1126	1136	1147	1157	1167	1178	1188	119.9	1210	1221
T_{15}	61.8	61.8	61.8	61.7	61.4	61.1	60.68	60.1	59.4	58.6
T_{20}	28.68	29.39	30.12	30.88	31.65	32.44	33.25	34.09	34.95	35.83
T_{26}	237.9	278.2	325.2	380.1	444.2	518.9	606.1	707.8	826.4	964.5
T_{33}	65.94	69.79	73.76	77.84	81.99	86.19	90.40	94.57	98.64	102.52

2.6. The neural network model

Based on the SD model, the following neural network model of traffic problem considered can be represented as

$$\begin{cases} \frac{d(T_{33}(t))}{dt} = a_1T_{33}(t) + b_{11}T_{33}(t) + b_{13}T_{15}(t)T_{33}(t) + b_4T_{26}^2(t) + c_1, \\ \frac{T_{15}(t)}{dt} = -a_2T_{15}(t) + b_{21}T_{15}(t) + b_{23}T_{26}(t) + b_{24}T_{15}(t) + b_{25}T_{33}(t)T_{15}(t) + \\ \qquad b_{26}T_{26}(t)T_{15}(t) + b_{27}T_{20}T_{15}(t) + c_2, \\ \frac{d(T_{26}(t))}{dt} = -a_3T_{26}(t) + b_{31}T_{26}(t) + b_{32}T_{15}(t)T_{26}(t), \\ \frac{d(T_{20}(t))}{dt} = -a_4T_{20}(t) + b_{41}T_{33}(t) + b_{42}T_{33}(t) + b_{43}T_{26}(t) + b_{44}T_{33}(t)T_{15}(t) + \\ \qquad b_{45}T_{20}(t)T_{15}(t)) + c_4, \\ \frac{d(T_9(t))}{dt} = -a_5T_9(t) + b_{51}T_{33}(t) + b_{52}T_{15}(t) + b_{53}T_{26}(t) + b_{54}T_{33}(t)T_{15}(t) + c_5, \\ \frac{d(T_1(t))}{dt} = -a_6T1_(t) + b_{62}T_{15}(t) + b_{63}T_{26}(t) + b_{64}T_{20}(t) + b_{65}T_{33}(t)T_{15}(t) + \\ \qquad b_{66}T_{26}(t)T_{15}(t) + b_{67}x_4(t)T_{15}(t) + c_6, \\ T(t) = a_{11}T_{33}(t) + a_{12}T_{15}(t) + a_{13}T_{26}(t) + a_{14}T_{20}(t) + a_{15}T_9(t) + a_{16}T_1(t), \end{cases} \tag{2.7}$$

where the meaning of the coefficients of (2.7) can be found in (A.1). The five neurons are represented by T_3, T_{14}, T_{19}, T_{25} and T_{28}. The capacitance of each neuron is fixed to be 1. $\gamma_i = \frac{1}{T_i}$ $(i = 3, 14, 19, 25, 28)$ are the resistances of per neuron [1, 2, 22]. If T_0 is the initial value, I signifies the expected value, S_i are the years to achieve I of each neuron, and T_s is T_0 after s years, then the neural network model is

$$T_s = T_0 + (I - T_0) \times (1 - \exp\frac{-S_i}{\gamma_i}),$$

further

$$S_i = \gamma_i \times \ln\frac{I - T_0}{I - T_s}, \tag{2.8}$$

which provides an algorithm to calculate I (see Example 2.1).

Example 2.1. If T in 2015 and 2018 are noted as $T_0 = 214.74$ and $T_s = 224.73$ respectively, I is set to be 300, then

$$S_i = \gamma_i \times \ln\frac{I - T_0}{I - T_s} = \gamma_i \times \ln\frac{85.27}{75.29}, \tag{2.9}$$

where

$$\gamma_3 = 10.34,\ \gamma_{14} = 3.43,\ \gamma_{19} = 99.80,\ \gamma_{25} = 7.40,\ \gamma_{28} = 14.94,$$

so we can get

$$S_3 = 1.29,\ S_{14} = 0.43,\ S_{19} = 12.42,\ S_{25} = 0.92,\ S_{28} = 1.86.$$

Model 2.9 shows that the length of time to achieve the expected value of each subsystem are $1.29, 0.43, 12.42, 0.17, 0.92$, and 1.86 years respectively. Obviously, the maximum is 12.42 years, which is the length of time to arrive the expected value. The SD and neural network models are advantageous to improve the speed and efficiency of the development of the traffic system.

3. Conclusions

This paper considers the development of the traffic system of Shandong Province, based on the system dynamics method, and the SD model of traffic system related to the six subsystem which contains the highway vehicle subsystem, the private car subsystem, the highway mileage subsystem, the rail transit subsystem and the traffic accident subsystem, in light of which prediction on the future situation of the traffic system of Shandong Province is derived. Furthermore, the neural network model of the problem considered in paper is also given. In terms of the SD model and neural network model, we can give some policies to force the traffic systems to develop to the situation we want. For example, the government need to develop rail transit and take some measures to reduce the number of traffic accidents. The model is helpful to improve the convenience of traffic services and to promote to development the stable traffic. Furthermore, based on the neural network model, we can investigate the dynamics of the traffic system, and we will pursue this line in the future.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work is supported by Natural Science Foundation of Shandong Province (No. ZR2020MA054) and Research Foundation for Talented Scholars of SDUT (No. 4041/419023).

Conflict of interest

The authors declare that there is no conflict of interest.

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Appendix

$$
\begin{aligned}
a_1 &= 0.125T_{34},\\
a_2 &= T_{19},\\
a_3 &= 0.135T_{30},\\
a_4 &= 3.1550e-05T_{30} + T_{25} - 0.245,\\
a_5 &= T_{14},\\
a_6 &= 0.223a_2 + 4.031T_3,\\
b_{11} &= 0.0047T_{40},\\
b_{12} &= 0.0040T_{19},\\
b_{13} &= -0.00801T_{19},\\
b_{14} &= -0.00561T_{30},\\
b_{21} &= 7.0176e-06T_{40},\\
b_{22} &= 0.010.751T_{17} + 0.233,\\
b_{23} &= 9.1428e-06T_{19} \times T_{30} + 9.3700e-05T_{30},\\
b_{24} &= -9.6500e-05,\\
b_{25} &= 0.000129,\\
b_{26} &= 7.3600e-05T_{19},\\
b_{27} &= 3.0500e-04a_2,\\
b_{31} &= 0.075T_{30}, b_{32} = 0.0344T_{19},\\
b_{41} &= -1.3375e-05T_{40},\\
b_{42} &= -4.4720e-04,\\
b_{43} &= -0.1801,\\
b_{44} &= -1.1172e-05T_{19},\\
b_{45} &= 0.205a_2,\\
b_{51} &= 7.0176e-04T_{40},\\
b_{52} &= 0.233,\ b_{52} = 0.0233,\\
b_{53} &= 0.0166T_{30},\\
b_{54} &= 5.9098e-04T_{19},\\
b_{25} &= 0.205a_2,\\
b_{51} &= 0.0070T_{40},\\
b_{53} &= 0.016644T_{30},\\
b_{54} &= 5.8824e-04T_{19},\\
b_{61} &= -5.2550e-04T_{40},\\
b_{62} &= 0.02935,\ b_{64} = -0.0331,
\end{aligned}
$$

$$
\begin{aligned}
b_{63} &= -0.0259T_{30},\\
b_{65} &= -4.4050e-04T_{19},\\
b_{66} &= 0.0108a_2, b_{67} = 0.104a_2,\\
a_{11} &= 0.0703,\ a_{12} = 0.298,\\
a_{13} &= 1.765,\ a_{14} = 0.226,\\
a_{15} &= 0.096,\ a_{16} = 0.011,\\
c_1 &= 4.6720e-04T_{28} - 0.569T_{38},\\
c_2 &= 0.316T_{10} + 1.3128e-06T_{28}\\
c_4 &= -6.0040e-04T_{10} + 1.3128e-06T_{28}\\
c_5 &= 0.316T_{10} + 6.9146e-05T_{28},\\
c_6 &= -5.8400e-05T_{28} + 0.0398T_{10} + 8.6899e-06T_{28},\\
a &= 0.456T_{19},\ \bar{a} = T_{19},\ b = 0.544T_{40}, d = 0.04672T_{28},\ \bar{d} = T_{30},\\
f &= 0.751T_{17},\ k = 5.8T_3, j = 0.569T_{38},\ h = 0.316T_{10},\ r = T_{25},\ q = T_{14}.
\end{aligned} \tag{A.1}
$$

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